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MEETING TECHNOLOGIES AND OPTIMAL TRADING
MECHANISMS IN COMPETITIVE SEARCH MARKETS

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Meeting Technologies and Optimal Trading Mechanisms in Competitive Search Markets*

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Abstract

In a market in which sellers compete by posting mechanisms, we allow for a general meeting technology and show that its properties crucially affect the mechanism that sellers select in equilibrium. In general, it is optimal for sellers to post an auction without a reserve price but with a fee, paid by all buyers who meet with the seller. However, we define a novel condition on meeting technologies, which we call *invariance*, and show that meeting fees are equal to zero if and only if this condition is satisfied. Finally, we discuss how invariance is related to other properties of meeting technologies identified in the literature.

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1 Introduction

What trading mechanism should a seller use to sell a good? This is a classic question in economics and the answer, of course, depends on the details of the environment. We study this question in an environment with three key ingredients.

First, there are a large number of sellers and a large number of buyers, each seller has one indivisible good, and sellers *compete* for buyers by posting the trading mechanism they will use to sell their good. Then, buyers observe the mechanisms that have been posted and choose the one that promises the maximal expected payoff, but the process by which buyers and sellers meet is *frictional*. In particular, when a buyer chooses to visit a seller, there may be uncertainty about whether he is actually able to meet with that seller, as well as the number of other buyers who also meet with that seller; we call the function that governs this process the “meeting technology.” Finally, once buyers meet with a seller, they learn their idiosyncratic *private valuation* for the seller’s good.

In this environment, we provide a complete characterization of the mechanism that sellers choose in equilibrium. More specifically, we show that sellers can do no better than a second-price auction with a reserve price equal to their own valuation and a fee (or subsidy) that is paid by (or to) each buyer that participates. We characterize this fee in closed form. Then, in the same spirit as Eeckhout & Kircher (2010), we study how this optimal mechanism depends on the properties of the meeting technology.¹

Our results suggest that conclusions drawn in the existing literature may depend quite heavily on the specific functional form that has typically been chosen for the meeting technology. In particular, previous studies of this basic environment (e.g., Peters (1997), Peters & Severinov (1997), Albrecht et al. (2012, 2013a,b) and Kim & Kircher (2013)) have shared three features in common. First, the equilibrium in these papers is constrained efficient, in the sense that the solution to the planner’s problem is implemented. Second, the trading mechanism that implements the constrained efficient allocation only requires transfers between the two agents that ultimately trade, so that agents who do not trade are not required to pay a fee nor are they offered a subsidy. Last, all of the papers cited above assume the same functional form for the meeting technology—what is often called the “urn-ball” meeting technology.

Taken together, the first two properties are somewhat surprising. After all, constrained efficiency places strict requirements on two distinct margins. First, from ex ante point of view, it

¹We view this paper as being complementary to Eeckhout & Kircher (2010), as there are several important differences between the two analyses. They provide a partial characterization of the equilibrium in an environment in which valuations are either high or low and buyers learn their realization before visiting a seller, while we provide a full characterization of the equilibrium in a world in which buyers draw a valuation from a continuous distribution after selecting a seller. Perhaps the most important difference is that the property of the meeting technology that they highlight is not the relevant concept for the results that we derive. We discuss this extensively in Section 4.2.

requires the proper allocation of buyers to sellers. Second, from an ex post point of view, it requires the proper allocation of the good once buyers arrive at a seller and learn their valuations. In order to implement an equilibrium that satisfies these two requirements, the payoffs received by buyers and sellers must inherit certain properties. In particular, to satisfy the first requirement, each buyer must receive an expected payoff exactly equal to his marginal contribution to the match surplus.² However, in order to satisfy the second requirement, payoffs must ensure that buyers truthfully report their valuation and that the good is allocated to the agent with the highest valuation. In principle, there is no reason to think that the payoffs satisfying the former requirement should coincide with the payoffs satisfying the latter.

We show that, in fact, these payoffs coincide—or, equivalently, that the meeting fee in the optimal mechanism is equal to zero—if, and only if, the meeting technology satisfies a novel condition that we call *invariance*. Loosely speaking, a meeting technology is invariant if the decision by one buyer to visit a seller does not interfere with the process by which the seller meets with other buyers. When invariance is violated, one buyer’s search decision exerts an externality on the meeting prospects of the seller with other buyers, and this externality needs to be taxed (or subsidized).

The urn-ball process is one example of a meeting technology that satisfies invariance. Hence, we learn that this functional form is critical for understanding why the optimal mechanism in previous studies was so simple, and why it did not require any additional (rarely observed) side payments. However, the urn-ball meeting technology also satisfies several other properties that previous studies have conjectured are important for determining the optimal trading mechanism. We close our analysis by comparing two of these properties with invariance; we show that one of them is a necessary (but not sufficient) condition for invariance, while the other is neither necessary nor sufficient.

2 Environment

Agents and Preferences. The economy is populated by a measure $\mu_B > 0$ of buyers and a measure $\mu_S > 0$ of sellers, with $\Lambda = \frac{\mu_B}{\mu_S}$.³ Each seller possesses one, indivisible good and each buyer has unit demand for this good. All agents are risk neutral.

Sellers value their own good at y , which is common knowledge, while buyers have to visit a seller in order to learn their valuation. In particular, once a buyer meets a seller, he learns his private valuation x for that seller’s good, which is drawn from a twice continuously-differentiable

²For detailed discussions of the relationship between the division of surplus and efficiency in search environments, see, e.g., Mortensen (1982), Hosios (1990) and Moen (1997).

