WORKING PAPERS
RESEARCH DEPARTMENT

WORKING PAPER NO. 13-9

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February 2013

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Worker Flows and Job Flows: A Quantitative Investigation*

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Abstract

This paper studies the quantitative properties of a multiple-worker firm matching model with on-the-job search where heterogeneous firms operate decreasing-returns-to-scale production technology. We focus on the model's ability to replicate the business cycle features of job flows, worker flows between employment and unemployment, and job-to-job transitions. The calibrated model successfully replicates (i) countercyclical worker flows between employment and unemployment, (ii) procyclical job-to-job transitions, and (iii) opposite movements of job creation and destruction rates over the business cycle. The cyclical properties of worker flows between employment and unemployment differ from those of job flows, partly because of the presence of job-to-job transitions. We also show, however, that job flows measured by net employment changes differ significantly from total worker separation and accession rates, because separations also occur at firms with positive net employment changes, and similarly firms that are shrinking on net may hire workers to partially offset attritions. The presence of job-to-job transitions is the key to producing these differences.

JEL Classification: E24, E32, J63, J64

Keywords: Job flows, worker flows, multiple-worker firm, and search and matching.

*For helpful comments, we thank seminar and conference participants at Bank of Japan, CEMFI, Cleveland Fed, Deutsche Bundesbank, European Central Bank, 2009 Far East and South Asia Econometric Society Meeting, Kansas City Fed/NYU-Stern Joint Workshop for Economists Working in Parallel, New York/Philadelphia Workshop on Quantitative Macroeconomics, 2009 North American Econometric Society Meeting, Simon Fraser University, 2009 Society for Computational Economics Meeting, University of British Columbia, University of Pennsylvania Wharton School, and ZEW conference on Recent Development in Macroeconomics. The views expressed in this paper are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Philadelphia or the Federal Reserve System. This paper is available free of charge at www.philadelphiafed.org/research-and-data/publications/working-papers/.

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1 Introduction

Worker flows and job flows behave differently over the business cycle. It is well known that worker flows between employment and unemployment are countercyclical. Separations into unemployment go up during recessions because the transition rate into unemployment increases; hires from unemployment also rise because the increases in the separation flow raise the unemployment pool rapidly, thus increasing hires as well.[1] The behavior of job flows is different. Job destruction is countercyclical, whereas job creation is procyclical.

In this paper, we investigate the sources of the differences in the cyclicality of worker flows and job flows using a search/matching model in which each firm operates a decreasing returns-to-scale production technology and is subject to aggregate as well as idiosyncratic productivity shocks. Each firm hires multiple workers, in contrast to the canonical matching model of Mortensen and Pissarides (1994) where a worker-firm match is taken to be the unit of analysis. One apparent reason that job flows behave differently from worker flows (between employment and unemployment) is the presence of job-to-job transitions.[2] Specifically, firms can hire workers not only from the unemployment pool but also from other employers, and similarly, workers can separate to other employers as well as into unemployment. The behavior of job flows is influenced by both types of worker flows. Mortensen (1994) addresses this issue in a single-worker matching model by adding on-the-job search (henceforth, OJS) into the Mortensen and Pissarides (1994) model. We introduce OJS and associated job-to-job transitions into the multiple-worker firm environment.

There are two reasons why we need a multiple-worker firm environment for our research interest. The first is that job flows are defined by establishment-level net employment changes within a certain period (say, a quarter). To be consistent with this measurement, we need a model with a meaningful notion of establishments that can hire many workers. Moreover, net employment changes over a quarterly period can be quite different from gross separations and hires that have occurred throughout the period. We address the time aggregation issue by solving the model at higher frequency and examining the cyclicality of job flow series constructed in the same way as in the actual data. The second reason is more substantive. That is, in order for us to distinguish between job flows and total hires/separations, we need a model with multiple-worker firms. Specifically, worker separations occur at establishments that are "creating" jobs (positive net employment changes), and similarly hires occur at establishments that are "destroying" jobs (negative net employment changes). In our model, when a firm aims to achieve the optimal employment level through laying off existing workers or hiring new workers, it explicitly takes into account the worker attritions through job-to-job transitions.[3]

[1] The job finding rate from unemployment drops significantly during recessions, thereby lowering hires, but this effect is dominated by the increase in the inflow.

[2] Flows between employment and the out-of-the-labor force are another reason, but we do not consider these flows in the present paper.

[3] Empirically speaking, net employment changes are indeed different from underlying hires and separations (see, for example, Burgress et al. (2000) and Davis et al. (2012)). But in a single-worker environment, one cannot distinguish between worker flows and job flows even after incorporating job-to-job transitions.

Our model is an extension of Cooper et al. (2007) and Elsby and Michaels (2013), who also consider the multiple-worker firm environment with random labor matching. Our extension is to incorporate OJS in a similar environment. In doing so, we follow Mortensen (2010), who analyzes the steady-state equilibrium of a multiple-worker firm model with OJS. We adopt his specifications about OJS with several minor changes and examine the quantitative properties under the presence of aggregate uncertainty. In our model, firms are subject to idiosyncratic and aggregate productivity shocks. Hiring workers is subject to search frictions and actively shedding workers (layoffs) requires the firm to pay a firing cost. The economic significance of job-to-job transitions in the model comes from the fact that, when workers leave the firm through job-to-job transitions, the firm avoids paying the firing cost. We abstract away from the employed workers' job search decision and simply assume that all employed workers participate in the job search with reduced intensity. Following Mortensen (2010), wages are determined by Stole and Zwiebel (1996) intra-firm bargaining that splits the marginal surplus from current-period output. We also assume that workers make their acceptance decision based on a comparison of the current wage and the offered wage. This is a shortcut necessary to solve the model. If instead the decision is based on the comparison of present discounted values (PDVs) of staying and moving, solving the model requires us to know the distributions of the PDVs of all firms as well as the vacancy posting firms. Even with our simplifying assumption, we need to keep track of the wage distribution of all workers and the wage offer distribution of the vacancy-posting firms. Moreover, when aggregate uncertainty is present, these distributions are time varying. The firm needs to know the information about the wage distribution because it tells the firm the acceptance probability of its wage offer (if the firm is hiring). The firm also needs to know (regardless of whether it is hiring or not) the wage offer distribution because it tells the firm how many workers will leave through job-to-job transitions for a given level of wage that the firm is currently paying to its employees. Note also that even without job-to-job transitions, the joint distribution over idiosyncratic productivity and employment is a state variable in the decreasing returns scale environment: the firm needs to forecast labor market tightness as a function of the aggregate states, which include the firm-type distribution. We solve for the dynamic stochastic equilibrium of this challenging environment by applying the standard tool developed for heterogeneous agent models (e.g., Krusell and Smith (1998)).

The calibrated model successfully matches key cyclical features of worker flows and transition rates. First, it matches the procyclicality of the job finding rate of unemployed workers (UE transition rate) and countercyclicality of the separation rate into unemployment (EU transition rate).[4] Second, the model also matches countercyclical worker flows between employment and unemployment. Third, the model also generates procyclical job-to-job transitions, which are consistent with the data by Fallick and Fleischman (2004).

The model is solved and calibrated at monthly frequency. However, as mentioned above, empirical measures of job flows are constructed from quarterly net employment changes. We construct job flows in the model using the same measurement procedure. Our model implies

[4]See, for example, Elsby et al. (2009), Fujita and Ramey (2009), and Shimer (2012) for empirical facts on the transition rates between employment and unemployment.

the procyclical job creation rate and countercyclical job destruction rate, with these two variables being strongly negatively correlated.[5]

Our key experiment, using the model, is to investigate how similarly or differently job creation and destruction rates behave compared to the total separation and accession rates.[6] Importantly, in our model, job flows are still different from gross separation and accession rates even at monthly frequency. On top of that, quarterly measurement of job flows introduces time aggregation effects. We find that, even when both worker flows and job flows are compared without time aggregation effects, the two sets of flows behave differently over the business cycle. Time aggregation causes additional differences. In particular, the difference in the cyclicality between the total separation rate and the quarterly job destruction rate in our model is large: the total separation rate is procyclical and its correlation with aggregate output is higher than 0.4. The reason for this procyclicality is that even though the EU transition rate is strongly countercyclical, the job-to-job transition rate is strongly procyclical, with the average volume of the latter being larger. The quarterly job destruction rate is countercyclical and its correlation with output is -0.38. The economic reason for the difference in the cyclicality is that job-to-job transitions (as a part of separations) occur not jut at firms that are "destroying" jobs but also at the firms that are "creating" jobs. But, by definition, the job destruction rate includes only the firms that reduced employment on net. Those establishments are more likely to be the ones that are using layoffs (EU separations). In other words, the share of layoffs out of total separations goes up as net employment growth declines. Accordingly, the contribution of the EU separation rate to the cyclical movement of the job destruction rate is stronger, which pushes its correlation with output into a negative direction. When time aggregation is taken into account, the negative correlation reaches a level consistent with the observed data. The discrepancy between the job creation rate and the accession rate arises in a situation where a firm loses its workers through job-to-job transitions and decides to offset the attritions only partially. Due to this effect, the job creation rate becomes less procyclical than the accession rate is. We also find that time aggregation further reduces the procyclicality of the job creation rate, but it remains weakly procyclical.

We also conduct several exercises that are useful to further demonstrate the importance of job-to-job transitions in our model. First, we show that presence of job-to-job transitions magnifies the business-cycle fluctuations of the EU separation rate. That is, when a firm needs to reduce employment, achieving it through job-to-job transitions is cheaper. In a recessionary period, lower aggregate productivity by itself raises aggregate layoffs, as in the standard model. However, an additional effect exists in a multiple-worker firm model with OJS: in a recession, the likelihood of needing to use layoffs further increases because reducing employment through job-to-job transitions becomes more difficult. The opposite is true during a boom. We also examine the quantitative properties of the model without OJS and show that the model fails to replicate the key cyclical features of job flows, although

[5]The procyclicality of the job creation rate is somewhat weaker in the model than in the data, but the overall cyclical patterns of job flows are consistent with the data.

[6]The total separation rate is simply a sum of the EU separation rate and the job-to-job transition rate. The accession rate is calculated as total hires normalized by aggregate employment.

worker flows and transition rates between employment and unemployment are in line with the data.

Let us now discuss where our paper stands in relation to the literature. As mentioned above, this paper is closely related to Cooper et al. (2007) and Elsby and Michaels (2013) who also analyze and quantitatively evaluate a multiple-worker firm matching model. The key difference between these papers and our paper is the presence of OJS in our model. In terms of modeling of OJS, we follow Mortensen (2010), who analyses the steady-state equilibrium of a similar setup. Wage bargaining in our model also follows Mortensen (2010), who adopts the bargaining framework developed by Stole and Zwiebel (1996) to his environment. There are several papers that use the same bargaining framework in the multiple-worker matching model (e.g., Smith (1999), Cahuc and Wasmer (2001), Krause and Lubik (2007), Cahuc et al. (2008), and Acemoglu and Hawkins (2011)). These papers consider a much simpler environment than ours, because in these papers job destruction is exogenous and OJS is not allowed.

