WORKING PAPERS
RESEARCH DEPARTMENT

WORKING PAPER NO. 15-08
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February 2015

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Credit Risk Modeling in Segmented Portfolios: An Application to Credit Cards[1]

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Abstract

The Great Recession offers a unique opportunity to analyze the performance of credit risk models under conditions of economic stress. We focus on the performance of models of credit risk applied to risk-segmented credit card portfolios. Specifically, we focus on models of default and loss and analyze three important sources of model risk: model selection, model specification, and sample selection. Forecast errors can be significant along any of these three model-risk dimensions. Simple linear regression models are not generally outperformed by more complex or stylized models. The impact of macroeconomic variables is heterogeneous across risk segments. Model specifications that do not consider this heterogeneity display large projection errors across risk segments. Prime segments are proportionally more severely impacted by a downturn in economic conditions relative to the subprime or near-prime segments. The sensitivity of modeled losses to macroeconomic factors is conditional on the model development sample. Models estimated over a period that does not incorporate a significant period of the Great Recession may fail to project default rates, or loss rates, consistent with those experienced during the Great Recession.

JEL classifications: G20, G32, G33

Keywords: Credit cards, credit risk, stress test, risk segmentation

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I. INTRODUCTION

In this paper, we analyze the relative loss-forecasting performance of credit risk models applied at an aggregated risk-segment level to a representative credit card portfolio. Our main emphasis is on the analysis of model development elements that may impact default/loss forecasting under stressed economic conditions, while the lessons learned may also apply more broadly to the modeling of credit risk in segmented portfolios.

Although the use of loan level models is widespread, risk segmentation continues to be widely used across banks and other financial companies with different levels of complexity. The use of segmentation is also an important component in the analysis of credit risk within the advanced Basel III framework. The reasons for using data at an aggregate segment level vary from data availability, to time constraints, to simplicity of approach, among others. In this paper, we analyze the performance of a variety of models and model specifications taking advantage of a panel data set of a representative credit card portfolio aggregated at a risk-segment level. Specifically, our primary focus is on the analysis of a one-year default rate and loss rate at the segment level. We focus our attention on three dimensions of model risk: model selection, model specification assumptions, and sample selection.

Our analysis employs a panel data set of credit card accounts from the Equifax credit bureau over the period 2001–2011. The data cover the Great Recession and afford us the opportunity to analyze the performance of credit risk models under conditions of significant economic stress. We consider a variety of approaches for the estimation of models of default and loss rate. The linear regression represents the most simple and straightforward approach. We also consider potential improvements on the simple regression that have been suggested in the recent empirical literature, relying on transformations of the dependent variable (Ferrari and Cribari–Neto, 2004; Ospina and Ferrari 2010, 2012a, 2012b). Finally, we consider flexible parametric and semiparametric model specifications (for a survey, see Li, Qi, Zhang, and Zhao, 2014). Interestingly, the linear regression projections compare favorably with other, more sophisticated models. The regression model seems to have a slight edge in the projection of the initial increase in default rates at the start of the recession. In contrast, flexible parametric models seem to have a slight edge in the projection of the decrease in default rates as the recession subsides. It is not possible to isolate what aspect of model design contributes to the better performance of a specific model in certain circumstances, but it seems that the simplicity and flexibility of the regression approach allows it to overcome some well-known limitations of the model.

Our analysis also indicates that loss projections can be significantly impacted by model specification assumptions. Models that overlook potential interactions between macroeconomic conditions and segment characteristics can result in significant under projection of prime portfolio losses under stressed economic conditions. Surprisingly, relative to the subprime borrowers, near-prime and prime borrowers in the portfolio are proportionally more severely impacted by economic downturns and model specification assumptions. In addition, models estimated over a time frame that does not include a representative economic downturn period may not project levels of credit loss consistent with losses experienced during the Great Recession of 2007–2009. Once again, the projection error seems to be much larger for the prime segment relative to the subprime segment.

Reliance on models as tools for effective supervision and risk surveillance has increased significantly within regulatory agencies over the years, with the advent of the Internal Ratings Based (IRB) approach to regulatory capital postulated in the Basel II accord. More recently, in the Comprehensive Capital Analysis and Review (CCAR) exercise, now mandated by the Dodd-Frank Wall Street Reform and Consumer Protection Act, the Federal Reserve conducts its own independent assessment of capital adequacy through periodic stress testing across participant banks as a benchmark to the banks' own stress-test analyses. As complex credit risk models are the foundation for these exercises, it becomes paramount to evaluate and mitigate risks associated with the development and implementation of these models. The recent regulatory guidance on model risk (OCC Bulletin 2011-12 and SR letter 11-7, "Guidance on Model Risk Management"), along with its antecedents (primarily OCC Bulletin 2000-16), stresses as much and highlights the important aspects of model risk management, including model development, implementation, use, validation, governance, policies, and controls structure.

Credit card portfolios constitute the largest unsecured loan portfolios at most large banks. The net charge-off rate for credit card portfolios increased more than twofold for a number of large banks during the peak of the financial crisis, and it is estimated that more than $160 billion in credit card debt has been charged off since 2008.[2] Further, recent stress tests conducted by banks and regulators showed the importance of credit card portfolios as a potential source of losses for the banks in the event of a severe economic downturn. In particular, projected losses from card portfolios of the largest banks participating in the 2012–2013 DFAST stress test exercise totaled $87 billion, or 19 percent of overall projected losses, over the projected stress period. Only trading and counterparty losses represented a larger share of projected losses, at $97 billion, while losses from first-mortgage portfolios were projected at $61 billion and losses for other types of mortgage exposures (junior liens, HELOC, etc.) were projected at $56 billion (Board of Governors of the Federal Reserve System, 2013).

Early studies on credit card portfolios over the business cycle did not find conclusive evidence of macroeconomic factors having an impact on portfolio losses. Gross and Souleles (2002) analyze credit card delinquency and personal bankruptcy in the 1990s, using panel data on credit card accounts. The authors conclude that the relation between default and economic fundamentals appears to have changed substantially over the period of study in ways not explained by their control variables; however, they did not find that unemployment had a significant impact on credit card default. Agarwal and Liu (2003) also examine credit card delinquency and bankruptcy behavior. They note that previous empirical studies did not consistently find a significant effect of macroeconomic factors on bankruptcy because of either inadequate data or a lack of sufficient variation in the unemployment variable during the period of analysis. Their analysis indicates that the level of unemployment appears to be a significant determinant of default, while the change in unemployment is usually insignificant. In contrast, Banerjee and Canals-Cerdá (2012), taking advantage of significant variations in macroeconomic variables, risk exposure, and performance outcomes experienced during the Great Recession, report that both the level and the change in unemployment are statistically and materially significant determinants of credit card defaults. Banerjee and Canals-Cerdá (2012) also report that unemployment was not a relevant factor in the account balance exposure at default (EAD) during the Great Recession.

