WORKING PAPERS
RESEARCH DEPARTMENT

WORKING PAPER NO. 15-07
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January 2015

RESEARCH DEPARTMENT, FEDERAL RESERVE BANK OF PHILADELPHIA
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Abstract

A sovereign's inability to commit to a course of action regarding future borrowing and default behavior makes long-term debt costly (the problem of debt dilution). One mechanism to mitigate the debt dilution problem is the inclusion of a seniority clause in sovereign debt contracts. In the event of default, creditors are to be paid off in the order in which they lent (the "absolute priority" or "first-in-time" rule). In this paper, we propose a modification of the absolute priority rule that is more suited to the sovereign debt context and analyze its positive and normative implications within a quantitatively realistic model of sovereign debt and default.

JEL Classifications: E44, F34, G12, G15

Keywords: debt dilution, seniority, sovereign default

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1 Introduction

External foreign debt is an important vehicle through which many sovereign countries finance investment and consumption. When the debt is long term, a sovereign acting in a discretionary fashion will choose to ignore the adverse impact of additional borrowing on the value of outstanding debt and, therefore, will tend to borrow too much and default too frequently.[1] This (so-called) *debt dilution problem* makes long-term debt costly and inflicts welfare losses on the sovereign.

The debt dilution problem has been viewed as an important reason for emerging-market borrowers to be crisis prone and, in the event of a crisis, experience a costly and protracted period of restructuring (Borensztein, Chamon, Jeanne, Mauro, and Zettelmeyer (2006)). The possibility of debt dilution (and the attendant higher interest cost of debt) is thought to induce sovereign borrowers to opt for debt structures that are hard to dilute, such as very short-term debt (Sachs and Cohen (1982) and Kletzer (1984)) or debt that cannot be easily restructured so that the costs of default are high and the likelihood of default correspondingly low (Shleifer (2002) and Dooley (2000)). The tendency toward short-maturity debt exposes the sovereign to the risk of a confidence-driven rollover crisis (Giavazzi and Pagano (1990) and Cole and Kehoe (1996)), and the tendency toward hard-to-restructure debt makes crises very costly when they happen, perhaps inefficiently so (Bolton and Jeanne (2009)).

Debt dilution also appears as the key friction in quantitative-theoretic models that focus on explaining the unique characteristics of emerging market business cycles (Neumeyer and Perri (2005), Aguiar and Gopinath (2006), and Arellano (2008), among others). This literature builds on the endogenous default model of Eaton and Gersovitz (1981). Extended to long-term debt, this class of models implies that debt dilution is an important force elevating the frequency of default and the volatility of spreads on emerging market sovereign debt (Chatterjee and Eyigungor (2012) and Hatchondo, Martinez, and Sosa-

[1] Argentina's decision to issue bonds to a wide base of borrowers in the late 1990s is an example of a sovereign choosing to dilute the value of debt in the hands of existing bondholders. Similarly, the decision by Russia and Ukraine to issue short-term debt in the months leading up to default in late 1998 is another example of dilution. In these instances, the possibility of dilution arguably encouraged these countries to take on more debt than they would have otherwise and made default more likely.

Padilla (2014)).[2]

It is well known that an explicit seniority structure on debt can mitigate the dilution problem. Seniority means that in the event of default, a creditor who lent earlier must be paid in full before a later creditor can be paid anything at all (the "first-in-time" or "absolute priority" rule). Such a rule makes it harder to dilute long-term debt because existing creditors (in the event of default) do not have to share default payments with new creditors.[3] Since sovereign defaults are followed by a settlement on the defaulted debt, the imposition of a "first-in-time" rule has been proposed as a (partial) solution to the debt dilution problem (Borensztein, Chamon, Jeanne, Mauro, and Zettelmeyer (2006), Bolton and Skeel (2004), Bolton and Jeanne (2009) and Gelpern (2004)).[4]

The goal of this paper is to advance our understanding of the role of seniority in ameliorating the costs of debt dilution by incorporating seniority in a quantitatively realistic, infinite-horizon model of sovereign debt. The seniority arrangement we study is in the spirit of a proposal put forth in Bolton and Skeel (2004). The core of their proposal is an arrangement in which, in the event of default, creditors as a group decide on the size of the settlement (equivalently, the haircut on the defaulted debt) with the proceeds distributed in the order of absolute priority (first to lend, first to be repaid).

We make three sets of contributions. First, on the methodological side, we develop a computationally tractable and analytical transparent way of keeping track of seniority. The idea is to index each bond by its *rank* at the time of issuance and then arrange the assignment of rank in such a way that a bond issued in period t has a higher rank than a bond issued in any period $\tau > t$. The scheme is generally applicable and can be used to study the effects of seniority in other contexts such as corporate bonds.

[2] Chatterjee and Eyigungor (2012) reach this conclusion by comparing the model outcome with long-term debt to the outcome with short-term debt that cannot be diluted. Hatchondo, Martinez, and Sosa-Padilla (2014) reach it more precisely by considering idealized debt contracts that exactly compensate existing creditors for losses resulting from additional borrowing by the sovereign.

[3] Fama and Miller (1972) gave an early discussion of how creditors of a firm can protect themselves from dilution by making their loans senior.

[4] It is important to note, however, that seniority is not a panacea. For instance, giving seniority to the most recently issued debt might lead to better outcomes if investment decisions are endogenous (Hennessey (2004)) or if there is the possibility of an inefficient default due to coordination difficulties among existing creditors (Saravia (2010)). Also, Bizer and DeMarzo (1992) show that if borrowers can influence the probability of default through their effort decisions, the ability to borrow sequentially can lead to inefficiently high levels of debt and default even if seniority among creditors is respected.

Second, we propose and investigate a *relative* priority rule that, we believe, is more appropriate to the sovereign debt context. We show that an absolute priority rule maximally intensifies intercreditor conflict regarding the acceptable "haircut" on the defaulted debt. In light of this, we propose a rule in which a contractually pre-determined fraction of each bond (regardless of seniority) is required to suffer the aggregate haircut on the defaulted debt, with only the remaining fraction eligible to receive payouts in accordance with the bond's seniority. Thus, our proposal combines the *pari passu* arrangement with the absolute priority rule. We show that such a hybrid arrangement has the potential to eliminate conflict between junior and senior creditors at the time of settlement.

Finally, we provide a quantitative exploration of the benefits of the proposed seniority arrangement for Argentina. We parametrize our model to match (for the current *pari passu* system) Argentina's average debt-to-GDP ratio, the average spread on its debt, the length of delay, and the level of repayment on its debt in the most recent (2001) default episode. Then, we investigate how equilibrium properties of the model change as the *pari passu* system is replaced with one that mixes absolute priority and *pari passu* in equal measure (i.e., one-half of each bond suffers the aggregate haircut, while the other half receives distributions according to seniority). We find that this change reduces default frequency by about 35 percent and on average spreads by about 50 percent, and it boosts average equilibrium debt by 25 percent. The welfare gain is about 2 percent of flow consumption, with the bulk of it coming from the expansion in debt. These effects are more pronounced if a larger fraction of each bond receives payouts according to seniority.

To put these findings in perspective, several background issues need to be addressed. First, does respecting a relative priority rule require additional commitment on the part of the sovereign? The legal literature suggests it does not. It is generally understood (see, for instance, Borensztein et al. (2006)) that for seniority to be effective, senior creditors must have the right to sue junior creditors who receive payments in contravention of their order of priority. It is sufficient that this right exists in jurisdiction where the debt is issued (such as New York State). In this sense, a seniority clause does not presuppose any additional commitment on the part of the sovereign.[5]

[5]To strengthen the effectiveness of the clause, it may need to give senior creditors an "exit option" in case

Second, what are the costs of implementing seniority? For a relative priority rule to work, it is necessary that investors be able to determine which outstanding bonds are ahead in the queue when contemplating the purchase of new bonds in the primary market or existing bonds in the secondary market. For this information to be easily available, there will need to be a "bond registry" that keeps track of which bonds issued by a sovereign are currently outstanding.[6] The setup and maintenance costs of this registry will have to be borne by the sovereign.[7]

Third, what are the advantages of seniority as a solution to the debt dilution problem over other mechanisms? An alternative mechanism, proposed in Hatchondo, Martinez, and Sosa-Padilla (2014) is a debt covenant that directly constrains the borrowing of the sovereign — such as a debt limit or a limit on the debt-to-output ratio. Debt limits or limits on the debt-to-output ratio will presumably require adjustments as fundamentals evolve; in contrast, a seniority arrangement is flexible, with the debt capacity of the sovereign adjusting to fundamentals as needed.

The rest of the paper is organized as follows. In Section 2, we lay out the basic assumptions regarding preferences, endowments, market arrangements, and the default option. Then, in Section 3, we briefly describe the sovereign's decision problem and equilibrium bond price function under the *pari passu* arrangement. In Section 4, we introduce our notion of rank to keep of track of seniority and then describe the seniority arrangement explored in this paper. In Section 5, we explain how the strength of seniority rights affects

the sovereign issues new debt whose priority in relation to existing debt is unclear. For instance, the "exit option" could take the form of an immediate demand for repayment of debt whose seniority is potentially compromised by a new issuance. It is also worth noting that in recent years, the developed-country courts (where the sovereign debt is typically issued) have handed down rulings that suggest seniority in sovereign debt contracts will be enforced by the courts. See, for instance, the discussion in Bolton and Skeel (2005) (pp. 188-189) of the *Elliott Associates v. Republic of Peru* case, wherein the court in Brussels permitted Elliott to compel EUROCLEAR — the entity responsible for disbursing payments to creditors who had accepted Peru's restructured debt in 2001 — to stop making these payments because Elliott's prior claim on the Republic of Peru had not been satisfied. Most recently, the U.S. Supreme Court similarly upheld the rights of creditors pursuing repayment on Argentina's 2001 defaulted debt to stop payments to creditors who accepted Argentina's restructured debt in 2005.