There are several papers that study the directed search environment with decreasing returns to scale (e.g., Kaas and Kircher (2011) and Schaal (2012)). Kaas and Kircher (2011) analyze the environment without OJS and Schaal (2012) adds OJS to the model. However, Schaal (2012) focuses more on the recent Great Recession episode in the presence of the uncertainty shock. On the other hand, our paper follows more closely the traditional random matching environment with continuous wage renegotiation and looks more generally at the model's cyclical features, paying particular attention to worker flows and job flows.[7]

In terms of the research interest, Mortensen (1994) attempts to replicate worker flows and job flows simultaneously, as does our paper. He does so, however, in a single-worker firm matching model with OJS and thus faces several limitations that we have discussed above. Veracierto (2009) provides a synthesis of the different strands of the literature (in particular, Mortensen-Pissarides random-matching framework and Lucas and Prescott (1974) island framework) and discusses the cyclical properties of worker transition rates and job flows. However, his model does not allow for OJS and thus no job-to-job transitions exist in his model. Our analysis on the differences between total separation/accession rates and job flows is an important part of our paper that is distinct from his paper.[8]

This paper proceeds as follows. In the following section, we summarize the business cycle features of worker flows and job flows by looking at standard business cycle statistics. In Section 3, we present our model. In Section 4, we briefly discuss the solution algorithms to solve for the steady-state equilibrium and the dynamic stochastic equilibrium. Details of the algorithms are presented in the Appendix. Section 5 discusses the calibration strategy. Section 6 is the main section of the paper where we discuss the quantitative results in

[7]A recent work by Moscarini and Postel-Vinay (forthcoming) analyzes and solves the random-matching, wage-posting model under the presence of the aggregate shock, but worker transitions into unemployment are exogenous. In addition, their research interest is different from ours.

[8]Note, however, that his paper looks at broader statistics that we do not consider in our paper. For example, his model is a full-fledged RBC model with physical capital and risk aversion. He can therefore assess the broarder macroeconomic implications of this model. See also Veracierto (2007) who studies normative aspects of a similar environment without aggregate uncertainty.

detail. After discussing the overall results under the benchmark calibration, we also present additional experiments (including some sensitivity analysis) that shed more light on the importance of job-to-job transitions. Section 7 concludes the paper. We discuss some micro-level counterfactual properties of the model and potentially useful extensions to overcome those problems.

2 Cyclicality of Worker Flows and Job Flows

This section reviews the cyclical properties of worker flows and job flows. While one can find the cyclical properties of worker flows and job flows in literature, the two sets of data are usually discussed in isolation. We discuss them together and highlight the differences of their cyclicality. Let us first review the definitions of the series.

2.1 Measurement

Job flows. The job flow series are measured from the Business Employment Dynamics (BED) data, which are based on the administrative records of the Quarterly Census of Employment and Wages (QCEW).[9] The coverage of the QCEW is very broad, representing 98% of employment on nonfarm payrolls. The administrative records are linked across quarters to provide a longitudinal history for each establishment. The linkage process allows the tracking of net employment changes at the establishment level, which in turn allows calculating net employment gains at opening and expanding establishments and net employment losses at closing and contracting establishments. The measures of job flows were originally developed by Davis et al. (1996): job creation (destruction) is defined as the sum of net employment gains (losses) over all establishments that expand (contract) or start up (shut down) between the two sampling dates. Since we are interested in business cycle fluctuations of the series, we use the series that trace net employment changes over a quarterly period. Normalizing creation and destruction by aggregate employment yields rates of job creation and destruction, respectively.[10] In this paper, we use the term "job flows" to represent "rates" unless otherwise explicitly mentioned. The sample period of the job flow series starts at 1992Q3 and ends at 2011Q4.

Worker flows and transition rates. Worker flows between employment and unemployment can be constructed based on changes in the labor market status of workers. We use the Current Population Survey (CPS) for the construction of worker flows. The CPS asks whether the worker is employed and, if nonemployed, whether or not he/she is engaged in active job search activities (i.e., unemployed) over the preceding month. While the CPS is designed to provide a snapshot of the U.S. labor market for each month, one can use its

[9]The BED series are available at www.bls.gov/bdm/.

[10]More precisely, average employment between the beginning and the end of the quarter is used for normalization.

longitudinal component to obtain measures of worker flows. We use the flow series constructed by the BLS.[11] Worker flows between employment and unemployment come from the comparison of the labor market status at each monthly survey. To be specific, transition rates between employment and unemployment are, respectively, measured by:

$$\frac{EU_t}{E_{t-1}} \text{ and } \frac{UE_t}{U_{t-1}}, \tag{1}$$

where EU_t (UE_t) refers to the number of workers who switch their labor market status from "employed" ("unemployed") to "unemployed" ("employed") between month $t-1$ and t. EU_t and UE_t represent separations into unemployment and hires from unemployment, respectively. The definition in Equation (1) gives the EU transition rate and UE transition rate, respectively. The sample period for the BLS data is between Jan. 1990−Dec. 2011.[12]

We also consider job-to-job transitions. Measuring job-to-job transitions in the CPS became feasible at the CPS redesign that took place in 1994. Specifically, the dependent coding, which asks the individual if he/she is employed by the same employer as in the previous month, made it possible to measure job-to-job transitions. Fallick and Fleischman (2004) are the first to exploit this data structure to measure job-to-job transitions in the CPS.[13] Denoting the worker flow corresponding to those who are employed at different employers between $t-1$ and t by EE_t, we can write the job-to-job transition rate as:

$$\frac{EE_t}{E_{t-1}}. \tag{2}$$

The data are updated regularly and the sample period for our analysis is Jan. 1994−Dec. 2011. All monthly worker flows and transition rates are converted into quarterly series by time averaging.[14]

2.2 Business-Cycle Statistics

Unimportance of entry and exit. First, consider Figure 1 where we plot the time series of job flows. In the figure, we show not only the total rates of job creation and destruction but also their breakdowns into expansion, entry, contraction, and exit. The intention of this figure is to show unimportance of the extensive margins for the business-cycle fluctuations

[11] The data are available at www.bls.gov/cps/cps_flows.htm. Fujita and Ramey (2006) also construct worker flow series that are comparable to the BLS series. The cyclicality of the two data sets is very similar. See Fujita and Ramey (2006) for the data construction details and measurement issues in the CPS.

[12] In the BLS data, the flow that occurs from $t-1$ and t is dated at t. Due to that convention, the BLS flow data start at Feb. 1990.

[13] Moscarini and Thompson (2007) explore several measurement issues of CPS-based job-to-job transitions and correct some measurement issues that existed in Fallick and Fleischman (2004). While their adjustments alter the overall level of job-to-job transitions somewhat, the time-series behavior is not significantly affected. We thus use the readily available series by Fallick and Fleischman (2004).

[14] Our analysis omits the flows into and out of the out-of-the-labor-force state. This is an important dimension that our analysis is silent about.

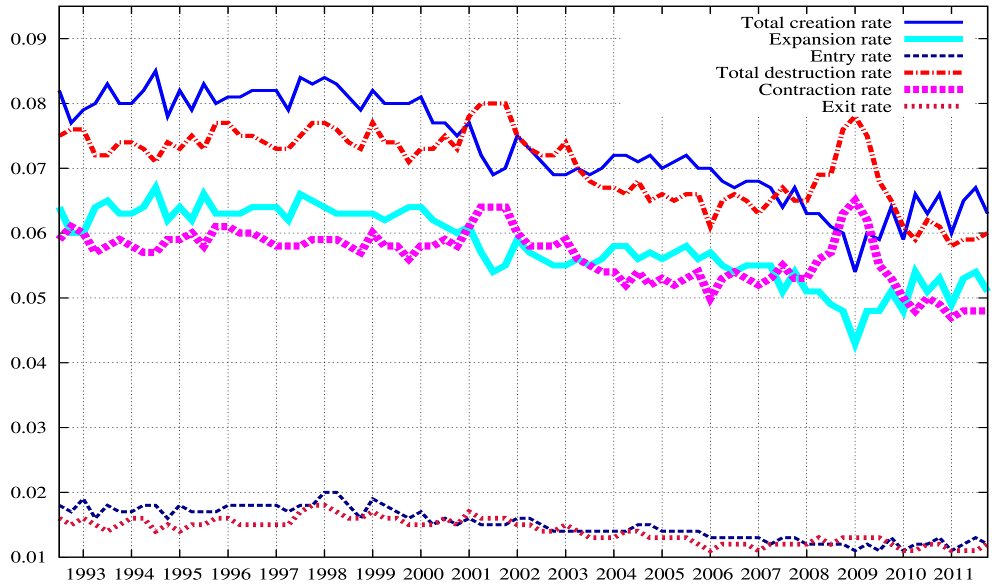

Figure 1: Job Creation and Destruction Rates

Notes: The data are taken from the BLS Business Employment Dynamics and cover the private business sector.

of job flows. According to the data, roughly 75% of total job flows come from expansion or contraction of the existing establishments at quarterly frequency. More important, cyclical fluctuations of job flows are mostly accounted for by expansion or contraction. For instance, the correlation between the total job creation (destruction) rate and the expansion (contraction) rate is higher than 0.95. It is important to recognize that these two facts do not imply the unimportance of entry and exit at lower frequency.[15] However, Figure 1 establishes our point that extensive margins are not important at the quarterly frequency.[16] We abstract away from the extensive margin in the model that we analyze below based on the quarterly data presented in Figure 1.

Cyclicality. Table 1 characterizes the cyclicality of worker flows and job flows using standard business-cycle statistics. The original series are logged and then detrended by using the HP filter with smoothing parameter of 1,600. As mentioned above, original worker flows and transition rates are monthly series. We render them quarterly by simple averaging, so that we can examine their cyclicality on an equal footing with job flow series. The real GDP series is used as a cyclical indicator to gauge each variable's volatility and cyclicality. We

[15]The frequency of the measurement is important because at quarterly frequency entrants become incumbents after a quarter, but the same entrants measured at annual frequency become incumbents only after one year. Thus, the share of job flows accounted for entrants and exits becomes larger when measured at lower frequency. They also become more important cyclicality wise.

[16]Schaal (2012)'s model includes entry and exit, but the model significantly overpredicts the fluctuations of these variables, given that those variables do not vary much in the observed data, as seen in the figure.

Table 1: Business Cycle Statistics for Worker Flows and Job Flows

	Standard Deviation	Relative Standard Deviation	Correlation With Output
Worker flows			
E to U	0.065	5.496	−0.800
E to E	0.060	4.950	0.744
U to E	0.046	3.824	−0.687
Transition rates			
EU transition rate	0.071	5.966	−0.840
EE transition rate	0.056	4.620	0.698
UE transition rate	0.080	6.731	0.860
Job flows			
Creation rate	0.036	3.060	0.447
Destruction rate	0.045	3.838	−0.450
Stocks			
Unemployment rate	0.116	9.712	−0.889
Vacancies	0.121	10.153	0.862

Notes: First column: standard deviation of logged and HP filtered series with smoothing parameter of 1,600. Middle column: standard deviation of each variable relative to that of real GDP. Sample periods: worker flows and transition rates between unemployment and employment: 1990Q1–2011Q4; job-to-job flow and transition rate: 1994Q1–2011Q4; job flows: 1992Q3–2011Q4. Unemployment and vacancies: 1990Q1–2011Q4. Worker flows and transition rates between employment and unemployment are calculated from the the BLS labor flow series available at www.bls.gov/cps/cps_flows.htm. Job-to-job worker flows and the transition rate are calculated by Fallick and Fleischman (2004). The series are downloaded from www.federalreserve.gov/pubs/feds/2004/200434/200434abs.html. Worker flows and transition rates are measured at monthly frequency but converted into quarterly series by time averaging. The sample period for real GDP is adjusted to match the sample period of each variable.

can summarize the characteristics of the labor market flows as follows.

- The EU transition (separation) rate is countercyclical, while UE (job finding) and EE (job-to-job) transition rates are procyclical.
- The UE transition rate is somewhat more volatile than the EU transition rate.[17]
- The EU flow is somewhat more volatile than the other two flows (UE and job-to-job flows).
- The job destruction rate is countercylical and the job creation rate is procyclical. But the correlations are weaker in general.
- The job destruction rate is somewhat more volatile than the job creation rate.

[17]Shimer (2012) and Hall (2005) argue that the separation rate into unemployment is roughly constant over the business cycle. Fujita and Ramey (2006, 2009), Fujita (2011), Elsby et al. (2009), and Yashiv (2007) argue otherwise.

