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On the Timing of Monetary Policy Reform*

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Abstract

This paper argues that there is a normative case for delaying policy reform. Policy design in dynamic economies typically faces a trade-off between the policy effects in the short and long term, and possibly across future states of nature. When the economy is in an atypical state or available policies are less flexible than ideal, this trade-off can be steep enough that retaining the status-quo policy in the short term and taking on the reform at a later date is welfare improving. In a simple New Keynesian economy, I consider monetary policy reform from discretion to the optimal targeting rule. I find that the policy reform should be postponed if a sharp drop in output drives the nominal interest rate to the zero lower bound but only modest deflation pressures are observed under the status-quo policy.

1 Introduction

When should a status-quo policy be replaced by its optimal counterpart? In this paper I show the answer is not a straightforward "as soon as possible." Delaying the policy reform—retaining the status-quo policy in the interim—can be welfare improving over immediately adopting the optimal policy in some states of the economy. There is thus a *normative* case for postponing policy reform.[1] The case is stronger whenever economic conditions are atypical—when calls for reform are often louder—or the policies available are less flexible than ideal. The possibility of a non-trivial timing of monetary policy reform arises quite naturally in an economic context resembling that of the United States circa 2012: a sharp drop in output driving the nominal interest rate to the zero lower bound, yet only relatively moderate deflation pressures are observed.

The normative case for delaying a policy reform starts with a theoretical observation: the design of optimal policy in dynamic economies typically faces a trade-off between the policy effects in the short and in the long term, and possibly across future states of nature. Even if complete history-contingent policies are possible, future policy actions determine both present

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[1] A large strand of the literature has instead sought to explain delays in policy reform as a feature of the political economy. See Alesina and Drazen (1991), among many others.

and future allocations. Typically, these welfare effects will not be identical at both horizons. The short-term outlook is tied to the initial state of the economy while the economy will converge back to the ergodic distribution in the long term. Quite intuitively, if the initial state of the economy is atypical—that is, unlikely from the point of view of the unconditional distribution—the trade-offs may be steeper. Similarly, the trade-offs can also be steeper if history- or state-contingent policies are not admissible, as then there are less degrees of freedom to tailor policy across horizons and future states of nature.

As a result of the aforementioned trade-offs, the conditional optimal policy is a compromise across horizons and possibly across future states of nature. This opens up the possibility that a delay in policy reform is welfare improving. First, the status-quo may actually outperform the conditional optimal policy in the short term. Second, postponing policy reform has an "option value," since the policy authority will choose the optimal policy conditional on the realization of the future state of nature—or will choose to delay the policy reform even further. In a general framework I show how to compute the optimal timing of reform recursively and formalize the link between the trade-offs in policy design and a welfare-improving reform delay. There are two important assumptions in my analysis worth spelling out upfront: first, the status-quo policy is itself an admissible choice at the time of reform and, second, the policy reform remains unanticipated even if delayed.[2]

The timing of *monetary* policy reform proves to be of particular interest. I consider a simple log-linearized New Keynesian model with persistent cost-push and real interest rate shocks. Policy is given by a targeting rule on inflation and output gap deviations, without backward-looking variables, and the nominal interest rate must observe its zero lower bound.[3] The targeting rule is flexible enough to encompass policy discretion, which is assumed to be the status-quo policy. In a benchmark model where the zero lower bound is ignored, the optimal policy problem does not actually face any trade-off across horizons or states. Thus immediate policy reform is always optimal. The benchmark case highlights the key role that the zero lower bound plays in the results.

Once the zero lower bound is observed, I find that delaying the policy reform improves welfare if the economy finds itself at the zero lower bound due to a sharp drop in output, yet inflation remains close to the target. These economic conditions can only be rationalized under a large, negative real interest rate shock accompanied by a mild positive cost-push shock. Under such an initial state, the design of optimal policy then faces a steep intertemporal trade-off. To provide relief in the short term, policy would ideally allow an upward shift in inflation expectations. To do so, the targeting rule would need to be more pro-active with output stabilization given the underlying upward price pressures. Such a policy, though, clashes with the optimal prescription for the long run: away from the zero lower bound, stabilization policy greatly benefits from anchoring medium-term inflation expectations. Delaying the policy reform allows the monetary

[2]The first assumption guarantees that the policy reform occurs in finite time. The second assumption implies that the private sector's expectations are proven wrong at the time of the policy reform. This is, of course, implicitly assumed whenever optimal policy is solved for at date $t = 0$. I should note that an anticipated reform may allow the policy authority to manipulate future expectations without adjusting present policy, which actually would deliver an undue advantage to a reform delay.

[3]This is the kind of higher-level policy description that central banks have actually adopted as their framework. A targeting rule also keeps the problem of the timing of the policy reform tractable and the results transparent.

authority to keep a more accommodating policy in the interim and revisit the choice of an optimal targeting rule at a later date, when trade-offs are not so steep. Assuming perfect foresight, I can actually provide sufficient conditions such that a delay in policy reform is optimal as well as a simple characterization of the conditional optimal policy problem.

In order to solve the stochastic economy I must resort to numerical methods.[4] The model is too simple for a complete quantitative analysis, so for the necessary choice of parameter values I turn to the literature as well as target some simple moments on output, inflation and the incidence of the zero lower bound.

The results in the stochastic economy confirm the insights from the perfect-foresight economy. In addition, I also find that the option value of a reform delay contributes to the enlargement of the range of states when delay is beneficial as well as to the lengthening of the expected time to reform. I can also characterize the impact of policy on the conditional and unconditional forecasts for the nominal interest rate, though I do not find large differences between forecasts. The possibility of delaying the policy reform is shown to be robust to alternative parameter values regarding the slope of the Phillips curve, the persistence of both shocks, and the output elasticity of the real interest rate.

To my knowledge, this paper is the first to show that there can be a normative case for delaying policy reform. From a positive standpoint, though, the apparent failure of policymakers to implement superior and readily available policies is one of the fundamental questions in political economy. Alesina and Drazen (1991) argue that the delay in policy reform is due to a war of attrition between political groups who disagree on how the burden of policy reform should be distributed. In another classic piece, Fernandez and Rodrik (1991) argue that uncertainty regarding the distribution of gains and losses from reform leads to a bias toward the status-quo. There is also a large research literature that attempts to capture the political constraints policymakers face: see Dewatripont and Roland (1991) for an early leading example.[5]

There is a large literature on optimal monetary policy as well on the zero lower bound; too large indeed to allow a meaningful summary of either here. Yet their intersection is clearly related to the issues discussed here. Eggertsson and Woodford (2003), developing several key themes from Krugman (1998), is a key contribution to understanding how expectations of future policy impact stabilization while the nominal interest rate is at the zero lower bound.[6] The main mechanism here operates on the same channel despite the substantial differences in the analysis. Adam and Billi (2006) and Nakov (2008) characterize the optimal policy in a stochastic economy where stays at the zero lower bound are recurrent. In particular, Adam and Billi (2007) analyze the performance of discretionary policy—assumed here to be the status-quo policy—in an economy with a zero lower bound.[7] More recent work has sought to analyze policy in

[4]The model is solved using adaptive expectations. The overall algorithm is quite intensive as I must solve the optimal policy problem for each possible realization of the state of the economy. Unfortunately I cannot use a sparse grid because it is crucial that atypical states are well represented despite having a low likelihood from the point of view of the ergodic distribution.

[5]A much larger literature is concerned instead with the set of policies that are available given an institutional arrangement: the question of the credibility of policy immediately springs to mind. See Volume 1 in Persson and Tabellini, eds (1994) for an overview.

[6]See also Jung et al. (2005).

[7]An important difference with these studies is that I consider persistent cost-push and real interest rate shocks.

medium-scale dynamic stochastic general equilibrium models. See Fernandez-Villaverde et al. (2012) for example.

This paper is organized as follows. Section 2 develops the theoretical underpinnings of the timing of policy reform in a general framework. The New Keynesian model is described in Section 3. The results are then presented sequentially: Section 4 analyzes the benchmark case, where the zero lower bound is ignored; Section 5 contains the analytic results regarding the perfect foresight economy; and finally the stochastic model is solved for in Section 6. Robustness exercises are collected in Section 7. If the reader wishes to go straight to the results on monetary policy, it is possible to do so: Sections 3 to 7 are self-contained. Finally, Section 8 offers some concluding remarks.

2 Design and timing of policy reform

I start my analysis by describing formally the optimal policy problem in a general setup, paying special attention to the trade-offs involved. I then introduce the possibility of delaying the policy reform and clarify the assumptions behind it, namely, that policy reform remains unanticipated and credible even if carried out at a later date. The optimal timing of the policy reform has a simple recursive formulation, which I use to show the tight link between an optimal delay and the trade-offs in the optimal policy design.

2.1 Setup

Time is infinite and discrete, indexed by $t = 0, 1, \ldots$. Let $s_t \in S$ be the exogenous state of the economy at date t, which evolves according to a first-order Markov stochastic process, $F(s'|s)$, with full support, that is, $dF(s'|s) > 0$ for all $s, s' \in S$.[8] I use the notation s^t and S^t to denote histories up to date t and their set, respectively, and extend F to be defined over set S^t. At each date the monetary authority undertakes a policy action, denoted $\xi(s^t) \in \Xi$. The set of possible actions Ξ is assumed to be time- and state-invariant.

A **policy plan** $\xi = \left\{\xi(s^t) \in \Xi : s^t \in S^t, t \geq 0\right\}$ is a complete description of policy actions at all dates and for all histories. Let Ξ^∞ be the set of all policy plans. I assume there are no endogenous state variables. Hence policy is the only possible source of history dependence. It is useful to have a notation of *continuation* plans of a node s^t, $\xi|s^t \equiv \left\{\xi(s^j) : s^j \in S^j | s^t \in s^j, j \geq t\right\}$. Note that $\xi|s^t \in \Xi^\infty$ for all $s^t \in S^t$.

An underlying set of equilibrium conditions maps any policy plan ξ into allocations and those in turn into a per-period loss function $l : S \times \Xi^\infty \to \Re_+$. The welfare loss given node s^t and policy plan ξ is given by

$$L_t\left(s^t;\xi\right) = \sum_{j=0}^{\infty} \beta^j \int_{S^j} l\left(s_j;\xi|s^j\right) dF(s^j|s_t) \tag{1}$$

Considering both shocks simultaneously is key to the characterization of the conditional optimal policy: I believe this point to be novel. Adam and Billi (2007) also assume that the cost-push shock is i.i.d., which removes the stabilization bias from discretionary policy.

[8] The state space S is assumed to be compact, and F is assumed to have a unique ergodic distribution.

where $\beta \in (0,1)$ is the discount factor, and $L_t : S^t \times \Xi^\infty \to \Re_+$ is assumed to be bounded for all nodes and policies. Note L_t is defined over histories of length t. From now on I will dispense with the explicit integration and use the expectation operator E instead.

It is important to make explicit what is implicit in the welfare function (1): the policy plan is evaluated as an invariant parameter in the economy. In other words, the policy plan ξ is assumed to be credible and the private sector expects it to remain in place at all times.

