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A Tale of Two Commitments: Equilibrium Default and Temptation

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Abstract

I construct the life-cycle model with equilibrium default and preferences featuring temptation and self-control. The model provides quantitatively similar answers to positive questions such as the causes of the observed rise in debt and bankruptcies and macroeconomic implications of the 2005 bankruptcy reform, as the standard model without temptation. However, the temptation model provides contrasting welfare implications, because of overborrowing when the borrowing constraint is relaxed. Specifically, the 2005 bankruptcy reform has an overall negative welfare effect, according to the temptation model, while the effect is positive in the no-temptation model. As for the optimal default punishment, welfare of the agents without temptation is maximized when defaulting results in severe punishment, which provides a strong commitment to repaying and thus a lower default premium. On the other hand, welfare of agents with temptation is maximized when weak punishment leads to a tight borrowing constraint, which provides a commitment against overborrowing.

JEL Classification: D91, E21, E44, G18, K35
Keywords: consumer bankruptcy, debt, default, borrowing constraint, temptation and self-control, hyperbolic-discounting, heterogeneous agents, incomplete markets

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1 Introduction

Preferences that exhibit present bias have become widely used in economics. Based on the success of the models with present bias in replicating various dimensions of borrowing behavior, White (2007) argues that present bias is an important feature in constructing a model of bankruptcies for policy evaluation. This paper revisits her claim using a model with equilibrium default. In particular, the goal of this paper is twofold. First, I investigate whether a model with preferences featuring temptation and self-control (Gul and Pesendorfer (2001)) and equilibrium default provides different implications with respect to the causes behind the observed rise in debt and bankruptcy filings or various bankruptcy law reforms as compared with the standard model without temptation. Second, I explore the differences in welfare implications between the two models.

This is the first paper that extends the quantitative macroeconomic model with equilibrium bankruptcy (Livshits et al. (2007) and Chatterjee et al. (2007)) by introducing preferences featuring temptation and self-control. I introduce the temptation preferences following the formulation provided by Krusell et al. (2010). The finite-horizon model with Gul-Pesendorfer preferences that Krusell et al. (2010) construct includes the hyperbolic-discounting model of Strotz (1956) and Laibson (1997) as a special case. I use this special case since estimates for the preference parameter that controls the degree of present bias are available for the hyperbolic-discounting model. I use the model with Gul-Pesendorfer preferences because the model allows straightforward welfare analysis. The model is calibrated to match the facts related to recent borrowing and bankruptcy in the U.S. economy and is used for a series of counterfactual experiments. The aim of this paper is to do the same set of exercises using both the standard model without temptation and the model with temptation and to compare the implications obtained from the two models.

There are four main findings. First, the calibrated temptation model exhibits some notable differences from the standard model without temptation. The temptation model generates a larger amount of total savings and total debt simultaneously, and more agents default due to poor draws of income shocks, compared with the no-temptation model. Second, regardless of these differences, the temptation model provides quantitatively similar causes for the observed rise in debt and bankruptcy, and the macroeconomic implications of various bankruptcy policy reforms. In other words, for positive questions, the temptation model does not provide significantly different answers than the standard model without temptation. Third, however, welfare implications of policy reforms are strikingly different between the models with and without temptation. Specifically, the 2005 bankruptcy reform has an overall negative welfare effect according to the temptation model, while the effect is slightly positive in the model without temptation. Behind this contrast are two kinds of commitments. Agents without temptation gain from the reform because the reform strengthens their commitment to repaying, and unsecured loan rates decrease, reflecting the decline in the default risk. However, at the same time, agents with temptation suffer from overborrowing, because they lose a commitment against overborrowing when the borrowing constraint becomes relaxed. Fourth, consequently, the two models have contrasting implications regarding the optimal degree of default punishment. While the welfare in the model without temptation is maximized when defaulting results in severe punishment, which provides a strong commitment to repaying and thus a low risk premium to borrowers, the welfare in the temptation model is maximized when weak punishment leads to a tight borrowing constraint, which provides a commitment against overborrowing.

Building on earlier studies, such as those by Strotz (1956) and Pollak (1968), Laibson (1996, 1997) introduces the hyperbolic-discounting preferences into standard macroeconomic models to investigate the role of present bias. Furthermore, Laibson et al. (2003) show that the hyperbolic-discounting model can explain why the majority of households with credit cards pay interest on the cards even if they have assets as well. On the other hand, Barro (1999) finds that the neoclassical growth model with hyperbolic-discounting preferences and log utility is observationally equivalent to the same model with the standard exponential-discounting preferences.

Welfare implications of macroeconomic models with preferences that exhibit present bias have been studied recently. Krusell et al. (2010) study a neoclassical growth model with Gul-Pesendorfer preferences that includes the hyperbolic-discounting model as its special case They find that the optimal long-run capital income tax rate in their temptation model is negative, as opposed to zero in the standard model because the agent undersaves. İmrohoroğlu et al. (2003) find that unfunded Social Security could be welfare-improving in an overlapping-generations model with hyperbolic discounting, by mitigating undersaving. By the same logic, compulsory savings floors are welfare-improving in Malin (2008). In Nakajima (2012), a relaxed borrowing constraint and associated increase in debt could imply lower welfare when agents are subject to temptation.

There has been extensive literature on the quantitative analysis of default. Athreya (2002) and Chatterjee et al. (2007) study the effects of introducing a means-testing requirement for bankruptcy. The latter find a positive welfare effect. Livshits et al. (2007) compare the model economy with bankruptcy, which provides a better consumption smoothing across states, and the model economy without bankruptcy, which provides a better consumption smoothing over the life cycle. Livshits et al. (2010) explore the causes of the observed rise in bankruptcies and debt. Narajabad (2012) and Athreya et al. (2012) study the same issue, with a focus on the role of the improved information technology used by credit card companies. Li and Sarte (2006) investigate the role of different chapters of bankruptcy. In a recent paper, Banjamin and Mateos-Planas (2013) investigate the role of informal default. As compared with existing literature, the model developed in this paper does not include imperfect information, general equilibrium, choice of default options, or informal default, but none of the existing work investigates the implications of present bias to debt and default.

The remaining parts of the paper are organized as follows. Section 2 gives an overview of the environment surrounding consumer bankruptcy in the U.S. Section 3 sets up the model. Section 4 describes how the model is calibrated. Section 5 comments on how the model is numerically solved. Section 6 covers the experiments associated with the observed rise in debt and bankruptcy filings and various policy reforms that affect borrowing and bankruptcy. Section 7 concludes. Appendix A.1 provides more details about calibration, while Appendix A.2 describes the computational algorithm. Appendix A.3 contains additional figures depicting the U.S. credit and default data.

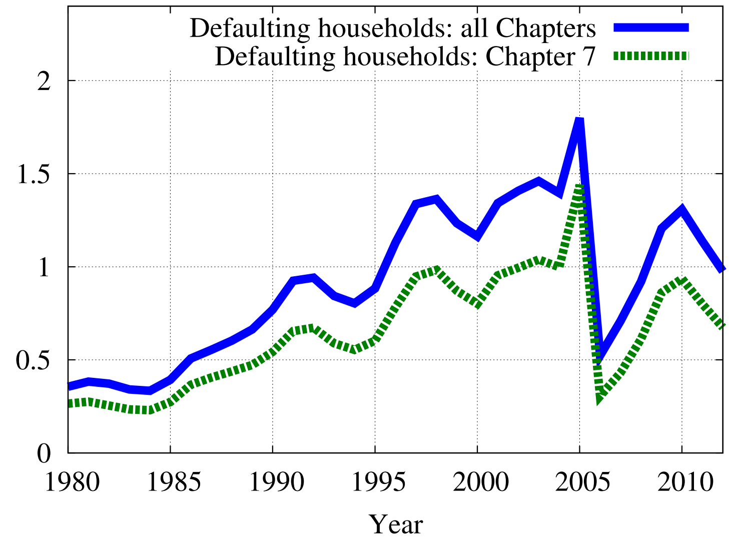


Figure 1: Percentage of Households Filing for Bankruptcy

2 Consumer Bankruptcy in the U.S.

This section provides an overview of the environment associated with consumer bankruptcy in the U.S.[1] When a borrower of unsecured debt fails to repay his debt on schedule, creditors take various measures, such as garnishing labor income, to recover the unrepaid amount.[2] When the borrower files for bankruptcy, these attempts to recover debt are stopped. There are two major types of consumer bankruptcy: Chapter 7 and Chapter 13. Chapter 7, which is also called liquidation, allows debtors to clean up the debt, after paying back a part of the existing debt using assets that are nonexempt. A debtor filing for Chapter 7 bankruptcy obtains a "fresh start" in the sense that once the Chapter 7 bankruptcy is in place, there is no future obligation to pay back the debt. The other major bankruptcy option is Chapter 13, an option of individual debt adjustment. Under Chapter 13, the bankrupt can draw his own repayment plan over three to five years and, upon approval by the judge, reschedule the repayment plan according to the proposed schedule.[3] The assets at the time of bankruptcy filing need not be used for immediate repayment as in Chapter 7, but the bankrupt has to use his future income for repayment. Once a debtor files for Chapter 7 bankruptcy, that debtor cannot file for Chapter 7 bankruptcy again for six years but can file under Chapter 13. Historically, the proportion of Chapter 7 bankruptcies remains stable at about 70 percent of total consumer bankruptcies. There is also a study reporting that many who filed for bankruptcy under Chapter 13 ended up also filing for Chapter 7 bankruptcy (Chatterjee et al. (2007)). The focus of this paper is Chapter 7 bankruptcy, and the "default" option in

[1] See Chatterjee et al. (2007) for more details.

[2] Banjamin and Mateos-Planas (2013) analyze the choice between informal default (to stop repaying debt) and formal default (to file for bankruptcy).

[3] Under the Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA), the bankrupt no longer draws the repayment plan himself. See Section 6.3.

the model resembles the Chapter 7 bankruptcy.[4]

Figure 1 shows the number of total (all types of) bankruptcy filings and Chapter 7 bankruptcy filings in the U.S. from 1980 to 2012. There are three notable features: First, the proportion of Chapter 7 bankruptcy filings has remained stable. Second, the number of bankruptcy filings increased dramatically from 1980 to the early 2000s; the number of Chapter 7 bankruptcy filings increased more than fivefold, from 213,983 in 1980 to 1,117,766 in 2004. Third, there was a significant spike in 2005 and a plunge in 2006. This is because of of the enactment of the Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA). The BAPCPA, which made filing for bankruptcy (especially Chapter 7 bankruptcy) more difficult, became effective in fall 2005, and a large number of debtors rushed to file before the new law took effect. The dip in 2006 was a rebound from that rush to file. Finally, the number seems to be rising again after the dip in 2006, but since this period coincides with the Great Recession, it is impossible to tell at which level the number of bankruptcy filings stabilizes.

In the background of the BAPCPA was a concern about the sharp increase in the number of consumer bankruptcies.[5] The main concern behind the bankruptcy reform was the fact that many people were abusing the bankruptcy law. Naturally, the reform is intended to transform the bankruptcy scheme from a debtor-friendly one, in which the cost of defaulting is low and anybody can file for bankruptcy, to a more creditor-friendly one, in which the cost of defaulting is high and defaulting is available only to low-income borrowers. More details about the BAPCPA will be provided later when I use the models to study the implications of the reform in Section 6.3.

3 Model

The key features of the model are overlapping generations, equilibrium default, and preferences featuring temptation and self-control (Gul and Pesendorfer (2001)). Livshits et al. (2007) feature overlapping generations and equilibrium default, while Nakajima (2012) introduces preferences with temptation and self-control into an overlapping-generations model, following the formulation by Krusell et al. (2010). The current paper combines all three features.

Although I use the preferences featuring temptation and self-control, Krusell et al. (2010) show that a special case of the preferences can be interpreted as the hyperbolic-discounting preferences that are developed by Strotz (1956) and Laibson (1997).

3.1 Demographics

Time is discrete. The economy is populated by I overlapping generations of agents. Each generation is populated by a mass of measure-zero agents. Agents are born at age 1 and live up to age I. Agents who die are replaced by the same measure of newborns, which make the total measure of agents constant over time. Agents retire at age $1 < I_R < I$. Agents with age $i \leq I_R$ are called *workers*, and those with age $i > I_R$ are called *retirees*. I_R is a parameter, implying that retirement is mandatory.

[4] Li and Sarte (2006) investigate the model with both chapters of bankruptcy.

[5] See White (2007) for details of the BAPCPA.

3.2 Preferences

The preferences of agents are time separable and characterized by a period utility function, two discount factors, δ and β, and another parameter, γ. The period utility function takes the following form:

$$u\left(\frac{c_i}{\nu_i}\right) \tag{1}$$

where $u(.)$ is assumed to be strictly increasing and strictly concave. ν_i is the size of a household of age-i in equivalent scale units.[6] δ and β are called the *self-control discount factor* and the *temptation discount factor*, respectively. γ represents the *strength of temptation.* δ is the only discount factor if the agent can exert perfect self-control and thus is not affected by temptation. In other words, in a special case in which the temptation is nonexistent (strength of temptation γ is zero), the model with temptation and self-control preferences reverts to the standard exponential-discounting model with δ as the only discount factor. $\beta < 1$ is the additional discount factor with which an agent is tempted to discount future utility when making a consumption-savings decision. In other words, β captures the degree of present bias.