³Though we focus our attention on an environment with an exogenous measure of sellers, our results do not change if we endogenize the measure of sellers through a free entry condition.

distribution $F(x)$ with support $[\underline{x}, \bar{x}] \subset \mathbb{R}_+$. An individual buyer's valuations are i.i.d. across sellers, as are the valuations of each buyer at an individual seller. We assume that $y \in [\underline{x}, \bar{x}]$.⁴

Mechanisms. In order to attract buyers, each seller posts and commits to a direct mechanism. A mechanism specifies an extensive form game that determines for each buyer i a probability of trade and an expected payment as a function of: (i) the total number n of buyers to meet with the seller; (ii) the valuation x_i that buyer i reports; and (iii) the valuations x_{-i} reported by the $n - 1$ other buyers. Formally, a mechanism is summarized by a probability of trade $\phi(x_i, x_{-i}, n)$ and a transfer $\tau(x_i, x_{-i}, n)$ for all $n \in \mathbb{N}_1 \equiv \{1, 2, 3, \dots\}$ and $i \in \{1, 2, \dots, n\}$.

Meeting Technology. After observing all posted mechanisms, each buyer chooses the mechanism at which he will attempt to trade. However, the meeting process between buyers and sellers is frictional, so that some buyers will not meet a seller and some sellers will not meet with any buyers.⁵ As is often done in the literature, we restrict attention to meeting technologies that exhibit constant returns to scale. More specifically, suppose a measure σ of sellers post the same mechanism and a measure β of buyers choose this mechanism. Then, letting $\lambda = \beta/\sigma$ denote the market tightness or *queue length* at that mechanism, a seller meets exactly $n \in \mathbb{N}_0 \equiv \{0, 1, 2, \dots\}$ buyers with probability $P_n(\lambda)$.⁶ We assume that $P_n(\lambda)$ is twice continuously-differentiable. We also require that, for any $\lambda \in (0, \infty)$, some sellers meet at least one buyer while other sellers do not, i.e. $0 < P_0(\lambda) < 1$. Since the number of meetings cannot exceed the number of buyers who attempt to trade at a particular mechanism, the probability distribution must satisfy $\sum_{n=0}^{\infty} nP_n(\lambda) \leq \lambda$.

We denote the probability for a buyer to meet a seller with exactly $n - 1$ other buyers by $Q_n(\lambda)$. This probability is related to $P_n(\lambda)$ through the consistency requirement

$$nP_n(\lambda) = \lambda Q_n(\lambda), \tag{1}$$

for all $n \in \mathbb{N}_1$. Intuitively, given market tightness $\lambda = \beta/\sigma$ at some mechanism, the measure of buyers at those sellers who end up with precisely n buyers, $\sigma nP_n(\lambda)$, must equal the measure of buyers who face $n - 1$ buyers at their seller, $\beta Q_n(\lambda)$. Finally, we define $Q_0(\lambda) = 1 - \sum_{n=1}^{\infty} Q_n(\lambda)$ as the probability that the buyer fails to meet with a seller.

⁴Since we do not impose much structure on $F(x)$, this is a fairly weak assumption, as the probability that a buyer's valuation x is smaller than y can be driven to zero without any loss of generality.

⁵Note that our language follows that of Eeckhout & Kircher (2010), who make the distinction between meetings and matches. All buyers who participate in a seller's trading mechanism are said to "meet" with the seller, whereas the buyer who acquires the seller's single, indivisible good is said to "match" with the seller.

⁶We follow most of the literature by assuming that the process through which buyers and sellers meet is exogenous. Only a small number of papers derive the meeting technology as the endogenous outcome of agents' optimal decisions. See Lester et al. (2013) for an example.

Examples. In this section, we briefly introduce two examples of meeting technologies in order to fix ideas about how meeting technologies are formulated in general, and familiarize the reader with some of the more specific technologies that have been used in the search literature. Later, in Section 4, we will introduce a few additional meeting technologies that will be useful in facilitating a discussion of our results.

1. *Urn-Ball.* A particularly popular choice in the directed search literature, this specification arises when a measure β of buyers randomize evenly across a measure σ of sellers who have posted the same mechanism.⁷ As a result, the number of buyers that ultimately visit a particular seller is determined by a Poisson distribution with mean equal to the queue length $\lambda = \beta/\sigma$. That is, $P_n(\lambda) = e^{-\lambda} \frac{\lambda^n}{n!}$.
2. *Bilateral.* This specification, often used in the literature that uses search-theoretic models to study monetary theory, has the following interpretation.⁸ Suppose there is a mass β and σ of buyers and sellers, and all agents are randomly matched in pairs. Then, given $\lambda = \beta/\sigma$, the probability distribution over the number of *buyers* that a seller meets is given by:

$$P_n(\lambda) = \begin{cases} \frac{1}{1+\lambda} & \text{for } n = 0 \\ \frac{\lambda}{1+\lambda} & \text{for } n = 1 \\ 0 & \text{for } n \in \{2, 3, \dots\}. \end{cases} \quad (2)$$

Probability Generating Function. It will prove useful to define the function

$$m(z; \lambda) \equiv E[z^n | \lambda] = \sum_{n=0}^{\infty} P_n(\lambda) z^n \quad (3)$$

for $z \in [0, 1]$ with, by convention, $m(0; \lambda) = P_0(\lambda)$. This function is the probability-generating function of n and provides an alternative representation of $P_n(\lambda)$, as $P_n(\lambda) = \frac{1}{n!} \frac{\partial^n}{\partial z^n} m(0; \lambda)$. Notice that, setting $z = F(x)$, $m(z; \lambda)$ can be interpreted as the probability that the maximum valuation at a seller with a queue length λ is less than x .

Given the properties of a probability-generating function, the expected number of meetings can then be expressed as

$$\sum_{n=0}^{\infty} n P_n(\lambda) = m_z(1; \lambda), \quad (4)$$

⁷This specification was first used by Butters (1977) and Hall (1977). See Burdett et al. (2001) for an explicit derivation of the micro-foundations of this meeting technology in a directed search model.

