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Abstract

We develop a new dynamic general equilibrium model to explain firm entry, exit, and relocation decisions in an urban economy with multiple locations and agglomeration externalities. We characterize the stationary distribution of firms that arises in equilibrium. We estimate the parameters of the model using a method of moments estimator. Using unique panel data collected by Dun and Bradstreet, we find that our model fits the moments used in estimation as well as a set of moments that we use for model validation. Agglomeration externalities increase the productivity of firms by about 8 percent. Economic policies that subsidize firm relocations to the central business district increase agglomeration externalities in that area. They also increase economic welfare in the urban economy.

JEL Classifications: R30, R13, L11, C51

KEYWORDS: Agglomeration Externalities, Firm Dynamics, Firm Sorting, Urban Economy, General Equilibrium, Estimation, Empirical Analysis.

1 Introduction

A key insight of Marshall (1920) is that geographic proximity of economic activity increases efficiency in production and trade. Over the past several decades, research has formalized this idea and developed general equilibrium models to study the impact of agglomeration externalities on firm choices and economic welfare in an urban economy. These models provide strong predictions about firm dynamics as well as entry, exit, and relocation decisions.[1] While there is some empirical evidence supporting the basic modeling assumptions, previous empirical papers have not structurally estimated these types of models. The main contribution of this paper is to provide an integrated approach for estimating a dynamic equilibrium model of firm locational choices in an urban economy.[2] We show that this approach provides new empirical insights into the sorting of firms within large metropolitan areas.

We consider a new model of an urban economy with two distinct locations that endogenously differ in the magnitude of their agglomeration externalities. Following Lucas and Rossi-Hansberg (2002), agglomeration externalities are a function of local employment density and, therefore, depend on firm sorting. We model firm dynamics and industry equilibrium as suggested by Hopenhayn (1992). Firms enter the urban economy with an initial productivity and must pay an entry cost. Productivity then

[1] Henderson (1974) formalized Marshall's idea in a Muth-Mills type equilibrium model. Krugman (1991) provided theoretical foundations for a two-location model of agglomeration in the presence of small transportation costs. Anas and Kim (1996) and Lucas and Rossi-Hansberg (2002) have developed equilibrium models of mono- and poly-centric urban land use with endogenous congestion and job agglomeration. Rossi-Hansberg (2004) studies optimal land use policies. Duranton and Puga (2001) focus on the effect of agglomeration externalities in innovation and the development of production processes. The literature of agglomeration theory is reviewed in Fujita and Thisse (2002) and Duranton and Puga (2004).

[2] From a purely methodological perspective, our paper is related to Davis, Fisher, and Whited (2011), who develop a growth model in which the total factor productivity of cities depends on the density of economic activity. They estimate the magnitude of this external effect and evaluate its importance for the growth rate of consumption per capita in the U.S. Similarly, Holmes (2011) estimates a dynamic model to study the expansion of Walmart and to quantify the importance of geographic proximity in designing distribution networks.

evolves according to a stochastic first-order Markov process.[3] Each period, firms compete in the product market, must pay a fixed cost of operating, and realize a profit.[4] Agglomeration externalities affect firm dynamics and the sorting of firms in at least three important ways. First, entry patterns depend on local land rental rates and location-specific externalities. This gives rise to an initial sorting of new firms. Second, the productivity of firms changes over time which implies that growth trajectories will differ by location, thus creating incentives to relocate within a city to exploit a better match with the agglomeration externalities. Finally, the continuation value for a firm is location specific, which implies that exit rates depend on location. We characterize the stationary equilibrium of the urban economy. While we can establish some properties of stationary equilibria, analytical solutions of equilibria do not exist. We develop an algorithm that can be used to compute equilibria. Our computational analysis suggests that equilibria are locally unique; that is, two different equilibria will have distinctly different local land rents.

We develop a new algorithm that can be used to estimate the parameters of our model. We show that a subset of the structural parameters of the model can be estimated using the observed input and output choices without solving for the equilibrium of the model. The remaining parameters of the model, which include the cost parameters, affect the equilibrium selection rules and can be estimated using a nested fixed point algorithm. The estimator is a simulated method of moments estimator that matches selected moments characterizing entry, exit and relocation of firms within the metropolitan area. When computing equilibria during estimation, we condition on the observed equilibrium land rental prices in both locations. Since equilibrium is locally unique, this approach allows us to avoid potential multiplicity

[3]Also related to our research is work by Rossi-Hansberg and Wright (2007), who examine the relationship of establishment scale and entry and exit dynamics.

[4]Our data set does not include any measure of product quality or investment in research and development. We, therefore, abstract from innovation, which is discussed in detail in Klette and Kortum (2004) and Duranton and Puga (2004).

of equilibria problems in estimation.

Our empirical analysis is based on a unique panel data collected by Dun and Bradstreet. Large U.S. cities often act as a hub for service sector industries for a larger region. We, therefore, focus on locational choices within the service sector, excluding industries in which proximity to the consumer is a key factor for firm location. Our analysis reveals a number of important empirical regularities that characterize firm sorting within metropolitan areas.[5] Firms located in the central business district (CBD) are older and larger than firms located outside the CBD. They use more land and labor in the production process. However, they face higher rental rates for office space, which implies that they operate with a higher employee per land ratio. Firms entering or exiting the city center are typically larger than firms outside the CBD. Firms that relocate to the CBD tend to be larger firms that have grown in the recent past. These facts are not specific to Pittsburgh, which is our main empirical application, but hold for most U.S. metropolitan areas. Our model is broadly consistent with these observations.