[6]Our paper abstracts from other forms of explicit government liabilities (bank loans, bilateral loans, loans advanced by multilateral institutions) and implicit liabilities (deposit guarantees, obligations to pension, and Social Security funds). See Roubini and Setser (2004) (Ch.7) and Gelpern (2004) for a wide-ranging discussion of the prospects and problems in enforcing seniority across the whole gamut of sovereign liabilities.

[7]Since this sort of information is a byproduct of sovereign borrowing and repayment activity, making it widely available would seem not to be an overly taxing requirement.

junior and senior creditors' incentives regarding the acceptable haircut on the defaulted debt and show how our relative priority scheme attenuates the intercreditor conflict along this dimension. In Section 6, we present and explain our quantitative findings. Section 7 collects concluding comments.

2 Environment

Time is discrete and denoted $t \in \{0, 1, 2, ...\}$. The sovereign receives a strictly positive endowment y_t each period. The stochastic evolution of y_t is governed by a first-order finite-state Markov chain with state space $Y \subset \mathbb{R}_{++}$ and transition law $\Pr\{y_{t+1} = y' | y_t = y\} = F(y', y) > 0,\ y$ and $y' \in Y$.

The sovereign maximizes expected utility over consumption sequences, where the utility from any given sequence c_t is given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t),\ 0 < \beta < 1. \quad (1)$$

The momentary utility function $u(\cdot) : [0, \infty) \to \mathbb{R}$ is continuous, strictly increasing, and strictly concave.

The sovereign can borrow in the international credit market with the option to default. We analyze long-term debt contracts that mature probabilistically (Leland (1998)). Specifically, each unit of outstanding debt matures next period with probability λ; if the unit does not mature, it gives out a coupon payment z. Unit bonds are infinitesimally small: b units outstanding at the start of the period implies a certain debt service of $[z \cdot (1 - \lambda) + \lambda]b$. As is customary in this literature, we view debt as negative assets and assume that $b \in [\underline{b}, 0] = B$, where $\underline{b}$ is some suitably large negative number (for simplicity, we abstract from asset accumulation by the sovereign).

The option to default means that the sovereign has the right to unilaterally stop servicing its debt obligations. Default is costly in two ways. First, as long as the sovereign does not reach settlement with creditors on the defaulted debt, the sovereign has no access to the international credit market. Second, as long as it remains in default, the country experiences a loss in output $\phi(y)$ each period. These assumptions mean that as long as the sovereign remains in default, the country consumes $y - \phi(y)$ units of goods. We assume that $y - \phi(y) > 0$ for all $y \in Y$ and that $y - \phi(y)$ is increasing in y.[8]

Since repayment on defaulted debt is a precondition for discussing the role of seniority, we incorporate a theory of repayment but keep it relatively simple.[9] We assume that settlement occurs with probability $\xi > 0$ in any period following default. The level of debt agreed to in the settlement depends on the output level prevailing at the time of settlement (and other parameters). The determination of the settlement size, G, is discussed in detail later in the paper (Section 5).

Within this framework, we analyze two different market arrangements. In one arrangement, all defaulted debt is treated equally in the settlement (which is the current system). In the other arrangement, the defaulted debt is classified into priority groups depending on their time of issuance, with earlier debts given higher priority at the time of settlement. For each market arrangement, we assume that lenders are risk-neutral and that the market for sovereign bonds is competitive.

3 The Model Without Seniority

With this market arrangement, the price of a unit bond will depend only on the current persistent component of output and on the level of outstanding debt. We will denote the price of a unit bond by $q(y, b')$.

3.1 Decision Problem of the Sovereign

Denote the lifetime utility of a sovereign in good standing (i.e., one that is not in a state of default) that enters a period with (y, b) by $W(y, b)$ and the lifetime utility of a sovereign

[8] In this paper, a function $f(x)$ is *increasing (decreasing)* in x if $x' > x$ implies $f(x') \geq (\leq) f(x)$ and is *strictly increasing (strictly decreasing)* in x if $f(x') > (<) f(x)$.

[9] The theory of post-default bargaining between a sovereign and its creditors is an active area of research. Our approach is closest in spirit to Yue (2010). Benjamin and Wright (2009) and D'Erasmo (2012) provide alternative approaches that attempt to explain why delays arise in reaching settlement.

in a state default as $X(y)$. Then,

$$X(y) = u(y - \phi(y)) + \beta E_{\{y'|y\}}\{(1-\xi)X(y') + \xi W(y', G(y'))\}, \tag{2}$$

where $G(y')$ is the debt level the sovereign agrees to in the settlement if it happens next period. If settlement does not happen, the sovereign continues in the state of default. The payoff from repaying the debt, denoted $V(y, b)$, is given by

$$\begin{aligned} &V(y,b) = \max_{b' \in B} u(c) + \beta E_{\{y'|y\}} W(y', b') \\ &\text{s.t.} \\ &c \leq y + (\lambda + [1-\lambda]z)b + q(y, b')[(1-\lambda)b - b']. \end{aligned} \tag{3}$$

The above assumes that the budget set under repayment is nonempty, meaning there is at least one choice of b' that leads to nonnegative consumption. But it is possible that all choices of b' may lead to negative consumption, in which case repayment is simply not an option. In this case the value of $V(y, b)$ is set to $-\infty$. In the event repayment is feasible, the optimal debt choice conditional on repayment is denoted $a(y, b)$. We assume that if the sovereign is indifferent between two distinct b's, it chooses the larger one (i.e., it chooses a lower debt level over a higher one).

Finally,

$$W(y, b) = \max\{V(y, b),\ X(y)\}. \tag{4}$$

Since W determines both X and V (via equations (2) and (3), respectively), equation (5) defines a Bellman equation in W. This equation implicitly determines the sovereign's default decision rule $d(y, b)$, where $d = 1$ if the default is optimal and 0 otherwise. We assume that if the sovereign is indifferent between repayment and default, it repays.

3.2 Equilibrium Bond Price

The world one-period risk-free rate r_f is taken as exogenous. Under competition, the price of a unit bond satisfies the following pricing equations:

$$q(y, b') = \tag{5}$$
$$E_{\{y'|y\}} \left[[1 - d(y', b')] \frac{\lambda + (1-\lambda)[z + q(y', a(y', b'))]}{1 + r_f} + d(y', b') \frac{P(y')}{b'} \frac{1}{(1 + r_f)} \right],$$

where $P(y)$ is the aggregate value of repayment expected on the defaulted debt conditional on output being y. Since this aggregate value is equally distributed across all bonds, each bond will, in expectation, receive $E_{y'|y} P(y')/b'$. The aggregate value of expected repayment is given by

$$P(y) = E_{\{y'|y\}} \left[(1 - \xi) \frac{P(y')}{1 + r_f} + \xi \frac{G(y') q(y', G(y'))}{1 + r_f} \right]. \tag{6}$$

In the event a settlement is reached next period, creditors as a group receive a settlement whose aggregate value is $q(y', G(y'))G(y')$.

The combination of repayment on defaulted debt and the possibility of debt dilution can lead to behavior that blurs the distinction between repayment and default. When default is imminent, the sovereign may choose to borrow as much as it can and then default with very high probability next period. By borrowing as much as it can today, the sovereign minimizes the share of the default payout going to existing creditors and promises as much of the payout as possible to investors who buy its new debt. In this way, the sovereign increases its current consumption at the expense of existing creditors and postpones the costs of default by one period.[10]

In models calibrated to resemble real economies, maximum dilution before default is a common outcome. Since we do not see maximum dilution in reality, the model needs

[10] Formally, suppose the sovereign services its current debt b and issues enough debt, M', so that default is certain next period. Servicing b costs $[\lambda + (1-\lambda)z]\, b$. Against this cost is the revenue from new issuances $-q(y, M')\,(M' - [1-\lambda]b)$. Since default is certain for M', $q(y, M')M' = E_{\{y'|y\}} P(y')/(1 + r_f)$ and the revenue from new issuances can be raised by issuing as much as debt as possible, namely, $\underline{b}$. If $q(y, \underline{b})$ is approximately zero, the payoff from issuing $\underline{b}$ (and defaulting for sure next period) is approximately $u(y + \Delta) + \beta E_{\{y'|y\}} X(y')$, where $\Delta = E_{\{y'|y\}} P(y')/(1 + r_f) - [\lambda + (1-\lambda)z]\,(-b)$. If $F(y, y) \approx 1$ and $\Delta > 0$, this payoff will exceed $X(y)$.

to be augmented with some force that counteracts such behavior. We assume this force is *underwriting standards* that impose an upper bound on the anticipated probability of immediate default on a new issue of bonds.[11] As noted in Flandreau, Flores, Giallard, and Nieto-Parra (2009), such standards have been common in sovereign debt markets for a long time, although this "gatekeeping" role appears to have weakened in recent times. We incorporate this force in our model in the form of a constraint requiring that if $[(1-\lambda)b - b'] > 0$, then $E_{y'|y}d(y', b')$ cannot exceed some $\delta \in (0, 1)$.

4 The Model with Seniority

In a market arrangement where seniority is enforced, it is necessary for investors to keep track of the seniority of the bonds they hold. If all debts issued in a given period have the same priority (i.e., are all equally senior), then, generally speaking, an investor needs to know the vector $\{i_{t-1}, i_{t-2}, \ldots\}$, where $i_{t-\tau}$ is the stock of debts issued $\tau \geq 1$ periods ago that is still outstanding, to correctly price the new debt issued in period t. This feature makes the quantitative exploration of a model with seniority subject to a severe "curse of dimensionality."

