WORKING PAPERS
RESEARCH DEPARTMENT

WORKING PAPER NO. 13-10
RISK, ECONOMIC GROWTH, AND THE VALUE OF U.S. CORPORATIONS

Luigi Bocola
University of Pennsylvania

Nils Gornemann
University of Pennsylvania

March 2013

RESEARCH DEPARTMENT, FEDERAL RESERVE BANK OF PHILADELPHIA
Ten Independence Mall, Philadelphia, PA 19106-1574 • www.philadelphiafed.org/research-and-data/

Risk, Economic Growth and the Value of U.S. Corporations*

Luigi Bocola[†] Nils Gornemann[‡]

Abstract

This paper documents a strong association between total factor productivity (TFP) growth and the value of U.S. corporations (measured as the value of equities and net debt for the U.S. corporate sector) throughout the postwar period. Persistent fluctuations in the first two moments of TFP growth predict two-thirds of the medium-term variation in the value of U.S. corporations relative to gross domestic product (henceforth value-output ratio). An increase in the conditional mean of TFP growth by 1% is associated to a 21% increase in the value-output ratio, while this indicator declines by 12% following a 1% increase in the standard deviation of TFP growth. A possible explanation for these findings is that movements in the first two moments of aggregate productivity affect the expectations that investors have regarding future corporate payouts as well as their perceived risk. We develop a dynamic stochastic general equilibrium model with the aim of verifying how sensible this interpretation is. The model features recursive preferences for the households, Markov-Switching regimes in the first two moments of TFP growth, incomplete information and monopolistic rents. Under a plausible calibration and including all these features, the model can account for a sizable fraction of the elasticity of the value-output ratio to the first two moments of TFP growth.

JEL Codes: E2, E3, G12.

Keywords: Productivity Growth, Asset Prices, Long-Run Risk, Learning

*First Draft: June 1, 2011. This Draft: March 4, 2013. We would like to thank Jesús Fernández-Villaverde and Frank Schorfheide for their invaluable guidance and advice. We are indebted to Dirk Krueger and Iourii Manovski for several very helpful conversations on our project. We would also like to thank Selman Erol, Jim Nason, Dongho Song and the seminar participants at the Penn Macro Club and the Philadelphia FRB for their thoughtful comments and encouragement. Any errors or omissions are our responsibilities.

[†]Department of Economics, University of Pennsylvania, 160 McNeil Building, 3718 Locust Walk, Philadelphia, PA, 19104-6297 USA. E-mail: lbocola@sas.upenn.edu.

[‡]Department of Economics, University of Pennsylvania, 160 McNeil Building, 3718 Locust Walk, Philadelphia, PA, 19104-6297 USA. E-mail: gonils@sas.upenn.edu.

1 Introduction

During the postwar period, the U.S. economy experienced large movements in the value of firms. The market value of U.S. corporations relative to gross domestic product (henceforth, *value-output ratio*) went through a slump during the 1970s followed by a large increase throughout the 1980s and 1990s until the marked decline of the last decade.[1] Researchers have devoted a great effort to understanding the origins of these *medium-term fluctuations*.[2] A widespread view among them is that the value of corporations should be particularly sensitive to variables that drive expectations of future corporate payouts and that influence the rate at which investors discount them. Within this context, research pioneered by Barsky and De Long (1993) and Bansal and Yaron (2004) suggests that economic fundamentals affecting the long-run growth of corporate payouts and its risk should be responsible for these large swings in the stock market.

Motivated by this view, recent studies have investigated the links between aggregate productivity and asset prices. While Beaudry and Portier (2006) and Croce (2010) have documented a strong sensitivity of stock prices to the mean of total factor productivity (TFP) growth, the theories mentioned above also suggest that variation in other moments of aggregate productivity may be relevant. Moreover, the documented empirical correlation between TFP growth and stock prices does not reveal a direction of causality.[3] This latter concern is strengthened by the finding that productivity driven standard macroeconomic models are not able to generate the medium-term fluctuations in the value of firms when calibrated with a realistic TFP process (see Boldrin and Peralta-Alva, 2009).

In this paper, we present new evidence on the relation between the value of U.S. corporations and aggregate productivity. First of all, we document empirically that changes in the volatility of TFP growth are important in predicting the medium-term movements in the value of U.S. corporations. We fit a Markov Switching model to TFP growth, detecting large and infrequent shifts in the mean and volatility of this series throughout the postwar period.[4] We then show that these shifts explain two-thirds of the medium-run variability

[1] We define the market value of the U.S. corporate sector to be the sum of outstanding equities and net debt liabilities. See Appendix A.1 for details in the construction of this series using *Flow of Funds* data.

[2] In this paper, we refer to medium-term fluctuations as the movements in the medium frequency component of a time series. As in Comin and Gertler (2006), the medium term consists of frequencies between 32 and 200 quarters.

[3] Changes in asset prices, for example, may feed-back into decisions of economic agents and therefore influence aggregate productivity. Jermann and Quadrini (2007) study an economy where increases in asset prices relax firms' credit constraints and endogenously generate an increase in measured TFP.

[4] In particular, we estimate that TFP was in a "high growth regime" between 1960Q1-1973Q1 and 1994Q1-2003Q4, while we estimate a "low volatility regime" during the period 1984Q1-2009Q3. These results are consistent with previous empirical analysis on the behavior of U.S. productivity; see for example Kahn and Rich (2007) and Benigno, Ricci, and Surico (2011).

in the value-output ratio measured using a band pass filter. In particular, a 1% increase in the conditional mean of productivity growth is associated with a 19% increase in the value-output ratio, while this indicator declines by 4% following a 1% increase in the standard deviation of TFP growth. The second contribution of this paper is to assess whether these elasticities can be interpreted as the response of asset prices to exogenous changes in the first two moments of TFP growth. We develop a stochastic growth model and show that, for reasonable calibrations, the model is consistent with a large response of the value-output ratio to shocks in the mean and volatility of TFP growth.

We build a stochastic growth model where households have Epstein-Zin preferences and where TFP growth is driven by two disturbances: a persistent Markov-Switching shock to its mean and a purely transitory shock. Consistent with our empirical analysis, we allow the volatility of the transitory component to vary over time. We assume a particular form of incomplete information: agents are aware of the underlying structure of the economy, and in every period they observe realized TFP growth, but they cannot tell whether movements in TFP growth come from the Markov-Switching mean or the transitory shock. We assume that they form beliefs about the mean of TFP growth using Bayes' rule.[5] The induced movements in beliefs about the growth regime influence agents' views over future corporate payouts. Beside the standard neoclassical channel, our model features monopolistic rents in production. This is intended to capture variation in dividends unrelated to the marginal product of physical capital, e.g., organizational capital, patents, etc.. These are factors that previous research identifies as important drivers of firms' valuation (see Hall, 2001). The interaction between incomplete information, monopolistic rents and Epstein-Zin preferences generates a strong sensitivity of the value-output ratio to the first two moments of TFP growth compared to a full information neoclassical benchmark. In the next two paragraphs, we briefly discuss the intuition underlying this result.

In the model, the behavior of the value of corporations conditional on a growth shock resembles *qualitatively* that of related production based asset pricing models, in particular Croce (2010). Indeed, under a plausible calibration of preferences, the model features a strong intertemporal substitution effect. A persistent increase in the mean of TFP growth is associated with expectations of a higher growth in corporate payouts, and households react to this change in expectations by demanding more assets. A higher demand for assets pushes up the value of corporations, therefore generating a positive association between TFP growth and asset prices. Our model, though, is less restrictive than Croce (2010) with respect to the *quantitative* association. In fact, and differently from a neoclassical setting,

[5]The resulting filtering problem implies that the model shifts in the mean of productivity growth are difficult to detect in real time, a fact that is well documented for the U.S. economy (see Edge, Laubach, and Williams, 2007).

the value of corporations in our model is the sum of two components: the market value of the physical capital stock *and* the present value of rents that firms are expected to generate. As we will discuss in the paper, the latter component is an order of magnitude more sensitive to the mean of TFP growth than the former. This aspect greatly improves the model's ability to generate an empirically plausible behavior for prices and quantities relative to its neoclassical benchmark.[6]

The model also has implications for the effects of volatility on the value of corporations. As in other production-based long-run risk models, agents in our economy are strongly averse to long-run fluctuations in the growth rate of corporate payouts, and they therefore ask for a sizable compensation when holding shares. An increase in the uncertainty over the long-run component of firms' productivity growth would accordingly generate a reduction in asset prices through its effects on risk premia. In our model, this channel is triggered by the interactions between incomplete information and volatility. Indeed, an increase in the volatility of the *transitory* component of TFP growth adds more noise to the filtering problem that agents are solving in real time. This makes them more uncertain about the long-run properties of corporate payouts. Depending on the calibration considered, asset prices can be very sensitive to the volatility of TFP growth, a feature that a model with full information would miss.[7]

We document that, under plausible calibrations, the model generates business cycle statistics for real and financial variables that are in line with postwar U.S. observations. Moreover, we compute the model implied elasticities of the value-output ratio to the mean and volatility of TFP growth and compare them with our empirical estimates. We find that a 1% increase in the mean of TFP growth is associated with a 4% increase in the value-output ratio. At the same time, this indicator falls by 0.4% after a 1% increase in the standard deviation of productivity growth. This represents, respectively, 20% and 9% of the magnitude observed in the data. We also show that, for less conservative calibrations of the TFP process, the growth elasticity can be reconciled with our empirical estimates, while the model accounts for 60% of the sensitivity of the value-output ratio to the volatility of TFP growth.

Related Literature. The idea that variations in risk and economic growth influence asset prices has a long tradition in economics, see for example Malkiel (1979), Pindyck (1984), Barsky (1989), Barsky and De Long (1993), Bansal and Yaron (2004) and references

[6]Indeed, absent monopolistic rents, strong frictions in the production of capital would be required by the model in order to generate a large elasticity of the value-output ratio to the mean of TFP growth. These frictions, while making asset prices more volatile, would reduce the relative volatility of investment to an empirical implausible level.

[7]See for example Naik (1994).

therein. In a recent paper, Lettau, Ludvigson, and Wachter (2008) present an endowment economy where shifts in the mean and volatility of consumption growth influence stock prices. Their model accounts for the 1990s "boom" in the U.S. stock market via a decline in the volatility of consumption growth, while they estimate that changes in the mean of consumption growth have small effects on stock prices. To the best of our knowledge, we are the first to look at more fundamental sources of variation in a general equilibrium model with production.

Our paper contributes to the production-based asset pricing literature (Jermann, 1998; Tallarini, 2000; Boldrin, Christiano, and Fisher, 2001; Gourio, 2012). Within this literature, our work is closely related to that of Croce (2010). He studies a neoclassical growth model with recursive preferences, adjustment costs and persistent variation in the mean of TFP growth. His model is consistent with the cyclical behavior of standard real and financial indicators of the U.S. economy. The analysis, though, is silent about the performance of the model regarding the elasticity of asset prices to the mean and volatility of TFP growth. One of our contributions is to show that two plausible mechanisms dramatically improve the model's ability to capture this conditional behavior of asset prices: incomplete information and monopolistic rents.

Incomplete information is important in our model to generate a quantitatively meaningful association between volatility and asset prices. The friction we consider is not new in the literature, see for example Kydland and Prescott (1982) and Edge, Laubach, and Williams (2007). Relative to the existing literature, we point out an interesting interaction between learning and time-varying volatility. An increase in the volatility of the transitory component of TFP growth dampens the ability of agents to learn about the persistent component of TFP growth.[8] In a model with Epstein-Zin preferences, this endogenous variation in uncertainty over long-run growth has strong asset pricing implications. We find in addition that monopolistic rents greatly enhance the performance of standard exogenous growth models regarding the volatility of asset prices without impairing their ability to account for variations in quantities. A similar point has been made recently by Comin, Gertler, and Santacreu (2009) and Iraola and Santos (2009) in a class of endogenous growth models.

We consider our empirical findings particularly relevant for the literature studying movements in asset prices over longer horizons. Several attempts have been put forth to explain the behavior of the U.S. stock market. Plausible explanations for the medium-term

[8]Bullard and Singh (2012) discuss a setting in which the opposite happens. They consider an RBC model with a similar signal extraction problem to ours, but they model an increase in the volatility of TFP growth by permanently increasing the gap between the two means of the TFP growth process. This change, while increasing the unconditional volatility of TFP growth, makes the signal extraction problem easier as agents can better distinguish between the two different growth regimes.

movements in the value of corporations include technological revolutions (Greenwood and Jovanovic, 1999; Laitner and Stolyarov, 2003; Pastor and Veronesi, 2009), variation in taxes and subsidies (McGrattan and Prescott, 2005), intangible investments (Hall, 2001) and the saving behavior of baby boomers (Abel, 2003). Our paper suggests that a successful theory should account for the *joint* evolution of productivity growth and asset prices, since these two series share common cycles in the medium run. This would help the profession with the task of measuring the contribution of each of these mechanisms.

Layout. The rest of the paper is organized as follows. In section 2 we document the medium-term association between productivity growth and the value-output ratio. Section 3 presents the model. In section 4 we calibrate the model, study its business cycle properties and analyze the behavior of the value-output ratio conditional on a persistent change in the mean and volatility of TFP growth. Section 5 concludes.

2 Productivity Growth and the Market Value of U.S. Corporations: Empirical Evidence

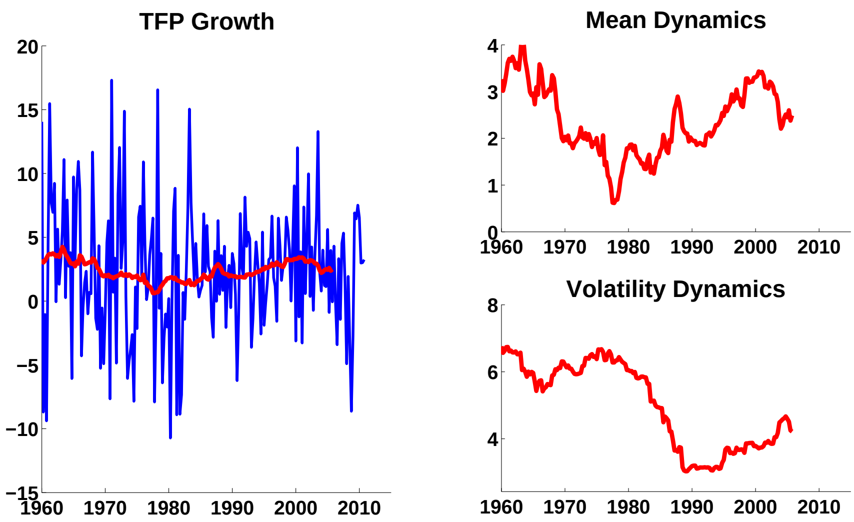
We begin by looking at simple indicators of time variation in the mean and volatility of productivity growth. We construct a quarterly series for *total factor productivity* (TFP) in the U.S. business sector using BLS and NIPA data. Our data cover the period from the first quarter of 1952 to the last quarter of 2010. In Figure 1, we plot 10 years centered rolling window estimates for the annualized mean (top-right panel) and standard deviation (bottom-right panel) of productivity growth. The left panel of the figure plots the TFP growth series.