A recent paper by Canals-Cerdá and Kerr (2014) explores the potential sources of model risks in loan level credit risk models. In their paper, the authors note that loan level credit losses are

[2] Data are from bank call reports and other regulatory filings (Figure 1); see also Hunt (2013). The net charge-off rate peaked at the time of the financial crisis, propelled by an increase in the net dollar charge-off rate and a decline in dollars outstanding.

decomposed into probability of default (PD) and EAD and modeled separately using survival analysis and linear regression. Their analysis shows that macroeconomic variables have a strong and heterogeneous impact on an account's likelihood and timing of default but play a much smaller role in an account's projected EAD. The proportionally larger impact of unemployment on the near-prime and prime segments in the Great Recession led them to conclude that model specifications matter more for prime segments than for subprime. Finally, they extend their analysis to alternative scenarios and show the dramatic importance of scenarios in loss projection. The analysis conducted in this paper complements research by Canals-Cerdá and Kerr (2014) by looking at the impact of model choice and model specification assumptions, as well as sample selection, with a focus on aggregated segment-level panel data models.

In the next section, we present the data and provide descriptive statistics for some of the key variables in our sample. Section 3 contains the empirical methodology. Section 4 presents the empirical results and analysis, while Section 5 concludes the paper. Tables and figures are presented in Section 7.

II. DATA AND DESCRIPTIVE ANALYSIS

We have access to a panel data set containing historical tradeline credit card information from a 5 percent random sample of individuals with a credit file in the consumer database from Equifax, one of the major credit bureaus in the United States. The data are updated biannually and include information on credit card account characteristics, such as account age, line, and utilization; account balances; current and past delinquencies; as well as an individual's Equifax credit score. Given the enormous size of the original data set, in our analysis, we employ a 1 percent random subsample from the sample described previously.

For simplicity of exposition, we focus our analysis on the one-year default experience of accounts current at the time of observation, and we also expand our research to the analysis of losses. The one-year default experience of accounts delinquent at the time of observation can also be modeled using the framework developed in this paper, but it is recommended to model delinquent accounts separately from current accounts due to their significantly higher default rate. Delinquent accounts represent a small percentage of accounts in a representative cards portfolio due to a low incidence of delinquency and a high rate of transition to default. Thus, delinquent accounts are likely to contribute significantly to portfolio losses in the short run, while current accounts are likely to contribute to portfolio losses over a longer time horizon.

Table 1 lists the primary risk drivers employed in our statistical analysis of credit risk. Relevant variables include a proprietary account-specific Equifax risk score, account balance, account line, and account utilization (defined as the percentage of the credit line that is being utilized). We consider the level and change in unemployment as the primary measures of economic activity driving credit card default. We also include a second quarter dummy in our models to account for potential patterns of seasonality in credit card defaults.

In our investigation, we employed segmented data; the unit of analysis is a segment of accounts where, at each observation point, segments are defined according to score, line, and utilization ranges described in Table 1, as well as regional variation in macroeconomic risk drivers. We employ a panel with information on credit card accounts performance from 2001 to 2011. We observe historical information on monthly account performance. The payment status of each loan in a cohort is followed for four consecutive quarters, and segment performance variables are derived from this information. The last performance period is December 2011.

Figure 1 provides information on the historical values of macroeconomic variables and credit card delinquencies. The macroeconomic variables considered in our study have experienced significant variation during the period of analysis. Unemployment remained relatively stable between 2000 and 2007, with a record low unemployment rate of 4 percent in 2000 and a maximum unemployment rate of 6.3 percent in June 2003. Unemployment increased significantly between 2007 and 2009, from about 4.4 percent to about 10 percent, and it recovered gradually thereafter. Unemployment and credit card charge-offs historically have moved together, although in the most recent period, charge-offs decreased at a more rapid pace.

Tables 2a and 2b present historical average performance across risk segments for the outcome variables analyzed. Outcome variables of interest considered are a measure of default and a measure of loss, both represented in the form of ratios. The default rate considered is defined as the ratio of accounts that became 120 days past due within a one-year period for each segment of accounts considered. The loss rate considered is defined as the ratio of account balances that become at least 60 days past due within a one-year period for each segment of accounts. We chose to compute the loss rate for accounts that become 60 days past due to avoid potential measurement error problems associated with banks' reporting of account information for more severe delinquencies. While our analysis was conducted at the segment level, any reported statistical results are weighted by the number of accounts per segment.

As expected, default and loss rates are higher for low score segments and low line segments and lower for low utilization segments. Default and loss rates are significantly higher during the most recent recession years. Interestingly, the increase in default and loss rates, from their low values of the 2004–2006 period to the highest values observed in 2009, are proportionally larger for the higher risk-score segments. Specifically, default rates for the lowest risk-score or subprime segment increased from 20.4 percent to 28.4 percent, a 40 percent increase, while default rates for the higher risk-score or super-prime segment increased from 0.2 percent to 0.6 percent, or a 300 percent increase.

III. EMPIRICAL METHODOLOGY

The use of loan-level statistical methods for the analysis of credit risk in retail portfolios is commonly used in the finance literature. However, the use of portfolio segmentation in the analysis of credit risk in retail portfolios continues to be employed regularly across financial companies, particularly among less sophisticated institutions. Furthermore, risk segmentation represents the basis for the analysis of regulatory capital for credit risk in the Basel II and III framework.

In this section, we describe the empirical framework for the analysis of default rate and loss rate defined at the segment level. We highlight technical difficulties associated with the analysis of this type of data and review alternative approaches that can be employed to deal with these problems. In the empirical section of the paper, we compare results from different methods.