3.3 Endowment

Agents are born with zero assets. Working agents receive labor income e each period. The labor income takes the following form:

$$e(i, p, t) = e_i \exp(p + t) \tag{2}$$

e_i captures the average life-cycle profile of labor income and is common across all age-i agents. Moreover, $e_i = 0$ for retired agents (i.e., $i > I_R$). p is the persistent shock to labor income and is assumed to follow a first-order Markov process with the transition probability $\pi^p_{i,p,p'}$.[7] t is the transitory shock to labor income. $\pi^t_{i,t}$ represents the probability that an age-i agent draws a shock t.[8] After retirement ($i > I_R$), an agent receives Social Security benefits $b(i, p, t)$. The amount of benefits does not change with age, but i is an argument so that $b(i, p, t) = 0$ for working agents ($i \leq I_R$). An agent also faces shocks to compulsory expenditure $x \geq 0$. $\pi^x_{i,x}$ represents the probability that an age-i agent faces a compulsory expenditure of amount x. x is independently and identically distributed, as in Livshits et al. (2007).

3.4 Bankruptcy

Agents have an option to default on their debt or bills associated with expenditure shocks. The default option is modeled as in Chatterjee et al. (2007) and Livshits et al. (2007). The default option in the model resembles in procedure and consequences a Chapter 7 bankruptcy filing, in particular, before the reform of the bankruptcy law in 2005.

[6] Changes in household size over the life cycle are found to be important in accounting for the hump-shaped life-cycle profile of consumption (Attanasio and Weber (1995)).

[7] i is attached to the Markov transition probability, in order to accommodate the case in which the agent is retired and p no longer changes.

[8] i is attached in order to accommodate the case in which the agent is retired and t is always zero.

Suppose an agent has debt (equivalently, a negative amount of assets) or receives an expenditure shock with which the asset position becomes negative, and the agent decides to default on the debt. The following things happen:

1. The defaulting agent has to pay for a fixed cost of filing, ξ.
2. There is a utility cost of filing, represented by a proportional reduction in consumption ζ.
3. The debt and the expenditure shock (think of a hospital bill) are wiped out, and the agent does not have an obligation to pay back the debt or the expenditure in the future (the *fresh start*).
4. The agent cannot save during the current period. If the agent tries to save, the savings will be completely garnished.
5. Proportion η of the current labor income is garnished. This is intended to capture the effort of the agent to repay until finally deciding to default within a period. The Social Security benefit is not subject to this garnishment.
6. The credit history of the agent turns bad. I use $h = 0$ and $h = 1$ to denote a good and bad credit history, respectively.
7. While the credit history is bad ($h = 1$), the agent is excluded from the loan market. In other words, the borrowing constraint is zero.
8. With probability λ, the agent's bad credit history is wiped out, or h turns from 1 to 0. After that, there is no longer a negative consequence of the past default.

The benefit of using the default option is to get away from debt or an expenditure shock. The default option is a means of partial insurance. The costs are (i) monetary cost of filing, (ii) utility cost of filing, (iii) the income garnishment in the period of default, (iv) inability to save in the period of default (due to asset garnishment), and (v) temporary exclusion from the loan market. (i) and (iii) are different since (ii) is received by credit card companies and thus affects (lowers) the interest rate of loans, while (i) does not directly affect the loan interest rate. Agents in debt or with an expenditure shock weigh the benefits and the costs of defaulting, and default if it is optimal to do so or if there is no other option. The former is called *voluntary default*, and the latter is called *involuntary default*. It is possible that an agent with a bad credit history cannot consume a positive consumption when the agent is hit by an expenditure shock. Only in this case (*involuntary default*) is default by agents with a bad credit history allowed. In other words, an agent with a bad credit history cannot choose *voluntary default*. In reality, a record of default remains on the credit record of an agent for 10 years. However, I use stochastic recovery of the credit status in order to reduce the size of state space. Thanks to the stochastic recovery, I only need to have $h \in \{0, 1\}$ instead of having 11 different possibilities of h, in the case one period is one year. For notational convenience, I use $\pi_0^h = \lambda$ and $\pi_1^h = 1 - \lambda$, which are the probabilities that a bad credit history is wiped out and not wiped out, respectively.

3.5 Agent's Problem

For a clean notation, I start by defining a recursive problem of an agent with an arbitrary discount factor, d. Once I finish characterizing the problem of an agent given d, I will define the problem featuring temptation and self-control.

The individual state variables are (i, h, p, t, x, a), where i is age, h is credit history, p and t are persistent and transitory components of individual productivity shocks, x is the compulsory expenditure shock, and a is asset position. I will start with the problem of an agent with a good credit history ($h = 0$). Given a discount factor d, an agent with a good credit history chooses whether to default or not. Formally:

$$V^*(i, 0, p, t, x, a; d) = \max\{V^*_{\text{non}}(i, 0, p, t, x, a; d), V^*_{\text{def}}(i, 0, p, t, x, a; d)\} \tag{3}$$

where $V^*_{\text{non}}(i, 0, p, t, x, a; d)$ and $V^*_{\text{def}}(i, 0, p, t, x, a; d)$ are values conditional on not defaulting and defaulting, respectively. The Bellman equation for an agent with a good credit history ($h = 0$), conditional on not defaulting, is as follows:

$$V^*_{\text{non}}(i, 0, p, t, x, a; d) = \begin{cases} -\infty & \text{if } B(i, 0, p, t, x, a) = \emptyset \\ \max_{a' \in B(i,0,p,t,x,a)} \left\{ u\left(\frac{c}{\nu_i}\right) + d\, \mathbb{E}V(i + 1, 0, p', t', x', a')\right\} & \text{if } B(i, 0, p, t, x, a) \neq \emptyset \end{cases} \tag{4}$$

subject to:

$$c + a'q(i, 0, p, t, x, a') + x = e(i, p, t) + b(i, p, t) + a \tag{5}$$

where $\mathbb{E}$ is an expectation operator, taken with respect to (p', t', x'). $B(.)$ characterizes the budget set in the case of not defaulting. For an agent with a good credit history ($h = 0$), $B(.)$ is defined as follows:

$$B(i, 0, p, t, x, a) = \{a' \in \mathbb{R} | c + a'q(i, 0, p, t, x, a') + x = e(i, p, t) + b(i, p, t) + a, c \geq 0\} \tag{6}$$

The first case in equation (4) takes care of the case in which the budget set is empty. In this case, since the utility from not defaulting is negative infinity, while the utility from filing is finite, the agent ends up defaulting *involuntarily*. Now, let me make three remarks. First, notice that the discount factor used here is an arbitrary discount factor d. Second, the optimal value characterized by equation (4) is different from the future value in the same equation. I will define the formula when I describe the problem featuring temptation and self-control. Third, $q(i, h, p, t, x, a')$ denotes the discount price of bonds and depends on the type of agent, and the amount saved ($a' \geq 0$) or borrowed ($a' < 0$). $q(.)$ depends on the individual type of borrower because I allow credit card companies to adjust the price of loans reflecting perfectly the risk associated with each loan. I will come back to the determination of $q(i, h, p, t, x, a')$ in Section 3.6.

Given a discount factor d, the Bellman equation for an agent, conditional on defaulting, is defined below. Notice that this Bellman equation is valid regardless of the current credit status (h), because the benefits and the costs of default are the same regardless of the current credit status of an agent. That is why the problem is defined for $\forall h$ and not only for $h = 0$:

$$V^*_{\text{def}}(i, h, p, t, x, a; d) = u\left(\frac{c(1-\zeta)}{\nu_i}\right) + d\, \mathbb{E}V(i+1, 1, p', t', x', 0) \tag{7}$$

$$c + \xi = e(i, p, t)(1 - \eta) + b(i, p, t) \tag{8}$$

Notice the following five differences from the previous case. First, the existing debt (a) and the expenditure shock (x) are wiped out from the budget constraint (8) as a result of default. Second, on the other hand, the agent has to pay for the default cost ξ, and the fraction η of the current labor income is garnished. Third, there is also a utility cost of defaulting, represented by a parameter ζ. In the baseline calibration, ζ is set at zero. I will use ζ in Section 6.2 to account for a lower number of bankruptcy filings in the 1980s. Fourth, the optimal saving level is $a' = 0$, since any assets above 0 would be garnished by assumption. Fifth, the credit history of the agent turns bad ($h' = 1$).

Finally, given a discount factor d, the problem of an agent with a bad credit history ($h = 1$) is defined as follows:

$$V^*(i, 1, p, t, x, a; d) = \\ \begin{cases} V^*_{\text{def}}(i, 1, p, t, x, a; d) & \text{if } B(i, 1, p, t, x, a) = \emptyset \\ \max_{a' \in B(i,1,p,t,x,a)} \left\{ u\left(\frac{c}{\nu_i}\right) + d\, \mathbb{E}V(i+1, \hat{h}', p', t', x', a') \right\} & \text{if } B(i, 1, p, t, x, a) \neq \emptyset \end{cases} \tag{9}$$

subject to the budget constraint (5). $\mathbb{E}$ is an expectation operator, taken with respect to $(\hat{h}', p', t', x')$.[9] $B(.)$ characterizes the budget set, as follows:

$$B(i, 1, p, t, x, a) = \{a' \in \mathbb{R}^+ | c + a'q(i, 1, p, t, x, a') + x = e(i, p, t) + b(i, p, t) + a, c \geq 0\} \tag{10}$$

Notice the following three differences from the problem of an agent with a good credit history. First, the agent can default only when the budget set is empty (i.e., involuntary default). In other words, there is no choice with respect to default for an agent with a bad credit history. Second, the agent with a bad credit history is excluded from the credit market (i.e., $a' \geq 0$). Third, although it is contained in the expectation operator $\mathbb{E}$ and thus is not explicit, a bad credit history will be wiped out with a probability $\pi^h_0 = \lambda$ and will remain with probability $\pi^h_1 = 1 - \lambda$.

We are ready to define the problem with temptation and self-control. First, denote the value conditional on a default decision h' and a saving decision a' as $\widetilde{V}(i, h, p, t, x, a, h', a'; d)$. Obviously, $V^*(i, h, p, t, x, a; d)$, which is the optimal value conditional on a discount factor d, is $\widetilde{V}(i, h, p, t, x, a, h', a'; d)$ associated with the optimal default and saving decision. Now, the problem of an agent with preferences featuring temptation and self-control can be defined as follows:

$$\begin{aligned} V(i, h, p, t, x, a) = \max_{h', a'} \Big\{ & \widetilde{V}(i, h, p, t, x, a, h', a'; \delta) \\ & + \gamma \left(\widetilde{V}(i, h, p, t, x, a, h', a'; \beta\delta) - V^*(i, h, p, t, x, a; \beta\delta) \right) \Big\} \end{aligned} \tag{11}$$

[9] Credit status in the next period has a hat ($\hat{h}'$) in order to distinguish the future credit history that changes stochastically from the default choice h'.

$g_h(i,h,p,t,x,a) \in \{0,1\}$ is the associated optimal default rule, and $g_a(i,h,p,t,x,a)$ is the associated optimal saving rule. The first part in the maximand, $\widetilde{V}(.;\delta)$, is called *self-control utility*, while the part in the maximand multiplied by γ, $(\widetilde{V}(,;\beta\delta) - V^*(.;\beta\delta))$ is called *temptation utility*. In order to understand why, let's assume $\gamma = 0$ for now. In this case, the temptation utility drops off from the maximand and the problem becomes standard: maximizing only the self-control utility using the discount factor δ. This situation is when the agent can exert perfect self-control and is not affected by the temptation to consume or borrow more, which is represented by the discount factor $\beta\delta$ in the temptation utility. In other words, when $\gamma = 0$, temptation drops out of the agent's problem, and the problem collapses back to the exponential-discounting model with the discount factor δ. Another special case is $\beta = 1$. When $\beta = 1$, even if the temptation utility is present ($\gamma > 0$), the problem collapses to the standard exponential-discounting model with a sole discount factor δ. This is because when the pair (h', a') is chosen to maximize $\widetilde{V}(.;\delta)$, the temptation utility is also maximized (at zero) as well.