We implement our estimator for a variety of different model specifications. We find that our model fits the moments used in estimation as well as a set of moments that we use for model validation. Our estimates imply that agglomeration externalities increase the productivity of firms by 8 percent. Economic policies that subsidize firm

[5]Most previous empirical studies have focused on sorting across cities or metropolitan areas. For example, Ellison and Glaeser (1997) argued that agglomeration externalities are important to understanding the geographic concentration of manufacturing in the U.S. Deckle and Eaton (1993) find that the geographic scale of agglomeration is mostly at the national level, while the financial sector is concentrated in specific metropolitan areas. Combes, Duranton, Gobillon, Puga, and Roux (2010) distinguish between selection effects and productivity externalities by estimating productivity distributions across cities. In contrast we focus on sorting within a metropolitan area, which is more consistent with the notion that agglomeration occurs on a local scale. This is consistent with findings by Rosenthal and Strange (2001, 2003). They report the level and type of agglomeration at different geographic scales, and also measure the attenuation of these externalities within metropolitan areas. Holmes and Stevens (2002) also find evidence of differences in plant scale in areas of high concentration, suggesting that production externalities act on individual establishments.

relocation to the CBD increase employment concentration in that area. The welfare gains in the CBD are typically larger than the losses outside the CBD, leading to an increase in overall welfare.

The rest of the paper is organized as follows. Section 2 describes the data set used in our empirical analysis and characterizes firm sorting within a metropolitan area. Section 3 develops our stochastic dynamic equilibrium model and discusses its properties. Section 4 describes identification and estimation of the parameters of our model. Section 5 presents the estimation results. Section 6 discusses the policy implications of our analysis, while Section 7 offers some conclusions that can be drawn from the analysis.

2 Data and Empirical Regularities

Our empirical application primarily focuses on firm location choices in Allegheny County, the second most populous county in Pennsylvania, and the nucleus of the Pittsburgh metropolitan area. In Appendix A of the paper we show that most other large metropolitan areas in the U.S. show sorting patterns of establishments that are similar to the one we observe in Pittsburgh.[6] We are interested in characterizing the observed sorting of establishments, as well as entry, relocation and exit rates.[7] For this purpose we use establishment-level data from Dun and Bradstreet's Million Dollar Database.[8] This database provides detailed information on establishments in Allegheny County in two years, 2008 and 2011. The coverage is nearly universal

[6] That comparison is based on aggregate census data, while the estimation of our model uses micro level data from Dun and Bradstreet.

[7] While we use the terms "firm" and "establishment" interchangeably, our unit of analysis in the empirical section is an establishment.

[8] Information on Dun and Bradstreet data is available on-line at http://www.dnbmdd.com/. An appendix is also available upon request, which discusses in detail how our data set was constructed from the raw data.

compared to census counts of establishments in the county. The database provides data on location, facility size, total employment, industry, year established, and sales.

We focus on service industries, given that there is strong evidence that U.S. cities have undergone a transformation during the past decades, moving from being centers of individual manufacturing sectors toward becoming hubs for service industries.[9]

We exclude wholesale and retail businesses from our analysis of services, since the locational decisions of these businesses are primarily driven by proximity to consumers (Hotelling, 1929).[10] For similar reasons, we also do not consider businesses in the entertainment sector. Finally, we omit businesses related to agriculture, forestry, mining, and fishing for fairly obvious reasons. We, therefore, define the service sector as consisting of businesses that operate in information technology, finance, real estate, professional services, management, administrative support, education, health care and related sectors. These sectors account for 51 percent of employment in Allegheny County.

In order to characterize the sorting patterns of establishments we define the central business district of our geographic area as the three zip codes in the center of Pittsburgh. These include the downtown central business district and the business district in Oakland, the two significant dense commercial areas of Pittsburgh.[11] The CBD, so defined, accounts for over 20 percent of total employment, about 28 percent of service sector employment, and 13 percent of establishments, but less than 1 percent of all the land in Allegheny County. The places in Allegheny County outside

[9]Duranton and Puga (2005), for example, show evidence that cities have become more functionally specialized, with larger cities, in particular, emerging as centers for headquarters and business services. They posit that this change is primarily related to industrial structure and a decrease in remote management costs in particular. Davis and Henderson (2008) provide further evidence that services and headquarters are indeed more concentrated in large cities relative to the entire economy and that headquarter concentration is linked to the availability of diverse services.

[10]Following Bresnahan and Reiss (1991), there is a large literature that explains entry and exit into markets with a small number of potential entrants.

[11]The zip codes are 15222, 15219, and 15213.

the CBD are treated in what follows as the alternate location, denoted by oCBD (mnemonic for outside the CBD).[12]

Comparing firms that are located inside the CBD with firms that are outside the CBD, we find some important patterns that hold for all service industries. Table 1 reports the total employment, the average employment and the facility space per employee for firms inside and outside the CBD for selected service industries in 2011. Notice that finance, education, and professional services are the industries that are most heavily concentrated in the CBD.