As a first step, we begin with the analysis of the standard linear regression framework. Specifically, consider the following linear specification for the relationship between the loss rate, or the rate of default, and a vector of risk drivers including macroeconomic variables,

$$r_{it} = \beta_0 + \beta_1 X_{it} + \varepsilon_{it}.$$

One problem with this specification is that the regression framework implicitly assumes that the dependent variable is defined in the real line, while in our case, r_{it} takes values in the unit interval. Specifically, the natural restriction of a ratio taking values in the unit interval is not embedded in the simple linear regression model. As a result, predictions using this specification may fall outside of the unit interval in certain cases. There is a growing econometric literature that proposes applying different types of transformations of the dependent variable as a first step toward resolving this problem. These transformations can be broadly described as nonlinear parametric functions from the open unit interval to the space of real numbers. A common example is the logit transformation, where logit(r) = log[r/(1-r)]. Typically, the transformations suggested in the literature are not well defined at the 0 or 1 boundaries; this is clearly the case for the logit transformation. To avoid the {0,1} boundary problem, researchers often proceed in two steps: First, they consider a transformation from the [0,1] interval into the (0,1) interval, and second, they consider a transformation from the open-unit-interval to the real line. For example, Smithson and Verkuilen (2006) propose a transformation of the form r' = (r(N-1) + 0.5)/N, with N representing the sample size. In general, if we define the compounded transformation from [0,1] into the real line as R(•), the proposed approach in this literature specifies a linear regression of the form

$$R_{it} = \beta_0 + \beta_1 X_{it} + \varepsilon_{it},$$

where $R_{it} = R(r_{it})$ represents a transformation of the original rate variable. This regression can be estimated using standard OLS techniques, and predicted values for the transformed dependent variable, denoted $\widehat{R_{it}}$, can be derived from the estimated regression. For the purposes of forecasting the original rate variable, some authors have suggested applying an inversion to the predicted value of the transformed variable after OLS estimation and obtaining a predicted value for the original rate variable as $\widehat{r_{it}} = R^{-1}\left(\widehat{R_{it}}\right)$. Unfortunately, due to the nonlinearity of the original transformation, it is generally the case that $E\left[R^{-1}\left(\widehat{R_{it}}\right)|X\right] \neq R^{-1}E\left[\widehat{R_{it}}|X\right]$. As a result, the prediction resulting from applying the transformation in this form is biased and inconsistent.

A review of the relevant literature and a discussion of potential pitfalls can be found in Li, Qi, Zhang, and Zhao (2014). Their paper describes an approach for correcting the bias associated with the transformation estimator by means of Monte Carlo integration techniques. The nonparametric version of this approach will be generally robust to the potential non-normality of residuals. However, this approach will not generally be robust to potential heteroscedasticity, or measurement error of the dependent variable. In contrast, the simple regression estimator without transformation is by construction robust to these potential data problems.

In the extensive empirical analysis we have conducted, we observe that naive predictions using transformations of the dependent variable result in substantial forecasting bias, even for in-sample forecasts. Also, after controlling for the bias using a Monte Carlo methodology, we still observe that the results generally underperform these from the simple linear regression model without transformation. Because of these findings, results from the standard linear regression equation are reported in this paper rather than transformations of linear models. Li, Qi, Zhang, and Zhao (2014) also report empirical results broadly supportive of the simple linear regression.

An alternative estimation approach uses parametric or semiparametric specifications to describe the associated underlying distribution of default or loss rate conditional on a finite vector of parameters that can be estimated using maximum likelihood, or related estimation techniques. This approach is broadly consistent with the simple linear regression framework, as the OLS regression can be represented easily in the form of a maximum likelihood estimator. In this framework, one can consider statistical models specifically designed to accommodate the natural restrictions of a default or loss ratio.

Some of the most flexible statistical models proposed in the recent literature for the analysis of ratios have yet to be incorporated into standard statistical packages. However, there is a well-developed and closely related literature in the area of count models that tackles the same problems that we encounter in our data. Specifically, our data suffer from an excess-zero problem, which is also a typical problem in count data. In simple terms, this problem is identified when the proportion of occurrence concentrated at zero in the data is more than what would be expected in a standard parametric model. In count data, the dependent variable takes on non-negative integer values only. Thus, to take advantage of the extensive suite of statistical models available for the analysis of count data, we transform our continuous default and loss rate data into count data. We accomplish this easily by dividing the unit interval into N separate segments of equal length and assigning an integer value n_{it} when the associated r_{it} takes a value in the range of $(n_{it}-1)/N$ to n_{it}/N.

Figure 2 presents the empirical distribution of default and loss rates in our data and illustrates the distribution concentration in the vicinity of zero. It is typical when facing an “excess-zero” problem in the data to consider extensions of standard parametric models. One approach explores the possibility that the excess-zero problem results from the presence of unobserved heterogeneity in the data. A popular count model used in this case is the negative binomial regression model. An alternative approach explores the possibility of resolving the excess-zero problem by modeling the zero event as a distinctive process. A popular count model used in this case is the zero-inflated Poisson regression model. A detailed description of these models is beyond the realm of this paper. A good source of information about these types of models is Cameron and Trivedi (2013), and earlier survey reviews of the literature (e.g., Gurmu and Trivedi, 1994). In our empirical analysis, we present estimation results from linear regression, negative binomial, and zero-inflated Poisson regression models.

IV. EMPIRICAL RESULTS AND ANALYSIS

After years of mild and short-lived recessions, a financial crisis led to the 2007–2009 recession and a rapid increase in the unemployment rate. The national unemployment rate topped at around 10 percent in October 2009, reaching levels of 14 percent in Michigan and Nevada and 12 percent in California. It remained above 8 percent nationwide until the third quarter of 2012. Using the impact of this recent economic downturn on credit card default and loss rates, we analyze the modeling framework described in the previous section under a variety of model assumptions, model specification assumptions, and sample estimation periods.

With the exception of two measures of unemployment that are treated as continuous variables, other variables included in our model specifications are represented as dummy variables reflecting specific segment characteristics. For example, credit score is divided into subprime, near-prime, prime, and super-prime, and the segmentation scheme for other relevant variables is described in Table 1. This approach allows us to estimate the potential nonlinear impact of particular segment characteristics without relying on specific functional form assumptions. We focus our attention on segments of accounts current at the time of observation and measure the default and loss rate at the segment level over a one-year time interval.

A. MODEL SELECTION

In the empirical methodology section, we discussed a variety of approaches for the estimation of models of default and loss rate. We introduced the linear regression as the most simple and straightforward approach and discussed potential weaknesses of this approach. We also discussed possible improvements on the simple linear regression by means of transformations of the dependent variable and discussed naive applications of this concept as well as more sophisticated applications using Monte Carlo techniques. Finally, we discussed the use of flexible parametric and semiparametric specifications along with maximum likelihood estimation. In this subsection, we analyze how these techniques perform in our particular application. To achieve this objective, we estimate each of the alternative models described previously. In all cases, we employ the same risk drivers and basic model specification structure over the full sample period observed in our data. After that, we compare the model fit performance of different approaches across time, with a special focus on the recent period of economic downturn.