To circumvent the "curse," we propose a way of keeping track of seniority that does not involve keeping track of the vector $\{i_{t-1}, i_{t-2}, \ldots\}$. The idea is to imagine that every unit bond outstanding has a rank and to arrange the assignment of rank is such a way that bonds issued in period t have higher rank than bonds issued in any subsequent period $\tau > t$. Then, one single state variable, the rank, can be used to keep track of seniority. With this in mind, let s denote the rank of a unit bond. If there are b units of the debt outstanding, the rank of any given unit bond is a unique number $s \in [b, 0]$. The closer s is to 0, the higher the bond's seniority is in the event of default.

The assignment of rank is done in the following way. Suppose that the sovereign arrives in some period with b units of bonds outstanding and issues new bonds, i.e., $b' < (1-\lambda)b$. Each member of the mass of newly issued bonds is identical at the time

[11] Sovereign bonds issued in financial centers (such as New York, London, Frankfurt, and Tokyo) have to be underwritten by some investment bank. Reputational concerns may make these banks wary of issuing bonds on which the probability of immediate default is very high.

of sale, but, following the sale (say at the end of the period), each newly issued bond is randomly assigned a unique rank s in the (semi-open) interval $[b', (1-\lambda)b)$. Thus, all newly issued bonds have lower rank than any outstanding bond and the set of rankings of all debt outstanding at the end of the period is $[0, b']$. If the sovereign buys back debt (i.e., $b' > (1-\lambda)b$), then it is safe to assume that it is the mass of the most junior bonds — namely, the unit bonds with $s \in [(1-\lambda)b, b']$ — that are bought back (this point is discussed in more detail below). Thus, in the event of a buyback, the set of rankings of all debt outstanding at the end of the period is also $[0, b']$.

How does a bond's rank evolve over time? Consider a unit bond with rank $s \in [b, 0]$. Since any unit bond has a probability λ of maturing next period, among the bonds that are higher ranked than s, there is a fraction λ that will mature. Thus, at the start of the next period, there will only be $(1-\lambda)s$ bonds with a rank higher than s. This means that we can preserve the current ranking among all bonds with rank greater than or equal to s if we reset the rank of each unit bond that survives into the next period to $(1-\lambda)s$.[12]

This setup implies that a bond with rank s could not have been issued earlier than a bond with rank $\tilde{s} > s$. In what follows, respecting seniority means that the payout to the bond with rank s cannot exceed the payout to the bond with rank $\tilde{s}$ in the event of default, but it can be less. This means, however, that two bonds issued in the same period (i.e., part of the same issue) may be assigned different payouts in the event of default. Actual proposals for implementing seniority do not have this feature (all bonds of the same issue are viewed as equally senior and treated equally in default). Thus, seniority based on s should be viewed as a computationally tractable approximation to seniority based on the time of issuance.

4.1 Pricing of New Debt and Buybacks

Since the payoff to the bondholder in the case of default depends on the seniority of the bond, the price of a unit bond with ranking $s \in [0, b]$ will depend on the triple (y, b, s).

[12]This resetting rule implies that the unit bond with rank 0 (the most senior unit bond) continues to have rank 0 as long as it survives, and any other unit bond's rank approaches 0 at a geometric rate the longer it survives.

Denote this price by $q(y, b, s)$.

In the event of new issuances, the revenue obtained from the sale is simply the sum (more precisely, the integral) of the price of each new unit issued once each unit has been assigned a rank. Since the assignment of rank following a sale is completely random, the expected market value of any given unit of new bonds is given by

$$Q(y, b', (1-\lambda)b) = \frac{\int_{b'}^{b(1-\lambda)} q(y, b', s)ds}{(1-\lambda)b - b'} \tag{7}$$

Given that investors are risk neutral, we may take $Q(y, b', (1-\lambda)b)$ as the competitive market price of each new bond issued.

In the event of a buyback, the sovereign buys the least senior bonds at a price that is equal to the price of the junior-most bond *after* the buyback. The reason for this specification is as follows. First, whether the sovereign buys senior or junior bonds will have no effect on the seniority structure of outstanding debt, since junior debt will rise in seniority if senior debt is bought back. Second, as we show later in the paper, the price function $q(y, b, s)$ is increasing in s (more senior bonds trade at a higher price). The combination means that the sovereign will minimize its purchase cost if it purchases junior-most debt. Third, observe that a bondholder with a bond with $s < b'$ knows that he will be in possession of a bond whose value following the buyback will be $q(y, b', b')$ and therefore will be unwilling to sell his bond at any price less than $q(y, b', b')$. The upshot is that a sovereign that wishes to buy back debt can implement its plans at the least cost if it announces that it will buy debt back at the price $q(y, b', b')$.[13] Since

$$\lim_{b \downarrow b'} Q(y, b', b) = \lim_{b \downarrow b'} \frac{\int_{b'}^{b} q(y, b', s)ds}{b - b'} = q(y, b', b'), \tag{8}$$

we may also say that the sovereign buys debt back at the price $Q(y, b', b')$.

[13] Bondholders who hold unit bonds with ranking $s > b'$ have bonds whose price, namely, $q(y, b', s)$, is at least as large as $q(y, b', b')$ and therefore will have no incentive to participate in the buyback.

4.2 Decision Problem of the Sovereign

With some abuse of notation, we will continue to use the same notation for lifetime utility $W(y,b)$ (and other value functions and decision rules), although there is no presumption that these functions are the same as in the case without seniority. Then, the payoff from default, $X(y)$, is given as in (2). The payoff from repayment is given by

$$V(y,b) = \max_{b' \in B} u(c) + \beta E_{y'|y} W(y', b') \tag{9}$$
$$\text{s.t.}$$
$$c = y + [\lambda + (1-\lambda)z]\, b + R(y,b,b'),$$

where $R(y,b,b')$ denotes the revenue received from changing the asset level from b to b' (it will be positive if new bonds are issued and negative if bonds are bought back). This function is given by

$$R(y,b,b') = \begin{cases} Q(y,b',(1-\lambda)b)[(1-\lambda)b - b'] & \text{if } (1-\lambda)\,b - b' > 0 \\ Q(y,b',b')\,[(1-\lambda)\,b - b'] & \text{if } (1-\lambda)\,b - b' \leq 0. \end{cases}$$

Finally, as before, $W(y,b)$ is given by (5).

4.3 Seniority Rules

We analyze seniority rules that involve a modest modification of the *pari passu* clause. The modification envisages a hybrid system in which all debts regardless of seniority receive a contractually predetermined fraction, $(1-\theta)$, of the aggregate payout on the defaulted debt with only the remaining portion, θ, of the payout distributed in accordance with seniority. The end result is that bonds end up in two tiers: those that receive only the base amount allocated to all bonds and those that also receive an additional payment because they are sufficiently senior.

Formally, suppose that at the time of settlement the sovereign issues debt $G_\theta(y) < 0$, where we have indexed the settlement debt by θ since that will be important. The aggregate value of this payout is $Q(y, G_\theta(y), 0)(-G_\theta(y))$.[14] All bonds, regardless of rank, receive a $(1-\theta)$ portion of this aggregate payout, pro-rata. Thus, in value terms, each unit bond receives

$$(1-\theta)\frac{Q(y, G_\theta(y), 0)(-G_\theta(y))}{(-b)} = (1-\theta)Q(y, G_\theta(y), 0)\frac{G_\theta(y)}{b}, \tag{10}$$

where $b < 0$ is the amount of debt defaulted on. The remaining θ portion of the aggregate payout is distributed only to bonds with rank $s \geq G(y)$, pro-rata. Thus, sufficiently senior bonds receive an additional

$$\theta\frac{Q(y, G_\theta(y), 0)(-G_\theta(y))}{-G_\theta(y)} = \theta Q(y, G_\theta(y), 0). \tag{11}$$

This settlement rule has the following simple interpretation in terms of quantities. It is as if each defaulted bond is made up of two parts: a portion $(1-\theta)$ that is treated *pari passu* and a portion θ that is treated according to the bond's seniority. If $G_\theta(y)$ units of bonds are issued in settlement, the *pari passu* portion of each bond must suffer the aggregate haircut and receive $(1-\theta)\times G_\theta(y)/b$ units of these bonds. If the defaulted bond is sufficiently senior, i.e., $s \geq G_\theta(y)$, its seniority-based portion receives $\theta \times 1$ unit of the settlement bond (otherwise, the seniority-based portion receives nothing). The parameter θ controls the degree to which seniority is maintained during settlement. If $\theta = 1$, bonds with rank less than $G_\theta(y)$ get nothing, while higher-ranked bonds receive exactly one unit of a settlement bond, which corresponds to an absolute priority rule. If $\theta = 0$, all bonds regardless of rank receive $G_\theta(y)/b$ units of the settlement bond, and the rule collapses to the *pari passu* arrangement.

Table 1 gives an example to illustrate how this arrangement would work in practice, when there is no distinction (in terms of seniority) between bonds issued in the same period. In the example, there are only four distinct issuances of amounts, 10, 20, 10, and 10 billion, listed in order of seniority (the face value of total defaulted debt is, thus, \$50 billion). We assume that the settlement is \$20 billion and θ is 0.90. The first row gives the amount owed by seniority. The second row records the same information, scaled

[14] At the time of settlement, the old defaulted debt is extinguished and outstanding *new* debt is zero.