Table 1: Employment and Facility Size by Industry in 2011

NAICS	Total Emp	% Emp CBD	Emp		Facility	
			oCBD	CBD	oCBD	CBD
Information (51)	8,487	23.8	8.4	18.2	613	428
Finance (52)	36,420	45.6	8.3	49.1	412	166
Real Estate (53)	13,749	21.9	4.5	12.1	1,290	1,207
Prof. Services (54)	52,521	30.5	6.7	10.8	420	355
Management (55)	995	13.1	5.1	8.1	499	345
Admin. Support (56)	31,613	11.9	5.1	14.0	551	304
Education (61)	23,328	63.4	19.9	224.0	458	106
Health Care (62)	98,144	18.0	14.7	25.2	387	357
Total	265,257	27.9	8.5	22.9	482	297

Notes: "Emp" denotes average employment. "Facility" is average facility size measured in square feet per employee. Source: Authors' calculations based on Dun and Bradstreet's data set.

Establishments' size. The average employment size of establishments is larger in the CBD. The average establishment in the CBD employs about 23 persons, while the average firm outside the CBD has fewer than 9 employees.

[12] None of the results reported in this paper rely on this definition of the alternate location. We can, for example, omit those parts of Allegheny County that have little economic development and obtain similar results regarding firm sorting.

Land use. Firms located in the CBD use less office space per employee; that is, office space per employee is 297 square feet in the CBD compared to 482 outside the CBD. This is partially due to the fact that rents for office space are approximately 12 percent higher in the CBD.

To get some additional insights into the firm sorting process, we analyze the full distribution of firms by location. Table 2 reports a number of percentiles of the age, facility size, employment size, and revenue distribution by location. Table 2 reveals two other important features of firms sorting across locations.

Age. Firms in the CBD tend to be older. The 75th percentile of the age distribution in the CBD is 26 years while it is 21 years outside the CBD.

Revenue per employee. Firms in the CBD have a higher revenue per employee. The latter fact is consistent with the notion that firms in the CBD may have higher productivity levels than firms located outside the CBD.

Table 2 also shows that there are significant differences among firms in the right tail of the distribution. Looking at the 90th and higher percentiles, we find large differences between firms inside and outside the CBD.

One potential concern of the analysis above is that differences between firms located inside and outside the CBD may be due to aggregation among different industries. In the lower panels of Table 2, we, therefore, report the same statistics for two industries. The middle panel reports the results for the information technology sector which is an "average" service industry in terms of its concentration of employment in the CBD. The lower panel reports the statistics for the finance industry which is the most heavily concentrated industry in our sample. We find that the qualitative differences between firms located inside and outside the CBD are not driven by aggregation across firms in the different service industries. If anything, the differences in the financial service industry are more pronounced than the differences in the sample

Table 2: Establishment Characteristics by Location and Industry in 2011

All Service Industries (NAICS 51-62)								
	CBD				oCBD			
Percentile	Age	Emp	Facility	Rev/Emp	Age	Emp	Facility	Rev/Emp
10th	2	2	1,432	47,481	2	1	1,565	40,000
25th	5	2	1,873	60,000	4	2	2,119	50,000
50th	13	3	2,499	70,000	10	2	2,474	64,000
75th	26	9	4,200	95,000	21	4	3,471	84,000
90th	42	28	8,470	140,000	34	11	5,276	116,077
95th	57	51	14,625	265,337	44	23	8,228	164,550
99th	108	288	53,563	890,257	76	99	22,841	495,803
Professional, Scientific, and Technical (NAICS 54)								
	CBD				oCBD			
Percentile	Age	Emp	Facility	Rev/Emp	Age	Emp	Facility	Rev/Emp
10th	2	1	1,432	55,000	3	1	1,227	46,105
25th	5	2	1,438	63,333	7	1	1,678	60,000
50th	12	2	2,184	75,000	13	2	2,184	75,000
75th	24	7	3,200	95,000	22	4	2,588	95,000
90th	40	25	6,459	127,589	33	10	4,000	125,000
95th	52	45	10,994	145,020	40	20	5,794	150,000
99th	88	138	33,348	342,633	62	74	15,982	400,000
Financial Services (NAICS 52)								
	CBD				oCBD			
Percentile	Age	Emp	Facility	Rev/Emp	Age	Emp	Facility	Rev/Emp
10th	2	2	1,638	60,000	2	2	1481	55,000
25th	5	2	2,159	80,000	3	2	2,041	70,000
50th	13	5	2,940	106,666	10	2	2,499	90,000
75th	29	13	5,000	283,779	23	5	3,106	120,000
90th	52	40	10,019	495,803	42	10	5,160	283,779
95th	77	114	15,023	874,940	59	20	6,628	316,056
99th	151	106	109,411	2,010,160	94	58	16,660	874,940

Notes: "Emp" is average employment, "Facility" is average facility size measured in square feet, "Rev/Emp" is revenue per employee. Source: Authors' calculations based on Dun and Bradstreet data.

of all service industries.

We observe firms at two points in time, 2008 and 2011. Table 3 reports firm-level characteristics for 2008 as well as 2011. Notice that the average firm characteristics are similar in both time periods.

Table 3: Establishment Characteristics in 2008 and 2011.

	2008	2011
Establishments count CBD	3,590	3,231
Establishments count oCBD	20,546	22,495
Average Employment CBD	23.77	22.93
Average Employment oCBD	9.32	8.50
Average Facility Size CBD	6,556	6,818
Average Facility Size oCBD	3,421	4,097
Source: Authors' calculations based on Dun and Bradstreet data set.		

Finally, we characterize differences in entry, exit, and relocation rates by location. Here we exploit the panel structure of our data set. Dun and Bradstreet collect detailed information about names, addresses and age of the firm. In addition, establishments are matched across time periods using the D-U-N-S number, which remains assigned to an establishment regardless of location.[13] Entrants were determined as establishments that were present in 2011 but not in 2008. Exits were calculated similarly. Also shown are measures of total movement (where the zip code of the establishment changed) as well as movement to and from the CBD. Table 4 reveals four more empirical regularities.

