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INVENTORIES, AND LEVERAGE**

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# Market Run-Ups, Market Freezes, Inventories, and Leverage\*

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## **Abstract**

We study trade between an informed seller and an uninformed buyer who have existing inventories of assets similar to those being traded. We show that these inventories may induce the buyer to increase the price (a “run-up”) but may also make trade impossible (a “freeze”) and hamper information dissemination. Competition may amplify the run-up by inducing buyers to purchase assets at a loss to prevent competitors from purchasing at lower prices and releasing bad news about inventories. In a dynamic extension, we show that a market freeze may be preceded by high prices. Finally, we discuss empirical and policy implications.

# 1 Introduction

Consider the sale of mortgages by a loan originator to a buyer. As widely noted, the originator has a natural information advantage and knows more about the quality of the underlying assets than do other market participants. One consequence, which has been much discussed, is that the originator will attempt to sell only the worst mortgages.<sup>1</sup> However, a second important feature of this transaction has received less attention. Both the buyer and the seller may hold significant inventories of mortgages similar to those being sold, and they may care about the market valuation of these inventories, which affects how much leverage they can take. Consequently, they may care about the dissemination of any information that affects market valuations of their inventories. In this paper, we analyze how inventories affect trade—in particular, prices and information dissemination. Our setting applies to the sale of mortgage-related products, but more broadly, to situations in which the seller has more information about the value of the asset being traded.

Our main result is that the effect of inventories on trade depends on the buyer’s and seller’s initial leverage, or more precisely, on how tight their capital constraints are. When capital constraints are moderately tight, concerns about the value of existing inventories lead to higher prices (a market “run-up”). However, when capital constraints are very tight (i.e., initial leverage is very high), trade becomes impossible (a market “freeze”) and information dissemination ceases.

Our results cast light on several features of the market for structured financial

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<sup>1</sup>See, for example, Ashcraft and Schuermann (2008), and Downing, Jaffee, and Wallace (2009).

products that have attracted much attention. First, it is widely believed that these products were overpriced in the period leading up to the financial crisis. This is illustrated by Fig. 1, which shows a sharp divergence between foreclosure rates and mortgage backed securities (MBS) prices.<sup>2</sup> It is also consistent with many anecdotal accounts. For example, Lewis (2010, page 164) suggests that “from mid-2005 until early 2007, there had been this growing disconnect between the price of subprime mortgage bonds and the value of the loans underpinning them.” Second, this market collapsed in the financial crisis, as illustrated by Fig. 2. Third, the collapse of this market attracted concern not just because of the associated fall in potentially socially beneficial trade, but also because it severely hampered information dissemination (see, e.g., Scott and Taylor, 2009).

In our basic model, there is one buyer and one seller, and the (uninformed) buyer makes a take-it-or-leave-it offer to the informed seller. The motive for trade is that the buyer values the asset by  $\Delta > 0$  more than the seller. The buyer has existing inventories of the traded assets, and he incurs a large cost if the market value of his inventories falls below some threshold. For example, creditors may not roll over the buyer’s debt and the buyer may go bankrupt. We refer to this as the buyer’s capital constraint and assume that, before trade begins, the capital constraint is satisfied.

The intuition for our main results is as follows. Whenever the seller agrees to sell at a price  $p$ , the market infers that the value of the asset is less than  $p$ , and

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<sup>2</sup>In 2006, MBS prices also diverged from other proxies for the health of the housing sector, and hence, the value of the underlying mortgages. Examples include construction spending, pending home sales, and the share price of large residential construction firms such as Toll Brothers.

so the value of the buyer's existing inventories drops. This may lead to a violation of the buyer's capital constraint. When the buyer's capital constraint has sufficient slack, the buyer can prevent a violation of his constraint by increasing the price while still maintaining positive profits; hence, we obtain a price run-up. However, when the capital constraint is tight, the buyer can no longer increase the price without losing money. At this point, the buyer prefers not to make any offer; hence, trade completely breaks down, and the market learns nothing about the value of the asset.<sup>3</sup> We obtain similar results when the seller is subject to a capital constraint and must retain a fraction of his assets on his balance sheet.

In a dynamic extension of our basic model (Section 4), we show that when the buyer has high leverage and holds inventories of two assets with independent values, trade in the first asset leads to a tightening of the buyer's capital constraint, which in turn leads the buyer to either increase the price he offers for the second asset, or else stop trading. One implication is that high leverage may lead to a run-up in prices that is followed by a market freeze.

We also study an extension in which two buyers compete by making offers to a single seller. In this case a buyer may be forced to acquire assets at a loss-making price, just to make sure that a competing buyer does not acquire them at a lower

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<sup>3</sup>Of course, asymmetric information by itself can lead to a reduction in trade, even absent inventories and capital constraints. However, when there are strictly positive gains from trade for even the lowest-valuation seller, as there are in our setting, some trade still survives absent inventories and capital constraints. In particular, sellers with sufficiently low valuations still trade. In contrast, inventories and capital constraints can lead to a complete market breakdown.

price (Proposition 3). The key insight is that a purchase by one buyer leads to the release of information that may cause a violation of a competing buyer's capital constraint, and this forces the competing buyer to increase the price.

In our model, prices are important because market participants infer information from prices about asset values. These inferences affect a market participant's ability to borrow against the value of his existing assets. In practice, transaction prices may also be important because of accounting rules that rely on the actual price rather than what can be inferred from it. Milbradt (2012) focuses on such an environment. In his setting, a regulated financial institution is subject to a capital constraint that is based on mark-to-market accounting. Milbradt shows that such an accounting rule may induce the financial institution to suspend trade so that losses are not reflected on its balance sheet.<sup>4</sup> One might be tempted to conclude that the problem is flawed regulation and/or flawed accounting rules. Our model, which is based on rational expectations, shows that the problem is more fundamental. Our model also addresses situations in which market participants can strategically choose ("manipulate") prices. In our model the price is endogenous and may not reflect the true value. In contrast, in Milbradt the price is exogenous and always reflects the true value, by assumption.

Endogenizing the price leads to new predictions that are absent in Milbradt. Under many conditions, these new predictions continue to hold even if the capital constraint is based on mark-to-market accounting rather than rational inferences.

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<sup>4</sup>Other papers that discuss problems arising from mark-to-market accounting rules include Allen and Carletti (2008), Heaton, Lucas, and McDonald (2010), and Plantin, Sapra, and Shin (2008).

The first new prediction is the price run-up and related implications. The second new prediction is the possibility of a complete market breakdown and lack of information dissemination. We discuss the robustness of these predictions in Section 5.

As noted above, one application of our analysis is to the market for structured financial products. Our analysis provides an explanation for prices that change over time without any change in the distribution of asset values; for a price run up; for a market breakdown; and for the lack of information revelation in the market breakdown. Related, our analysis predicts that tight capital constraints are associated both with a market breakdown and high asset prices immediately before the breakdown.

A separate application is to the effects of broker-dealer inventories on prices. By interpreting buyers in our model as market-makers, our model predicts that higher inventories may lead to higher prices, consistent with the empirical findings in Manaster and Mann (1996). In contrast, previous models of the effect of market-maker inventories on prices, such as Amihud and Mendelson (1980) and Ho and Stoll (1981, 1983), assume symmetric information and predict that prices fall as inventories increase. Prices fall in these models either because the dealer is risk averse and concerned about future price movements, or because he is not allowed to carry too much inventory.

We also use our model to discuss implications for regulatory intervention in illiquid markets (Section 7). On the buyer's side, our analysis highlights the potential role of a large investor unencumbered by existing inventories (the government, for example). One implication is that by purchasing assets, the government may impose a cost on potential buyers who choose not to trade. On the seller's side, our analysis

suggests potential limitations to the standard prescription that sellers should retain a stake in the assets they sell. Under some circumstances, this prescription may lead to a market breakdown.

### 1.1 *Related literature*

Our paper relates to the literature on trade under asymmetric information, in which the seller is better informed (e.g., Akerlof, 1970; Samuelson, 1984). We show that adding inventories and capital constraints to this standard problem can lead to a complete market breakdown even if the gains from trade for the lowest-valuation seller are strictly positive.<sup>5</sup> Moreover, we show that high leverage may lead to higher prices.<sup>6</sup> In our setting, trade is always efficient, so increasing the price increases welfare (as the probability that the seller will accept the buyer's offer increases). In this sense, our paper differs from papers in which price manipulation creates

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<sup>5</sup>In Akerlof's (1970) example, the market breaks down completely, but there are no gains from trade between the buyer and the seller with the lowest possible valuation. Glode, Green, and Lowery (2012) endogenize the extent of adverse selection and show that overinvestment in financial expertise may lead to a reduction in the probability of efficient trade, but not a complete breakdown. Kremer and Skrzypacz (2007) and Daley and Green (2012) obtain periods of a complete breakdown of trade in a *dynamic* lemons problem, but they add the assumption that some noisy information about the asset quality is revealed at some future date. An alternative explanation for a complete market breakdown involves Knightian uncertainty (e.g., Easley and O'Hara, 2010).

<sup>6</sup>Clayton and Ravid (2002) study how leverage affects bidding behaviors in private-value auctions. They show that higher leverage *reduces* bid prices. However, they focus on a debt-overhang problem rather than the effect of trade on inventory valuation.

distortions that are suboptimal from a social point of view.<sup>7</sup>

Our competing-buyers result illustrates a situation in which trade between the seller and one buyer has externalities for other buyers, and hence relates to existing auction-theoretic papers dealing with externalities (e.g., Jehiel, Moldovanu, and Stacchetti, 1996). In contrast to this literature, the externality in our paper depends on the price paid rather than on simply whether another buyer obtains the asset.

Our paper also relates to the literature that explores the link between leverage and trade. For example, Shleifer and Vishny (1992) show that high leverage may force firms to sell assets at fire-sale prices, while Diamond and Rajan (2011) show that the prospect of fire sales may lead to a market freeze. Other papers explore feedback effects between asset prices and leverage: Low prices reduce borrowing capacity, and hence asset holdings and prices also (see, e.g., Kiyotaki and Moore, 1997). In contrast, we model a situation in which firms can meet their financial needs by staying with the status quo. Hence, there is no need for fire sales or “cash in the market” pricing, as in Allen and Gale (1994). In addition, in our setting, prices may increase even after valuations of existing inventories fall.

As noted earlier, our paper also relates to Milbradt (2012), to the literature on mark-to-market accounting, and to the market microstructure literature that links market-maker inventories to prices.

Finally, our paper relates to the literature on equity issuance, in which an informed issuer cares about the market valuation of its remaining equity.<sup>8</sup> However,

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<sup>7</sup>E.g., Allen and Gale (1992); Brunnermeier and Pedersen (2005); Goldstein and Guembel (2008).

<sup>8</sup>E.g., Allen and Faulhaber (1989); Grinblatt and Hwang (1989); Welch (1989).

our main focus is on the bidding behavior of uninformed buyers rather than signaling. Section 5 discusses the case in which informed sellers make offers.

## 2 The basic model

The model is based on a simple variant of Akerlof's (1970) "lemons problem." The new feature is that the buyer has an inventory of the traded asset and is subject to a capital constraint. In the basic model, an uninformed buyer makes a take-it-or-leave-it offer to buy one unit of an asset from an informed seller. The buyer and seller are risk neutral. The value of the asset is  $v$  to the seller and  $v + \Delta$  to the buyer, where  $\Delta > 0$  denotes the gains from trade. Since  $\Delta > 0$ , trade is always efficient. Both  $\Delta$  and the distribution of  $v$  are common knowledge, but the realization of  $v$  is the seller's private information. Consequently, trade affects posterior beliefs about  $v$  and hence the market value of each unit of the asset. For simplicity,  $v$  is drawn from a uniform distribution on  $[0, 1]$ . The buyer's offer and the seller's response are publicly observable (see also footnote 15).

In one interpretation, the seller is a loan originator, and the gains from trade reflect the fact that the buyer has a lower cost than the seller of retaining risky assets on the balance sheet. For example, the buyer may face lower borrowing costs or less stringent regulation. Alternatively, the buyer might be a broker-dealer who helps with the matching process between the seller and other investors who have higher valuations for the asset.

The buyer has an inventory of  $M$  units of the asset, which he acquired earlier.<sup>9</sup> The buyer also has cash and a short-term debt liability. The liability net of cash holdings is  $L$ , so the buyer can roll over his liabilities only if the value of his noncash assets exceeds  $L$ . Assume, for simplicity, that the buyer holds only the traded asset and that the purchase of additional units is financed out of existing cash holdings and/or new short-term borrowing. Specifically, suppose the buyer purchases  $q \in \{0, 1\}$  additional units at a price per unit  $p$ , and let  $h$  denote the “market value” of the asset, defined as the expected value of  $v + \Delta$ , conditional on the trading outcome, using Bayes’ rule. Then the buyer can roll over his debt if

$$h(M + q) \geq L + pq, \tag{1}$$

where  $M + q$  is the buyer’s total inventory of assets net of trade, and  $L + pq$  is the buyer’s total liabilities net of trade. Equation (1) is the buyer’s *capital constraint*.<sup>10</sup>

If the buyer violates his capital constraint, he defaults and incurs a cost  $\kappa$ , which represents lost growth opportunities due to bankruptcy or closure by a regulator. Alternatively, one can think of a situation in which the buyer must raise  $L$  dollars to invest in a profitable opportunity with cash flows that cannot be promised to others (e.g., because of nonobservability). In this case, the cost of violating the capital

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<sup>9</sup>We obtain qualitatively similar results if, instead, the buyer’s inventory consists of assets whose values are correlated with the value of the asset being traded.

<sup>10</sup>Our results generalize to the case in which  $h$  is the expected value of  $\alpha(v + \gamma\Delta)$ , where  $\alpha \leq 1$  reflects constraints on the buyer’s ability to pledge future cash flows, and  $\gamma \in [0, 1]$  allows the capital constraint to be based on creditors’ valuations of the asset instead of the buyer’s. See Appendix B in a working paper version of this paper.

constraint is that the buyer cannot take advantage of the investment opportunity.