There are a host of factors that may constraint the design of monetary policy. Concerns regarding communication or transparency may rule out complex policy descriptions, or perhaps there are limits to which policies the monetary authority can credibly commit to. I capture these constraints without modeling them explicitly by restricting the policy plan to belong to a subset $\Psi \subseteq \Xi^\infty$. If $\Psi = \Xi^\infty$ we retain complete flexibility with commitment in the policy design. Though at this point I am not ruling any possibility out, I will be particularly interested in policy plans spanned by simple rules. For example, Ψ may be the set of policy plans spanned by a broad collection of interest-rate policy or targeting rules; each member $\psi \in \Psi$ would correspond to a particular parameter vector within the collection. Another possibility is that $\psi \in \Psi$ indexes a particular objective function for the monetary authority, which retains discretion.[9]

2.2 Optimal policy design

I start with the problem of choosing a policy $\psi \in \Psi$ when the current state of the economy is $s_t \in S$. The history up to the reform date is irrelevant, and thus the optimal policy design solves $\min_{\psi\in\Psi} L_0(s_t;\psi)$, or

$$\min_{\psi\in\Psi} l(s_t;\psi) + \beta E\left\{L_1(\{s_t, s_{t+1}\};\psi)\,|s_t\right\}. \tag{2}$$

A function $\psi^* : S \to \Psi$ is an **optimal policy** if it achieves the minimum (2)—which is denoted $L^*(s)$—for all states $s \in S$.[10] To be clear, $\psi^*(s)$ denotes the complete policy plan chosen when the optimal policy design problem is solved with s being the initial state, that is, the conditional optimal policy.[11]

The crucial observation here is that the optimal policy problem will typically face a trade-off between the policy effects in the short and the long term, and possibly across future states of nature as well. Consider first the purely inter-temporal trade-off across horizons. In a dynamic economy, future policy actions will affect both present and future allocations—but we should not expect the welfare effect to be identical at both horizons. After all, the short-term outlook is tied to the current state of the economy, while in the long term, the economy will converge back to the ergodic distribution. There is little guarantee that the welfare effects over the conditional and

[9] I do require some assumptions regarding Ψ to ensure that no new policies become available arbitrarily at later dates—or previously available policy choices stop being so. Concisely, the set Ψ must be closed with respect to history truncation, that is, $\psi|s^t \in \Psi$ for all $\psi \in \Psi$ and all $s^t \in S^t, t \geq 0$.

[10] The optimal policy may not be unique as the policy actions prescribed for nodes that are not a continuation of s_t are completely irrelevant from the point of view of the welfare loss conditional on current state s_t. It may be unique if Ψ imposes a constraint across states, e.g., $\psi \in \Psi$ indexes a single parameter in a Taylor rule.

[11] An alternative definition, which is often in use in the literature, is the unconditional optimal policy, where the choice of policy is based on the unconditional loss function, $EL_0(s;\psi)$. This is not a well-defined problem unless some additional restrictions are imposed on Ψ that are more restrictive than needed here. However, the concept is simple enough that I will use it occasionally as a benchmark in the discussion.

unconditional forecast line up with each other. For example, medium-term inflation expectations will determine price setting both in the short and medium term. Unless some strong conditions are in place, neither the output-price elasticity nor the welfare effect of output and inflation at each horizon will coincide. Moreover, the set Ψ may exclude time-dependent policies for whatever reason, limiting the degrees of freedom available to tailor policy at all horizons.

Perhaps the most straightforward way to formalize the intertemporal trade-off is to say that the joint minimization does not achieve the minimum loss at both horizons simultaneously,

$$l(s; \psi^*(s)) > \min_{\psi \in \Psi} l(s; \psi),$$
$$E\left\{L_1(\{s, s'\}; \psi^*(s))|s\right\} > \min_{\psi \in \Psi} E\left\{L_0(s'; \psi)|s\right\}.$$

That is, from the narrow perspective of either the short or the long term, the optimal policy is actually sub-optimal.[12] For convenience, I took the current period to be the "short term" and all periods thereafter as the "long term." The frequency at which the model is evaluated is irrelevant for the discussion here, so the distinction is only for ease of exposition.

In addition, the design of the optimal policy may face a trade-off across future realizations of the state of nature. For example, the set Ψ may constrain the flexibility of the policy plan with respect to future eventualities by ruling out state-contingent policy actions. The optimal policy will thus balance the welfare effects along a particular realization against those in alternative realizations. Even if the optimal policy plan manages to minimize the welfare loss at both horizons, it may not achieve the minimum welfare loss for *every* future realization $s' \in S$,

$$\min_{\psi \in \Psi} E\left\{L_0(s'; \psi)|s\right\} > E\left\{L^*(s')|s\right\}.$$

Whenever either trade-off is present, the optimal policy will be a compromise between the distinct welfare effects across horizons and/or across future states of nature.

2.3 Delaying policy reform

Let $\psi_0 \in \Psi$ be the status-quo policy, which will be in place until the policy reform occurs. Since $\psi_0 \in \Psi$, immediate policy reform cannot be worse than no policy reform at all. The question is whether delaying the policy reform achieves a lower efficiency loss than switching to the optimal policy at the initial date.

It is important to emphasize that the policy reform is always assumed to be an unanticipated event by the private sector—even if the reform occurs at a later period. That is, in the interim period the private sector believes the status-quo policy will remain in place indefinitely. They will thus be proven wrong at the date of the policy reform. By doing so I put policy reforms at all dates on an equal footing: otherwise we would be comparing unanticipated with anticipated reforms.[13]

[12]If the underlying loss function is not differentiable with respect to policy, or ψ^* lies at a non-interior point of Ψ, one of the two conditions could hold with strict equality.

[13]I should also note that an anticipated reform can actually be more effective than an unanticipated one. For example, in a New Keynesian economy an anticipated reform pins down the medium-term expectations while allowing for aggressive output stabilization in the short term.

Let us start by evaluating a one-period delay to the policy reform. Given initial state s_0, the economy will attain $L^*(s_0)$ if the policy reform is carried in the first period. By delaying the policy reform one period, the status-quo policy remains in place at date $t = 0$, and the conditional optimal policy is then chosen at date $t = 1$, after observing the realization of the next period's state, s_1:

$$l(s_0; \psi_0) + \beta E \{L^*(s_1)|s_0\} \tag{3}$$

By comparing the welfare loss under a one-period delay (3) with the welfare loss under immediate reform, $L^*(s_0)$, we find that the delay is beneficial if

$$l(s_0; \psi_0) - l(s_0; \psi^*(s_0)) \leq \beta E \left\{ L_1(s^1; \psi^*(s_0)) - L^*(s_1)|s_0 \right\}. \tag{4}$$

The left-hand side is the difference in the per-period welfare loss at the initial date: if positive, the conditional optimal policy is outperforming the status quo. The right-hand side is the difference in the expected continuation welfare loss. Note that the order of terms is inverted.

The inequality (4) illustrates the connection with the trade-offs the optimal policy design faces. As discussed earlier, the optimal policy will typically compromise between its performance in the short and the long term, and thus will not achieve a minimum loss at both horizons. Thus, the left-hand side of (4) can have either sign, for we are comparing two sub-optimal policies. The right-hand side is always non-negative, capturing the option value of waiting to reform once the next period state is known. In the trivial case where policy has no effect on the initial period, $l(s_0; \psi_0) = l(s_0; \psi^*(s_0))$, delaying the policy reform is always optimal, as it would allow the monetary authority to pick the conditional optimal policy given next period's state s_1.

2.4 Optimal reform timing

It is easy to characterize the optimal timing for the policy reform. Let $J : S \times \Psi \to \Re_+$ be the welfare loss prior to policy reform, in state $s \in S$, given status-quo policy $\psi_0 \in \Psi$. The value of policy reform is simply $L^*(s)$. Delaying the reform implies that the status-quo policy is in effect for the current period, $l(s; \psi^0)$ and then in the next period the question of whether to reform or not will be revisited, $E \left\{ J(s'; \psi^0)|s \right\}$. The loss function J is then given by the Bellman equation:

$$J(s; \psi^0) = \min \left\{ l(s; \psi^0) + \beta E \left\{ J(s'; \psi^0)|s \right\}, L^*(s) \right\}. \tag{5}$$

The above equation describes a contraction mapping within the set of non-negative functions such that $J(s; \psi^0) \leq L^*(s)$ for all $s \in S$. Hence there is a unique solution J.

The optimal reform timing is the stopping time to event $\left\{ J(s; \psi^0) = L^*(s) \right\}$. Since $L(\psi_0; s) \geq L^*(s)$, equation (5) implies that $J(s; \psi^0) = L^*(s)$ on a measurable subset $S_0 \subseteq S$, and the expected stopping time $T : S \to \Re_+$ is bounded above.

2.5 What makes an optimal delay more likely?

The steeper the trade-offs faced by the optimal policy problem, the more likely that delaying the policy reform will be beneficial. Indeed, the presence of trade-offs—across horizons and/or across states—in the optimal policy design is a necessary condition for the desirability of a policy reform delay. Assume the optimal policy problem does not face any trade-off across horizons and states, and thus the joint minimization does not impede the optimal policy from achieving the minimum loss at each horizon and future state, that is,

$$l(s;\psi^*(s)) = \min_{\psi\in\Psi} l(s;\psi),$$
$$E\left\{L_1(\{s,s'\};\psi^*(s))|s\right\} = E\left\{L^*(s')|s\right\},$$

for all $s \in S$. Simply by substituting in (2) I obtain that

$$L^*(s) = \min_{\psi\in\Psi} l(s;\psi) + \beta E\left\{L^*(s')|s\right\} \tag{6}$$

for all $s \in S$. From (6) it is then quite clear that delaying the policy reform cannot deliver strictly higher welfare in any state, that is, $J(s;\psi^0) = L^*(s)$ for all $s \in S$. For any status-quo policy ψ^0, $l(s;\psi^0) \geq l(s;\psi^*(s))$ by (6). Thus

$$l(s;\psi^0) + \beta E\left\{L^*(s')|s\right\} \geq L^*(s)$$

and $J = L^*$ is a fixed point for (5). It is possible to show that if the optimal policy problem does not face any trade-offs, then the conditional optimal policy is invariant to the state of the economy at the time of the policy reform, and thus it is trivially equal to the unconditional optimal policy. Among dynamic economies, this seems to be satisfied only by linear-quadratic models in which the effects of policy and the shocks are multiplicatively separable. I would actually provide an example of such an economy as a benchmark later.

Next I argue that an optimal delay is more likely if the state of the economy is more atypical, that is, less likely from the point of view of the ergodic distribution; or the set of admissible policies Ψ is more restrictive, e.g., state-contingent policy plans are not available to the policy authority. Before proceeding further, though, I must give a more precise content to the assertion that a beneficial delay is more likely. To this end, I use the status-quo policy as the dimension along which I evaluate the likelihood that a policy delay is optimal. Let Ψ_D be the set of status-quo policies such that delay is strictly optimal at the initial date, $\Psi_D = \{\psi \in \Psi | J(s_0;\psi) < L^*(s_0)\}$. If I find that $\Psi_D \subset \Psi'_D$ when comparing two economies, I can assert unambiguously in which economy a delay is optimal for a larger set of status-quo policies.

The connection with the trade-offs in the optimal policy design is quite clear. A natural metric for the trade-offs faced by the optimal policy design problem is the distance from a (counterfactual) no trade-off world: $C_1 = |l(s_0;\psi^*(s_0)) - \min_{\psi\in\Psi} l(s_0;\psi)|$ for the intertemporal dimension and $C_2 = |E\{L_1(\{s,s'\};\psi^*(s))|s\} - E\{L^*(s')|s\}|$ for the across-future-states dimension. Given an initial state, an increase in C_1 and C_2 weakly expands the set Ψ_D.[14] To see this, it is sufficient to show that a one-step delay is now more likely to be optimal. The condition (4) is clearly more relaxed, as the performance of the conditional optimal policy in the first period decreases and the option value of delaying increases.

In the long run, the economy is expected to converge to its ergodic distribution, while in the short term the outlook will be driven by the current state of the economy. If the current state is atypical, that is, quite unlikely from the point of view of the unconditional forecast,

[14]It is of course possible to scale proportionally the loss functions and change the magnitude in C_1 and C_2 without changing Ψ_D.

the conditional forecast is bound to be quite different—and probably policies that provide relief in the short run are damaging in the long run and vice versa. In contrast, short- and long-term forecasts should line up quite well in a typical state since, by definition, the latter has a significant probability mass in the ergodic distribution. Thus the more atypical the state, the steeper the trade-offs the optimal policy problem faces—worsening both metrics C_1 and C_2.