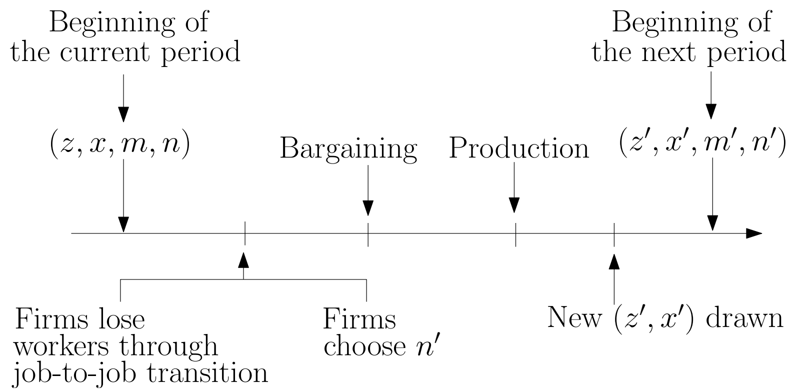

Figure 2: Timing of events

- Worker flows are more volatile than job flows.

Table 1 also shows volatilities of the unemployment rate and vacancies. As is well known in the literature, these two variables are quite volatile when compared with the volatility of labor productivity. The same is true with respect to output volatility. Lastly, one can also see that the model replicates a well-known fact about the cyclicality of unemployment and vacancies, i.e., the Beveridge curve. This is indicated by each variable's correlation with output.

3 Model

Our model is an extension of the models developed by Cooper et al. (2007), Elsby and Michaels (2013), and Mortensen (2010). Time is discrete. There are two types of agents: firms and workers. Both are infinitely lived. The total measure of firms is normalized to one. The total measure of workers is denoted by L.

3.1 Timing

The timing of events is summarized in Figure 2. At the beginning of the period, a firm's idiosyncratic states are characterized by (x, n), where x represents idiosyncratic productivity and n represents the number of its workers. In addition, there is aggregate uncertainty in the economy in the form of a shock to aggregate productivity z. As will be clear later, each firm's decision is also influenced by the economy-wide joint distribution of x and n and is written as $m(x, n)$. We summarize the aggregate states by $\mathbf{s} = \{z, m\}$. The stochastic processes for z and x are, respectively, denoted by $G_z(z'|z)$ and $G_x(x'|x)$. Since we formulate the model recursively, we drop time subscripts from all variables and follow the convention that the primed variable denotes the variable at the beginning of the next period. Note, however, as indicated in Figure 2, the firm enters into the current period with the employment level n and produces with n', after labor turnover is completed in the current period. It then starts the next period with n'.

After the realization of productivities, firms make the separation or hiring decision. The hiring decision is subject to a search friction, which is discussed below. Hires include those from other firms (job-to-job transitions) as well as those from unemployment. Similarly, separations include worker flows to other hiring firms and to unemployment. As described in Figure 2, worker turnover occurs within a period. For example, vacancies posted at the beginning of the period after the realizations of productivities can be filled in the same period before production. After all worker flows are completed, wage negotiation between the employer and employees takes place and then the firm produces. These timing assumptions greatly simplify our analysis.

3.2 Wage Bargaining and Production

We first describe wage bargaining and production. The following decreasing-returns-to-scale production technology is available to all firms:

$$y = zxn'^{\alpha}. \tag{3}$$

Again, n' corresponds to the number of workers who engage in production at the firm, whose beginning-of-the-period employment level is n. For wage bargaining, we follow Mortensen (2010) who applies the framework developed by Stole and Zwiebel (1996) to an environment similar to ours. As in Mortensen (2010), the possibility of a long-term contract between the firm and workers is excluded. Specifically, the bargaining outcome is such that the current-period marginal surplus is split between each marginal worker and the employer, according to each party's bargaining power. See Stole and Zwiebel (1996) for the explicit strategic bilateral bargaining game between the employer and the marginal worker. The default position of the firm is no production and that of the worker is the flow value of home production b. The outcome is characterized by the following rule:

$$\eta\left[\alpha zxn'^{\alpha-1} - w - \frac{\partial w}{\partial n'}\right] = (1-\eta)\Big[w - b\Big],$$

where η is the bargaining power of the employee. The three terms in the square brackets on the left-hand side give the marginal surplus that the employer obtains by having one more worker. The third term captures the firm's incentive of "overemployment" as pointed out by Stole and Zwiebel (1996), which comes from the fact that employing more workers reduces the wage. The solution to the above differential equation is given by:

$$w = (1-\eta)b + \left(\frac{\eta\alpha}{1-\eta+\eta\alpha}\right)zxn'^{\alpha-1}. \tag{4}$$

Note that having this simple expression for wage, which depends only on n' (as well as exogenous state variables), greatly simplifies our quantitative analysis, particularly because we allow for on-the-job search.[18]

[18] Elsby and Michaels (2013) derive the closed-form expression for wage when the employer and each marginal worker bargain over the present discounted value of the marginal surplus. Extending their analysis to the model with on-the-job search is difficult.

3.3 Search and Matching

Due to the search friction, only a fraction of job openings are filled every period. There is a flow vacancy posting cost, as in the standard model. However, we assume that the marginal cost of posting a vacancy is increasing in the number of vacancies posted at the firm level. We also introduce the firing cost, which applies when the firm decides to shed workers above and beyond job-to-job transitions. The specific formulation of these features is described below.

The meeting technology takes the following Cobb-Douglas form:

$$M = \mu S^{\psi} V^{1-\psi},$$

where S is the efficiency-weighted number of job seekers, V is the aggregate number of job openings, and μ is a scaling parameter. We normalize search efficiency of each unemployed worker at 1 and assume that each on-the-job seeker searches for a job at a reduced efficiency of $\gamma \in [0, 1]$. This specification allows us to abstract away from the search decision of the employed workers, while giving us the flexibility of matching the volume of job-to-job transitions in our quantitative exercise.[19]

Recall that L is the fixed measure of the labor force. Thus S can be written as:

$$S = \gamma L(1 - U) + LU,$$

where U is the unemployment rate. Given the meeting technology, the contact probability for each vacancy posted $q(\theta)$ is:

$$q(\theta) = \frac{\mu S^{\psi} V^{1-\psi}}{V} = \mu\theta^{-\psi},$$

where $\theta = \frac{V}{S}$ is labor market tightness in this economy. Similarly for workers, the contact probability per unit of search $f(\theta)$ is written as:

$$f(\theta) = \frac{\mu S^{\psi} V^{1-\psi}}{S} = \mu\theta^{1-\psi}.$$

While unemployed workers meet with a potential employer with this probability each period, the contact probability of employed workers, denoted by $f_e(\theta)$, is reduced by a factor of γ, as in:

$$f_e(\theta) = \gamma f(\theta).$$

We consider an equilibrium where the worker's job acceptance decision is based on the offered wage. This is another simplification necessary for us to proceed. We will come back to the discussion on this simplification later in a few paragraphs. Let the CDF of wages of all employed workers be $H(w)$ with $H(\underline{w}) = 0$ and $H(\overline{w}) = 1$. Next, let $K(w)$ be the CDF

[19] Mortensen (2010)'s focus is on theoretical properties of the model. He therefore assumes that employed workers search at the same intensity as unemployed workers do, which is a natural outcome in his environment with no search cost.

of wages offered by hiring (vacancy posting) firms with $K(\underline{w}) = 0$ and $K(\overline{w}) = 1$. Given these objects, we can express the vacancy filling rate (also known as the vacancy yield) of a firm offering the wage w as:

$$h(w, \mathbf{s}) = q(\theta)\frac{U + \gamma(1-U)H(w)}{U + (1-U)\gamma},$$

and the quit rate from a firm paying the wage w as:

$$k(w, \mathbf{s}) = f_e(\theta)(1 - K(w)).$$

When a firm is posting vacancies, each job opening receives an application at rate $q(\theta)$. If the worker is unemployed, the acceptation rate is 1, and if the worker is employed, she accepts the wage offer with probability $H(w)$, the probability that the worker is currently employed at a firm paying the wage less than w. The quit rate for a firm can be expressed as a product of the contact probability $f_e(\theta)$ and the probability that the wage offered by the poaching firm is higher than the wage paid by the current employer w.

As mentioned, we adopt the assumption that the worker's acceptance decision is based on the current-period wage. In principle, the decision should be based on the values. However, as is clear from the discussion in the previous paragraph, incorporating this feature into our analysis entails solving for the distributions of these values. Moreover, in the presence of aggregate uncertainty, these objects are time varying. Our simplifying assumption allows us to sidestep this complication. Note also that even in our simplified economy, the wage and wage offer distributions are endogenous objects and time varying when the aggregate shock is present.

3.4 Optimal Employment Decision

The firm makes hiring and separation decisions by maximizing the discounted present value of flow profits:

$$\begin{aligned}\Pi(x, n, \mathbf{s}) = \max_{n' \geq 0} \Bigg\{ & zxn'^{\alpha} - wn' - \max\left[\kappa_0\left(v + \frac{\kappa_1}{2}v^2\right), 0\right] \\ & - \tau \max\left[(1 - k(w, \mathbf{s}))n - n', 0\right] + \beta \int\int \Pi(x', n', \mathbf{s}')dG_x(x'|x)dG_z(z'|z)\Bigg\}, \end{aligned} \tag{5}$$

where v represents the number of vacancies posted by the firm and is written as:

$$v = \frac{n' - (1 - k(w, \mathbf{s}))n}{h(w, \mathbf{s})}.$$

The present discounted value of profits $\Pi(.)$ is a function of the four state variables, including the type distribution $m(x, n)$. The first two terms on the right-hand side correspond to the flow profits to the firm. The wage function takes the form derived earlier, Equation (4). The third term captures the fact that expanding employment incurs the cost which is convex in the number of vacancies posted. Specifically, we assume that this cost is quadratic in the number of vacancies with $\kappa_0 > 0$ and $\kappa_1 > 0$.[20]

Note that the firm loses $k(w, \mathbf{s})n$ workers through job-to-job transitions. Paying higher wage has the effect of saving the hiring cost since it reduces attritions. Observe also that the firm posts $1/h(w, \mathbf{s})$ vacancies per hire, given that each vacancy is filled with probability $h(w, \mathbf{s})$. Thus paying higher wage has another effect that the offered wage is more likely to be accepted by on-the-job seekers at other firms. The last term of (5) gives the expected value of the next-period PDV of profits, discounted by β. The fourth term captures the firing cost that is linear in the reduction of the number of employees after quits. The quit rate $k(w, \mathbf{s})$ also affects the layoff decision because separations through quits allow firms to avoid paying the firing cost. Moreover, in reducing employment to n', the firm knows that it has an offsetting upward effect on wage, thus reducing the quit rate $k(w, \mathbf{s})$. We will come back to the discussion on this effect.

The optimal employment decision of the firm is characterized by an (s, S) rule, with the inaction region $(\underline{n}^*, \overline{n}^*)$ characterized by the following first-order conditions:

$$\alpha z x \underline{n}^{*\alpha-1} - w - w_n \underline{n}^* - \frac{\kappa_0(1+\kappa_1 v)}{h(w,\mathbf{s})}\left[1 + w_n\Big(k_w(w,\mathbf{s})n - v h_w(w,\mathbf{s})\Big)\right] + \beta \int\int \Pi_n(x', \underline{n}^*, \mathbf{s}') dG_x(x'|x) dG_z(z'|z) = 0, \quad (6)$$

$$\alpha z x \overline{n}^{*\alpha-1} - w - w_n \overline{n}^* + \tau[1 + k_w(w,\mathbf{s}) w_n n] + \beta \int\int \Pi_n(x', \overline{n}^*, \mathbf{s}') dG_x(x'|x) dG_z(z'|z) = 0. \quad (7)$$

Note that under the assumptions of no job-to-job transitions ($\gamma = 0$), no firing cost ($\tau = 0$), and the linear vacancy posting cost ($\kappa_1 = 0$), these two equations reduce to those presented in Elsby and Michaels (2013), save for some minor differences.