**Assumption 1**  $(\frac{1}{2} + \Delta)M \geq L$ .

**Assumption 2** *The cost  $\kappa$  of violating the capital constraint is very high.*

**Assumption 3**  $\Delta < \frac{1}{2}$ .

With the exception of a brief analysis of the benchmark case  $M = 0$  in Section 3.1, we also assume:

**Assumption 4**  $M > 1$ .

Assumption 1 says that the capital constraint is satisfied before trading begins (i.e.,  $q = 0$  and  $h = \frac{1}{2} + \Delta$ , so assets are evaluated at the prior). This allows us to focus on how the buyer changes his behavior to avoid violating the capital constraint, rather than on the much studied fire sales that follow when the constraint is violated.

Assumption 2 implies that the buyer's first priority is to satisfy his constraint.<sup>11</sup> Consequently, the buyer maximizes expected trading profits subject to the constraint that his offer satisfies the capital constraint.

Assumption 3 says that the gains from trade are not too high. Without this assumption, the market freeze result does not hold because a transaction price of 1 satisfies the seller's participation constraint for each seller type and gives the buyer positive profits, since the buyer acquires an asset with an expected value of  $\frac{1}{2} + \Delta > 1$ . However, Assumption 3 is not crucial for price run ups.

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<sup>11</sup>For example, for the results in Section 3, it is enough to assume that  $\kappa \geq 1 + \Delta$ .

Finally, Assumption 4 says that the quantity of the asset available for trade is smaller than the buyer's existing asset holdings. This assumption ensures that inventory valuation effects are important and, in particular, implies that the buyer's capital constraint puts a lower bound on the price (see below).

The assumption that the buyer can purchase only one unit is made for simplicity. The nature of the results remains if the asset is divisible and the buyer can choose a quantity in addition to a price.<sup>12</sup>

We discuss extensions of our basic model and robustness of our main results to other trading environments in Sections 3.4, 4, and 5.

### 3 Analysis

We start with the benchmark case  $M = 0$ , in which the buyer has no inventories. Then we analyze the main case with inventories,  $M > 0$ .

#### 3.1 *Benchmark: Buyer does not have inventories*

Absent inventories, the buyer offers to purchase the asset at a price  $p$ , which maximizes his expected profits. The seller accepts the offer if and only if  $v \leq p$ , which happens with probability  $p$ , since  $v$  is uniform on  $[0, 1]$ .<sup>13</sup> Conditional on the seller accepting the offer, the expected value of the asset to the buyer is  $\frac{1}{2}p + \Delta$ . Since the

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<sup>12</sup>See Appendix B in a working paper version of this paper.

<sup>13</sup>If  $p > 1$ , the acceptance probability is simply 1. However, by Assumption 3, offers of  $p > 1$  generate negative profits.

buyer pays  $p$ , his expected profit per unit bought is  $\Delta - \frac{1}{2}p$ . Taking into account the probability of trade, the buyer's expected profit is  $\pi(p) \equiv p(\Delta - \frac{1}{2}p)$ . The buyer's profit-maximizing bid is  $p = \Delta$ .

**Lemma 1** *In the benchmark case of no inventories, the buyer offers to buy the asset at a price  $\Delta$ . The seller accepts this offer if and only if  $v \leq \Delta$ .*

### 3.2 Buyer cares about the value of his inventory

When a seller accepts an offer, the market infers that  $v$  is below  $p$ . Hence, the market value of existing inventories falls from the prior of  $(\frac{1}{2} + \Delta)M$  to  $(\frac{1}{2}p + \Delta)M$ . To ensure that the capital constraint continues to hold after the offer is accepted, the offer  $p$  must satisfy

$$\left(\frac{1}{2}p + \Delta\right)(M + 1) \geq L + p, \quad (2)$$

which follows from equation (1) with  $h = \frac{1}{2}p + \Delta$  and  $q = 1$ .

Define  $\delta \equiv \frac{L}{(\frac{1}{2} + \Delta)M}$ , a measure of both the buyer's initial leverage and the tightness of his capital constraint before trade begins. Assumption 1 is equivalent to  $\delta \leq 1$ . By Assumption 4, the capital constraint (2) puts a lower bound on the price. Specifically, by defining

$$P(\delta) \equiv \frac{\delta M(1 + 2\Delta) - 2\Delta(M + 1)}{M - 1}, \quad (3)$$

the capital constraint (2) simplifies to

$$p \geq P(\delta). \quad (4)$$

If instead the seller rejects the offer, the market infers that  $v$  is above  $p$  and the market value of inventories rises to  $(\frac{1}{2}p + \frac{1}{2} + \Delta)M$ , relaxing the capital constraint.

Hence, the buyer faces a constrained optimization problem: namely, choose a price  $p$  to maximize expected profits  $\pi(p)$  subject to the constraint that either he makes no offer,  $p = 0$ , or else the offer satisfies equation (4).<sup>14,15</sup> Observe that the lower bound on the price  $P(\delta)$  is increasing in the buyer's initial leverage  $\delta$ .

Fig. 3 illustrates the solution to the buyer's problem. The parabola represents profits  $\pi(p)$ , and the vertical lines show  $P(\delta)$  for three different values of  $\delta$ . If leverage is low, i.e.,  $P(\delta) \leq \Delta$ , the capital constraint is not binding, and the buyer makes his benchmark offer  $\Delta$ . If leverage is intermediate,  $P(\delta) \in (\Delta, 2\Delta)$ , the capital constraint binds, and it is optimal to offer  $p = P(\delta)$ , since a higher price reduces profits. Finally, if leverage is high,  $P(\delta) > 2\Delta$ , the buyer will lose money if he offers  $p \geq P(\delta)$ . In this case, the buyer prefers not to trade; i.e., the market "freezes."

The condition  $P(\delta) \leq 2\Delta$  reduces to  $\delta \leq \frac{4\Delta}{1+2\Delta}$ . Hence, we have established:

**Proposition 1** *In the basic model of a monopolist buyer who cares about the value of his inventories, trade can happen if and only if the buyer's initial leverage is not*

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<sup>14</sup>From the law of iterated expectations, the value of inventoried assets equals its prior. Hence, maximizing  $\pi(p)$  is the same as maximizing the expected value of the buyer's assets.

<sup>15</sup>We obtain similar results if the market observes only the terms of accepted offers but not the terms of rejected offers. Since there is a positive probability that the buyer's offer is rejected along the equilibrium path, inferences regarding  $v$  are the same in both cases. Moreover, if the capital constraint is satisfied after accepted offers, it is also satisfied after rejected offers. Hence, the analysis below remains unchanged.

too high, i.e.,  $\delta \leq \frac{4\Delta}{1+2\Delta}$ . In this case, the buyer offers to purchase the asset at a price  $\max\{\Delta, P(\delta)\}$ , which increases in his initial leverage.

Proposition 1 says that when leverage is intermediate, the price, and hence the probability of trade, is increasing in leverage and is above the benchmark bid  $\Delta$ . However, when leverage is high, trade completely breaks down.

An immediate corollary concerns the effect of high leverage and the corresponding market breakdown on the revelation of the seller's information about asset values:

**Corollary 1** *If initial leverage is high,  $\delta > \frac{4\Delta}{1+2\Delta}$ , market participants learn nothing about the value  $v$  of the asset.*

### 3.3 Implications

Although simple, the basic model delivers many of the main implications. First, the market price is not determined solely by the distribution of asset values, but is also affected by the tightness of the buyer's capital constraint. In particular, the price increases in the tightness of this constraint. Second, and related, the buyer's expected return from holding the asset is decreasing in the tightness of his capital constraint. Third, very tight capital constraints lead to market breakdown and prevent information dissemination.

### 3.4 Remarks

*Remark 1:* The nature of the results above remains if the seller is capital constrained and can sell only a fraction of his assets. In particular, since accepting a low offer

reduces the market value of the units that the seller retains, a seller who is highly leveraged will accept an offer  $p \geq v$  only if the price is sufficiently high.<sup>16</sup>

*Remark 2:* The nature of the results above remains if the market value  $h$  in the capital constraint is a transaction price, as in mark-to-market accounting, rather than an expected value. To see this, observe that substituting  $h = p$  into the capital constraint (1) yields  $p \geq \frac{L}{M}$ , so the capital constraint continues to put a lower bound on the price, and the lower bound increases in the buyer's leverage.

## 4 Dynamic run-ups and breakdowns

The static model is suggestive of a dynamic process in which the buyer increases leverage and prices until the market breaks down eventually. To model this explicitly, we extend our single-period model to a two-period model in which the monopolist buyer trades sequentially with two potential sellers. Each seller sells a different asset, and the values of the two assets are independent.<sup>17</sup> Hence, one cannot infer anything

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<sup>16</sup>Specifically, if the seller has  $M_s$  units of the asset but can sell only  $x < M_s$ , then  $p$  must satisfy  $h(M_s - x) \geq L_s - px$ , where  $L_s$  denotes the seller's liabilities. This constraint puts a lower bound on  $p$ , and this lower bound increases in the seller's initial leverage. In particular, if  $h$  is the expected value of  $v + \gamma\Delta$  for some  $\gamma \in [0, 1]$ , then the seller's initial leverage is  $\delta_s \equiv \frac{L_s}{(\frac{1}{2} + \gamma\Delta)M_s}$  and the capital constraint reduces to  $p \geq P_s(\delta_s) \equiv \frac{\delta_s(\frac{1}{2} + \gamma\Delta)M_s - \gamma\Delta(M_s - x)}{\frac{1}{2}(M_s + x)}$ .

<sup>17</sup>One can think of the two assets as idiosyncratic components of the same class of assets, e.g., mortgage-backed securities. Moreover, a positive correlation between the two values would only strengthen our results below, because trade in the first period would reduce not only the market value of the first asset but also the market value of the second asset. Hence, the buyer's capital

about the value of one asset by observing trade in the other asset. This allows us to focus on the effect of leverage, which changes endogenously.

Specifically, seller  $i$  ( $i = 1, 2$ ) can sell one unit of asset  $i$  and can trade only in period  $i$ .<sup>18</sup> Before trading begins, the buyer has inventories of  $M$  units of each asset. The value of asset  $i$  is  $v_i$  to the seller and  $v_i + \Delta$  to the buyer, where  $v_1, v_2$  are both distributed uniformly over  $[0, 1]$ . In period  $i = 1, 2$ , the buyer makes a take-it-or-leave-it offer  $p_i$  to seller  $i$ . The discount rate is 1.

The two-period capital constraint is analogous to the one-period constraint (1):

$$h_1(M + q_1) + h_2(M + q_2) \geq L + p_1q_1 + p_2q_2, \quad (5)$$

where  $q_i \in \{0, 1\}$  is the amount of asset  $i$  that the buyer purchases, and  $h_i$  is the market value of asset  $i$ . Parallel to Assumption 1, assume that the capital constraint (5) is satisfied absent new trade (i.e., if  $q_1 = q_2 = 0$  and  $h_1 = h_2 = \frac{1}{2} + \Delta$ ). The buyer's initial leverage is  $\bar{\delta} \equiv \frac{L}{(\frac{1}{2} + \Delta)(2M)}$ . The cost  $\kappa$  (Assumption 2) is incurred if the capital constraint is violated at the end of the second period. We obtain the same outcome if the buyer also incurs a cost  $\kappa$  for violating the constraint after the first period. (The proof of Proposition 2 is in the Appendix and contains more details.)

The buyer's second-period offer  $p_2$  can depend on the outcome of trade with the first seller. Denote by  $p_a$  (respectively,  $p_r$ ) the second-period offer after the first seller accepts (rejects) the offer. Since sellers' acceptance decisions are simple cutoff rules

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constraint would be tightened by a larger amount after his first offer is accepted.

<sup>18</sup>See Frenkel (2013) for a model in which the seller cares about the release of information and can choose in which period to trade.

(seller  $i$  accepts offer  $p_i$  if and only if  $v_i \leq p_i$ ), the buyer's expected profits are

$$\pi(p_1) + p_1\pi(p_a) + (1 - p_1)\pi(p_r). \quad (6)$$

The buyer's problem is to choose prices  $(p_1, p_a, p_r)$  to maximize his expected profits subject to satisfying the capital constraint with probability 1.<sup>19</sup>

We show in the Appendix that if the first offer is rejected, the capital constraint becomes slack enough that the buyer can make his unconstrained optimal bid of  $\Delta$  in the second period (i.e.,  $p_r = \Delta$ ). It remains to characterize  $p_1$  and  $p_a$ . Define

$$H \equiv \frac{(\frac{1}{2} + \Delta)2M\bar{\delta} - 2\Delta(M + 1)}{\frac{1}{2}(M - 1)}, \quad (7)$$

which is an increasing function of the buyer's initial leverage  $\bar{\delta}$ , and let  $\sigma \equiv \frac{\frac{1}{2}M - \Delta}{\frac{1}{2}(M - 1)}$ .

Our main result in this section is:

**Proposition 2** *In the two-period model, there exist  $\hat{H} \in (2\Delta, 4\Delta)$  and  $\check{H} > 2\Delta + \sigma > \hat{H}$  (that depend on  $M$  and  $\Delta$ ) such that:*

- (1) *If  $H \leq 2\Delta$ , the buyer bids  $(p_1, p_a) = (\Delta, \Delta)$ .*
- (2) *If  $H \in (2\Delta, \hat{H})$ , the buyer bids  $(p_1, p_a)$  with  $p_a > p_1 > \Delta$  (i.e., the buyer increases his bid if his first offer is accepted).*
- (3) *If  $H \in (\hat{H}, \check{H})$ , the buyer bids  $p_1 = \max\{H - \sigma, \Delta - \frac{1}{2}\Delta^2\}$  and then withdraws from the market if this offer is accepted, i.e.,  $p_a = 0$ .*
- (4) *If  $H > \check{H}$ , the buyer does not bid at all.*

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<sup>19</sup>The solution to this constrained maximization problem does not depend on whether the buyer can commit to his date 2 offers  $p_a$  and  $p_r$ . If  $(p_1, p_a, p_r)$  maximizes profits when the buyer can commit, the buyer has no incentive to deviate from  $p_a$  and  $p_r$  at date 2.