Figure 3 presents realized loss rates as well as projected loss rates from a regression model, a negative binomial regression model, and a zero-inflated Poisson regression model. Interestingly, the linear regression projections compare favorably with those of the more sophisticated models. All the approaches are able to accurately project the increase in default rates during the last recession, with a small lag. But the regression model seems to have a slight edge in the projection of the initial increase in default rates. In contrast, both the negative binomial regression model and a zero-inflated Poisson regression model seem to have a slight edge in the projection of the decrease in default rates as the recession subsides. Models using transformations of the dependent variable, even those that explicitly control for potential forecasting bias, generally underperform these from the simple linear regression framework without transformation. Therefore, transformation models are not reported.

From the practitioner's perspective, our investigation suggests that using regression analysis as a benchmark to more complex models is a reasonable and fruitful strategy. Furthermore, a simple regression analysis seems to be able to compete in projection accuracy with more sophisticated methods. These results are also consistent with further analysis of out-of-sample projections not reported in this paper. Comparable findings reported in recent research by Li, Qi, Zhang, and Zhao (2014) in a different context add validity to our analysis.

B. MODEL SPECIFICATION

In this subsection, we analyze the impact of different model specification assumptions on credit card default rates and loss rates. Recent studies have shown that both the level and change in the unemployment rate have a significant impact on credit card default rates. However, this impact is not uniform across the portfolio. The impact of a rising unemployment rate environment on prime borrowers is different from its impact on subprime borrowers. Our analysis indicates that models that do not account for this heterogeneous impact are likely to exhibit large forecast errors, especially for subsegments of the portfolio. Further, these errors can become larger due to portfolio migration during the course of a business cycle.

We analyze model specifications in which the impact of macroeconomic factors is homogeneous across the entire credit card portfolio and compare these with model specifications that allow the model parameters to vary across risk-score segments, thus allowing for a heterogeneous impact of the macroeconomic factors. Risk-score segments are based on the Equifax credit score and are defined in Table 1. The lowest segment, referred to as subprime, consists of accounts with scores below 650. The highest segment, referred to as super-prime, consists of accounts with scores above 770. Other intermediate risk-score segments represent the near-prime and prime borrowers, respectively, as identified in Table 1.

Table 3 reports estimation results for the negative binomial regression models that allow for the heterogeneous impact of macroeconomic variables and other risk drivers. The table presents results for both loss rate and default rate models. We report large differences in the impact of utilization and macroeconomic factors across risk-score groups. Low utilized accounts among subprime borrowers are 46 percent less likely to default (odds ratio of 0.54) relative to high utilized accounts among the same group of borrowers, but this effect is even stronger among the near-prime and prime borrowers, in which the likelihood of default for low utilized accounts is 67 percent and 73 percent lower, respectively. The heterogeneous impact of utilization is also seen in the loss rate model, where low utilized accounts among subprime borrowers show a mere 1 percent decline in the likelihood of loss, but it has a much stronger effect among prime and super-prime borrowers. For credit limit, there is a greater default probability associated with higher lines, after controlling for score, but this effect is not consistent across risk-score segments. However, the most interesting results are related to the level and change in the unemployment rate. We find that the unemployment rate has almost no impact on default and loss rates in the subprime segment of the portfolio. Looking at the combined impact, both level and change, of an increase in unemployment on default rates, we find that a one-unit increase in the unemployment rate has a relatively small positive impact on the likelihood of default in the subprime segment when compared with its impact on the near-prime segment. In contrast, the same increase in the level of unemployment rate results in a much larger increase in the likelihood of default in the high score segments. These results show that the prime segments of the portfolio are more severely impacted by a downturn in economic conditions and model specification assumptions relative to the subprime or near-prime segments.

In Figure 4, we present the results of the relative projection accuracy of both model specifications over the sample period. The solid line at each calendar time shows the realized one-year default rates over the 2001–2011 period. The dotted lines represent the model predictions from the two model specifications reported. The Great Recession, or years 2007–2009, is of particular interest. We see that both models fit the actual default rates for the overall portfolio well but lag the actual rise in default rates by about six months, especially when the unemployment rate rose rapidly during the 2007–2008 period. Furthermore, when we evaluate the relative performance of different model specifications across portfolio segments, we observe that the homogeneous model specification significantly overpredicts in the subprime segment while also underpredicting in the near-prime and prime segments of the portfolio. The magnitude of error is particularly severe for the prime segment in 2008 and 2009, when it underpredicts the actual default rates by 45 percent (1.00 percent point relative to an actual default rate of 1.82 percent point). By contrast, the alternative model specification that allows for a heterogeneous impact of macroeconomic variables across risk segments exhibits a much smaller forecast error on the prime segment of about 10 percent during the same period. The reverse results are seen for the subprime segment where the homogeneous model specification overpredicts default rates by 18 percent, whereas the alternative model specification has a much smaller overprediction rate of 6 percent.

Thus, our analysis indicates that the impact of macroeconomic factors is heterogeneous across portfolio risk segments. These results follow naturally from the regression results where we show that the subprime segment is proportionally less sensitive to changes in macroeconomic conditions than higher risk-score segments. This finding has important implications for portfolio risk management as it indicates that banks that do not consider the heterogeneous response to macro factors in their models may encounter large loss projection errors, especially during recessions when the portfolio composition changes because of account migration or changes in credit policy.

C. SAMPLE SELECTION

Even a well-specified model that accounts for the heterogeneous impact of the macroeconomic variables on portfolio default rates may have large forecast errors if the development sample period is constrained by the lack of a complete business cycle with a significant downturn. This source of model risk can affect the identification and estimation of model parameters and compromise a model's ability to forecast accurately when the postulated stress environment falls outside the stress levels experienced in the data.