Entry and exit rates. They are of similar magnitudes in the CBD and outside.

[13]See Neumark, Zhang, and Wall (2007) for a discussion of this matching method, as well as a more general discussion of the Dun and Bradstreet data set.

Table 4: Entry, Exit and Relocation

Establishments that:	Count	Percent**	Average Employment	
			2008	2011
Relocate*	554	2.2	43.7	49.1
Move from CBD to oCBD	84	0.33	12.1	8.2
Move from oCBD to CBD	57	0.22	15.7	20.1
Entry in the CBD	743	2.9	n.a.	7.1
Entry oCBD	7,982	31.0	n.a.	4.1
Exit from CBD	1,075	4.2	17.6	n.a.
Exit from oCBD	6,060	23.6	6.2	n.a.

Notes: *All establishments that changed zip code between 2008 and 2011.
**Percent out of total number of establishments in 2011.
Source: Authors' calculations based on Dun and Bradstreet data set.

Average employment of new entrants. New establishments in the CBD are larger than new establishments outside the CBD. Entrants in the CBD hire on average 7.1 employees, while entrants outside the CBD employ 4.1 workers.

Average employment of exiting firms. Firms that exit the CBD are much larger than firms that exit outside the CBD. The average employment of exiting firms in the CBD is 17.6 compared to 6.2 for firms outside the CBD.

Relocation. There is substantial relocation measured at the zip code level; that is, approximately 2.2 percent of all firms relocate during the three-year period. Firms that relocate to the CBD are expanding, while firms relocating to outside the CBD are shrinking.

3 A Dynamic Model of Firm Location Choices

We develop a new dynamic general equilibrium model of firm location choice that can explain the empirical regularities described in the previous section. We consider a model with two distinct locations, denoted by $j = 1, 2$, which can be interpreted as inside ($j = 1$) and outside ($j = 2$) a CBD.

3.1 Technologies and Markets

There is a continuum of firms that produce a single output good and compete in a product market. In each period, a firm chooses to stay where it is, relocate to the other location, or shut down. Firms are heterogeneous and productivity evolves according to a stochastic law of motion.

Assumption 1 *In each period, a firm is subject to an exogenous probability of exiting. We denote by ξ the complement probability of a firm surviving into the next period. If the firm survives, it draws a new productivity shock, φ', each time period. The productivity shock evolves over time according to a Markov process with a conditional distribution $F(\varphi'|\varphi)$. A firm located in j also faces an exogenous probability λ_j of relocating to the other location.*

Each firm produces a single output good using labor, land and capital as input factors.[14] The technology that is available to the firms in the economy satisfies the following assumption.

[14] Note that for convenience we use the word "land" in the description of the model. In the empirical analysis we use data on establishments' facility size as the empirical counterpart of this input.

Assumption 2 *The production function of a firm in location j can be written as:*

$$y = q(\varphi, n, l, k; e_j) \tag{1}$$

where y is output, n is labor, l is land, k is capital, and e_j is the agglomeration externality in location j. The production function satisfies standard regularity conditions.

The agglomeration externality arises due to a concentration of employees operating in the same location (Lucas and Rossi-Hansberg (2002)).

Assumption 3 *The agglomeration externality can be written as*

$$e_j = e_j(N_j/L_j) \tag{2}$$

where N_j and L_j are aggregate measures of labor and land, respectively. The function e_j is increasing in its argument.

The urban economy is part of a larger economic system that determines output prices and wages.[15]

Assumption 4 *The output price, p, the capital rental price, p_k, and the wage, w, are constant over time, independent of location, and determined exogenously.*

Rental prices for land, r_j, however, are equilibrium outcomes. The supply of land is determined by an inverse land supply function in each location.

Assumption 5 *The inverse land supply function is given by:*

$$r_j = r_j(L_j), \quad j = 1, 2 \tag{3}$$

[15]It is straightforward to endogenize wages and output prices by adding a local labor market and an aggregate demand for output to our model.

which is increasing in the aggregate amount of land used in j.

Since rental prices for land must be higher in equilibrium in locations with high externalities, the agglomeration externality is, at least, partially capitalized in land rents.

We can break down the decision problem of a firm into a static and a dynamic problem. Consider the static part first. This problem arises because firms compete in the product market each period.[16]

Assumption 6 *The product market is competitive and firms behave as price takers. Firms make decisions on land, labor, and capital usage after they have observed their productivity shock, φ, for that period.*

Let π_j denote a firm's one period profit in location j. The static profit maximization problem can be written as:

$$\max_{\{n,l,k\}} \pi_j\left(n,l,k;\varphi\right), \tag{4}$$

where the profit function is given by:

$$\pi_j\left(n,l,k;\varphi\right) = p\; q\left(\varphi,n,l,k;e_j\right) - w\; n - p_k\; k - r_j\; l - f_j. \tag{5}$$

The parameter f_j denotes a fixed cost of operation that may depend on location. Solving this problem, we obtain the demand for inputs as a function of φ, denoted by $n_j(\varphi)$, $l_j(\varphi)$ and $k_j(\varphi)$, as well as an indirect profit function, denoted by $\pi_j(\varphi)$.[17]

[16] We abstract from oligopolistic competition, which is studied, for example, in Ericson and Pakes (1995). That framework is more appropriate when there are few competitors in the industry. Pakes and McGuire (1994, 2001) discuss how to solve models with oligopolistic competition. Doraszelski and Pakes (2006) provide a survey of that literature.