A restrictive set of available policies that, say, does not include history- or state-contingent policy actions will also worsen the trade-offs in the optimal policy problem, as there are fewer degrees of freedom to uncouple the policy effect across the eventualities the policy cannot be made contingent to. Naturally the chances of an optimal delay look even better if the policy reform will impose a more restrictive framework than the current one.

At the present level of generality, though, there is no meaningful sufficient condition for delay to be optimal given a status-quo policy. Obviously, if the current policy is very ineffective, then policy reform should be implemented as soon as possible. More interestingly, a status-quo policy that is biased toward the short term makes an optimal delay more likely. Recall that an unambiguous advantage of delaying the policy reform is the option value allowing the policy choice to be tailored to the future state of the economy. If, as expected, the economy converges back to its ergodic distribution, the status-quo policy will not remain for long as the policy reform will be enacted soon. Thus the long-term costs of a status-quo policy are irrelevant for the decision to delay the policy reform.

3 A simple New Keynesian model

I briefly describe here what is possibly the simplest New Keynesian model around that takes the zero lower bound on the nominal interest rate into account. The model has well-known structural foundations that will not be discussed here; see Woodford (2003) for all the details. Policy is given by a simple targeting rule on inflation and output.

3.1 The economy

Let π_t and y_t be the inflation rate and the output gap, in log-deviations from the steady state, at date $t = 0, 1, \ldots$. The standard New Keynesian Phillips curve (NKPC henceforth) is

$$\pi_t = \kappa y_t + \beta E_t \pi_{t+1} + u_t \tag{7}$$

where $\beta \in (0, 1)$ is the intertemporal discount factor, $\kappa > 0$ governs the slope of the NKPC, and u_t is a so-called cost-push shock. I assume the latter follows an autoregressive process of the first-order,

$$u_t = \rho_u u_{t-1} + \varepsilon_t \tag{8}$$

where $\rho_u \in [0, 1)$ is the autocorrelation coefficient and ε_t is a Gaussian innovation with standard deviation σ_ε.

The second equation of the trinity is the log-linear approximation to the interrtemporal Euler equation, equating the nominal interest rate R_t to the real interest rate plus expected inflation,

$$R_t = \nu \left(E_t y_{t+1} - y_t \right) + E_t \pi_{t+1} + v_t. \tag{9}$$

Parameter $\nu > 0$ is equal to the intertemporal elasticity of substitution, and v_t is a real-rate shock also following an autoregressive process of the first order,

$$v_t = \rho_v v_{t-1} + \zeta_t \tag{10}$$

with $\rho_u \in [0, 1)$ and ζ_t distributed normally with zero mean and standard deviation σ_ζ. The nominal interest rate must respect the zero lower bound (ZLB henceforth),

$$R_t \geq R^0 \tag{11}$$

where $R^0 < 0$ is the ZLB in terms of log-deviations from the steady state.

3.2 Policy

Monetary policy is given by a simple targeting rule of the form:

$$\pi_t + \psi y_t = 0, \tag{12}$$

at all dates t such that $R_t > R^0$. The set of admissible policies is $\Psi = \{\psi > 0\}$. Under a targeting rule, the monetary authority is committed to adjusting its policy instrument to satisfy—whenever possible—a particular criterion.[15] The target specified by (12) is a weighted sum of output and inflation deviations, where the parameter ψ governs the weight given to the output gap. I label the targeting rule as "simple" because it has no form of history-dependence, that is, it contains no backward-looking terms. Such a limitation is not without loss: policy rules with history dependence will outperform the targeting rule (12). In addition, the ZLB implies that the monetary authority will occasionally fail to achieve its target, namely, when doing so would require a negative nominal interest rate.

Why specify policy as a simple targeting rule rather than, say, an interest-based rule? The first and foremost rationale is that actual monetary policy frameworks, as well as most of the alternatives considered in practice, are higher-level policy descriptions. A simple targeting criterion can be easily communicated and/or explicitly coded in legislation. As such, it captures the kind of commitment that central banks are capable of. A targeting rule also leaves a lot of room for judgment on how to achieve the targets—a practical necessity given the uncertainty regarding the economy. Inflation-targeting regimes of various kinds fit the definition, as their descriptions acknowledge a role for output stabilization. Incidentally, I should also note that we have yet to see a central bank adopting any form of an explicit history-dependent interest-rate rule.

There are some additional considerations behind my choice of (12). Being a single-parameter policy equation, the targeting rule keeps the problem of the timing of the policy reform tractable and transparent. It has the added advantage that it is flexible enough to encompass policy discretion, which will be my preferred choice for the status-quo policy. I will also show later that if the ZLB is ignored, there is no case for delaying policy reform for any status-quo policy

[15] The normative case for targeting rules as an alternative to interest-rate rules is developed in Svensson (2003). Woodford (2003) argues that the rule should be viewed as a criterion "projected to be satisfied, according to the central bank's forecast of the economy" (page 521).

belonging to Ψ. In other words, the existence of the ZLB is a necessary condition for an optimal delay in this economy.[16]

3.3 Equilibrium equations

Taking as given policy ψ, the equilibrium can be concisely defined by two systems of stochastic difference equations, which I label "regimes," and a third equation that determines whether the ZLB binds or not at a given date, and thus which regime governs the equilibrium.

At every date t the economy is in one of two regimes. The first regime corresponds to dates such that the ZLB is not binding. It is termed **LQ** as the system of equations is identical to the linear-quadratic model that arises when the ZLB is ignored at all dates:

$$\begin{pmatrix} \pi_t \\ y_t \end{pmatrix} = \begin{pmatrix} 1 & -\kappa \\ 1 & \psi \end{pmatrix}^{-1} \begin{pmatrix} \beta & 0 \\ 0 & 0 \end{pmatrix} E_t \begin{pmatrix} \pi_{t+1} \\ y_{t+1} \end{pmatrix} + \begin{pmatrix} 1 & -\kappa \\ 1 & \psi \end{pmatrix}^{-1} \begin{pmatrix} u_t \\ 0 \end{pmatrix}. \tag{13}$$

Note that conditional on being in the LQ regime, the real interest rate shock v_t has no contemporaneous effect.

The second regime determines allocations at dates when the ZLB is binding and it is rather unimaginatively named the **ZLB** regime.

$$\begin{pmatrix} \pi_t \\ y_t \end{pmatrix} = \begin{pmatrix} 1 & -\kappa \\ 0 & \nu \end{pmatrix}^{-1} \begin{pmatrix} \beta & 0 \\ 1 & \nu \end{pmatrix} E_t \begin{pmatrix} \pi_{t+1} \\ y_{t+1} \end{pmatrix} + \begin{pmatrix} 1 & -\kappa \\ 0 & \nu \end{pmatrix}^{-1} \begin{pmatrix} u_t \\ v_t - R^0 \end{pmatrix}. \tag{14}$$

Finally the Euler equation (9), together with the system (13), is used to determine whether the ZLB is binding or not. Let R_t^{lq} be the underlying, unbounded nominal interest rate in regime LQ. Then if

$$R_t^{lq} \equiv \nu E_t y_{t+1} + \left(1 + \frac{\nu\beta}{\kappa + \psi}\right) E_t \pi_{t+1} + \frac{\nu}{\kappa + \psi} u_t + v_t < R^0 \tag{15}$$

then regime ZLB applies at date t. If $R_t^{lq} \geq R^0$ then quite trivially $R_t = R_t^{lq}$. The stochastic processes for cost-push and real-rate shocks, (8) and (10), close the set of equilibrium equations.

3.4 Equilibrium definition

The state of this economy at date t is given by $s_t \equiv (u_t, v_t)$, and the corresponding state space is $S \equiv \Re^2$. There are no endogenous state variables. In particular, neither the policy equation nor the ZLB condition introduces a form of persistence.

The equilibrium definition takes as given the targeting rule, indexed by $\psi \in \Psi$. That is, the private sector expects *no variation*—either deterministic or stochastic—in policy. Thus policy reforms will be unanticipated, yet credible. I also impose a minimum state solution, so inflation, output and the nominal rate are functions of the state.

[16] On a further note, it is well known that the targeting rule has a counterpart in the form of an interest-rate rule, but not necessarily the other way around. The targeting rule can also be easily implemented by modifying the monetary authority objective.

Definition 1 *A* ***competitive equilibrium*** *given* $\psi \in \Psi$ *and initial condition* s_0 *is a triple of functions* $\{\pi(s;\psi), y(s;\psi), R(s;\psi)\}$ *such that (13)-(15) are satisfied at all nodes* S^t *and dates* $t \geq 0$.

It should be noted that the log-linearized set of equations may be a poor approximation of the underlying non-linear economy, as argued by Braun et al. (2012). It is hard to gauge whether this is the case for the present analysis: as far as I know, the optimal policy problem—or the discretionary equilibrium—remains computationally too taxing for the fully non-linear economy.

3.5 Welfare

As is commonplace in the literature, I use a quadratic loss function for welfare evaluation. The period loss function is given by

$$l_t = \pi_t^2 + \lambda y_t^2 \tag{16}$$

with $\lambda > 0$. Agents have time-separable preferences, and discount future periods at rate $\beta > 0$. Thus welfare loss is given by $L_t = E_t \sum_{j=0}^{\infty} \beta^j l_{t+j}$, where E_t is the conditional expectation given the state of the economy at date t, s_t. In terms of the notation in Section 2, $l(s;\psi)$ and $L(s;\psi)$ are the period and welfare loss functions, respectively, under state s and $\psi \in \Psi$.

3.6 Status-quo policy

Finally, I need to endow the economy with a status-quo policy, ψ^0. I choose to go with policy discretion, which, as I show below, actually takes the form of a targeting rule, $\psi^0 \in \Psi$, in a Markov-perfect equilibrium. This property is very convenient, so the status-quo policy does not have an undue advantage against a policy reform. From Section 2, if $\psi^0 \in \Psi$ then the policy reform will occur in finite time. Discretion is actually strictly sub-optimal due to the stabilization bias.[17] Leaving convenience aside, discretion seems an acceptable description of the dual mandate of the Federal Reserve Act, which does not stipulate a pecking order between the goals of full employment and price stabilization.

Under discretion, the monetary authority sets the nominal interest rate to minimize the welfare loss,

$$\min_{R_t \geq R^0} E_t \sum_{j=0}^{\infty} \beta^j \left(\pi_{t+j}^2 + \lambda y_{t+j}^2\right)$$

taking as given the private sector's expectations regarding inflation and output, $E_t\pi_{t+1}, E_t y_{t+1}$, and the equilibrium conditions (7) and (9). I am looking for a Markov-perfect equilibrium, where allocations and policy are a function of the state. Thus the monetary authority understands that welfare in future periods is beyond its reach. As long as the lower bound on the interest rate (11) is not binding, the necessary and sufficient condition for the Markov policy is

$$\pi_t = -\frac{\lambda}{\kappa} y_t.$$

[17] There is no inflationary bias in this economy.

This is a targeting rule indeed, with $\psi^0 = \lambda/\kappa$. Of course, if the ZLB is binding, then policy is simply given by $R_t = R^0$, as with all targeting rules. I assume that $\nu\kappa > \lambda$, that is, the weight on output deviations in the welfare loss function is bounded above. This ensures that the monetary authority's response to output is not so strong that it overturns the usual dynamics of the real interest rate.

4 Ignoring the zero lower bound

The first logical step in highlighting the role of the ZLB is, of course, to drop it and use the resulting economy as a benchmark case. This turns out to work nicely for targeting rules, as the optimal policy problem does not face any trade-off across horizons or future states in this case. Thus, per the results in Section 2, delaying the policy reform is never optimal.