Table 1: Settlement Allocations for $B = 50$, $G = 20$, $\theta = 0.9$

Owed & Allocated	Priority 1	Priority 2	Priority 3	Priority 4	Row Total
Owed	10	20	10	10	50
$\theta\times$ Owed	9	18	9	9	45
Pari Passu Allocation	0.40	0.80	0.40	0.40	2
Seniority Allocation	9	9	0	0	18
Combined Allocation	9.4	9.8	0.4	0.4	20
New/Old Face Value	0.94	0.49	0.04	0.04	

down by θ. The third row records what each issue receives due to the *pari passu* portion. In the aggregate, each dollar of debt receives 20/50 or 40 cents. Since the *pari passu* portion of each bond is 10 percent, each bond receives 4 cents on the dollar. Thus, the first issue receives 0.4 billion, the second issue 0.8 billion, and so on. The fourth row records the priority allocation. Since there is \$18 billion to allocate according to priority, the senior-most issue receives the entire θ-adjusted amount owed to it, namely, \$9 billion. The remaining \$9 billion is distributed entirely to the second senior-most issue since the adjusted amount owed to it is \$18 billion. The two junior-most issues receive nothing from the priority allocation. The end result of this allocation is shown in the fifth row. Finally, the last row records what each issue receives on the dollar (new versus old face values).

4.4 Equilibrium Bond Price Function

Let $p(y, b, s)$ be the expected default payout on a bond with rank $s \geq b$. Then, the the price of a unit bond of rank $s \geq b'$ satisfies

$$\begin{aligned} & q(y, b', s) = \\ & E_{y'|y}\left[[1 - d(y', b')]\frac{\lambda + [1-\lambda][z + q(y', a(y', b'), \max\left([1-\lambda]s, a(y', b')\right))]}{1 + r_f}\right. \\ & \qquad\qquad \left. +\ d(y', b')\frac{p(y', b', s)}{1 + r_f}\right]. \end{aligned} \tag{12}$$

In the event there is repayment and the bond does not mature, the rank of bond rises to $(1 - \lambda)s$ if the bond is not bought back, i.e., $a(y', b') \leq (1 - \lambda)s$. If the bond is bought back, the bondholder receives the price of a unit bond with rank $a(y', b')$, as explained earlier. In the event of default, the investor receives the expected value of the settlement payout $p(y', b', s)$.

The expected payout on a unit defaulted bond of rank $s \geq b$, denoted, $p(y, b, s)$, is given by the recursion

$$\begin{aligned} p(y,b,s) &= (1-\xi)E_{\{y'|y\}}p(y',b,s) + \\ &\xi E_{\{y'|y\}} \begin{cases} (1-\theta)Q(y',G_\theta(y'),0)\frac{G_\theta(y')}{b} & \text{if } s < G_\theta(y'), \\ \theta Q(y',G_\theta(y'),0) + (1-\theta)Q(y',G_\theta(y'),0)\frac{G_\theta(y')}{b} & \text{if } s \geq G_\theta(y'). \end{cases} \end{aligned} \tag{13}$$

We give a characterization result regarding the behavior of the equilibrium price schedule with respect to s and confirm our earlier claim that, all else remaining the same, the price of a unit bond is increasing in seniority (or rank) s. The proof of this claim essentially follows from the observation that the payoff to a bond of rank s cannot be any less (under any state of the world) than the payoff to a bond of rank $s' < s$.

Proposition 1 *For any $G_\theta(y)$, the equilibrium price function $q^*(y, b', s)$ is increasing in s.*

5 Settlement

We turn to the determinants of $G_\theta(y)$. Since $\theta = 0$ leads to the model without seniority, we may discuss the determination of G for both models in an integrated fashion.

Actual renegotiations on sovereign debt following default is a complex and potentially protracted affair. As discussed in Sturzenegger and Zettelmeyer (2007), creditors may join ad hoc committees that strive to form a united front against the sovereign. The sovereign, in turn, may seek to undermine such committees by making offers directly to individual creditors that are conditional on (super) majority acceptance. And, in turn, creditors reserve the right to reject majority-accepted offers and litigate, singly or in a group, for full repayment.

As there is no settled theory of such renegotiations, we take a simple approach. We assume that when the time for settlement arrives (recall that it arrives each period following a default with probability ξ), creditors as a group make a take-it-or-leave-it offer to the sovereign. Thus, if negotiations fail, the sovereign is condemned to autarchy. The lifetime value in this state is given by

$$A(y) = u(y + \alpha) + \beta E_{y'|y} A(y'), \tag{14}$$

where α is any additional net benefit or cost of autarchy and is a parameter that allows the model to match the debt recovery rate. Define $\overline{G}_\theta(y)$ as the level of debt that makes the sovereign indifferent between settling and remaining forever in autarchy:

$$E_{y'|y} W(y', \overline{G}_\theta(y)) = E_{y'|y} A(y'). \tag{15}$$

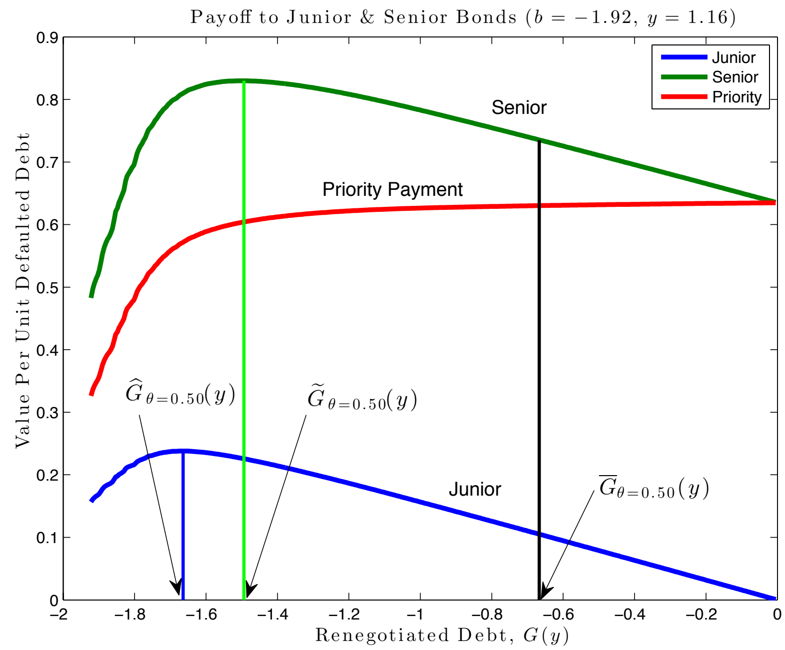
The current level of output does not appear directly in this equation because what the sovereign receives in the current period under either alternative is the same, namely, $u(y - \phi(y))$.

Consider, first, the case where $\theta = 0$. Since creditors share equally in the settlement payout, every creditor, regardless of his or her holdings of defaulted debt, would prefer a settlement that maximizes the aggregate value of bonds given in repayment, subject to the sovereign's participation constraint. Therefore, the creditors' take-it-or-leave-it offer, $G(y)$, satisfies:

$$G_{\theta=0}(y) = \text{argmax}_{G \geq \overline{G}_{\theta=0}(y)} \; q(y, G)(-G). \tag{16}$$

When seniority is enforced, however, creditor interests at the time of settlement will generally not be aligned. To see this, consider the case where $\theta = 1$ (i.e., strict absolute priority is enforced). If G units of bonds are issued in settlement, bonds with rank $s \geq G$ get exchanged for exactly one unit of the settlement bond. Thus, the interests of these senior creditors is to maximize the *price* of the settlement bond, not the aggregate value of the settlement. But the price of a defaultable bond generally declines with total issuance, which implies that a creditor with a bond of rank s would prefer the settlement to be large enough to include his bond, but no larger (i.e., he would prefer $G = s$). Similarly, a creditor with a bond of rank $\tilde{s} \neq s$ would prefer $G = \tilde{s}$.

Figure 1



When $\theta > 0$, however, intercreditor conflicts regarding the size of the settlement are attenuated. Figure 1 illustrates this point for the case where $\theta = 0.5$, $y = 1.16$, and defaulted debt is $b = -1.92$. In Figure 1, the bottom curve, labeled "Junior," plots $(1-\theta)Q(y, G, 0)G/B$, the payout to junior bonds (i.e., bonds with $s < G$) for each G. Since θ and B are given, the shape of this plot inherits the shape of the revenue curve $Q(y, G, 0)(-G)$. Thus, the payout to junior bonds rises until the level of debt $\widehat{G}_{\theta=0.5}(y)$ and then declines. The middle curve, labeled "Priority Payment," plots $\theta Q(y, G, 0)$. This is the value of the seniority-based payment that each senior bond receives when the settlement is G. Observe that this payment is simply a scaled-down version of the price of debt. For the usual reasons, this curve is declining in G. Finally, the top curve, labeled "Senior," plots $\theta Q(y, G, 0) + (1-\theta)Q(y, G, 0)G/B$, the total payout to senior bonds (i.e., bonds with $s \geq G$). This curve is simply the sum of the bottom and middle curves. It, too, inherits the shape of the total revenue curve but attains its maximum at $\widetilde{G}_{\theta=0.5}(y) > \widehat{G}_{\theta=0.5}(y)$.

The key implication of the inverted-U shape for the payoff to senior bonds is that the payout to *all* bonds, regardless of rank, is increasing in debt issued until G reaches

$\widetilde{G}_{\theta=0.5}(y)$. This is evident from the fact that the curves labeled "Senior" and "Junior" are both rising until $\widetilde{G}_{\theta=0.5}(y)$ and from the fact that the payout to a bond that switches from being junior to senior jumps *up* (from the bottom to the top curve). For $G < \widetilde{G}_{\theta=0.5}(y)$, however, conflicts between senior and junior bonds reappear. The payouts to junior bonds continue to rise until G reaches $\widehat{G}_{\theta=0.5}(y)$, and all bonds that switch to being senior gain, while the payout to all other bonds (i.e., senior bonds with $s \geq \widetilde{G}_{\theta=0.5}(y)$) decline. Beyond $\widehat{G}_{\theta=0.5}(y)$, the payout to all junior bonds declines as well; only bonds that switch from being junior to senior gain.