Observe that in (6) and (7), wage and wage offer distributions, $K(w, \mathbf{s})$ and $H(w, \mathbf{s})$ are endogenous objects that are moving along with the aggregate shock. The first-order conditions above imply that firms need to know the derivatives of these distributions with respect to their employment, denoted by $k_w(w, \mathbf{s})$ and $h_w(w, \mathbf{s})$. In our numerical exercises, we assume that these derivatives are zero, and avoid significant complications that arise due to these terms. Intuitively speaking, setting these derivatives to zero means that the firm's employment decision ignores the effects discussed above that higher wage reduces the quit rate and also raises the acceptance rate of job-to-job movers. Making these assumptions

[20] The presence of the linear portion in the hiring cost implies the nonconvexity of the cost: the marginal cost jumps to a positive value when the firm decides to hire. In the absence of the quadratic term, it is the same as the standard linear cost model, where the marginal cost is constant at the positive value κ_0.

allows us to write (6) and (7) as follows:

$$\alpha z x \underline{n}^{*\alpha-1} - w - w_n \underline{n}^* - \frac{\kappa_0(1+\kappa_1 v)}{h(w,\mathbf{s})} + \beta \int\int \Pi_n(x', \underline{n}^*, \mathbf{s}') dG_x(x'|x) dG_z(z'|z) = 0, \quad (8)$$

$$\alpha z x \overline{n}^{*\alpha-1} - w - w_n \overline{n}^* + \tau + \beta \int\int \Pi_n(x', \overline{n}^*, \mathbf{s}') dG_x(x'|x) dG_z(z'|z) = 0. \quad (9)$$

The envelope condition implies that the derivative of the profit function is written as:

$$\Pi_n(x,n,\mathbf{s}) = \begin{cases} \frac{1}{h(w,\mathbf{s})}\kappa_0(1+\kappa_1 v)(1-k(w,\mathbf{s})) & \text{if } \tilde{n} < \underline{n}^*, \\ (1-k(w,\mathbf{s}))[\alpha z x \tilde{n}^{\alpha-1} - w - w_n \tilde{n} + \beta \int\int \Pi_n(x', \tilde{n}, \mathbf{s}') dG_x dG_z] & \text{if } \tilde{n} \in [\underline{n}^*, \overline{n}^*], \\ -\tau(1-k(w,\mathbf{s})) & \text{if } \tilde{n} > \overline{n}^*, \end{cases} \quad (10)$$

where $\tilde{n} = (1-k(w,\mathbf{s}))n$.

4 Computation

The details of the computational algorithms are presented in the Appendix. Here we summarize the algorithms used to solve for the steady-state equilibrium and the dynamic stochastic equilibrium. As mentioned, one complication in solving the model is that the wage and wage offer distributions are endogenous objects that go into the decision problem. A conceptually straightforward way of dealing with the situation is to check the convergence on the mass of workers at all grid points of wage. We propose a more efficient parametric method without sacrificing the accuracy: we use the beta function to approximate these functions.[21]

To ease the notation, let us define the expected marginal profit function after the employment decision is completed in the current period:

$$D(x, n', z, m') = \int\int \Pi_n(x', n', z', m') dG_x(x'|x) dG_z(z'|z). \quad (11)$$

To approximate this function, we replace the type distribution m by the aggregate unemployment rate. The idea is the same as the solution technique used to solve heterogeneous agent models with the uninsurable income risk, where the information in the wealth distribution is captured well with its mean. The D function is approximated by a piecewise linear function of the continuous state variables n' z and U for each discretized value of idiosyncratic productivity x.

4.1 Steady-State Equilibrium

In the steady-state equilibrium, the aggregate state variable $\mathbf{s}$ is time-invariant and thus can be dropped. The first stage to solve for the steady-state equilibrium is to iterate on $\overline{n}^*(x,n)$,

[21]The beta function is characterized by two parameters over an interval $[0,1]$. We transform wage values into this interval. The bound of the interval in the wage space needs to be determined endogenously, and thus this procedure requires the convergence on four parameters for each distribution function.

$\underline{n}^*(x,n)$, and $D(x,n')$ for given guesses of $H(w)$ and $K(w)$ (which are parameterized by the beta function), and market tightness θ, using the first-order conditions (8) and (9).[22] The reason that this step requires the iteration also on $\overline{n}^*$ and $\underline{n}^*$ is because the firm makes the employment adjustment decision taking into account its effect on the current-period wage, which in turn affects the quit rates and the acceptance rate of job-to-job movers, thereby having a feedback into the employment adjustment decision.

Once we obtain the convergence on the optimal employment adjustment function and the $D(x,n')$ function, the second stage of the algorithm simulates the economy to obtain the invariant distribution of $m(x,n)$. Using the invariant distribution and the wage function, we can actually compute $H(w)$ and $K(w)$, from which we update the parameters of the beta functions. The labor market variables such as vacancies posted and the number job seekers are also obtainable at this point, given that we have the information on the employment adjustment policy and the invariant distribution of $m(x,n)$. The entire process repeats until the convergence on (i) the employment policy function, (ii) the expected marginal profit function $D(x,n')$, and (iii) the parameters of the beta functions are achieved.

4.2 Dynamic Stochastic Equilibrium

One difficulty of solving the model under the presence of aggregate uncertainty is that current-period market tightness θ depends not only on realized aggregate productivity z but also on the type distribution $m(x,n)$. In making the employment adjustment decision, each firm therefore needs to know the relationship between θ and $m(x,n)$ as well as z. As mentioned above, it is assumed that the firms use only the mean of the distribution (aggregate employment and thus equivalently unemployment) to summarize the information in $m(x,n)$. Further, calculating and updating the $D(x,n',z,m')$ function requires the firms to form the forecast for the next-period distribution. Given our assumption about the approximate equilibrium, this entails forecasting next-period aggregate unemployment using current period unemployment and realized aggregate productivity. It is important to also recognize that calculation of $D(x,n',z,U')$ requires the prediction of $H(w,\mathbf{s})$ and $K(w,\mathbf{s})$ as well. This process is carried out by postulating the forecasting rule for each parameter of the two beta functions.

The algorithm starts with guessing a set of coefficient values of the forecasting rules we just described. Given these rules, we can solve the individual firms' problem, following the procedure used to solve for the steady-state equilibrium (except that those functions now depend on the aggregate state variables). Once we achieve the convergence on the employment policy function, we simulate a large panel dataset from which we can obtain a long time series of $\{z,U,\theta,H(w),K(w)\}$. By using these objects, we can update the forecasting rules by running regressions. The algorithm stops when the convergence on the coefficients on those forecasting rules is achieved.

[22] Strictly speaking, $\overline{n}^*(x,n)$ actually does not depend on n because the marginal firing cost is constant. But we are using a more notation here.

5 Calibration

One period in the model is assumed to be one month. The exogenous productivity processes follow standard AR(1) processes:

$$\ln z' = \rho_z \ln z + \varepsilon'_z,$$
$$\ln x' = \rho_x \ln x + \varepsilon'_x.$$

where $\varepsilon_x \sim N(0, \sigma_x^2)$ and $\varepsilon_z \sim N(0, \sigma_z^2)$. These processes are then approximated by a finite-state first-order Markov chain.[23]

Note that some of the statistics used to calibrate the model are available only at quarterly frequency or annual frequency. In particular, job flows are measured by taking net employment changes over a quarterly period. It is important for us to construct the model-based statistics in the same way as in the observed data. The details will be discussed below.

We partition the model parameters into two groups, the one determined exogenously to the model without solving the model and the other determined by matching the empirical moments.

5.1 Parameters Set Exogenously

First, the time discount factor β is set to 0.996, which implies the quarterly discount factor of 0.99, a standard value used in the business cycle literature. The curvature of the production function α is set to 0.67. This appears to be a value commonly used in the literature, for example, by Cooper et al. (2007). We will later consider a somewhat higher value to check the sensitivity of the results with respect to this parameter. Worker bargaining power η and the elasticity of the matching function with respect to unemployment $1-\psi$ are set to 0.5. We consider an alternative value for η later. The persistence parameter of aggregate productivity is set to 0.983, which implies a quarterly autocorrelation of 0.95, following the convention of the business cycle literature. The parameter values so far are relatively uncontroversial.

The flow outside option value b is set to 0.4. It is well known that this parameter plays an important role for the amplification in the standard model with linear production technology (Shimer (2005),Costain and Reiter (2008), and Hagedorn and Manovskii (2008)). However, one can no longer use this parameter to control the model's amplification in the environment with decreasing returns to scale, because the firm's employment decision internalizes the level of this parameter.[24] Moreover, we adopt the wage setting mechanism that is based on the current-period output.

In our calibration procedure, we keep this parameter at 0.4 and adjust other parameters to match the targeted statistics. In this sense, this value of b is simply a normalization. Intu-

[23] While we use the finite-state approximation in calculating the conditional expectation with respect to aggregate uncertainty (when we solve the firm's problem), we maintain the original AR(1) process in the simulation stage so that the process has a continuous state space. This enables us to generate smooth impulse response functions presented below.

[24] Elsby and Michaels (2013) extensively discuss the role played by this parameter in the decreasing-returns-to-scale environment.

Table 2: Model Parameters

Symbol	Description
ψ	Elasticity of matching function with respect to job seekers
α	Curvature of production function
β	Time discount factor
η	Worker bargaining power
μ	Scale parameter of matching function
κ_0	Parameter of the vacancy posting cost
κ_1	Parameter of the vacancy posting cost
τ	Firing cost
γ	Search intensity of on-the-job seekers
b	Flow outside benefit (normalization)
ρ_x	Persistence of idiosyncratic productivity process
σ_x	Standard deviation of idiosyncratic shock
ρ_z	Persistence of aggregate productivity process
σ_z	Standard deviation of aggregate shock
L	Labor force (population) size

itively speaking, for a given level of b, each firm determines its employment level from which we obtain aggregate employment. We then set the level of the total labor force (equivalently, population) to match the steady-state EU transition rate in the model. The quantitative properties of the model are unaffected by the level of b in our calibration procedure.

Lastly, κ_1, one of the two parameters that characterize the vacancy posting cost, is set to 0.1 with no reference to the data. Let us defer the discussion on this parameter to the next subsection, because it is easier to discuss the choice of this value together with the determination of κ_0.

5.2 Parameters Set Endogenously

First, consider the scale parameter of the matching function μ. We target the steady-state UE transition rate $f(\theta)$ and the job filling rate $q(\theta)$ at 0.20 and 0.9 per month, respectively. The former number is based on the time series data on the unemployment-to-employment transition rate computed from the CPS labor force status flows data over the period between Jan. 1990 and Dec. 2011.[25]

The latter is based on the evidence by Davis et al. (forthcoming) who show that the

[25]Note that the model includes job-to-job transitions to which a different transition rate applies because employed workers do not necessarily accept all offers. Because unemployed workers accept all offers in our model, $f(\theta)$ in the model corresponds to the UE transition rate in the data. Also note that the steady-state UE transition rate of 0.2 is somewhat lower than the time-series average of the observed series. But the lower target in the steady state is due to the fact that, when aggregate shock is present, the UE transition rate fluctuates around a somewhat higher value, i.e., 0.23, which is roughly consistent with the data.