[17] Note that the sub-index j summarizes the dependence of the profit and input demand functions on location j's rent and externality.

Let μ_j denote the measure of firms located in j. Given the static choices for land and labor use for each firm, we can use this measure to calculate the aggregate demand for land and labor:

$$L_j = \int l_j(\varphi)\mu_j(d\varphi), \tag{6}$$

$$N_j = \int n_j(\varphi)\mu_j(d\varphi). \tag{7}$$

Notice that the agglomeration effect e_j in equation (2) depends on the ratio of these two quantities.

Consider now the dynamic aspect of a firm's problem. The following Bellman equations formalize the decision problem of a firm that begins the period in location 1 or 2 with a productivity shock φ:

$$\begin{aligned} V_1(\varphi) &= \pi_1(\varphi) + \beta\xi(1-\lambda_1)\max\left\{0, \int V_1(\varphi')F(d\varphi'|\varphi), \int V_2(\varphi')F(d\varphi'|\varphi) - c_r\right\} + \\ &\quad \beta\xi\lambda_1\left\{\int V_2(\varphi')F(d\varphi'|\varphi) - c_r\right\} \qquad (8) \\ V_2(\varphi) &= \pi_2(\varphi) + \beta\xi(1-\lambda_2)\max\left\{0, \int V_2(\varphi')F(d\varphi'|\varphi), \int V_1(\varphi')F(d\varphi'|\varphi) - c_r\right\} + \\ &\quad \beta\xi\lambda_2\left\{\int V_1(\varphi')F(d\varphi'|\varphi) - c_r\right\} \end{aligned}$$

where β is the discount factor, and c_r is the cost of relocating from one location to another. The first term on the right-hand side of the Bellman equations represents the flow of profits from operating in the location; the second term reflects the choice that each firm faces at the end of each period among three alternative choices: exit and get a liquidation value normalized to zero; continue operating in the current location; move to the other location and continue operations there. The third term on the right-hand side of each Bellman equation represents the value associated with being exogenously relocated.

Solving the dynamic decision problem above implies decision rules of the following form for firms currently in location j:

$$x_j(\varphi) = \begin{cases} 0 \text{ if the firm exits at the end of the period} \\ 1 \text{ if the firm chooses location 1 at the end of the period} \\ 2 \text{ if the firm chooses location 2 at the end of the period} \end{cases} \tag{9}$$

To close the model, we need to specify the process of entry.

Assumption 7 *Firms are free to enter in both locations. All prospective entrants are ex-ante identical. Upon entering, a new firm incurs a cost c_j and draws a productivity shock φ from a distribution $\nu_j(\varphi)$.*

Note that we allow the entry cost to vary by location. This assumption guarantees that the expected discounted profits of a prospective firm are always less than or equal to the entry cost:

$$c_j \geq \int V_j(\varphi)\nu_j(d\varphi), \quad j = 1, 2 \tag{10}$$

If there is positive entry of firms, then this condition holds with equality.

We focus on a stationary economy in which the measures of firms μ_j are time-invariant. Formally, (μ_1, μ_2) must satisfy the following recursive equations:

$$\begin{aligned} \int_0^{\varphi'} \mu_1(dz) = {} & \xi\,(1-\lambda_1) \int F(\varphi'|z) 1\{x_1(z)=1\}\,\mu_1(dz) \\ & +\xi\,(1-\lambda_2) \int F(\varphi'|z) 1\{x_2(z)=1\}\,\mu_2(dz) \\ & +\xi\lambda_2 \int F(\varphi'|z)\mu_2(dz) + M_1 \int_0^{\varphi'} \nu_1(dz) \end{aligned} \tag{11}$$

and

$$\int_0^{\varphi'} \mu_2(dz) = \xi\left(1-\lambda_1\right)\int F(\varphi'|z)1\left\{x_1(z)=2\right\}\mu_1(dz) \tag{12}$$
$$+\xi\left(1-\lambda_2\right)\int F(\varphi'|z)1\left\{x_2(z)=2\right\}\mu_2(dz)$$
$$+\xi\lambda_1\int F(\varphi'|z)\mu_1(dz)+M_2\int_0^{\varphi'}\nu_2(dz)$$

where $1\{x_j(\varphi)=l\}$ is an indicator function equal to 1 if x_j equals l and 0 otherwise and M_j denotes the measure of firms entering in location j.

3.2 Equilibrium

We are now in a position to define a stationary equilibrium for our economy.

Definition 1 *A stationary equilibrium for this economy consists of rents, r_j^* , externalities, e_j^*, masses of entrants, M_j^*, aggregate quantities of labor N_j^* and land L_j^*, stationary distributions of firms, $\mu_j^*(\varphi)$, land, labor, and capital demand functions, $l_j^*(\varphi)$, $n_j^*(\varphi)$, and $k_j^*(\varphi)$ respectively, value functions, $V_j^*(\varphi)$, and decision rules, $x_j^*(\varphi)$, for each location $j = 1, 2$, such that:*

1. *The demand functions for labor, land, and capital inputs solve the firm's static problem in (4).*
2. *The decision rule (9) for a firm's location is optimal, in the sense that it maximizes the right-hand side of equations (8).*
3. *The free entry conditions (10) are satisfied in each location, with equality if and only if $M_j^* > 0$.*
4. *The aggregate quantities of labor and land are given by equations (6) and (7);*