The data show substantial time variation in the mean and volatility of TFP growth. In the top-right panel of Figure 1, we can observe the slowdown in growth during the late 1960s/early 1970s and the subsequent resurgence during the 1990s, facts that have been extensively discussed in narrative and econometric studies on the U.S. economy.[9] Regarding volatility, the bottom-right panel of Figure 1 shows the drastic decline during the 1980s, consistent with the "great moderation" in macroeconomic aggregates after 1984 (see Kim and Nelson, 1999; McConnell and Quiros, 2000). It is also important to notice that shifts in these two series are *large* and *infrequent*. For instance, the standard deviation of productivity

[9] One issue of the *Journal of Economic Perspectives* is devoted to the productivity growth slowdown of the 1970s (*Volume 2, Number 4*) and another issue to the resurgence in productivity growth of the 1990s (*Volume 14, Number 4*). Econometric studies that have recently analyzed shifts in the trend growth rate of productivity include Roberts (2000), French (2001), Kahn and Rich (2007), Croce (2010) and Benigno, Ricci, and Surico (2011).

growth fluctuates very little during the period 1960-1980, and it experienced a sudden reduction of about 50% in the early 1980s. Similarly, productivity growth fell by 2% within a few years in the late 1960s and then fluctuated very little around a value of 1.5% for about 20 years.

Figure 1: **Growth and Volatility: Rolling Windows Estimates**



Note: The figure reports 10 years centered rolling windows estimates for the mean and volatility of TFP growth (right panel). For example, the mean of TFP growth in 1980 is calculated as the arithmetic mean of productivity growth within the period 1977.II-1983.II. The left panel reports the TFP growth series (blue line), along with the rolling windows estimates for the mean (red line). The TFP growth series is annualized.

2.1 Identifying Shifts in Growth and Volatility: A Markov-Switching Approach

We now describe a parametric model that we use to fit the shifts in the mean and volatility of TFP growth documented in the previous section. We model the growth rate of TFP as follows:

$$\begin{aligned} \Delta Z_t &= \mu_t + \phi\left[\Delta Z_{t-1} - \mu_{t-1}\right] + \sigma_t \varepsilon_t \\ \mu_t &= \mu_0 + \mu_1 s_{1,t} \qquad \mu_1 > 0 \\ \sigma_t &= \sigma_0 + \sigma_1 s_{2,t} \qquad \sigma_1 > 0 \\ \varepsilon_t \sim \mathcal{N}(0,1) &\qquad s_{1,t} \sim \mathcal{MP}(P_\mu) \qquad s_{2,t} \sim \mathcal{MP}(P_\sigma) \end{aligned} \tag{1}$$

The variable μ_t captures the different "growth regimes" characterizing the postwar behavior of aggregate productivity. In particular, we assume that μ_t alternates between two regimes driven by the Markov process $s_{1,t} \in \{0, 1\}$ whose law of motion is governed by the transition matrix P_μ. Since $\mu_1 > 0$, we interpret $s_{1,t} = 1$ as the "high growth" regime during which productivity grows on average at the rate $\mu_0 + \mu_1$, while $s_{1,t} = 0$ implies that TFP grows at the rate μ_0 ("low growth"). Transitory fluctuations around μ_t are modeled via the white noise ε_t and an autoregressive component. The volatility of ε_t is allowed to fluctuate between a "high volatility" regime and a "low volatility" regime driven by the Markov process $s_{2,t} \in \{0, 1\}$.[10]

Markov-Switching models are commonly used in the literature to fit large and infrequent changes of the type observed in Figure 1, and they have already been used to fit changes in the trend growth rate of productivity (see French, 2001; Kahn and Rich, 2007).[11] Our approach is different from existing ones in that we do not model transitory fluctuations in the *level* of TFP. This is done mainly because the process in equation (1) makes the general equilibrium model of Section 3 more tractable.

We estimate the model's parameters using Bayesian methods. Appendix A.2 describes in detail the selection of the prior as well as the sampler adopted to conduct inference. Table 1 reports posterior statistics for the model's parameters under the header *Univariate Model*, while the top panel of Figure 2 plots posterior estimates of μ_t and σ_t.

The model clearly identifies movements in the volatility of productivity growth. From the top right panel of Figure 2, we can observe a decline in σ_t of 44% in the mid-1980s, with little uncertainty regarding this event. The model also identifies a slight increase in the volatility of TFP growth toward the end of the sample, although credible sets are large. On the contrary, shifts in the mean are poorly identified with this approach, as shown by the large credible sets on μ_t and on the parameters governing its behavior.

2.2 Identifying Shifts in Growth and Volatility: Multivariate Analysis

High uncertainty in our estimates for μ_t reflects the difficulties in detecting changes in the trend growth rate of TFP. As the left panel of Figure 1 shows, transitory fluctuations in

[10] Our choice regarding the number of regimes is suggested by the nonparametric analysis in the previous section and is confirmed by formal posterior odds comparisons.

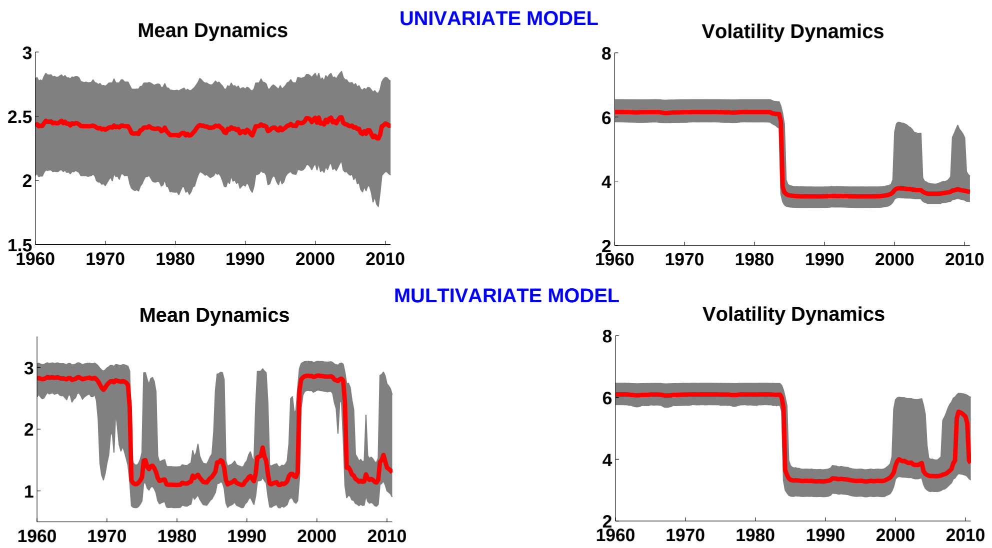
[11] An equally plausible specification would be that of a random coefficients model in which variations in μ_t and σ_t are represented by continuous stochastic processes. This approach has been followed in a similar context by Cogley (2005). We have formally compared our specification with one in which μ_t follows an AR(1) process, and the marginal data density slightly favors our model. Results are available upon request.

Table 1: **Markov-Switching Model: Prior Choices and Posterior Distribution**

		Univariate Model		Multivariate Model	
Parameter	Prior	Median	90% Credible Set	Median	90% Credible Set
μ_0	$\mathcal{N}(1.5, 1)$	1.68	[0.30,2.67]	1.12	[0.59,2.67]
μ_1	$\mathbf{1}(x > 0)\mathcal{N}(1, 1)$	1.19	[0.29,2.48]	1.75	[0.32,2.57]
σ_0	$\mathcal{IG}(5, 2)$	3.63	[3.40,3.83]	3.30	[2.60,3.99]
σ_1	$\mathcal{IG}(5, 2)$	2.51	[2.02,2.79]	2.92	[2.00,3.84]
ϕ	$\mathcal{N}(0, 1)$	0.10	[-0.01,0.20]	0.04	[-0.06,0.12]
$P_{1,1,\mu}$	$\mathbf{1}(\lvert x\rvert < 1)\mathcal{N}(0.98, 0.3)$	0.953	[0.543,0.998]	0.971	[0.861,0.997]
$P_{2,2,\mu}$	$\mathbf{1}(\lvert x\rvert < 1)\mathcal{N}(0.98, 0.3)$	0.975	[0.570,0.998]	0.969	[0.890,0.996]
$P_{1,1,\sigma}$	$\mathbf{1}(\lvert x\rvert < 1)\mathcal{N}(0.98, 0.3)$	0.995	[0.965,0.999]	0.990	[0.901,0.999]
$P_{2,2,\sigma}$	$\mathbf{1}(\lvert x\rvert < 1)\mathcal{N}(0.98, 0.3)$	0.992	[0.971,0.998]	0.990	[0.954,0.998]

Note: The column "Univariate Model" reports posterior statistics for the parameters of the model in equation (1). The column "Multivariate Model" reports posterior statistics for selected parameters of the model in equation (2). Posterior statistics are based on 80,000 draws from the posterior distribution. See Appendix A.2 and A.3 for details concerning the estimation procedure.

Figure 2: **Growth and Volatility: Markov-Switching Model**



Note: In the left panels, we report smoothed posterior estimates of μ_t. In particular, the solid line represents the posterior mean $\mathbb{E}[\mu_t|\mathcal{I}^{\mathbf{T}}]$, while the shaded area denotes a 60% pointwise credible set. The right panels report the same information for σ_t.

TFP growth are large compared to the changes in the conditional mean that we wish to isolate, and this complicates the filtering problem significantly. A remedy suggested in the literature consists of introducing additional variables that carry information on μ_t. We follow this insight and augment the model in equation (1) as follows:

$$\begin{aligned} \begin{bmatrix} \Delta Z_t \\ \Delta \mathbf{Y_t} \end{bmatrix} &= \begin{bmatrix} \mu_t \\ \mu_t \end{bmatrix} + \Phi \begin{bmatrix} \Delta Z_{t-1} - \mu_{t-1} \\ \Delta \mathbf{Y_{t-1}} - \mu_{t-1} \end{bmatrix} + \Sigma_t \mathbf{e_t} \\ \mu_t &= \mu_0 + \mu_1 s_{1,t} \\ \Sigma_t &= \Sigma_0 + \Sigma_1 s_{2,t} \\ \varepsilon_t \sim \mathcal{N}(0,1) \qquad & s_{1,t} \sim \mathcal{MP}(P_\mu) \qquad s_{2,t} \sim \mathcal{MP}(P_\sigma) \end{aligned} \tag{2}$$

This specification introduces a set of variables $\Delta \mathbf{Y_t}$ that share the same growth rate as TFP. This formulation is rooted in economic theory. Indeed, under balanced growth restrictions, equilibrium models predict that several economic ratios share a common trend with TFP. This justifies the introduction of $\Delta \mathbf{Y_t}$ into the analysis, as one can pool these time series with TFP growth in order to obtain a sharper estimate of μ_t. Following Kahn and Rich (2007), we include the growth rate of consumption per hour and compensation per hour in $\Delta \mathbf{Y_t}$.[12]

The law of motion for μ_t and Σ_t has the same Markov-Switching structure described in the previous section. For tractability and parsimony, we allow the variance of the innovations to have common switches while keeping their correlation structure constant over time.[13] The model is estimated via Bayesian techniques as discussed in Appendix A.3, and the results are reported under the header *Multivariate Model* in Table 1 and in the bottom panel of Figure 2. These estimates are consistent with the univariate analysis presented in the previous section. The multivariate approach allows us to identify the shifts in the mean of productivity growth more precisely. Credible sets on the parameters governing μ_t are considerably tighter and this is reflected in increased precision of our estimates for μ_t. As Figure 2 shows, we estimate that the trend growth rate of TFP was about 3% in the periods 1960Q1:1973Q1 and 1997Q3:2004Q1, while growth was around 1.3% in the remaining periods.

[12]Consumption and wages are scaled by total hours in order to account for the unit root behavior of hours worked that, under preferences consistent with balanced growth, is unrelated to TFP dynamics. See Chang, Doh, and Schorfheide (2007) for a discussion of this issue.

[13]This is accomplished by reparametrizing Σ_t, see Appendix A.3 for details.

2.3 Productivity Growth and the Market Value of U.S. Corporations

We now consider the relation between productivity growth and the market value of U.S. firms. Following Hall (2001), Wright (2004) and McGrattan and Prescott (2005), we use *Flow of Funds* data and define the market value of the U.S. corporate sector to be the sum of outstanding equities and net debt liabilities.[14] This indicator has the advantage of including the market value of closely held firms, thus being a more reliable measure for trends in the value of firms relative to standard indicators that are based only on publicly held corporations.

We summarize the relation between the value-output ratio and our estimates of μ_t and σ_t via linear projections. Our benchmark specification is the following:

$$MV_t = \mathbf{a} + \mathbf{b}\hat{\mu}_t + \mathbf{c}\hat{\sigma}_t + e_t \tag{3}$$

Table 2 reports the results. From Column 3, we can observe a positive relation between trend growth and the value-output ratio. An increase of 1% in the trend growth rate of TFP is associated with an increase in the value-output ratio of 21%. Volatility, on the other hand, is negatively associated with the value of U.S. corporations. An increase of 1% in the standard deviation of TFP growth is associated with a reduction in the value-output ratio of 12%.

The linear projection also shows that the association between productivity growth and the value-output ratio is sizable. Indeed, fluctuations in the first two moments of productivity jointly predict more than half of the variation in the value-output ratio at quarterly frequencies. In order to gain more insights into this association, we plot in Figure 3 the value-output ratio along with the fitted values of the linear projection. We can verify that the decline in growth in the early 1970s is closely followed by a sharp decline in the value-output ratio and that the growth resurgence is associated with a boom in this indicator, while the subsequent decline in productivity growth is associated with a fall in the valuation of U.S. corporations. The great moderation occurred during a period of a rising value-output ratio, while the surge in aggregate volatility observed toward the end of the sample is associated with a decline in this indicator. Moreover, from the figure we can see that most of the association between productivity growth and the value-output ratio occurs over horizons longer than the business cycle. The fitted values of equation (3) closely track the medium term component of the value-output ratio constructed using the band-pass filter (32-200 quarters).