Table 3: Summary of Calibration Strategy and Parameter Values

Parameter	Value	Source/Target Statistic
		Exogenously Chosen
ψ	0.5	Petrongolo and Pissarides (2001)
α	0.67	Cooper et al. (2007)
β	0.996	Quarterly discount factor of 0.95
η	0.5	Often used in the literature; equal to ψ
ρ_z	0.983	Quarterly autocorrelation of 0.95
κ_0	0.1	See text for explanations
b	0.4	Normalization
		Endogenously Chosen
μ	0.4243	Labor market tightness
γ	0.1145	Monthly job-to-job transition rate of 2.5%
κ_0	0.028	Equilibrium labor market tightness of 0.222
τ	0.125	Volatility of UE transition rate
L	10.3603	EU separation rate of 1.5%
ρ_x	0.93	Quarterly job-creation persistence of 0.7
σ_x	0.1011	Standard deviation of annual employment growth rate of 0.60
σ_z	0.0026	Quarterly output volatility of 1.18%

CPS-based daily job filling rate fluctuates at around 7%, which translates into the monthly filling rate of 0.9.[26] These two targeted transition rates imply that steady-state labor market tightness of $0.20/0.9 = 0.222$. This value, together with the targeted UE transition rate of 0.2 and the elasticity parameter of the matching function, implies $\mu = 0.424$.

To achieve the target level of the EU transition rate, we adjust one of the two parameters of the vacancy posting cost κ_0. This procedure yields the value $\kappa_0 = 0.028$.[27]

The search intensity parameter of employed workers γ is selected to match the average job-to-job transition rate in the Fallick and Fleischman (2004) data that cover the period between Jan. 1994 and Dec. 2011. The average job-to-job transition rate over this period is 2.5% in the data, and we roughly match this number by setting $\gamma = 0.115$.

Next, we calibrate the two parameters of the firm-level productivity process, ρ_x and σ_x, by referring to the following two statistics. First, the persistence parameter is selected to match the average "one-quarter persistence measure" of the job creation rate. This statistic is proposed by Davis et al. (1996) and measures the percentage of newly created jobs at time t that remain filled at the next sampling date one quarter later. They report the historical average of this measure for the manufacturing sector over the period of 1972Q2 through 1988Q4 to be 0.68.[28] Our choice $\rho_x = 0.93$ allows us to roughly match this statistic. The standard de-

[26]Fujita and Ramey (2012) also use the same target based on the evidence by Barron et al. (1997).

[27]Note that, once we match the target level of tightness and the EU transition rate, the target for the job filling rate is automatically fulfilled.

[28]Unfortunately, empirical evidence on this measure is available only for the manufacturing sector. The

viation σ_x is assigned to match the dispersion of the employment growth distribution. Davis et al. (2006) calculate employment-weighted cross-sectional dispersion (standard deviation) of annual employment growth rates using the Longitudinal Business Database (LBD) over the period between 1978 through 2001. The standard deviation is roughly around 0.60. The model generates the value close to this target by setting $\sigma_x = 0.101$. Note that the original measure is based on net employment changes over an annual interval. We therefore construct the corresponding model-based statistic, taking net employment changes over a 12-month period.

Next, the labor-force size L is set to the level that is consistent with the average EU separation rate of 1.5%. The idea is as follows. Once we solve for the employment decisions of individual firms, we can obtain aggregate employment and worker flows into and out of unemployment. Thus we can choose L such that we achieve the target aggregate EU transition rate.

The size of the aggregate shock σ_z is set to 0.0026 and selected by matching the standard deviation of the aggregate output series. Over the sample period 1990-2011, the standard deviation of the logged and HP-filtered real GDP series is 0.0119.[29] $\sigma_z = 0.0026$ allows us to roughly match this level of volatility. The fact that we can match the output volatility with such a small standard deviation of the shock (relative to the value usually used in the RBC literature) highlights the strong amplification mechanism embedded in our model. We will come back to this issue in the next section.

Let us now discuss how we chose κ_1 (parameter determining the slope of the marginal cost of a vacancy). As mentioned in the previous subsection, κ_1 is set to 0.1 with no reference to the data (we keep this value throughout our calibration procedure). Recall that the marginal cost of vacancies is $\kappa_0 + \kappa_0\kappa_1 v$. Our parameter values for κ_0 and κ_1 imply a fairly small slope ($\kappa_0 \times \kappa_1 = 0.00028$) and thus the calibration makes our model close to the model with a linear vacancy posting cost. We introduce the curvature for a technical reason that, when the cost is linear (i.e., $\kappa_1 = 0$), the wage distribution is too compressed, so that it becomes difficult to obtain the stable numerical solution to our model featuring OJS, which directly links the wage distribution to job-to-job transitions.[30]

Lastly, the firing cost is set to 0.125. We use this parameter to match the volatility of the UE transition rate. One may think that this parameter is most effective to control the volatility of the EU transition rate. This is true in the model without OJS. To understand our calibration procedure, first note that higher τ implies that we need to lower κ_0 (the level parameter of the marginal vacancy posting cost) to match the target level of labor market tightness. When firing becomes more costly, firms are less willing to post vacancies, and thus in order to compensate for this effect, we need to lower κ_0. In the model without OJS, a

persistence measure of job destruction is defined similarly as the percentage of newly destroyed jobs at time t that do not reappear at the next sampling date. Davis et al. (1996) report that job destruction persistence in manufacturing is 0.723 over the same period.

[29] We use the post-1990 output series simply because the other series we use are available from or shortly after 1990. See notes to Table 1.

[30] We also considered a calibration with a higher value of this parameter (e.g., $\kappa_1 = 0.15$), and the results are little affected.

Table 4: First-Moment Properties: Benchmark Calibration

	Data collection frequency	Model	Empirical target
Worker-side data			
EU transition rate	Monthly	0.018	0.015
EE transition rate	Monthly	0.022	0.023
UE transition rate	Monthly	0.220	0.250
Unemployment rate	Monthly	0.081	–
Establishment-side data			
Job flow rates	Quarterly	0.090	0.080
Job flow persistence measure	Quarterly	0.732	0.700
b/(average labor productivity)	–	0.785	–

Notes: See text for the explanation of the job flow persistence measure. To calculate the model-based moments, we follow the same data-collection procedures as those used in actual surveys. The unemployment rate is not directly targeted in our calibration. Instead, the average EU and UE transition rates are used as targets.

higher firing cost and a lower vacancy posting cost would only shift the balance of employment adjustments to the hiring margin without affecting overall employment volatility. However, in the presence of OJS, its effect is not that straightforward. When a firm loses workers through job-to-job transitions, it allows them to save the firing cost when their idiosyncratic productivity is low, and thus the firm wants to reduce employment. The important thing is that this margin is time varying. Because job-to-job transitions are procyclical, this saving of the firing cost is larger (smaller) in the boom (recession) period. Thus, this channel by itself lowers the EU transition rate in a boom because attritions will take care of the needs to reduce the workforce (i.e., less needs for costly layoffs for firms) when the firm is facing a low idiosyncratic productivity while the aggregate economy is in a boom. A symmetric argument can be made to illustrate the increases in the EU transition rate in a recession. In the next section, we will discuss this channel more extensively. But the bottom line is that, by changing τ (also changing κ_0 to match the target average level of labor market tightness), we can control the volatility of the UE transition rate easily and that is what we do.

This parameter τ represents the per-worker resource cost associated with layoffs, and $\tau = 0.125$ in the our benchmark calibration implies that this resource cost amounts to roughly 30% of the monthly wage. It is small, as is consistent with the empirical evidence that firing costs are small in the U.S.

6 Results

This section presents the results of the paper. We first assess the model's capability of replicating the basic business cycle properties of the data that we discussed in Section 2. After making sure the model matches the first moments of the observed data reasonably well, we will first focus on the discussion on the cyclicality of worker flows and transition rates. We then discuss the cyclicality of job flows and investigate the sources of the differences in the cyclicality of the two sets of flow variables. We consider several quantitative experiments (including the sensitivity analysis to alternative calibrations) in order to demonstrate the importance of job-to-job transitions for our results.

6.1 First-Moment Properties

The first-moment properties of the model under the benchmark calibration are summarized in Table 4. Recall that we calibrate the model to achieve the average levels of the three worker transition rates and job flow persistence. While we are unable to match the first moments exactly, the model-based average levels are reasonably close to the targeted level. The unemployment rate in the model is somewhat higher than the average level used in the literature (e.g., 6%) but it is because in our calibration, the EU transition rate is on the high side and the UE transition rate on the low side, both of which raise the average unemployment rate. These results do not have any material impacts on our results. Job creation and destruction rates in the model, which are not directly targeted in our calibration, fluctuate around 9%. Overall, the calibrated model successfully replicates the features of the observed data in terms of the first moments.

In the last row, we also report the ratio of the flow outside option value to average labor productivity. We report this number only because of the interest in the literature about the volatility puzzle of Shimer (2005). Elsby and Michaels (2013) recently show that their model generates a larger magnification because of curvature of the production function. Our number (0.785) is somewhat higher than their corresponding number (0.61) but not far from it. Note, however, that our model differs significantly from theirs due to the presence of OJS and the different wage setting mechanism. Their model is a natural extension of Mortensen and Pissarides (1994), and thus the comparison to the literature on the volatility puzzle is more direct. Our focus is different and we introduce OJS, which is made possible only by adopting a simpler wage setting. Adopting different wage setting in our paper, on the other hand, makes the direct comparison difficult. However, we will show below that our model generates large volatilities.

6.2 Cyclicality of Worker Flows and Transition Rates

Table 5 presents the same second-moment statistics discussed earlier in Table 1. As we mentioned in the calibration section, we choose τ to match the volatility of the UE transition rate (often known as the "job finding rate"). In the observed data, its standard deviation is 6.7 times as large as the output volatility. We roughly match this value in our simulated data. Other cyclical measures in this table are not directly targeted. First, observe that the EU transition rate in our model is somewhat too volatile. In the data, the EU separation rate is less volatile than the UE transition rate is, whereas in our model, it is more volatile. Intuitively, one may think that we can reduce the volatility of this variable by raising the firing cost parameter. However, this is not possible in our model because, in our model,

Table 5: Second-Moment Properties of the Model: Benchmark Calibration

	Standard Deviation	Relative Standard Deviation	Correlation With Output
Worker flows			
E to U	0.097	8.215	−0.771
E to E	0.088	7.379	0.992
U to E	0.076	6.396	−0.599
Transition rates			
EU transition rate	0.104	8.735	−0.821
EE transition rate	0.078	6.576	0.986
UE transition rate	0.081	6.858	0.983
Job flows			
Creation rate	0.029	2.464	0.088
Destruction rate	0.031	2.595	−0.375
Stocks			
Unemployment rate	0.139	11.677	−0.905
Vacancies	0.123	10.386	0.943

Notes: Based on the simulation panel of 1 million establishments over 1,200 (monthly) periods. Worker flows, worker transition rates, unemployment rate, and vacancies are converted into quarterly data by time averaging. Job flows are based on net employment changes over a quarter. All observations are logged and HP filtered with smoothing parameter of 1,600.

firms' incentive to laying off workers is directly affected by how many workers leave through job-to-job transitions and thus the behavior of $f(\theta)$.

Note that the large volatility of the EU transition rate makes not only EU worker flow more volatile but also UE flow more volatile. Large fluctuations in the flow into unemployment also implies large fluctuations in the flow out of unemployment with a lag (due to search frictions). The larger volatility of separations into unemployment (together with the fluctuations in $f(\theta)$ comparable to the data) produce the volatility of the unemployment rate that is somewhat larger than the observed data.

The model replicates very well the correlation patterns with respect to output. We documented earlier the countercyclicality of the EU separation rate and the procyclicality of the UE and EE transition rates. The model naturally reproduces this pattern. Next, the model also replicates the cyclical pattern of worker flows: countercyclicality of flows between employment and unemployment, and procyclicality of the job-to-job flow. As discussed in Section 2, hires from unemployment are countercyclical because of the larger unemployment pool in recessions due to the increases in the EU flow. This effect is counteracted by the decline in the UE transition rate, and therefore the countercyclicality of the hiring flow is weaker (although this flow is still strongly countercyclical both in the model and the data).