5. *The market for land clears in each location consistent with equation (3).*

6. *The externalities are consistent with (2).*

7. *The distributions of firms μ_j^* satisfy equations (11) and (12).*

3.3 Discussion

Any stationary equilibrium of our model can be characterized by a vector of equilibrium values for rents, masses of entrants, and externalities in each location: $(r_1, r_2, M_1, M_2, e_1, e_2)$. Finding an equilibrium for this model is equivalent to the problem of finding the root of a nonlinear system of six equations. For any vector $(r_1, r_2, M_1, M_2, e_1, e_2)$, we can:

1. Solve the firms' static profit maximization problem and obtain the indirect profit functions for each location;

2. Solve the dynamic programming problem in equations (8) and obtain the optimal decision rules for exit, entry, and relocation;

3. Iterate on equations (11) and (12) to obtain the stationary distributions of firms, μ_j;

4. Calculate the aggregate land and labor demands using equations (6) and (7);

5. Check whether the following six equations are satisfied: the two land market clearing conditions (equation 3), the two free-entry conditions (equation 10), and the two equations that define the externalities in each location (equation 2).

If the equilibrium conditions are not satisfied, the vector of scalars can be updated and the process repeated until all of the conditions for equilibrium are satisfied. If this algorithm converges, an equilibrium of the model has been computed.[18]

Note that the mapping described above is not a simple contraction mapping. As a consequence we cannot apply standard techniques to prove the existence of equilibrium. In Appendix C, we provide a proof of the existence of equilibrium for a simplified version of our model in which firm productivity remains constant over time upon entry. Moreover, we have computed equilibria for a large number of different specifications of our general model. We, therefore, conclude that equilibria exist for reasonable parameterizations of the model.

With respect to the uniqueness of the stationary equilibrium, there are four issues. First, an equilibrium typically exists with communities that are ex-post identical if the land supply functions are the same across locations. These "non-sorting" equilibria are not interesting and are easily rejected empirically. We analyze sorting equilibria here. Second, the non-convexities in the model associated with community choice preclude the use of standard techniques to establish the uniqueness of sorting equilibria. Third, entry conditions may not hold with equality, which can give rise to equilibria with entry in only one of the two locations. In the quantitative analysis, we only focus on equilibria with entry in both locations. Last, the endogenous firm productivity distributions in a stationary equilibrium may not be unique. While there are several sources of potential multiplicity, we find in our computational analysis that stationary (sorting) equilibria are robust. When we perturb an equilibrium that we have computed, the algorithm converges back to the original equilibrium. These computational experiments suggest that equilibrium is, at least, locally unique.[19]

[18] In Section 4.1 we adopt a number of fairly standard functional forms for the firms' production function and the agglomeration externality that allow us to express each agglomeration effect as a function of the land rent in the same location. This allows us to simplify the computation of equilibrium, as described in detail in Appendix B.

[19] We do not have a formal proof of local uniqueness of the sorting equilibrium.

Characterizing the properties of the equilibrium requires additional assumptions, and additional insights can be gained only by using numerical methods. Before describing the estimation of the model's parameters it is useful to discuss the model's potential in accounting for the empirical regularities documented in Section 2.

In the Dun and Bradstreet data, establishments located in the CBD are on average larger (in terms of employment) than establishments located outside the CBD. There are two types of mechanisms within the model that help to account for this fact. First, stronger agglomeration effects (i.e., $e_1^* > e_2^*$) in the CBD will contribute to increased productivity and size there. The second mechanism is firms' selection. In turn, selection comes in two flavors. On the one hand, the entrants' distribution v_j is allowed to vary across locations, so that new establishments that enter in the CBD might be larger from the beginning of their lives, as the data suggest. On the other hand, relocation to the CBD can also play a role if relocating firms are drawn from the right tail of the productivity distribution of firms outside the CBD. Relocation from outside the CBD to the CBD can also explain why in the data firms in the CBD tend to be older than firms outside the CBD.

In the data, firms in the CBD economize on the use of land, in the sense that they have higher employee to facility size ratios on average. In the model firms optimally choose to use less facility space per employee in the CBD if the equilibrium rent is higher there, i.e., $r_1^* > r_2^*$.

Firms located in the CBD are on average older than firms located outside the CBD. Relocation plays an important role in accounting for this fact as the largest firms outside the CBD also tend to be the oldest firms there, and these firms choose to relocate to the CBD.

Finally, the model can account for the fact that exiting firms in the CBD tend to be larger than exiting firms outside the CBD. The model explains this fact through the exogenous exit of firms. Notice that while the exogenous exit rate ξ is independent of location, if on average firms in the CBD are larger than those outside the CBD, the average exiting firm in the CBD is also larger than its counterpart outside the CBD.

In order to gain additional intuition about the workings of the model, it is useful to consider an example that is broadly consistent with the parameter estimates reported. We plot the optimal decision rules and the stationary distribution of incumbents and entrants in Figure 1. To make the picture clearer we rescaled all the measures of establishments in the top panel so that they have unit mass.

The lower panel of Figure 1 plots the optimal decision rules. The equilibrium implies that firms with high productivity shocks relocate to the CBD, while low productivity firms leave the CBD to operate in cheaper locations.