By removing its only non-linearity, the economy boils down to a linear-quadratic model given by (13). Fluctuations in the output gap and the inflation rate are entirely driven by cost-push shocks: in the absence of a ZLB, the targeting rule leaves enough leeway for the monetary authority to completely undo real-interest rate shocks by adjusting the nominal interest rate appropriately. The model can be solved analytically for any targeting rule $\psi > 0$:

$$\begin{aligned}
\pi_t &= \frac{\psi}{\kappa + \psi - \beta\psi\rho_u} u_t, \\
y_t &= \frac{-1}{\kappa + \psi - \beta\psi\rho_u} u_t, \\
l_t &= \frac{\psi^2 + \lambda}{(\kappa + \psi - \beta\psi\rho_u)^2} u_t^2.
\end{aligned}$$

Note the per-period loss is multiplicatively separable in u_t^2, $l(s_t;\psi) = B(\psi)u_t^2$. The separability carries over to the welfare loss, which can be written as $L(s_t;\psi) = A(s_t)B(\psi)$ where A is a strictly positive function. By iterating on the j-step forward conditional variance, $E_t u_{t+j}^2$, I obtain

$$A(s_t) = u_t^2 + \frac{\sigma_\varepsilon^2}{1-\rho_u^2}\frac{\beta}{1-\beta} + \left(u_t^2 - \frac{\sigma_\varepsilon^2}{1-\rho_u^2}\right)\frac{\rho_u^2\beta}{1-\rho_u^2\beta}.$$

It is easy to see that the optimal policy problem faces no trade-offs. At any state or horizon, the optimal policy simply solves $\min_{\psi\in\Psi} B(\psi)$, and thus

$$\psi^*(s_t) = \psi^* = (1-\beta\rho_u)\frac{\lambda}{\kappa}.$$

Obviously, the unconditional optimal policy is also ψ^*. Loosely speaking, the economy is homothetic in policy, so the state of the economy does not alter the relative benefits and cost of any targeting rule. To be clear, both the linear-quadratic form of the economy and the targeting rule are behind this result. Other policy rules, like a Taylor rule, may imply that real interest rate shocks influence output and inflation. Similarly, if the NKPC equation had two independent shocks with different persistence, the welfare loss would no longer be multiplicatively separable, and the optimal policy problem would face a trade-off across horizons.

Since the optimal policy can achieve the minimum loss at all horizons and all states, immediate policy reform is optimal. It is still worthwhile to ask when the policy reform is most *valuable*. For any $\psi^0 \neq \psi^*$, the difference $L(s_t;\psi^0) - L^*(s_t)$ is strictly increasing in $|u_t|$ as $B(\psi^0) - B(\psi^*) > 0$ by the optimality of policy and $A(s_t)$ is strictly increasing in u_t^2. The latter is due to the fact that conditional j-step ahead variance of the cost-push shock is minimized at $u_t = 0$ and is increasing in u_t^2. The very same calculation applies to comparing the per-period loss function, $l(s_t;\psi^0) - l^*(s_t)$. In the benchmark case, thus, the larger the magnitude of the cost-push shock, the larger the efficiency improvement in the event of a policy reform. Comparing with the baseline choice for the status-quo policy, the discretion policy is more permissive with inflation variation, $\psi^m \geq \psi^*$, with strict inequality whenever the cost-push shock has any persistence $\rho_u > 0$.

Also worth noting is the role played by the persistence of the cost-push shock, ρ_u. In a linear-quadratic economy, the only channel which through future policy affects present welfare is the expectation regarding the response to future cost-push shocks, $E_t u_{t+1}$. If the latter is expected to be zero—as would be the case with i.i.d. shocks—then next period inflation expectations will be zero as well. This is, of course, also at the bottom of the stabilization bias: discretion and the optimal policy coincide if the cost-push shock is i.i.d.

To close this short section, I note that an anticipated reform that takes place, say, after one period can actually lead to better outcomes. The reason is that by tying down the medium-term expectations, policy discretion is capable of stabilizing output at a much more favorable sacrifice ratio.

5 Optimal delay and the short-term outlook

A common approach in the early work on the ZLB was to assume that the ZLB is binding initially and to assume perfect foresight on the equilibrium path back to the non-stochastic steady state.[18] This approach turns out to be quite useful here, as it allows a clear look at the optimal policy design problem. It does require us, though, to make a call regarding the initial state of the economy—as the ZLB may be binding under different states. Fortunately, it is possible to provide sufficient conditions on the state of the economy—in terms of observable variables—for a policy reform delay to be optimal. The main insights obtained here are largely confirmed when I solve for the stochastic economy in the next section.

5.1 When is the ZLB binding?

Assume there is perfect foresight regarding the path of the shocks $\{u_t, v_t\}$. The economy starts at date $t = 0$ with an initial state $s_0 = \{u_0, v_0\}$ and innovations from date $t = 1$ onward are zero, $\varepsilon_t, \zeta_t = 0$ for $t \geq 1$, with probability one. The first question is under which states of the economy s_0 the nominal interest rate is initially at the ZLB. I also assume that the ZLB does not bind from date $t = 1$ onward for simplicity, as the analysis carries over for any arbitrary number of periods: the key simplification brought in by perfect foresight is that, once the economy leaves

[18] See Jung et al. (2005). Eggertsson and Woodford (2003) assume that the real interest rate shock is initially at a large, negative value and reverts to an absorbing zero state with an exogenous probability.

the ZLB, it never returns to regime ZLB. Indeed, the results in this section do not depend on the frequency at which the model is evaluated.

The equilibrium conditions from the LQ regime for output and inflation expectations imply that the nominal interest rate at date $t = 0$ is

$$R^{lq} = b_\psi u_0 + v_0 \tag{17}$$

where

$$b_\psi = \frac{\nu(1-\rho_u) + \rho_u \psi}{\kappa + (1-\beta\rho_u)\psi}.$$

The condition $R^{lq} \leq R^0$ has thus a simple typology. The economy may have been driven to the ZLB by a large negative real-rate shock, $v_0 < 0$, or a large negative cost-push shock, $u_0 < 0$.

What are the implications for output and inflation in each scenario? From (14), knowing that next period output and inflation expectations are given by (13) instead, I can substitute in the Euler equation to obtain a simple expression for output

$$y_0 = -d_\psi u_0 + \frac{1}{\nu}\left(v_0 - R^0\right) \tag{18}$$

where

$$d_\psi = \frac{\nu - \psi}{\kappa + (1-\beta\rho_u)\psi}\frac{\rho_u}{\nu},$$

which is strictly positive under the previous assumption $\nu > \psi$. Inflation is pinned down by the NKPC (7), which can be simplified to

$$\pi_0 = \kappa y_0 + c_\psi u_0 \tag{19}$$

where

$$c_\psi = \frac{\kappa + \psi}{\kappa + (1-\beta\rho_u)\psi}.$$

With equations (18) and (19) I am now in place to describe each scenario in terms of observable variables. For reasons that will soon become clear, the effect of both shocks in both scenarios is assessed.

Real-rate shock scenario. The targeting rule naturally accommodates positive real-rate shocks by raising the nominal rate. Similarly, small negative shocks do not present a problem. However, the ZLB will be binding under a large negative real-rate shock. Once the nominal interest rate can no longer absorb the shock, current output must contract in order to restore the equilibrium real interest rate. Everything else constant, the fall in output will bring inflation down as well. This is the classic deflation scenario outlined in Krugman (1998).

If the negative real-rate shock is large enough, then a binding ZLB can co-exist with a mild but positive cost-push shock. The latter would ease somewhat the upward pressure on the real interest rate but, by the everyday logic of cost-push shocks, will push inflation up and output further down. This combination of shocks can rationalize the joint observation of mild downward price pressures and the nominal interest rate being at the ZLB.

Cost-push shock scenario. A large, negative cost-push shock can drive the economy into deflation and, if the shock is persistent enough, into the ZLB due to inflation expectations falling sharply. The outlook for output, though, is now expansionary, $y_0 > 0$. The targeting rule is calling the monetary authority to fight deflation by stimulating the economy further, that is, lowering the nominal interest rate. Once at the ZLB, this is no longer possible, and the real rate must absorb part of the cost-push shock. Needless to say, this scenario does not conform with the experience of any developed economy at the ZLB.

A small, positive real interest rate shock may not be enough to get the economy out of the ZLB. Such a shock would actually bring welcome stabilization to the economy, as it achieves what the monetary authority would like to do: boost output in order to help inflation drift back to target.

5.2 The initial state of the economy

The distinction between conditional and unconditional forecasts are at the crux of my analysis. Before proceeding further, we must make a call on what the initial state of the economy is. I will compute numerically the stochastic economy later: for now, only the *qualitative* properties of the initial shocks are relevant. In particular, the sign of the cost-push shock plays a key role and its implications will be discussed extensively.

My aim is to replicate the U.S. experience at the ZLB—as modestly as a simple model as the present one allows. The Federal funds rate was set to virtually zero in December 2008, and so far it has remained there. Output contracted sharply in 2009, falling more than 3 percent in real terms: the unemployment rate reached 10 percent. Both 2010 and 2011 have seen positive but measly growth: real GDP just regained its 2007 level in the fourth quarter of 2011. The CBO currently estimates that the U.S. output gap over the period 2009-2011 was consistently above 6 percent and as high as 7.7 percent (second quarter of 2009).

There is thus little question that the scenario with a large, negative real interest rate shock fits the picture much better than the cost-push shock scenario. However, the description of the state of the economy cannot end here. As discussed earlier, such a shock is usually associated with deflation or, at the very least, a sharp and persistent fall in inflation. Yet inflation did little more than briefly bulge down. Core PCE did touch below 1 percent (annualized rate) for two quarters at the beginning of the crisis and once in 2010. However, it averaged over 1.7 percent both in 2009 and 2011. Consumer price indexes tell a similar story.

How can we reconcile the output drop with the small response of inflation? A positive cost-push shock implies a countering upward price pressure and helps to explain the sharp drop in output. Provided that the cost-push shock is small enough, the economy can still be at the ZLB. Actually, the case for a cost-push shock in addition to the real interest rate shock is quite forceful, as it is the only combination that can rationalize the observation that $y_0 < 0$ and $\pi_0 \approx 0$.

5.3 Conditional optimal policy

Let us consider first the optimal policy problem at date $t = 0$, that is, the case of immediate policy reform. As I argued earlier, the key to an optimal delay actually resides in the trade-offs the optimal policy problem would face if enacted in the current period.

In short, the ZLB brings a steep intertemporal trade-off in the optimal policy problem. To relieve the current situation, policy would ideally allow an upward drift in inflation expectations. To do so, the targeting rule would need to be more pro-active with output given that the underlying price pressures are upward, that is, $u_0 > 0$. Such a policy, though, clashes with the optimal prescription for the long run: away from the ZLB, stabilization policy greatly benefits from anchoring medium-term inflation expectations.

Under perfect foresight, once the economy exits the ZLB it will converge back to the steady state according to the LQ-regime equations (13). The long-term outlook is thus identical to the benchmark case presented in the previous section. There is no secret then about the optimal policy from the perspective of the long run: the targeting rule should emphasize price stability over full employment. I shall use the unconditional optimal policy in the benchmark economy, ψ^*, and the status-quo policy, ψ^0, as reference points to guide the discussion on the intertemporal trade-off that the optimal policy problem faces at date $t = 0$, with the understanding that the conditional optimal policy will not be exactly equal to ψ^*.

Regarding the short term, the first observation is that policy impacts current inflation and output only through their respective expectations for the next period.[19] As usual, inflation expectations shift the NKPC as firms price in some of the expected future price-level increases. Since the nominal interest rate is fixed at zero, inflation expectations require a one-to-one adjustment in the real interest rate. Output expectations also play a role in the latter, forcing current output to adjust to restore the equilibrium growth rate of output.

To disentangle these effects as well as the comparative statics with ψ, I return to equations (18) and (19). The latter is simply the NKPC once inflation expectations have been substituted by their corresponding equilibrium equation at date $t = 1$, that is,

$$E_0\pi_1 = \frac{\psi}{\kappa + \psi - \beta\psi\rho_u}\rho_u u_0.$$

The coefficient c_ψ simply adds up the direct impact of the cost-push shock. Since $u_0 > 0$, there is upward pressure on prices and downward pressure on output, as we would expect. The impact of policy here is the standard one. As is well known, medium-term inflation expectations undo output stabilization. The more accommodating the targeting rule—that is, a higher ψ—the bigger the shift in the NKPC—that is, a higher coefficient c_ψ. This is, in a nutshell, why the optimal policy in the benchmark economy emphasizes inflation stabilization. Under a positive cost-push shock, inflation is too high and output too low under the status-quo policy compared with the unconditional optimal policy.