[14] See Appendix A.1 for details on the calculation of this indicator.

Table 2: **Growth, Volatility and the Value of U.S. Corporations**

	[1]	[2]	[3]	[4]
Constant	-0.34	0.36	0.17	-0.13
	(0.04)	(0.07)	(0.05)	(0.03)
$\hat{\mu}_t$	0.19		0.21	0.19
	(0.02)		(0.02)	(0.01)
$\hat{\sigma}_t$		-0.07	-0.12	-0.04
		(0.01)	(0.01)	(0.005)
R^2	0.23	0.13	0.56	0.68

Note: Column [1] reports the results of a linear projection of $\mathbb{E}[\mu_t|\mathcal{I}^T]$ on the value-output ratio. Column [2] reports the results of a linear projection of $\mathbb{E}[\sigma_t|\mathcal{I}^T]$ on the value-output ratio. Column [3] reports the results from the estimation of equation (3). Column [4] reports the results of a linear projection of $\mathbb{E}[\mu_t|\mathcal{I}^T]$ and $\mathbb{E}[\sigma_t|\mathcal{I}^T]$ on the medium frequency component of the the value-output ratio isolated using the band-pass filter between 32 and 200 quarters. The value-output ratio is demeaned prior to running the projections. Robust standard errors are in parentheses.

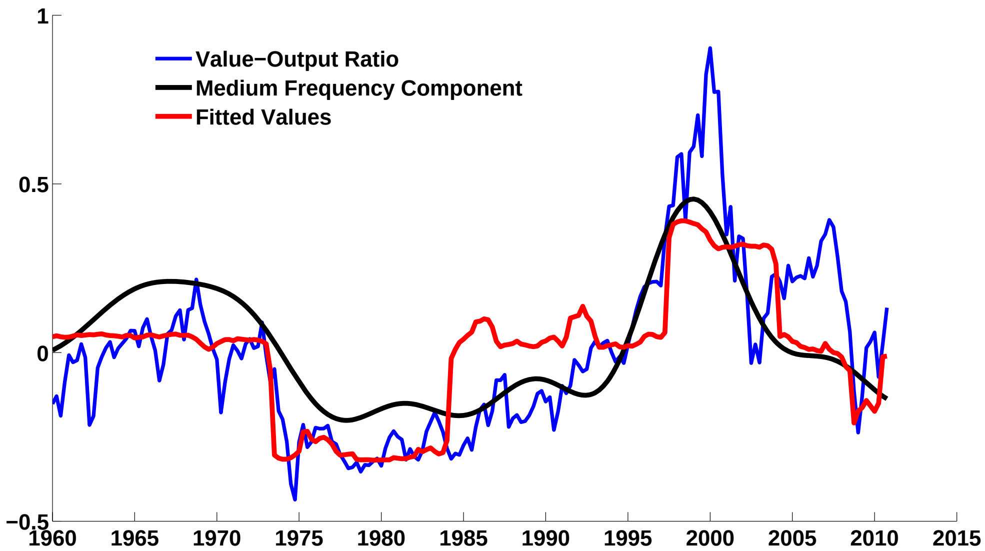
This point is confirmed by column [4] of Table 2, where we project the medium frequency component of the value-output ratio on $\hat{\mu}_t$ and $\hat{\sigma}_t$. Relative to column [3], the R^2 increases substantially, suggesting that the movements in TFP growth and volatility that we identify are mainly relevant for predicting the medium-term fluctuations in the value-output ratio.

3 Model

So far, we have documented a strong relationship between persistent innovations to the mean and standard deviation of TFP growth and medium-term fluctuations in the value-output ratio. As mentioned in the Introduction, this reduced form association may have several interpretations. In what follows, we set up a quantitative model with the aim of measuring the fraction of this association that can be explained by exogenous variation in the mean and volatility of TFP growth. We consider a fairly standard growth model with four major ingredients:

1. Markov-Switching regimes in the mean and volatility of technological growth
2. Recursive preferences

Figure 3: **Growth, Volatility and the Value of U.S. Corporations**



Note: The blue solid line plots the value of corporations scaled by gross domestic product. The red solid line reports the fitted values of Equation (3). The black dotted line reports the medium frequency component of the value-output ratio isolated using the band-pass filter between 32 and 200 quarters.

3. Capital adjustment costs and monopolistic rents

4. Incomplete information about the drivers of technological growth

In the model, infinitely lived households supply labor inelastically to firms and own shares of the corporate sector. They use their dividend and labor income to consume the final good and accumulate shares of the corporate sector. The final good is sold in a competitive market by firms that aggregate a set of imperfectly substitutable intermediate goods. Each variety is produced by an intermediate good firm using capital and labor. Those firms rent the capital stock and labor in competitive markets. They are monopolists in producing their variety. Capital services are supplied by capital producers in a competitive market. These firms own the capital stock and make optimal capital accumulation plans by maximizing the present discounted value of profits.

Below we describe the major ingredients of our model, while Appendix B contains a detailed account of the agents' decision problems and of the equilibrium concept adopted. In terms of notation, the level of variable X at time t is denoted by X_t. Even though every endogenous variable depends on the history of shocks, we keep the notation simple and omit this explicit dependence.

3.1 Preferences

Households have Epstein-Zin preferences over streams of consumption. Given a continuation value U_{t+1} and consumption c_t today, the agent's utility is given by:

$$U_t = ((1-\beta)c_t^{\frac{1-\gamma}{\eta}} + \beta \mathbb{E}_t[((U_{t+1})^{1-\gamma})]^{\frac{1}{\eta}})^{\frac{\eta}{1-\gamma}}.$$

γ controls the degree of risk aversion, η is equal to $\frac{1-\gamma}{1-\frac{1}{\Psi}}$ and Ψ parametrizes the elasticity of intertemporal substitution in consumption. The operator $\mathbb{E}_t[.]$ is interpreted as the expectation conditional on all the observations made by the agents up to period t.

3.2 Production

A fixed variety of intermediate goods is produced in the economy. Intermediate goods are indexed by $j \in [0,1]$, and each variety is produced by an intermediate good producer. He uses capital services $k_{j,t}$ and labor services $l_{j,t}$ to produce $y_{j,t}$ units of the good according to the production function:

$$y_{j,t} = (e^{Z_t} l_{j,t})^{1-\alpha} k_{j,t}^{\alpha}.$$

Z_t is the log of TFP common to all firms. Intermediate goods are aggregated by final good producers into units of a final good using the production function

$$y_t = \left(\int_0^1 y_{j,t}^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}.$$

The final output is consumed by households or purchased by capital producers to invest in capital. In particular, if a capital producer with k_t units of capital invests i_t, his stock of capital in period $t+1$ will increase by $G\left(\frac{i_t}{k_t}\right) k_t$. For the quantitative analysis, we parametrize $G(.)$ as[15]

$$G(.) = a(.)^{1-\tau} + b.$$

[15] This functional form is quite standard in the literature. Jermann (1998) points out that the inverse of τ is equal to the elasticity of the investment-capital ratio to Tobin's Q in a wide class of models.

Capital depreciates every period at the rate δ. Therefore, the stock of capital for a producer evolves according to the following law of motion:

$$k_t = (1-\delta)k_t + G\left(\frac{i_t}{k_t}\right)k_t$$

3.3 Total Factor Productivity and Information Structure

We model the logarithm of TFP as a random walk with time varying drift and volatility:[16]

$$\Delta Z_t = \mu_t + \sigma_t \varepsilon_t \qquad \varepsilon_t \sim \mathcal{N}(0,1),$$

where the drift and the volatility follow the Markov processes:

$$\begin{aligned} \mu_t &= \mu_0 + \mu_1 s_{1,t} \qquad & \mu_1 > 0 \\ \sigma_t &= \sigma_0 + \sigma_1 s_{2,t} \qquad & \sigma_1 > 0. \end{aligned}$$

The variable $s_{j,t} \in \{0,1\}$ is a state whose probabilistic law of motion is governed by the transition matrix $\mathcal{P}_j$.

Household and firms know the parameters governing the stochastic process and use it to form expectations about future periods. They are, however, imperfectly informed about the drivers of technological progress. In particular, we assume that they learn, at every point in time, the realization of TFP growth ΔZ_t while not observing its components μ_t and ε_t separately. Therefore, the information that agents can use to update their beliefs about the current state of the stochastic process is given by the history of TFP growth realizations, the state governing volatility and an unbiased Gaussian signal $g_t = \mu_t + \sigma_g e_t$ that they receive in every period. They are fully rational and form their beliefs about $s_{1,t}$ via Bayes' rule. We denote the probability that any agent assigns to being in growth regime $s_{1,t}$ in period t by $p_t(s_{1,t})$. Similarly, we label the likelihood that he attaches to being in state $s_{1,t+1}$ in period $t+1$ by $p_{t+1|t}(s_{1,t+1})$. Bayes' rule implies the following updating equations:

$$p_{t+1|t}(s_{1,t+1}) = \frac{\sum_{i=1}^{2} p_{t-1}(i)\mathcal{P}_1(s_{1,t}|i)}{\sum_{j=1}^{2}\sum_{i=1}^{2} p_{t-1}(i)\mathcal{P}_1(j|i)};$$

and

$$p_t(s_{1,t}) = \frac{p_{t|t-1}(s_{1,t})p_N(\Delta Z_t, g_t|s_{1,t}, s_{2,t})}{\sum_{i=1}^{2} p_{t|t-1}(i)p_N(\Delta Z_t, g_t|i, s_{2,t})}$$

[16] This is different from Section 2 in that we do not include a autoregressive component, as we did not find a strong contribution of this component in the estimation and as the omission simplifies the numerical solution of the model.

$p_N(.|j,\hat{j})$ is the pdf of a two dimensional normal random variable with mean $\mu_0 + \mu_1 j$ and standard deviation $\sigma_0 + \sigma_1 \hat{j}$ for the first component and mean $\mu_0 + \mu_1 j$ and standard deviation σ_g for the second one. Both components are assumed to be independent. The first equation updates the beliefs about the state today into beliefs over the expected state tomorrow using the known probabilities of a state transition. The second equation captures how those probabilities are updated after observing the realizations of the growth rate and the signal. As we see, given the structure of the stochastic process considered, $(\Delta Z_t, s_{2,t}, g_t)$ are sufficient to update the beliefs of the household about the underlying state in the last period to the beliefs in the current period.

3.4 Equilibrium and Numerical Solution

We focus on a symmetric equilibrium in which all capital good producers initially own the same amount of capital. This assumption implies that capital good producers make the same investment choices and that intermediate good producers charge the same price and sell the same quantity to the final good producers. In appendix B, we argue that the equilibrium law of motion for aggregate variables can be derived from a planner's problem, which we describe below.

The planner maximizes lifetime utility of the representative household by selecting a sequence for investment, consumption, the capital stock and the value function $(i_t, c_t, k_{t+1}, V_{t+1})_{t=0}^{\infty}$,[17] subject to the same information restriction as the households, initial conditions and a function that maps the observed realizations of shocks into an aggregate capital stock $(\overline{k}_t)_{t=0}^{\infty}$:[18]

$$\max_{(i_t,c_t,k_{t+1},V_{t+1})_{t=0}^{\infty}} V_0$$

$$\textbf{s.t.} \quad c_t + i_t = \frac{\nu - 1}{\nu} Z_t k_t^{\alpha} + \frac{1}{\nu} Z_t \overline{k}_t^{\alpha}$$

$$V_t = [(1-\beta) c_t^{\frac{1-\gamma}{\eta}} + \beta \mathbb{E}_t [V_{t+1}^{1-\gamma}]^{\frac{1}{\eta}}]^{\frac{\eta}{1-\gamma}}$$

$$k_{t+1} = (1-\delta) k_t + G\left(\frac{i_t}{k_t}\right) k_t.$$

[17]These are functions of the realization of the stochastic process subject to the measurability restrictions implied by the information structure.

[18]The aggregate capital is therefore measurable with respect to the households' information set and does not add new information to the signal extraction problem.

In addition, the choice of V_t has to be finite for all t. An equilibrium is fully characterized when $\overline{k}_t = k_t$. We solve the model numerically using global methods as described in Appendix C.

3.5 Asset Prices

We can express the market value of firms as the present discounted value of corporate payouts to households. In our economy, there are two types of firms making nonzero profits: the capital good producers and the intermediate good producers. The per period profits of a capital good producer are given by:

$$d_t^{cp} = r_t^k k_t - i_t,$$

where r_t^k stands for the return to capital. The per period profits of an intermediate good producer are given in equilibrium by:

$$d_t^{ip} = \frac{1}{\nu} y_t.$$

Profits are a fixed fraction of the revenues of an intermediate good producing firm. Both types of producers distribute these profits to households in every period. As a result, one can express the market value of these two types of firms as follows:

$$p_t^s = \mathbb{E}_t \left[\sum_{j=1}^{\infty} \Lambda_{t,t+j} d_{t+j}^s \right] \qquad s = \{cp, ip\}.$$

Here we denote by $\Lambda_{t,t+s}$ the stochastic discount factor of the household between period t and period $t+s$. The market value of the corporate sector is then the sum of these two components. For future reference, it is convenient to further characterize this object. Based on Hayashi (1982) it is easy to show that the equilibrium value of the corporate sector is given by

$$p_t = \frac{1}{(1-\tau)a} \left(\frac{i_t}{k_t} \right)^{\tau} k_{t+1} + \frac{1}{\nu} \mathbb{E}_t \left[\sum_{j=1}^{\infty} \Lambda_{t,t+j} y_{t+j} \right]. \tag{4}$$

Indeed, one can easily verify that in our model the equilibrium value of capital good producers can be expressed as the product of marginal Q and the capital stock. The decomposition of equation (4) has an intuitive interpretation. It tells us that, in equilibrium, the value of the corporate sector is the sum of two components: the value of the capital stock and the present discounted value of monopolistic rents.[19] In the next section, we will calibrate the model and study how these two components respond to fluctuations in μ_t and σ_t.