On the other hand, when $\gamma > 0$ and $\beta \in [0,1)$, the agent's optimization problem includes two considerations. First, the agent still benefits by maximizing the self-control utility as before. Second, at the same time, the agent suffers from deviating from the optimal decision associated with the discount factor $\beta\delta$. Remember again, $V^*(.;\beta\delta)$ is the optimal value associated with the discount factor $\beta\delta$. When the agent chooses (h', a') that are different from the optimal pair associated with $V^*(.;\beta\delta)$, the agent suffers a negative temptation utility, which is multiplied γ. In this sense, γ represents the strength of the temptation. When γ is larger, the agent is more strongly tempted to choose (h', a') that are closer to the optimal pair under the discount factor $\beta\delta$ and make the utility loss from the temptation utility smaller. In an extreme case in which $\gamma \to \infty$, it becomes optimal for an agent to minimize the utility loss from the temptation utility by choosing (h', a') that are optimal under the discount factor $\beta\delta$. I use this special case throughout the paper because this special case is shown to be equivalent to the hyperbolic-discounting preferences with the short-term discount factor β and the long-term discount factor δ, and estimates of β are available for the hyperbolic-discounting model. See Krusell et al. (2010) and Nakajima (2012) for a discussion about the equivalence. Notice that when $\gamma \to \infty$, equation (11) becomes simplified as follows:

$$V(i,h,p,t,x,a) = \widetilde{V}(i,h,p,t,x,a,h',a';\delta) \tag{12}$$

where $h' = g_h(i,h,p,t,x,a) \in \{0,1\}$ and $a' = g_a(i,h,p,t,x,a)$ are the optimal decision rules associated with the value $V^*(i,h,p,t,x,a;\beta\delta)$, which maximizes the temptation utility. In other words, when an agent completely succumbs to temptation, the agent chooses the optimal default decision h' and the optimal saving decision a' by discounting the future with a discount factor $\beta\delta$. However, the actual value is evaluated with the discount factor δ.

3.6 Credit Card Companies

The only assets available in the model are one-period discount bonds. This is a common assumption, used in Chatterjee et al. (2007) and Livshits et al. (2007), but it is less innocuous in the case with temptation preferences. When agents with preferences featuring temptation and self-control can restrict future borrowing, they might want to trade bonds for more than one period ahead. Basically, multi-period bonds could be used as a commitment device against overborrowing in the future. By assuming that only one-period bonds are traded, such possibility is assumed away. I also assume that retired agents cannot borrow, following Livshits et al. (2007).

The saving interest rate is fixed at r. Since the only financial assets available in the model are discount bonds issued by agents, the bond price of the saving agents in equilibrium is $q(i, h, p, y, x, a' \geq 0) = 1/(1+r)$. Notice that this is the only bond price for agents with a bad credit history, as they are excluded from the loan market (i.e., $a' \geq 0$). When an agent borrows, it is assumed that the agent has to pay for the interest premium ι in addition to the interest rate. If there is no default premium, the borrowing interest rate is $r + \iota$ and the price of discount bonds issued by an agent who does not default is $1/(1 + r + \iota)$. However, the only loans available in the model are unsecured loans, and the default premium is added depending on the riskiness of loans. The unsecured loans are provided by a competitive credit sector that consists of a large number of credit card companies. Free entry is assumed. Credit card companies can target agents of one particular type with one particular level of debt. Since the credit sector is competitive, free entry is assumed, and each credit card company can target one specific level of debt, it is impossible in equilibrium to *cross-subsidize*, that is, offer agents of one type an interest rate implying a negative profit while offering agents of another type an interest rate implying a positive profit so that, in sum, the credit card company makes a positive total profit. In this case, there is always an incentive for another credit card company to offer a lower interest rate for agents of the second type and steal the profitable customers away. In equilibrium, any loans to any type of agents and any level of debt make zero profit.

Suppose that a credit card company makes loans to type-$(i, 0, p, t, x)$ agents who borrow a' each.[10] Remember that the current asset position of the agents, a, does not matter for the pricing of loans. By making loans to a mass of agents of the same type, the credit card company can exploit the law of large numbers and insure away the idiosyncratic default risks, even if the individual loans are defaultable. In other words, the credit sector provides a partial insurance, by pooling risk of default across agents of the same type. Now, assume the credit card company makes loans to measure m agents of the same type. The zero profit condition associated with the loans made to type-$(i, 0, p, t, x)$ agents whose measure is m and who borrow a' each can be expressed as follows:

$$\begin{aligned} m(-a')\mathbb{E}\mathbb{1}_{g_h(i+1,0,p',t',x',a')=0} + m\mathbb{E}\mathbb{1}_{g_h(i+1,0,p',t',x',a')=1} e(i+1, p', t') \frac{\eta(-a')}{x' - a'} \\ = m(-a' q(i, 0, p, t, x, a'))(1 + r + \iota) \quad (13) \end{aligned}$$

where $\mathbb{1}$ is an indicator function that takes the value of one, if the logical statement attached to it is true, or zero otherwise. $\mathbb{E}$ is an expectation operator and is taken with respect to (p', t', x'). The two terms on the left-hand side represent the total income from the loans. In particular, if an agent repays the loan ($g_h(.) = 0$), the credit card company receives the amount $-a$. If an agent defaults on its loan, $\eta e(i, p, t)$ is garnished, but the garnished amount is shared proportionally between the issuer of the bill x' and the credit card company

[10] Notice that $h = 0$. I only need to consider the case $h = 0$, as agents with a bad credit history ($h = 1$) cannot borrow.

that extended the loan of amount $-a'$. The right-hand side is the total cost of the loans. Specifically, the discount value of a loan $-a'q(.)$ is the principal, and the credit card company has to pay for the interest and the premium $r+\iota$. By solving equation (13) for $q(i,0,p,t,x,a')$, one can obtain the formula for the equilibrium discount price of loans, as follows:

$$q(i,0,p,t,x,a') = \frac{\mathbb{E}\left\{\mathbb{1}_{g_h=0} + \mathbb{1}_{g_h(i+1,0,p',t',x',a')=1}\frac{\eta e(i+1,p',t')}{x'-a'}\right\}}{1+r+\iota} \tag{14}$$

Finally, I assume there is a maximum limit on the interest rate charged by credit card companies, which is denoted by $\overline{r}$. Since the price of the bond $q(.)$ is used instead of interest rate $r(.)$ for loans, the upper bound of the interest rate $\overline{r}$ is converted into the lower bound of the bond price by $\underline{q} = \frac{1}{1+\overline{r}}$. In the U.S., since the *Marquette* decision in 1978, which basically eliminated the usury law, nationally operating credit card companies are no longer subject to the usury law of the states they are operating in.[11] In other words, currently, there is no effective limit on the interest rate. Therefore, I will set $\overline{r}$ at a level that is non-binding in the baseline calibration and later investigate macroeconomic and welfare implications of introducing a binding interest rate ceiling in Section 6.4.

In order to better understand the pricing of unsecured loans, let's look at some of the special cases. In case the default probability is zero, the price of loans will be:

$$q(i,0,p,t,x,a') = \frac{1}{1+r+\iota} \tag{15}$$

In case all agents default on the debt in the next period, the price of loans will be:

$$q(i,0,p,t,x,a') = \frac{\mathbb{E}\frac{\eta e(i+1,p',t')}{x'-a'}}{1+r+\iota} \tag{16}$$

Consider the special case in which there is no garnishment (i.e., $\eta = 0$). If the loan is defaulted with probability one, $q(i,0,p,t,x,a') = 0$. This is because, when $\eta = 0$, credit card companies cannot receive anything from defaulters. In this case, if $q(i,0,p,t,x,a')$ is monotonically decreasing with respect to a', one can define $\underline{a}(i,0,p,t,x)$, which satisfies:

$$\underline{a}(i,0,p,t,x) = \max\{a'|q(i,0,p,t,x,a') = 0\} \tag{17}$$

$\underline{a}(i,0,p,t,x)$ is the endogenous borrowing constraint for agents of type $(i,0,p,t,x)$. For an agent with a bad credit history, $\underline{a}(i,1,p,t,x) = 0$. By construction, the constraint is less strict than the not-too-tight borrowing constraint by Alvarez and Jermann (2000). This is because the not-too-tight borrowing constraint is associated with no default in equilibrium, while the constraint here allows default in equilibrium. See Chatterjee et al. (2007) for further characterization of the equilibrium loan price function.

3.7 Equilibrium

I define the steady-state recursive equilibrium next. Let $\mathbf{M}$ be the space of the individual state. $(i,h,p,t,x,a) \in \mathbf{M}$. Let $\mathcal{M}$ be the Borel σ-*algebra* generated by $\mathbf{M}$, and μ a probability measure defined over $\mathcal{M}$. I will use a probability space $(\mathbf{M},\mathcal{M},\mu)$ to represent a type distribution of agents.

[11] Supreme Court decision on *Marquette National Bank of Minneapolis v. First of Omaha Service Corp.*

Definition 1 (Steady-state recursive equilibrium) *A steady-state recursive equilibrium consists of loan pricing function $q(i,h,p,t,x,a')$, value function $V(i,h,p,t,x,a)$, optimal decision rules $g_a(i,h,p,t,x,a)$ and $g_h(i,h,p,t,x,a)$, and the stationary measure after normalization μ, such that:*

1. *Given the loan price function, $V(i,h,p,t,x,a)$ is a solution to the agent's optimization problem defined in Section 3.5, and $g_a(i,h,p,t,x,a)$ and $g_h(i,h,p,t,x,a)$ are the associated optimal decision rules.*

2. *Loan price function $q(i,h,p,t,x,a')$ satisfies the zero-profit conditions for all types. Specifically, the loan price function is characterized by equation (14).*

3. *Measure of agents μ is time-invariant and consistent with the demographic transition, stochastic process of shocks, and optimal decision rules.*

4 Calibration

This section describes how the baseline models are calibrated. Table 1 summarizes the parameter values. The top panel of Table 1 contains parameters common across all models. The remaining three panels show the parameters that are independently calibrated for different models (the no-temptation model and the temptation models with different values of the temptation discount factor β).

4.1 Demographics

One period is set as one year in the model. Age 1 in the model corresponds to the actual age of 20. I is set at 54, as in Livshits et al. (2007), meaning that the maximum actual age is 73. I_R is set at 45, implying that the agents become retired at the actual age of 65.

4.2 Preferences

For the period utility function, the following constant relative risk aversion (CRRA) functional form is used:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \tag{18}$$

σ is set at 2.0, which is the commonly used value in macroeconomics. The household size in equivalent scale units, $\{\nu_i\}$, is constructed using the average household size in the 2006 Current Population Survey (CPS), converted into equivalence scale units following Fernández-Villaverde and Krueger (2007). Figure 6 in Appendix A.1 shows $\{\nu_i\}$ used here.

The two discount factors, β and δ, and the parameter controlling the strength of temptation, γ, are calibrated differently for different economies. For the model economy without temptation, $\gamma = 0$ by definition, and δ is calibrated, jointly with other parameters (see Section 4.6), to match the aggregate debt-to-income ratio, which is 9 percent in recent years. When $\gamma = 0$, β is irrelevant. For the economy with temptation, I set $\gamma = \infty$, which makes the model equivalent to the hyperbolic-discounting model, and use the temptation discount factor β of 0.70 and 0.55. The temptation discount factor of 0.7 corresponds to the discount

Table 1: Summary of Calibration

Parameter	Value	Remark
Common Parameters		
I	54	Maximum age (corresponding to 73 years old).
I_R	45	Last working age (corresponding to 64 years old).
σ	2.000	Coefficient of relative risk aversion.
$\{\nu_i\}$	Fig 6	Household size in family equivalence scale.
$\{e_i\}$	Fig 7	Average labor income profile. Following Gourinchas and Parker (2002).
ρ_p	0.9500	Persistence of persistent shocks to earnings. From Livshits et al. (2010).
σ_p^2	0.0250	Variance for persistent shocks to earnings. From Livshits et al. (2010).
σ_t^2	0.0500	Variance of transitory shock to earnings. From Livshits et al. (2010).
ψ_e	0.2000	Parameter for Social Security benefits. From Livshits et al. (2010).
ψ_p	0.3500	Parameter for Social Security benefits. From Livshits et al. (2010).
π_1^x	0.02367	Probability of a small expenditure shock. From Livshits et al. (2007).
π_2^x	0.00153	Probability of a large expenditure shock. From Livshits et al. (2007).
x_1	0.3960	Magnitude of a small expenditure shock. From Livshits et al. (2007).
x_2	1.2327	Magnitude of a large expenditure shock. From Livshits et al. (2007).
λ	0.1000	On average, 10 years of exclusion from loan market upon default.
ξ	0.0280	Cost of a bankruptcy filing is \$600.
r	0.0400	Annual interest rate.
ι	0.0400	Transaction cost of loans.
$\overline{r}$	∞	No ceiling for interest rate in the baseline.
ζ	0	No utility cost of default in the baseline.
No-Temptation Model		
γ	0.0000	Strength of temptation.
β	–	Temptation discount factor.
δ	0.8995	Self-control discount factor. Chosen to match $D/Y = 0.09$.
η	0.2610	Garnishment ratio. Chosen to match number of bankruptcies=0.84 percent.
Temptation Model ($\beta = 0.70$)		
γ	∞	Strength of temptation.
β	0.7000	Temptation discount factor.
δ	0.9641	Self-control discount factor. Chosen to match $D/Y = 0.09$.
η	0.3125	Garnishment ratio. Chosen to match number of bankruptcies=0.84 percent.
Temptation Model ($\beta = 0.55$)		
γ	∞	Strength of temptation.
β	0.5500	Temptation discount factor.
δ	0.9932	Self-control discount factor. Chosen to match $D/Y = 0.09$.
η	0.4660	Garnishment ratio. Chosen to match number of bankruptcies=0.84 percent.

rate of 40 percent, which is the point estimate obtained by Laibson et al. (2007) with the hyperbolic-discounting model. Angeletos et al. (2001) argue that $\beta = 0.7$ "corresponds to the one-year discount factor typically measured in laboratory experiments." The discount factor of 0.55 corresponds to the 80-percent discount rate, which is twice as large as the baseline discount rate. I use β of 0.55 for robustness analysis. In all cases with the temptation model, the self-control discount factor δ is calibrated to match the same target as in the no-temptation model – the debt-to-income ratio of 9 percent. Of course, δ will be different for different economies, but all the models are calibrated to match the same set of targets so that all models with different preference parameters are observationally equivalent with respect to the chosen targets.