Figure 1 provides some visual guidance to the discussion. Current output y_0 is on the horizontal axis and current inflation π_0 on the vertical axis. For policies ψ^* and ψ^0 I plot equations (18) and (19): their intersection determines the equilibrium allocation at date $t = 0$ for each policy. The NKPC (19) is the line labeled "NK": solid for the status-quo policy, dashed for ψ^*. It is upward sloping and intersects the horizontal axis—zero inflation—on the negative quadrant for output, reflecting that $u_0 > 0$: attempts to completely stabilize inflation

[19] It is very unlikely that the optimal policy lifts the economy from the ZLB in the current period. In any case, an analysis of the endogenous duration of the ZLB is provided in the next section.

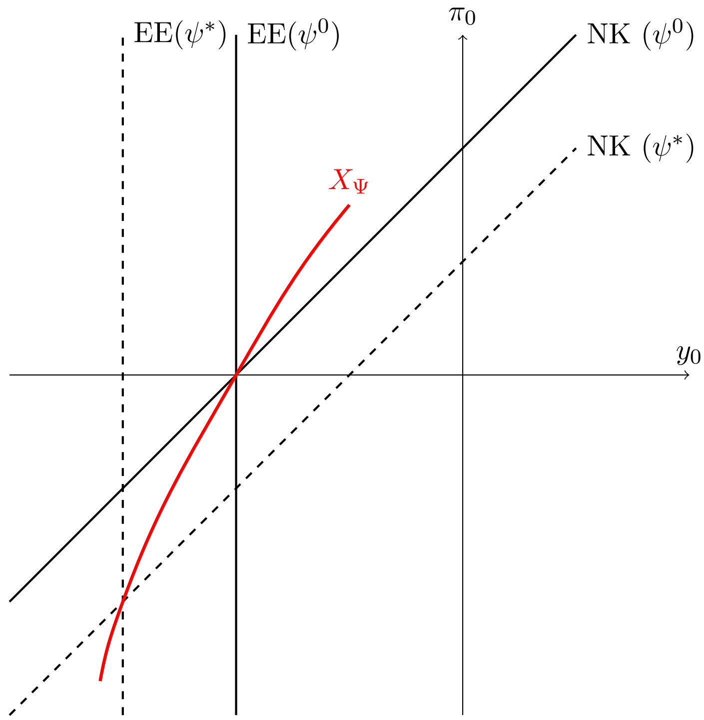

Figure 1: Short-term outlook and optimal policy

(output) against a positive cost-push will result in low output (high inflation). A switch to the unconditional optimal policy ψ^* brings the NKPC closer to the origin, as it tames the cost-push shock amplification created by the medium-term inflation expectations.

Turning to the Euler equation (18), now both inflation and output expectations play a role and have been substituted by their equilibrium conditions at $t = 1$. Note that current inflation does not appear in (18), and thus the Euler equation—in conjunction with the ZLB, of course—fully determines output. It is thus depicted as a vertical line in Figure 1, unimaginatively labeled "EE." Since $u_0 > 0$ and $v_0 < 0$, clearly $y_0 < 0$.

What is the impact of policy? The coefficient d_ψ is strictly decreasing in ψ, and thus the unconditional optimal policy ψ^* amplifies the negative effect of a positive cost-push shock. This is a key comparative static result and deserves a careful discussion. First, given a constant nominal interest rate, an increase in inflation expectations must bring the real interest rate down. The latter is a function of the expected growth rate in the output gap, $E_1 y_1 - y_0$. Let us now evaluate a more accommodating targeting rule—higher ψ. By definition, it seeks to temper output fluctuations at the cost of higher inflation volatility. Now, *conditional on a positive cost-push shock*, this means that inflation and output expectations drift up. As a result, the real interest rate will need to fall despite output expectations being higher, and thus current output increases.

We are finally in place to evaluate the impact of policy on the short-term outlook. Figure

1 depicts the solid lines—the equations belonging to the status-quo policy—intersecting right at the horizontal axis: under the current conditions and policy, we have $y_0 < 0$ and $\pi_0 \approx 0$. The dashed lines correspond to the equations under the unconditional optimal policy ψ^*: the output would contract further and inflation would drop sharply below target. Connecting both intersections, the locus labeled X_ψ plots the output-inflation pairs spanned by the set Ψ.

To recap, the design of the optimal policy at date $t = 0$ faces a steep intertemporal trade-off: emphasizing price stability works wonders for the long term but would dramatically worsen the short-term outlook, exacerbating the output drop and leading to deflationary pressures. Conversely, a more accommodating policy would stabilize output in the short run but would not be effective at all as soon as the economy left the ZLB. The actual optimal policy lies somewhere in between ψ^* and ψ^0, the exact value depending on parameters as well as the frequency at which the model is evaluated.

5.4 Optimal delay

I now turn to the possibility of delaying the policy reform. Since after one period the economy is identical to the benchmark case, there is no point in delaying the reform past $t = 1$. It is thus sufficient to evaluate the condition for a one-period delay (4). It is also clear that ψ^* would be the optimal policy if chosen at date $t = 1$.

It is now possible to provide a sufficient condition for a policy reform delay to be unambiguously beneficial. If the current status-quo policy is such that output is below the target, inflation is close to the target, and the nominal interest rate is at the ZLB, then delaying the policy reform will be optimal. The reason is quite simple. The observation that $y_0 < 0, \pi_0 \approx 0$ can only be rationalized by a state of the economy satisfying $v_0 < 0, u_0 > 0$, and a status-quo policy that is more accommodating than ψ^*. We then know that $\psi^*(s_0)$ lies in between ψ^* and ψ^0. From the previous discussion, under any policy $\psi^*(s_0) < \psi^0$, output and inflation will be strictly further from the target. Therefore,

$$l(s_0; \psi^*(s_0)) > l(s_0; \psi^0).$$

This is sufficient to establish that delay is optimal, though it is worth noting that there is an option value of postponing the policy reform even under perfect foresight,

$$L(s_1; \psi^*(s_0)) > L^*(s_1)$$

since $\psi^*(s_0) > \psi^*(s_1) = \psi^*$.

The sufficient condition above has the advantage of being stated in terms of observable variables. I should emphasize, though, that it is the condition that $u_0 > 0$ and $\psi^0 > \psi^*$ that drives the results. If we are confident that this is the case, even if inflation is substantially below target we should postpone the policy reform. Once again the switch to the conditional optimal policy will entangle a downward movement along the locus X_Ψ, which unambiguously worsens the short-term outlook. Notably, the result does not depend on β or ρ_u (beyond being strictly positive) and thus applies at any frequency we wish to evaluate the model.

Figure 2 plots, in a solid black line, the frontier of possible loss pairs at date $t = 0$ and beyond as spanned by Ψ at date $t = 0$. On the horizontal axis there is the period-loss function,

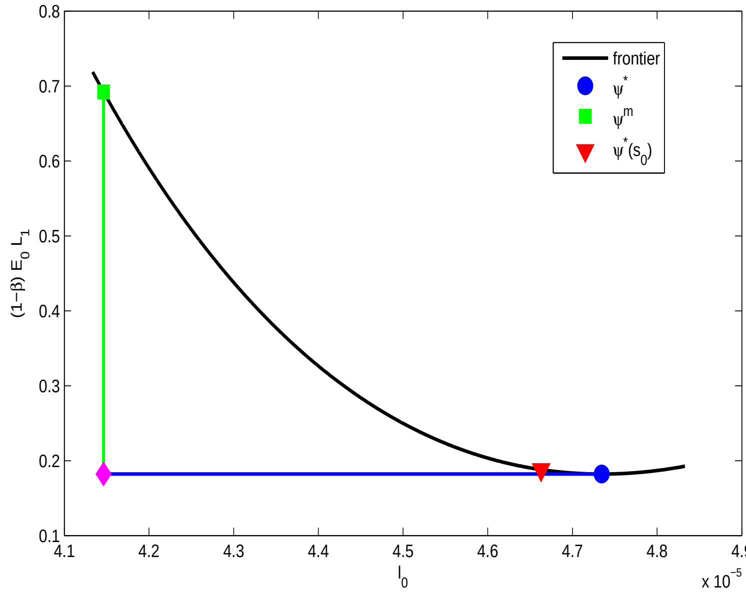

Figure 2: Short- and long-term loss frontier: Delay is optimal

$l(s_0;\psi)$, plotted against the continuation loss (normalized to per-period terms), $(1-\beta)L(s_1;\psi)$. On the frontier I display the location of the unconditional optimal policy ψ^*, as a circle marker, as well as the conditional optimal policy, $\psi^*(s_0)$, as a triangle marker, and the status-quo policy, set at discretion $\psi^0 = \psi^m$, as a square marker.[20]

The unconditional optimal policy minimizes the future loss, as expected. The conditional optimal policy is very close by, given the large weight the long-term allocations have versus the short-term outlook. The performance of both the conditional and unconditional optimal policy is very poor in the short term, when they are clearly dominated by the status-quo policy. Note that the latter does not achieve the minimal per-period loss at $t = 0$—and even more accommodating policy would improve the short-term outlook further.

Figure 2 also displays the outcome of delaying policy reform, as a diamond marker. In the short term the status-quo policy stays in place, while, in the long term, policy switches to the unconditional optimal policy. Delaying the policy reform thus achieves a better outcome than any policy at date $t = 0$, as it introduces a time-dependence that is not available in Ψ while simultaneously exploiting the unanticipated nature of the reform to re-adjust inflation expectations in the long run.

[20]The numerical values used are provided in the next section; here I adjusted the frequency of the model to one year. The initial state of the economy is such that inflation is on target and the output gap is about minus 4 percent. Only the relevant subset in Ψ is displayed. The location of the conditional optimal policy, $\psi^*(s_0)$, is slightly displaced to the left for visibility.

5.5 The case for immediate policy reform

There is quite a lot to learn about the determinants of an optimal delay by actually asking what supports the case for immediate policy reform. I briefly discuss here three deviations from the previous analysis that do the trick: an initial state of the economy with a negative or zero cost-push shock, an alternative status-quo policy, and a larger set of policy options. The first two assumptions break the premise that $\pi_0 \approx 0$ under the status-quo policy and thus they can be settled empirically. The last one, though, concerns what policies the monetary authority can effectively and credibly implement, a subject on which it is possible to speculate much.

Different initial state. The previous analysis was based on the premise that $y_0 < 0$ and $\pi_0 \approx 0$, which, in the context of the model, led us to conclude that $v_0 < 0$ and $u_0 > 0$. Here I consider the possibility that $u_0 \leq 0$, which would imply that inflation is substantially below target.[21] The first thing to note is that if the cost-push shock is exactly zero, policy has absolutely no impact in the short term.[22] This highlights the role played by expectations: if $E_0 u_1 = 0$, policy simply has no ability to influence the next period's forecast for inflation and output, and it is therefore unable to affect current variables.

Going one step further, the comparative statics are simply reversed if the cost-push shock is negative, $u_0 < 0$. Switching to the unconditional optimal policy now increases inflation and output expectations, which provides relief in the short-term. Immediate policy reform is thus advised.[23]

Worse status-quo policy. Consider the possibly far-fetched scenario that the ZLB is binding yet inflation is above the target, $\pi_0 > 0$, due to a very accommodating status-quo policy. The comparative statics remain unchanged: switching to the unconditional optimal policy will lower both inflation and output at date $t = 0$. Now, however, reduced inflation is welcome: whether it compensates the further drop of output or not depends on the particular parameters. Figure 3 displays an example where the status-quo policy has been set above ψ^m but I have retained the previous parameter configuration elsewhere and the initial state satisfies $u_0 > 0, v_0 < 0$. The frontier bends backwards, reflecting that once the policy is too accommodating, inflation deviations overcome the gains from output stabilization. The status-quo policy is then outperformed by the unconditional optimal policy even in the short term.