4 Risk, Economic Growth and the Value of Corporations

4.1 Calibration

We set a model period to be a quarter. The parameters of our model are:

$$\theta = [\delta, a, b, \tau, \beta, \Psi, \gamma, \mu_0, \mu_1, P^{\mu}_{0|0}, P^{\mu}_{1|1}, \sigma_0, \sigma_1, P^{\sigma}_{0|0}, P^{\sigma}_{1|1}, \alpha, \nu, \sigma_g]$$

The depreciation parameter δ is set to 0.025, leading to an annual depreciation rate of roughly 10%. We follow the literature in calibrating a and b so that the deterministic balanced growth path of our model coincides with that of an economy without adjustment costs (Van Binsbergen, Fernandez-Villaverde, Koijen, and Rubio-Ramirez, 2010).[20] The parameter controlling the capital adjustment (τ) is set to 0.5, in the range of values considered in the literature. The discount factor β is set to 0.994, a value that implies an average annualized risk free-rate of 2.07%.[21] Following Croce (2010), we set Ψ to 2 and γ to 10.[22]

We use the empirical results in Section 2 to calibrate the parameters of the shock process Z_t.[23] In accordance with our estimates, we set $\mu_0 = 0.003$, $\mu_1 = 0.0045$, $\sigma_0 = 0.0082$ and $\sigma_1 = 0.0073$. As a benchmark, we restrict the transition matrices of the two Markov processes to be symmetric, and we assume that $\mathcal{P}^{\mu}_{j|j} = \mathcal{P}^{\sigma}_{j|j}$. Thus, the two transition matrices can be represented by a single parameter, denoted by ρ. We set $\rho = 0.99$, a value that implies an average state duration of 25 years. The unbiased signal in our model stands for all additional information that agents use to infer shifts in the mean of productivity growth. We calibrate its precision so that the average speed of learning is 16 quarters.[24]

[19] Indeed, in our decentralization, the price of capital equals $\frac{1}{(1-\tau)a}\left(\frac{i_t}{k_t}\right)^{\tau}$.

[20] We construct a deterministic balanced growth path around the average growth rate of TFP, which we denote by μ.

[21] We solve the model repeatedly for different values of β until the average risk-free rate computed on simulated data matches the target value. See Table 3 for additional details.

[22] A value of 2 for the IES and a coefficient of relative risk aversion of 10 are, for example, consistent with the estimates obtained by Attanasio and Vissing-Jorgensen (2003).

[23] In order to be consistent, we reestimate the model in Section 2 restricting the autoregressive component of TFP growth to be equal to zero.

[24] We simulate the signal extraction problem 100,000 times. We keep the mean growth rate fixed in the

This number is consistent with the results in Edge, Laubach, and Williams (2007) and Jorgenson, Ho, and Stiroh (2008).

We calibrate ν to 10, implying a markup of 10%. This value is in the range typically considered in the business cycle literature for the whole U.S. economy.[25] The remaining parameter to be calibrated is α. In our model, the labor income share is given by

$$\frac{w_t l_t}{y_t} = \frac{(\nu - 1)}{\nu}(1 - \alpha), \tag{5}$$

Because pure economic profits are treated as a reward to capital in U.S. national accounts, we can calibrate α by matching a labor income share of 70% in line with U.S. data. This strategy results in $\alpha = 0.22$.

4.2 Unconditional Moments

Table 3 reports a set of model implied statistics for selected real and financial variables along with their data counterparts. For comparison, we also report the results for two natural benchmarks. We consider a version of our model in which agents have perfect information over the TFP process (Full Info) and a version in which intermediate firms operate in a competitive environment (No Rents).[26] Under the calibration considered, our model generates business cycle fluctuations for consumption, output and investment that are not too far from the data. In particular, we obtain that consumption growth is less volatile than output growth, while investment growth is more volatile, with relative magnitudes in line with data observations. The model predicts a high degree of comovement of consumption and investment growth with output growth. However it differs from the standard Real Business Cycle model in that we obtain a relatively small correlation between consumption and investment growth. This happens because changes in the beliefs over the trend growth rate of TFP induce differential movements in aggregate investment and consumption.[27] Finally, the model implies a small autocorrelation for the variables in growth rates, which

high regime for 100 periods. We then switch the regime to the low state and count the number of periods it takes the filter to attach a probability of 0.9 to the low regime for the first time. We keep changing σ_g until the average time it takes over the simulations is 16. The resulting value for σ_g is 0.0074.

[25]See for example Altig, Christiano, Eichenbaum, and Linde (2011) and their references.

[26]We recalibrate β in each case to keep the risk-free rate at 2.07 in order to make it easier to contrast the three examples with regard to their asset pricing behavior.

[27]Indeed, a shock to the growth rate of TFP induces offsetting wealth and substitution effects on the part of households. On the one hand, higher growth signals households' higher income in the future, which makes them more willing to reduce their savings and increase their consumption level today. On the other hand, higher TFP growth implies a higher reward to savings today, which motivates households to save more. Irrespective of which of these two effects prevails in equilibrium, consumption and investment growth move in opposite directions conditional on a TFP "growth" shock.

Table 3: **Model Implied Moments for Selected Variables**

	$\sigma(\Delta y)$	$\frac{\sigma(\Delta c)}{\sigma(\Delta y)}$	$\frac{\sigma(\Delta i)}{\sigma(\Delta y)}$	$\rho(\Delta y, \Delta c)$	$\rho(\Delta y, \Delta i)$	$\rho(\Delta i, \Delta c)$	$\rho_{-1}(\Delta y)$	$\rho_{-1}(\Delta c)$	$\rho_{-1}(\Delta i)$
Data	1.00	0.79	2.69	0.70	0.69	0.60	0.34	0.23	0.50
Model	1.00	0.90	1.96	0.96	0.80	0.60	0.05	0.05	-0.04
Full Info	0.97	0.91	1.47	0.99	0.94	0.90	0.05	0.07	0.02
No Rents	0.90	0.89	1.94	0.89	0.82	0.47	0.06	0.05	-0.03

	$\mathbb{E}[R^e - R^f]$	$\sigma[R^e - R^f]$	$\rho_{-1}[R^e - R^f]$	$\mathbb{E}[R^f - 1]$	$\sigma[R^f - 1]$	$\rho_{-1}[R^f - 1]$
Data	4.49	15.89	0.02	2.07	2.6	0.61
Model	3.34	8.28	-0.06	2.07	1.63	0.87
Full Info	1.70	5.06	0.04	2.07	0.25	0.96
No Rents	3.52	5.00	0.43	2.07	2.12	0.87

Note: ΔX_t stands for the quarterly growth rate of variable X. R^e is the annualized gross return on equity, while R^f is the annualized gross return on a risk-free bond. We assume that equity is leveraged using a debt-to-equity ratio of 1. The data figure for the volatility of the value-output ratio stands for the standard deviation of the fitted values in equation (3). Means and standard deviations are reported in percentage terms. Model statistics are based on a long simulation (T=1000000). The data column is based on quarterly observations (1960Q1-2010Q4). Statistics on the equity premium and on the risk-free rate are calculated using annual data from 1948 to 2010, which we downloaded from Robert Shiller's website http://www.econ.yale.edu/~shiller/data.htm.

is not surprising given its lack of a strong internal propagation mechanism.[28]

The model is also consistent with the first two moments of the equity premium in postwar U.S. data. However, the mechanism through which we achieve a large and volatile equity premium differs from that of other production-based versions of the long-run risk model. Croce (2010), for example, generates a sizable equity premium by introducing a persistent random component into the growth rate of productivity. This leads to covariation at low frequencies between consumption growth and corporate payouts, therefore triggering the long run-risk channel discussed in Bansal and Yaron (2004). In our model, this channel is triggered by incomplete information. The Markov-Switching structure, in fact, imposes a trade-off between the persistence and the volatility of μ_t. Under complete information, the changes in the growth rate of corporate payouts would be too rare for agents to require large premia on stocks.[29] With incomplete information, though, what matters for the eq-

[28]This problem is shared by many simple business cycle models as discussed in Cogley and Nason (1995).

[29]As we use realized postwar growth rates in output to discipline our calibration, the model cannot generate a high risk premium by triggering the rare disaster risk channel as in Gourio (2012).

uity premium are the beliefs of agents regarding μ_t. In our model, these beliefs are more volatile than μ_t because the learning process is influenced by high frequency variations in TFP growth, in its volatility and in the signal. This generates additional risk from the perspective of investors. The resulting effect on prices stands out clearly when comparing the performance of our model to its full information benchmark since the latter generates a sensibly lower equity premium (1.70% versus 3.34% implied by our model).

While still falling short on the volatility of the equity premium in the data, the calibrated model substantially improves relative to the Full Information and the No Rents model. As will be argued in more depth in the next section, incomplete information raises the sensitivity of asset prices to σ_t, while monopolistic rents make them more sensitive to μ_t. A stronger response of asset prices to economic fundamentals contribute to raising the unconditional volatility of the equity premium. Finally, the model is able to generate the low volatility and high persistence of the risk-free rate observed in U.S. postwar data. The mechanisms through which this happens are well understood in the literature on the production-based long-run risk model, see for example Croce (2010).

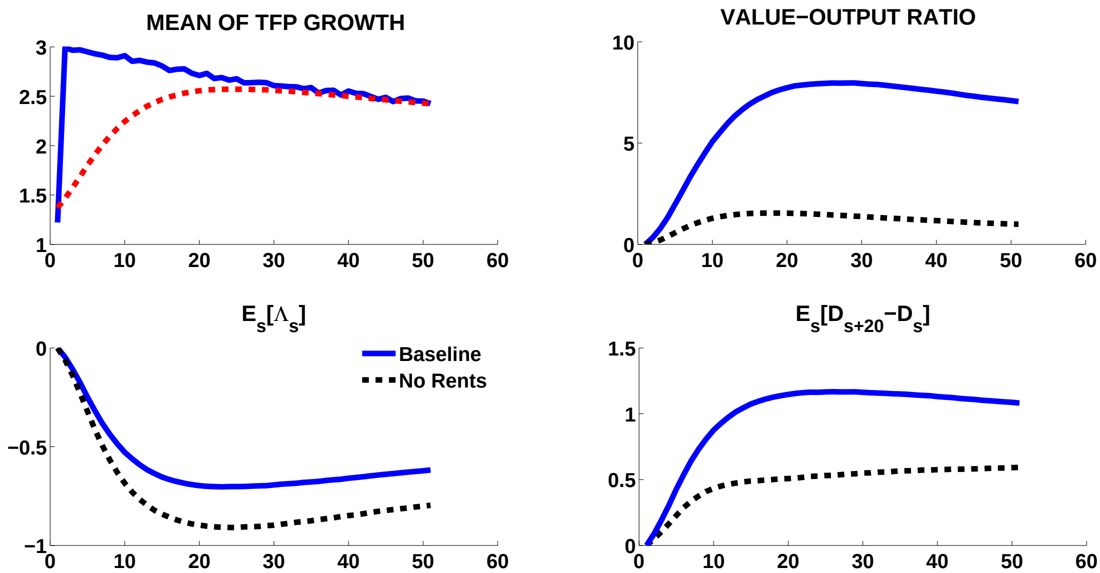
4.3 Growth, Volatility and the Value of Corporations

In the previous section, we discussed the performance of the model in reproducing key unconditional moments. We now study the sensitivity of the value-output ratio to the first two moments of TFP growth. For this purpose, and in view of the analysis of Section 2, it is natural to study the model implied elasticities of the value-output ratio to the mean and volatility of TFP growth. In what follows, we analyze the economic mechanisms through which μ_t and σ_t influence the value-output ratio by means of impulse response functions (IRFs) and through extensive sensitivity analysis. In Section 4.5, we will ask how far the model goes in matching quantitatively these elasticities.

Figure 4 shows IRFs of selected variables to a positive change in μ_t. The top panel reports the dynamics of TFP growth and the value-output ratio, while the bottom panel plots the response of the expected stochastic discount factor and the expected average 5-year growth rate of corporate payouts. The annualized growth rate of TFP increases by 1.5% and reverts back to trend thereafter. After the switch, agents slowly learn about the transition to the high-growth regime. Agents' beliefs about μ_t, represented by the dotted line in the top-left panel of the figure, steadily increase from quarter 1 to quarter 20, after which agents become almost sure that a change in regime has occurred. During this period, we observe a protracted increase in the value-output ratio. From a quantitative point of view, a 1.5% increase in TFP growth is associated, at peak, with an 8% increase in the value-output ratio.

The bottom panel of Figure 4 captures the mechanism through which higher economic growth induces an increase in the value-output ratio. As households learn about the switch to the high-growth regime, they anticipate higher consumption growth for an extended period of time. This positive wealth effect lowers the rate at which households discount corporate payouts, as the bottom-left panel of the figure shows. *Ceteris paribus*, the decline in the stochastic discount factor has a depressive effect on the value of corporations. However, higher TFP growth changes the expectations that households have regarding future corporate payouts. Indeed, the bottom-right panel of the figure shows that long-run expectations regarding the growth rate of corporate payouts slowly increase after the switch to the high-growth regime. Households have thus an incentive to substitute from current to future consumption by acquiring more securities, generating upward pressures on the value of firms. Since under our calibration agents are not too averse to intertemporal substitution, the latter effect dominates and the value-output ratio rises after a switch to the high-growth regime.