⁸See, e.g., Kiyotaki & Wright (1993) and Trejos & Wright (1995). This technology has also been used more recently to study trading dynamics in over-the-counter financial markets; see, e.g., Duffie et al. (2007).

where m_z and m_λ denote the partial derivative of $m(\cdot)$ with respect to the first and second arguments, respectively. Note that $m_z(\cdot) > 0$, with $m(\cdot)$ ranging from $m(0; \lambda) = P_0(\lambda)$ to $m(1; \lambda) = 1$. In addition, we impose that $m_\lambda(\cdot) < 0$, with $m(\cdot)$ ranging from $m(z; 0) = 1$ to $m(z; \infty) = 0$ for all $z \in [0, 1)$. Again, letting $z = F(x)$, this assumption implies that a longer queue length reduces the probability that the maximum valuation at a seller is below x .⁹ Finally, we assume that $m_{\lambda\lambda}(z; \lambda) \equiv \frac{\partial^2}{\partial \lambda^2} m(z; \lambda) > 0$. This implies that the effect of a marginal increase in the queue length on the probability that the maximum valuation is below $x = F^{-1}(z)$ is smaller when the queue length is longer.¹⁰ All meeting technologies discussed in this paper share the properties described above.

3 The Planner's Problem and Market Equilibrium

In this section, we first characterize the solution to the planner's problem, and then characterize the decentralized market equilibrium.

3.1 The Planner's Problem

Consider the problem of a benevolent social planner whose objective is to maximize net social surplus, subject to the frictions of the physical environment (e.g., the meeting frictions). The planner's decision rule can be broken down into two components. First, the planner has to assign queue lengths to each seller, subject to the constraint that the sum of these queue lengths across all sellers cannot exceed the total measure of available buyers. Second, the planner has to specify a trading rule for agents to follow after buyers arrive at sellers. We solve these stages in reverse order. As the derivation is standard for each component, we keep the exposition brief.

Trading Rule. Consider a seller who is visited by n buyers. Clearly, the surplus at this seller is maximized by instructing the seller to trade with the buyer who has the highest valuation x , so long as this valuation exceeds y . If, instead, the highest valuation among the buyers is below y , then the seller should keep the good himself. Therefore, the expected net surplus at a seller with queue length λ is

$$S(\lambda) = \sum_{n=0}^{\infty} P_n(\lambda) \int_y^{\bar{x}} (x - y) dF^n(x) = \bar{x} - y - \int_y^{\bar{x}} m(F(x); \lambda) dx. \quad (5)$$

⁹Monotonicity of $m(\cdot)$ in λ is implied if a change in λ causes a first-order stochastic dominance shift in $P_n(\lambda)$.

¹⁰This last assumption is also common in the literature; see, e.g., Eeckhout & Kircher (2010) for a similar assumption.

Allocation of Buyers. Now consider the allocation of buyers to sellers. At this stage, the objective of the planner is to choose queue lengths at each seller, i.e., λ_j for $j \in [0, \mu_S]$, to maximize the sum of $S(\lambda_j)$ across all sellers. Hence, he solves $\max_{\{\lambda_j\}} \int_0^{\mu_S} S(\lambda_j) dj$ subject to the constraint that $\int_0^{\mu_S} \lambda_j dj = \mu_B$. As $S''(\lambda) = -m_{\lambda\lambda}(F(x); \lambda) < 0$, it follows immediately that the planner maximizes total surplus by assigning equal queue lengths across all sellers, so that $\lambda_j = \Lambda = \frac{\mu_B}{\mu_S}$ for all j and total surplus equals $\int_0^{\mu_S} S(\Lambda) dj = \mu_S S(\Lambda)$. The following proposition summarizes.

Proposition 1. *A unique solution to the planner's problem exists. In this solution, each seller is assigned a queue length Λ . After buyers arrive and learn their valuation, the planner assigns the good to the agent with the highest valuation.*

3.2 Market Equilibrium

To characterize the relevant properties of the mechanism that sellers select in equilibrium, we proceed in two steps. First, we restrict attention to second-price auctions with a reserve price r and a meeting fee t that is paid by (or to) each buyer before learning his type. Within this restricted set of mechanisms, we characterize the profit-maximizing values of r and t . Then, we establish that sellers can do no better than this second-price auction, even with access to the set of all direct mechanisms. More specifically, we show that any direct mechanism chosen by a seller in equilibrium will be payoff-equivalent to the second-price auction we characterize.

Equilibrium with Second-Price Auctions and Meeting Fees. As a first step, we calculate the expected revenue of a seller who sets r and t and attracts a queue length λ , along with the expected payoff of each buyer that visits this seller.¹¹

Lemma 1. *A seller who posts a second-price auction with reserve price r and meeting fee t who attracts a queue λ obtains an expected payoff equal to*

$$\begin{aligned} R(r, t, \lambda) &= \bar{x} - (r - y) m(F(r); \lambda) - \int_r^{\bar{x}} m(F(x); \lambda) dx \\ &\quad - \int_r^{\bar{x}} (1 - F(x)) m_z(F(x); \lambda) dx + t m_z(1; \lambda). \end{aligned}$$

The expected payoff for each buyer in the seller's queue equals

$$U(r, t, \lambda) = \frac{1}{\lambda} \int_r^{\bar{x}} (1 - F(x)) m_z(F(x); \lambda) dx - \frac{t}{\lambda} m_z(1; \lambda). \quad (6)$$

¹¹It is well known that a buyer's optimal strategy is to bid his true valuation, independent of the number of other buyers who arrive. Since a seller's expected revenue and buyers' expected payoffs are standard calculations when the number of buyers n is fixed, deriving the results in Lemma 1 only requires taking expectations across n , and hence are relegated to the Appendix.

A seller's objective is to maximize revenue, taking into account that his choice of r and t affects the queue λ that he attracts. In particular, optimal search behavior implies that buyers must be indifferent between all sellers who receive a strictly positive queue. Formally, let \bar{U} denote the highest level of utility that buyers can obtain in this market, or what is often called the “market utility.” Then, the equilibrium relationship between r , t , and λ is determined by $U(r, t, \lambda) = \bar{U}$.