4.3 Endowment

The average life-cycle profile of the individual labor productivity $\{e_i\}_{i=1}^{I}$ is taken from the estimates of Gourinchas and Parker (2002). Figure 7 in Appendix A.1 shows the life-cycle profile of the average labor productivity used in the model. Since mandatory retirement at the model age is I_R, $e_i = 0$ for $i > I_R$. The persistent shock to labor income, p, is constructed by discretizing an AR(1) process with the persistence parameter of $\rho_p = 0.95$ and the variance of the normally distributed innovation of $\sigma_p^2 = 0.025$. I use the discretization method of Adda and Cooper (2003) with 11 grid points to approximate the AR(1) process using a first-order Markov process. As for the transitory shock to labor income, I discretize a normal distribution with variance of $\sigma_t^2 = 0.05$, again using the method of Adda and Cooper (2003), with three grid points. These parameter values are within the range of values estimated in the literature and also used in Livshits et al. (2010). As for the Social Security benefits, I use the same formula as Livshits et al. (2010), which is the sum of $\psi_e = 0.2$ of the average labor income of the economy and $\psi_p = 0.35$ of the persistent component of the individual labor income just before retirement ($i = I_R$).

Livshits et al. (2007) construct the compulsory expenditure shocks using a three-point distribution, characterized by the three different sizes of expenditures $\{x_0, x_1, x_2\}$ and the probabilities attached to each size $\{1 - \pi_1^x - \pi_2^x, \pi_1^x, \pi_2^x\}$. The first point is associated with zero expenditure ($x_0 = 0$). The second point is a smaller expenditure shock and captures three kinds of events: unwanted births, divorces, and smaller medical expenditures. The size of the shock (x_1) is calibrated to be 26.4 percent of the average income, and the probability attached to the shock is 7.1 percent. However, since the model period in Livshits et al. (2007) is three years, I use $\pi_1^x = 0.0237$, which is one-third of the probability they used. As for the size of the shock, I use half of the value used by Livshits et al. (2007). The adjustment to the size of the shock is not straightforward, since the size of the shock is computed by calculating the expenditures across a three-year period when an agent is hit by one of the shocks. Dividing the size of the shock used by Livshits et al. (2007) ignores the persistence of the expenditures, while not dividing by anything overstates the size of expenditures per year. Dividing by two is a compromise between the two considerations. The large shock (x_2) captures a large medical expenditure. Livshits et al. (2007) calculated that the size of such a shock is 82.2 percent of the average income, and the probability of such an occurrence is 0.46 percent. I adjusted their parameter values in the same way as I did for the smaller expenditure shock (x_1).

4.4 Bankruptcy

There are four parameters associated with defaulting: λ, which is associated with the average length of punishment; ξ, which represents the filing cost of defaulting; η, which defines the amount of labor income garnished during the period of filing; and ζ, which characterizes the utility cost of defaulting. λ is set at 0.1, implying that, on average, defaulters cannot obtain new debt for 10 years after defaulting. This average punishment period corresponds to a 10-year period during which a bankruptcy filing stays on a person's credit record, in accordance with the Fair Credit Reporting Act. According to White (2007), the average cost of filing for Chapter 7 bankruptcy was \$600 before the BAPCPA was introduced in 2005. ξ is pinned down by converting \$600 into the unit in the model. I obtain $\xi = 0.028$, meaning 2.8 percent of the average annual labor income. η is chosen such that the number of defaults in the model matches the same number in the U.S. economy (0.84 percent of households per year, according to Livshits et al. (2007)). However, notice that the parameter will be chosen jointly with other parameters. I will come back to the calibration of η, together with other parameters jointly calibrated. ζ, which is the utility cost of defaulting, is set at zero in the baseline calibration but will be used in exploring the role of declining default costs in Section 6.2.

4.5 Credit Card Companies

The interest rate is set at 4 percent ($r = 0.04$). The cost of making loans, ι, is set at 4 percent, following Livshits et al. (2007). The upper bound of the lending interest rate is set at infinity ($\overline{r} = \infty$), so that it is not binding in the baseline model. I will lower $\overline{r}$ to investigate the effects of the usury law in Section 6.4.

4.6 Simultaneously Calibrated Parameters

As mentioned, there are two parameters, δ and η, which cannot be pinned down independently from the model. I calibrate the two parameters such that two closely related targets – the aggregate debt-to-income ratio is 9 percent and the proportion of defaulters each year is 0.84 percent – are achieved in the steady-state equilibrium of the model. Notice two things. First, in order to find such parameter values, it is necessary to run the model many times while trying different combinations of (δ, η). Basically, this is a simulated method of moments with exact identification. Second, the values of (δ, η) are different depending on the model specification. At the end, parameter values are different depending on the preference specifications of the model, but the targets are the same across different versions of the model.

The bottom three panels of Table 1 summarize calibration for the three versions of the model, one without temptation and the other two with temptation. In the no-temptation model, the self-control discount factor δ is calibrated to be 0.8995. As for the temptation model with the baseline value of $\beta = 0.70$, δ is calibrated to be 0.9641, which is close to 0.9588, the point estimate of Laibson et al. (2007). δ for the no-temptation model is lower than the values commonly used in existing literature, but it is a result of targeting a large amount of loans regardless of high default risks and thus high loan interest rates. For the temptation model with $\beta = 0.55$, which implies a discount rate twice as high as that in the baseline case of $\beta = 0.70$, δ is calibrated to be 0.9932.

The garnishment parameter η is calibrated to be 0.2610 for the no-temptation model and 0.3125 for the temptation model with $\beta = 0.70$. For the temptation model with $\beta = 0.55$, I obtain $\eta = 0.466$. In order to match the number of defaults in the data, it is necessary to assume a higher garnishment rate for the temptation models, since agents tend to default more often with temptation, all other things being equal.

5 Computation

Since the model cannot be solved analytically, numerical methods are employed. I solve the individual agent's problem using backward induction, starting from the last period of life, with discretized state space. Details about the solution algorithm can be found in Appendix A.2, but one feature of the model is worth pointing out. The equilibrium price of loans, $q(i, h, p, t, x, a')$, is solved simultaneously with the agent's optimization problem. Once the optimal decision rules for agents of age i are obtained, the price of debt for age $i-1$ agents, $q(i-1, h, p, t, x, a')$, can be computed, using the optimal default policy $g_h(i, h, p, t, x, a)$. $q(i-1, h, p, t, x, a')$ in turn is used to solve the optimization problem of agents of age $i-1$. In short, there is no need to use iteration to find an equilibrium loan price $q(i, h, p, t, x, a')$ as in Chatterjee et al. (2007).

6 Results

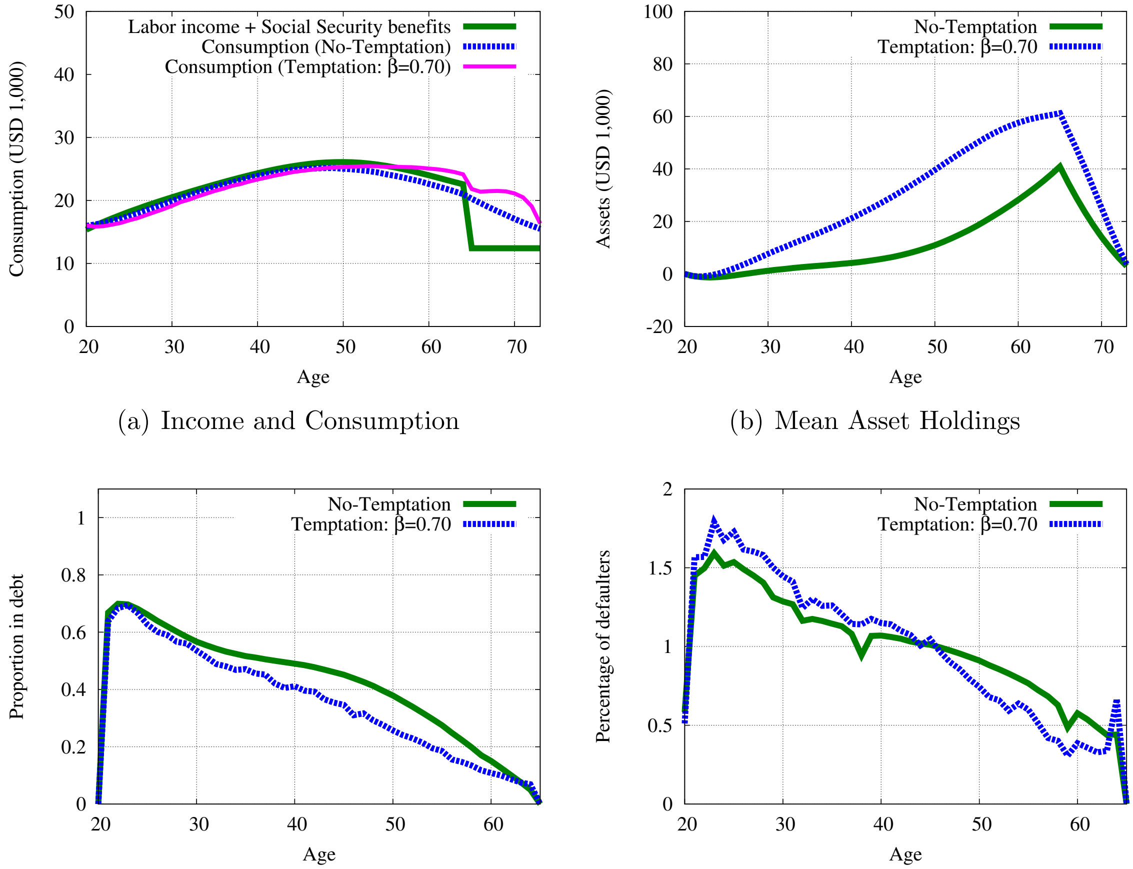
6.1 Comparison of the Baseline Models

In showing the results, I focus on comparing the model with and without temptation. For most of the results with the temptation model, the baseline temptation discount factor $\beta = 0.70$ is used. The results of the model with a lower discount factor of $\beta = 0.55$ are mainly shown in Section 6.6.

Figure 2 compares the properties of the baseline models with and without temptation. Table 2 compares selected statistics between the two models. As explained in Section 4.6, both models are calibrated such that the total amount of debt (9 percent of annual income) as well as the number of defaulters (0.84 percent of the population per year) are the same as in the U.S. data for the late 1990s (see Table 2). Going back to Figure 2, panel (a) shows the average nonfinancial income (labor income and Social Security benefits) and consumption over the life cycle. As usual in a life-cycle model, consumption profiles are smoother than income profiles. Moreover, the differences between the models with and without temptation are minor.

However, panel (b) shows that there is a significant difference between the two models: Average savings in the temptation model are significantly higher than average savings in the no-temptation model. Table 2 confirms that aggregate savings are 52 percent of total income in the no-temptation model, whereas the ratio is 126 percent in the temptation model with $\beta = 0.70$. Aggregate savings are even higher, at 190 percent of aggregate income, with $\beta = 0.55$.

Why does this divergence happen? Harris and Laibson (2001) theoretically provide an



(a) Income and Consumption

(b) Mean Asset Holdings

(c) Proportion of Debtors

(d) Proportion of Defaulters

Figure 2: Comparison of No-Temptation and Temptation Models

Table 2: Comparison of No-Temptation and Temptation Models

(Percent)	U.S. 1995-1999[1]	No-Temptation	Temptation $\beta = 0.70$	Temptation $\beta = 0.55$
Total assets over income	–	52.4	126.1	189.8
Proportion in debt	–	35.1	29.8	22.6
Total debt over income	9.0	9.0	9.0	9.0
Charge-off rate	4.8	4.9	4.4	4.1
Average borrowing rate	10.9-12.8	9.9	10.1	11.1
Total defaults	0.84	0.84	0.84	0.84
Due to expenditure shock	–	0.83	0.73	0.41
Due to income shock only	–	0.01	0.11	0.44

[1] Sources: Livshits et al. (2007, 2010).

intuition.[12] The temptation model generates endogenously divergence of the discount factor;

[12]Both Harris and Laibson (2001) and Tobacman (2009) use the hyperbolic-discounting model, which is

agents that are closer to the borrowing constraint exhibit an effective discount factor that is closer to $\beta\delta$ and lower, while agents with ample savings exhibit an effective discount factor that is closer to δ and higher. Tobacman (2009) shows, using a calibrated life-cycle model, that the model with temptation exhibits a larger wealth inequality than the standard model without temptation.