Figure 4: **IRFs to a Growth Switch**



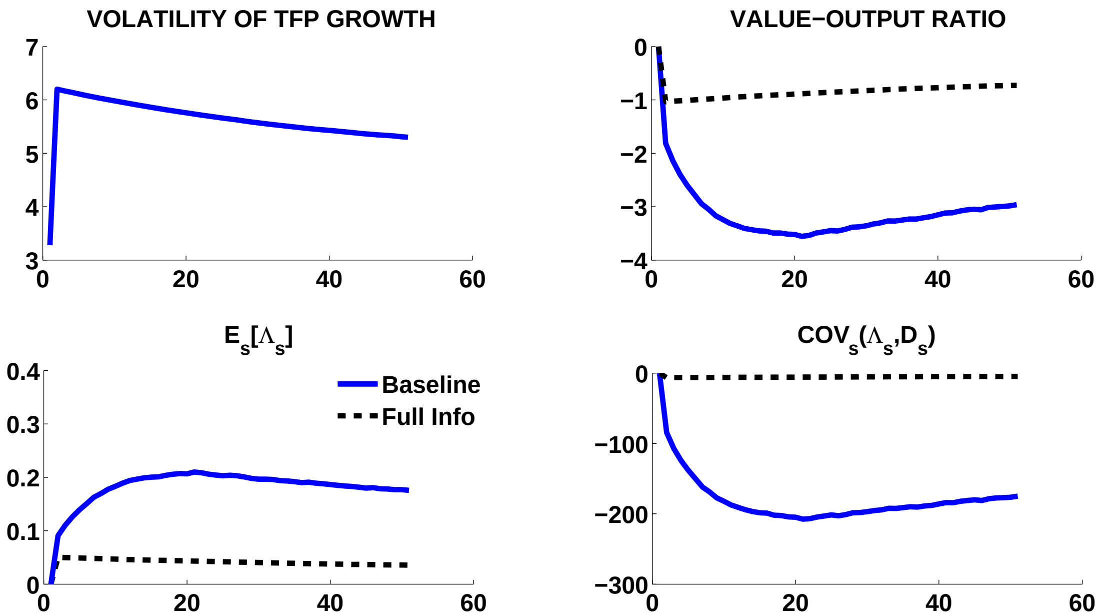
Note: IRFs are calculated via simulation techniques. We simulate $M = 25000$ different realization of length $T = 500$. Each simulation has the characteristic that the trend growth rate of TFP is in the low state between period 1 and period 400 and switches to the high regime in period 401. After that, the simulations are not restricted with regard to the mean. The volatility state is fixed to its low state throughout the simulations. The above figure report the mean across the Monte Carlo replications as percentages with respect to period 400 for the expected discount factor and the value output ratio. The growth rate of TFP and corporate payouts is reported in annualized terms. The average expected growth of corporate payouts over 20 periods is shown relative to the average dividends in period 400. The black dotted lines report the IRFs for a model with perfect competition in the markets for intermediate goods ($\nu = \infty$).

We can also observe that imperfect competition significantly raises the sensitivity of asset prices to μ_t. As we can see from the dotted line in Figure 4, the No Rents model implies a response of the value-output ratio of only 1% at peak, sensibly smaller with respect to that in our benchmark model. We can rationalize this difference across models by looking at the behavior of expected corporate payouts growth in the bottom-right panel of the figure. In our model, corporate payout growth is more responsive to μ_t relative to what happens in the No Rents model. This result is best understood in terms of the decentralization of the economy discussed in Section 3. In the No Rents model, the value of corporations equals the value of capital good firms, while in our model the value of corporations also includes the market value of intermediate good producers. These two sectors of the economy differ in terms of their competitiveness. Capital good producers are identical to each other, while intermediate good firms have traits that partly shield them from competitive pressures. Once the growth rate of TFP increases, capital good producers have an incentive to invest. As more producers invest, the marginal product of existing capital for every firms declines, therefore eroding part of the profits induced by higher TFP growth. Firms operating in the intermediate good sector, instead, are not affected by these competitive pressures. Thus, their payouts growth is more responsive to changes in μ_t.

Figure 5 reports the response of the value-output ratio when the economy transits from the low to the high volatility regime. Higher volatility of TFP growth is associated with declining asset prices. In particular, the value-output ratio in our model falls by 3.5%. The bottom panel of Figure 5 captures the major trade-off that higher volatility brings. An increase in σ_t is associated by agents with more aggregate risk. As individuals are risk averse, they have stronger incentives to demand shares in order to insure consumption fluctuations. The expected stochastic discount factor, therefore, increases. However, households also realize that corporate shares are now riskier securities. Indeed, as the bottom-right panel of Figure 5 shows, the covariance between the stochastic discount factor and equity returns declines.Therefore, households have an incentive to substitute corporate shares with current consumption, and this puts downward pressure on share prices. Since the IES is sufficiently large in our economy, this latter effect dominates, resulting in a negative association between volatility and asset prices. It is also clear from the figure that incomplete information is the key model element governing the sensitivity of asset prices to σ_t. Indeed, in the full information model, the switch to the high volatility regime is associated with a 1% decline in the value-output ratio, 3.5 times smaller with respect to our benchmark specification. This happens because a change in the volatility is perceived differently by the agents in the two models. In the full information set-up, the increase in the volatility of the transitory component of TFP growth has almost no influence on risk, since the stochastic discount factor is hardly affected by transitory TFP growth shocks. In our model, instead, an increase in σ_t makes learning over μ_t more difficult and raises agents' uncertainty over long-run growth.

As a result, households demand a higher compensation to hold assets whose expected discounted payouts are strongly influenced by μ_t. This variation in risk premia, absent in the full information model, is the major driver of the response of the value-output ratio to σ_t.

Figure 5: **IRFs to a Volatility Switch**



Note: IRFs are calculated via simulation techniques. We simulate $M = 25000$ different realization of length $T = 500$. Each simulation has the characteristic that the volatility of TFP is in the low state between period 1 and period 400 and switches to the high regime in period 401. After that, the simulations are not restricted with regard to the volatility. The mean growth rate is fixed to its high state throughout the simulations. The above figure reports the mean across the Monte Carlo replications as percentages with respect to period 400 for all series but the volatility. The black dotted lines report the IRFs for a model with perfect information ($\sigma_g = 0$).

4.4 Sensitivity Analysis

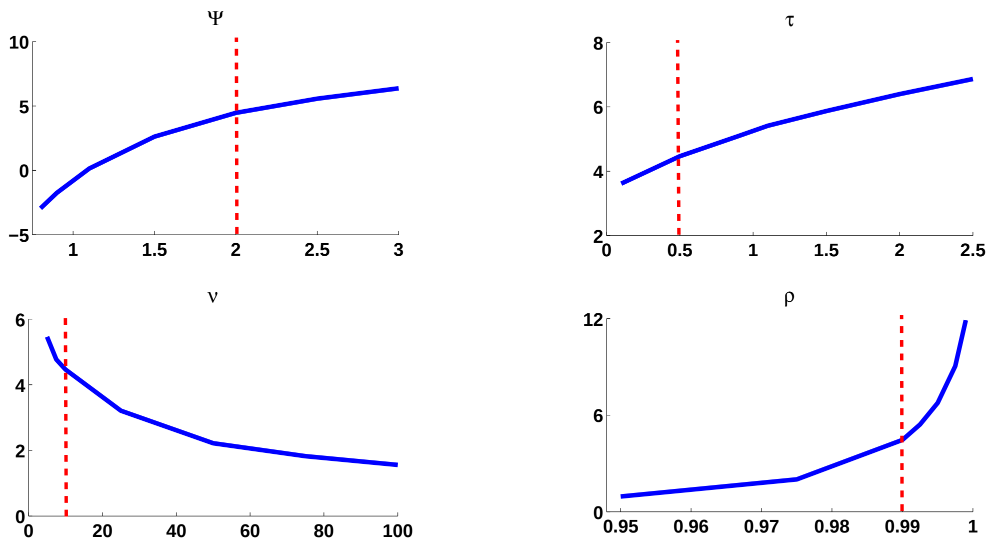
We now briefly discuss the sensitivity of the results presented in the previous section to our calibration. In order to do so, we construct the model implied elasticities of the value-output ratio to μ_t and σ_t and study how these elasticities are affected when changing some key parameters of the model. Let θ' be a given value for our parameter vector. Conditional on θ', we simulate realizations of length $\mathbf{T}$ for μ_t, σ_t and for the value-output ratio. Given these simulated series, we run the following linear projection:

$$\frac{p_t}{y_t} = \mathbf{a} + \mathbf{b}\hat{\mu}_t + \mathbf{c}\hat{\sigma}_t + e_t.$$

The coefficients of these linear projections, $\{\mathbf{b}(\theta'), \mathbf{c}(\theta')\}_{\theta'}$, are the model counterparts of the elasticities computed in Section 2 using U.S. data.[30] Moreover, they are an interesting object to base our sensitivity analysis on since they give information on the *sign* and *size* of the association between the value-output ratio, economic growth and volatility.

We organize our discussion around four key parameters: i) the elasticity of intertemporal substitution (Ψ); ii) the elasticity of marginal Q with respect to the investment-capital ratio (τ); iii) the elasticity of substitution between intermediate goods (ν); and iv) the persistence of the Markov processes (ρ). Figure 6 reports the value of $\mathbf{b}$ when varying these four parameters one at a time, with the red dotted line marking our benchmark calibration.

Figure 6: **Sensitivity Analysis: Elasticity of Value-Output Ratio to μ_t**



Note: For each point in the parameter space, the elasticity of the value output ratio to μ_t is calculated according to the procedure in Section 4.4. In the simulation, $\mathbf{T}$ is set to 25000000. The red dotted line marks the benchmark calibration.

The top-left panel shows that $\mathbf{b}$ is increasing in Ψ. This is in line with our discussion in the previous section. Indeed, we have seen how a switch from the low- to the high-growth regime brings in offsetting wealth and substitution effects on households. As agents become less averse to intertemporal substitution (Ψ increases), the substitution effect becomes stronger, and their demand of corporate shares becomes more sensitive to fluctuations in μ_t. As a

[30] In our simulations, $\hat{\mu}_t$ and $\hat{\sigma}_t$ are the retrospective estimates of households. These differ, in principle, from what an econometrician would obtain by estimating the system in (2) using data simulated from our model. We have verified, though, that in practice the two produce almost identical quantitative results. We have therefore decided to use households' retrospective estimates when calculating the model implied elasticities, since it substantially reduces the computational burden of the procedure.

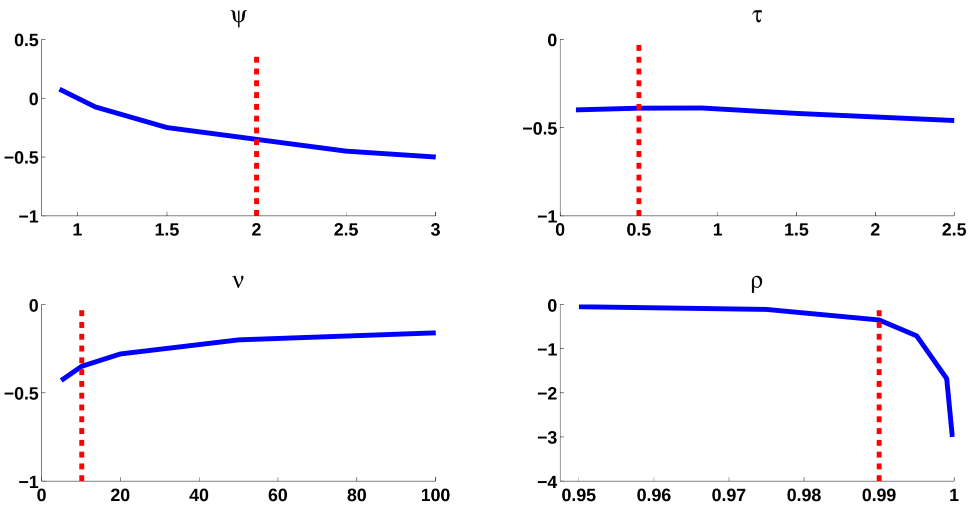
result, a 1% increase in the trend growth rate of TFP is associated with a stronger increase in the value-output ratio. Notice that when Ψ is sufficiently small, $\mathbf{b}$ becomes negative. In these situations, the wealth effect dominates the substitution effect, leading to a negative association between economic growth and the value of corporations.

The next two panels of Figure 6 report the sensitivity of $\mathbf{b}$ with respect to τ and ν. We can see that a higher τ is associated with a stronger response of asset prices to fluctuations in μ_t, while a higher ν is associated with a smaller $\mathbf{b}$. When τ is large, adjusting capital is more costly from the perspective of capital good producers, who then have less incentives to invest. Thus, after a shift in μ_t, their profits on existing capital are eroded less from the process of capital accumulation. This implies that the value of capital good firms is more sensitive to μ_t: when τ equals 2.5, a 1% increase in the trend growth rate of the economy is associated with a 7% increase in the value-output ratio, almost double with respect to what we obtain in our benchmark calibration. A similar phenomenon occurs when decreasing ν. Indeed, we have seen that the present value of rents is the most volatile component of asset prices in our model. As ν declines, the share of this component on the total value of corporation increases, thus raising the sensitivity of the value-output ratio to μ_t.

The last panel of the Figure shows the sensitivity of $\mathbf{b}$ with respect to ρ. This is by far the most important parameter in determining quantitatively the response of the value-output ratio to fluctuations in economic growth. When the switch from the low to the high TFP growth regime is perceived to be almost permanent ($\rho = 0.999$), a 1% increase in μ_t is associated with a 12% increase in the value-output ratio. On the contrary, $\mathbf{b}$ is almost 0 when ρ is equal to 0.95. This is in line with our previous discussion. When ρ is high, households expect the growth rate of corporate payouts to be high for a long period of time. Since households are forward looking, they have now stronger incentives to buy corporate shares, and this raises the response of the value-output ratio. Notice also that $\mathbf{b}$ is highly nonlinear in ρ around our benchmark parametrization. Even a small increase in this parameter results in the elasticity of the value-output ratio to double or triple with respect to our benchmark calibration.

Figure 7 reports the same experiment for the elasticity of the value-output ratio to σ_t. The Figure confirms the above discussion. Higher Ψ is associated with a decline in $\mathbf{c}$. Asset prices are marginally more sensitive to volatility fluctuations when the supply of capital is less elastic (τ is large) or when monopolistic rents are more relevant (ν small). Again, ρ is the most important parameter governing the elasticity of the value-output ratio to σ_t. As ρ passes from 0.99 to 0.999, the absolute value of $\mathbf{c}$ increases by almost 10 times.

Figure 7: **Sensitivity Analysis: Elasticity of Value-Output Ratio to σ_t**



Note: See Figure 6

4.5 Posterior Predictive Analysis

After having analyzed the economic mechanisms governing the relation between growth, volatility and asset prices, we now assess the model's quantitative performance along this dimension. For this purpose, we will ask how far the model implied elasticities **b** and **c** are from the ones we estimated for the U.S. economy (Column 3 of Table 2). Because of the extreme sensitivity of these elasticities to the value chosen for the persistence of μ_t and σ_t, we will rely on posterior predictive analysis. In particular, let $\theta_1 = [\mu_0, \mu_1, P^{\mu}_{0|0}, P^{\mu}_{1|1}, \sigma_0, \sigma_1, P^{\sigma}_{0|0}, P^{\sigma}_{1|1}]$ and let θ_{-1} be the vector collecting the remaining structural parameters of our model, fixed at their calibration values. Given a series of posterior draws for the TFP process parameters, $\{\theta_1^m\}_{m=1}^M$, one can calculate $\{\mathbf{b}^{\text{model}}(\theta_1^m, \theta_{-1}), \mathbf{c}^{\text{model}}(\theta_1^m, \theta_{-1})\}_{m=1}^M$ and use those values to characterize the posterior distribution of the model implied value-output ratio elasticities. Because of the high computational burden involved when solving our equilibrium model repeatedly, we evaluate the model implied elasticities at $\{\theta_1^m\}_{m=1}^M$ using the following procedure:[31]

Posterior Draws for Model Implied Elasticities Let $\{\theta_1^m\}_{m=1}^M$ be a set of posterior draws for the TFP growth process.