*(At the boundaries  $H \in \{\hat{H}, \check{H}\}$  the buyer is indifferent between the two alternate strategies.)*

Proposition 2 captures a few aspects of a dynamic behavior. If the buyer's initial leverage is not too high (Parts 1 and 2), the buyer has enough slack in his capital constraint to make two rounds of offers. In Part 1 (low leverage), the capital constraint is not binding and the buyer makes the benchmark bid  $\Delta$  in both periods. In Part 2 (moderate leverage), the capital constraint is binding. This induces the buyer to bid more than the benchmark price in the first period ( $p_1 > \Delta$ ), and if the first bid is accepted, the capital constraint is tightened, forcing the buyer to bid even more in the second period ( $p_a > p_1$ ). Hence, prices increase even though, by assumption, there is no change in the distribution of asset values.

If instead initial leverage is higher (Part 3), there are two consequences. First, the buyer prefers to have at most one offer accepted because if two offers are accepted, the bid prices must be very high to satisfy the capital constraint, and the buyer is left with very low or even negative profits. Hence, if the buyer's first offer is accepted, there is no trade in the second period. Second, a tight capital constraint pushes the initial bid high, for the same reasons as in the benchmark case. Consequently, high leverage generates a market freeze that is preceded by high prices. Specifically:

**Corollary 2** *If  $H \in (\max\{\hat{H}, \Delta + \sigma\}, \check{H})$ , the buyer bids more than the benchmark in the first period ( $p_1 = H - \sigma > \Delta$ ), and if the first offer is accepted, the buyer does not bid in the second period. That is, a market freeze is preceded by high prices.*

Note that while the buyer never bids above his valuation in the second period, it

might be optimal for him to do so in the first period. In particular, if  $H \in (2\Delta + \sigma, \check{H})$ , the buyer bids  $p_1 = H - \sigma > 2\Delta$  and  $p_a = 0$ . The advantage of offering such  $p_1$  is that, if the offer is rejected, the market valuation of the buyer's inventory rises, thereby relaxing the buyer's capital constraint. This allows the buyer to make a profitable offer in the second period.<sup>20</sup>

Finally, if initial leverage is too high (Part 4), the market completely breaks down.

*Remark 3:* The nature of the results in Proposition 2 remains if the capital constraint is based on actual transaction prices rather than inferences, but only if initial book values are sufficiently high (e.g., more than  $2\Delta$ ). In this case, the capital constraint is tightened after the first offer is accepted, which forces the buyer to either increase his next bid or else stop bidding. If instead initial book values are low, an accepted offer may relax rather than tighten the capital constraint, and we may obtain different predictions. In particular, the buyer will neither stop bidding after the first offer is accepted nor increase his bid.

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<sup>20</sup>If  $\hat{H} < \Delta + \sigma$ , Part 3 of Proposition 2 includes an interval of  $H$  in which  $p_1$  is below the benchmark bid  $\Delta$ . Intuitively, since the buyer wants to have only one offer accepted, the sellers in periods 1 and 2 are effectively in competition, which puts downward pressure on the price. However, this interval does not exist if  $\Delta$  is sufficiently large (see Appendix). This interval would also not exist in perturbations of the environment that weaken the competition effect. Two examples are introducing more buyers to restore “competitive balance” between the two sides of the market; and changing the dynamic model to one in which there are two buyers (who have inventories of both assets), with each buyer active only in one period.

## 5 Discussion

In this section, we discuss the extent to which our main results, namely price run-ups and market freezes, extend beyond our basic model.

Our run-up result hinges on two important properties of our basic model. First, the surplus the buyer gets from trade varies with leverage. Second, and related, the asset does not trade at its true value  $v$ . If the buyer obtains zero surplus, our run-up result may not hold. For example, if the seller (rather than the buyer) makes a take-it-or-leave-it offer and the condition for trade in our basic setting holds, there is an equilibrium in which the seller sells the asset for a price  $2\Delta$  if  $v \leq 2\Delta$  and does not sell if  $v > 2\Delta$ .<sup>21</sup> In this equilibrium the buyer makes zero expected profits, and the price is independent of the buyer's initial leverage.

However, when there are competing buyers, the run up result may be amplified because a highly leveraged buyer may engage in a negative-surplus trade. To see this, consider an extension of our basic model in which two buyers compete by making simultaneous offers to the informed seller. The two buyers have different liabilities ( $L_1$  and  $L_2$ , respectively) but the same inventory ( $M$ ) and the same valuation for the asset ( $v + \Delta$ ); hence, buyer  $i$ 's leverage is  $\delta_i = \frac{L_i}{(\frac{1}{2} + \Delta)M}$ . Then:

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<sup>21</sup>Because the seller is the informed party, the result is a signaling game with many equilibria. Any equilibrium with trade is characterized by a price  $p$  such that, if  $v \leq p$ , the seller sells at price  $p$  and, if  $v > p$ , the seller does not sell. In the equilibrium that is most preferred by the seller,  $p = 2\Delta$ . Equilibria in which sellers with different valuations sell for different prices do not exist because a seller with a low valuation would deviate by acting as if he has a high valuation.

**Proposition 3** *Suppose there are two competing buyers.*

(1) *If  $\max\{P(\delta_1), P(\delta_2)\} \leq 2\Delta$ , the unique equilibrium price is  $2\Delta$ .*

(2) *If  $P(\delta_1) \leq 2\Delta < P(\delta_2) \leq 1$ , the unique equilibrium price is  $P(\delta_2)$ , which increases in buyer 2's leverage. In this case, whenever the seller sells the asset, he sells it to buyer 2, who makes negative profits.*

In Part (1), both buyers have low leverage, and competition drives the price to the zero-profit price. In Part (2), buyer 1 has low leverage and buyer 2 has high leverage. Buyer 2 is forced to make a high offer,  $P(\delta_2)$ , since otherwise the seller would trade with buyer 1 at a low price, which would lead to a violation of buyer 2's capital constraint. Buyer 2 makes this loss-making offer, even though he would not bid at all if he were the only buyer.<sup>22</sup> More generally, if two buyers have different valuations for the asset, it is possible that the seller sells the asset to the less leveraged buyer at a price that is determined by the leverage of the highly-leveraged buyer.<sup>23</sup>

Clearly our run-up result does not hold if trade always occurs at a price equal to the asset's true value  $v$ .<sup>24</sup> When  $v$  is private information to the seller, as in our

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<sup>22</sup>For this result, it is important that buyer 2 can purchase the entire amount for sale without violating his capital constraint. If buyer 2 is too leveraged ( $P(\delta_2) > 1$ ) or if the aggregate amount for sale is too high (e.g., there are many informed sellers), buyer 2 cannot prevent a violation of his capital constraint by making a high bid. If he purchases the entire amount, his losses are too large and lead to a violation of the constraint. But if he purchases less than the full amount, buyer 1 will purchase the remaining units at a low price.

<sup>23</sup>See Appendix C in a working paper version of this paper, which analyzes the general case in which  $\Delta$  and  $M$  vary across the two buyers, in addition to  $L$ .

<sup>24</sup>For example, this is a feature of Milbradt (2012).

setting, the most natural force that could push the price to equal  $v$  is when multiple informed sellers (with assets of equal value  $v$ ) compete to sell to the buyer. This is a signaling game with multiple equilibria, so it is hard to draw conclusions on how prices vary with leverage. But note that if the capital constraint is sufficiently tight, then if the buyer can choose whether to participate in the game (and the sellers make no offers if the buyer chooses not to participate), there is no equilibrium in which sellers with low valuations sell at the true value. Whenever these sellers sell, the price is strictly higher than the true value, so that the buyer's capital constraint is not violated. In this sense, high leverage still leads to a price run-up.<sup>25</sup>

Another property of our model is that trade tightens the capital constraint. This property is important for our market freeze result and is also used to obtain our dynamic run-up result. As discussed in Remark 3, this property does not hold when the capital constraint is based on mark-to-market accounting and the initial price is low. This property may also not hold when there are competing sellers, as discussed above. If sellers with valuations  $v > \frac{1}{2}$  sell at the true value, then this relaxes capital constraints and eliminates the force pushing up prices over time.

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<sup>25</sup>There may also be equilibria in which trade occurs when  $v$  is high but does not occur when  $v$  is low. These equilibria have some features that resemble the equilibrium outcome in Milbradt (2012) but are not plausible in our setting with endogenous prices, because sellers with low valuations have a strong incentive to deviate and act as if they have high valuations. Such a deviation poses no risk for a deviating seller, since sellers already receive the minimal payoff of zero, and the deviation can lead to strictly positive gains if all sellers coordinate on the same price (so the buyer cannot detect the deviation). Moreover, one can show that no equilibrium of this type exists under our benchmark assumption that rejected offers are observed.

Finally, our market freeze result uses the property that abstaining from trade leads the capital constraint to be satisfied. Given Assumption 1, this property is likely to hold in any setting with a single buyer. However, with multiple competing buyers, even if *all* buyers are very leveraged and would each prefer to abstain from trade, there are equilibria in which the market does not freeze. In these equilibria, one buyer makes a low (latent) offer, while another buyer makes a high and loss-making offer to prevent trade from occurring at the low price.<sup>26</sup> However, these equilibria are not robust in the sense that they do not survive iterated elimination of weakly dominated strategies. Formally, in the notation of Proposition 3, when  $\min\{P(\delta_1), P(\delta_1)\} > 2\Delta$  and  $P(\delta_1) \neq P(\delta_1)$ , the unique equilibrium that survives iterated elimination of weakly dominated strategies is a market freeze.<sup>27</sup>

## 6 Empirical implications

In this section we collect our model's main predictions and, where possible, provide supporting evidence.

*Prices increase even though valuations do not increase:* A basic implication of our analysis is that the price of an asset may respond to variables other than beliefs about its true value. In particular, as the buyer's capital constraint tightens, he offers to

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<sup>26</sup>In our model, latent offers induce entry. This is different from existing literature on nonexclusive contracting, in which latent offers deter entry (e.g., Bisin and Guaitoli, 2004; Attar, Mariotti, and Salanie, 2011; Ales and Maziero, 2011).

<sup>27</sup>See Appendix C in a working paper version of this paper.

pay a higher price even though his beliefs regarding the asset value are unchanged. In Section 4 we show how this force can lead to prices that increase over time even without a change in the “fundamentals” (i.e., the value distribution). Although hard to definitively test, there is at least some evidence consistent with the view that prices in the market for structured financial products were partially divorced from fundamentals in the run-up to the financial crisis. We discussed some evidence in the introduction (see also Coval et al., 2009).<sup>28</sup>

*Tight capital constraints lead to market breakdown:* This prediction is consistent with the sharp increase in dealers’ leverage prior to the recent financial crisis (Adrian and Shin, 2010) and Fig. 2 in the introduction. More anecdotally, many market observers expressed the view that concerns about the value of inventories induced firms not to sell their assets. For example, an analyst was quoted in *American Banker*<sup>29</sup> as saying that “[other companies] may be wary of selling assets for fear of establishing a market-clearing price that could force them to mark down the carrying value of their nonperforming portfolio.” Also related is the view expressed in Lewis (2010, pages 184-185) that dealers who sold credit default swaps on subprime mortgage bonds did

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<sup>28</sup>Less direct evidence is provided by Faltin-Traeger et al. (2010), who show that prices of asset-backed securities failed to reflect relevant and observable information about sponsor credit quality, and by Ashcraft et al. (2010) and Rajan et al. (2012), who show that subordination levels failed to reflect the riskiness of underlying cash flows. Finally, the claim that structured financial products were overpriced is closely related to the widely held view that these same products received excessively favorable credit ratings. See, for example, Griffin and Tang (2012) for evidence.

<sup>29</sup>“Nonperformance Space: Risky Assets Find Market” (*American Banker*, August 19, 2009).

not make a market in these securities to prevent information from being revealed and a subsequent “mark down” of their positions.

*Market breakdown is associated with a loss of information:* In our analysis, trade is a source of information, so when the market freezes, information dissemination ceases. Loss of information was a primary concern expressed by observers during the financial crisis. For just one example, see Scott and Taylor (2009).

*The tightness of capital constraints affects expected holding returns:* This prediction relates to the prediction that the tightness of capital constraints affects prices. Corollary 2 implies that when capital constraints are sufficiently tight, the market freezes and prices before the freeze are high; so one should see low average returns on assets purchased shortly before a market freeze.

*In broker-dealer markets, prices are increasing in dealer inventories:* This prediction is essentially a special case of the first implication above, and as discussed in the introduction, is consistent with the empirical findings of Manaster and Mann (1996), which are not easily explained by previous models of market-maker inventories. More formally, this implication is a corollary of our Proposition 1. To see this, observe that the derivative of  $P(\delta)$  with respect to inventories  $M$  has the same sign as  $4\Delta - \delta(1 + 2\Delta)$ , which by Proposition 1 is positive whenever trade occurs.<sup>30</sup>

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<sup>30</sup>Here we are characterizing only the direct effect of inventories, since we are holding leverage  $\delta$  constant. If changes in inventory levels also affect leverage, there is an additional indirect effect on prices. This indirect effect reinforces the direct effect if higher inventories are associated with greater leverage (as seems likely).

*Competition may induce highly leveraged buyers to purchase assets at a loss.* This prediction follows from Proposition 3. More generally, the leverage of one buyer may affect the price offered by another buyer.

## 7 Policy implications

Our analysis has implications for government attempts to defrost markets and for regulatory proposals aimed at improving market functioning.