To illustrate this, in Figure 5, we compare the results from our most general model specification when it is estimated with data up to 2008, 2010, and 2011. This model specification captures the heterogeneous impact of the macroeconomic variables across the portfolio segments. However, it still results in significant overprediction when estimated with data only up to 2008. The model forecasts default rates that are 16 percent higher than the actual default rates for the overall portfolio during the 2008–2011 period,[3] when unemployment rose sharply first and declined thereafter. This overprediction for the model estimated with data up to 2008 can be attributed to an estimation period that does not capture the complete effect of the downturn as the

[3] The years 2009–2011 can be considered as the out-of-time validation sample for this model because it is estimated with data up to 2008.

unemployment rate was still rising during that period. The magnitude of overprediction falls from 16 percent to 9 percent as we expand the estimation sample to include data up to 2010, and to 4 percent when we include data up to 2011 that includes a more complete business cycle in which unemployment has come down to 6.5 percent. This analysis illustrates that even the most general model specification can have significant forecast errors when it is estimated over a period that does not include a significant downturn or, conversely, a large enough recovery period. Additional analysis, not reported here, shows that the primary impact of our model specification and sample selection on portfolio loss is channeled through the process of transition to default rather than the drawdown behavior of the borrowers that affects the exposure at default.

D. LOSS RATES

For most of the paper, we have focused on the impact of model specification and sample selection on forecasting default rates for the portfolio. However, our discussion of default rate models is generalizable to loss rates as well. In Figure 6, we recreate for loss rates the same analysis displayed in Figure 5 for default rates. The most general model specification considered in Figure 6 overpredicts the loss rate by 22 percent over the 2009–2011 period when the estimation sample is cut off at 2008 and does not include the effect of the complete downturn and recovery. The magnitude of this overprediction is even greater than the overprediction seen in the case of default rates. As in the case of default rates, the forecast error declines considerably as the estimation window is expanded to include 2010 and 2011, when overprediction falls to 14 percent and finally to 8 percent. Even after including all the years in our sample, some overprediction remains; this may be attributed to the model not capturing the entire recovery period.

V. CONCLUSIONS

This paper focuses on the impact of model selection, model specification, and sample selection on the relative loss-forecasting performance of credit risk models applied at an aggregated risk-segment level to a representative credit card portfolio. The lessons learned may be applicable more broadly to the modeling of credit risk.

Our loss-projection framework fits the loss experienced in our representative portfolio well when models are estimated over the full sample. The results indicate that macroeconomic conditions have a significant and sizable impact on the default and loss ratios. We observe that the impact of macroeconomic factors is heterogeneous across portfolio risk segments, in particular along the credit score dimension. Segments of accounts classified as subprime according to their low credit score have the highest loss rate in all economic environments considered but are proportionally less impacted by changes in macroeconomic conditions than the near-prime and prime segments.

The econometric literature offers a variety of alternatives for modeling default and loss rates. Simple regression analysis can be challenging because ratios are naturally defined in the unit interval and because of the prevalence of segments with zero loss, or near zero loss. Several alternatives to the simple linear regression have been proposed in the literature; we evaluate the performance of dependent variable transformations and parametric models specifically designed for ratios and the excess-zero problem observed in the data. Interestingly, the linear regression projections compare favorably with these more sophisticated models. The naive application of nonlinear transformations generates significant bias; attempts to correct bias using a Monte Carlo approach do not eliminate the problem in our particular application. Of our preferred models, linear regression seems to have a slight edge in the projection of the initial increase in default rates at the beginning of the economic downturn. In contrast, parametric, more refined models seem to have a slight edge in the projection of the decrease in default rates as the recession subsides. From the perspective of a practitioner, this analysis suggests that using regression analysis as a benchmark to more complex models is a reasonable and fruitful strategy. Findings are broadly consistent whether the models considered refer to default ratio or loss ratio.

Our analysis indicates that loss projections are significantly impacted by model specification assumptions. Models with controls for macroeconomic variables that do not consider interactions between these variables and measures of creditworthiness can significantly underpredict or overpredict loss under stress for segments of the portfolio. The effect of macroeconomic variables and model specification assumptions is heterogeneous across portfolio segments. In particular, the impact of the significant and rapid increase in unemployment experienced during the Great Recession was proportionally much larger for the near-prime and prime segments in contrast with the subprime segment. The lack of a sufficiently severe stress period in the estimation sample represents another potential source of model risk and results in a significant under-prediction of portfolio losses under stressed economic conditions in our application. Model performance improves significantly once a sufficiently large downturn period is included in the data, particularly in the most flexible model specification considered.

VI. REFERENCES

Agarwal, S., and C. Liu (2003). “Determinants of Credit Card Delinquency and Bankruptcy: Macroeconomic Factors,” *Journal of Economics and Finance*, 27(1), 75–85.

Banerjee, P., and J.J. Canals-Cerdá (2012). “Credit Risk Analysis of Credit Cards Portfolios Under Economic Stress Conditions,” Federal Reserve Bank of Philadelphia, Working Paper 2012-18.

Board of Governors of the Federal Reserve System (2013). “Dodd-Frank Act Stress Test 2013: Supervisory Stress Test Methodology and Results, March 2013,” http://www.federalreserve.gov/newsevents/press/bcreg/dfast_2013_results_20130314.pdf.

Cameron, A.C., and P.K. Trivedi (2013). *Regression Analysis of Count Data*, 2nd edition, Econometric Society Monograph 53, Cambridge University Press, 1998.

Canals-Cerdá, J.J., and S. Kerr (2014). “Forecasting Credit Card Portfolio Losses in the Great Recession: A Study in Model Risk,” Federal Reserve Bank of Philadelphia, Working Paper 2014-10.

Ferrari, S.L.P., and F. Cribari–Neto (2004). “Beta Regression for Modelling Rates and Proportions,” *Journal of Applied Statistics*, 31, 799–815.

Gross, D.B., and N.S. Souleles (2002). “An Empirical Analysis of Personal Bankruptcy and Delinquency,” *Review of Financial Studies*, 15, 319–347.

Gurmu, S., and P.K. Trivedi (1994). “Recent Developments in Event Count Models: A Survey,” University of Virginia, Thomas Jefferson Center Discussion Paper 261.

Hunt, R. (2013). “Understanding the Model: The Life Cycle of a Debt,” prepared presentation for the FTC-CFPB Roundtable.

Li, P., M. Qi, X. Zhang, and X. Zhao (2014). “Further Investigation of Parametric Loss Given Default Modeling,” Office of the Comptroller of the Currency, Economics Working Paper 2014-2.

Ospina, R., and S.L.P. Ferrari (2010). “Inflated Beta Distributions,” *Statistical Papers*, 51, 111–126.

Ospina, R., and S.L.P. Ferrari (2012a). “A General Class of Zero-or-One Inflated Beta Regression Models,” *Computational Statistics and Data Analysis*, 56, 1609–1623.