1. Given $\{\theta_1^m\}_{m=1}^M$ we obtain bounds on each parameter so that all elements of the se-

[31] In order to reduce the dimensionality of the problem, we estimate the system in (2) by imposing symmetry on the transition matrices and by fixing $\mu_0 + \frac{1}{2}\mu_1$ and $\sigma_0 + \frac{1}{2}\sigma_1$ to their sample means. Thus, θ_1 is a 4-dimensional object.

quence lie in the set defined by those bounds. We denote this set by Θ_1.

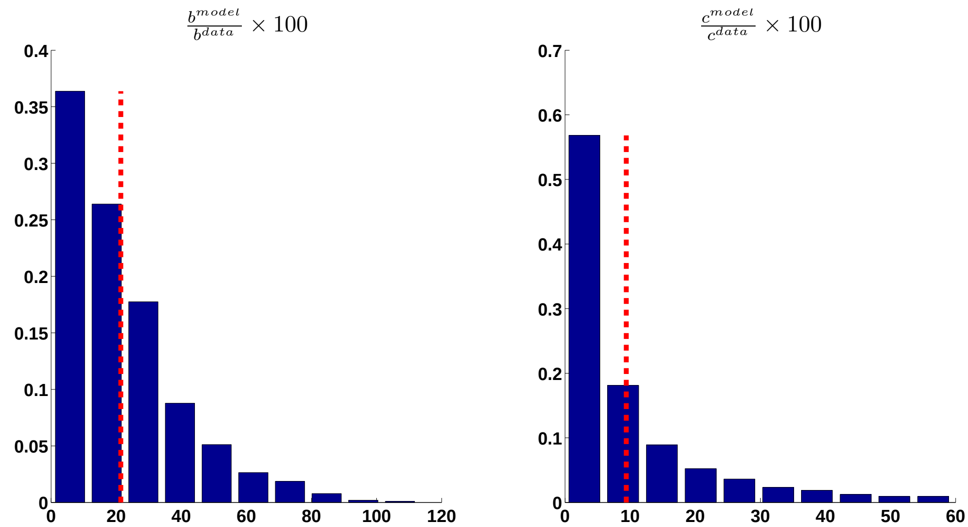
2. We compute the Smolyak collocation points for Θ_1 as described in Krueger and Kuebler (2003). We denote these collocation points by $\{\theta_1^s\}_{s=1}^S$.

3. For each element in $\{\theta_1^s\}_{s=1}^S$, we compute the model implied elasticities $\mathbf{b}(\theta_1^s, \theta_{-1})$ and $\mathbf{c}(\theta_1^s, \theta_{-1})$ using the simulation procedure described in Section 4.4.

4. We fit a polynomial through the computed $\{\mathbf{b}^{\text{model}}(\theta_1^s, \theta_{-1}), \mathbf{c}^{\text{model}}(\theta_1^s, \theta_{-1})\}_{s=1}^S$. We then use this polynomial to evaluate the model implied elasticities at the sequence $\{\theta_1^m\}_{m=1}^M$.

Our exercise consists of assessing how far the coefficients of the linear projection in Table 2, obtained from actual U.S. data, lie in the tails of the model implied distributions for the same objects.

Figure 8: **Growth, Volatility and the Value of U.S. Corporations: Model vs. Data**



Note: The histogram reports the model implied elasticities of the value-output ratio to the mean and volatility of TFP growth relative to their data counterparts. The red dotted lines report the posterior mean of these statistics. The figures are constructed using 50000 draws from the posterior distribution of θ_1.

The left panel of Figure 8 reports the posterior distribution of $\frac{\mathbf{b}^{\text{model}}}{\mathbf{b}^{\text{data}}} \times 100$. A value of this statistic equal to 100 tells us that the model predicts the same elasticity estimated in the data, while values smaller than 100 would imply a weaker association between economic growth and the value-output ratio with respect to what we have estimated in the data. We verify from the figure that the model is broadly consistent with the data along this dimension. Indeed, on average the model captures 20% of the relation between economic growth and the value-output ratio (red vertical line in the Figure). Moreover, we can also see that for reasonable parametrizations of the TFP process,[32] the model is able to deliver elasticities of the value-output ratio to the trend growth rate of TFP that are consistent with the estimates in Table 2.

The right panel of Figure 8 plots the posterior distribution of $\frac{\mathbf{c}^{\text{model}}}{\mathbf{c}^{\text{data}}} \times 100$. The graph shows that the model is less successful in accounting for the association between the value-output ratio and the volatility of TFP growth. On average, in fact, it predicts that the value-output ratio falls by 0.4% following a 1% increase in the standard deviation of TFP growth, roughly 10% of what we have estimated in the data. Moreover, the histogram shows that the model can account for at most 60% of the association between the value-output ratio and the volatility of TFP growth.

5 Conclusion

In this paper we have uncovered a striking association between the first two moments of TFP growth and the value of corporations in postwar U.S. data. Persistent fluctuations in the mean and volatility of TFP growth predict two-thirds of the medium-term variation in the value-output ratio. This indicator rises strongly after an increase in the trend growth rate of TFP, while it declines substantially following an increase in the volatility of TFP growth. A possible explanation for this association, suggested elsewhere in the literature, is that movements in aggregate productivity influence investors' expectations of future corporate payouts as well as the rate at which they discount them. This explanation is put under scrutiny by us. We developed a general equilibrium model with production featuring Markov-Switching fluctuations in the mean and volatility of TFP growth, incomplete information, capital adjustment costs, monopolistic competition and recursive preferences. Under plausible calibrations, the model is consistent with the behavior of several U.S. real and financial indicators during the postwar period. It accounts on average for roughly 20% (9%) of the association between the mean (volatility) of TFP growth and the value-output ratio. For reasonable parametrizations of the TFP process, the model predicts an elasticity of the value-output ratio to economic growth that is in line with the data, while it predicts an elasticity of the value-output ratio to the volatility of TFP growth that is 60% of the data observation.

It is important to stress the *ex-post* nature of our analysis. This has at least two important

[32] By reasonable parametrization, we mean regions of the parameter space that have positive mass in the posterior distribution.

implications. First of all, our estimates for the TFP process are retrospective and this may contribute to muting some of the channels analyzed in this paper. This is surely the case for the implications of incomplete information. Indeed, while a two-state process for the mean of TFP growth fits postwar U.S. data well, agents may still consider states that never occurred during this period when forming their expectations. If that was the case, a model restricted to two states necessarily bounds the amount of perceived risk over long-run growth, which dampens the response of risk premia to a change in the volatility of TFP growth. This particular aspect may explain why the model does not generate a strong sensitivity of the value-output ratio to second moments. Secondly, agents in our model have perfect knowledge of the parameters governing the growth and volatility regimes. This assumption rules out important sources of history dependence. With uncertainty on the persistence of the Markov processes, for example, agents' perception about these parameters depends on previous realizations of the process. Our approach may thus misrepresent how agents interpreted these fluctuations when solving their decision problem in real time. This may severely affect our results, since the sensitivity of asset prices to shifts in the first two moments of TFP growth depends crucially on their perceived duration.

While difficult to discipline empirically, we believe that an analysis that relaxes these two types of restrictions would further enhance our understanding of the medium-term movements in the value of corporations.

References

Abel, A. (2003): "The Effects of a Baby Boom on Stock Prices and Capital Accumulation in the Presence of Social Security," *Econometrica*, 71.

Altig, D., L. Christiano, M. Eichenbaum, and J. Linde (2011): "Firm-Specific Capital, Nominal Rigidities and the Business Cycle," *Review of Economic Dynamics*, 14(2), 225–247.

Attanasio, O., and A. Vissing-Jorgensen (2003): "Stock Market Partecipation, Intertemporal Substitution and Risk Aversion," *American Economic Review Papers and Proceedings*, 93(2), 383–391.

Bansal, R., and A. Yaron (2004): "Risk for the Long Run: A Potential Resolution of Asset Pricing Puzzles," *The Journal of Finance*, 59(4), 1481–1509.

Barsky, R. B. (1989): "Why Don't the Prices of Stocks and Bonds Move Together?," *The American Economic Review*, 79(5), 1132–1145.

Barsky, R. B., and J. B. De Long (1993): "Why Does the Stock Market Fluctuate?," *The Quarterly Journal of Economics*, 108(2), 291–311.

Beaudry, P., and F. Portier (2006): "Stock Prices, News and Economic Fluctuations," *American Economic Review*, 96(4), 1293–1307.

Benigno, P., L. Ricci, and P. Surico (2011): "Unemployment and Productivity in The Long Run: The Role of Macroeconomic Volatility," NBER Working Paper No. 15842.

Boldrin, M., L. Christiano, and J. Fisher (2001): "Habit Persistence, Asset Returns and the Business Cycle," *American Economic Review*, 91(1), 149–166.

Boldrin, M., and A. Peralta-Alva (2009): "What Happened to the U.S. Stock Market? Accounting for the Past 50 Years," *Federal Reserve Bank of St. Louis Review*, 91(6), 627–646.

Bullard, J., and A. Singh (2012): "Learning and the Great Moderation," *International Economic Review*, 53(2), 375–397.

Chang, Y., T. Doh, and F. Schorfheide (2007): "Non-stationary Hours in a DSGE Model," *Journal of Money, Credit and Banking*, 39(6), 1357–1373.

Cogley, T. (2005): "How Fast Can the New Economy Grow? A Bayesian Analysis of the Evolution of Trend Growth," *Journal of Macroeconomics*, 27, 179–207.

Cogley, T., and J. Nason (1995): "Output Dynamics in Real-Business-Cycle Models," *The American Economic Review*, 85(3), 492–511.

Cogley, T., and J. Sargent, T (2005): "Drifts and Volatilities: Monetary Policies and Outcomes in Post WWII US," *Review of Economic Dynamics*, 8, 262–302.

Comin, D., and M. Gertler (2006): "Medium-Term Business Cycles," *American Economic Review*, 96(3), 523–551.

Comin, D., M. Gertler, and A. M. Santacreu (2009): "Technology Innovation and Diffusion as Sources of Output and Asset Price Fluctuations," Harvard Business School WP, 09-134.

Croce, M. M. (2010): "Long-Run Productivity Risk: A New Hope for Production-Based Asset Pricing?," Unpublished Manuscript.

Edge, R., T. Laubach, and J. Williams (2007): "Learning and Shifts in Long-Run Productivity Growth," *Journal of Monetary Economics*, 54(8), 2421–2438.

French, M. (2001): "Estimating Changes in Trend Growth of Total Factor Productivity: Kalman and H-P Filter versus a Markov-Switching Framework," Federal Reserve Board Staff Working Paper, N0. 2001-44.

Gourio, F. (2012): "Disaster Risk and the Business Cycle," *American Economic Review*, 102(6), 2734–2766.

Greenwood, J., and B. Jovanovic (1999): "The Information-Technology Revolution and the Stock Market," *American Economic Review*, 89(2), 116–122.

Hall, R. (2001): "The Stock Market and Capital Accumulation," *American Economic Review*, 91(5), 1185–1202.

Hayashi, F. (1982): "Tobin's Marginal q and Average q: A Neoclassical Interpretation," *Econometrica*, 50, 213–24.

Iraola, M. A., and M. S. Santos (2009): "Long-Term Asset Price Volatility and Macroeconomic Fluctuations," ITAM Working Paper WP, 09-09.

Jermann, U. (1998): "Asset Pricing in Production Economies," *Journal of Monetary Economics*, 41, 257–275.

Jermann, U., and V. Quadrini (2007): "Stock Market Boom and the Productivity Gains of the 1990s," *Journal of Monetary Economics*, 54(2), 413–432.

Jorgenson, D. W., M. S. Ho, and K. J. Stiroh (2008): "A Retrospective Look at the U.S. Productivity Growth Resurgence," *Journal of Economic Perspective*, 22(1), 3–24.

Judd, K. (1998): *Numerical Methods in Economics*. MIT Press.

Kahn, J. A., and R. W. Rich (2007): "Tracking the New Economy: Using Growth Theory to Detect Changes in Trend Productivity," *Journal of Monetary Economics*, 54, 1670–1701.

Kim, C., and C. Nelson (1999): "Has the U.S. Economy Become More Stable? A Bayesian Approach Based on a Markov-Switching Model of the Business Cycle," *The Review of Economics and Statistics*, 81, 608–616.

Krueger, D., and F. Kuebler (2003): "Computing Equilibrium in OLG Models with Stochastic Production," *Journal of Economic Dynamics and Control*, 28(7), 1411–1436.

Kydland, F., and E. Prescott (1982): "Time to Build and Aggregate Fluctuations," *Econometrica*, 50(6), 1345–1370.

Laitner, J., and D. Stolyarov (2003): "Technological Change and the Stock Market," *American Economic Review*, 93(4), 1240–1267.

Lettau, M., S. C. Ludvigson, and J. A. Wachter (2008): "The Declining Equity Premium: What Role Does Macroeconomic Risk Play?," *The Review of Financial Studies*, 21(4), 1653–1686.

Malkiel, B. (1979): "The Capital Formation Problem in the United States," *Journal of Finance*, 34.

McConnell, M. M., and G. P. Quiros (2000): "Output Fluctuations in the United States: What Has Changed Since the Early 1980s?," *American Economic Review*, 90, 1464–1476.

McGrattan, E., and E. Prescott (2005): "Taxes, Regulations and the Value of U.S. and U.K. Corporations," *Review of Economic Studies*, 72.

Naik, V. (1994): "Asset Prices in Dynamic Production Economies with Time-Varying Risk," *Review of Financial Studies*, 7(4), 781–801.

Pastor, L., and P. Veronesi (2009): "Technological Revolutions and Stock Prices," *The American Economic Review*, 99(4), 1451–1483.

Pindyck, R. S. (1984): "Risk, Inflation and the Stock Market," *American Economic Review*, 74.

Roberts, J. (2000): "Estimates of the Productivity Trend Using Time-Varying Parameter Techniques," Federal Reserve Board Staff Working Paper, N0. 2001-8.