More policy options. Finally, I ask what policies would need to be available for the immediate policy reform to be a forgone conclusion. Clearly the monetary authority would like to have some time-dependent policies in order to decouple the effects in the short and long term. One possibility would be to literally commit to a moving targeting rule or to enlarge the target criterion with some backward-looking variables, e.g., the price level or past output gap values. As the literature has extensively established, the accommodating stage must outlast the economy's

[21] I do not see any value in discussing the case where the economy is at the ZLB because u_0 is positive and large enough: this scenario would feature an output expansion.

[22] This is no longer true in the stochastic economy, as policy shifts the probability that the economy re-enters the ZLB. In the next section I show that the set of states such that delay is optimal actually expands once I drop the perfect foresight assumption.

[23] Delaying the reform retains the option value, as the conditional optimal policy will now be below the unconditional, $\psi^*(s_0) < \psi^*$. However, if $\psi^0 > \psi^*$, the status-quo policy is strictly dominated by the unconditional optimal policy, and thus immediate policy reform is optimal.

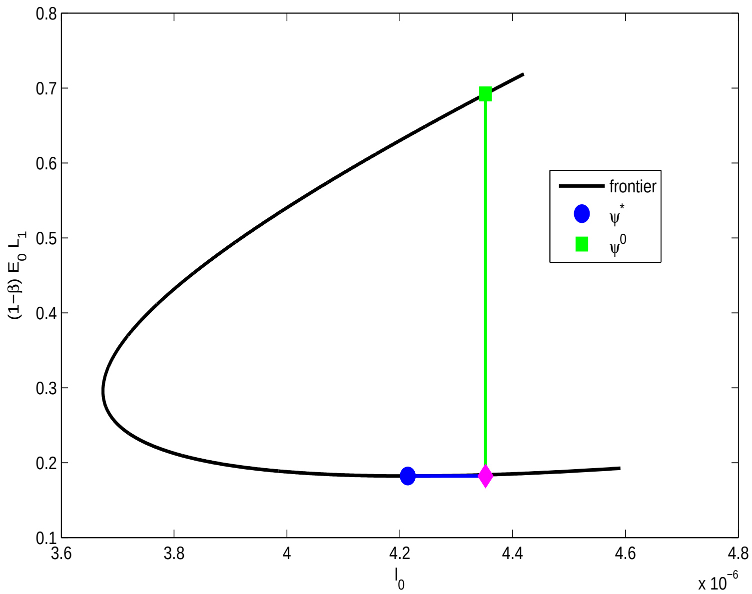

Figure 3: Short- and long-term loss frontier: Immediate reform is optimal

stay at the ZLB. Theoretically, delay retains some chance of remaining optimal even when history-dependent policies are available due to the option value.

6 Stochastic economy

It is finally time to evaluate the possibility of an optimal delay in the stochastic economy. To do so I must resort to numerical methods, which in turn demand a choice of parameters. The latter are documented in the first subsection, yet the present model is not suited to a full-fledged quantitative analysis. Rather my aim is to check that the results from the previous section hold in the stochastic, non-linear economy, look for additional insights, and explore several comparative statics of interest.

6.1 Calibration

The model is evaluated at quarterly frequencies. Let me start documenting the parameters set at standard values according to the literature. The intertemporal elasticity of substitution ν is set at 2 and the intertemporal discount rate β at .996 such that the annual real interest rate is 1.5 percent in the steady state. The third parameter borrowed from the literature is the slope of the NKPC, κ, which I set to .024. This is the value used in Rotemberg and Woodford (1997) and followed countless times for small-scale New Keynesian models. There is, though, enough dissenting opinions to return to the slope of the NKPC for robustness analysis.

The remaining parameters are chosen to match some basic moments for output and inflation, as well as the ZLB event, under the status-quo policy. I target the standard deviation of both

Parameter	Value	Source/Target
κ	.024	*Literature*
β	.996	*S.s. real interest rate*
ν	2	*Literature*
λ	.038	*Ratio output-inflation volatility*
ρ_u	.85	*Persistence output gap*
ρ_v	.93	*Average ZLB stay*
σ_ε	.09	*Inflation volatility*
σ_ζ	.63	*Prob ZLB binding*
R^0	-.035	$R_{ss} = 3.5\%$
ψ^0	.43	*Discretion equilibrium*

Table 1: Parameters and calibration targets

inflation (core PCE) and the output gap (as computed by the CBO) as well as the persistence of the latter. These are tightly linked to the cost-push shock process parameters and the status-quo policy: the real interest rate shocks contribute to inflation and output fluctuations only when the ZLB is binding or close to binding, that is to say, rarely. Regarding policy, I follow the previous sections and use discretion as the status-quo policy. I then obtain a link between the weight of the output gap in the loss function, λ, and policy, $\psi^0 = \lambda/\kappa$, which is set such that the ratio of output and inflation standard deviations match the data.

Finally, I use the real interest rate shock parameters to target the prevalence and persistence of the ZLB. As discussed in Section 5, the ZLB may be binding under an output expansion or recession. As the former is yet to be observed, I calibrate the parameters using the latter scenario only.[24] There is some disagreement on how likely it is that the nominal interest rate is binding at the ZLB: see Chung et al. (2012) for a discussion. My calibration renders the unconditional probability of the preferred ZLB scenario at just above 5%. It is remarkably difficult to generate a long expected ZLB duration, the longest being slightly more than three quarters.[25] It should be noted, though, that this is the duration of an average spell or, in other words, the unconditional expected duration. There are states of the economy for which the conditional forecast features a high probability of the nominal interest rate remaining at the ZLB past six and even eight quarters.

It is worth contrasting my choice of parameters with Adam and Billi (2007), who calibrate a similar model with discretionary monetary policy and the ZLB on nominal interest rates. Both the slope of the NKPC and the weight on the output gap in the loss function are very close; similarly Adam and Billi (2007) feature a quite persistent real-rate shock. The key distinction concerns the persistence of the cost-push shock: Adam and Billi (2007) assume it is i.i.d., while it has an autocorrelation of .85 for my chosen parameters. I will return to this point again in

[24]The (so far) counterfactual ZLB scenario is pervasive in New Keynesian models, large or small, though it is very often just ignored. See Fernandez-Villaverde et al. (2012) for an honest exception.

[25]What ties down the expected duration is the low probability of being at the ZLB: any increase in the persistence of the real interest rate shock must be accompanied with a reduction in its volatility—otherwise, the probability of being at the ZLB shoots up.

the robustness analysis, but it is worth noting here that a higher autocorrelation makes the status-quo policy perform worse.

The model is solved using adaptive expectations. The state space is discretized using the Rouwenhorst method on a square grid of 900 hundred points.[26] The overall algorithm is quite intensive, as I must solve the optimal policy problem for each possible realization of the state of the economy: to ease the computational burden, I approximate the conditional loss function with cubic splines.

6.2 Optimal policy

There are some noteworthy results regarding the conditional and unconditional optimal policy. First, I confirm previous work that argues that the benchmark economy does *not* provide a good approximation of the stochastic economy for normative purposes. Second, the conditional optimal policy is usually very close to the unconditional optimal policy—the exception being large, negative real interest rate shocks that drive the economy to the ZLB. In the latter case the conditional optimal policy is sensitive to the cost-push shock.

The first result is hardly surprising. Both Adam and Billi (2006) and Nakov (2008) show how the ZLB has a substantial impact in the design of optimal policy, whether the latter is in the form of a simple rule or a full history-contingent plan. I find that the properly computed unconditional optimal policy weighs price stability more than the benchmark case would suggest, reducing inflation volatility by more than 10 percent in the calibrated economy. The main reason behind the renewed emphasis on price stability is to drop the probability of the ZLB binding when output is below target.[27] Due to the non-linearity of the model, the targeting rule actually changes the ergodic mean for both output and inflation. However, I find that the deviations from the non-stochastic steady state are very small and do not drive the design of the optimal policy.

Let us turn our attention to the conditional optimal policy. Unlike in Section 4, the conditional optimal policy is not invariant to the state of the economy at the time of the policy reform. The variation, though, is quite small for most of the probability mass at the ergodic distribution. The main reason is a simple one: at a discount rate of virtually one, $\beta = .996$, the policy authority weighs the long run quite heavily. There is persistence in both shocks, but mean-reversion remains relatively strong. In particular, stays at the ZLB tend to be rare and short. The exception are states associated with a large, negative real interest rate shock that drives the nominal interest rate to the ZLB for a substantial spell. In these states, the conditional optimal policy departs substantially, signaling the presence of steep trade-offs in the optimal policy design problem.

Interestingly, the conditional optimal policy under a large negative real interest rate shock becomes quite sensitive to the cost-push shock. Figure 4 plots the conditional optimal policy,

[26] See Kopecky and Suen (2010) for a discussion of the Rouwenhorst method. I cannot use a sparse grid because it is crucial that atypical states are well represented despite having a low likelihood from the point of view of the ergodic distribution.

[27] Interestingly, the probability of the alternative scenario with the ZLB binding—featuring an output expansion—actually increases. This scenario is not so costly, as the expectations channel tends to act as an automatic stabilizer rather than an amplification mechanism.

$\psi^*(s_t)$, as a function of the cost-push shock, u_t, in standard deviations, and averaging across real interest rate shocks for three scenarios: unconditional and conditional to a negative and a positive real interest rate shock. The dotted line is the unconditional optimal policy.

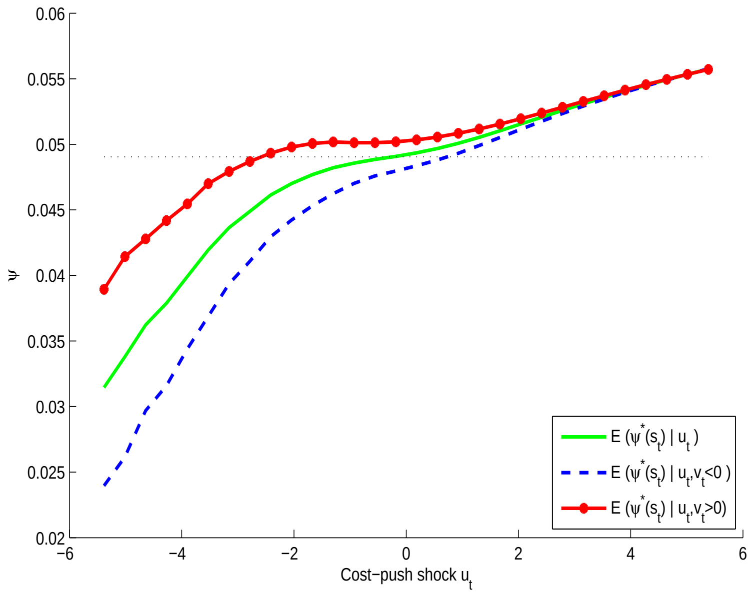

Figure 4: Conditional optimal policy

Figure 4 clearly shows that the optimal policy problem encounters a trade-off in most states, as it is willing to deviate from the unconditional optimal policy. The conditional optimal policy is always increasing in the cost-push shock, reflecting the stabilization role inflation expectations play if the ZLB is binding. It also shows *when* the trade-off is steeper. In particular, when both shocks are negative and large, the conditional optimal policy practically becomes an "inflation nutter." It is instead slightly more accommodating when cost-push shocks are large and positive. Recall that a large, negative cost-push shock drives the economy to the ZLB even if there is no real interest rate shock—the scenario with an output expansion. Not surprisingly then, the conditional optimal policy veers further toward price stabilization. For positive cost-push shocks, the ZLB becomes much less likely, so the conditional optimal policy is less sensitive. Nevertheless, it shifts slightly toward output stabilization, as in Section 5.