Ospina, R., and S.L.P. Ferrari (2012b). “On Bias Correction in a Class of Inflated Beta Regression Models,” *International Journal of Statistics and Probability*, 1. doi:10.5539/ijsp.v1n2p269.

Smithson, M., and J. Verkuilen (2006). “A Better Lemon Squeezer? Maximum-Likelihood Regression with Beta-Distributed Dependent Variables,” *Psychological Methods*, 11, 54–71.

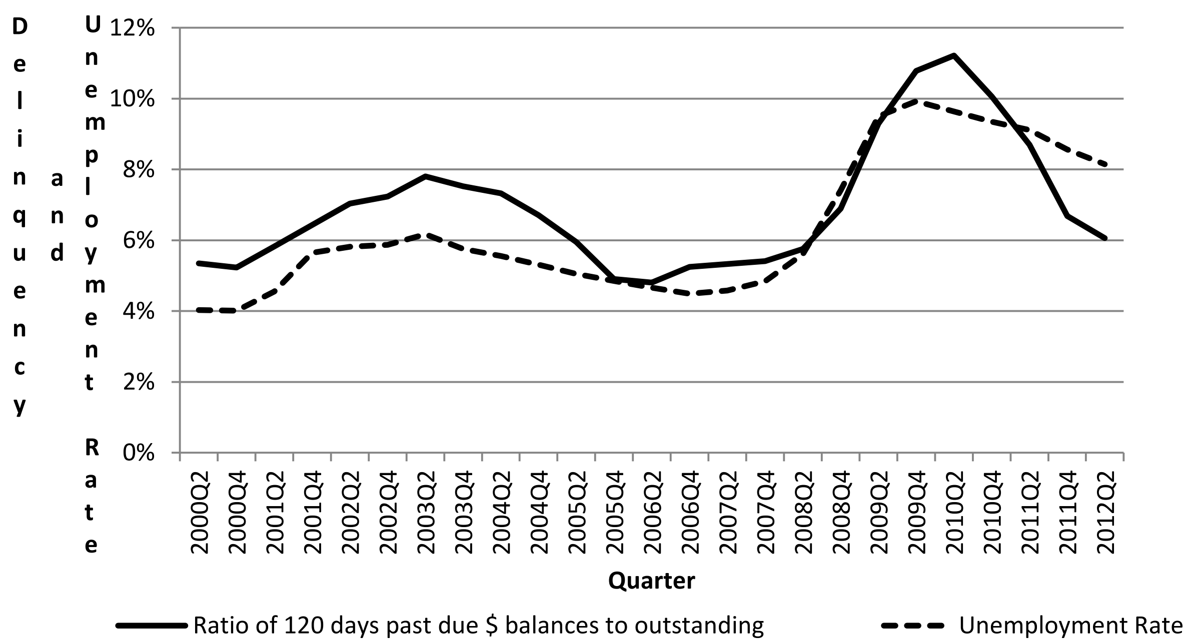
VII. Tables and Figures

Table 1: Variable Definitions

Risk Score (sp; np; p; spp)	*Borrowers' Credit score dummies for ranges 250-650, 650-710, 710-770, 770+*
Line (low, medium, high)	*Credit limit dummies for ranges less than $1,500, $1,500-$10,000, and $10,000+*
Util. (low, medium, high)	*Utilization dummies for ranges 0%-25%, 25%-90%, and 90%+*
Seasonality	*Indicator for first and second half of the year.*
Ur.	*State-level unemployment rate with a 6-month lag*
Chg. Ur. 12m.	*Change in unemployment rate over the last 12 months with a 6-month lag*

Note: "sp" refers to subprime, "np" refers to near-prime, "p" refers to prime, and "spp" refers to super-prime.

Figure 1: Credit Card 120 Days Past Due Balance to Outstanding Ratio and Unemployment Rate Over Time



Sources: Aggregated Call Report data for insured commercial banks, Bureau of Labor Statistics

Table 2a: Default Rates Across Risk Segments

Variable	2001-03	2004-06	2007	2008	2009	2010	2011
Risk Score							
Subprime	15.3%	13.1%	13.6%	16.1%	17.3%	15.5%	11.7%
Near-prime	2.5%	2.4%	2.6%	3.5%	4.8%	4.3%	3.0%
Prime	0.5%	0.6%	0.7%	0.9%	1.6%	1.6%	1.0%
Super-prime	0.1%	0.1%	0.1%	0.1%	0.2%	0.2%	0.2%
Line							
Low	9.9%	8.2%	8.9%	11.2%	11.2%	8.9%	6.0%
Medium	3.8%	3.2%	3.1%	3.8%	4.8%	4.1%	2.8%
High	1.1%	1.1%	1.0%	1.4%	2.1%	2.0%	1.4%
Utilization							
Low	1.3%	1.1%	1.1%	1.2%	1.4%	1.1%	0.8%
Medium	5.5%	4.5%	4.4%	5.2%	5.9%	5.3%	3.9%
High	13.0%	11.6%	12.4%	15.4%	16.4%	13.7%	8.4%

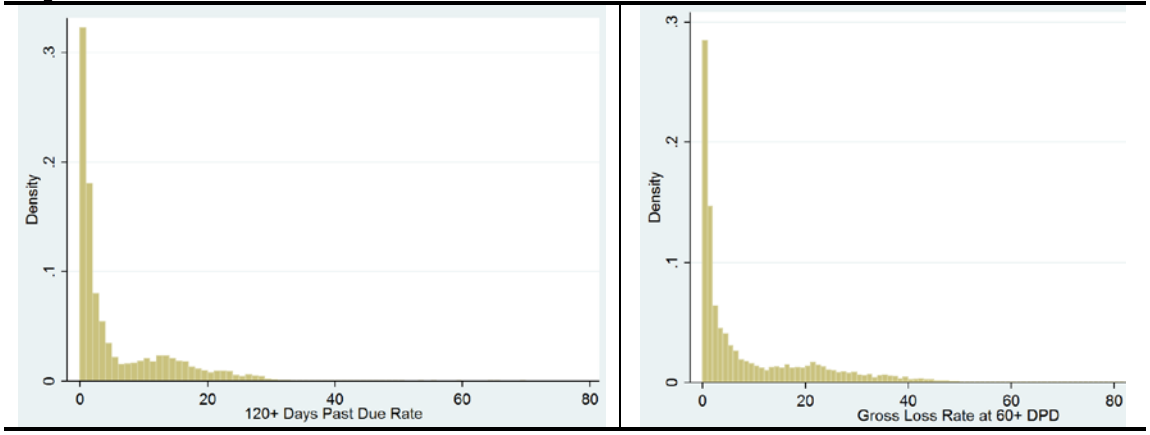
Note: Descriptive statistics are account weighted at the segment level. Data source: Equifax