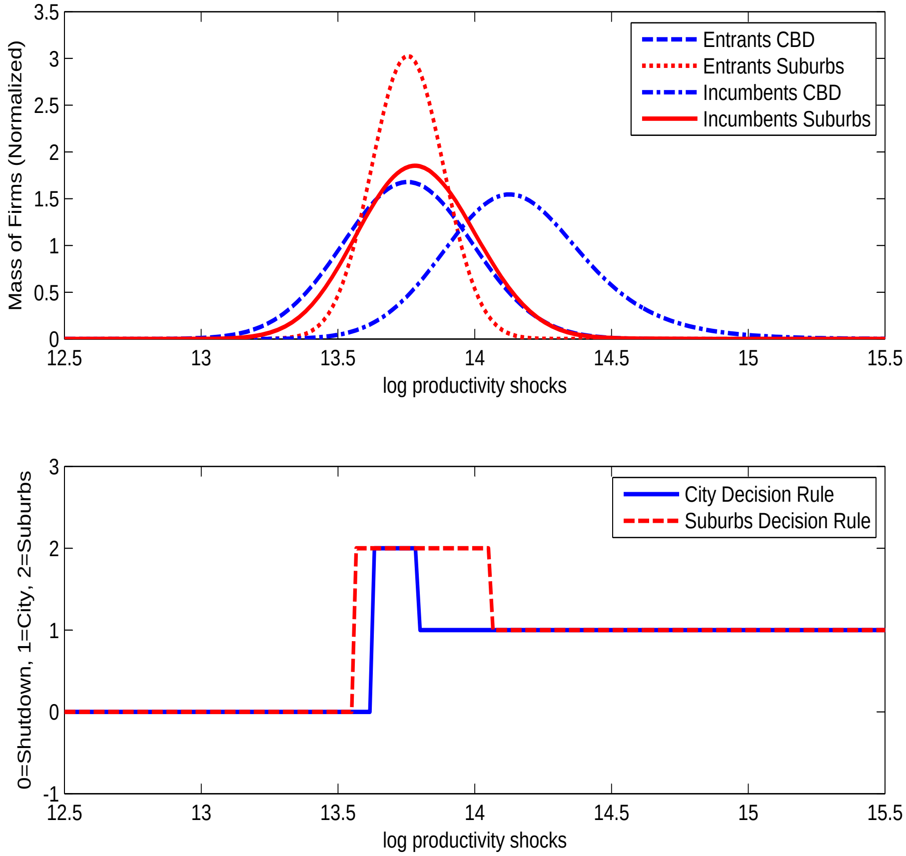
4 Identification and Estimation

This section describes the estimation procedure. We start by making a number of functional form assumptions described in Section 4.1 and then describe the identification and estimation of the model's parameters in Sections 4.2 and 4.3.

4.1 Parameterization of the Model

In our parameterized model, we assume that the logarithm of the productivity shock follows an autoregressive process of order one, i.e., $\log(\varphi)' = \rho \log(\varphi) + \varepsilon'$, where ρ is the autocorrelation coefficient and ε' is a normally distributed random variable with mean μ_ϵ and variance σ_ϵ^2 . The entrants' distributions ν_j of productivity shocks are assumed to be both lognormal with the same location parameter μ_e and potentially location-dependent scale parameters σ_{ej}^2. The dependence of σ_{ej}^2 on j gives us enough

Figure 1: Stationary Distributions and Decision Rules



flexibility to be able to match the mean and variance of the employment distribution of entrants in both locations.

Rosenthal and Strange (2003) suggest that the externality acts as a multiplier on the production function. In our computational model we use a standard Cobb-Douglas function: $y = \varphi e_j n^\alpha l^\gamma k^\eta$. The externality is assumed to be an iso-elastic function of employment density in a location: $e_j = (N_j/L_j)^\theta$, with $\theta > 0$.

The land inverse supply function in each location is also iso-elastic: $r_j = A_j L_j^\delta$, where A_j and δ are parameters. Notice that the elasticity parameter δ is the same across locations, while differences in the scale parameter A_j allow us to capture differences in the scale of economic activity in and outside the CBD.

Our strategy for identification and estimation of the model's parameters involves two steps. First, we show that a subset of parameters can be identified and estimated without computing the equilibrium of the model. Second, we construct a nested fixed point algorithm to estimate the remaining parameters.

4.2 Parameters Identified Without Solving the Model

Consider first the subset of parameters that can be identified and estimated without solving the model. They are the production function parameters (α, γ), the externality parameter θ, the parameters of the law of motion for productivity $(\mu_\epsilon, \sigma_\epsilon, \rho)$, and the parameters of the entrants' productivity distributions $(\mu_e, \sigma_{e1}, \sigma_{e2})$. We make the following assumption about variables that are observed by the econometrician and that allow us to identify these parameters.[20]

[20] We do not observe any measures of the capital stock in the Dun and Bradstreet data. We restrict the capital share so that the sum of capital and land shares amount to half of the labor share: $\eta + \gamma = 0.5\alpha$. This is consistent with the evidence reported in Valentinyi and Herrendorf (2008).

Assumption 8

1. *w, r_1 and r_2 are observed without error.*

2. *We observe for a large random sample of firms output, labor and land inputs as well as locational choices.*

3. *We observe entry, exit and relocation decisions of all firms in the sample.*

4. *Output is measured with error, i.e., we observe*

$$\tilde{y}_{ij} = y_{ij} - u_{ij} \tag{13}$$

where y_{ij} denote true output of firm i in location j, $\tilde{y}_{ij}$ denotes observed output, and the error term satisfies $E[u_{ij}] = 0$.