On the other hand, this result is in contrast to the observational equivalence result of Barro (1999) and the observational similarity in Nakajima (2012). In the latter, the average life-cycle profiles of savings in the two models are similar as long as δ is calibrated to the aggregate amount of savings in the two models. To the contrary, since the discount factor is calibrated to match the total amount of debt in Section 4, the two models do not necessarily generate a comparable amount of savings. Moreover, in Nakajima (2012), the ad-hoc borrowing constraint is independently calibrated for the two models so that they generate the same amount of debt, but a parameter that governs the tightness of the borrowing constraint in the current model, which is the garnishment parameter η, is used to match the number of defaults. Technically speaking, the life-cycle profile of savings is different because there are only two parameters, δ and η, which are used to match the amount of debt and the number of defaults, leaving total savings unmatched. I calibrate the discount factor δ to match the amount of debt instead of the amount of savings, because the amount of debt is more relevant in the model for debt and default.

Panel (c) compares the life-cycle profile of the proportion of debtors, and panel (d) shows the life-cycle profile of the number of defaulters in the two models. Let me make three remarks. First, they are similar, which is not surprising because the total amount of debt as well as the total number of defaulters are matched to the data in the two models. Second, because of the life-cycle motive of savings, fewer agents are in debt as they age. This is typical in a life-cycle model. Third, the number of defaulters closely tracks the proportion of debtors; there are more defaults among younger agents, many of whom are in debt.

Finally, let me point out two more results in Table 2. First, although the charge-off rate and the average borrowing rate are not targeted when the models are calibrated, both the models with and without temptation match these data well. Second, even though the three models are similar in many ways regarding borrowing and default, there is one notable difference: While almost all defaults in the no-temptation model are due to expenditure shocks (0.83 percent out of 0.84 percent), a sizable fraction of defaults (0.11 percent out of 0.84 percent) in the temptation model with $\beta = 0.70$ is due to income shocks. The proportion is even higher (0.41 percent out of 0.84 percent) in the model with a stronger present bias ($\beta = 0.55$). In the no-temptation model, an agent does not borrow a large amount and subsequently default because of the high borrowing rate unless forced by an expenditure shock. On the other hand, a tempted agent borrows and defaults without an expenditure shock more often, because of the temptation to borrow even in the face of a high borrowing rate. The temptation model is more consistent with the survey evidence constructed by Chakravarty and Rhee (1999), who use the Panel Study of Income Dynamics (PSID) and find that a large fraction of defaults is due to credit misuse (41.3 percent) or job loss (12.2 percent). These reasons for default seem to be closely related to bad draws of income shocks.

equivalent to the version of the temptation model employed in this paper.

6.2 Accounting for the Rise in Bankruptcies

Starting in this section, I will revisit important questions surrounding unsecured consumer credit and bankruptcies with the temptation model. The focus is on whether the temptation model provides answers different than those of the standard no-temptation model used in existing literature. This section seeks the reasons behind the observed rise in debt and the number of bankruptcy filings. Sections 6.3 investigates the macroeconomic and welfare effects of the 2005 bankruptcy law reform (the BAPCPA). Section 6.4 compares the implications of the ceiling on the loan interest rate between the models with and without temptation. In Section 6.5, I show how the optimal degree of punishment for defaulting is significantly different between the two models. Finally, in Section 6.6, I show the results with $\beta = 0.55$ as the robustness analysis.

I follow the approach of Livshits et al. (2010) and use the calibrated model to evaluate the contributions of different elements to the observed rise in consumer bankruptcy filings and the amount of unsecured debt between the early 1980s and the late 1990s. In particular, I make the following changes to the baseline model economy (which I call the 2000 economy) so that the modified model economy (which I call the 1980 economy) captures the relevant features of the U.S. economy in the early 1980s:

(1) Variance of persistent income shocks is 60 percent lower.

(2) Variance of transitory income shocks is 25 percent lower.

(3) Probability of receiving a positive expenditure shock is 15 percent lower.

(4) Credit card companies cannot distinguish among borrowers, and thus they pool all the risks and charge the risk premium based on the average default risk of the pool of borrowers.

(5) Cost of extending a loan (ι) is higher.

(6) Utility cost of defaulting (ζ) is higher.

(7) There is an ad-hoc borrowing constraint ($\underline{a}$).

Elements (1) to (3) are taken from Livshits et al. (2010).[13] In particular, the size of the income shocks is obtained from the estimates of Heathcote et al. (2010). Element (4) is motivated by Narajabad (2012) and Athreya et al. (2012). The latter reports that, according to the Survey of Consumer Finances (SCF), there was little dispersion of unsecured loan interest rates in the 1980s, while the dispersion widened by the 2000s. Complete pooling of all borrowers in the 1980 economy is a stylistic way to capture the fact that credit card companies can separate borrowers and charge different loan rates according to the riskiness of different borrowers in the 2000s, but this technology was unavailable back in the 1980s. Finally, I use elements (5) to (7) in the same spirit as Livshits et al. (2010); namely, I

[13] However, Livshits et al. (2010) do not use the decline in the variance of the persistent income shocks because they find it difficult to replicate the U.S. economy in the early 1980s if a lower variance of persistent shocks is introduced.

Table 3: Comparison of Causes for the Rising Number of Bankruptcy Filings[1]

	% Default	% in Debt	D/Y %	Charge-off %	Avg r %	Welfare %
No-Temptation Model						
2000 (Model)	0.84	35.1	9.0	4.9	9.9	–
2000 (U.S.)[2]	0.84	–	9.0	4.8	11.9	–
1980 (Model)	0.25	33.8	5.0	3.1	10.6	+13.5
1980 (U.S.)[2]	0.25	–	5.0	1.9	11.5	–
(Composition)						
Expenditure shock	0.29	32.8	5.0	3.6	10.8	+13.1
Income shock	0.32	24.0	3.7	4.8	11.1	−2.0
Cost of loan	0.26	36.9	5.5	2.9	8.6	+13.8
Cost of default	0.83	34.7	5.4	5.8	11.4	+14.3
Risk rating	0.34	53.2	23.1	1.5	10.9	+15.9
Temptation Model ($\beta = 0.70$)						
2000 (Model)	0.84	29.8	9.0	4.4	10.1	–
2000 (U.S.)	0.84	–	9.0	4.8	11.9	–
1980 (Model)	0.25	37.9	5.0	2.7	10.5	+13.2
1980 (U.S.)	0.25	–	5.0	1.9	11.5	–
(Composition)						
Expenditure shock	0.29	37.5	5.0	3.1	10.6	+12.9
Income shock	0.33	25.3	3.4	4.6	11.0	−0.7
Cost of loan	0.25	39.7	5.3	2.6	8.5	+13.3
Cost of default	0.79	38.3	5.2	4.9	11.2	+13.8
Risk rating	0.28	45.5	20.9	1.4	10.9	+13.5

[1] The six columns show proportion defaulting, proportion in debt, debt-to-income ratio, charge-off rate, average borrowing rate, and welfare gain from policy reform, represented as the rate of permanent increase in consumption.

[2] Sources: Livshits et al. (2007, 2010).

calibrate the three parameters (ι, ζ, and $\underline{a}$) so that the 1980 model can replicate (i) the number of defaults of 0.25 percent, (ii) the debt-to-income ratio of 5.0 percent, and (iii) the loan interest rate of around 11.0 to 12.1 percent, in the early 1980s. As discussed in Livshits et al. (2010), the decline in ι might reflect a lower cost of funds due to financial innovation, or lower margins charged owing to an increased level of competition in the credit card industry. As for ζ, several studies argue that the rise in bankruptcy filings is a result of the declining cost of filing, possibly associated with less stigma attached to bankruptcy filings (Gross and Souleles (2002), Fay et al. (2002)). A decline in ζ is a parsimonious way to capture such changes. The ad-hoc borrowing constraint $\underline{a}$ could be due to the adverse selection problem, which is not modeled explicitly.[14]

Once the three parameters are recalibrated, I can bring back one element at a time to evaluate the contribution of each element to the observed increase in bankruptcies and debt. Importantly, I implement this exercise separately for the models with and without temptation. For the no-temptation model, I obtain $\iota = 0.06$, $\zeta = 0.353$, and $\underline{a} = -0.207$. For the temptation model, I keep the same parameter controlling the cost of loans as in the no-temptation model ($\iota = 0.06$) and obtain $\zeta = 0.379$ and $\underline{a} = -0.148$. For the temptation model with $\beta = 0.55$, covered in Section 6.6, I obtain $\iota = 0.06$, $\zeta = 0.22$, and $\underline{a} = 0.152$.

There are other potential changes between the early 1980s and the late 1990s that might have contributed to the rising debt and defaults. As for demographic changes, as Livshits et al. (2010) argue, the changes were too gradual to explain the rapid change in the number of defaults and the amount of debt. Therefore, I abstract from changes in demographics. I also abstract from the effects of the usury law, which was effective in the early 1980s but became non-binding after the early 1980s. As will be shown in Section 6.4, when there is a binding upper bound for the loan interest rate that credit companies charge, the number of defaults is lower. Moreover, the timing is correct in the sense that the usury law became ineffective when the amount of debt and number of defaults started increasing. However, the effective interest rate varied across states, which makes it difficult to put the realistic usury law into the experiment. Moreover, Livshits et al. (2010) find that the repeal of the usury law does not contribute to a significant portion of the observed changes in debt and defaults.

Table 3 summarizes the results. There are two panels: The top panel shows the results of the no-temptation model, and the bottom panel is associated with the temptation model with $\beta = 0.70$. For each panel, the first two rows compare the baseline model (the 2000 economy) and the U.S. economy in the late 1990s. The next two rows compare the recalibrated model economy (the 1980 economy) and the U.S. economy in the early 1980s. The remaining five rows show the decomposition of the total effects into effects from different elements. Specifically, in the fifth row (labeled "Expenditure shock"), I use the 1980 economy but change the probabilities of the expenditure shocks back to the 2000 levels. In the sixth row ("Income shock"), I change back only the variances of the persistent and transitory income shocks to their 2000 levels. In the seventh row ("Cost of loan"), I set ι back to the baseline level of 0.04, leaving all the other parameters at the 2000 levels. In the eighth row ("Cost of default"), I change only the utility cost of default ζ back to its baseline value of zero. Finally, in the ninth row ("Risk rating"), I enable credit card companies in the model to adjust the loan rate according to the individual risk characteristics of borrowers but leave all the parameters at their 1980 levels. Notice that since this change makes the borrowing constraint endogenous, $\underline{a}$ becomes irrelevant. The six columns of Table 3 show the proportion defaulting per year, the proportion of agents in debt, the aggregate debt-to-income ratio, the charge-off rate, the average loan interest rate (weighted by the number of borrowers as

[14] Livshits et al. (2010) do not assume pooling in their 1980s economy, and recalibrate ι and ζ to match the number of defaults and the debt-to-income ratio. However, since I assume lower variances of both persistent and transitory income shocks, without setting ι to a counterfactually high level, it is impossible to achieve the debt-to-income ratio of 5 percent. Therefore, I introduce pooling, which is consistent with the U.S. data on interest rate dispersion, and use $\underline{a}$ to match the low debt-to-income ratio in the early 1980s.

well as the loan size), and changes in welfare from the baseline (the 2000 economy).

Throughout the paper, welfare is evaluated as the ex-ante expected lifetime utility. Formally, social welfare $\mathbb{E}V$ is defined as:

$$\mathbb{E}V = \sum_p \sum_t \sum_x \pi_p^0 \; \pi_t^t \; \pi_x^x \; V(1, 0, p, t, x, 0) \tag{19}$$

where π_p^0, π_t^t, and π_x^x denote the initial distribution of the persistent income shock p, the distribution of the transitory income shock t, and the distribution of the expenditure shock x, respectively. Also notice that an agent is born into the model economy with a good credit history ($h = 0$) and zero assets ($a = 0$). This definition of welfare is the standard in the life-cycle model with heterogeneous agents in evaluating the welfare effects of policy changes (e.g., Conesa et al. (2009)). The difference in welfare between two economies is measured by the percentage change in lifetime consumption. For example, the number +13.5 percent in the third row of the upper panel in Table 3 means that an unborn agent in the 2000 (baseline) economy has to be compensated by 13.5 percent of consumption in every period and state in his lifetime in order to make him indifferent to being born into the 1980 economy.

Notice that although the model with temptation and self-control has the same behavioral implications as the hyperbolic-discounting model, welfare analysis is more straightforward with the former. On the one hand, a hyperbolic-discounting agent is dynamically inconsistent. In the hyperbolic-discounting model, the same agents in different time periods are considered as different selves. Naturally, the decision problem of an agent is interpreted as a dynamic game played among different selves within the same agent. This structure makes it difficult to define the welfare of an agent. On the other hand, as argued in Gul and Pesendorfer (2001, 2004), an agent with the preferences featuring temptation and self-control is dynamically consistent. Therefore, the welfare of an agent can be defined in a straightforward manner.