### *7.1 Defrosting frozen markets*

Suppose trade has completely broken down because the buyer's capital constraint is too tight. One option for the government is to offer to buy the seller's assets. A central question is whether such government purchases can succeed without taxpayer subsidies (in expectation). Our model has two implications in this respect. First, if the government faces the same lemons problem that potential buyers do, a subsidy-free purchase scheme is possible only if the asset is worth more to the government than to the seller. Second, even if this condition holds, a government purchase may impose a cost on the original buyer. In particular, unless the government has a strictly higher valuation than the original buyer, then any subsidy-free purchase scheme either leads to a violation of the buyer's capital constraint, or else it forces him to purchase the asset at a loss. This is similar to the negative externality that one buyer imposes on another buyer in Proposition 3.

Another option is to remove assets from the buyer's balance sheet: that is, to

replace assets with cash. If the buyer can borrow against the full value of his assets, as assumed in our analysis so far, then again, purchasing assets from the buyer can relax his capital constraint only if the purchase involves a taxpayer subsidy. If instead the buyer has limited borrowing capacity, purchasing assets from the buyer might relax his capital constraint even if the purchase does not involve a taxpayer subsidy.

## 7.2 *Should regulation mandate some retention of the asset by the seller?*

A commonly voiced regulatory proposal is that sellers of assets subject to asymmetric information problems, such as issuers of asset-backed securities, should be required to retain some stake in the assets they sell.<sup>31</sup> Our analysis identifies a potential cost to this proposal: namely, that under some circumstances it leads to a market breakdown. Specifically, if the seller is highly leveraged and must retain a large fraction of his assets on his balance sheet, the seller may prefer not to trade because trade may reduce the market value of the assets that he retains and this may lead to a violation of his capital constraint.<sup>32</sup>

The goal that regulators appear to have in mind with this regulation is to reduce moral hazard on the part of loan originators. Our analysis does not speak to this issue, and it seems likely that the regulation will have its intended effect in this

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<sup>31</sup>See, for example, section 15G of the Investor Protection and Securities Reform Act of 2010.

<sup>32</sup>Formally, it follows from footnote 16 that trade is possible only if  $P_s(\delta_s) \leq 2\Delta$ . If  $\gamma < 1$ , this condition reduces to  $x \geq \frac{\delta_s(\frac{1}{2} + \gamma\Delta)M_s - (1 + \gamma)\Delta M_s}{(1 - \gamma)\Delta}$ .

regard. Our point here is instead to draw attention to a potentially significant cost of this regulation: namely, that it can lead to the breakdown of socially efficient trade.

## 8 Summary

We analyze how existing stocks of assets—inventories—affect prices and information dissemination. When market participants are highly leveraged, concerns about the revelation of bad news prevent socially beneficial trade and information dissemination. However, when market participants are only moderately leveraged, inventories lead to higher equilibrium prices and stimulate socially beneficial trade. Competition may amplify this effect by inducing highly leveraged buyers to purchase assets at a loss. We also analyze a dynamic extension and show that a market freeze may be preceded by a run-up in prices.

The model's predictions are consistent with several features of the market for structured financial products during the recent financial crisis. The model also predicts that asset prices are increasing in broker-dealer inventories, consistent with empirical evidence but different from the predictions of existing market microstructure models. Our model also provides policy implications for government interventions in illiquid markets.

## Appendix

**Proof of Proposition 2.** For use throughout, note that  $\pi'(p) = \Delta - p$ ,  $\pi''(p) = -1$ ,  $\pi(\Delta) = \frac{1}{2}\Delta^2$ , and  $\pi(p)$  can be written as

$$\pi(p) = \pi(\Delta) + \frac{1}{2}\pi'(p)(p - \Delta). \quad (8)$$

We first show that  $p_r = \Delta$ , and that  $p_1$  and  $p_a$  satisfy the capital constraint (5) if and only if

$$p_1 \geq \begin{cases} H - p_a & \text{if } p_a > 0 \\ H - \sigma & \text{if } p_a = 0. \end{cases} \quad (9)$$

Consider a buyer's strategy  $(p_1, p_a, p_r)$  that satisfies (5) with probability 1. Then (5) must be satisfied when offers are accepted. In particular, if  $p_a = 0$  (and  $p_1 > 0$ ), we must have

$$\left(\frac{1}{2}p_1 + \Delta\right)(M + 1) + \left(\frac{1}{2} + \Delta\right)M \geq L + p_1, \quad (10)$$

and if  $p_a > 0$ , we must have

$$\left(\frac{1}{2}p_1 + \Delta\right)(M + 1) + \left(\frac{1}{2}p_a + \Delta\right)(M + 1) \geq L + p_1 + p_a. \quad (11)$$

By Assumption 4, and using the definitions for  $H$  and  $\sigma$ , (10) reduces to  $p_1 \geq H - \sigma$ , and (11) reduces to  $p_1 \geq H - p_a$ .

Without loss of generality, assume  $p_1 \leq 1$  and  $p_a \leq 1$ . By Assumption 3,  $\sigma \geq 1$ . Hence, (11) implies (10). (As an aside, note that (10) means that the capital constraint is satisfied at the end of the first period, as claimed in the main text.)

By Assumption 4, (10) implies that  $\left(\frac{1}{2}p_1 + \Delta\right)(M + 1) + \left(\frac{1}{2} + \Delta\right)M + \frac{1}{2}\Delta(M - 1) + \frac{1}{2}p_1 \geq L + p_1$ , or equivalently,  $\left(\frac{1}{2} + \frac{1}{2}p_1 + \Delta\right)M + \left(\frac{1}{2}\Delta + \Delta\right)(M + 1) \geq L + \Delta$ . Hence,

if  $(p_1, p_a, p_r)$  satisfies (5) then  $(p_1, p_a, \Delta)$  does also. Since  $p_r = \Delta$  maximizes single-period profits, offering  $p_r = \Delta$  is optimal. Finally, since rejected offers increase the value of existing assets, if  $(p_1, p_a)$  satisfies (9) and  $p_r = \Delta$ , then the capital constraint (5) is satisfied with probability 1.

Given  $p_r = \Delta$ , the buyer's expected two-period profits are

$$V(p_1, p_a) \equiv \pi(p_1) + p_1\pi(p_a) + (1 - p_1)\pi(\Delta). \quad (12)$$

Hence, the buyer's problem reduces to choosing  $(p_1, p_a) \in [0, 1] \times [0, 1]$  to maximize  $V(p_1, p_a)$  such that either  $p_1 = 0$  or else (9) is satisfied. The solution to this problem must have  $p_a \leq 2\Delta$ . If, to the contrary, the solution is  $(p_1, p_a)$  with  $p_a \in (2\Delta, 1]$ , then  $V(p_1, p_a) < V(p_1, 0)$  and  $p_1 \geq H - p_a > H - \sigma$ . But this contradicts the optimality of  $(p_1, p_a)$ .

*Part 1.* If  $H \leq 2\Delta$ , then  $p_1 = p_a = \Delta$  is uniquely optimal, since this satisfies the capital constraint (9) and attains the unconstrained maximum profits of  $2\pi(\Delta)$ .

*Part 2.* Suppose  $H \in (2\Delta, 4\Delta]$ . Since  $\sigma > 1 > 2\Delta$  (Assumption 3), it follows that  $H - \sigma < 2\Delta$ , so the pair  $(p_1 = H - \sigma, p_a = 0)$  is feasible and leads to positive profits. Hence,  $p_1 > 0$ . The heart of the proof is in comparing strategies with  $p_a = 0$  and  $p_a > 0$ . Accordingly, define maximized profits from each of these two strategies respectively by

$$v_1(H) = \max_{p_1 \in [\max\{H - \sigma, 0\}, 1]} V(p_1, 0) \quad (13)$$

$$v_2(H) = \max_{p_1, p_a \in [0, 1] \text{ s.t. } p_1 + p_a \geq H} V(p_1, p_a). \quad (14)$$

Note that both  $v_1$  and  $v_2$  are continuous.

The profit function  $v_1$  is straightforward to evaluate. The derivative of  $V(p_1, 0)$  with respect to  $p_1$  is  $\pi'(p_1) - \pi(\Delta) = \Delta - p_1 - \pi(\Delta)$ . Hence  $V(p_1, 0)$  is increasing in  $p_1$  up to  $\Delta - \pi(\Delta)$  and decreasing thereafter. So  $v_1(H) = V(\max\{H - \sigma, \Delta - \pi(\Delta)\}, 0)$ , and

$$v_1'(H) = \begin{cases} 0 & \text{if } H \leq \Delta - \pi(\Delta) + \sigma \\ \pi'(H - \sigma) - \pi(\Delta) & \text{if } H \geq \Delta - \pi(\Delta) + \sigma \end{cases}. \quad (15)$$

Note that this characterization of  $v_1$  makes no use of  $H \in (2\Delta, 4\Delta]$ .

Next, we evaluate the profit function  $v_2$  when  $H \in (2\Delta, 4\Delta]$ . First observe that

$$v_2(H) = \max_{p_a \in [\max\{H-1, 0\}, \min\{H, 1\}]} V(H - p_a, p_a), \quad (16)$$

as follows. Since  $H > 2\Delta$ , if  $p_a > 0$  then  $p_1 > \Delta$  and/or  $p_a > \Delta$ . Note that  $\frac{\partial}{\partial p_a} V(p_1, p_a) = p_1 \pi'(p_a)$  and  $\frac{\partial}{\partial p_1} V(p_1, p_a) \leq \pi'(p_1)$ . Hence at least one of  $\frac{\partial}{\partial p_a} V(p_1, p_a)$  and  $\frac{\partial}{\partial p_1} V(p_1, p_a)$  is strictly negative, implying that the solution to the maximization problem that defines  $v_2(H)$  must have a binding capital constraint, i.e.,  $p_1 + p_a = H$ .

Next we show that  $V(H - p_a, p_a)$  has a unique local maximum, which we denote by  $p_a(H)$ . We also show that the local maximum lies in the interval  $(\frac{H}{2}, H - \Delta]$ , approaches  $\Delta$  as  $H \rightarrow 2\Delta$ , and is strictly increasing in  $H$ . The proof is as follows.  $V(H - p_a, p_a)$  is a cubic in  $p_a$  with a positive coefficient on the cubic term. Hence, if a local maximum exists, it is the smaller root of the following convex quadratic in  $p_a$ :

$$F(p_a, H) \equiv \frac{d}{dp_a} V(H - p_a, p_a) = -\pi'(H - p_a) - (\pi(p_a) - \pi(\Delta)) + (H - p_a) \pi'(p_a). \quad (17)$$

By (8), this simplifies to  $F(p_a, H) = -\pi'(H - p_a) + (H - p_a - \frac{1}{2}(p_a - \Delta)) \pi'(p_a)$ .

It follows that for  $H \in (2\Delta, 4\Delta]$ ,

$$F\left(\frac{H}{2}, H\right) = \left(\frac{H}{4} + \frac{1}{2}\Delta - 1\right) \pi'\left(\frac{H}{2}\right) > 0 \quad (18)$$

$$F(H - \Delta, H) = \left(\Delta - \frac{1}{2}(H - 2\Delta)\right) \pi'(H - \Delta) \leq 0, \quad (19)$$

where (18) uses Assumption 3.

Hence  $p_a(H)$  is well defined, with

$$p_a(H) \in \left(\frac{H}{2}, H - \Delta\right], \quad (20)$$

and  $p_a(H) \rightarrow \Delta$  as  $H \rightarrow 2\Delta$ .

To establish that  $p_a(H)$  is strictly increasing in  $H$ , note that  $\frac{\partial F}{\partial p_a} < 0$ , and so  $\text{sign}(p'_a(H)) = \text{sign}\left(\frac{\partial F}{\partial H}\right) = \text{sign}(-\pi'' + \pi'(p_a(H))) = \text{sign}(1 + \Delta - p_a(H)) > 0$ .

Next, we show that if  $v_2(H) \geq v_1(H)$ , then  $v_2(H) = V(H - p_a(H), p_a(H))$ . Specifically, from (20) and Assumption 3,  $p_a(H) \geq \frac{H}{2} \geq \max\{H - 1, 0\}$ . Hence, the solution to problem (16) is either  $p_a(H)$  or  $\min\{H, 1\}$ . Since  $\min\{H, 1\} > 2\Delta$  (by Assumption 3), and since we know that the solution to the buyer's problem satisfies  $p_a \leq 2\Delta$ , it follows that the solution is  $p_a(H)$ .

Hence, it follows from (20) that whenever  $H \in (2\Delta, 4\Delta)$  and  $v_2(H) > v_1(H)$ , the optimal solution to the buyer's maximization problem satisfies  $p_a > p_1 > \Delta$ .

To complete the proof of Part 2, we need to show that there exists  $\hat{H} \in (2\Delta, 4\Delta)$  such that  $v_2(H) > v_1(H)$  if  $H \in (2\Delta, \hat{H})$  and  $v_1(H) > v_2(H)$  if  $H \in (\hat{H}, 4\Delta]$ . This result follows from the continuity of  $v_1$  and  $v_2$  and the following observations, which are proved below: (i)  $v_2(H) > v_1(H)$  for all  $H$  sufficiently close to  $2\Delta$ ; (ii)

$v_1(H) > v_2(H)$  at  $H = 4\Delta$ ; and (iii) if  $v_2(H) \geq v_1(H)$  over some interval, then  $v_2(H) - v_1(H)$  is either monotone strictly decreasing; or strictly decreasing then strictly increasing; or monotone strictly increasing. The first observation follows since as  $H \rightarrow 2\Delta$ ,  $v_2(H)$  approaches the unconstrained optimum  $V(\Delta, \Delta)$  while  $v_1(H)$  does not. The second observation follows since  $p_a(4\Delta) > 2\Delta$ , by (20). Finally, the third observation is proved as follows: First, if  $H \leq \Delta - \pi(\Delta) + \sigma$ , then  $v_1(H)$  is constant, from (15), while  $v_2(H)$  is decreasing, since the constraint  $p_1 + p_a \geq H$  is binding. Second, consider the case  $H > \Delta - \pi(\Delta) + \sigma$ . Since  $v_2(H) \geq v_1(H)$ , we know from above that  $v_2(H) = V(H - p_a(H), p_a(H))$ . Given this, standard envelope arguments imply  $v_2'(H) = \pi'(H - p_a(H)) + \pi(p_a(H)) - \pi(\Delta)$ , so that from (15),  $v_2'(H) - v_1'(H) = \pi'(H - p_a(H)) + \pi(p_a(H)) - \pi'(H - \sigma) = p_a(H) - \sigma + \pi(p_a(H))$ , and hence  $v_2''(H) - v_1''(H) = (1 + \pi'(p_a(H)))p_a'(H) > 0$ , where the inequality follows since  $p_a(H) \leq 1$  and  $p_a'(H) > 0$  as shown above. Hence, the observation follows.