6.3 When is delay optimal?

Delaying a policy reform is optimal for a non-negligible subset of the state space, adding to more than 12 percent probability from the point of view of the ergodic distribution. The insights from Section 5 hold: delay is optimal whenever there is a large, negative real interest rate shock together with a positive cost-push shock. In these states output is substantially low, inflation is not too far from target, and the ZLB is binding under the status-quo policy. However, this characterization proves too narrow in the stochastic model: postponing policy reform may also be optimal when the economy is close to, but not at, the ZLB.

Figure 5 provides a more careful look at the states and allocations when immediate policy reform is not optimal. The left plot displays the full support for the shocks, in small black dots; and the states when delay is optimal, in red circles. Both axes are in standard deviations for each shock. The delay states are clearly concentrated in the Northwest quadrant. The range of cost-push shocks increases for larger negative real interest rate shocks, as a large positive cost-push shock can effectively extract the economy from the ZLB. Note how the delay states "spill" pretty close to the unconditional mean. Delay in these states shows that the option value can play a key role. Close to the origin, policy has close to no impact on inflation expectations in the short term and thus removes the key upside to switching to the optimal policy.[28] Delaying the policy reform then allows the policy authority to "wait and see": if the economy moves toward the ZLB, the policy reform will be postponed further.

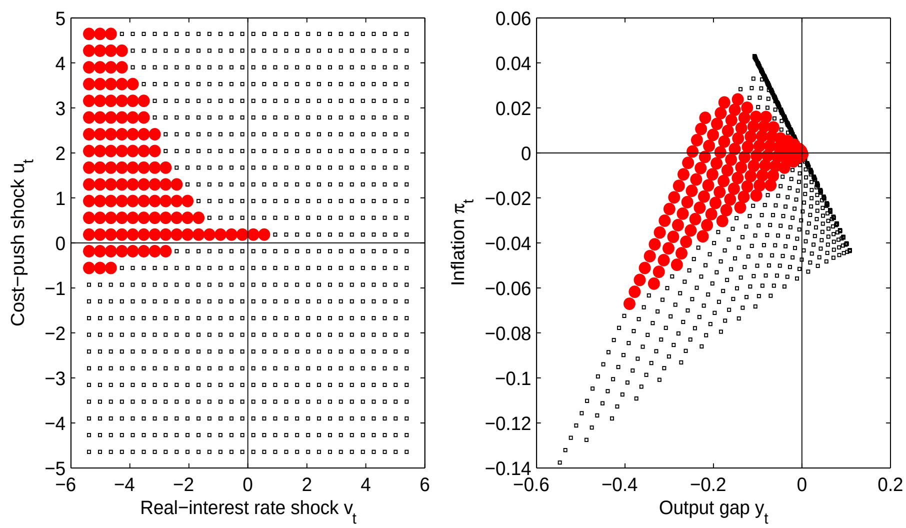

Figure 5: States and allocations when delay is optimal

The right plot in Figure 5 depicts the support for output and inflation in the same fashion as for the states of the economy. Most realizations of output and inflation lie in the locus $\pi_t + \psi y_t = 0$, which is visible as a downard-sloping straight line going through the origin. Additional output-inflation pairs fan toward the Southwest corner: these correspond to states such that the ZLB is binding or close to binding. As argued in Section 5, the telltale sign that delay is optimal is that output is clearly below target and inflation is not too far from it—all while the ZLB is binding. Figure 5 shows that the range of inflation values consistent with delay is actually quite broad and actually would encompass deflation (about 2 percent below target) if the output drop were sharp enough.

Another way to present the results is to offer the conditional probability that a delay is optimal. As mentioned before, the unconditional probability is just above 12 percent. If the ZLB is binding and output below target, the probability increases to 40 percent. The probability increases to 80 percent if we go one step further and condition on output being more than one

[28] At the extreme, if cost-push shocks are i.i.d. in the benchmark model, discretion and optimal policy coincide.

standard deviation below target and inflation less than one standard deviation above or below target.

6.4 Short-term forecast

Retracing the steps in Section 5, I now look at the short-term forecast under the status-quo policy and the conditional optimal policy. To do this, I have to pick an initial state when delay is optimal. I do so by targeting an inflation rate slightly below target—about a quarter of a percentage point—and an output-gap drop of 6 percent. As expected, these allocations correspond to a severe negative real interest rate shock coupled with a positive cost-push shock.

The results largely confirm the previous results: a switch to the conditional optimal policy actually worsens the short-term outlook substantially—and thus sticking with the status-quo policy in the interim is beneficial. Figure 6 plots the forecast for up to twelve quarters ahead for the inflation rate and the output gap in the top two panels. The solid line corresponds to the conditional optimal policy, $\psi^*(s_0)$. The dashed line is the forecast under the status-quo policy. It is pretty clear which one is the most benign forecast. A switch to the unconditional optimal policy brings a substantial worsening of the output gap compared to the status-quo policy, falling more than two full percentage points to close to -8 percent. The drop is also persistent, with output under the unconditional optimal policy being below that under the status-quo policy for more than ten quarters. An immediate policy reform would also bring a fall in inflation, of about two fifths of a percentage point. Here the gap between policies closes faster, because inflation is a little bit more persistent under the status-quo policy. This is, though, good news for output stabilization, as higher inflation expectations in the medium term help to stabilize output.

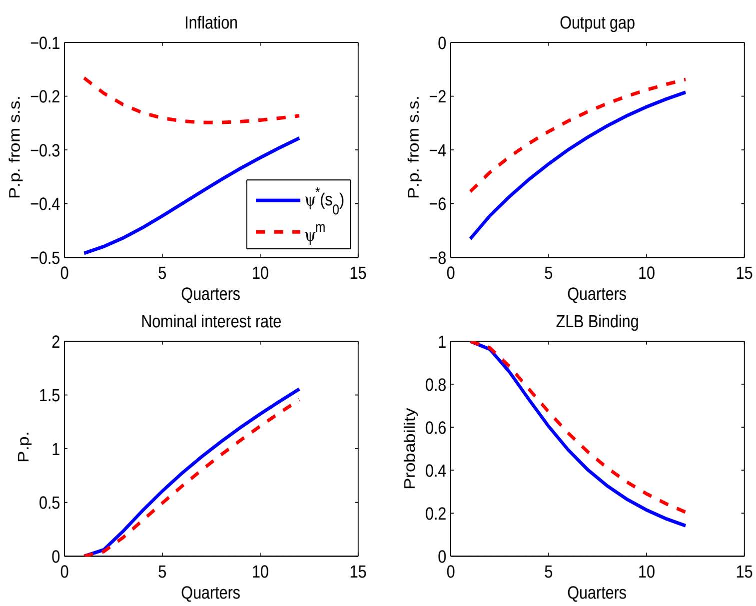

Figure 6: Conditional short-term forecasts under ψ^0 and $\psi^*(s_0)$

I can also now take a look at the forecast for the nominal interest rate, accounting for the possibility of re-entry into the ZLB. As the lower-left panel in Figure 6 shows, though, the forecast path for the nominal interest rate is very similar under both policies, dragging just a little bit behind under the status-quo policy. The lower-right panel displays more information, plotting the probability that the nominal interest rate will remain at the ZLB after n quarters.[29] Again, the differences are quite small, with the economy being just a little bit more likely to leave the ZLB under the conditional optimal policy. The expected duration of the ZLB is just above two years and varies very little according to policy.[30]

If immediate policy reform is not optimal, when is it? The expected optimal delay—conditional on the initial state—is just past five quarters, with the median duration being four quarters. The central moments, though, do not tell the whole story as the distribution has quite a long tail. There is a 16 percent chance that the delay exceeds two years: actually, conditional on the policy reform being postponed after one year, the probability of reform stays at 30 percent for the next three years, and the additional expected delay stays constant at about three quarters.

The mapping between the exit from the ZLB and the optimal reform is not as straightforward as the similar expected duration times would suggest. First, the economy may exit the ZLB—under the status-quo policy—but policy reform may be further delayed. This is actually what happens along the point-forecast path for the shocks: the cost-push shock remains positive, albeit small; and the real interest rate shock eventually is small enough that the economy just exits the ZLB. At that point, though, the possibility of re-entry is very high and the option value of delaying the policy reform is high—while, as discussed earlier, the short-term benefits of a policy reform are smaller. Second, the converse can also happen: the economy remains at the ZLB, yet the policy reform is enacted. The trigger in this scenario is a switch in the sign of the cost-push shock. This is quite likely, since the cost-push shock is small to start with. Inflation would then drop below target and output would contract further. More important, now an emphasis on price stability brings relief by stabilizing inflation expectations: the status-quo policy is then dominated at both horizons.

6.5 Ins and outs from the ZLB

Finally, I take a closer look at the probability of being at the ZLB in the short- and long-term horizons. To this end, I compare two conditional forecasts. The first forecast uses the initial state picked above. As the economy starts at the ZLB, I use this forecast to characterize the distribution over exit times, as well as the exit hazard function. The second forecast is conditional on the initial state being at the origin, $u_0 = v_0 = 0$, and is used to compute the entry times and the associated hazard function. In both forecasts, I compare the status-quo policy with the unconditional optimal policy.[31] As in the previous subsections, I focus on the

[29] I compute the probability that the nominal interest rate has stayed at the ZLB continuously, that is, I do not include the probability of re-entry.

[30] Note the difference with the unconditional expected duration, which is substantially shorter.

[31] Using the unconditional optimal policy rather than the conditional optimal policy for each state helps the exposition by limiting the source of variation across exit and entry rates to the forecast itself, and not different policy. In any case, the differences between the conditional optimal policy and the unconditional optimal policy

recession scenario for the ZLB binding.

Figure 7 documents the model predictions over a 24-quarter horizon. The left column displays the exit hazard and the probability distribution over exit times.[32] As we already saw in Figure 6, there are no big differences across policies. The exit hazard is only substantially lower for the status-quo policy well past the year horizon, without having much of an impact on the probability distribution over exit times. The difference in the expected exit time is less than a quarter.

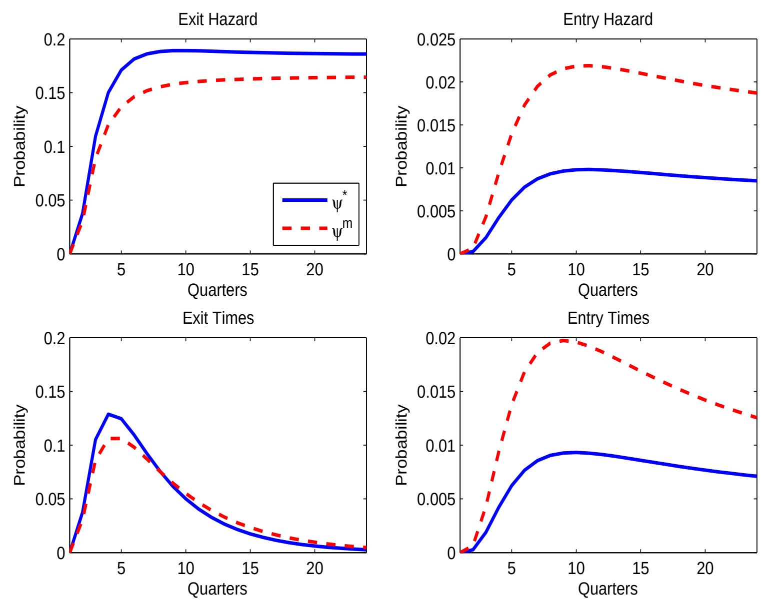

Figure 7: Entry and exit hazards and times under ψ^m and ψ^*

Forecast exit times can behave very differently for seemingly small changes in the state of the economy. In particular, the sign of the cost-push shock can switch the sign of the comparative statics with respect to policy. It is possible to get a good look at this by ignoring the probability of re-entry at the ZLB. In this case, the condition $R_t^{lq} \leq R^0$ in the benchmark economy is sufficient to determine the exit time. Recalling equation (17),

$$R_t^{lq} = b_\psi u_t + v_t,$$

it is clear that policy will impact the probability of the ZLB binding through the sensitivity of the nominal interest rate to cost-push shocks. Unfortunately, it is not possible to unambiguously establish how the coefficient b_ψ varies with ψ for all parameters. However, the policy effect on the coefficient is monotone: to fix ideas, the calibration implies that b_ψ is decreasing in ψ for all $\psi \in \Psi$.

are very small.