The model also replicates the Beveridge curve, a strong negative correlation between unemployment and vacancies, which is indicated by the countercyclicality of unemployment and procyclicality of vacancies shown in the last rows in the table. We discuss the correlations

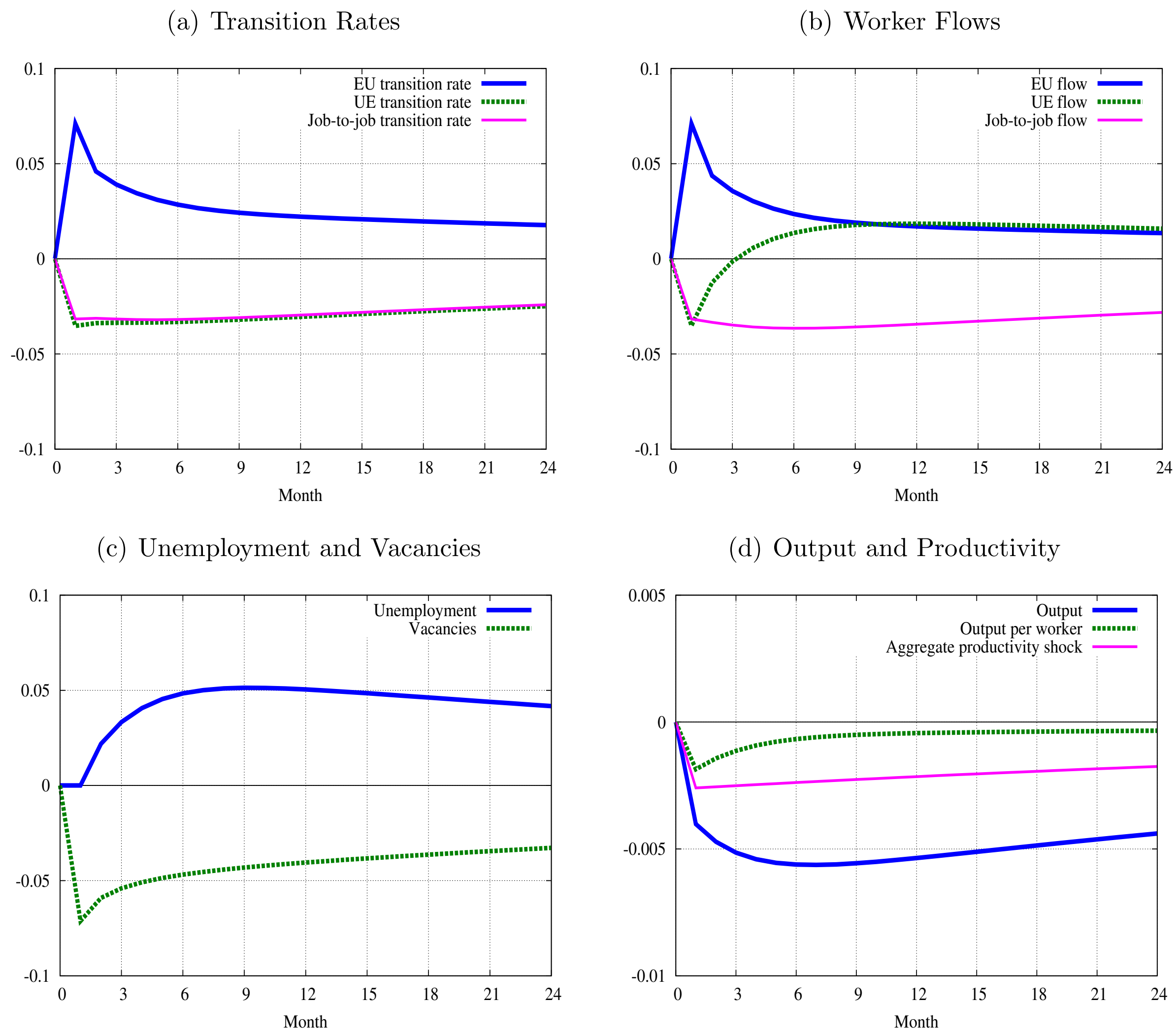

Figure 3: Impulse Responses Functions

Notes: Plotted are responses to one standard deviation negative aggregate shock, expressed as log deviations from the steady-state levels.

of job flows with output using Table 6. This table is meant to explain the differences in the cyclicality of worker flows and job flows.

Figure 3 presents impulse response functions to a one standard-deviation negative aggregate productivity shock.[31] Panel (a) presents the responses of worker transition rates. The EU transition rate sharply increases on impact. While a substantial part of the initial increase is reversed in the following period, it stays at the level higher than the steady-state

[31]Strictly speaking, impulse responses are not symmetric because the model is nonlinear. We checked if the model inherits serious asymmetry by comparing the responses to the positive and negative shocks and found that only minor asymmetry exists in the model.

Table 6: Job Flows vs. Worker Flows

		Standard Deviation	Relative Standard Deviation	Correlation With Output
Creation rate	(Q)	0.029	2.464	0.088
Creation rate	(M)	0.031	2.588	0.327
Accession rate	(M)	0.035	2.953	0.562
Destruction rate	(Q)	0.031	2.595	−0.375
Destruction rate	(M)	0.023	1.937	−0.078
Separation rate	(M)	0.020	1.673	0.414

Notes: The letter in parentheses indicates the data collection frequency (Quarterly or Monthly). Monthly job flows are constructed by applying the same idea as the quarterly job flows to monthly employment changes: job creation (destruction) = sum of employment changes at expanding (shrinking) establishments over a monthly period (normalized by employment). Accession rate: all hires (sum of UE and EE worker flows) as a fraction to employment. Separation rate: all separations (sum of EU and EE worker flows) as a fraction to employment.

level for an extended period. The UE and the job-to-job transition rates decline on impact and stay persistently low. Panel (b) presents the responses of worker flows. Not surprisingly, the behavior of the EU flow is very similar to that of the EU transition rate. The hiring flow from unemployment declines initially, reflecting the drop in the UE transition rate, but quickly reverses its course and goes up to the level higher than the steady-state level. The latter movements are responsible for the UE flow's negative correlation with output. [32] In contrast, the job-to-job flow stays below the steady-state level, exhibiting a mild hump-shaped response followed by the initial sharp drop. This is because the pool that generates this flow (i.e., employment) shrinks while the transition rate is also declining (observe that the decline in the job-to-job flow in terms of log deviation is somewhat larger than that in the job-to-job transition rate).

Panel (c) shows that the model is capable of generating the Beveridge curve. In Panel (d), we present responses of aggregate output and labor productivity along with the exogenous driving process. Reflecting the persistent hump-shaped increases in the unemployment rate, aggregate output also exhibits the hump-shaped response, hitting the lowest level in the seventh month after the shock. In our model, labor productivity, measured by aggregate output divided by aggregate employment, is an endogenous variable and is influenced by the reallocation of workers across establishments. Labor productivity does not decline as much as the driving process and recovers more quickly, as indicated by the difference between the pink solid line and the green dashed line in panel (d).

6.3 Cyclicality of Job Flows

In Table 5, we also report that job creation and destruction rates fluctuate much less than worker flows do. One can also see that the job creation rate is weakly procyclical and the destruction rate is countercyclical. To better understand these results, Table 6 reports the same second-moment statistics for quarterly job flows (as shown in the previous table), monthly job flows, and rates of total separations and accessions. Note that we can construct "monthly job flows" using the monthly interval in lieu of the quarterly interval. Even at the monthly frequency, job creation and destruction rates are not the same as total separation and accession rates. The differences arise because of the presence of job-to-job transitions. First, note that hires can occur at the firms that are "destroying" jobs in the sense that net employment change is negative. Firms lose workers due to job-to-job transitions and their optimal decision could be to hire workers to only partially offset the attritions.[33] Second, take the firms that are expanding their employment size on net. Job-to-job transitions occur at those firms, implying that separations are not equal to job destruction.[34] In sum, the comparison presented in Table 6 is meant to capture the time aggregation effect (i.e., difference between quarterly and monthly job flows) and the attrition effects (i.e., difference between monthly job flows and total separation/accession rates).

One can see that job flows fluctuate much less than the three components of worker flows do, as shown in Table 5. Table 6 indicates that the same is true, when one considers total separation/accession rates instead of job flows. This result is easily understood by noticing that the countercyclicality of EU and UE flows is countered by the procyclicality of the job-to-job flow. In Figure 4, we plot impulse responses of job flows and separation/accession rates. In panel (b) of the previous figure (Figure 3), we plot responses of three worker flows. One can clearly see EU and job-to-job flows are offsetting each other's movements and thus creating a less volatile total separation flow. On the hiring side, although UE and job-to-job flows are moving in the same direction in the first few months after the shock, they move in the opposite directions after that, again offsetting each other's movements.[35]

While volatilities are of similar magnitude for all three rows in Table 6, the correlation with output differs considerably. First, consider the difference between the job destruction rate and the total separation rate. Surprisingly, the total separation rate is procyclical even though the job destruction rate is countercyclical. The procyclicality of the separation rate

[32]Fujita (2011) presents impulse response functions of worker transition rates and flows based on the identified VARs, and they are similar to those presented in Figure 3.

[33]In the model, this happens when a firm was in an inactive region at the beginning of the period and loses its workers through quits, pushing the firm outside the (s, S) band. If the idiosyncratic as well as aggregate productivity levels are the same as (or close to) the previous levels, then the firm would hire new workers only to go back to the lower band of the inactive region.

[34]Note that the logical difference between worker flows and job flows also arises in the model with exogenous worker attritions. But, in the model where firms' active shedding of workers is the only source of separations (such as Elsby and Michaels (2013)), the two concepts are identical at least at the frequency of the model.

[35]In panel (b) of Figure 4, both separations and accessions are normalized by employment, to be consistent with job creation and destruction rates, while worker flows in panel (b) of Figure 3 are not normalized by employment, but this does not have a large impact on the results.

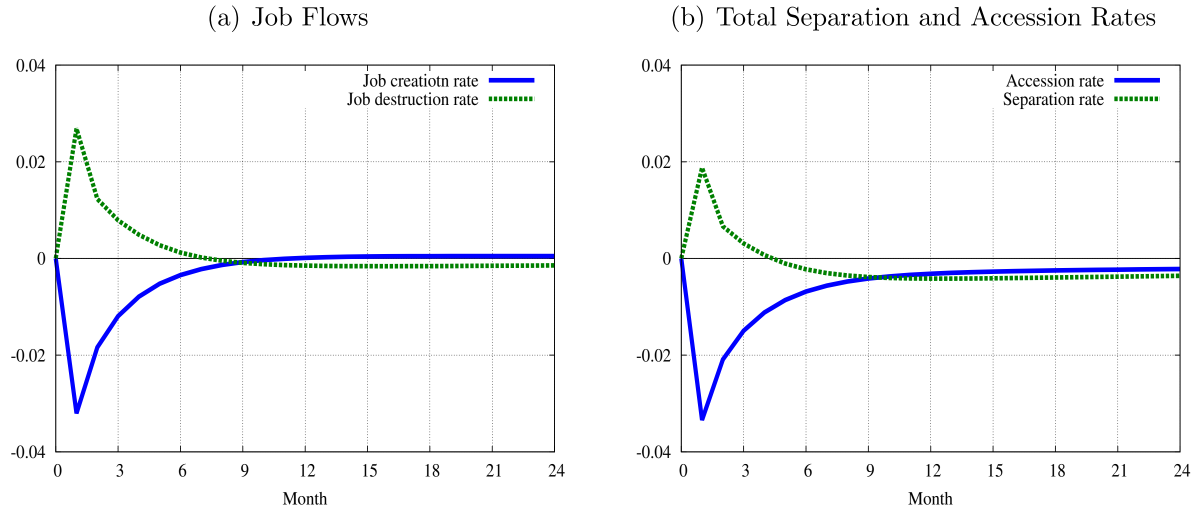

Figure 4: Impulse Responses Functions: Job Flows vs. Worker Flows

Notes: Plotted are responses to one standard deviation negative aggregate shock, expressed as log deviations from the steady-state levels.

implies that the procyclicality of job-to-job transitions dominates the countercyclicality of the EU separation rate on net. Note that the volatility of the EU separation rate is somewhat larger than that of the job-to-job transition rate (see Table 5) when the two transition rates are examined separately. However, the average level of the job-to-job transition rate is larger and thus it carries a larger weight when the total separation rate is concerned. Panel (b) of Figure 4 shows that the total separation rate initially increases due to the increase in the EU transition rate, in response to the negative shock. However, from the fourth month on, the total separation rate stays below the steady-state level for extended periods. This latter part makes the correlation with output positive.