Looking at the first four rows of the upper panel of Table 3, which is associated with model without temptation, the model replicates perfectly the number of defaults and the debt-to-income ratio in the U.S. data. The average interest rate of the 1980 model is also close to the average U.S. interest rate of the 1980s. According to the no-temptation model, which element contributed to the changes in the number of bankruptcies and the amount of debt? As for the number of bankruptcies, utility cost of default played by far the biggest role in its rise. Just by eliminating the utility cost of default in the 1980 economy, the number of defaults (0.83 percent) comes back to the high level in the late 1990s (0.84 percent). However, lowering the cost of default does not explain the increase in the debt-to-income ratio, which remains at 5.4 percent in the economy where only the cost of default is eliminated from the 1980 economy. Instead, eliminating the pooling of risks and relaxing the borrowing constraint play a significant role in the rising amount of debt. In fact, the amount of debt overshoots significantly (to 23.1 percent of total income) only if the pooling of risks and the borrowing constraint are eliminated from the 1980 economy. What brings the amount of debt to the 2000 level (9 percent) is the larger variation of income shocks. With higher variances of income shocks, loans to agents become riskier, and thus credit card companies charge higher interest rates, inducing a reduction of the total amount of debt in the economy. This also explains why, without a tight borrowing constraint, the economy with small variances of income shocks cannot generate a relatively small debt-to-income ratio, which Livshits et al. (2010) find as well.

As for welfare, the no-temptation model indicates that an unborn agent in the 1980 economy is significantly better off than an unborn agent in the 2000 economy, by as much as 13.5 percent of lifetime consumption. Where is this large welfare gain coming from? The decomposition indicates it is basically due to the smaller volatility of income shocks in the 1980s. Indeed, if the variances of income shocks revert back to their 2000 (baseline) levels in the 1980 economy, the welfare gain disappears, and an unborn agent is worse off by 2 percent of lifetime consumption as compared with an unborn agent in the 2000 economy. If the pooling of loan risks is eliminated and the borrowing constraint is relaxed in the 1980 economy, the welfare gain as compared with an unborn agent in the 2000 economy goes up further, to 15.9 percent of lifetime consumption.

Now, looking at the bottom panel of Table 3, the most important message is that the results are similar in the temptation model. The recalibrated temptation model successfully replicates the number of defaults, the debt-to-income ratio, and the average loan interest rate. The single most important reason for the observed increase in the number of defaults is the decline in the utility cost of defaulting; the single most important reason behind the observed increase in debt between 1980 and 2000 is the adoption of a risk-adjusted interest rate and the relaxation of the borrowing constraint; and the large welfare gain for an unborn agent in the 1980 model economy as compared with an unborn agent in the 2000 (baseline) economy is due to the smaller volatility of individual labor income shocks. In sum, in terms of the reasons behind the observed increase in debt and defaults, the temptation model provides qualitatively the same and quantitatively similar answers as the standard model without temptation preferences.

6.3 2005 Bankruptcy Reform

In 2005, the government enacted the Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA) in response to the increase in consumer bankruptcy filings. According to White (2007), the main elements of the BAPCPA are:

(1) Means-testing: Under the BAPCPA, a debtor whose household income over the past six months prior to the filing is over the median income of the state in which the debtor lives, the borrower cannot file for Chapter 7 (fresh start) bankruptcy and can only file for Chapter 13 bankruptcy, which is basically debt restructuring and repayment rescheduling.

(2) Higher cost of filing: Under the BAPCPA, in order for his debt to be discharged, a debtor is required to take credit counseling, complete a financial management course, and submit detailed financial information that has to be certified by a lawyer. The typical cost of filing for Chapter 7 bankruptcy is raised from $600 to $2,500.

(3) Repayment schedule: Under the BAPCPA, a debtor filing for a Chapter 13 bankruptcy can no longer propose a repayment schedule. Instead, the law determines how much a filer has to pay back.

Since I do not explicitly model the Chapter 13, I focus on the effects of (1) means-testing and (2) the higher cost of defaulting. As for the means-testing, a borrower cannot default if his current income is above the median income of the model economy. The exception is when the budget set is empty, i.e., the borrower cannot consume a positive amount without defaulting. As for the higher cost of defaulting, I change ξ, the fixed cost of defaulting, from \$600 (converted into the model unit) to \$2,500. I also implement exercises in which only one of the two components of the BAPCPA is enacted, in order to evaluate separately the effects of each of the two components. The key questions are whether and how the effects of the BAPCPA evaluated using the temptation model are different from those obtained using the standard no-temptation model.

I also compare the model predictions with the observed changes in the U.S. However, the numbers for the U.S. have to be taken cautiously, especially when the numbers of the U.S. are compared to the numbers of a steady-state of the model. Because 2005 saw a surge in bankruptcy filings before the BAPCPA became effective, and 2006 observed a rebound from the spike in 2005, I will not use the data from 2005 to 2006. However, since the Great Recession started at the end of 2007, it is impossible to disentangle the effects from the BAPCPA and the cyclical effects from the Great Recession. In order to keep the data from being affected by the Great Recession, I use only the data in 2007 to represent the data after the BAPCPA. But the data are noisy, and most likely the economy was still on its transition to the new steady-state, both of which make the data in 2006 not the most desirable. In my opinion, however, this is the best among feasible options. Appendix A.3 contains figures of the default rate, the debt-to-income ratio, the charge-off rate of credit card loans, and the average credit card interest rate of the U.S. economy.

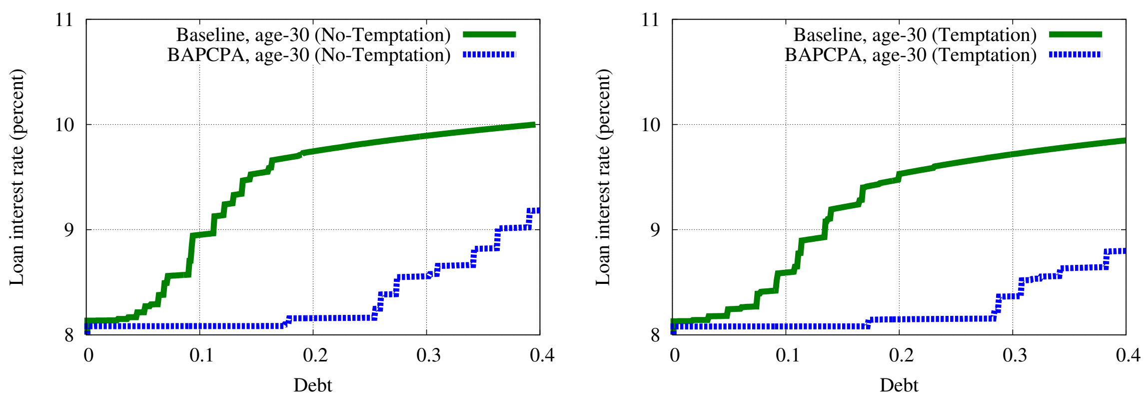
Table 4 compares the U.S. data (top panel) with the effects of the BAPCPA that are implied by the no-temptation model (middle panel) and the temptation model with $\beta = 0.70$ (bottom panel). Comparing the middle and the bottom panels, one sees that the implications of the BAPCPA are similar for the two models. In both models, the number of defaults declines, as the default cost rises and agents with above-median income are prohibited from defaulting. The number of defaults drops from 0.84 percent of the population to 0.41 percent in the no-temptation model, and from 0.84 percent to 0.44 percent in the temptation model. The charge-off rate declines, because less debt is defaulted on. The average borrowing rate also declines, reflecting lower risk of loans on average. These are the direct effects, but there are indirect effects as well, in both models. Since loan rates decline in general, agents can and do borrow more. The debt-to-income ratio rises from the baseline level of 9.0 percent to 12.4 percent in the no-temptation model, while it rises from 9.0 percent to 10.7 percent in the temptation model. The proportion of agents in debt rises in both models. If the two key components of the BAPCPA are investigated separately, in both models, the effects of the higher default cost are stronger than the effects of introducing the means-testing, but qualitatively the effects are similar. This is reasonable. Although both changes make defaults more difficult, a higher default cost more strongly affects lower-income agents, who tend to default more often, while the means-testing mainly affects agents with higher current income, who tend to default less often.

If the model implications are compared with the U.S. data, the predictions of both models are generally consistent with the observed changes, except for the aggregate amount of debt,

Table 4: Comparison of the Effects of the 2005 Bankruptcy Reform[1]

	% Default	% in Debt	D/Y %	Charge-off %	Avg r %	Welfare %
U.S. Economy						
Avg of 1999-2004	0.96	–	8.3	5.4	13.8	–
2007	0.43	–	8.4	4.0	13.3	–
No-Temptation Model						
Baseline	0.84	35.1	9.0	4.9	9.9	–
BAPCPA	0.41	39.1	12.4	2.5	9.3	+0.02
Means-testing	0.62	36.0	9.5	3.7	9.7	−0.06
Default cost	0.59	37.5	11.4	3.5	9.7	+0.01
Temptation Model ($\beta = 0.70$)						
Baseline	0.84	29.8	9.0	4.4	10.1	–
BAPCPA	0.44	31.5	10.7	2.8	9.7	−0.33
Means-testing	0.67	30.4	9.2	3.9	10.2	−0.03
Default cost	0.55	31.1	10.4	3.4	9.9	−0.30

[1] The six columns show proportion defaulting, proportion in debt, debt-to-income ratio, charge-off rate, average borrowing rate, and welfare gain from policy reform, represented as the rate of permanent increase in consumption.



(a) No-Temptation Model

(b) Temptation Model

Figure 3: Loan Interest Rate: Baseline and BAPCPA

whose changes in the models are stronger than in the data. In the U.S., the proportion of households defaulting declined from 0.96 percent to 0.43 percent. The level after the reform is close to the predictions of the models. The charge-off rate of credit card loans dropped from 5.4 percent to 4.0 percent in the U.S., and the size of the drop is comparable to the model predictions. The average credit card interest rate declined slightly in the U.S., as in the model economies. The debt-to-income ratio picked up slightly in the U.S. economy, while it went up sharply in the model economies, especially in the no-temptation model.

Table 5: Comparison of the Effects of Usury Law[1]

	% Default	% in Debt	D/Y %	Charge-off %	Avg r %	Welfare %
No-Temptation Model						
Baseline	0.84	35.1	9.0	4.9	9.9	–
Usury law: 15%	0.84	35.1	9.0	4.9	9.9	−0.00
Usury law: 10%	0.78	19.1	3.0	9.6	9.5	−2.90
Temptation Model ($\beta = 0.70$)						
Baseline	0.84	29.8	9.0	4.4	10.1	–
Usury law: 15%	0.83	30.2	9.1	4.4	10.1	−0.01
Usury law: 10%	0.73	25.8	4.4	6.2	9.6	−0.65

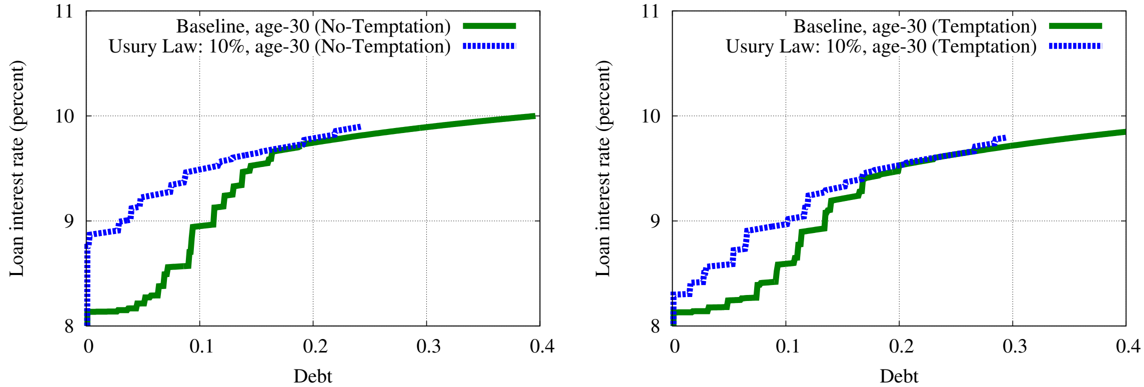
[1] The six columns show proportion defaulting, proportion in debt, debt-to-income ratio, charge-off rate, average borrowing rate, and welfare gain from policy reform, represented as the rate of permanent increase in consumption.

Figure 3 confirms the finding that the behavioral implications of the BAPCPA are similar between the two models. The figure compares the loan rate schedules under the baseline model economy and the alternative economy under the BAPCPA, with the no-temptation case on the left (panel (a)) and the temptation case on the right (panel (b)). The loan rate schedules for age-30 agents with the median productivity shock and zero expenditure shock are drawn. Clearly, in both models, the BAPCPA lowers the loan interest rate since the BAPCPA makes defaulting more difficult, either by charging a higher cost of defaulting or imposing a means-testing requirement.