The key point we take from Figure 1 is that if $\overline{G}_\theta(y) \geq \widetilde{G}_\theta(y)$, then *all* creditors regardless of their specific holdings of bonds would agree to $\overline{G}_\theta(y)$ as the settlement size since a settlement with less debt will make all creditors worse off while a settlement with more debt is not feasible. Since creditors make a take-it-or-leave-it offer, it is in the interest of the sovereign to accept any offer as long as the offer gives it at least as much utility as "autarchy." Therefore, for this case, our theory unambiguously predicts that $G_\theta(y) = \overline{G}_\theta(y)$. This is the case shown in Figure 1, where the vertical line corresponding to $\overline{G}_\theta(y)$ has been drawn to the right of $\widetilde{G}_\theta(y)$. This, in fact, is true of the equilibrium underlying this figure and is true for all of the simulations reported in the next section.

Although not relevant for our simulations, we next consider what happens when $\overline{G}_\theta(y) < \widetilde{G}_\theta(y)$. In this case, all outcomes in $[\overline{G}_\theta(y), \widetilde{G}_\theta(y)]$ are in the bargaining set. Without additional information on specific bond holdings of each creditor, it is not possible to say which settlement size will be selected. Since any outcome in this set is acceptable to the sovereign, we assume that all outcomes are equally probable and set the settlement size equal to its expected value $[\overline{G}_\theta(y) + \widetilde{G}_\theta(y)]/2$.

To summarize, we assume

$$G_\theta(y) = \begin{cases} \overline{G}_\theta(y) & \text{if } \widetilde{G}_\theta(y) \leq \overline{G}_\theta(y) \\ (\overline{G}_\theta(y) + \widetilde{G}_\theta(y))/2 & \text{if } \widetilde{G}_\theta(y) > \overline{G}_\theta(y). \end{cases} \tag{17}$$

We close this section with a comment on the role of θ. We have already noted that θ controls the degree to which seniority is maintained during settlement. The upshot of this section is that θ also controls the degree of intercreditor conflict regarding the size of the settlement. When $\theta = 1$, the arrangement enforces strict absolute priority, which mitigates the debt dilution problem as much as possible but maximally intensifies the conflict between junior and senior creditors. In contrast, when $\theta = 0$, all creditors are treated equally in default, which eliminates intercreditor conflict regarding settlement size, but there is no mitigation of debt dilution at all. An intermediate value of θ is, thus, a compromise between mitigation of debt dilution and the degree of intercreditor conflict engendered by priority rules.

6 Welfare and Seniority: The Argentine Case

In this section, we explore the quantitative implications, both positive and normative, of enforcing seniority. We focus on Argentina, the country most intensively studied in the quantitative sovereign debt literature. To make the model quantitative, we assume that $u(c)$ is a CRRA function with curvature parameter $(1-\gamma)$ and that $\ln(y)$ follows an AR1 process with parameters $(\rho, \sigma_\epsilon^2)$. Following Chatterjee and Eyigungor (2012), we assume that the form of the default cost function $\phi(y)$ is given by $\max\{0, d_0 y + d_1 y^2\}$.

6.1 Calibration

The value of γ is set at 2, which is the standard value used in this literature. The parameters of the output process are estimated on linearly detrended quarterly real GDP data for the period 1980:1 to 2001:4.[15] The estimated values of ρ and σ^2 are 0.948503 and 0.027092^2, respectively. The risk-free rate r_f is set to 0.01, which corresponds to an annual rate of 4.0 percent.[16] The parameters describing the bond were determined to match the maturity and coupon information for Argentina reported in Broner, Lorenzoni,

[15]To avoid convergence issues, computation of the model requires a small transitory shock to endowments (see Chatterjee and Eyigungor (2012) for details). We assume the shock is uniformly distributed with variance 0.006^2, which is less than 1 percent of the variance of detrended GDP over this period. The quarterly data series on real GDP and the (nominal) yield on Argentine sovereign debt are taken from Neumeyer and Perri (2005). The GDP data were deseasonalized using the multiplicative X-12 routine in EViews.

[16]This is roughly the average nominal yield on 3-month U.S. Treasury bills over the period 1980:1 to 2001:4. The T-bill rate series used is the TB3MS series available at http://research.stlouisfed.org/fred2/categories/116.

Table 2: Parameters Selected Independently

Parameter	Description	Value
γ	Risk Aversion	2
σ	Standard Deviation of Output Innovation	0.027
ρ	Autocorrelation of Output Process	0.949
r_f	Risk-free Return	0.01
λ	Reciprocal of Avg. Maturity	0.05
z	Coupon Payments	0.03
ξ	Prob. of Settlement & Reentry	0.069
δ	Upper bound on default probability	0.75

and Schmukler (2007). The median maturity of Argentine bonds is 20 quarters, so $\lambda = 1/20 = 0.05$. We set $z = 0.03$, corresponding to an annual coupon rate of 12 percent.[17] The value of ξ is chosen to deliver an average delay of 3.6 years, which is the length of time it took for Argentina to reach settlement with a majority of creditors following the 2001 default. The value of δ was set at 0.75, which prohibits issuing new debt on which default is almost certain within the next year.[18] These parameter selections are summarized in Table 2.

The four remaining parameters, namely, β, d_0, d_1, and α, are determined jointly to match four target statistics. The first statistic is the average external-debt-to-output ratio for Argentina over the period 1993:Q1 to 2001:Q4, which is 1.0.[19] The model analog of this ratio is b/y.[20] The next two statistics are the average annualized spread on Argentine

[17]In the data, the value-weighted average coupon rate is about 11 percent. We chose 12 percent because, with an annual risk-free rate of 4 percent and an average spread of around 8 percent, a bond with a coupon of 12 percent will trade roughly at par. So, whether the debt is recorded at face value (which is the accounting practice) or at market prices (which is economically more sensible) will not matter for the calibration of the model.

[18]Since δ is the probability of default in the next quarter, the implied probability of default over the next four quarters $1 - (0.25)^4 = 0.9961$, which is a relatively weak underwriting standard. We show later in the paper that even if the standard was tightened significantly (δ is lowered to 0.50, say), our results are unaffected. Thus, while the presence of the standard is important in eliminating maximum debt dilution, our numerical results are not sensitive to its precise strength within some range.

[19]Debt is total long-term public and publicly guaranteed external debt outstanding that is disbursed and owed to private and official creditors at the end of each year, as reported in the World Bank's Global Development Finance (GDF) database (series DT.DOD.DPPG.CD).

[20]In the GDF database, the external commitments of a country are reported on a cash-accounting basis, which means that commitments are recorded at their face value, i.e., they are recorded as the undiscounted sum of future promised payments of principal (see "Coverage and Accounting Rules" in Section 3 of the World Bank Statistical Manual on External Debt). The agreed-upon coupon payments do not figure directly in this accounting because they are not viewed as obligations until they are past due. Given this valuation principle, the model analog of debt as reported in the data is simply b. To see this, observe that each bond can be viewed as a combination of unit bonds with varying maturities. For instance, a

Table 3: Parameters Selected Jointly

Parameter	Description	Value
β	Discount Factor	0.938
d_0	Default Cost Parameter	-0.194
d_1	Default Cost Parameter	0.278
α	Autarchy Parameter	0.011

sovereign debt over the same period and its standard deviation, which are 0.0815 and 0.0443, respectively.[21] The final statistic is the average recovery rate in the event of default. The model analog of this statistic is $G(y_{t+k})/(1 + r_f)^k b_t$ averaged over default episodes, where t is the period of default and $t + k$ is the period of settlement. Since defaults are relatively rare events, we target the recovery rate observed in the most recent Argentine default, which is 0.30.[22] Table 3 reports the parameter values that jointly match the four targets. Importantly, the default cost parameters imply that the output loss from default, on average, is 7 percent of output.

6.2 Welfare Gain from Enforcing Seniority

For our baseline comparisons, we set $\theta = 0.50$, a relatively low value. As we will see, there is considerable benefit from enforcing this rather weak form of seniority. In our sensitivity analysis, we explore higher values of θ, which generate higher welfare gains.

Table 4 reports the effects of moving from an environment without seniority to one in which there is seniority when the initial debt level b is zero. There is a significant increase in the lifetime utility of the sovereign from this move. In the absence of seniority, the value of constant consumption that gives the same average lifetime utility starting

measure λ of unit bonds is due in the next period, a measure $(1-\lambda)\lambda$ is due in two periods, ..., a measure $(1-\lambda)^{j-1}\lambda$ is due in j periods, and so on. Since each of these obligations has a face value of 1, each would be recorded as a unit obligation. Thus, the total obligation is simply $\sum_{j=1} \lambda(1-\lambda)^{j-1} = 1$.

[21]The default spread in the model is calculated as in the data. We compute an internal rate of return $r(y, b')$, which makes the present discounted value of the promised sequence of future payments on a unit bond equal to the unit price, that is, $q(y, b') = [\lambda + (1 - \lambda)z]/[\lambda + r(y, b')]$. The difference between $(1 + r(y, b'))^4 - 1$ and $(1 + r_f)^4 - 1$ is the annualized default spread in the model. If there is no possibility of default, the unit price would be a constant $\bar{q}$ such that $\bar{q} = [\lambda + (1 - \lambda)(z + \bar{q})]/[1 + r_f]$, which implies $\bar{q} = [\lambda + (1 - \lambda)z]/[\lambda + r_f]$. Since $q(y, b') \leq \bar{q}$, it follows that $r(y, b') \geq r_f$. Furthermore, the higher the probability of default, the lower $q(y, b')$ is and the higher $r(y, b')$ is.