*Parts 3 and 4.* From Part 2, we know that whenever  $H \in (\hat{H}, 4\Delta]$ , the buyer chooses  $p_1 > 0$  and  $p_a = 0$ , and that whenever  $p_1 > 0$ , the buyer bids  $p_1 = \max\{H - \sigma, \Delta - \frac{1}{2}\Delta^2\}$ . The rest of this part focuses on  $H > 4\Delta$ . First, observe that  $v_1(H) > v_2(H)$ , as follows. Consider any offer  $(p_1, p_a)$  such that  $p_1 + p_a \geq H > 4\Delta$ . Since  $p_a \leq 2\Delta$ , we know  $p_1 > 2\Delta \geq p_a$ , but then the offer  $(p_1, p_a)$  is strictly dominated by  $(p_a, 0)$ , since  $\pi(p_1) + p_1\pi(p_a) + (1 - p_1)\pi(\Delta) - \pi(p_a) - (1 - p_a)\pi(\Delta) = \pi(p_1) - (1 - p_1)\pi(p_a) - (p_1 - p_a)\pi(\Delta)$  is strictly negative; and moreover,  $(p_a, 0)$  satisfies the capital constraint. Hence, when  $H > \hat{H}$ , the buyer chooses  $p_a = 0$ . If  $v_1(H) > 0$ ,

it is optimal to choose  $p_1 > 0$ , and if  $v_1(H) < 0$  and/or  $H - \sigma > 1$ , it is optimal to choose  $p_1 = 0$ . The offer  $(p_1, p_a) = (2\Delta, 0)$  yields strictly positive profits. Hence, for all  $H \leq 2\Delta + \sigma$  the buyer can make strictly positive profits while satisfying the capital constraint. To show that there exists some  $\check{H} \in (2\Delta + \sigma, 1 + \sigma)$ , such that if  $H > \check{H}$  the buyer bids  $p_1 = 0$  and if  $H \in (\hat{H}, \check{H})$  the buyer bids  $p_1 > 0$ , observe that  $v_1(H)$  is strictly decreasing when  $H > 2\Delta + \sigma$ , from (15), and that  $v_1(1 + \sigma) < 0$ . Finally,  $v_1(H)$  and  $v_2(H)$  do not depend on  $L$ , so  $\check{H}$  and  $\hat{H}$  depend only on  $M$  and  $\Delta$ .

Remarks for footnote 20. We show that if  $\Delta$  is sufficiently high (but satisfies Assumption 3) then  $\hat{H} > \Delta + \sigma$ , so an interval on which  $p_1 < \Delta$  does not exist. Consider first  $\Delta = \frac{1}{2}$ . Note that  $\sigma = 2\Delta = 1$ . Hence at  $H = \Delta + \sigma$ , we know  $v_2(H) > V(\Delta, 2\Delta) = V(\Delta, 0) > v_1(H)$ , since the offer  $(p_1, p_1) = (\Delta, 2\Delta)$  satisfies (9) and is strictly dominated by the offer  $(\Delta + \varepsilon, 2\Delta - \varepsilon)$  when  $\varepsilon$  is sufficiently small. So, by continuity,  $v_2(\Delta + \sigma) > v_1(\Delta + \sigma)$  for all  $\Delta$  sufficiently close to  $\frac{1}{2}$ . Hence for all  $\Delta$  sufficiently close to  $\frac{1}{2}$ ,  $\hat{H} > \Delta + \sigma$ .

**Proof of Proposition 3.** As a preliminary, note that if buyer  $-i$  offers price  $p_{-i}$  and the seller accepts the offer, the market learns that  $v \leq p_{-i}$ . Hence, the capital constraint of buyer  $i$ , whose offer is not accepted, becomes  $(\frac{1}{2}p_{-i} + \Delta)M \geq L_i$  (i.e., constraint (1) with  $q = 0$  and  $h = \frac{1}{2}p_{-i} + \Delta$ ). This is equivalent to  $p_{-i} \geq \delta_i(1 + 2\Delta) - 2\Delta$ . Hence, if  $p_{-i} \geq 2\Delta$ , buyer  $i$ 's capital constraint is satisfied if  $\delta_i \leq \frac{4\Delta}{1+2\Delta}$ , which is equivalent to  $P(\delta_i) \leq 2\Delta$ . Conversely, if  $P(\delta_i) > 2\Delta$ , buyer  $i$ 's capital constraint is satisfied only if  $p_{-i} > 2\Delta$ .

Part (1). There is an equilibrium in which both buyers offer  $p = 2\Delta$  because, in equilibrium, the capital constraint of both buyers is satisfied and, given that one buyer offers  $2\Delta$ , the other buyer cannot gain by offering  $p \neq 2\Delta$ . There is no equilibrium in which the equilibrium price is less than  $2\Delta$  because if the buyer that offers the highest price offers  $p < 2\Delta$ , the other buyer (with leverage  $\delta_i$ ) can strictly increase his expected profits without violating his capital constraint by offering  $\max\{p + \varepsilon, P(\delta_i)\}$ , where  $\varepsilon$  is sufficiently small. There is also no equilibrium in which the equilibrium price is more than  $2\Delta$ . To see why, suppose in contradiction that there is an equilibrium in which buyer  $i$  offers  $p_i > 2\Delta$  and buyer  $-i$  offers  $p_{-i} \leq p_i$ . Then buyer  $i$  makes negative profits. If instead buyer  $i$  offers  $2\Delta$ , he makes zero profits and guarantees that his capital constraint will be satisfied, as follows. If  $p_{-i} \geq 2\Delta$  and the seller accepts the offer of buyer  $-i$ , buyer  $i$ 's capital constraint is satisfied by the preliminary argument above. If  $p_{-i} \leq 2\Delta$ , and the seller accepts the offer of buyer  $i$ , buyer  $i$ 's capital constraint is satisfied since  $P(\delta_i) \leq 2\Delta$ .

Part (2). There is an equilibrium in which buyer 1 offers  $P(\delta_1)$  and buyer 2 offers  $P(\delta_2)$ . In this equilibrium, the capital constraints of both buyers are satisfied, buyer 1 makes zero profits (since his offer is never accepted), and buyer 2 makes negative expected profits. Buyer 1 cannot gain by offering  $p \geq P(\delta_2)$  because he will make negative profits. Buyer 1 cannot gain by offering  $p < P(\delta_2)$ , since his offer will not be accepted. Buyer 2 cannot gain by reducing the price because his capital constraint will be violated. In particular, if the seller accepts an offer  $p' < P(\delta_2)$ , buyer 2's capital constraint is violated by (3), and if the seller accepts the offer from buyer 1, buyer 2's capital constraint is violated from the preliminary observations

above. Finally, buyer 2 cannot gain by offering more than  $P(\delta_2)$  because doing so increases his expected losses. There is no equilibrium in which the equilibrium price is more than  $P(\delta_2)$  because the buyer who offers the highest price can reduce his expected losses by reducing the price slightly. There is also no equilibrium in which the equilibrium price is less than  $P(\delta_2)$ , as follows. If the highest price is from buyer 2, buyer 2's capital constraint is violated, and buyer 2 can strictly gain by increasing the price to  $P(\delta_2)$ . If the highest price is from buyer 1 and the price is more than  $2\Delta$ , buyer 1 can reduce his losses without violating his constraint by reducing the price to  $2\Delta$ . If the highest price is from buyer 1 and it is less than  $2\Delta$ , buyer 2's capital constraint is violated, and buyer 2 can gain by increasing the price to  $P(\delta_2)$ , so that his capital constraint is never violated.

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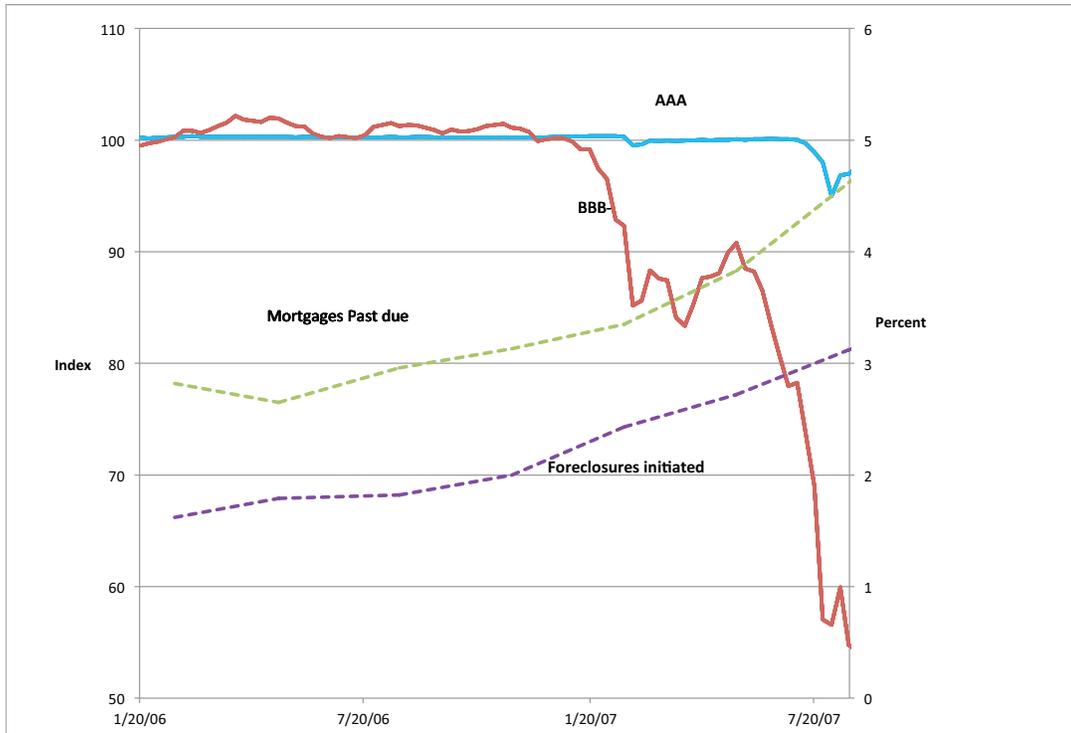
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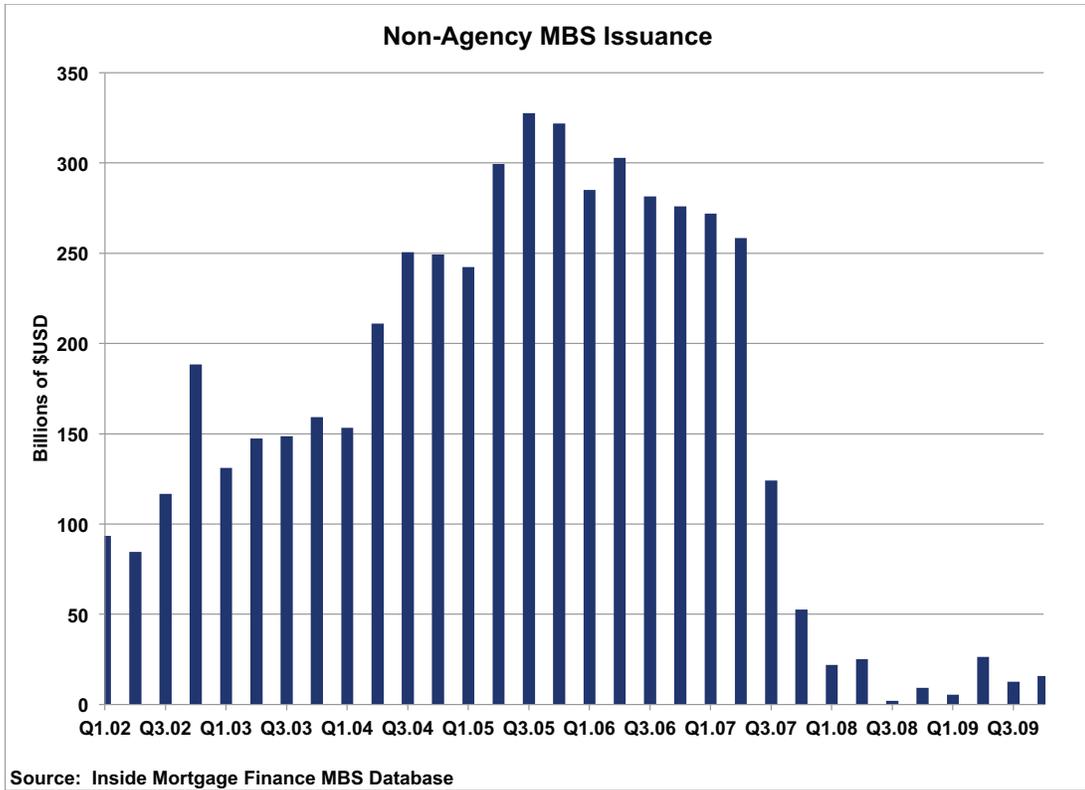
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**Figure 1:** The solid lines plot the Markit ABX.HE 2006-1 indexes for subprime residential MBS rated AAA and BBB-. The dashed lines plot quarterly data from the Mortgage Bankers Association on subprime mortgage payments: specifically, the percentage rate of mortgages that are past due 90 days or more, and the percentage rate of loans for which a foreclosure has been initiated during the quarter (both are seasonably adjusted).