Tallarini, T. D. J. (2000): "Risk-Sensitive Real Business Cycles," *The Journal of Monetary Economics*, 45(3), 507–532.

Van Binsbergen, J. H., J. Fernandez-Villaverde, R. S. J. Koijen, and J. Rubio-Ramirez (2010): "The Term Structure of Interest Rates in a DSGE Model with Recursive Preferences," Unpublished Manuscript.

Wright, S. (2004): "Measures of Stock Market Value and Returns for the U.S. Nonfinancial Corporate Sector, 1900-2002," *Review of Income and Wealth*, 50(4), 561–584.

Online Appendix

A Empirical Analysis

A.1 Data Definition and Sources

A.1.1 TFP Growth

Quarterly data on output growth and hours growth for the U.S. Business Sector are from the BLS, respectively Series id PRS85006042 and PRS85006032. We construct an annual series for the growth rate of the U.S. Business Sector capital stock using NIPA Tables. In particular, from Table 6.2, we calculate the growth rate of real private fixed assets for the following sectors:

- Corporate
- Sole Proprietorships
- Partnerships

We use Table 7.2A and 7.2B from NIPA to construct the growth rate of real fixed assets of government sponsored enterprises (GSE). We construct the growth rate of real fixed assets for the Business Sector as follows:

$$\Delta k_{bus} = \theta_{corp}\Delta k_{corp} + \theta_{propr}\Delta k_{propr} + \theta_{part}\Delta k_{part} + \theta_{gse}\Delta k_{gse},$$

where θ_j is the share of subsector j in total fixed assets of the Business Sector.[33]
Next, we use the following formula to calculate TFP growth for the Business Sector:[34]

$$\Delta z_t = \frac{\Delta y_t - \alpha\Delta k_t - (1-\alpha)\Delta l_t}{1-\alpha}$$

We choose a value of α equal to 0.30, as it is customary in the macroeconomic literature.[35]

[33] That is, we take fixed assets measured at current cost in sector j, divide by the same figure for the entire Business Sector, and average over the time period considered.

[34] In order to reconcile the frequency of output and hours with those of capital, we linearly interpolate the growth rate of capital and convert it at quarterly frequencies.

[35] We also considered a version of TFP growth with time-varying factor shares. Data on labor shares for the Business Sector are from BLS, Series id PRS85006173. Our results did not change, both from a qualitative and quantitative standpoint.

A.1.2 Market Value of U.S. Corporations

Our indicator of value is the sum of two components, the value of equities and the value of net debt for the U.S. corporate sector. We use data from the Federal Reserve's Flow of Funds Accounts to obtain these two time series. In particular:

- Market value of Corporate Equities: We take the data from Table L.213 of the Flow of Funds ("Market Value of Domestic Corporations").
- Net Debt: We construct a net debt series for all domestic sectors issuing corporate equities. The sectors issuing corporate equities can be obtained from Table F.213 of the Flow of Funds. These are:
  - Non Financial Corporate Business (Table L.201).
  - Domestic Financial Corporations (Tables L.110, L.114, L.115, L.121, L.126, L.122, L.127, L.128, L.129).[36]

  We define net debt as the difference between total debt liabilities and total debt assets, where "debt" includes any financial instrument that is not corporate equity, mutual funds holdings that are equity and the equity component of "miscellaneous claims."[37] We then aggregate to obtain the net debt series. Notice that the net debt series computed consists of instruments that are recorded mainly at book value in the Flow of Funds.[38]

[36]Domestic financial corporations issuing equities are, in order: i) U.S. chartered depository institutions; ii) property-casualty insurance companies; iii) life insurance companies; iv) close-end funds and exchange traded funds; v) REITS; vi) government-sponsored enterprises; vii) brokers and dealers; viii) holding companies; and ix) funding companies.

[37]We follow the procedure described in the online appendix of McGrattan and Prescott (2005) in order to deduce the equity component of mutual funds holdings and miscellaneous claims. For this purpose, we use Flow of Funds Tables L.229, L.230, L.231.

[38]Hall (2001) proposes a procedure to correct for this issue. McGrattan and Prescott (2005) show that correcting for this issue results in only minor changes to the sample 1960-2001 (see Figure A.3 in their online appendix).

A.2 Univariate model

We model TFP growth as follows:

$$\begin{aligned}
\Delta Z_t &= \mu_t + \phi\left[\Delta Z_{t-1} - \mu_{t-1}\right] + \sigma_t \varepsilon_t \\
\mu_t &= \mu_0 + \mu_1 s_{1,t} \\
\sigma_t &= \sigma_0 + \sigma_1 s_{2,t}
\end{aligned}$$

$$\varepsilon_t \sim \mathcal{N}(0,1) \qquad s_{1,t} \sim \mathcal{MP}(P_\mu) \qquad s_{2,t} \sim \mathcal{MP}(P_\sigma)$$

We collect in $\theta = [\mu_0, \mu_1, \sigma_0, \sigma_1, \phi, P_{1,1,\mu}, P_{2,2,\mu}, P_{1,1,\sigma}, P_{2,2,\sigma}]$ the parameters to be estimated. We use Bayesian methods to conduct inference over θ. Given prior information on the parameters, represented by the distribution $p(\theta)$, and given the likelihood function $p(\{\Delta Z_t\}_{t=2}^{\mathbb{T}}|\theta, \Delta Z_1)$, the posterior distribution of θ is found using Bayes' rule:

$$p(\theta|\{\Delta Z_t\}_{t=1}^{T}) = \frac{p(\theta)p(\{\Delta Z_t\}_{t=1}^{T}|\theta)}{p(\{\Delta Z_t\}_{t=1}^{T})}$$

The parametrization of the prior distribution is given in Table 1. Functional forms are chosen for tractability. We center the prior on μ_0 and μ_1 so that, on average, the growth rate of TFP is 2% at an annual level. In the low-growth regime, TFP growth is 60% of the high-growth regime. We center the prior on σ_0 and σ_1 so that the the standard deviation of TFP growth is on average 4%. In the low volatility regime, the standard deviation of TFP growth is on average 60% that of the high volatility regime. We center the prior of ϕ at zero, reflecting beliefs of low autocorrelation of TFP growth. Finally, the prior on the transition probabilities is centered so that the expected duration of a regime is approximately 15 years. A better way of interpreting our prior is to look at Table 4, which reports prior predictive checks. Essentially, our prior is that TFP growth has very low autocorrelation, with small and very persistent time variation in the mean. Moreover, we can verify from the table that our prior is quite diffused, as the standard deviations of the statistics show.

We use a Random Walk Metropolis Hastings (RWMH) algorithm to sample from the posterior of θ. We follow common practice in choosing the following proposal density:

$$\theta^{proposal} \sim \mathcal{N}(\theta^i, c\mathbf{H^{-1}}),$$

where θ^i is the state of the Markov Chain at iteration i and $\mathbf{H^{-1}}$ is the inverse Hessian of the log-posterior density evaluated at the posterior mode. The scaling factor c is chosen so that our RWMH algorithm has an acceptance rate of approximately 30%. We generate $N = 100000$ draws and discard the first 20000 when computing posterior statistics.[39]

Table 4: **Prior Predictive Checks**

Statistic	MS-Model
Mean(ΔZ_t)	2.00 (1.10)
Stdev(ΔZ_t)	4.42 (4.50)
Acorr(ΔZ_t)	0.05 (0.53)
Expected Duration of Regimes	10 (80)

Note: Prior Predictive Checks are calculated as follows: 1) generate a random draw of the model's parameters θ^m from $p(\theta)$; 2) given θ^m, use the Markov-Switching model to compute a realization ($T = 10000$) for ΔZ_t; 3) compute statistics on the generated sample; 4) repeat this procedure $M = 10000$ times and report mean and standard deviation (in parenthesis) of each statistic computed in 3). Expected duration of a regime is reported in years.

[39] We perform several tests confirming that our choice of N yields an accurate posterior approximation.

A.3 Multivariate Model

The multivariate analysis is based on the following model:

$$\begin{aligned}
\begin{bmatrix} \Delta Z_t \\ \Delta \mathbf{Y_t} \end{bmatrix} &= \begin{bmatrix} \mu_t \\ \mu_t \end{bmatrix} + \Phi \left(\begin{bmatrix} \Delta Z_{t-1} - \mu_{t-1} \\ \Delta \mathbf{Y_{t-1}} - \mu_{t-1} \end{bmatrix} \right) + \Sigma_t \mathbf{e_t} \\
\mu_t &= \mu_0 + \mu_1 s_{1,t} \\
\Sigma_t &= \Sigma_0 + \Sigma_1 s_{2,t}
\end{aligned}$$

$$\varepsilon_t \sim \mathcal{N}(0,1) \qquad s_{1,t} \sim \mathcal{MP}(P_\mu) \qquad s_{2,t} \sim \mathcal{MP}(P_\sigma)$$

We include in $\Delta \mathbf{Y_t}$ the growth rate of aggregate consumption per hour and the growth rate of real compensation per hour in the Non Farm Business Sector. We follow Cogley and Sargent (2005) in parametrizing the matrix Σ_t as follows:

$$\Sigma_t = \mathbf{B}\hat{\Sigma}(s_{2,t})$$

with $\mathbf{B}$ being a lower triangular matrix with ones on the main diagonal and $\hat{\Sigma}(s_{2,t})$ a diagonal matrix whose (j,j) element evolves as follows:

$$\sigma_{j,t} = \sigma_{j,0} + \sigma_{j,1} s_{2,t}$$

Given these restrictions, the parameters to be estimated are 25. As in the previous section, we use Bayesian methods to estimate the above model. In particular, we sample from the posterior distribution of the model's parameter using a Metropolis-within-Gibbs algorithm. Our posterior simulator has four main steps:

1. Sample $\{s_{1,t}, s_{2,t}\}_{t=1}^{T}$ given the data and the model's parameters using the Kim-Hamilton smoother (Kim and Nelson, 1999);

2. Sample Φ conditional on $\{s_{1,t}, s_{2,t}\}_{t=1}^{T}$ and the other model's parameters from a standard linear Bayesian regression with conjugate priors;

3. Sample the lower diagonal elements of $\mathbf{B}$ conditional on $\{s_{1,t}, s_{2,t}\}_{t=1}^{T}$ and the other model's parameters from a system of unrelated regressions with conjugate priors, see Cogley and Sargent (2005);

4. Sample the parameters $[\mu_0, \mu_1, \{\sigma_{j,0}, \sigma_{j,1}\}_{j=1}^{3}, P_{1,1,\mu}, P_{2,2,\mu}, P_{1,1,\sigma}, P_{2,2,\sigma}]$ using a Metropolis step, with proposal density constructed in the same way as in Appendix A.2

The prior for the parameters governing μ_t and $\sigma_{j,t}$ is the same as the one described in the previous section, while the priors on the remaining parameters are fairly diffuse. We generate 100000 draws from the posterior and discard the first 20000 when computing posterior statistics.

B Equilibrium and Auxiliary Planner's Problem

In this appendix, we are going to

- formally define an equilibrium for our model;
- characterize some of its properties that are useful for the computation;
- describe the Auxiliary Planner's Problem utilized in the numerical solution;
- explain how we choose the state variables during computation to minimize the computational burden.

B.1 Equilibrium

An equilibrium of our economy are sequences (depending on realizations of the stochastic process) of quantities [40]

$$(k_t, (k_{j,t})_{j\in[0,1]}, i_t, (l_{j,t})_{j\in[0,1]}, (d_{j,t})_{j\in[0,1]}, (y_{j,t})_{j\in[0,1]}, (\hat{y}_{j,t})_{j\in[0,1]}, (\omega_{j,t})_{j\in[0,1]}, y_t, c_t, \omega_t)_{t=0}^{\infty},$$

prices $(r_t, w_t, (\mathcal{P}_{j,t})_{j\in[0,1]}, (p_{j,t})_{j\in[0,1]}, \bar{p}_t, p_t^k, d_t)_{t=0}^{\infty}$, value functions $(V_t)_{t=0}^{\infty}$ and discount factors $(\Lambda_{0,t})_{t=0}^{\infty}$ such that

- $(V_t)_{t=0}^{\infty}$, $((\omega_{j,t})_{j\in[0,1]}, \omega_t)_{t=0}^{\infty}$, $(c_t)_{t=0}^{\infty}$ solves the household's problem given $(w_t, (d_{j,t})_{j\in[0,1]}, (\mathcal{P}_{j,t})_{j\in[0,1]}, p_t^k, d_t)_{t=0}^{\infty}$:

$$\max_{(\tilde{V}_t)_{t=0}^{\infty}, ((\tilde{\omega}_{j,t})_{j\in[0,1]})_{t=0}^{\infty}, (\tilde{c}_t)_{t=0}^{\infty}, (\tilde{d}_t)_{t=0}^{\infty}} \tilde{V}_0$$

$$\text{s.t. } \tilde{V}_t = [(1-\beta)\tilde{c}_t^{\frac{1-\gamma}{\eta}} + \beta\mathbb{E}_t[\tilde{V}_{t+1}^{1-\gamma}]^{\frac{1}{\eta}}]^{\frac{\eta}{1-\gamma}}$$

$$\tilde{c}_t + \int \mathcal{P}_{j,t}\tilde{\omega}_{j,t+1}dj + \omega_{t+1}p_t^k = w_t + \int (\mathcal{P}_{j,t} + d_{j,t})\tilde{\omega}_{j,t}dj + \omega_t(d_t + p_t^k)$$

  and a no Ponzi condition for ω and finiteness for $\tilde{V}_t$;

- $\forall j$ $(d_{j,t}, k_{j,t}, l_{j,t}, p_{j,t})_{t=0}^{\infty}$ solve intermediate good producer j's problem given $(r_t, w_t, p_{i,t})_{i\in[0,1]\setminus\{j\}}, \bar{p}_t)_{t=0}^{\infty}$, $(\Lambda_{0,t})_{t=0}^{\infty}$ and $(\hat{y}_{j,t})_{j\in[0,1]}$:

$$\max_{(\tilde{d}_{j,t}, \tilde{k}_{j,t}, \tilde{l}_{j,t}, \tilde{p}_{j,t})_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_{0,t}\tilde{d}_{j,t}$$

[40] $\hat{y}_{j,t}$ is the demand function for good j and not a number.

$$\text{s.t. } \tilde{d}_{j,t} = \hat{y}_{j,t}(\tilde{p}_{j,t})\frac{\tilde{p}_{j,t}}{\bar{p}_t} - w_t\tilde{l}_{j,t} - r_t\tilde{k}_{j,t}$$