Table 2b: Loss Rates, for 60 Days Past Due Balance over Outstanding Ratio, Across Risk Segments

Variable	2001-03	2004-06	2007	2008	2009	2010	2011
Risk Score							
Subprime	23.5%	20.4%	22.5%	27.3%	28.4%	21.6%	16.7%
Near-prime	5.4%	5.6%	6.7%	8.2%	9.8%	8.2%	5.7%
Prime	1.4%	1.5%	1.7%	2.4%	3.9%	3.6%	2.1%
Super-prime	0.3%	0.2%	0.2%	0.4%	0.6%	0.7%	0.3%
Line							
Low	11.1%	10.0%	11.3%	14.1%	14.1%	10.7%	7.1%
Medium	6.3%	6.2%	7.0%	8.4%	9.6%	7.2%	5.5%
High	3.7%	3.4%	3.8%	5.8%	7.8%	6.8%	4.7%
Utilization							
Low	6.8%	6.0%	7.1%	9.2%	10.2%	7.0%	4.5%
Medium	5.4%	4.8%	5.4%	6.7%	7.3%	6.3%	5.0%
High	10.9%	10.5%	11.5%	13.9%	15.4%	12.6%	8.5%

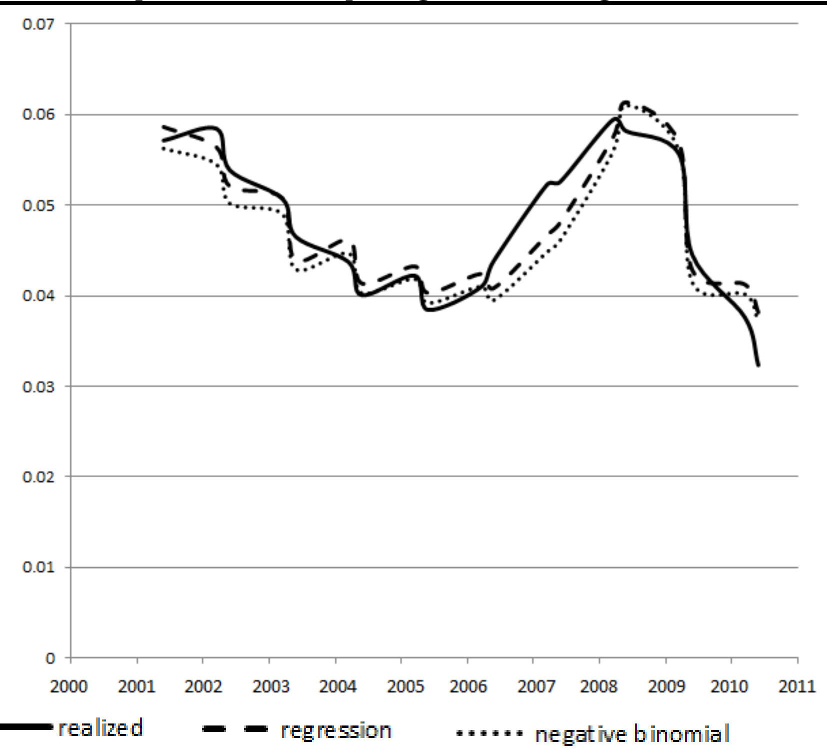
Note: Descriptive statistics are account weighted at the segment level. Data source: Equifax

Figure 2: Distribution of Default and Loss Rates



Note: Distributions are account weighted at the segment level. Data source: Equifax

Figure 3: In Sample Model Fit, Simple Regression and Negative Binomial Model



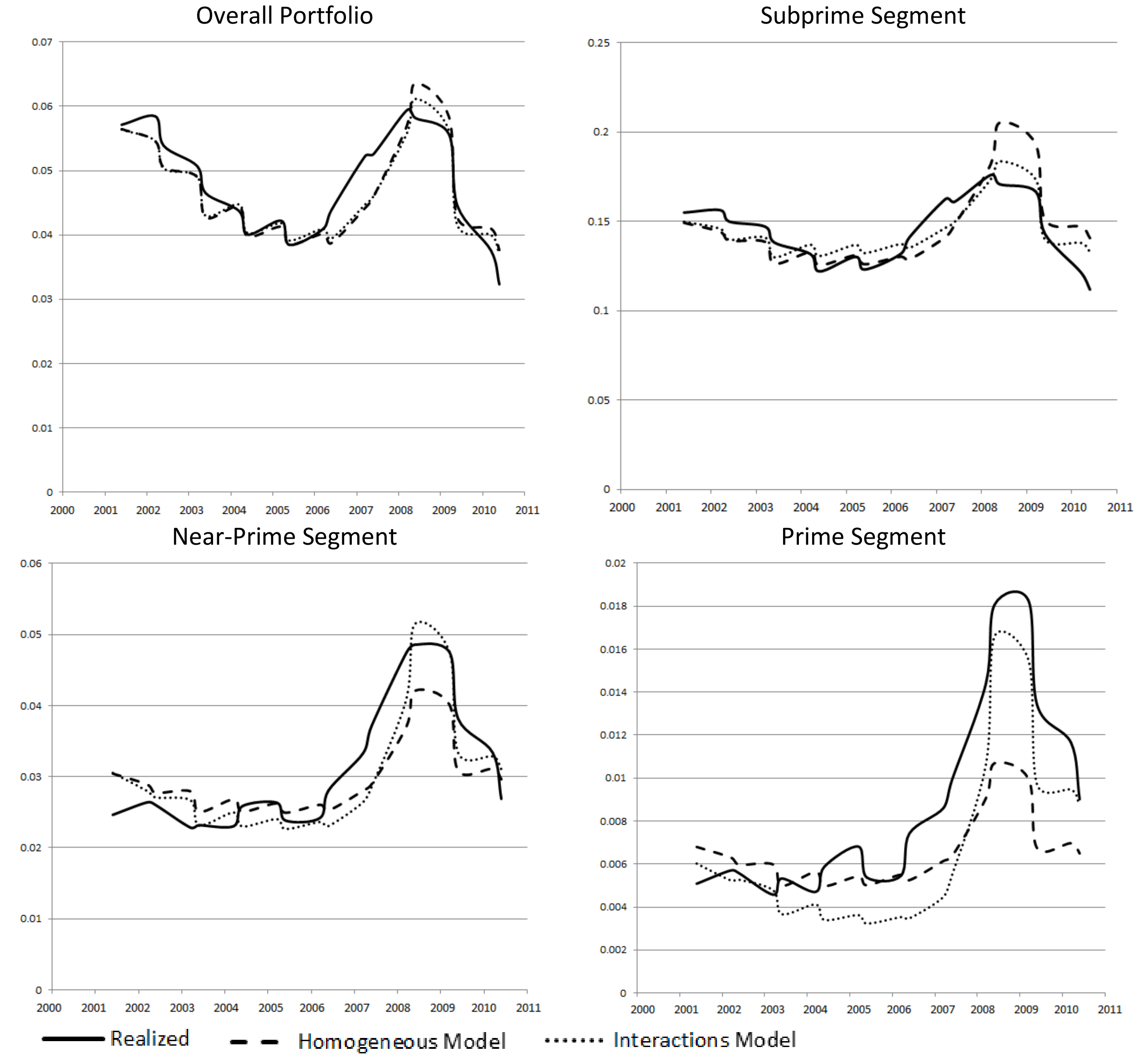
Note: The zero-inflated Poisson model provides an identical fit to the negative binomial model and has been excluded from the figure. Projections are account weighted at the segment level. Data source: Equifax