[22]Estimates of the recovery rate (alternatively, 1 minus the haircut) on Argentine debt vary, with the lowest being 22 percent and the highest 37 percent (Cruces and Trebesch (2013), Table 2, p. 97); we calibrate to (roughly) the midpoint of this range.

Table 4: Impact of Seniority, $\theta = 0.50$

Statistic	Baseline	Seniority
Welfare ($b = 0$)	1.0276	1.0463
Avg. Spread	0.0812	0.0393
S.D of Spreads	0.0430	0.0180
Default Probability	0.0844	0.0554
Recovery Rate	0.30	0.47
Average b/y	1.00	1.25

from zero debt is 1.0276, where the average is computed over the invariant distribution of y.[23] If the sovereign were to issue bonds that partially respect seniority, the constant consumption equivalent of the new average lifetime utility, similarly computed, would be 1.0463. Thus, in constant consumption equivalents, the sovereign is willing to pay an additional 1.8 percent of consumption in perpetuity for this arrangement.

The proximate sources of the welfare gain are evident in Table 4. First, the cost of borrowing is substantially lower with seniority; average spreads decline from 8.12 percent to 3.93 percent, and the volatility of spreads declines substantially as well. The lower spreads result from lower default frequency, which falls from 8.44 percent to 5.54 percent, and from a higher recovery rate, which rises from 0.30 to 0.47. Second, the lower default frequency contributes to the gain in welfare directly as well by reducing output losses due to default. Finally, there is a significant expansion in the amount of debt that can be sustained in equilibrium; the debt-to-output ratio rises from 1 to 1.25. Since the sovereign is relatively impatient, the expansion in debt capacity contributes positively to welfare as well.

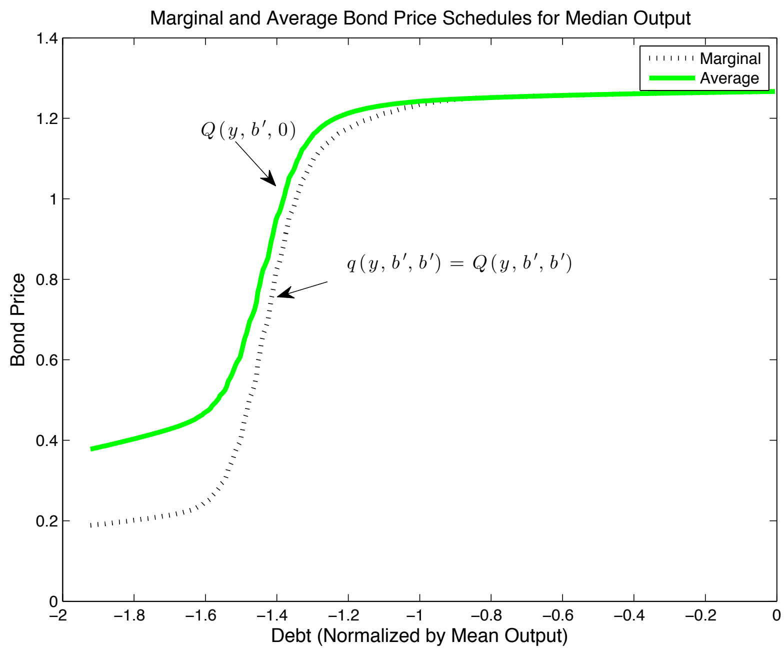
These effects stem, fundamentally, from the fact that seniority rights (partially) protect existing creditors from the adverse consequences of future borrowing by the sovereign. Because of this protection, the marginal issuances of debt are more exposed to losses arising from default than inframarginal units. This makes the price of marginal debt more sensitive to the amount of debt issued and induces the sovereign to restrain its borrowings.

This effect is displayed in Figure 2, which plots the $Q(y, b', 0)$ schedule along with $q(y, b', b')$ schedule (or, equivalently, $Q(y, b', b)$ schedule). For low levels of debt, the

[23] More precisely, the value for consumption reported is the value for which $c^{1-\gamma}/(1-\gamma) = (1-\beta)\sum_y W(0, y)\Phi(y)$, where $\Phi(y)$ is the invariant distribution of the transition law $F(y, y')$.

marginal and the average price of debt are virtually identical because all bonds issued are effectively senior (and all senior bonds get treated equally). Once the debt level is sufficiently high, though, the price of the marginal debt (the $q(y, b', b')$ schedule) begins to decline rapidly as the likelihood that the marginal unit will be senior in the event of default begins to fall.

Figure 2



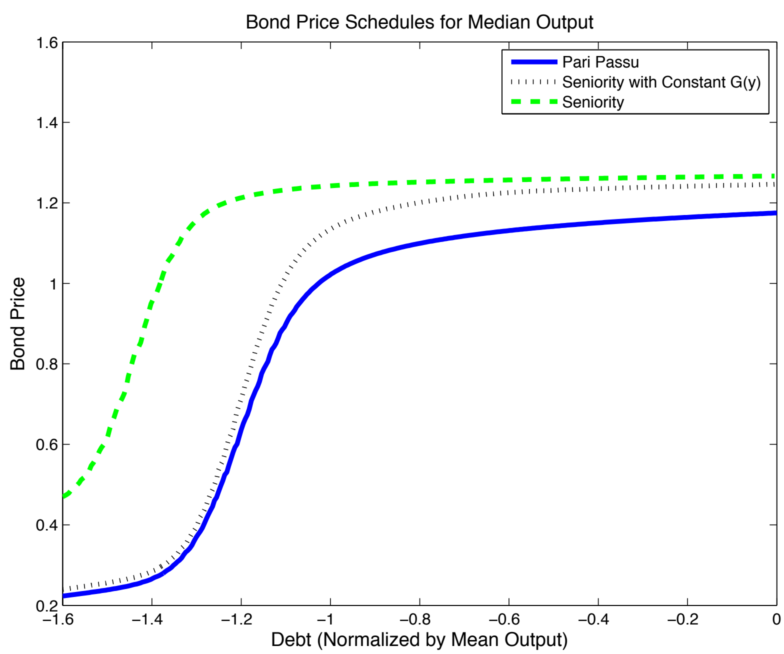
The increase in debt capacity makes the largest contribution to the welfare gain from enforcing seniority. This can be seen from Table 5, where we decompose the effect of enforcing seniority into two parts. In the first part, we determine how equilibrium behavior changes when we enforce seniority but keep the $G(y)$ function the same as in the case where seniority is not enforced. The results are shown in the third column. Welfare rises by 0.6 percent of consumption, which comes about because of a decline in average spreads (and spread volatility) and default frequency.

To understand why there is a decline in average spreads, holding $G(y)$ fixed, one can look at Figure 3. The blue (solid) line plots the $q(y, b')$ schedule and the black (dotted) line plots the $Q(y, b', 0)$ schedule, assuming the same $G(y)$ function as in the case of no seniority. Observe that the $Q(y, b', 0)$ schedule lies entirely above the $q(y, b')$ schedule,

Table 5: Decomposing the Effects of Seniority

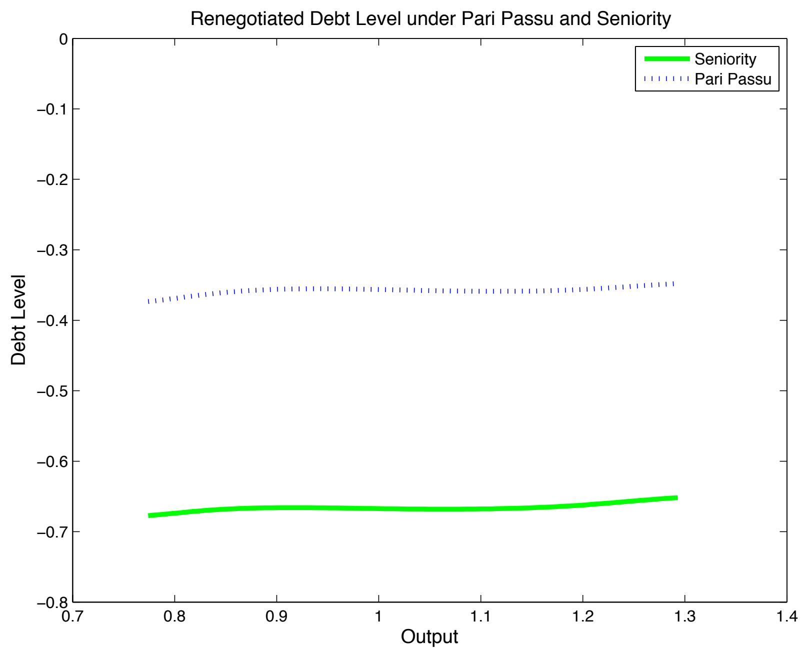
Statistic	w/o Seniority	w/Seniority, $G(y)$ fixed	w/Seniority
Welfare ($b = 0$)	1.0276	1.0334	1.0463
Avg. Spread	0.0812	0.0614	0.0393
S.D. of Spreads	0.0430	0.0291	0.0180
Default Probability	0.0844	0.0638	0.0554
Recovery Rate	0.30	0.31	0.47
Avg. b/y	-1.00	-1.01	-1.25

Figure 3



indicating that spreads are considerably lower when seniority is enforced than when it is not. The gap is large when new debt issued is low, as most of the issued debt will be senior in the event of default and, therefore, less subject to dilution. On the other hand, when debt issuance is high, and default is almost certain, the two schedules are close to each other because the total expected payout on the debt is similar (they are not identical even though $G(y)$ is the same in both cases because the price of debt is higher in the model with seniority). As is evident in the figure, these features together imply that $Q(y, b', 0)$ falls more steeply than $q(y, b')$ as debt issuance rises. The steeper decline in the price of debt, which comes about because the value of the marginal (junior-most) debt falls rapidly, gives the sovereign the incentive to constrain its borrowing. As a result, there is less overall dilution and less frequent default. Both forces lead to a lower and less volatile spread.