**Figure 2:** Total issuance of non-agency MBS, 2002-2009.

Figure 3 – panel a (low leverage)

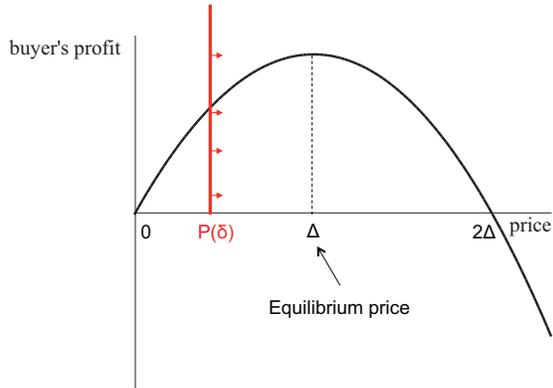


Figure 3 – panel b (intermediate leverage)

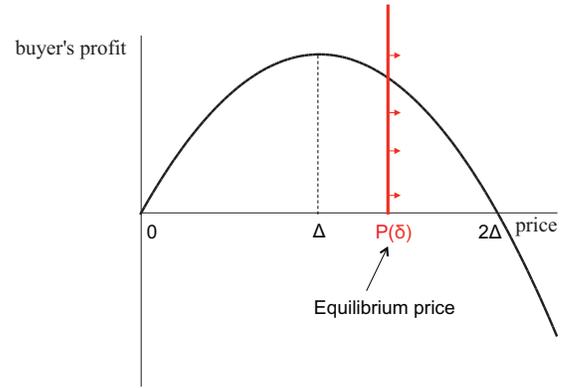
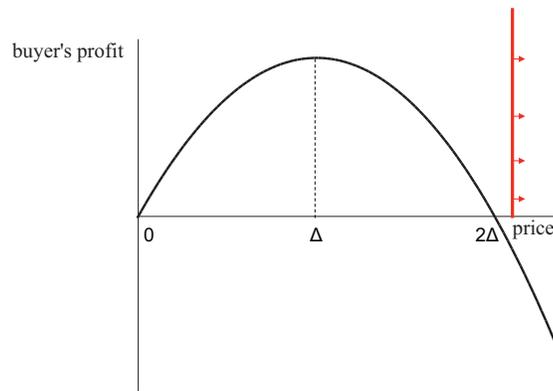


Figure 3 – panel c (high leverage)



**Figure 3:** Each panel shows the buyer's profit function as a function of the buyer's bid price. The vertical line represents the capital constraint. In panel a, the capital constraint is not binding, in panel b the capital constraint binds, and in panel c, the capital constraint is so tight that the buyer prefers not to bid and the market freezes.

## Appendix B: A monopolist buyer: a more general case

In this appendix, we characterize the buyer's optimal bidding strategy when  $h$  in the capital constraint (1) is the expected value of  $\alpha(v + \gamma\Delta)$ , as in footnote 10, and the buyer's offer includes a quantity  $q \in [0, 1]$  in addition to a price  $p$ . (In an earlier draft, available upon request, we show that the buyer cannot increase his profits by offering a nonlinear price schedule.) We replace Assumption 4 with

**Assumption B-1**  $x < \frac{\alpha}{2-\alpha}M$ ,

and define leverage as  $\delta = \frac{L}{(\frac{1}{2} + \gamma\Delta)M}$ , which generalizes the definition in the text. For use below, define  $p(q) \equiv \frac{2\theta - cq}{1 - \beta q}$ , where  $\theta \equiv \frac{L}{\alpha M} - \gamma\Delta$ ,  $\beta \equiv \frac{2-\alpha}{\alpha M}$ , and  $c \equiv \frac{2\gamma\Delta}{M}$ .

**Proposition B-1** (i) *If  $\frac{\alpha\gamma}{2-\alpha} < 1$ , trade can happen if and only if leverage is not too high; that is,  $\delta < \frac{2\alpha(1+\gamma)\Delta}{1+2\gamma\Delta}$ . If leverage is sufficiently low, the buyer offers to buy the entire quantity  $x$  for a price  $\Delta$ . As leverage increases, the buyer increases the price; and as leverage increases further, the buyer also reduces the quantity. Both price and quantity are continuous in leverage. As leverage approaches  $\frac{2\alpha(1+\gamma)\Delta}{1+2\gamma\Delta}$ , the price approaches  $2\Delta$  and the quantity approaches zero. Expected volume is continuous in leverage; it first increases and then drops to zero.*

(ii) *If  $\frac{\alpha\gamma}{2-\alpha} \geq 1$ , trade can happen if and only if leverage is sufficiently low, so that  $p(x) \leq 2\Delta$ . If trade happens, the buyer offers to buy the entire quantity  $x$  for a price per unit  $\max\{\Delta, p(x)\}$ , which is weakly increasing in initial leverage  $\delta$ .*

(When  $\gamma = \alpha = 1$ ,  $p(x) = P(\delta)$ , and we obtain Proposition 1 as a special case.)

**Proof of Proposition B-1.**

For use below, note that  $\theta = \delta \frac{1+2\gamma\Delta}{2\alpha} - \gamma\Delta$ ; that is,  $\theta$  is a monotone increasing function of leverage. Since  $1 - \beta q > 0$  (Assumption B-1), the capital constraint can be written as  $p \geq p(q)$ . Observe that  $p'(q) = \frac{2\beta(\theta - \frac{c}{2\beta})}{(1-\beta q)^2} = \frac{2\beta(\theta - \frac{\alpha\gamma\Delta}{2-\alpha})}{(1-\beta q)^2}$ . Thus,

$$\min_{q \in [0, x]} p(q) = \begin{cases} p(0) & \text{if } \theta \geq \frac{\alpha\gamma\Delta}{2-\alpha} \\ p(x) & \text{if } \theta \leq \frac{\alpha\gamma\Delta}{2-\alpha}. \end{cases}$$

Trade is possible if and only if there exists a quantity  $q \in (0, x]$ , such that  $p(q) \leq 2\Delta$ , so that the buyer makes nonnegative profits; that is, if either  $p(x) \leq 2\Delta$  or  $p(0) = 2\theta < 2\Delta$ . Hence, trade is possible if and only if  $\theta$  falls in some lower interval.

*Case 1:*  $\frac{\alpha\gamma}{2-\alpha} < 1$ . At  $\theta = \Delta$ ,  $p(x) > p(0) = 2\Delta$ . Thus, trade is impossible for  $\theta \geq \Delta$  but is possible for all  $\theta < \Delta$ , or equivalently,  $\delta < \frac{2\alpha(1+\gamma)\Delta}{1+2\gamma\Delta}$ . Next, we characterize the buyer's offer when  $\theta < \Delta$ . If  $p(x) \leq \Delta$ , which is equivalent to  $\theta \leq \frac{1}{2}(\Delta - \beta\Delta x + cx)$ , the capital constraint does not bind and the buyer makes the benchmark offer  $(x, \Delta)$ . If  $\theta \leq \frac{\alpha\gamma}{2-\alpha}\Delta$ , increasing  $q$  relaxes the capital constraint, and since  $\theta < \Delta$  we know the buyer has a strictly profitable trade. Consequently, the buyer bids for the entire amount  $x$  available and chooses a price  $\max\{\Delta, p(x)\}$ . This price is weakly increasing in  $\theta$ , and hence in initial leverage  $\delta$ .

The remainder of the proof of this case deals with the open interval of  $\theta$  values above  $\max\{\frac{\alpha\gamma}{2-\alpha}\Delta, \frac{1}{2}(\Delta - \beta\Delta x + cx)\}$  but below  $\Delta$ . Since  $\theta < \Delta$ , we know that the buyer has a strictly profitable trade. Moreover, any strictly profitable trade in which the capital constraint is slack is strictly dominated by one in which it binds: either  $q < x$ , in which case  $q$  can be increased, or  $q = x$  and  $p > p(x) > \Delta$ , in

which case  $p$  can be decreased. Consequently, the buyer's best offer is the solution to the more constrained maximization problem in which he must keep the capital constraint binding; that is,

$$\max_{q \in [0, x]} q\pi(p(q)). \quad (\text{B-1})$$

Observe that  $\frac{\partial}{\partial q} q\pi(p(q)) = \pi(p(q)) + qp'(q)\pi'(p(q))$ , and recall  $p'(q) > 0$  in the interval under consideration. Hence, (B-1) has a unique solution, as follows: if  $q\pi(p(q)) > 0$  and  $\frac{\partial}{\partial q} q\pi(p(q)) \leq 0$  for some  $q$ , then  $\pi'(p(q)) < 0$ , and so by the strict concavity of  $\pi$  and the strict convexity of  $p$ , it follows that  $\frac{\partial}{\partial q} q\pi(p(q)) < 0$  for all higher  $q$ . Moreover, the maximizer of (B-1) must be such that  $\pi'(p(q)) < 0$  (if the maximizer is the corner  $q = x$ , this follows from  $p(x) > \Delta$ ). Hence, in the interval from which the maximizer of (B-1) is drawn,  $\frac{\partial}{\partial q} q\pi(p(q))$  strictly decreases in  $\theta$ . Hence, the buyer's choice of  $q$  weakly decreases as  $\theta$  (and hence initial leverage  $\delta$ ) increases. Note also that if  $q$  is strictly decreasing at any  $\theta$ , the same is true for all higher  $\theta$ .

For the effect of  $\theta$  (and hence leverage) on the price offered, note first that if the buyer offers to buy everything ( $q = x$ ) for price  $p(x)$ , it follows immediately that the price increases. If instead the buyer offers to buy  $q < x$ , the price satisfies  $p = p(q)$ , and the optimal  $p$  solves  $\max_{p \in [0, 1]} q(p)\pi(p)$ , where  $q(p) = \frac{p-2\theta}{\beta p-c}$  is the inverse function of  $p(q)$ . Observe that  $\frac{\partial}{\partial p} q(p)\pi(p) = q'(p)\pi(p) + q(p)\pi'(p)$ ;  $q'(p) = \frac{2\theta\beta-c}{(\beta p-c)^2} > 0$ ; and recall that the optimal  $p$  satisfies  $\pi'(p) < 0$ . Hence, in the interval in which the (unique) optimal  $p$  is drawn,  $\frac{\partial}{\partial p} q(p)\pi(p)$  strictly increases in  $\theta$  and strictly decreases in  $p$  (the last part follows from the concavity of  $\pi$  and  $q$ ). Hence, the optimal  $p$

increases in  $\theta$ .

From the analysis above, the buyer's offer is continuous as a function of  $\theta$ . Finally, as  $\theta$  approaches  $\Delta$ , only offers with  $q$  close to 0 can satisfy the capital constraint (with a price below  $2\Delta$ ). It follows easily that as  $\theta$  approaches  $\Delta$ , the buyer's offer converges to  $(q, p) = (0, 2\Delta)$ . The expected volume  $(pq)$  converges to 0.

To show that expected volume first increases in leverage, it is enough to show that there exists some interval to the right of  $\frac{1}{2}(\Delta - \beta\Delta x + cx)$  such that when  $\theta$  falls in this interval, the buyer offers to buy everything,  $q = x$ . If  $\frac{\alpha\gamma}{2-\alpha}\Delta > \frac{1}{2}(\Delta - \beta\Delta x + cx)$ , this is immediate from the analysis above. Otherwise, note that at  $\theta = \frac{1}{2}(\Delta - \beta\Delta x + cx)$ , we know  $p(x) = \Delta$ ; thus,  $\left. \frac{\partial}{\partial q} q\pi(p(q)) \right|_{q=x} = \pi(\Delta) + xp'(x)\pi'(\Delta) = \pi(\Delta) > 0$ . By continuity, it follows that  $\left. \frac{\partial}{\partial q} q\pi(p(q)) \right|_{q=x} > 0$  over some interval to the right of  $\frac{1}{2}(\Delta - \beta\Delta x + cx)$ , implying that the buyer offers to buy  $q = x$  in this interval.

From the analysis above,  $q$  must eventually be strictly decreasing in  $\theta$ , (and if it is strictly decreasing at some  $\theta$ , the same is true for all higher  $\theta$  up to  $\Delta$ , when trade becomes impossible). In this case, expected volume changes by  $\frac{\partial(qp(q))}{\partial\theta} = \frac{\partial q}{\partial\theta}p(q) + qp'(q)\frac{\partial q}{\partial\theta} = \frac{\partial q}{\partial\theta}[p(q) + qp'(q)]$ , which is strictly negative in the interval under consideration.

*Case 2:*  $\frac{\alpha\gamma}{2-\alpha} \geq 1$  (contains Proposition 1 as a special case). In this case, at  $\theta = \Delta$ ,  $p(x) \leq p(0) = 2\theta$ , and so  $p(x) \leq 2\Delta$ . Hence, trade is certainly possible up to  $\theta = \Delta$ . Hence, trade is possible for all  $\theta$  weakly below the cutoff value of  $\theta$  such that  $p(x) = 2\Delta$ . The characterization of the buyer's offer for  $\theta$  below this cutoff is the same as for the first part of the case  $\frac{\alpha\gamma}{2-\alpha} < 1$ . Q.E.D.

*Comparative statics with respect to  $\alpha$ :* Increasing  $\alpha$  increases the region in which trade can happen. However, once  $\alpha$  is large enough so that trade can occur, but not too large so that the benchmark solution is not achieved, a further increase in  $\alpha$  reduces the bid price and the probability of trade. Specifically, observe that  $\beta$ ,  $p(q)$ , and  $p'(q)$  strictly decrease in  $\alpha$ . Hence, following similar steps as above, one can show that increasing  $\alpha$  has a similar effect on the price and quantity as does reducing  $\theta$ .