Table 3: Estimated Negative Binomial Regression for the Overall Sample

	Default Rate Model				Loss Rate Model			
Credit Score:	SP	NP	P	SPP	SP	NP	P	SPP
Line								
Medium	0.76	0.87	1.11	2.19	0.69	0.86	1.05	1.11
High	0.87	0.90	1.08	2.33	0.73	0.82	1.03	1.28
Utilization								
Low	0.54	0.33	0.27	0.49	0.99	0.90	0.57	0.65
Medium	0.60	0.51	0.48	0.45	0.63	0.60	0.55	0.52
Unemployment								
Ur.	1.00	1.04	1.10	1.09	0.98	1.03	1.08	1.08
Chg. Ur. 12m.	1.07	1.12	1.11	1.04	1.10	1.14	1.16	1.04
Q2 dummy	1.05	1.05	1.05	1.01	1.04	1.06	0.98	1.07
Constant	21.04	4.64	1.22	0.18	34.34	7.37	1.98	0.47

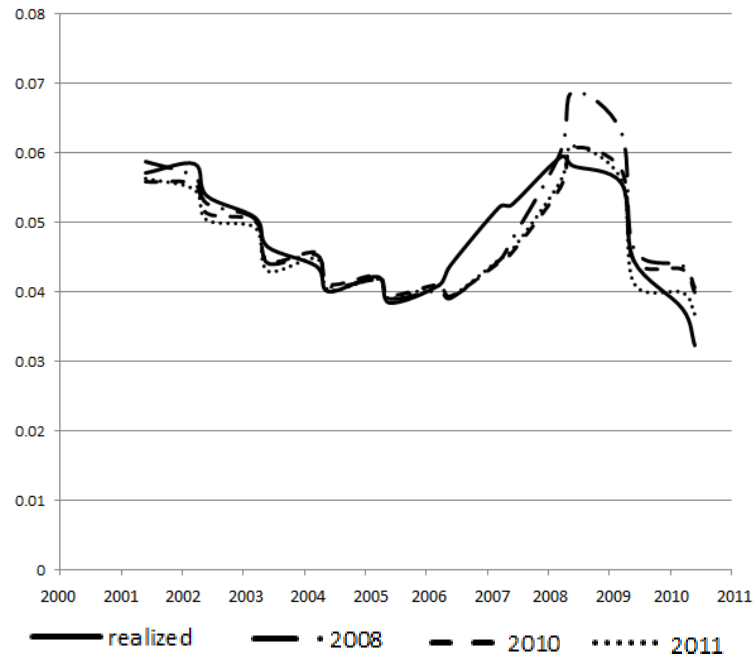
Note: “sp” refers to subprime, “np” refers to near-prime, “p” refers to prime, and “spp” refers to super-prime. Models are estimated using weights for the number of accounts at the segment level. All values reported are statistically significant at the 99% level. Data source: Equifax

Figure 4: Portfolio Projections and Projections Across Risk-Score Segments



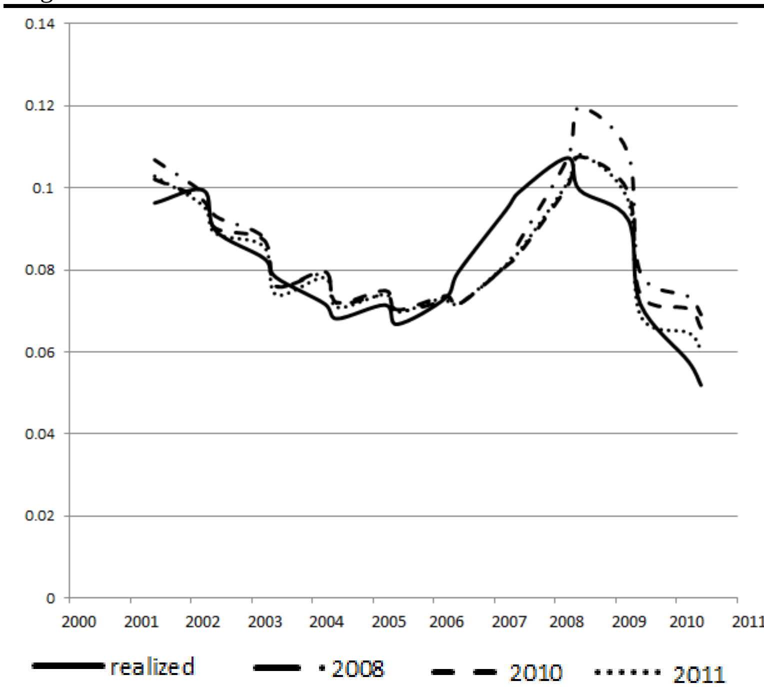
Note: Models are estimated using weights for the number of accounts at the segment level. Data source: Equifax

Figure 5: Default Rates Model Fit



Note: Models are estimated using weights for the number of accounts at the segment level.
Data source: Equifax

Figure 6: Loss Rates Model Fit



Note: Models are estimated using weights for the number of accounts at the segment level.
Data source: Equifax