Hence, an equilibrium in this environment is formally a distribution $G(r, t, \lambda)$ of reserve prices, meeting fees, and queue lengths across sellers, and a market utility \bar{U} , such that (i) given \bar{U} , each triple (r, t, λ) in the support of G maximizes revenue $R(r, t, \lambda)$ subject to the constraint $U(r, t, \lambda) = \bar{U}$; and (ii) aggregating queue lengths across all sellers yields the total measure of buyers, μ_B . However, one can prove that there exists a unique equilibrium in which all sellers post the same reserve price r^* and meeting fee t^* , and attract the same queue $\lambda^* = \Lambda$. Moreover, it turns out that the optimal reserve price is equal to the sellers' valuation ($r^* = y$), so that the good is allocated to the agent (one of the buyers or the seller) with the highest valuation. Hence, the decentralized equilibrium is constrained efficient in the sense that it creates the same amount of surplus as the planner's solution. The following proposition summarizes and derives the meeting fee in closed form.

Proposition 2. *A unique equilibrium exists and it is constrained efficient. In equilibrium, the reservation price equals $r^* = y$, the meeting fee equals*

$$t^* = \frac{\Lambda}{m_z(1; \Lambda)} \int_y^{\bar{x}} \left[\frac{1}{\Lambda} (1 - F(x)) m_z(F(x); \Lambda) + m_\lambda(F(x); \Lambda) \right] dx, \quad (7)$$

and the queue length equals $\lambda^ = \Lambda$ at all sellers, with buyers receiving market utility $\bar{U} = U(r^*, t^*, \Lambda)$.*

Equilibrium with General Mechanisms. Proposition 2 describes the equilibrium outcome when sellers have access to a restricted set of mechanisms and shows that the equilibrium coincides with the solution to the planner's problem. In Proposition 3 below, we establish that sellers would not benefit from having access to a larger set of mechanisms; both the expected payoffs and the allocations will be identical in any other equilibrium. The reasoning is simple: Since buyers' equilibrium payoffs are equal to the market utility \bar{U} , an individual seller is the residual claimant on any additional surplus he creates. Hence, he has incentive to select an efficient mechanism in order to maximize this surplus. The mechanism that we consider—a second-price auction with a reserve price r and a meeting fee t —is sufficiently flexible that it allows the seller to maximize the “size of the pie” without placing restrictions on how the pie is divided. Therefore, access to additional mechanisms is largely irrelevant.

Proposition 3. *A mechanism $\{\phi(x_i, x_{-i}, n), \tau(x_i, x_{-i}, n)\}$ is an equilibrium if, and only if, it is payoff-equivalent to the equilibrium characterized in Proposition 2.*

4 Properties of the Optimal Mechanism

In the previous section, we characterized the equilibrium mechanism that arises in a fairly general environment. An important feature of the optimal mechanism is a meeting fee, t^* , that allows sellers to manipulate expected revenue without distorting the efficient allocation. Somewhat surprisingly, this meeting fee does not play a role in the optimal mechanism that has been identified in the previous literature. Rather, these papers find that the optimal mechanism is simply an auction with reserve price $r^* = y$.

We now reconcile our results with the existing literature by showing that previous studies had focused on a special class of meeting functions. In particular, we introduce a novel property of meeting technologies that we call *invariance* and show that this property is necessary and sufficient for $t^* = 0$. We also review some other properties of meeting technologies that have been identified in the literature and discuss their relationship to invariance.

4.1 Invariance

In order to formally introduce the invariance property, suppose that each buyer has a certain characteristic with probability $\gamma \in [0, 1]$, which is independent of the realizations of other buyers. Then, the probability that a seller meets exactly n buyers with this characteristic equals¹²

$$\tilde{P}_n(\lambda, \gamma) \equiv \sum_{N=n}^{\infty} P_N(\lambda) \binom{N}{n} \gamma^n (1 - \gamma)^{N-n}. \quad (8)$$

Definition 1. *A meeting technology is invariant if, and only if,*

$$\tilde{P}_n(\lambda, \gamma) = P_n(\gamma\lambda) \quad (9)$$

for all $\gamma \in (0, 1)$, $\lambda \in (0, \infty)$, and all $n \in \mathbb{N}_0$.

To understand the intuition, consider a sub-market in which the ratio of buyers to sellers is λ , and suppose that a fraction γ of buyers are red and the remainder are green.¹³ Equation (9),

¹²This transformation, introduced by Rényi (1956), is known as “binomial thinning.” See Harremoës et al. (2010).

¹³For the purpose of this definition, whether the buyer’s color is realized before or after meeting the seller is irrelevant, so long as a buyer’s color does not make him more or less able to meet a seller (i.e., red buyers are not faster or stronger than green buyers). It does, however, slightly change the intuition in some of our explanations below. In what follows, we find it easiest to convey the intuition by treating a buyer’s color as being realized ex ante, but the

then, says that the ratio of red buyers to sellers must be a sufficient statistic for the distribution that governs the number of red buyers who arrive at a seller. In particular, changing the measure of green buyers should not change the probability that n red buyers arrive at a seller, for any $n \in \mathbb{N}_0$.

In other words, a marginal buyer choosing to visit the mechanism a seller has posted does not interfere with the process by which that seller meets with *other* buyers. When the invariance condition is violated, the seller either wants to tax or subsidize buyers for choosing his mechanism, depending on whether they exert a negative or positive externality on his prospects for meeting other buyers, respectively. The proposition below formalizes this argument: we establish that meeting fees are set to zero in equilibrium if, and only if, the meeting technology is invariant, and we derive a simple condition for verifying invariance for arbitrary technologies.