Figure 4



When we endogenize renegotiation, the higher value of market access (reflected in higher welfare, holding $G(y)$ fixed) implies a higher willingness to pay to regain market access and, so, a lower $G(y)$ function (higher debt level following renegotiation). This results in a positive feedback between the value of regaining market access and the implied $G(y)$ function: When market access is more valuable, the level of the renegotiated debt rises, which increases debt capacity further and, in turn, increases the value of regaining market access. This leads to a pronounced shift down in the $G(y)$ function, shown in Figure 4, a substantial increase in debt capacity, and a further 1.2 percent increase in welfare. The green (dashed) line in Figure 3 shows the full effect on the bond price schedule of enforcing seniority.

In the rest of this section, we examine how the estimate of the welfare gain from enforcing seniority depends on several important underlying factors. We study how the welfare gain is affected by the strength of seniority rights (controlled by θ), by the level of prior debt b at the time seniority is put in place, and by the strictness of the underwriting standard (controlled by δ). As we will show, the first two factors strongly affect the welfare estimate, while the last has no effect (within certain limits).

Table 6: Strength of Seniority Rights and Welfare

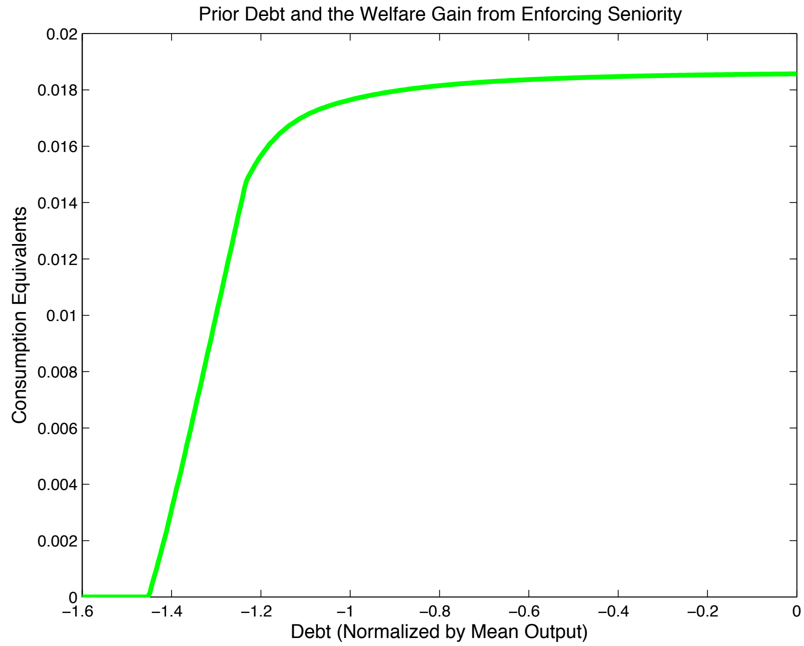
Statistic	$\theta = 0.50$	$\theta = 0.80$	$\theta = 0.90$	$\theta = 0.95$	$\theta = 0.99$
Welfare (b=0)	1.0463	1.0544	1.0562	1.0570	1.0576
Avg. Spread	0.0393	0.0264	0.0239	0.0227	0.0219
S.D. of Spreads	0.01798	0.0128	0.0119	0.0115	0.0112
Default Probability	0.0554	0.0411	0.0373	0.0356	0.0344
Avg. b/y	-1.25	-1.36	-1.39	-1.40	-1.41
Debt recovery	0.47	0.52	0.53	0.54	0.54
Agreement on Haircut	yes	yes	yes	yes	yes

Turning first to the role of θ, recall that $(1 - \theta)$ is the portion of each outstanding bond that is contractually required to suffer the aggregate haircut on the defaulted debt. Thus, higher values of θ correspond to stronger seniority rights since they leave a larger portion of each bond subject to payments based on seniority. Table 6 displays the welfare gain and associated statistics from the baseline ($\theta = 0.50$) to higher values of θ. Higher values are associated with higher welfare gains, lower average spread, lower volatility of spreads, lower probability of default, higher debt recovery and higher equilibrium debt (on average). All these effects flow from increased mitigation of debt dilution brought about by stronger seniority rights. The surprising finding is in the final row. This row records whether the welfare gains from stronger seniority rights come at the expense of intercreditor conflict. It turns out that they do not, unless θ is chosen to be very high. When θ is 0.99 or less, for every (y, b) active at the time of settlement in the simulations, the value of $\widetilde{G}_\theta(y) < \overline{G}_\theta(y)$. Hence, for all such θ values, creditors unanimously agree to push the sovereign to its participation constraint (so $G_\theta(y) = \overline{G}_\theta(y)$) every time there is a settlement.[24] Thus, if the goal is to harvest the benefits of seniority without engendering intercreditor conflicts at the time of settlement, a value of $\theta = 0.50$ seems conservative. Substantially higher values of θ can deliver still greater welfare gains and seem just as effective in avoiding intercreditor conflict at the time of settlement.

Turning next to the role of prior debt, suppose that at the time of the switch to a

[24]It is only when θ is raised to 0.995 that some (y, b) combinations active at the time of settlement feature $\overline{G}_\theta(y) < \widetilde{G}_\theta(y)$, and unanimity among creditors regarding settlement size at these nodes is lost.

Figure 5



seniority arrangement, the sovereign has prior debt b. Since existing bonds are governed by the *pari passu* clause, switching to a seniority arrangement would require making existing bonds senior to all new issuances of debt. Assume, then, that at the time of the switch, each outstanding bond is randomly assigned a rank between 0 and b and all *new* issuances of bonds have rank below b.[25] Figure 5 shows the welfare gains to Argentina from switching to a seniority regime for different levels of prior debt when current endowment is at its median (mean) value. As the level of prior debt increases, the welfare gain from imposing seniority drops. It drops because imposing seniority makes existing debt more valuable, which is to say that the sovereign promises to pay out more on these bonds in the future. Since the sovereign's real resources haven't changed, this promise is, in effect, a transfer of wealth from the sovereign to its existing creditors.[26]

[25] An alternative would be to treat all existing bonds as equally senior. This would lead to a common price for existing bonds when seniority is imposed. However, if under either arrangement the sovereign never buys back its existing debt, its behavior going forward will be the same regardless of which alternative is followed. This is because what matters then for the price of new bonds is only that they are junior to existing bonds. Accordingly, the total value of existing debt will be the same regardless of which arrangement is followed, and risk-neutral lenders will be indifferent between the two arrangements.

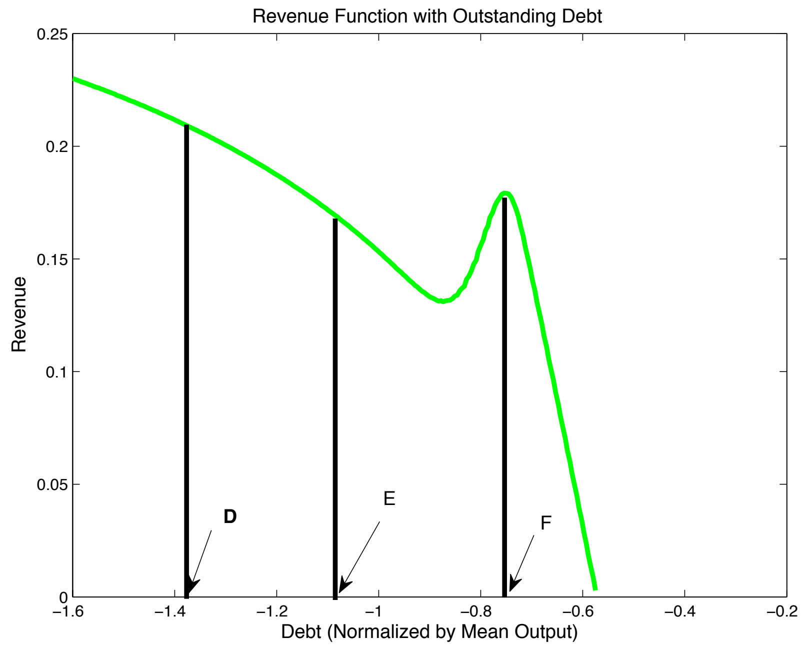
[26] Initially, the drop in welfare is concave with respect to the level of prior debt, but then it becomes almost linear. The linearly dropping portion of the graphs begins at the point where, in the absence of seniority, the sovereign wishes to default. The point where the graph becomes flat is where the sovereign wishes to default even when seniority is enforced.