## Appendix C: Competing buyers

This appendix contains a detailed analysis of two competing buyers when  $\Delta$  and  $M$  vary across the buyers, in addition to  $L$ . Buyer  $i$  has an inventory of  $M_i$  units of the asset and a debt liability  $L_i$ . The gain from trade with buyer  $i$  is  $\Delta_i$ , and buyer  $i$ 's initial leverage is  $\delta_i = \frac{L_i}{(\frac{1}{2} + \Delta_i)M_i}$ . Everything is common knowledge, except for the true value of the asset ( $v$ ), which is private information to the seller. Assumptions 1-4 hold for each buyer.

Both buyers make offers simultaneously, and we denote buyer  $i$ 's offer by  $p_i$ . The seller has one unit for sale and can accept at most one offer.<sup>33</sup> Hence, if  $v > \max\{p_1, p_2\}$ , the seller rejects both offers and trade does not take place. Otherwise, the seller accepts the offer with the highest price, and if prices coincide,  $p_1 = p_2$ , the

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<sup>33</sup>An earlier draft, available upon request, contains a full analysis of the case in which the asset is divisible and trade is nonexclusive in the sense that the seller can choose quantities  $q_i \in [0, 1]$  to sell to each buyer, such that  $q_1 + q_2 \leq 1$ . The nature of the results remains.

seller chooses one buyer randomly.

As in the monopolist buyer case, when the seller accepts the offer of buyer  $i$ , one can infer that  $v \leq p_i$ . Hence, the market value of buyer  $i$ 's inventories falls, and to ensure that buyer  $i$ 's capital constraint remains satisfied, the offer  $p_i$  must satisfy  $p_i \geq P_i(\delta_i)$ , where  $P_i(\delta_i)$  is defined parallel to equation (3); that is,

$$P_i(\delta_i) \equiv \frac{\delta_i M_i (1 + 2\Delta_i) - 2\Delta_i (M_i + 1)}{M_i - 1}. \quad (\text{C-1})$$

However, now the acceptance of offer  $p_i$  may also lead to a violation of the capital constraint of buyer  $-i$ , who did not purchase the asset, since the market value of his inventories falls as well. Specifically, buyer  $-i$ 's capital constraint is violated when the seller accepts buyer  $i$ 's offer if

$$\left(\frac{1}{2}p_i + \Delta_{-i}\right)M_{-i} < L_{-i}. \quad (\text{C-2})$$

Defining

$$P_i^0(\delta_i) \equiv \delta_i(1 + 2\Delta_i) - 2\Delta_i, \quad (\text{C-3})$$

equation (C-2) reduces to  $p_i < P_{-i}^0(\delta_{-i})$ ; that is, buyer  $-i$ 's capital constraint is violated when the seller accepts buyer  $i$ 's offer if  $p_i < P_{-i}^0(\delta_{-i})$ , and is satisfied otherwise.

Observe that both  $P_i(\delta_i)$  and  $P_i^0(\delta_i)$  increase in buyer  $i$ 's initial leverage. For use below, we omit the variable  $\delta_i$  and simply write  $P_i$  and  $P_i^0$ . To avoid technical issues associated with continuous-action games, we also assume that the price space is finite, and the values  $\{P_i, P_i \pm \varepsilon, \Delta_i, \Delta_i \pm \varepsilon, 2\Delta_i, 2\Delta_i \pm \varepsilon\}_{i \in \{1,2\}}$  lie within this space. The “tick” size  $\varepsilon$  is assumed to be close to zero, and for clarity, we exclude it from the statements of the results.

Because of the externalities generated by each buyer's bid on other buyers, there are typically Nash equilibria in which buyer  $i$  makes a bid that violates buyer  $-i$ 's capital constraint, if accepted, and forces buyer  $-i$  to make a higher bid himself. However, not all equilibria of this type are robust, in the sense that there is no good reason for buyer  $i$  to make such a bid in the first place. Accordingly, we focus on equilibria that are robust in the sense of not entailing dominated strategies. Specifically, we characterize equilibria that survive the following iterated process of elimination of weakly dominated strategies. In the first stage we eliminate all strategies that are weakly dominated. In the second stage, we consider the game remaining after the first stage and eliminate strategies that are weakly dominated in this new game. And so on. Lemma C-2 characterizes offers that survive the first elimination round. The lemma makes use of the following observation (all proofs are at the end of this appendix):

**Lemma C-1** *For every  $i \in \{1, 2\}$ , one of the following is true: (i)  $P_i = P_i^0 = 2\Delta_i$ ; (ii)  $P_i > P_i^0 > 2\Delta_i$ ; or (iii)  $P_i < P_i^0 < 2\Delta_i$ .*

**Lemma C-2** (A) *If  $P_i < 2\Delta_i$ , the offer  $p_i$  survives the first round of elimination of weakly dominated strategies if and only if  $\max\{\Delta_i, P_i\} \leq p_i < 2\Delta_i$ .*

(B) *If  $P_i = 2\Delta_i$ , the unique offer to survive the first round of elimination of weakly dominated strategies is  $p_i = 2\Delta_i$ .*

(C) *If  $P_i \in (2\Delta_i, 1]$ , the offers  $p_i = 0$  and  $p_i = P_i$  survive the first round of elimination of weakly dominated strategies. In contrast, any offer  $p_i \neq P_i$  such that  $p_i \geq 2\Delta_i$  is eliminated.*

(D) If  $P_i > 1$ , the only offer that survives the first round of elimination of weakly dominated strategies is the offer  $p_i = 0$ .

Part (A) says that when a buyer has a profitable trade, he always tries to exploit it by making an offer that yields positive profits and does not violate his capital constraint. This behavior is similar to the single-buyer case previously analyzed. The result in part (C) that the loss-making offer  $P_i \in (2\Delta_i, 1]$  is undominated reflects the fact that, with competition, a buyer may wish to make a “preemptive” bid to ensure that his capital constraint is not violated should the other buyer make an offer at a low price.<sup>34</sup>

### *Equilibrium outcomes*

We start with the case in which both buyers have sufficiently low leverage that each would offer to purchase the asset if he were the only buyer; formally,  $P_1 \leq 2\Delta_1$  and  $P_2 \leq 2\Delta_2$ . From Lemma C-2, we know that buyer  $i$  offers to purchase the asset at a price which is between his monopoly offer  $\max\{\Delta_i, P_i\}$  and his zero-profits offer  $2\Delta_i$ . If the two buyers have very different valuations (i.e.,  $\max\{\Delta_1, P_1\} > 2\Delta_2$  or  $\max\{\Delta_2, P_2\} > 2\Delta_1$ ), the buyer with the highest valuation can continue to make his monopoly offer, so competition has no effect on the equilibrium price.

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<sup>34</sup>Part (D) reflects the fact that such a preemptive bid is possible only if the buyer has sufficient slack in his capital constraint. The reason is that while a preemptive bid can ensure that the value of the buyer’s existing inventories remains at the prior, the purchase of an additional unit at a loss tightens the constraint and can lead to its violation.

Otherwise, competition drives the equilibrium price to  $\min \{2\Delta_1, 2\Delta_2\}$ . This is a standard outcome for settings with public buyer valuations: The buyer with the highest valuation acquires the asset at a price determined by the zero-profit condition of the buyer with the second-highest valuation.<sup>35</sup>

**Proposition C-1** *If both buyers have low leverage, i.e.,  $P_1 \leq 2\Delta_1$  and  $P_2 \leq 2\Delta_2$ , then the only equilibrium outcome that survives iterated elimination of weakly dominated strategies is such that whenever the seller agrees to sell, he sells the asset to the buyer with the higher valuation for price equal to the maximum of the two monopoly prices,  $\max \{\Delta_1, P_1\}$  and  $\max \{\Delta_2, P_2\}$ , and the zero-leverage competition price,  $\min \{2\Delta_1, 2\Delta_2\}$ .*

Next, we consider the case in which buyer 1 has low leverage, but buyer 2 has sufficiently high leverage that he would not offer to purchase the asset if he were the only buyer; formally,  $P_1 \leq 2\Delta_1$  and  $P_2 > 2\Delta_2$ . The key observation is that if buyer 1 offers to purchase the asset at a price  $p_1 < P_2^0$ , and the seller accepts buyer 1's offer, the expected value of  $v$  drops to  $\frac{p_1}{2}$ , and this causes buyer 2's capital constraint to be violated. Consequently, when buyer 2 is highly leveraged, so  $P_2^0$  is high, buyer 2 may bid more aggressively to ensure that the seller does not accept a lower bid from buyer 1.<sup>36</sup>

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<sup>35</sup>See, for example, Ho and Stoll (1983).

<sup>36</sup>From Lemma C-2, we know that such a preemptive bid is possible only if  $P_2 \leq 1$ . If instead,  $P_2 > 1$ , buyer 2 bids  $p_2 = 0$ , and buyer 1's unique best response is to act as if he were a monopolist; that is  $p_1 = \max \{\Delta_1, P_1\}$ . Accordingly, Proposition C-2 focuses on the case  $P_2 \leq 1$ .

**Proposition C-2** *If buyer 1 has low leverage and buyer 2 has high leverage (i.e.,  $P_1 \leq 2\Delta_1$  and  $P_2 \in (2\Delta_2, 1]$ ), then the only equilibrium outcome that survives iterated elimination of weakly dominated strategies is as follows:*

(A) *If buyer 2's leverage is not too high, i.e.,  $P_2^0 \leq \max\{\Delta_1, P_1\}$ , then whenever the seller agrees to sell, he sells the asset to buyer 1 for a price  $\max\{\Delta_1, P_1\}$ .*

(B) *If buyer 2's leverage is higher, i.e.,  $P_2^0 > \max\{\Delta_1, P_1\}$ , then whenever the seller agrees to sell, he sells the asset for a price  $P_2$ . If  $P_2 < 2\Delta_1$ , the seller sells to buyer 1, and if  $P_2 \geq 2\Delta_1$ , the seller sells to buyer 2 who makes negative profits.*

Proposition C-2 shows that the buyers' capital constraints continue to affect prices even when there are multiple competing buyers. Part (A) reflects the simple intuition that if buyer 2 prefers not to trade, then buyer 1 can act as a monopolist. In this case, there is basically no interaction between the buyers.

In part (B), in contrast, the buyers interact in a non-degenerate way. If buyer 2's leverage is relatively high so that  $P_2 \in (2\Delta_2, 2\Delta_1)$ , buyer 2's capital constraint leads him to compete more aggressively with buyer 1, and consequently buyer 1 ends up paying an amount  $P_2$  that is determined by buyer 2's capital constraint. If  $P_2 \geq \max\{2\Delta_1, 2\Delta_2\}$ , buyer 1 has no incentive to compete at such a high price; therefore, whenever trade occurs, buyer 2 acquires the asset at a price  $P_2$ . In the latter case, buyer 2 makes negative profits, even though he would not bid at all if he were the only buyer. Buyer 2 is forced to make this bid, since otherwise the seller trades with buyer 1 and buyer 2's capital constraint is violated.

Finally, consider the case in which both buyers are so leveraged, that, if bidding individually, trade collapses in the sense that no one makes an offer. Clearly, no trade is an equilibrium that survives iterated elimination of weakly dominated strategies, since given that one buyer is unwilling to make an offer, the unique best response for the other buyer is also not to make an offer. Moreover, no trade is the only outcome to survive iterated elimination of weakly dominated strategies when  $P_1 \neq P_2$ .<sup>37</sup>

**Proposition C-3** *If both buyers are highly leveraged (i.e.,  $P_i > 2\Delta_i$  for  $i \in \{1, 2\}$ ), then a no-trade equilibrium survives iterated elimination of weakly dominated strategies. When  $P_1 \neq P_2$ , this is the unique equilibrium that survives iterated elimination.*

Proposition C-3 shows that when both buyers have tight capital constraints, the conclusions of the single-buyer case continue to hold: trade collapses, and price dissemination stops. Indeed, the condition  $P_i > 2\Delta_i$  in Proposition C-3 is equivalent to the condition for no trade ( $\delta_i > \frac{4\Delta_i}{1+2\Delta_i}$ ) in Proposition 1.

Note that in the special case in which buyers differ only in their inventory positions, but not in their valuations of the asset, Propositions C-1-C-3 can be summarized as:

**Corollary C-1** *If  $\Delta_1 = \Delta_2 = \Delta$ , then: (i) if  $\max\{P_1, P_2\} \leq 2\Delta$ , the price is  $2\Delta$ ; (ii) if  $P_1 \leq 2\Delta < P_2 \leq 1$ , the price is  $P_2$ , and buyer 2 purchases the asset at a loss;*

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<sup>37</sup>In the nongeneric case  $P_1 = P_2 > \max\{P_1^0, P_2^0\}$ , we cannot rule out other equilibria in which one buyer makes a latent offer, knowing that it will not be accepted in equilibrium, and the second buyer makes a loss-making offer to rule out a situation in which the seller trades with the first buyer and the capital constraint of the second buyer is violated.

and (iii) if  $\min\{P_1, P_2\} > 2\Delta$ , trades completely breaks down.

### *Proofs*

**Proof of Lemma C-1.** From equations (C-1) and (C-3),  $P_i = \frac{M_i P_i^0 - 2\Delta_i}{M_i - 1}$ . Hence, the result follows. Q.E.D.

**Proof of Lemma C-2.** *Part (A):* From Lemma C-1,  $P_i < P_i^0 < 2\Delta_i$ . The offer  $p_i \in [\max\{\Delta_i, P_i\}, 2\Delta_i)$  survives the first stage of elimination, since it is a unique best response for buyer  $i$  when buyer  $-i$  bids  $p_{-i} = p_i - \varepsilon$ . Any offer with  $p_i < \Delta_i$  is weakly dominated by raising the offer to  $\Delta_i$ . Finally, any offer with  $p_i \geq 2\Delta_i$  or  $p_i < P_i$  is weakly dominated by the offer  $P_i^0$  as follows. The offer  $P_i^0$  produces positive profits whenever it is accepted (e.g., if  $p_{-i} = 0$ ) and guarantees that buyer  $i$ 's capital constraint is satisfied, regardless of  $-i$ 's offer. In contrast, offering  $p_i \geq 2\Delta_i$  never leads to positive profits, and offering  $p_i < P_i$  violates  $i$ 's capital constraint if it is accepted (e.g., if  $p_{-i} = 0$ ).