Proposition 4. *Consider a meeting technology $P_n(\lambda)$. The following statements are equivalent:*

1. *Meeting fees are not used in equilibrium, i.e., $t^* = 0$ for any distribution $F(x)$ with support $[\underline{x}, \bar{x}] \subset [0, \infty)$, $y \in [\underline{x}, \bar{x}]$ and $\Lambda \in (0, \infty)$;*
2. *The meeting technology is invariant;*
3. *The meeting technology satisfies both*

$$P_n(\lambda) = \frac{(-\lambda)^n M^{(n)}(\lambda)}{n!} \quad (10)$$

and

$$m(z; \lambda) = M(\lambda(1-z)),$$

where $M^{(n)}(\cdot)$ denotes the n^{th} -order derivative of a smooth function $M : [0, \infty) \rightarrow [0, 1]$ satisfying $M(0) = 1$, $M(\infty) = 0$, $M'(0) \in [-1, 0)$ and $\frac{(-\lambda)^n M^{(n)}(\lambda)}{n!} \in (0, 1)$ for all $n \in \mathbb{N}_0$ and $\lambda \in (0, \infty)$.

Note that the urn-ball technology introduced earlier in the text satisfies the invariance condition: when all buyers are randomizing across a group of sellers, the presence of more or fewer green buyers has no effect on the number of red buyers to arrive at a particular seller. Since the existing literature has focused primarily on the case of urn-ball matching, it should then come as no surprise that a simple second-price auction with $r^* = y$ was identified as the optimal mechanism.

Also note that the bilateral meeting technology is *not* invariant: since sellers only meet with one randomly selected buyer under this meeting technology, an increase in the measure of green buyers will decrease the probability that a seller meets one red buyer (and increase the probability he meets zero red buyers). Hence, in a competitive environment with this meeting technology, sellers would optimally employ a meeting fee to tax this congestion externality appropriately.

discussion could easily be adapted to treat the alternative case as well.

4.2 Relationship to Other Properties

Much of the existing literature on competing mechanisms assumes a specific meeting technology (often urn-ball) and studies how other features of the environment affect the optimal mechanism. Notable exceptions include Julien et al. (2000, 2011), who provide some informal speculation about the properties of the meeting technology that are important for their results, and Eeckhout & Kircher (2010) and Albrecht et al. (2013b), who provide more systematic analyses of how the assumed meeting technology affects the optimal mechanism that emerges in equilibrium. In this section, we formalize two properties of meeting technologies that have been discussed in the existing literature—either formally or informally—and illustrate how these properties are related to the invariance property introduced above.

A Few More Meeting Technologies. To facilitate the discussion below, it will be helpful to define a few additional meeting technologies.

3. *Pairwise Urn-Ball.* If buyers are first grouped in pairs and then allocated to sellers according to the urn-ball technology described above, the meeting technology can be described by

$$P_n(\lambda) = \begin{cases} 0 & \text{for } n \in \{1, 3, 5, \dots\} \\ e^{-\lambda/2} \frac{(\lambda/2)^{n/2}}{(n/2)!} & \text{for } n \in \{0, 2, 4, \dots\}. \end{cases} \quad (11)$$

4. *Urn-Ball with Congestion Effects.* A second variation on the urn-ball technology is obtained when a longer queue length reduces each buyers' chances of meeting a seller. That is,¹⁴

$$P_n(\lambda) = e^{-\Phi(\lambda)\lambda} \frac{[\Phi(\lambda)\lambda]^n}{n!}, \quad (12)$$

where $\Phi(\lambda) : [0, \infty) \rightarrow [0, 1]$ satisfies $\Phi(0) = 1$ and $\Phi'(\lambda) < 0$. To understand this meeting technology, imagine that buyers and sellers in each sub-market begin on separate islands. The measure σ of sellers each send a boat to transport the measure β of buyers, so that each boat carries $\lambda = \beta/\sigma$ buyers to the sellers' island. However, each boat sinks with probability $1 - \Phi(\lambda)$, so that heavier boats are more likely to sink. Then, the buyers that arrive safely at the sellers' island randomly select a seller, as in the urn-ball specification.

5. *Geometric.* The geometric meeting technology specifies that

$$P_n(\lambda) = \frac{\lambda^n}{(1 + \lambda)^{n+1}}. \quad (13)$$

¹⁴See, e.g., Kaas (2010) or Galenianos & Kircher (2012) for related examples of this class of meeting technologies.

This is the outcome of a process in which all buyers and sellers in a submarket are randomly positioned on a circle, after which the buyers walk to the nearest seller on their right-hand side.

Non-Rivalry. The first property of meeting technologies that we compare to our invariance condition is what Eeckhout & Kircher (2010) call “non-rivalry.” In words, they describe a (purely) non-rival meeting technology as one in which “the meeting probability for a buyer is not affected by the presence of other buyers in the market.” That is, $1 - Q_0(\lambda)$ is independent of λ . Using the definition of $Q_0(\lambda)$, it is easy to verify that this condition is equivalent to

$$\sum_{n=0}^{\infty} nP_n(\lambda) = \gamma\lambda, \quad (14)$$

for some constant $\gamma \in [0, 1]$ and all λ .¹⁵

Clearly the urn-ball technology described earlier in the text is non-rival because each buyer meets a seller with probability one. The bilateral meeting technology, however, violates non-rivalry. Since each seller can meet at most one buyer, an increase in the ratio of buyers to sellers implies that each buyer is more likely to be crowded out. As a result, the probability that each buyer meets a seller, $1 - Q_0(\lambda) = \frac{1-e^{-\lambda}}{\lambda}$, is decreasing in λ . Given these observations, a natural question is how non-rivalry relates to invariance. The following lemma establishes that non-rivalry is a necessary, but not sufficient, condition for invariance.