The procyclicality disappears when the job destruction rate is considered. The large part of the change comes from the difference between the monthly job destruction rate and the total separation rate (the correlation coefficient drops from 0.414 to -0.078). The selection of firms in the job destruction rate plays an important role here. That is, when only the firms with negative net employment changes are considered, the role of layoffs (EU flow) becomes more important. Note that separations at expanding firms (positive net growth) will consist of only job-to-job transitions, which are procyclical. In contrast, when the firms with negative growth are selected, the share of layoffs gets larger. The comparison of the responses of the job destruction rate and the total separation rate shows that (i) the initial increase in the job destruction rate is larger, that (ii) the job destruction rate stays above the steady-state level for longer periods, and that (iii) the negative deviation from the steady-state level is smaller for the job destruction rate, after the response turns into negative territory. These three factors contribute to making the job destruction rate a countercyclical variable. Table 6 also shows that time aggregation plays an important role for the countercyclicality of the job destruction rate. This effect comes from the fact that, when the three-month interval is considered, the selection effect discussed above becomes stronger. That is, the selection of the firms with negative net employment growth over the longer-run period is more likely to include those that needed to appeal to layoffs.

Turning to the hiring/job creation side, one can first notice that we do not match the magnitude of the positive correlation with output: the model generates the correlation 0.09, while the correlation of the empirical series is higher than 0.4. However, the issue here has more to do with a timing of the comovement. As we discussed above, the output response exhibits a clear hump shape, while changes in the job creation rate are concentrated in the short run. We therefore view that the model's underlying economic forces are largely consistent with the empirical evidence.

In terms of correlations with output, the (monthly) job creation rate and the accession rate behave similarly. The correlation coefficients of the monthly job creation rate and the accession rate with respect to output are 0.56 and 0.33, respectively. As mentioned earlier, these two series can differ because hires occur at the firms that are shrinking on net. However, this occurs only when firms hire workers to partially offset the worker attritions due to job-to-job transitions. This phenomenon is necessarily concentrated among the firms that are making only small employment adjustments and therefore its quantitative effect is relatively minor.[36] As can be seen in Figure 4, the impulse response functions of the two variables look similar to each other. However, time aggregation also makes a difference. At the quarterly level, even the firms that are laying off workers in one month could end up increasing employment on net over the three-month period (note, however, that the shock is very persistent and therefore this possibility cannot be too common). At the quarterly level, the correlation of the job creation is further reduced to 0.09.

6.4 Effects of Job-to-Job Transitions

Our model differs from Cooper et al. (2007) and Elsby and Michaels (2013) mainly because our model explicitly incorporates job-to-job transitions. In this subsection, we further investigate the implications of incorporating job-to-job transitions into the model in two ways. First, we discuss how the model's quantitative properties are affected when the firing cost parameter is set to a lower level. This exercise is useful, because the need for laying off workers is *directly* influenced by how many workers leave the firm through job-to-job transitions. Second, we examine the quantitative properties of the model without job-to-job transitions by setting $\gamma = 0$. That model reduces to the model of Elsby and Michaels (2013) except for the differences in wage setting and the specifications of the shock processes, and the presence of the firing cost in our model. The second exercise will clearly show that incorporating job-to-job transitions is crucial to match the cyclicality of worker flows and job flows simultaneously.

[36] On the other hand, the wedge between the job destruction rate and the separation rate (i.e., job-to-job separations occur at expanding firms) applies to all expanding expanding firms except for the firms at the top of the wage distribution.

Table 7: Second-Moment Properties: Lower Firing Cost and No OJS

	Benchmark		Lower τ		No OJS	
	Relative SD	Corr. w/ Output	Relative SD	Corr. w/ Output	Relative SD	Corr. w/ Output
Worker flows						
E to U	8.215	−0.771	8.411	−0.790	6.543	−0.647
E to E	7.379	0.992	6.500	0.995	n.a.	n.a.
U to E	6.396	−0.599	6.377	−0.662	4.166	−0.504
Transition rates						
EU transition rate	8.735	−0.821	8.929	−0.835	6.873	−0.726
EE transition rate	6.576	0.986	5.708	0.991	n.a.	n.a.
UE transition rate	6.858	0.983	5.984	0.987	7.340	0.980
Job flows						
Creation rate	2.464	0.088	2.289	−0.122	4.633	−0.588
Destruction rate	2.595	−0.375	2.943	−0.506	6.896	−0.719
Stocks						
Unemp. rate	11.677	−0.905	11.111	−0.911	10.390	−0.894
Vacancies	10.386	0.943	8.743	0.941	5.966	0.860

Notes: See notes to Table 5. The model with a low firing cost is recalibrated to satisfy the same steady-state moment conditions and the output volatility condition as in the benchmark calibration. Recalibrated parameters are set to the following values: $\tau = 0.010$, $\kappa_0 = 0.0478$, $\gamma = 0.1144$, $\sigma_x = 0.1021$, $\sigma_z = 0.0026$, and $L = 10.2592$. The model with no OJS is solved and calibrated by setting $\gamma = 0$. Recalibrated parameters are set to the following values: $\kappa_0 = 0.1433$, $L = 9.7560$, and $\sigma_z = 0.0032$. The first two parameters are used to match average market tightness and the EU separation rate. The last parameter is used to match the output volatility. Parameters not listed are unchanged from the benchmark calibration.

6.4.1 Effects of a Lower Firing Cost

The first two columns of Table 7 repeat the results from the benchmark calibration and the next two columns report the results under the alternative calibration with $\tau = 0.1$ (instead of 0.125). In the calibration with lower τ, we recalibrate the model by matching all the moment conditions except for the condition for the volatility of the UE transition rate. Recall that the benchmark calibration implies the volatility of the EU transition rate that is too large relative to that of the empirical series. Observe that in the case of $\tau = 0.1$, the volatility of the UE transition rate becomes smaller, whereas the volatility of the EU transition rate increases, making the distance from the observed volatility even larger. By comparing the results between the first and third column, one can see that, for any level of τ, the model is unable to simultaneously match the volatilities of the EU and UE transition rates. Although this result is an undesirable feature of the model, it nevertheless reflects an interesting economic mechanism. To see the underlying cause, note that the boom period is the time when firms' need for laying off workers declines on average simply because of higher productivity. However, the boom is also the time when more workers leave, further reducing the likelihood of layoffs. During the recession, in contrast, the pace of job-to-job transitions slows down, and thus it becomes harder for the firm (which needs to reduce employment) to use the attrition as a means for its employment adjustment.

Next, as mentioned in the calibration section, with a lower value for τ, we need to raise the value of κ_0 (from 0.028 to 0.048) so that we can maintain the first-moment condition for labor market tightness. With a higher cost of hiring, the volatility of vacancies is reduced, which causes the volatilities of UE and job-to-job transition rates to drop. This volatility effect on the hiring side has an undesirable effect on the correlation pattern of job flows. That is, both the job creation rate and job destruction rate are now negatively correlated with output. When τ is lowered, the cyclical adjustments of employment shifts more to layoffs. We already saw that the EU transition rate becomes more volatile, while the volatilities of UE and job-to-job transition rates decline. These changes imply that the correlation of the accession rate with output is influenced more by the UE flow, which is strongly countercyclical, nudging the cyclicality of the job creation rate into the countercyclical side. For a similar reason, the countercyclicality of the job destruction rate becomes stronger.

6.4.2 Model Without OJS

The last two columns of Table 7 present the cyclical properties of the model when the search intensity of employed workers is set to zero. In calibrating the model, we match the average levels of UE and EU transition rates by using κ_0 and L, as in the benchmark calibration. We also adjust the size of the standard deviation of the aggregate shock so that the model generates the output volatility of the same magnitude as before. The remaining parameters remain the same. In particular, we do not adjust the level of τ, and thus the model does not exactly match the volatility of the EU transition rate or the UE transition rate. However, their volatilities are within reasonable ranges.

First, note that the model now replicates the fact that the UE transition rate fluctuates somewhat more than the EU transition rate. Only in this dimension does the model without OJS perform better. With respect to the correlation patterns concerning transitions between employment and unemployment, both models with and without OJS do equally well. Further, the model with OJS generates much larger volatility of vacancies. One important channel behind this result is that with OJS, the cost of separation (per worker) is *effectively* countercyclical. The attrition (job-to-job transition) rate increases (decreases) in the boom (recession) and thus the cost of laying off workers for a given target level of employment is lower (higher). This time variation of the effective cost of worker separations, in turn, magnifies the incentive of posting vacancies.

Without OJS, the model fails to replicate the cyclical patterns in job flows. In the absence of job-to-job transitions, the job destruction rate is equivalent to the EU transition rate at monthly frequency. Thus, the difference in the correlation coefficients with output arises solely due to time aggregation in the measurement of the job destruction rate (-0.726 vs. -0.719). In the model without OJS, the effect of time aggregation is small. The job creation rate is now strongly negatively correlated with output. This result is quite intuitive because job creation now needs to happen only through hires from unemployment, which

Table 8: Second-Moment Properties: Sensitivity Analysis

	Benchmark		$\alpha = 0.72$		$\psi = 0.6$	
	Relative SD	Corr. w/ Output	Relative SD	Corr. w/ Output	Relative SD	Corr. w/ Output
Worker flows						
E to U	8.215	−0.771	8.694	−0.743	8.427	−0.779
E to E	7.379	0.992	8.972	0.983	6.515	0.996
U to E	6.396	−0.599	7.088	−0.552	6.404	−0.644
Transition rates						
EU transition rate	8.735	−0.821	9.257	−0.797	8.934	−0.825
EE transition rate	6.576	0.986	8.111	0.975	5.733	0.991
UE transition rate	6.858	0.983	8.464	0.972	5.988	0.987
Job flows						
Creation rate	2.464	0.088	2.922	0.250	2.310	−0.119
Destruction rate	2.595	−0.375	2.385	−0.149	2.908	−0.507
Stocks						
Unemp. rate	11.677	−0.905	13.481	−0.900	11.074	−0.906
Vacancies	10.386	0.943	13.252	0.935	11.741	0.958

Notes: See notes to Table 5. The model is recalibrated for both cases to match the steady-state statistics and output volatility. (τ is kept at the same value and thus the volatility of the UE transition rate is not targeted in the two alternative calibrations). Recalibrated parameters are set to the following values (those not listed are set to the same values as in the benchmark calibration). $\alpha = 0.72$: $\kappa_0 = 0.0081$, $\gamma = 0.115$, $\sigma_x = 0.0852$, $\sigma_z = 0.0020$, and $L = 17.9878$. $\psi = 0.6$: $\mu = 0.365$ and $\sigma_z = 0.0028$

is countercyclical. The correlation between the accession rate (again equivalent of the job creation rate at monthly frequency) and output is −0.594 (not reported in the table). When measured by net increases in employment at quarterly frequency, the correlation becomes somewhat weaker, but not by much (−0.588).