[32]To be clear, the exit hazard at n quarters is the probability of exiting at exactly n quarters conditional on having been at the ZLB during *all* previous quarters.

Consider the horizon at which the point forecast for the nominal interest rate first exits the ZLB,

$$T(s_0;\psi) = \min\left\{t : E\{b_\psi u_t + v_t | s_0\} \geq R^0\right\}.$$

The comparative statics of T with respect to ψ clearly depend on the sign of u_0.[33] Under a positive cost-push shock, an accommodating policy is expected to delay the exit time; but if the cost-push shock is negative, then it will actually speed up the exit time. This is a reminder that the target rule affects current allocations only through future expectations, and how the latter react to policy depends *qualitatively* on the current state.

It is harder to obtain results for the complete conditional distribution, because even in the benchmark economy R_t^{lq} is not Gaussian. We can gain some further insight into the stochastic model by assuming first that there is no cost-push shock innovation at date $t = 1$, $\varepsilon_1 = 0$, with probability one—and thus all the uncertainty is due to the real interest rate shock. The probability that $R_1^{lq} \leq R^0$ is

$$\Pr\left(b_\psi u_1 + v_1 \leq R^0 | u_0, v_0\right) = \Phi\left(\frac{R^0 - \rho_v v_0 - b_\psi \rho_u u_0}{\sigma_\zeta}\right)$$

where Φ is the standard normal c.d.f. For $u_0 > 0$, this is clearly decreasing in b_ψ—but increasing in b_ψ if instead $u_0 < 0$. Next consider the assumption that there is no real interest rate shock innovation, and thus now the uncertainty is exclusively due to the cost-push shock innovation. The same steps lead to

$$\Pr\left(b_\psi u_1 + v_1 \leq R^0 | u_0, v_0\right) = \Phi\left(\frac{R^0 - \rho_v v_0 - b_\psi \rho_u u_0}{b_\psi \sigma_\varepsilon}\right)$$

Recalling that $v_0 < 0$ and the ZLB is binding, the term $R^0 - \rho_v v_0$ must be positive, and thus the probability that $R_1^{lq} \leq R^0$ is decreasing in b_ψ for any sign of u_0.

The right column in Figure 7 displays the entry hazard and probability over entry times. The differences here are more substantial. The entry hazard is small for both policies, but substantially higher for the status-quo policy. Now the differences in the distribution of entry times are clear. The expected time to hit the ZLB under the status quo is a little bit more than 14 years—under the unconditional optimal policy, it is seven years more. The unconditional probability of being at the ZLB, it must be noted, is just 6 percent under the status-quo policy—a choice in the calibration. This probability is cut in half under the unconditional optimal policy.

I close this section noting that the probability of being at the ZLB under an output *expansion* displays completely different comparative statics. This alternative ZLB scenario is substantially more likely under the unconditional optimal policy than under the status quo: the total probability of being at the ZLB—independently of the sign of the output gap—actually increases under the unconditional optimal policy. It is not clear what to make of this result. The ZLB scenario with an output expansion is yet to be observed in the U.S. or elsewhere.

[33] To be clear, T is not the expected duration of the stay at the ZLB, though it proves to be a quite close approximation.

7 Robustness

In this section I vary some of the key parameters in the model and then ask whether there are still states such that a policy-reform delay is optimal and, if so, whether they are fundamentally different from those in Section 6. When changing one parameter, it is necessary to adjust some additional parameters if the calibration is still going to match the moments of the data reported before. As much as possible, I use the parameters governing the stochastic shock processes for this purpose.

7.1 A flatter Phillips curve

The slope of the Phillips curve κ has been traditionally subject to a fair amount of controversy. This parameter plays a key role in the transmission from inflation to output, and thus the real effects of policy. For the purposes here, a lower slope would also challenge the argument behind my characterization of the initial state of the economy as one with a positive cost-push shock: perhaps the small inflation dip only reflects a very flat Phillips curve.

In order to maintain the model's predictions regarding inflation and output volatility in line with the data, I adjust the standard deviation for cost-push and real interest rate shocks, as well as the weight of the output gap in the loss function. Note that the latter also changes the status-quo policy. It may seem misleading to tweak the welfare function and policy but actually both changes are a must. First, there is a structural link between the slope of the Phillips curve and the weight of the output gap in the loss function, $\lambda = \kappa/\theta$, where θ is the elasticity of substitution. Second, if the status-quo policy is not adjusted correspondingly, the ratio of inflation and output volatility drifts quite far from the data.

I set $\kappa = .01$, well below most calibrations in the literature.[34] I find that the probability that a delay is optimal increases substantially to 20 percent using the ergodic distribution. More important, the range of inflation-output pairs such that the delay is optimal expands: even under deflation—that is, with inflation more than 2 percent below target—an immediate policy reform is not optimal. Similar results are obtained with intermediate and lower values of κ.

Why does a flatter Phillips curve help the case of delay? The key observation is that a flatter Phillips curve renders the output *more* responsive to a cost-push shock when the economy is at the ZLB. The channel, of course, is inflation expectations. This added sensitivity makes the trade-off between the short and long term more acute. More subtly, it also raises the option value of a policy delay, which is key to explaining the expanded range of output-inflation pairs when delay is optimal. This also explains why, despite no change in the persistence of the shock process, expected delay times are definitively longer.

7.2 Less persistent cost-push shock

Next I turn to consider the possibility that the cost-push shock is less persistent than under the baseline calibration. There is no consensus on this parameter in the literature, and some researchers assume the cost-push shocks are i.i.d. In the simple model here, both inflation and the output gap essentially inherit the persistence of the cost-push shock as there are no

[34]The adjusted parameters are $\lambda = .04$, $\sigma_\varepsilon = .076$, and $\sigma_\zeta = .66$.

endogenous propagation mechanisms.[35] Thus, in the calibrations to follow, both inflation and output are substantially less autocorrelated than in the data. The only adjustment needed corresponds to the standard deviation of the cost-push and real interest rate innovations—the latter in order to restore the unconditional probability of the ZLB at 5 percent.

I find that even a moderate reduction in the persistence of the cost-push shock $\rho_u = .75$ massively expands the set of states where delay is optimal.[36] The main reason is actually quite trivial: the unconditional optimal policy is now much closer to the status-quo policy. In the benchmark equilibrium, the relationship between discretion and the optimal policy is simply $\psi^* = (1 - \beta\rho_u)\psi^m$ and while it is far from exact, it accurately shows how the gap closes between both policies as the persistence falls.

In short, while a lower cost-push shock persistence short-circuits the connection between policy and inflation expectations, the persistence is also the sole reason the status-quo policy is sub-optimal compared to the optimal targeting rule, and thus lowering it reduces the case for policy reform.

7.3 Worse status-quo policy

It is quite obvious that a worse status-quo policy will strengthen the case for immediate policy reform. The exercise is still worth doing as it will spot "robust" delay states—namely, those for which delay is still optimal even if the status-quo policy is worse than before. To this end, I fix the status-quo policy at .43, the value needed to match the relative volatility of inflation and output, but explore lower values of the weight of the output gap in the loss function, λ. This automatically increases the distance between the unconditional optimal policy and the status-quo policy—which is no longer tied down to the discretion equilibrium.

I reduce λ to .028, which approximately means that inflation volatility, under the unconditional optimal policy, should be 25 percent lower. As predicted, the probability of a delay falls to a little bit less than 6 percent—about half the previous value. Delay remains optimal for large negative real interest rate shocks and positive cost-push shocks, but the range is narrower. In terms of output and inflation pairs, delay remains optimal for sharp output drops coupled with the ZLB binding and inflation below, but close to, the target. The condition, though, is tighter. For example, delay is almost never optimal if there is deflation or the economy is out of the ZLB. Interestingly, it is the option value part of the delay decision that seems to be most diminished by the increase in the distance between the optimal and the status-quo policy.

Splitting λ once more by half makes delay very rare. It must be said, though, that at very low levels of λ the gap between the current policy and the optimal policy is very large, with inflation volatility falling five-fold and output volatility increasing three-fold compared with the data.

[35]The persistence of the real interest rate shock is only relevant when the economy is at the ZLB or very close to it.

[36]The standard deviation of cost-push shock innovations is now $\sigma_\varepsilon = .16$, and the standard deviation of the real interest rate shock needed only a small adjustment to $\sigma_\zeta = .6$.

7.4 Other parameters

I also explored alternative values for several other parameters. Below I include a short discussion regarding the intertemporal elasticity of substitution and the persistence of the real interest rate shock.

Several researchers use a higher intertemporal elasticity of substitution, mainly in order to reduce the sensitivity of the real interest rate to output growth. I find that higher values of ν increase the set of states in which delay is optimal.[37] The main reason is that, at the ZLB, we should not be talking about the elasticity of the real interest rate to output growth: the real interest rate is fixed by the ZLB and the real-rate shock, so what matters is how much output growth needs to adjust to restore equilibrium. Higher sensitivity means that inflation expectations play a more important role.

An interesting source of sensitivity is the persistence of the real interest rate shock. I explore a parameter value of $\rho_v = .75$, which requires me to increase substantially the standard deviation of the real interest rate to restore the unconditional probability of the ZLB binding. In this calibration, though, virtually all the spells at the ZLB are very short-lived. The probability of delay being optimal falls substantially, though it still remains optimal in the very few states where the conditional probability of the nominal rate being at the ZLB in the medium term is not very low.

8 Conclusions

It seems that calls for policy reform proliferate when the economy is under stress: there is undoubtedly added pressure on policymakers to "do something." Hopefully my analysis is viewed as a word of caution against any rush to reform: there is a normative case for delaying a policy reform, especially when economic conditions are atypical. The theory behind a policy reform delay is quite general but the possibility arises naturally in the context of monetary policy and economic conditions loosely resembling those of the United States circa 2012.

I foresee two objections of merit to my analysis. The first one is the assumption that a policy reform remains unanticipated even if delayed; second, policymakers may be able to circumvent the policy design trade-offs in atypical situations by using interim measures or richer policy designs.

Regarding the first objection, my assumption puts policy reforms, no matter when they occur, on equal footing. It is perhaps more constructive to elaborate on what an anticipated policy reform actually implies. One possibility is that the policy authority announces at date $t = 0$ the timing—and perhaps the terms as well—of a policy reform. If such an announcement were credible, the case for policy reform delay would be stronger in many cases. For example, in the context of the New Keynesian model, it would allow an aggressive output stabilization policy without minimal variation in future inflation expectations—a win-win situation. Another possibility is to approach the timing of the policy reform as the result of a game between the private sector and policymakers. This seems a promising avenue for a positive theory of reform

[37] I explored parameter values of $\nu = 4, 6$. In each case the standard deviation of the real interest rate shock needs to be adjusted upwards in other to keep the probability of the ZLB binding constant.

indeed but the focus of the present paper is exclusively normative.

Turning to the second objection, I should emphasize that even if a complete history-contingent set of policies is available, optimal delay is possible in dynamic economies. But I should equally acknowledge that my analysis of monetary policy relies on a restricted set of policies, that of "simple" targeting rules. In the context of my model, price-level targeting offers a superior outcome—especially if the economy is stuck at the zero lower bound. Similarly, interim policies can effectively stabilize the current situation without tying down policy objectives in the long run. There is a question of whether the central bank can credibly decouple short- and long-term policies, and what challenges a backward-looking target such as the price level would offer. But I have no qualms about the assertion that if more flexible policy designs are available, the case for a delay in policy reform delay is weaker.

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