Lemma 2. *Invariance implies non-rivalry, but non-rivalry does not imply invariance.*

To understand the first statement in the lemma, it is helpful to recall our previous example, where buyers are either red or green. If the meeting technology does not satisfy non-rivalry, then increasing the measure of green buyers, while holding constant the measure of red buyers and sellers, will decrease the probability that each red buyer meets a seller. This implies that increasing the measure of green buyers will decrease the probability that a seller meets with $n \geq 1$ red buyers, which violates the invariance condition.

The second statement in the lemma follows from considering the pairwise urn-ball technology. This technology is non-rival since each buyer meets a seller with probability one, regardless of the ratio of buyers to sellers. It is not invariant, however. Intuitively, when the measure of green buyers is insignificant, most red buyers are matched in pairs with other red buyers, so that it is highly likely that a seller is visited by an even number of red buyers. As the measure of green buyers gets large, however, it becomes increasingly likely that red buyers are paired with green

¹⁵Note that $\sum_{n=1}^{\infty} Q_n(\lambda) = \frac{1}{\lambda} \sum_{n=0}^{\infty} nP_n(\lambda)$.

buyers, which implies greater weights on the probability that a seller is visited by an odd number of red buyers.

Independence. In the introduction of their paper, Eeckhout & Kircher (2010) describe non-rivalry in a slightly different way: “Each additional meeting by another buyer does not affect one’s chances of meeting with the seller.”¹⁶ Even though the difference may seem small, the distinction between meeting with *a* seller and meeting with *the* seller is important. In particular, there exist meeting technologies for which the meeting between a buyer and a seller makes it more difficult for other buyers to meet *that specific seller*, but does not affect other buyers’ chances of meeting *a (different) seller* in the same sub-market.

To highlight this difference and keep terminology transparent, we will say that a meeting technology exhibits “independence” when a buyer who meets with a seller has no effect on the distribution (and thus the expectation) of the number of other buyers at the same seller. Formally, independence means $Q_n(\lambda) = (1 - Q_0(\lambda)) P_{n-1}(\lambda)$ for all λ and $n \in \mathbb{N}_1$. This property can be shown to be satisfied if, and only if, the meeting technology is of the following form:¹⁷

$$P_n(\lambda) = e^{-(1-Q_0(\lambda))\lambda} \frac{[(1 - Q_0(\lambda)) \lambda]^n}{n!}. \quad (15)$$

The following lemma establishes that independence is neither a necessary nor a sufficient condition for invariance.

Lemma 3. *Invariance does not imply independence and independence does not imply invariance.*

To see why invariance does not imply independence, consider the geometric technology (13). This meeting technology is invariant: by construction, the presence of green buyers has no effect whatsoever on the likelihood of a seller meeting n red buyers.¹⁸ However, this technology is not independent, as the fact that an individual buyer meets with a seller changes the probability distribution over the number of other buyers who meet with the seller. In particular, when a buyer meets with a seller, he learns that the seller has only met with buyers thus far, which increases the expected number of other buyers that this seller will ultimately meet; that is, the conditional mean of the queue length is greater than the unconditional mean.

Finally, to understand why independence does not imply invariance, consider the urn-ball meeting technology with congestion effects. Since the probability of each boat’s safe passage depends

¹⁶Albrecht et al. (2013b) use a similar definition: “The fact that one or more buyers choose to visit a particular seller does not make it more difficult for any other buyer to visit that seller.”

¹⁷The definition is equivalent to $\sum_{n=0}^{\infty} \frac{\lambda^n}{n!} [1 - Q_0(\lambda)]^n P_0(\lambda) = 1$. As $\sum_{n=0}^{\infty} \frac{q^n}{n!} = e^{-q}$, equation (15) follows.

¹⁸The geometric technology satisfies condition (10) for $M(\lambda) = \left(\frac{\theta}{\theta+\lambda}\right)^\theta$ and $\theta = 1$. Other values of $\theta \in \mathbb{N}_1$ correspond with negative-binomial distributions for $P_n(\lambda)$, which therefore are invariant as well.

on the ratio of buyers to sellers in the sub-market, this technology does not satisfy the requirements of non-rivalry, and hence is not invariant. However, once buyers arrive, the meeting process ensues according to the standard urn-ball technology, so that the arrival of an individual buyer has no effect on the distribution of other buyers to arrive. Hence, this meeting process satisfies independence.¹⁹

5 Conclusion

We consider an environment in which sellers compete by posting mechanisms and buyers direct their search toward the mechanism offering the maximal expected payoff. We characterize the trading mechanism that sellers choose in equilibrium, given a fairly general class of meeting technologies. We show that, in general, sellers can do no better than posting an auction with no reserve price, but with a meeting fee (or subsidy) that is paid by (or to) all buyers who participate in the auction. Then, in order to reconcile our results with the existing literature, we completely characterize a subset of meeting technologies for which the meeting fee is set to zero, and show that the meeting technology that previous studies have used was contained in this subset. We call the meeting technologies in this subset *invariant*.

Though this paper establishes the importance of invariance in a particular context, there is good reason to believe that this is an important property of meeting technologies in other environments as well. After all, invariance is a statement about whether buyers crowd each other out at a particular seller, which is a relevant consideration in a variety of different settings. For example, when buyers know their valuation before selecting a seller, existing results suggest that the degree of crowding out determines whether different buyer types pool or separate in equilibrium: an urn-ball technology leads to perfect pooling, while a bilateral technology leads to perfect separation.²⁰ Given our results, a natural conjecture is that the pooling equilibrium without meeting fees extends to all invariant meeting technologies and that non-invariant technologies give rise to an equilibrium with meeting fees and at least some separation of types.

Similarly, in a labor market model in which workers send multiple applications, firms are concerned that applicants who will reject their job offer (because they have a better offer from a different firm) crowd out applicants who will accept their offer. As a firm's matching probability in this environment equals $1 - \tilde{P}_0(\lambda, \gamma)$, where γ is the fraction of applicants that will accept the offer, our results suggest that meeting fees are needed to price this externality if the meeting

¹⁹This highlights the difference between non-rivalry and independence. Using our previous analogy, non-rivalry is primarily a requirement on the probability that “a boat arrives safely” (i.e., it must be independent of λ), while independence is primarily a condition on the meeting process that occurs “after boats arrive on shore.”