- $(k_{t+1}, i_t)_{t=0}^{\infty}$ solves the capital good producers problem given r_t:

$$\underset{(\tilde{i}_t,\tilde{k}_{t+1})_{t=0}^{\infty}}{max} \mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_{0,t} r_t \tilde{k}_t - \tilde{i}_t$$

$$\tilde{k}_{t+1} = (1-\delta)\tilde{k}_t + G\left(\frac{\tilde{i}_t}{\tilde{k}_t}\right)$$

- $\forall t$ $(y_t, y_{j,t})_{j\in[0,1]}$ given $\bar{p}_t$, $(p_{j,t})_{j\in[0,1]}$ solve the final good producers problem

$$\underset{\tilde{y}_t,(\tilde{y}_{j,t})_{j\in[0,1]}}{max} \bar{p}_t\tilde{y}_t - \int p_{j,t}\tilde{y}_{j,t}dj$$

$$\tilde{y}_t \leq \left[\int y_{j,t}^{\frac{\nu-1}{\nu}} dj\right]^{\frac{\nu}{\nu-1}}$$

and

$$(\hat{y}_{j,t})_{j\in[0,1]}$$

are consistent with pointwise maximization of the final good producer given any chosen price $p \in \mathbb{R}_t$ by intermediate producer j given $\bar{p}_t$, $(p_{j,t})_{j\in[0,1]\setminus\{j\}}$

- markets clear: $\forall t, h$

$$\int l_{j,t}dj = 1$$

$$\int \omega_{j,t}dj = 1$$

$$\omega_t = 1$$

$$\hat{y}_{j,t}(p_{j,t}) = y_{j,t}$$

$$c_t + \int i_{j,t}dj = y_t$$

$$k_t = \int k_{j,t}dj$$

- The discount factor of the firm fulfills $\Lambda_{0,t} = \Pi_{s=0}^{t-1}\Lambda_{s,s+1}$ where $\Lambda_{t,t+1} = \beta\left(\frac{c_{t+1}}{c_t}\right)^{-\frac{1}{\psi}} \frac{V_{t+1}^{(1-\gamma)(1-\frac{1}{\eta})}}{\mathbb{E}_t[V_{t+1}^{1-\gamma}]^{1-\frac{1}{\eta}}}$.[41]

[41] This condition could be easily derived from assuming there is a full set of Arrow securities in zero net supply. In order not to further expend the notation, we directly impose the condition on the discount factor.

We will focus on a symmetric equilibrium in the following. It then implies that prices and capital service choices are the same for all intermediate good producers.

B.2 Partial Characterization and Auxiliary Planner's Problem

It is well known in the literature and easy to check that the final good producer problem results in the following demand for intermediate good i in equilibrium: $\frac{p_{j,t}}{\bar{p}_t}\left(\frac{p_{j,t}}{\bar{p}_t}\right)^{-\nu}\bar{Y}_t$, where $\bar{Y}_t$ is demand and equal to output in equilibrium and $\bar{p}_t = \left[\int_0^1 p_{j,t}^{1-\nu}dj\right]^{\frac{1}{1-\nu}}$. Imposing this in any intermediate good producing firm's problem and combining it with the problem of a capital good producer,[42] we get as a representative firm's problem

$$\underset{k_{j,t},i_{j,t},l_{j,t},P_{j,t}}{max} \mathbb{E}_0 \sum_{t=0} \Lambda_{0,t}[\frac{p_{j,t}}{\bar{p}_t}\left(\frac{p_{j,t}}{\bar{p}_t}\right)^{-\nu}\bar{y} - i_{j,t} - w_t l_{j,t}]$$

$$\text{s.t.} \left(\frac{p_{j,t}}{\bar{p}_t}\right)^{-\nu}\bar{y}_t = F(k_{j,t}, Z_t l_{j,t})$$

$$k_{j,t+1} = (1-\delta)k_{j,t} + G(\frac{i_{j,t}}{k_{j,t}})k_{j,t}.$$

We continue by taking first-order conditions (We drop the j index for now.). Let λ_t be the multiplier on the first constraint and μ_t on the second.
FOC:

$$(p_t)\ \mathbb{E}_t[\Lambda_{0,t}(1-\nu)\left(\frac{p_t}{\bar{p}_t}\right)^{-\nu}\frac{\bar{y}_t}{\bar{p}_t} - \lambda_t(-\nu p_t\left(\frac{p_t}{\bar{p}_t}\right)^{-\nu}\bar{y}_t)] = 0$$

$$(i_t)\ \mathbb{E}_t[-\Lambda_{0,t} + \mu_t G'(\frac{i_t}{k_t})] = 0$$

$$(k_t)\ \mathbb{E}_{t-1}[-\{\mu_{t-1}\}] + E_{t-1}[\lambda_t F_{K,t} + \mu_t(1-\delta + G(\frac{i_t}{k_t}) - G'(\frac{i_t}{k_t}))\frac{i_t}{k_t}] = 0$$

$$(l_t)\ \mathbb{E}_t[\Lambda_{0,t}\{-w_t\} + \lambda_t F_{L,t}] = 0$$

$F_{K,t}$ and $F_{L,t}$ denote the derivatives of F with respect to K and L given period t inputs. Imposing a symmetric equilibrium (all intermediate prices are the same) and using $q_t = \frac{1}{G'_t}$:

$$(p_t)\ \mathbb{E}_t[-\frac{(1-\nu)}{\nu}\Lambda_{0,t}] = \mathbb{E}_t[\lambda_t]$$

$$(i_t)\ \mathbb{E}_t[-\Lambda_{0,t} + \frac{\mu_t}{q_t}] = 0$$

[42] A intermediate good producer's problem is static and the market for capital services is competitive. In addition, we assume a symmetric equilibrium so that the capital stock and capital and labor services are the same across firms. Therefore, we can combine the two problems without changing equilibrium allocations.

$$(k_t)\ \mathbb{E}_{t-1}[-\{\mu_{t-1}\}] + E_{t-1}[\lambda_t F_{K,t} + \mu_t(1-\delta+G(\frac{i_t}{k_t}) - \frac{\frac{i_t}{k_t}}{q_t}] = 0$$

$$(l_t)\ \mathbb{E}_t[\{-w_t\} - \frac{(1-\nu)}{\nu}F_{L,t}] = 0$$

Combining further and dropping the expectation operators where not necessary:

$$w_t = \frac{(\nu-1)}{\nu}F_{L,t}$$

$$\Lambda_{0,t}q_t = \mu_t$$

$$-\frac{(1-\nu)}{\nu}\Lambda_{0,t} = \lambda_t$$

$$q_{t-1} = \mathbb{E}_{t-1}[\Lambda_{t-1,t}[\frac{\nu-1}{\nu}F_{K,t} + q_t(1-\delta+G(\frac{i_t}{k_t})) - \frac{i_t}{k_t}]]$$

The last equation is the Euler equation for capital accumulation which we will target in the computation. The reader should notice the distortion terms $\frac{\nu-1}{\nu}$ for capital and labor compared to the "'standard"' Euler equation in a competitive model. The price of the final good in each period is set to one for simplicity. The total profits of the two types of firms combined can be seen to be (assuming a Cobb-Douglas production function) $F(k_t, Z_t l_t) * (\alpha + \frac{1-\alpha}{\nu}) - I_t$. It is easy to see that $\frac{1-\alpha}{\nu}F(k_t, Z_t l_t) - I_t$ is the share of the profit collected by the capital good producers while the remaining part of the $\frac{1}{\nu}F(k_t, Z_t l_t)$ is the profit of each intermediate good producer.

After these derivations it is easy to see now that the auxiliary planner's problem defined in the main text leads to the same Euler equation as the one of the firm if we impose symmetry. Given that also the resource constraints are the same we see that the symmetric equilibrium and the planner's problem result in the same allocations. If we solve the latter we can as usual use first order conditions to obtain prices.

B.3 State Space Selection for the Computation

In order to solve the model numerically using the planner's problem, we normalize all quantities that grow over time - consumption, investment and capital - and the value function in period t by Z_t. To be more specific, if X_t is the value of $X \in \{c, i, \bar{k}, k\}$ in period t, we define $\tilde{X}_t = \frac{X_t}{Z_t}$. Normalizing this way we get the following first-order conditions for the planner's problem, where we also normalize the multipliers accordingly.[43]

$$w_t = \frac{(\nu - 1)}{\nu} F_L(\tilde{K}_t, L_t)$$

$$\Lambda_{0,t} q_t = \mu_t$$

$$-\frac{(1-\nu)}{\nu}\tilde{\Lambda}_{0,t} = \tilde{\lambda}_t$$

$$q_{t-1} = E_{t-1}\tilde{\Lambda}_{t-1,t}[\frac{\nu - 1}{\nu} F_K + q_t(1 - \delta + G(\frac{\tilde{i}_t}{\tilde{k}_t})) - \frac{\tilde{i}_t}{\tilde{k}_t}]$$

$$\tilde{\Lambda}_{t,t+1} = \beta \left(\frac{\frac{Z_{t+1}}{Z_t}\tilde{c}_{t+1}}{\tilde{c}_t} \right)^{-\frac{1}{\psi}} \frac{\tilde{V}_{t+1}^{(1-\gamma)(1-\frac{1}{\eta})}}{\mathbb{E}_t[\tilde{V}_{t+1}^{1-\gamma}]^{1-\frac{1}{\eta}}}$$

$$\tilde{k}_{t+1}\frac{Z_{t+1}}{Z_t} = (1-\delta)\tilde{k}_t + G(\frac{\tilde{i}_t}{\tilde{k}_t})$$

$$\tilde{y}_t = \frac{1}{\nu}\tilde{\tilde{k}}_t^{\alpha} + \frac{\nu - 1}{\nu}\tilde{k}_t^{\alpha}$$

$$\tilde{V}_t = [(1-\beta)\tilde{c}_t^{\frac{1-\gamma}{\eta}} + \beta\mathbb{E}_t[\frac{Z_{t+1}}{Z_t}\tilde{V}_{t+1}^{1-\gamma}]^{\frac{1}{\eta}}]^{\frac{\eta}{1-\gamma}}.$$

It should be noted that Z_t drops out everywhere beside in the ratio $\Delta Z_{t+1} = \frac{Z_{t+1}}{Z_t}$. Furthermore, ΔZ_{t+1} factors out in the discount factor and the value function. It follows that Z_t does not influence the value of the normalized variables solving the equations beyond the effect it might have on the expectation for ΔZ_{t+1}. We therefore use the beliefs after updating using Z_t, g_t and $s_{2,t}$ as a state variable. This allows us to drop Z_t and g_t as states when solving the model in its normalized form. We have to incorporate the possible realizations of the shocks and beliefs into the transition probabilities only when taking expectations. We are left with both capital stocks $\tilde{\tilde{k}}_t, \tilde{k}_t$, updated beliefs $\hat{\mu}_t$ and the state of the volatility shock $s_{2,t}$. But once the Euler equation for capital was derived while correctly distinguishing the two capital stocks, we can impose $\tilde{K}_t = \tilde{k}_t$ without changing the results. This allows us to drop one capital stock in computing the solution, leaving us with two continuous and one discrete state variable for the purpose of numerically solving the normalized problem of the planner.

[43] Assuming F is Cobb-Douglas and of degree 1, F_L, F_K are of degree 0. It is also the case that $\frac{\tilde{i}_t}{\tilde{k}_t} = \frac{i_t}{k_t}$, which takes care of normalizing q.

C Computation - Projection

We follow Krueger and Kuebler (2003) by applying a Smolyak collocation method. Our system of state variables contain the capital stock in the beginning of a period and the belief about the state of the long run component of technology growth after observing the shock and the signal. In addition, we have a state variable taking two values for the variance, and we therefore use two sets of polynomials, one for the high and one for the low volatility regime, to approximate policy and value functions.[44] We outline the applied procedure in the following with references to more detailed descriptions. To solve for the coefficients of the polynomials, we use a time iteration procedure for the investment decision and the value function.

- Step 0: Define a tolerance (we used 10^{-5} but checked for the benchmark calibration that the results do not change significantly, if we use 10^{-6} instead). Select an upper and lower bound for capital and the belief to define the intervals the approximation should focus on. Compute the collocation points as described in Krueger and Kuebler (2003). For the baseline problem, we need to fit the investment policy function and the value function. As an initial guess, we use constant functions at the deterministic steady-state levels of these variables (we later used the approximations obtained from previous solutions as an initial guess). Denote this initial guess by $V_{p,i}^0$ and $I_{p,i}^0$ for $i = 1, 2$, where $i = 1$ is the low variance state and $i = 2$ the high variance state. Set n=1, Norm=10. Go to Step 1.

- Step 1: Enter iteration n. Calculate $I_{p,i}^n(x)$ by using the normalized Euler equation for capital of the planner imposing equilibrium conditions (especially $k = K$) in each point x of the grid and using $I_{p,i}^{n-1}$ to determine policy choices in the next period. For the computation of the expectation, see below. Go to step 2.

- Step 2: Use $I_{p,i}^n(x)$ to obtain $V_{p,i}^n(x)$ using the budget constraint and the definition of V and $V_{p,i}^{n-1}$ for the values in the next period. Go to step 3.

- Step 3: Use the interpolation rule as described in Krueger and Kuebler (2003) to obtain the approximation polynomials $V_{p,i}^n$ and $I_{p,i}^n$ for $i = 1, 2$. Go to step 4.

- Step 4: Compute Norm as the maximum norm of the difference of the values in the collocation for this and the previous approximation normalizing by the previous

[44] We refer the reader to the model appendix for the arguments on why these states suffice.

approximation. If the Norm is smaller than the tolerance end, otherwise set n=n+1 and go to Step 1.

After solving for the investment policy function, the value function and the law of motion for K, we compute Euler equation errors on a grid different from the one used to solve the model to check the quality of the approximation. For taking the expectation, we used that conditional on the beliefs and the realization of the volatility state the distribution of productivity growth shocks is normal and applied a five point Gauss Hermite Quadrature formula ((Judd, 1998) pages 261-263). We also checked with more points resulting only in very mild changes in the results. To obtain the full approximation, we use the transition matrix for the stochastic volatility and Bayes' rule to update beliefs given technology growth shocks and signals. In practice we use an accelerator method as described in Judd (1998) to speed up the computation.

Finally, we also need to compute asset pricing functions. In order to do this, we follow the same procedure as before for investment and value functions. We fix those solutions and then iterate on the price using the Euler equations for the specific asset we want to value until convergence.