*Part (B):* From Lemma C-1,  $P_i = P_i^0 = 2\Delta_i$ . The offer  $p_i = 2\Delta_i$  weakly dominates any other, as follows: The offer  $p_i = 2\Delta_i$  produces zero profits and guarantees that buyer  $i$ 's capital constraint is satisfied, regardless of  $-i$ 's offer. In contrast, the offer  $p_i > 2\Delta_i$  produces negative profits whenever it is accepted (e.g., if  $p_{-i} = 0$ ), and the offer  $p_i < 2\Delta_i$  violates  $i$ 's capital constraint if accepted (e.g.,  $p_{-i} = 0$ ).

*Part (C):* From Lemma C-1,  $P_i > P_i^0 > 2\Delta_i$ . The offer  $p_i = 0$  survives the first elimination round, since it is the unique best response for buyer  $i$  when buyer  $-i$  bids  $p_{-i} = 0$ . Specifically, if buyer  $i$  bids  $p_i = 0$ , he obtains zero profits and his

capital constraint is satisfied; if he bids  $p_i > 0$  and his offer is accepted, he either makes negative profits or his capital constraint is violated.

The offer  $p_i = P_i$  survives the first elimination round, since it is a unique best response when buyer  $-i$  bids  $p_{-i} \in (0, P_i^0 - \varepsilon)$ . Specifically, the offer  $p_i = P_i$  guarantees that buyer  $i$ 's capital constraint is satisfied, while if he bids  $p_i \in [P_i^0, P_i)$  his capital constraint is violated whenever the seller accepts; if he bids  $p_i \in [0, P_i^0 - \varepsilon]$  his capital constraint is violated whenever the seller accepts buyer  $-i$ 's offer; and if he bids  $p_i > P_i$ , his expected profits are reduced.

Any offer  $p_i > P_i$  is weakly dominated by the alternate offer  $P_i$ . The alternate offer weakly increases buyer  $i$ 's profits, with a strict increase whenever  $-i$ 's offer is lower, and does not affect whether the capital constraint is satisfied.

Any offer  $p_i \in [2\Delta, P_i)$  is weakly dominated by  $\tilde{p}_i = 0$ , as follows. If  $p_{-i} > p_i$ , the outcome is the same under both offers, since the seller accepts the other buyer's offer. If  $p_{-i} = p_i$ , the offer  $\tilde{p}_i = 0$  is at least as good since under both offers the capital constraint of buyer  $i$  is violated with probability  $p_{-i}$ , but the offer  $p_i$  leads to zero or negative profits while  $\tilde{p}_i = 0$  leads to zero profits. If  $p_{-i} < p_i$ , the offer  $\tilde{p}_i$  is strictly preferred since under  $\tilde{p}_i = 0$ , the capital constraint of buyer  $i$  is violated with probability 0 or  $p_{-i}$ , while under  $p_i$ , it is violated with probability  $p_i$ .<sup>38</sup>

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<sup>38</sup>Note that an offer  $p_i \in (0, 2\Delta_i)$  is not eliminated, as follows: Offering  $p_i \in (0, 2\Delta_i)_i$  is strictly preferred to offering 0 if the other buyer offers  $p_{-i} = p_i$ , since the capital constraint is violated in both cases, but  $p_i$  leads to positive profits. Offering  $p_i \in (0, 2\Delta_i)$  is strictly preferred to  $P_i$ , if the other buyer offers  $p_{-i} = 0$ , since the capital constraint is satisfied in both cases, but  $p_i$  strictly increases profits.

*Part (D):* Any offer  $p_i > 0$  is weakly dominated by  $\tilde{p}_i = 0$ , as follows. If the offer  $p_i < p_{-i}$ , buyer  $i$ 's utility is the same under the alternate offer  $\tilde{p}_i = 0$ . If instead  $p_i \geq p_{-i}$ , buyer  $i$ 's capital constraint is violated (since  $P_i > 1$ ) whenever the seller accepts offer  $p_i$ , whereas the offer  $\tilde{p}_i = 0$  satisfies the capital constraint if  $p_{-i} = 0$ . Q.E.D.

**Proof of Proposition C-1.** First, consider the case  $\Delta_1 \neq \Delta_2$ , and assume, without loss of generality, that  $\Delta_1 > \Delta_2$ . If  $P_2 = 2\Delta_2$  and  $P_1 = 2\Delta_1$ , the result follows immediately from Lemma C-2. If  $P_2 = 2\Delta_2$  and  $P_1 < 2\Delta_1$ , we know from Lemma C-2 that buyer 2 bids  $p_2 = 2\Delta_2$ ; therefore, if  $\max\{\Delta_1, P_1\} > 2\Delta_2$ , buyer 1's best response is to bid  $p_1 = \max\{\Delta_1, P_1\}$ , and if  $\max\{\Delta_1, P_1\} \leq 2\Delta_2$ , buyer 1's best response is to bid  $\min\{2\Delta_2 + \varepsilon, 2\Delta_1 - \varepsilon\}$ . If  $P_2 < 2\Delta_2$ , then from Lemma C-2,  $p_2 < 2\Delta_2$  therefore, if  $\max\{\Delta_1, P_1\} \geq 2\Delta_2$ , buyer 1's best response is to bid  $p_1 = \max\{\Delta_1, P_1\}$ , and if  $\max\{\Delta_1, P_1\} < 2\Delta_2$ , standard competition arguments imply that buyer 2 bids  $p_2 = 2\Delta_2 - \varepsilon$  and buyer 1 bids  $p_1 = 2\Delta_2$ .

Next, consider the case  $\Delta_1 = \Delta_2 = \Delta$ . If  $\max\{P_1, P_2\} = 2\Delta$ , then by Lemma C-2, the equilibrium price is  $2\Delta$  and is offered by the buyer with the highest  $P_i$ . If  $\max\{P_1, P_2\} < 2\Delta$ , Lemma C-1 and standard competition arguments imply that both buyers offer the price  $2\Delta - \varepsilon$ . Q.E.D.

**Proof of Proposition C-2:** From the first elimination round (Lemma C-2), we know that  $p_1 \in [\max\{\Delta_1, P_1\}, 2\Delta_1]$  and  $p_2 \in [0, 2\Delta_2) \cup \{P_2\}$ . From Lemma C-1,  $P_2 > P_2^0 > 2\Delta_2$ .

*Part (A):*  $P_2^0 \leq \max\{\Delta_1, P_1\}$ . Hence, any offer  $p_2 \geq \max\{\Delta_1, P_1\}$  is above  $2\Delta_2$

and so gives negative profits if accepted (e.g., if  $p_1 = \max\{\Delta_1, P_1\}$ ), whereas the offer  $p_2 = 0$  gives zero profits and guarantees that the capital constraint of buyer 2 is satisfied given any  $p_1$  that survives the first round. Hence, any offer that survives the second elimination round must satisfy  $p_2 < \max\{\Delta_1, P_1\}$ . Buyer 1's unique best response to any such offer is to bid  $p_1 = \max\{\Delta_1, P_1\}$ .

*Part (B):*  $P_2^0 > \max\{\Delta_1, P_1\}$ . There is no equilibrium in which buyer 1 bids  $p_1 < P_2^0$  and buyer 2 bids  $p_2 < 2\Delta_2$  because in such an equilibrium buyer 2's capital constraint is violated; but then buyer 2 would deviate to offer  $p_2 = P_2$ , which wins since  $P_2 > P_2^0$ . Nor is there an equilibrium in which buyer 1 bids  $p_1 \geq P_2^0$  and buyer 2 bids  $p_2 < 2\Delta_2$  because buyer 1 can increase his profits by deviating to  $p_1 = P_2^0 - \varepsilon \geq 2\Delta_2 > p_2$ . Consequently, in any candidate equilibrium  $p_2 = P_2$ . Hence, if  $P_1 = 2\Delta_1$ , the unique equilibrium is that buyer 2 bids  $p_2 = P_2$  and buyer 1 bids  $p_1 = 2\Delta_1$ ; and if  $P_1 < 2\Delta_1$  and  $P_2 < 2\Delta_1$ , the unique equilibrium is that buyer 2 bids  $p_2 = P_2$  and buyer 1 bids  $p_1 = \min\{P_2 + \varepsilon, 2\Delta_1 - \varepsilon\}$ . If instead,  $P_1 < 2\Delta_1$  and  $P_2 \geq 2\Delta_1$ , the unique equilibrium outcome is that buyer 2 bids  $p_2 = P_2$ , buyer 1 bids a lower price  $p_1 < 2\Delta_1 \leq P_2$ , and the equilibrium price is  $P_2$ . Q.E.D

**Proof of Proposition C-3.** From Lemma C-1,  $P_i > P_i^0 > 2\Delta_i$  for  $i \in \{1, 2\}$ . For use throughout the proof, note that this means that neither buyer can make an offer that is accepted with positive probability, makes non-negative profits, and satisfies the buyer's capital constraint upon acceptance.

From the first elimination round (Lemma C-2),  $p_i \in [0, 2\Delta_i) \cup P_i$ , for  $i \in \{1, 2\}$ . In addition, we know that for  $i \in \{1, 2\}$ , the offers  $p_i = 0$  and  $p_i = P_i$  survive the

first round. In fact, the offers  $p_1 = 0$  and  $p_2 = 0$  cannot be eliminated in any round, since each one is a unique best response against the other. Hence, the no trade equilibrium survives the elimination process. The remainder of the proof establishes that, provided  $P_1 \neq P_2$ , this is the unique equilibrium to survive the elimination process.

Start with the case  $P_1^0 > P_2$ . The offer  $p_1 = P_1$  survives the second elimination round, since it is a unique best response if buyer 2 bids  $p_2 = P_2$ . However, any offer  $p_1 \in (0, 2\Delta_1)$  is eliminated, since it violates buyer 1's capital constraint with a positive probability, while the offer  $p_1 = P_1$  never violates the constraint.<sup>39</sup> Hence, buyer 1 bids either  $p_1 = 0$  or  $p_1 = P_1$ . In the third elimination round, the offer  $p_2 = 0$  weakly dominates any other remaining offer for buyer 2, since  $p_2 = 0$  gives buyer 2 zero profits and guarantees that his capital constraint is satisfied, while any other remaining offer gives zero profits if  $p_1 = P_1$  and leads to negative profits and/or violates buyer 2's capital constraint, if  $p_1 = 0$ . Hence, the unique equilibrium that survives the third elimination round is that both buyers bid nothing. The case  $P_2^0 > P_1$  is similar.

The rest of the proof deals with the case in which both  $P_1^0 \leq P_2$  and  $P_2^0 \leq P_1$ ; and without loss, assume  $P_1 < P_2$ .

Note first that if every offer  $p_i \in (0, P_{-i}^0)$  is eliminated in the first elimination round, then (from the second elimination round) the other buyer  $-i$  must offer  $p_{-i} = 0$ , and the unique equilibrium is no trade, and the proof is complete. The

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<sup>39</sup>If  $p_2 < p_1$ , the offer  $p_1$  violates with probability  $p_1$ . If  $p_2 \geq p_1$ , we know from the first round that  $p_2 \leq P_2 < P_1^0$ , and so the offer  $p_1$  violates the constraint with probability of at least  $p_1$ .

rest of the proof deals with the case in which for both buyers some offer  $\bar{p}_i \in (0, P_{-i}^0)$  survives the first round. Consequently, for each buyer  $i$  the offer  $P_i$  survives the second round, since it is a unique best response when the buyer  $-i$  bids  $p_{-i} = \bar{p}_{-i}$ .

Next, we show that if  $P_1^0 < 2\Delta_2$ , no offer  $p_2 \in [P_1^0, 2\Delta_2)$  survives the second elimination round. In particular, any such offer is weakly dominated by  $\tilde{p}_2 = 0$ , as follows. If  $p_1 = P_1$ , buyer 2 is indifferent between  $\tilde{p}_2 = 0$  and  $p_2$ , since  $2\Delta_2 \leq P_2^0 \leq P_1$  and so in both cases he obtains zero profits and his capital constraint is satisfied. If  $p_1 = 0$ , buyer 2 strictly prefers  $\tilde{p}_2 = 0$ . It remains to show that buyer 2 weakly prefers  $\tilde{p}_2 = 0$  to  $p_2$  given all other potential offers from buyer 1 that survive the first round; any such offer would have  $p_1 \in (0, 2\Delta_1)$ . Given such offer, if buyer 2 offers  $\tilde{p}_2 = 0$ , his capital constraint is violated with probability at most  $p_1$ , but if he chooses  $p_2 \in [P_1^0, 2\Delta_2)$ , his capital constraint is violated with a higher probability, namely  $p_2$ .

In the third elimination round, any offer  $p_1 \in (0, 2\Delta_1)$  is eliminated, since it is weakly dominated by the offer  $\tilde{p}_1 = P_1$ , as follows. If  $p_2 = P_2$ , both offers provide the same utility, since  $P_2 > P_1 > 2\Delta_1$ . If instead,  $p_2 < P_1^0$ , then buyer 1's capital constraint is violated with a positive probability under the offer  $p_1$  but is never violated under the offer  $\tilde{p}_1$ . Hence, buyer 1 bids either  $p_1 = 0$  or  $p_1 = P_1$ . If  $p_1 = P_1$  is eliminated in the third round, we are done and the unique equilibrium is no trade; otherwise, since  $P_2^0 \leq P_1$ , the only offer for buyer 2 that survives the fourth round of elimination is  $p_2 = 0$ , and the unique equilibrium is no trade. Q.E.D.

**Proof of Corollary C-1.** Parts (i) and (iii) follow immediately. For part (ii),

observe that  $2\Delta < P_2^0 < P_2$  (by Lemma C-1), and so part (B) in Proposition C-2 applies. Q.E.D.