²⁰Peters & Severinov (1997), Virág (2010), Albrecht et al. (2012, 2013b) and Peters (2013) study the urn-ball case, while Eeckhout & Kircher (2010) analyze both urn-ball and bilateral meeting technologies. Shi (2001) and Shimer (2005) derive related results in a labor market setting for bilateral and urn-ball meeting technologies, respectively.

technology is not invariant.²¹ We leave a detailed investigation of these conjectures for future research.

Proofs

Proof of Proposition 1. The proof of this result is provided in the main text.

Proof of Lemma 1. From the description of the auction in the main text, it follows immediately that a buyer's expected payoff equals

$$U(r, t, \lambda) = \sum_{n=1}^{\infty} Q_n(\lambda) \left[\int_r^{\bar{x}} \int_{\underline{x}}^x (x - \max\{\tilde{x}, r\}) dF^{n-1}(\tilde{x}) dF(x) - t \right].$$

The inner integral simplifies to $\int_r^x F^{n-1}(\tilde{x}) d\tilde{x}$ after integration by parts. Changing the order of integration and substituting equations (1) and (3) then yields equation (6).

To calculate the payoff of a seller, consider the expected valuation $V(r, \lambda)$ of the agent that holds the good at the end of the period, given a queue length λ and reserve price r . As the maximum valuation among n buyers is distributed according to $F^n(x)$, $V(r, \lambda)$ equals

$$\begin{aligned} V(r, \lambda) &= P_0(\lambda)y + \sum_{n=1}^{\infty} P_n(\lambda) \left[F^n(r)y + \int_r^{\bar{x}} x dF^n(x) \right] \\ &= \bar{x} - (r - y)m(F(r); \lambda) - \int_r^{\bar{x}} m(F(x); \lambda) dx, \end{aligned}$$

where the second line follows after integration by parts and substitution of (3). As the payoff of a seller must satisfy $R(r, t, \lambda) = V(r, \lambda) - \lambda U(r, t, \lambda)$, the desired expression follows.

Proof of Proposition 2. The Lagrangian of the seller's maximization problem equals

$$L(r, t, \lambda, \zeta) = R(r, t, \lambda) + \zeta (U(r, t, \lambda) - \bar{U}),$$

where ζ denotes the multiplier on the market utility constraint. The first-order condition with respect to t equals

$$\left(1 - \frac{\zeta^*}{\lambda^*} \right) m_z(1; \lambda^*) = 0.$$

²¹This is consistent with the results in this literature that wage posting (without meeting fees) decentralizes the planner's solution when the meeting technology is urn-ball (Kircher, 2009), but not when firms are constrained in the number of applicants that they can meet (Galenianos & Kircher, 2009; Wolthoff, 2012).

As $m_z(1; \lambda) > 0$, this implies that $\zeta^* = \lambda^*$ in any equilibrium. Substituting this into the first-order condition with respect to r gives $(y - r^*) m_z(F(r^*); \lambda^*) F'(r^*) = 0$. Hence, $r^* = y$ in any equilibrium. In combination with these results, the first-order condition with respect to λ subsequently implies that the optimal meeting fee must satisfy

$$t^* = \frac{\lambda^*}{m_z(1; \lambda^*)} \int_y^{\bar{x}} \left[\frac{1}{\lambda^*} (1 - F(x)) m_z(F(x); \lambda^*) + m_\lambda(F(x); \lambda^*) \right] dx.$$

In combination with the first-order condition with respect to ζ , this implies that the optimal queue length λ^* is determined by

$$- \int_y^{\bar{x}} m_\lambda(F(x); \lambda^*) dx = \bar{U}. \quad (16)$$

Since $m(z; \lambda)$ is convex in λ , a unique solution exists for any \bar{U} . In equilibrium, the market utility has to be such that $\lambda^* = \Lambda$, or otherwise the market clearing constraint $\int_0^{\mu_S} \lambda^* dj = \mu_B$ is violated. It immediately follows that the equilibrium is efficient.

Proof of Proposition 3. This proof closely resembles the corresponding one in Lester et al. (2013) and suppresses the arguments of ϕ and τ to keep notation concise. Suppose there exists an equilibrium in which one or more sellers post a particular mechanism $\{\phi_0, \tau_0\}$ and attract a queue length λ_0 , which yields them a payoff $\mathbf{R}(\phi_0, \tau_0, \lambda_0)$, yields buyers a market utility $\bar{U} = \mathbf{U}(\phi_0, \tau_0, \lambda_0)$, and creates a surplus $\mathbf{S}(\phi_0, \lambda_0) = \mathbf{R}(\phi_0, \tau_0, \lambda_0) + \lambda_0 \mathbf{U}(\phi_0, \tau_0, \lambda_0) - y$.²² A deviant seller who posts a mechanism $\{\phi, \tau\}$ and attracts a queue λ , determined by the market utility condition $\mathbf{U}(\phi, \tau, \lambda) = \bar{U}$, obtains a payoff

$$\mathbf{R}(\phi, \tau, \lambda) = \mathbf{S}(\phi, \lambda) - \lambda \bar{U} + y.$$

This payoff is maximized when the deviant chooses trading probabilities ϕ which correspond with the planner's solution ϕ^* , and τ such that

$$\frac{\partial}{\partial \lambda} \mathbf{S}(\phi^*, \lambda) \equiv \frac{d}{d\lambda} S(\lambda) = \bar{U}. \quad (17)$$

Note that $\{\phi_0, \tau_0\}$ can be part of an equilibrium if, and only if, it is a solution to the deviant's maximization problem, i.e., $\phi_0 = \phi^*$ and λ_0 solves (17). Strict concavity of $S(\lambda)$ implies that a unique solution exists to (17) for each level of market utility. Hence, all sellers must attract the same queue length and this queue length must equal $\lambda = \Lambda$, as any other value is inconsistent with the aggregate buyer-seller ratio. It is easy to verify that $\frac{d}{d\lambda} S(\lambda)|_{\lambda=\Lambda} = U(r^*, t^*, \Lambda)$, which implies

²²Naturally, the amount of surplus created by a seller does not depend on the payments.

that $\{\phi_0, \tau_0\}$ is an equilibrium strategy if and only if it is pay-off equivalent to the equilibrium characterized in Proposition 2.