Lastly, the volatility of job flows is too high in this model relative to what the data indicate. Recall that, in the model with OJS, the procyclical movements of job-to-job transitions are offset by the countercyclicality of worker flows between employment and unemployment, thereby making the volatility of total worker flows smaller. Even though we showed earlier that separation and accession rates differ from job flows in terms of their comovements with output, this offsetting effect clearly is the reason that job flows are much less volatile than each component of worker flows, and without job-to-job transitions, this effect disappears.

6.5 Sensitivity

In this subsection, we examine the sensitivity of the model's quantitative properties with respect to the following two parameters: the curvature parameter of the production function

α and the elasticity parameter (with respect to job seekers) of the matching function ψ.[37] Specifically, we raise α from 0.67 to 0.72. Changing this parameter appears to be a sensible thing to examine not only because the original value is only loosely chosen but also because it is a priori conceivable that the change in the marginal product schedule may make significant differences in our results, for example, through the changes in the wage schedule and the firm-size distribution. Next, we change ψ from 0.5 to 0.6, given that the literature often uses different values within a certain range for this parameter.[38] For these two changes, we recalibrate the model by matching all steady-state statistics and the output volatility. Importantly, we keep the value of the firing cost parameter at the same value as in the benchmark model. In other words, we do not match the volatility of the UE transition rate in these calibrations. See notes to Table 8 for the parameter values.

Table 8 presents the results. The model under a higher α performs reasonably well: all correlation patterns are very close to those under the benchmark calibration. One can notice, however, that the model is somewhat "too volatile." In particular, the relative volatility of the UE transition rate now goes up to 8.5, while in the benchmark calibration, it was 6.9 which is calibrated to match the observed one. Matching the volatility of the UE transition rate requires lowering τ and raising κ_0. In this case, the model's overall behavior will become closer to the one under the benchmark calibration.[39]

The properties of the model with $\psi = 0.6$ remain relatively close to those under the benchmark calibration. The main differences lie in the higher volatility of the EU transition rate and the lower volatility of the UE transition rate (compared to the benchmark case). These changes make both job creation and destruction rates countercyclical, given that the effect of the EU transition rate on these variables becomes stronger. However, the correlation patterns of the remaining variables stay intact and consistent with the empirical evidence. Also note that raising the value of τ will make the volatilities of the EU and UE transition rates closer to those under the benchmark calibration. Accordingly, the behavior of job flows will be closer to the observed data. Overall, the sensitivity analysis here demonstrates that the model's overall features are not significantly affected with the alternative parameter values.

7 Conclusion

In this paper, we studied the quantitative properties of multiple-worker matching model with on-the-job search. We show that the model is capable of replicating the overall cyclical patterns of worker flows and job flows simultaneously. Procyclical job-to-job transitions,

[37] We also considered the effect of changing the worker bargaining power η from 0.5 to 0.6. The basic conclusion from this exercise is the same as the two cases and thus we focus on the effects of the two parameters.

[38] See Brügemann (2008) for the discussion on this issue.

[39] The combination of a lower τ and higher κ_0 results in a lower volatility of the UE and EE transition rates, which is not surprising, and also a slightly higher volatility of the EU transition rate. The latter result would make the volatility of the EU transition rate even greater than its empirical counterpart. See the discussion in section 6.4.

coupled with countercyclical worker flows between unemployment and employment, are important for this success. However, this paper also shows that the cyclical features of total separation and accession rates significantly differ from those of job flows. This is because job-to-job transitions (as a part of separations) occur at the firms that are “creating” jobs on net, and similarly, hires occur at firms that are “destroying” jobs on net.

Our results, however, are by no means perfect. We find that the separation rate into unemployment is too volatile relative to the data. We find that it is not possible to contain this problem with a higher firing cost because the layoff decision is strongly influenced by the pace of attritions (i.e., job-to-job transitions). Another issue that is not discussed in the main text is that the model misses important features of the cross-sectional employment growth distribution studied by Davis et al. (2012) (our calibration only matches the dispersion of the annual growth rate distribution). Davis et al. (2012) show that, at quarterly frequency, roughly 15% of establishments report no net employment change. Our model is simply unable to capture this pattern because the firm always loses some workers through job-to-job transitions even when the firm’s idiosyncratic state does not change. More generally speaking, firm-level employment adjustments in our model are too responsive to the shock. That is, there are more firms that are making relatively large adjustments than are implied by the observed data.

Modifying our model to accommodate the more realistic micro-level heterogeneity is a fruitful future research topic. For example, we abstract away from the employed workers’ search decision by assuming they are looking for a job with a fixed reduced search intensity. However, it is clearly the case in reality that the search intensity of workers at struggling (and thus shrinking) firms is higher.[40] Another promising dimension could be an integration with lumpy investment literature. Nonconvex capital adjustment costs could work as a device to slow down employment adjustments, thus making the model closer to the data.

A Computation

Details of the numerical procedure to solve for the steady-state equilibrium and the dynamic stochastic equilibrium are as follows.

A.1 Steady-State Equilibrium

1. Guess equilibrium market tightness θ, which immediately gives $q(\theta)$ and $f(\theta)$. Let Ω_H and Ω_K be the parameters of the beta function that characterizes $H(w)$ and $K(w)$, respectively. Each parameter vector includes four parameters; two of them determine the upper and lower bound of the distribution.[41] By guessing Ω_H and Ω_K, obtain approximate $H(w)$ and $K(w)$. Using these pieces of information, compute $h(w)$ and $k(w)$.

[40] Davis et al. (2012) call this the “abandon-ship” effect. See also Faberman and Nagypál (2008).

[41] Note that the beta distribution is defined on $[0, 1]$ and that we map wages into this interval.

2. Guess $D(x, n')$, the expected marginal profit function of a firm with type (x, n'). Guess also the optimal employment adjustment rule $n' = g(x, n)$. This function is used in the wage function and thus is used to calculate the quit rate $k(w)$ and the acceptance rate $h(w)$ if the firm is posting vacancies.

3. Use the first-order conditions (8) and (9) to obtain the (s, S) band, $\underline{n}^*(x, n)$ and $\overline{n}^*(x, n)$, of the firm's employment adjustment rule. These two functions are used to update the firm's optimal employment adjustment rule $n' = g(x, n)$.

4. Using the updated optimal employment adjustment rule $n' = g(x, n)$, (10), and (11), update the firm's marginal profit function $D(x, n')$.

5. Check convergence of $D(x, n')$ and $n' = g(x, n)$, based on the distance between the old and updated functions. Otherwise, update $D(x, n')$ and $n' = g(x, n)$ and go back to step 4.

6. Using this optimal employment adjustment rule and the stochastic process for x, simulate the economy until the invariant type distribution $m^*(x, n)$ is obtained.

7. Using $m^*(x, n)$, update $H(w)$ and $K(w)$ as follows:

$$H(\bar{w}) = \frac{\int \mathbb{1}[\bar{w} > w(x, g(x, n))] g(x, n) dm^*}{\int g(x, n) dm^*} \tag{12}$$

$$K(\bar{w}) = \frac{\int \mathbb{1}[\bar{w} > w(x, g(x, n))] \max[g(x, n) - (1 - k(w(x, g(x, n))))n, 0] dm^*}{\int \max[g(x, n) - (1 - k(w(x, g(x, n))))n, 0] dm^*}. \tag{13}$$

   where $\mathbb{1}$ is an indicator function. These distributions can be used to update Ω_H and Ω_K.

8. Compute the total labor force (population) consistent with the stationary distribution m^* and the unemployment rate U as:

$$L = \frac{\int g(x, n) dm^*}{1 - U}.$$

   Note that U is fixed at a targeted level. Efficiency-weighted number of searchers S, the total number of vacancies (normalized by the labor force) V, and the labor market tightness θ are calculated by:

$$S = \gamma(1 - U) + U, \tag{14}$$

$$V = \frac{1}{L} \int \frac{\max[g(x, n) - (1 - k(w(x, n'(x, n))))n, 0]}{h(w(x, g(x, n)))} dm^*, \tag{15}$$

$$\theta = \frac{V}{S}. \tag{16}$$

9. Check convergence of $\{\theta, \Omega_H, \Omega_K\}$. Update $\{\theta, \Omega_H, \Omega_K\}$ and go back to step 2 if the distance between the guess and the updated numbers is larger than the pre-specified value.

A.2 Dynamic Stochastic Equilibrium

1. Parameterize $H(w)$ and $K(w)$. Note that the type distribution m is replaced by U. Let $\Omega_H = \{\omega_i^H\}_{i=1,\dots,4}$ and $\Omega_K = \{\omega_i^K\}_{i=1,\dots,4}$ be a set of parameters associated with $H(w)$ and $K(w)$, respectively. $\{U', \theta, \Omega_H, \Omega_K\}$ is the list of variables to be forecast through the following forecasting rules:

$$\log U' = \phi_0^1 + \phi_1^1 \log z + \phi_2^1 \log U \tag{17}$$

$$\log \theta = \phi_0^2 + \phi_1^2 \log z + \phi_2^2 \log U \tag{18}$$

$$\log \omega_i^H = \phi_0^{2+i} + \phi_1^{2+i} \log z + \phi_2^{2+i} \log U \quad i = 1, ..., 4 \tag{19}$$

$$\log \omega_i^K = \phi_0^{6+i} + \phi_1^{6+i} \log z + \phi_2^{6+i} \log U \quad i = 1, ..., 4 \tag{20}$$

   Let Φ be the vector of the parameters $\{\phi_0^i, \phi_1^i, \phi_2^i\}_{i=1,\dots,10}$. With a guess for Φ, and current z and U, one can compute q, f, $H(w)$, and $K(w)$.

2. Guess the expected marginal profit of the firm with a type (x, n') under the aggregate state (z, U'), $D(x, n', z, U')$.

3. Using the first-order conditions (8) and (9), compute (s, S) band $\underline{n}^*(x, n, z, U)$ and $\overline{n}^*(x, n, z, U)$. These two functions characterize the firm's optimal employment adjustment rule $n' = g(x, n, z, U)$.

4. Using the optimal employment adjustment rule $n' = g(x, n, z, U)$ and the envelope conditions (10), update the firm's marginal profit function $D(x, n', z, U')$ from (11).

5. Check convergence of D. It is assumed that convergence is reached if the distance between the initial and updated functions is smaller than a pre-determined tolerance level. Otherwise, update D and go back to step 3.

6. Using the optimal employment adjustment rule $n' = g(x, n, z, U)$ and the stochastic processes for z and x, simulate the economy for $T = T_0 + T_1$ periods. The economy consists of a panel of one million establishments over 1,320 periods with $T_0 = 120$ and $T_1 = 1,200$. The simulation starts with the steady-state distribution of $m^*(x, n)$. The unemployment rate in the initial period can be obtained from the distribution. In each period, compute $H(w)$ and $K(w)$ following the formulas (12) and (13). The unemployment rate in each period is calculated as:

$$U = \frac{L - \int g(x, n, z, U) dm(x, n)}{L}. \tag{21}$$

   The number of vacancies V, the number of job seekers, and the labor market tightness in each period are calculated by the formulas (14), (15), and (16).

7. Using the sequence $\{z_t, U_t, \theta_t, H_t(w), K_t(w)\}_{t=T_0+1,\dots,T}$, run OLS regressions (17) through (20) and obtain the new set of coefficients Φ.

8. Check convergence of Φ. If the distance between the old and new Φ is smaller than a pre-determined tolerance level, then stop. Otherwise, update Φ and go back to step 3.

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