What is different between the two models is the welfare implication of introducing the BAPCPA, shown in the last column. The effect on social welfare of introducing the BAPCPA is small and positive (+0.02 percent of lifetime consumption) in the no-temptation model while it is larger and negative in the temptation model (−0.33 percent of lifetime consumption). Why? Both the negative direct effect of some agents not being able to default even if they want to, and the positive indirect effect of expanded credit due to a lower default premium are operative in the two models. However, as emphasized in Nakajima (2012), the relaxed borrowing constraint induces overborrowing among agents with temptation, which is an additional negative effect of the BAPCPA. The larger negative welfare effect implies that the effect of the induced overborrowing overwhelms the smaller effects also present in the temptation model.

6.4 Usury Law

Until the early 1980s, banks and other lending institutions were subject to limits on the interest rate they could charge. This usury law was imposed by the state in which each loan was made. However, the so-called *Marquette* decision in 1978 and the Depository Institutions Deregulation and Monetary Control Act virtually freed banks and lending institutions of interest rate limits and allowed them to charge any rate they chose. What are the effects of this usury law? More importantly, how are the effects of this law different between the



(a) No-Temptation Model (b) Temptation Model

Figure 4: Loan Interest Rate: Baseline and Under Usury Law

models with and without temptation? In order to answer these questions, I introduce a usury law with various interest rate limits to the two models.

Table 5 summarizes the results. First of all, when the usury law sets the upper bound of the interest rate at 15 percent, the effect is almost nil. In both model economies, not many agents borrow at such a high interest rate, so having an interest rate ceiling of 15 percent does not affect the behavior of agents. However, with an interest rate ceiling of 10 percent, the amount of loans, and thus the number of defaults, drops, in both the no-temptation and temptation models. The number of defaults goes down to 0.78 percent in the no-temptation model, while it drops to 0.73 percent in the temptation model. This drop happens as a result of tightened credit, which is apparent from the decline in the debt-to-income ratio, which falls from 9.0 percent to 3.0 percent in the no-temptation model and from 9.0 percent to 4.4 percent in the temptation model. The average loan rate drops slightly in both models, reflecting the decline in risky loans.

The tightening credit can be seen in Figure 4, which compares the loan rate schedules between the baseline model economy and the alternative economy with the interest rate ceiling of 10 percent. Panel (a) shows the loan rate schedules in the no-temptation model, while panel (b) is for the temptation model. There are two significant differences in each panel. First, because of the interest rate ceiling, a large amount of loans become unavailable; it is simply too risky to offer such large loans. In Figure 4, the interest rate schedule under the usury law disappears at around the loan size of 2.5 for the no-temptation model and at around 3.0 in the temptation model. This is because loans of such a large amount require an interest rate of above 10 percent to be profitable, which violates the usury law. Second, because of the tightening credit, agents default with smaller loan amounts, which pushes the interest rate schedule upward in general and makes borrowing more costly even for a loan. The interest rate schedule shifts up significantly more in the no-temptation model, which results in more tightening, as seen in Table 5.

Naturally, agents suffer from the tightening borrowing constraint. In the model without temptation, agents in the economy with the usury law's 10-percent ceiling are worse off,

Table 6: Comparison of Effects of Different η[1]

	% Default	% in Debt	D/Y %	Charge-off %	Avg r %	Welfare %
No-Temptation Model						
$\eta = 0.00$	1.21	22.8	2.8	20.9	11.1	-0.89
$\eta = 0.26$ (baseline)	0.84	35.1	9.0	4.9	9.9	–
$\eta = 0.85$ (optimal)	0.04	52.4	46.2	0.1	8.1	$+4.41$
Temptation Model ($\beta = 0.70$)						
$\eta = 0.00$ (optimal)	1.20	25.9	2.9	17.5	11.9	$+0.39$
$\eta = 0.31$ (baseline)	0.84	29.8	9.0	4.4	10.1	–
$\eta = 0.85$	0.02	38.2	24.9	0.2	8.1	$+0.11$

[1] The six columns show proportion defaulting, proportion in debt, debt-to-income ratio, charge-off rate, average borrowing rate, and welfare gain from policy reform, represented as the rate of permanent increase in consumption.

by as much as 2.9 percent of lifetime consumption. On the other hand, the welfare loss is not as large in the temptation model. This is because a tightening borrowing constraint has the additional positive effect of providing agents with a stronger commitment against overborrowing.

6.5 Optimal Degree of Default Punishment

Previous experiments show that even when the macroeconomic implications of policy changes are similar between the models with and without temptation, welfare implications could be very different. This implies that the optimal bankruptcy law could be greatly different, depending on which model is employed. I investigate this issue by comparing the optimal garnishment rate, η, between the two models. I use the word *optimal* in a very specific manner, in the sense that I change η without changing other elements of the bankruptcy law and I employ the ex-ante expected utility as the welfare measure. I leave the problem of designing the optimal bankruptcy law in a less restricted policy space for future research. Besides, the general equilibrium effect is not considered here.[15] However, as I will show, the contrasting optimal garnishment rate between the two models is worth pointing out.

Table 6 compares the calibrated baseline model with alternative economies with different values of the garnishment rate (η), for both the model without temptation (upper panel) and the model with temptation (lower panel). For each model, the baseline economy with the calibrated η, and the economies with the boundary values of η, are presented. The upper bound of η is 0.85 in both models, because of the fixed cost of filing, ξ; with $\eta > 0.85$, the budget set for a filing agent with the lowest income shock becomes empty. The lower bound is $\eta = 0$.

For the no-temptation model, the optimal level of the garnishment rate turns out to be its

[15] Nakajima (2012) considers the general equilibrium effect in a similar experiment. As expected, the general equilibrium effect lowers (increases) the welfare when the borrowing constraint is relaxed (tightened), due to capital decumulation (accumulation).

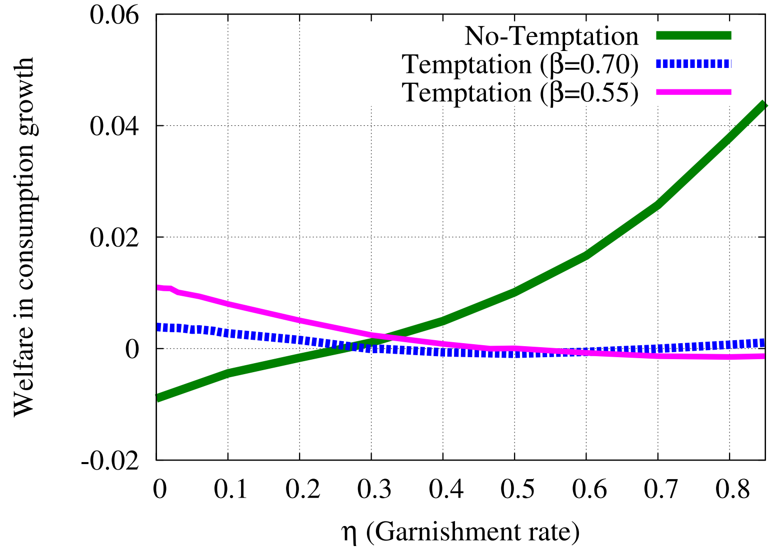


Figure 5: Welfare Effects of Different Garnishment Rates

upper bound, $\eta = 0.85$. Basically, in the model without temptation, the harshest punishment with the highest η gives agents the strongest commitment to repaying, which reduces both the probability of defaulting and the risk premium for unsecured loans, thus benefiting the borrowers. With the optimal level of η, the number of defaults declines significantly (0.84 percent to 0.04 percent per year), while the total amount of debt increases substantially (9 percent to 46 percent of aggregate income). Consistent with the low default rate, the charge-off rate is significantly lower than the baseline level (0.1 percent as compared with 4.9 percent), and the average loan interest rate is close to the risk-free rate of 8 percent (the saving rate of 4 percent plus the loan cost of 4 percent).

In the temptation model, the optimal rate of income garnishment turns out to be the lower bound, i.e., $\eta = 0$. At the optimal garnishment rate of zero, there are more defaults than in the baseline model (1.2 percent as compared with the baseline level of 0.84 percent), while the outstanding balance of debt is smaller (2.9 percent of income as compared with 9 percent). Since there are more defaults, the charge-off rate increases (from 4.4 to 17.5 percent) and the average interest rate rises, too (from 10.1 percent to 11.9 percent). It is interesting to note that while the economy with the optimal degree of punishment is associated with a large amount of consumer loans in the standard model without temptation, the model economy with the optimal degree of punishment is associated with a small amount of consumer credit in the temptation model.

Figure 5 shows how welfare changes from the baseline value of zero when the income garnishment rate upon default, η, is changed to various values. The figure contains the welfare changes of the no-temptation model as well as the temptation models with $\beta = 0.70$ and $\beta = 0.55$. Two points are worth mentioning. First, the size of the welfare effect is very different between the models with and without temptation. For the no-temptation model, the welfare effect at the optimal level of $\eta = 0.85$ is as high as 4.4 percent of flow consumption, while the welfare effect associated with the optimal $\eta = 0.0$ is only 0.39 percent in the temptation model. Second, the welfare is increasing with η in the no-temptation model, while it is decreasing in the large part of the space of η in the temptation model.

Why are there such differences? In the absence of expenditure shocks that might make the budget set empty and make consumption negative without defaulting, it is easy to see that agents are better off if they can commit to repaying their loans. If the agents have perfect commitment, they never default, the loan interest rate goes down to the risk-free level regardless of loan size, and the endogenous borrowing limit becomes loosened to the natural borrowing limit. This channel is still dominant with the model without temptation, in which the optimal rate of income garnishment is at the highest feasible level (85 percent). However, as discussed in other experiments, the welfare gain from stronger commitment might be overwhelmed by the other type of welfare effect in the temptation model: namely, loss of a commitment against overborrowing and the consequent welfare loss from overborrowing. Figure 5 implies that the welfare gain from a stronger commitment to repaying is almost canceled out by the welfare loss from a weaker commitment against overborrowing in the temptation model.

Worth emphasizing is that as in the previous experiments, the macroeconomic effects of changing η are similar between the two models, as shown in Table 6. However, since the tempted agents suffer from the extra channel of overborrowing, the welfare effects, and thus the optimal level of η, are significantly different between the two models. In this sense, the results in this paper support the view of White (2007) that it is necessary to use the model with temptation for the optimal design of the bankruptcy law, if the temptation model is the *right* model, although it is difficult to tell the models with and without temptation apart based on how agents in the two models respond to changes in environment related to borrowing and defaulting. As shown in this paper, the two models are observationally similar in many dimensions regarding debt and default.

6.6 Robustness: Model with a Stronger Present Bias

Table 7 shows the results of all the experiments, comparing the no-temptation model, the temptation model with the baseline value of the temptation discount factor ($\beta = 0.70$), and the temptation model with a lower temptation discount factor ($\beta = 0.55$). $\beta = 0.55$ corresponds to the discount rate of 80 percent, which is twice as large as the discount rate (40 percent) associated with the baseline discount factor of $\beta = 0.70$. Next is a summary of the results based on $\beta = 0.55$. The basic message is that the results of the temptation model with the baseline temptation discount factor of $\beta = 0.70$ are strengthened if the temptation discount factor is further lowered to $\beta = 0.55$.

1. The welfare in the 1980 economy is significantly higher (12 percent) as compared with that in the 2000 (baseline) economy, as in the no-temptation model and the temptation model with $\beta = 0.70$. The reason is the same: The variances of income shocks are significantly smaller in the 1980 economy.

2. The model with $\beta = 0.55$ implies a smaller contribution of the cost of default to the increase in the number of defaults. The number of defaults increases to 0.43 percent (not shown in the table), from 0.25 percent, if the cost of default reverts back to the baseline value of zero. In the model with $\beta = 0.70$, the number of defaults goes up to

Table 7: Comparison of Temptation Models with Different β[1]

	% Default	% in Debt	D/Y %	Charge-off %	Avg r %	Welfare %
No-Temptation Model						
Baseline	0.84	35.1	9.0	4.9	9.9	–
1980 Economy	0.25	33.8	5.0	3.1	10.6	+13.5
BAPCPA	0.41	39.1	12.4	2.5	9.3	+0.02
Usury law: 10%	0.78	19.1	3.0	9.6	9.5	−2.90
Optimal $\eta = 0.85$	0.00	52.4	46.2	0.1	8.1	+4.41
Temptation Model ($\beta = 0.70$)						
Baseline	0.84	29.8	9.0	4.4	10.1	–
1980 Economy	0.25	37.9	5.0	2.7	10.5	+13.2
BAPCPA	0.44	31.5	10.7	2.8	9.7	−0.33
Usury law: 10%	0.73	25.8	4.4	6.2	9.6	−0.65
Optimal $\eta = 0.0$	1.15	26.8	3.5	14.1	11.4	+0.39
Temptation Model ($\beta = 0.55$)						
Baseline	0.84	22.6	9.0	4.1	11.1	–
1980 Economy	0.25	33.4	5.0	2.1	10.5	+12.0
BAPCPA	0.23	26.3	11.1	1.7	9.3	−0.22
Usury law: 10%	0.48	24.8	7.0	3.2	9.5	0.10
Optimal $\eta = 0.0$	1.21	23.2	2.6	16.1	13.2	+1.10

[1] The six columns show proportion defaulting, proportion in debt, debt-to-income ratio, charge-off rate, average borrowing rate, and welfare gain from policy reform, represented as the rate of permanent increase in consumption.