Proof of Proposition 4. First, we show that invariance implies a zero meeting fee (i.e., statement 2 implies statement 1). Note that $\tilde{P}_n(\lambda, \gamma)$ is a compound distribution describing the sum of $N \sim P_n(\lambda)$ independent Bernoulli variables. The probability-generating function of a Bernoulli variable is $1 - \gamma + \gamma z$, so the probability-generating function of $\tilde{P}_n(\lambda, \gamma)$ equals $m(1 - \gamma + \gamma z; \lambda)$. The invariance condition is then equivalent to

$$m(1 - \gamma + \gamma z; \lambda) = m(z; \gamma \lambda). \quad (18)$$

Taking the derivative of this condition with respect to γ and evaluating the result in $\gamma = 1$ yields

$$-(1 - z) m_z(z; \lambda) = \lambda m_\lambda(z; \lambda), \quad (19)$$

for all z and λ . If we substitute this expression into (7) for $z = F(x)$ and $\lambda = \Lambda$, then $t^* = 0$ follows immediately.

Second, we show that a meeting technology that does not need a meeting fee for all distributions $F(x)$, y and Λ must be invariant (i.e., statement 1 implies statement 2). Note that since y affects the lower bound of integration in (7), expression (19) is not only a sufficient condition, but it is also a necessary condition. This expression is a first-order partial differential equation with solution

$$m(z; \lambda) = M(\lambda(1 - z)), \quad (20)$$

for arbitrary differentiable M . Clearly, (20) satisfies (18) and the corresponding meeting technology must be invariant.

Third, to see that statement 1 implies statement 3, note that interpreting $m(\cdot)$ as a probability-generating function requires a number of restrictions on M :

- (i) $P_n(\lambda) = \frac{1}{n!} \frac{\partial^n}{\partial z^n} m(0; \lambda)$ for all n and λ requires that M is a smooth function satisfying $\frac{(-\lambda)^n}{n!} M^{(n)}(\lambda) \in [0, 1]$ for all n and λ ;
- (ii) $m(1; \lambda) = 1$ for all λ requires $M(0) = 1$.
- (iii) $m(z; \infty) = 0$ for all z requires $\lim_{\lambda \rightarrow \infty} M(\lambda) = 0$.
- (iv) $m_z(1; \lambda) \in (0, \lambda]$ requires $M'(0) \in [-1, 0)$;

Note that restriction (iii) allows us to tighten restriction (i). To see this, realize that if $P_n(\lambda) = 0$ for some n and arbitrary λ , then $M^{(n)}(\lambda) = 0$ such that $P_{n'}(\lambda) = 0$ for all $n' \in \{n + 1, n + 2, \dots\}$.

In that case, it must be true that

$$\sum_{i=0}^{n-1} P_i(\lambda) = \sum_{i=0}^{n-1} \frac{(-\lambda)^i}{i!} M^{(i)}(\lambda) = 1,$$

which is a differential equation with solution $M(\lambda) = 1 + \sum_{i=1}^{n-1} c_i \lambda^i$, for some coefficients c_i . For any finite n , this polynomial implies either $\lim_{\lambda \rightarrow \infty} M(\lambda) = \infty$ or $\lim_{\lambda \rightarrow \infty} M(\lambda) = -\infty$, which contradicts restriction (iii). Hence, it must be true that $\frac{(-\lambda)^n}{n!} M^{(n)}(\lambda) \in (0, 1)$ for all n and λ .

Finally, it is straightforward to establish that statement 3 implies statement 2, which completes the proof.

Proof of Lemma 2. The necessity of non-rivalry follows from combining equation (4) and equation (20), which reveals that invariant meeting technologies satisfy $\sum_{n=0}^{\infty} n P_n(\lambda) = -M'(0) \lambda$, with $M'(0) \in [-1, 0)$. Hence, meeting technologies that are invariant are also non-rival.

To prove that non-rivalry is not sufficient, we show that the pairwise urn-ball technology (11) is a counterexample. Note that by the properties of the Poisson distribution, $\sum_{n=0}^{\infty} n P_n(\lambda) = \lambda$, which satisfies (14) for $\gamma = 1$. Hence, pairwise urn-ball is non-rival. At the same time, pairwise urn-ball violates invariance, because $\tilde{P}_1(\lambda, \gamma) > 0$ for all $\gamma > 0$, while $P_1(\gamma\lambda) = 0$. Hence, non-rivalry does not imply invariance.

Proof of Lemma 3. To establish that invariance does not imply independence, consider the geometric technology (13). Its probability-generating function is

$$m(z; \lambda) = \frac{1}{1 + \lambda(1 - z)},$$

which satisfies (18), implying invariance. However, the technology is not independent, as

$$Q_n(\lambda) = \frac{n\lambda^{n-1}}{(1 + \lambda)^{n+1}} \neq \frac{\lambda^{n-1}}{(1 + \lambda)^n} = (1 - Q_0(\lambda)) P_{n-1}(\lambda). \quad (21)$$

To demonstrate that independence does not imply invariance, we use the urn-ball technology with congestion (12). From (15), it is immediate that this technology satisfies independence. The technology is however not invariant, as $m(z; \lambda) = e^{-\Phi(\lambda)\lambda(1-z)}$, which does not satisfy condition (18).

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