Two features of Figure 5 are noteworthy. First, whether seniority is introduced when prior debt is 0 or 1 (the average equilibrium debt for our calibration) does not affect the welfare gain very much: The welfare gain from enforcing seniority when $b = -1$ and y is at its mean value is only slightly less than 1.8 percent. Second, the gain appears to be quite sensitive to higher levels of prior debt. For instance, if the prior debt at the time of the switch is 35 percent of annual GDP instead of 25 percent, which would correspond to $b = -1.4$ instead of $b = -1$ in Figure 5, the welfare gain drops to below two-tenths of a percentage point (from almost 2 percent). This might suggest that the net benefit from introducing seniority (taking into account some of the setup and maintenance costs of a seniority arrangement) may be relatively small when the sovereign is burdened with above-average debt levels. However, existing bondholders — all of whom gain from being made senior to future creditors — have an incentive to share a portion of their gain with the sovereign to get it to adopt a seniority arrangement. The sharing could take the form of a voluntary debt exchange wherein old debt is exchanged for new debt at less than par (in other words, a holder of a bond with a face value of 1 would receive a new bond with a face value of, say, 0.95). Our analysis suggests, then, that the existence of prior debt is unlikely to create a major hurdle to introducing seniority in sovereign debt contracts, provided a "surplus-sharing" agreement between the sovereign and its existing creditors can be worked out.

Finally, we turn to the role of underwriting standards in our welfare gain estimate. It is reasonable to conjecture that the strength of the underwriting standard should matter because such a standard, in effect, imposes state-contingent debt limits. Stricter underwriting standards will impose more stringent (state-contingent) debt limits and reduce the severity of the debt dilution problem and thus lower the gain from switching to seniority. It turns out, however, that making underwriting standards substantially tougher does not affect our welfare findings.

To understand this (initially) surprising result, it helps to look at how the revenue curve from new bond sales, namely $q(y, b')[(1 - \lambda)b - b']$, when there is prior debt b. This is shown in Figure 6 for a given level of y. For an initial range of debt levels, the curve has the familiar inverted-U shape. Beyond that initial stage, however, revenue is *increasing*

Figure 6



in debt again due to the logic explained earlier: When default is expected to occur with very high probability — as it is for debt in this range — the sovereign can expropriate existing creditors by increasing b'.

We can now see how δ affects the sovereign's choice set. For $\delta < 1$, the sovereign is constrained in how much debt it can issue, or, alternatively, how far out (to the left) it can go on the revenue curve. In Figure 6, the maximum amount of debt the sovereign can issue for the given δ (and y) is denoted D. If the revenue generated at D is larger than the revenue generated at the top of the revenue curve, corresponding to the debt level F, then D could be the optimal choice of the sovereign: The sovereign may find it in its interest to engage in maximum dilution and push bond sales all to the point where δ is binding and default with very high probability in the next period. On the other hand, if revenue generated at D is less than that generated at F, then F must dominate D because the sovereign gets more consumption and carries over less debt at F than at D. If δ is lowered enough so that the maximum debt level — now denoted E — generates less revenue than debt level F, the incentive to engage in maximal dilution goes away. Importantly, lowering δ further will not change the sovereign's choice set (and, therefore, behavior), unless it is lowered so much that the maximum debt limit lies between 0 and F.

Thus, there is a range of δ values for which the incentive to engage in maximal dilution is absent for all debt levels reached in equilibrium *and* the sovereign never chooses a debt level for which the constraint on default probability is binding. For our calibration, this range is $[0.33, 0.75]$. Our choice of δ, then, is the highest allowable probability of default consistent with there being no incentive to engage in maximal dilution and equilibrium default probabilities being strictly less than δ. Choosing a substantially tighter underwriting standard would not affect our findings as long as the implied δ remains above 0.33.

7 Conclusion

A sovereign's inability to commit to a course of action regarding future borrowing and default behavior makes long-term debt costly (the problem of debt dilution). Trading arrangements that mitigate the problems arising out of this lack of commitment are potentially valuable. In this paper, we quantitatively explored one such mechanism that has received attention in the policy literature, namely, a system of "seniority rights" that (partially) protects existing creditors from the adverse consequences of future borrowing by the sovereign. Because of this protection, the marginal issuances of debt are more exposed to losses arising from default than inframarginal units. This makes the price of marginal debt more sensitive to the amount of debt issued and induces the sovereign to restrain its borrowings. The restraint, working through several channels, lowers the cost of debt and improves the sovereign's welfare.

Our exploration required innovation along two dimensions. First, we devised a computationally tractable way to model seniority via the fiction of a *rank* associated with each bond, an innovation that may prove useful in other contexts as well. Second, we proposed a modification of the absolute priority rule that recognizes the realities of a sovereign debt restructuring. Our *relative* priority rule specifies that a contractually predetermined fraction of the aggregate settlement be paid out pro-rata to all creditors (regardless of seniority), with only the remaining portion of the aggregate settlement distributed according to seniority. This arrangement allows the sovereign to mitigate the adverse effects of debt dilution while attenuating inter-creditor conflicts at the time of settlement.

The quantitative work assessed the potential welfare gain to Argentina from introducing seniority, given its pattern of borrowing, output, and interest rate (on its sovereign bonds) during the decades preceding its default in 2001. We found the gain to be substantial even when only one-half of the settlement payout is distributed according to seniority. The bulk of the gain comes from an expansion in debt capacity permitted by seniority rights. We also found that the fraction of the settlement payout distributed pro-rata can be reduced substantially (with consequent strengthening of seniority rights and further improvements in welfare) without generating conflict between junior and senior creditors at the time of settlement.

Future research efforts could be usefully directed toward a better understanding of the nature and costs of the institutional preconditions for enforcing seniority (the setup of a bond registry and the legal design of sovereign debt contracts with a seniority clause being the two most important) and of bargaining protocols when players possess seniority rights.

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A Appendix: Proof of Proposition 1

In this appendix, we give the proof of Proposition 1 in the text. We first give some definitions and establish two preliminary results.

Definition. Let W be the set of all $(y, b', s) \in Y \times B \times B$ such that $b' \leq s$ and let w denote an element of W. Let F denote the set of all bounded and nonnegative functions $f(w) : W \to \mathbb{R}^+$. Let $\rho(f, \tilde{f}) = \sup_w |f(w) - \tilde{f}(w)|$ be the metric on F. Then (F, ρ) is a complete metric space (for a proof, see, for instance, Harris (1987), Lemma 2.1, p. 22).

Definition. Let $(Hq)(y, b', s) : F \to F$ be the operator defined by the r.h.s. of (12), given $p(y, b, s) \in F$ and decision rules $d(y, b)$ and $a(y, b)$.

Definition. Let $(Tp)(y, b, s) : F \to F$ be the operator defined by the r.h.s of (13), given $q(y, b, s) \in F$ and settlement payout function $G_\theta(y)$.

Definition. Let $Q^*(y, b', b)$, $G^*_\theta(y)$, $p^*(y, b, s)$, $d^*(y, b)$ and $a^*(y, b)$ be the equilibrium bond price function, settlement function, expected default payout function, default decision rule and asset decision rule, respectively.

Lemma 1 *$p^*(y, b, s)$ is increasing in s.*

Proof. Let T^* denote T when bond pricing function and the settlement function are fixed at their equilibrium values. Then the following three properties of T^* can be verified by inspection: (i) T^* is monotone: If $p^1(y, b, s) \leq p^0(y, b, s)$, and both are members of F, then $(T^*p^1) \leq (T^*p^0)$; (ii) T^* display shrinkage: For any $\kappa > 0$ and any $p \in F$, $(T^*(p + \kappa)) = (T^*p) + \kappa/(1 + r_f)$; and (iii) T^* preserves monotonicity w.r.t. s: If $p(y, b, s)$ is increasing in s, $(T^*p)(y, b, s)$ is increasing in s.

Let $\bar{F} = \{f \in F : f \text{ is increasing in } s\}$. Properties (i)-(iii) imply that $(T^*(\bar{F})) \subset \bar{F}$ and T^* is a contraction map with modulus $1/(1 + r_f)$. It follows from Theorem 3.1 and Corollary 1 to Theorem 3.1 of Stokey and Lucas (1989)(pp. 50-52) that there is a *unique* $\bar{f} \in \bar{F}$ such that $(T^*\bar{f}) = \bar{f}$. Since $p^*(y, b, s)$ satisfies $T^*(p^*) = p^*$, $p^*(y, b, s)$ must be $\bar{f}$ and, hence, $p^*(y, b, s)$ must be increasing in s. ■

Proof of Proposition 1. Let H^* be the H operator when decision rules and the expected default payout function are fixed at their equilibrium (starred) values. Again, the following three properties of H^* can be verified: (i) H^* is monotone: If $q^1(y,b,s) \leq q^0(y,b,s)$, both members of F, then $(H^*q^1) \leq (H^*q^0)$; (ii) H^* displays shrinkage: For any $\kappa > 0$ and any $q \in F$, $(H(q+\kappa)) = (Hq) + \kappa/(1+r_f)\{(1-\lambda)E_{y'|y}[1-d(y',b')]\}$ (since the term multiplying $\kappa/(1+r_f)$ is strictly less than 1, $H(q+\kappa) < H(q) + \kappa/(1+r_f)$); and (iii) H^* preserves monotonicity w.r.t. s: By Lemma 1, $p^*(y,b',s)$ is increasing in s. Then, given a $q(y,b',s)$ increasing in s, $(H^*q)(y,b',s)$ is increasing in s.

Again, let $\bar{F} = \{f \in F : f \text{ is increasing in } s\}$. Properties (i)-(iii) imply that $(H^*(\bar{F})) \subset \bar{F}$ and H^* is a contraction map with modulus $1/(1+r_f)$. It follows from Theorem 3.1 and Corollary 1 to Theorem 3.1 of Stokey and Lucas (1989)(pp. 50-52) that there is a *unique* $\bar{f} \in \bar{F}$ such that $(H^*\bar{f}) = \bar{f}$. Since $q^*(y,b',s)$ satisfies $H^*(q^*) = q^*$, $q^*(y,b',s)$ must be $\bar{f}$ and, hence, $q^*(y,b',s)$ must be increasing in s. ■