0.79 percent (Table 3). In the model with $\beta = 0.55$, the combination of a lower cost of default with the risk-adjusted interest rate contributes to the rise in the number of defaults.

3. The BAPCPA lowers the number of defaults to 0.23 percent.[16] This is even lower than the prediction of the model with $\beta = 0.70$ (0.44 percent). The welfare effect of introducing the BAPCPA is negative (−0.22 percent), as it is in the model with $\beta = 0.70$ (−0.33 percent). The negative effect is mainly due to the higher cost of defaulting, as it is in the temptation model with $\beta = 0.70$.

4. A tight usury law with the interest rate ceiling of 10 percent reduces the number of defaults (0.48 percent) more than in the model with $\beta = 0.70$ (0.73 percent), while the decline in the debt (debt-to-income ratio of 7.0 percent) is smaller than it is in the model with $\beta = 0.70$ (4.4 percent). The welfare effect is now positive, at +0.1 percent, while it was a small negative in the temptation model with $\beta = 0.70$ (−0.92 percent)

[16] Since a higher cost of default makes consumption negative for defaulters with very low labor income in the model with $\beta = 0.55$, I set a low consumption floor for defaulters. This low consumption floor does not bind in any other experiments with $\beta = 0.55$.

and a large negative in the no-temptation model (−2.9 percent).

5. The optimal income garnishment rate is $\eta = 0.0$, which is the same as it is in the temptation economy with $\beta = 0.70$. As shown in Figure 5, the welfare gain from lowering η is even higher than it is in the temptation model with $\beta = 0.70$. Since the present bias is even stronger in the temptation model with $\beta = 0.55$, the welfare gain from a low η and thus a stronger commitment against overborrowing is larger. The welfare gain from implementing the optimal $\eta = 0.0$ is 1.1 percent of flow consumption, as compared with 0.39 percent in the case of the temptation model with the baseline $\beta = 0.70$.

7 Conclusion

I extend a life-cycle model with equilibrium default by introducing preferences featuring temptation and self-control and revisit four important issues regarding bankruptcy: (1) the causes of the observed increase in debt and bankruptcy filings since the early 1980s, (2) macroeconomics effects of the 2005 bankruptcy reform, (3) welfare implications of the reform, and (4) the optimal design of the bankruptcy law. I find that the temptation model provides quantitatively similar answers to (1) and (2), which are the positive questions, but the model provides a contrasting answer to (3) and (4), because of the overborrowing that agents with temptation suffer when the commitment against overborrowing is weakened. Specifically, the 2005 bankruptcy reform has an overall negative welfare effect according to the temptation model, while the effect is positive in the standard no-temptation model. As for the optimal default punishment, while the welfare of agents without temptation is maximized when punishment for defaulting is severe, and therefore provides a strong commitment to repaying and thus a low risk premium to borrowers, the welfare of agents with temptation is maximized when a weak punishment leads to a tight borrowing constraint, which basically provides them with a stronger commitment against overborrowing.

One interesting path for future research would be to introduce preferences with temptation and self-control into the model of sovereign default (e.g., Arellano (2008)). The standard model of sovereign default has difficulty in generating both a large amount of debt and an observed high frequency of defaults. By extending the model of sovereign default with temptation preferences, it might be possible to overcome this problem.

A Appendix

The appendix includes details about calibration (A.1) and computation (A.2), as well as additional figures showing U.S. data (A.3).

A.1 Calibration Appendix

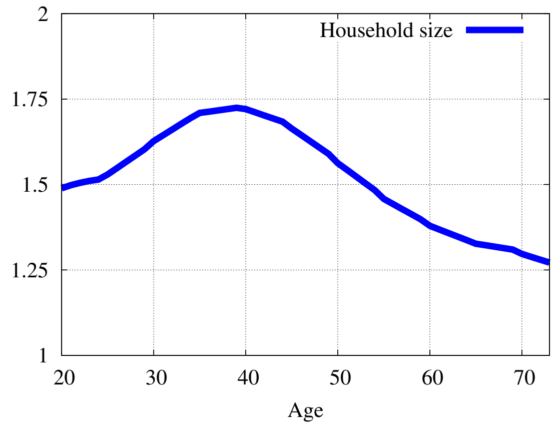


Figure 6: Household Size in Family Equivalence Scale

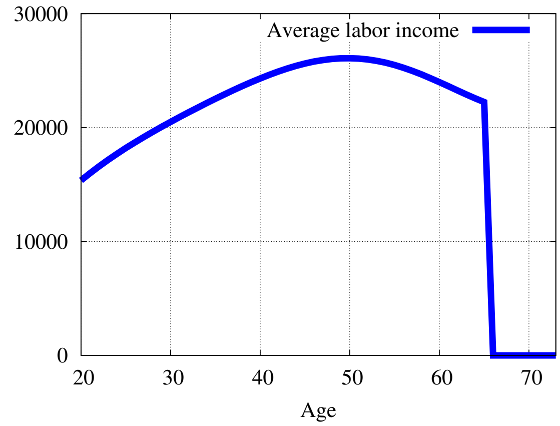


Figure 7: Average Life-Cycle Profile of Labor Productivity

A.2 Computational Appendix

I describe below the computational algorithm to solve the steady-state equilibrium of the model with temptation. The solution method for the standard model without temptation can be obtained with a straightforward modification.

Algorithm 1 (computation algorithm for solving steady-state equilibrium)

1. *Obtain the optimal value function $V(i, h, p, t, x, a)$ and the optimal decision rules $g_h(i, h, p, t, x, a)$ and $g_a(i, h, p, t, x, a)$ by solving the optimization problem backwards.*

   (a) *Start from the problem of age-I agents.*

   (b) *If $i = I$, set $V(i + 1, h, p, t, x, a) = 0$ for all (h, p, t, x, a). In the case $i < I$, $V(i + 1, h, p, t, x, a)$ is already obtained in the previous step.*

   (c) *If $i = I$, set $q(i, h, p, t, x, a') = 0$ for all (h, p, t, x, a). If $i < I$, $q(i, h, p, t, x, a')$ is already obtained in the previous step.*

   (d) *The temptation problem is solved first. In case of $h = 0$ (clean credit history), using the discount factor $d = \beta\delta$, and given $V(i+1, h, p, t, x, a)$ and $q(i, h, p, t, x, a')$, value conditional on non-defaulting and defaulting are obtained from Bellman equations (4) and (7). The optimal default decision $g_h(i, h, p, t, x, a)$ is characterized by equation (3). The optimal saving decision $g_a(i, h, p, t, x, a)$ is the one conditional on not defaulting if $g_h(i, h, p, t, x, a) = 0$ and is zero if $g_h(i, h, p, t, x, a) = 1$. The optimal value of the temptation problem $V^*(i, 0, p, t, x, a; \beta\delta)$ is obtained.*

(e) *In case of $h = 1$ (bad credit history), using the discount factor $\beta\delta$ and given $V(i+1, h, p, t, x, a)$ and $q(i, h, p, t, x, a')$, the optimal default decision $g_h(i, h, p, t, x, a)$ and the optimal saving decision $g_a(i, h, p, t, x, a)$ are obtained from equations (9) and (7). Notice that there is no optimal default decision because only involuntary default is allowed. The optimal value of the temptation problem $V^*(i, 1, p, t, x, a; \beta\delta)$ is obtained.*

(f) *Solving the self-control problem. In general, this step requires solving equation (11). However, this step becomes trivial because, by assumption, an agent completely succumbs to the temptation. Formally, this step only requires updating the value function using equation (12).*

(g) *Once the optimal default decision rule for age-i agents is obtained, the loan price for age-$i-1$, $q(i-1, h, p, t, x, a')$, can be computed using equation (14).*

(h) *If $i > 1$, go back to step (b) and solve the problem of age-$i-1$ agents. If $i = 1$ (initial age), this step is over.*

2. *Using the obtained optimal decision rules $g_h(i, h, p, t, x, a)$ and $g_a(i, h, p, t, x, a)$, simulate the model forward, starting from the type distribution of age-1 agents.*

   (a) *Set the type distribution for the newborns, which is exogenously given. In particular, all newborns have $i = 1$ and $a = 0$. The initial distribution of p and the distribution of t and x are also exogenously given.*

   (b) *Update the type distribution using the stochastic process for (p, t, x) and the optimal decision rules $g_h(i, h, p, t, x, a)$ and $g_a(i, h, p, t, x, a)$.*

   (c) *Keep updating until age I (last age).*

3. *Once the type distribution of agents is obtained, aggregate data can be computed by aggregating up individual data.*

A.3 Data Appendix

This appendix contains figures representing the number of bankruptcies, the debt-to-income ratio, the charge-off rate, and the average interest rate. Figure 8 replicates Figure 1. The number of bankruptcies is computed by dividing the number of total consumer bankruptcy filings and the Chapter 7 bankruptcy filings by the number of households in respective years. The data on bankruptcy filings are obtained from the U.S. Courts. The total number of households is from the U.S. Bureau of the Census. The number of bankruptcy filings can be considered as the upper bound because multiple persons in a single household could file for a bankruptcy separately. The debt-to-income ratio is computed by dividing the balance of the revolving credit by personal income. The former is constructed by the Federal Reserve Board (FRB, G.19). The revolving credit differs from unsecured credit in the sense that the revolving credit does not capture nonauto nonrevolving credit. However, after constructing the corrected measure of unsecured credit, Livshits et al. (2010) find that the gap between the two measures has been shrinking (see Figure 3 of their paper). Personal income is obtained from the Bureau of Economic Analysis. The charge-off rate for credit card loans is obtained

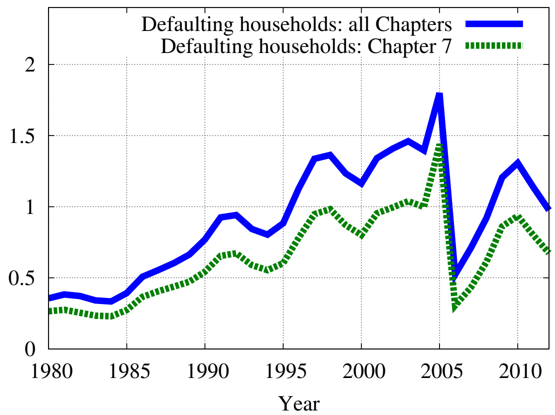


Figure 8: Proportion of Households Defaulting

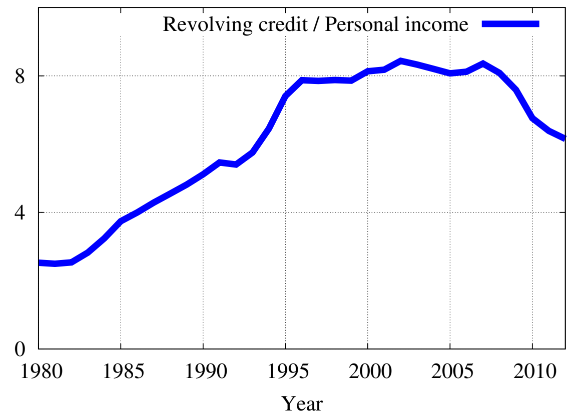


Figure 9: Revolving Debt Balance over Personal Income

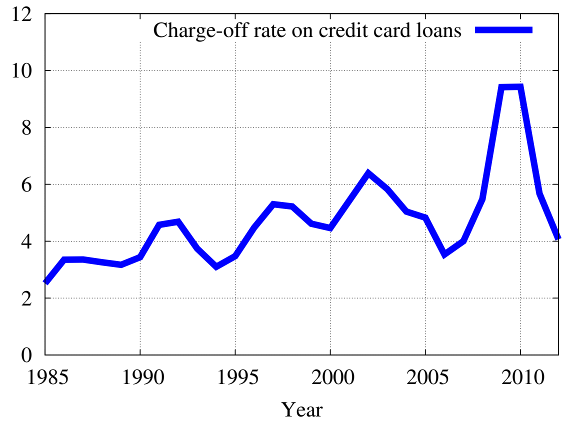


Figure 10: Charge-Off Rate for Credit Card Loans

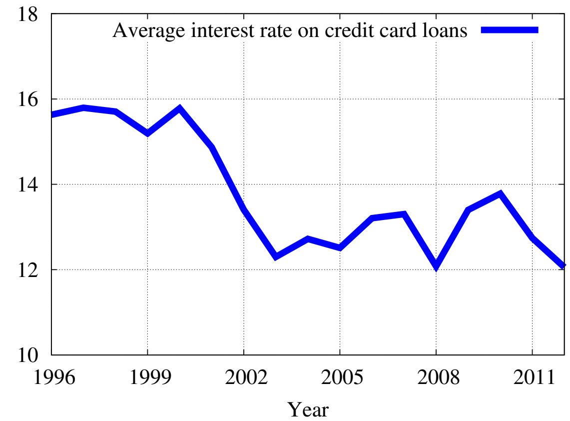


Figure 11: Average Interest Rate on Credit Card Loans

from the FRB (G.19). The average interest rate on credit card loans is also obtained from the FRB (G.19).

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