5. *A model period corresponds to three years in the data.*

Given the Cobb-Douglas production function, the first order conditions for optimal labor demand imply the standard factor share results:

$$\alpha \tilde{y}_{ij} - w n_{ij} = \alpha u_{ij} \qquad j = 1, 2. \tag{14}$$

The assumption that $E[u_{ij}] = 0$ then identifies α. Similarly, the land input satisfies:

$$\gamma \tilde{y}_{ij} - r_j l_{ij} = \gamma u_{ij} \qquad j = 1, 2 \tag{15}$$

and γ is, therefore, identified as well.

To show that the parameters of the distribution of productivity of entrants are identified, note that the log labor demand equation is given by

$$\ln(n_{ij}) = \frac{1}{1 - \alpha - \gamma - \eta} \left(\chi_j(\theta) + \ln(\varphi_{ij})\right) \qquad j = 1, 2 \tag{16}$$

where the term $\chi_j(\theta)$ is known up to the externality parameter θ:[21]

$$\chi_j(\theta) = (\theta - \gamma)\ln\left(\frac{r_j}{\gamma}\right) + (\gamma + \eta - \theta - 1)\ln\left(\frac{w}{\alpha}\right) - \eta \ln\left(\frac{p_k}{\eta}\right). \tag{17}$$

Taking the expectation of log employment among establishments that enter in location j yields the following moment:

$$E[\ln(n_{ij})|\text{entrants}] = \frac{1}{1-\alpha-\gamma-\eta}\left(\chi_j(\theta) + \mu_e\right) \qquad j = 1, 2 \tag{18}$$

These two equations identify μ_e and θ.[22] Moreover, the second moments of the entrants' employment distributions identify the scale parameters σ_{ej}:

$$V[\ln(n_{ij})|\text{entrants}] = \left(\frac{1}{1-\alpha-\gamma-\eta}\right)^2 \sigma_{ej}^2 \qquad j = 1, 2. \tag{19}$$

Finally, consider identification of the law of motion of productivity. Assumption 8 implies that we observe labor inputs for all firms that are active in period t including those that will exit at the end of that period. The law of motion for labor inputs can be written as:

$$\ln(n_{ijt}) - \chi_j - \rho(\ln(n_{ij't-1}) - \chi_{j'}) = \frac{1}{1-\alpha-\gamma-\eta}\epsilon_{ijt} \tag{20}$$

if establishment i is observed in location j' in period $t-1$ and in location j in period t. Hence, we can identify μ_ϵ , σ_ϵ and ρ based on the equation above.

To summarize, all parameters above are identified and can, therefore, be consistently estimated without computing the equilibrium of the model. Using the orthogo-

[21]Notice that the rental price of capital is not identified separately from μ_e. We normalize it to one in the rest of the analysis.

[22]Note that θ is identified by the differences in mean labor inputs among the two locations since it operates as a shifter of the intercept in the labor demand equation.

nality conditions above, we can define a GMM estimator to estimate these structural parameters.

4.3 Nested Fixed Point Algorithm

The remaining parameters that need to be estimated are the fixed cost parameters f_1, f_2, the relocation costs c_r, the entry costs c_1, c_2, the exogenous probability of exit, ξ, and the parameters of the land supply functions A_1, A_2.[23] To identify these parameters, we need additional moment restrictions. While we do not have a formal argument for identification, we can provide an intuitive explanation for identification. The fixed cost largely affects the expected life span of a firm.[24] Alternatively, an increase in fixed costs increases exits and reduces the number of incumbents and increases entry in equilibrium. We, therefore, identify these parameters using moments that capture the relative mass of entrants and incumbents in both locations.

Relocation costs affect the amount of relocation in equilibrium. We can, therefore, identify and estimate this parameter based on moments that characterize relocating firms. To identify the probability of exogenous exit, we can use the characteristics of firms that exit. Setting $\xi = 0$, the model predicts that only small firms will exit the economy. In the data, we observed that exit is not just limited to small firms. This type of exit is captured by the shock ξ, which leads to exogenous exits by establishments. We include the mean log employment of exiting firms to help us identify this parameter. The ratio of A_1/A_2 determines the relative size of both locations, which can be measured by M_1/M_2.[25]

[23] We set the discount factor β equal to 0.86 (or about 5 percent per year) and the land supply elasticity δ equal to 0.5 since we do not have access to data that would allow us to estimate the supply elasticity. Estimates vary for this rent elasticity of supply for office space but are generally accepted to be significantly greater than unity (Wheaton, 1999).

[24] This suggests using average age of the incumbents in both locations to identify the fixed costs. Unfortunately, the age data in Dun and Bradstreet have many missing observations.

[25] We normalize the level of A_2 so that in equilibrium $M_1 + M_2 = 1$.

We condition our analysis on the observed rental prices (r_1, r_2). As a consequence, we can exploit the local uniqueness of equilibrium. We know that at the observed values of (r_1, r_2), there can only be one equilibrium. By conditioning on the observed equilibrium rents, we, therefore, deal with the multiplicity of equilibria problem and do not need to rely on a potentially arbitrary equilibrium selection rule. Another consequence of this approach is that the entry cost parameters, c_1 and c_2, in both locations are then implicitly determined by the two free-entry conditions.[26]

We adopt a simulated method of moments approach to estimate all parameters of our model. Denote the parameter vector by ϕ. Let ϕ_0 denote the vector that characterizes the data-generating process. Let N denote the sample size of the Dun and Bradstreet sample. Combine all empirical moments used in the estimation procedure into one vector m_N and denote with $m_S(\phi)$ their simulated counterparts where S denotes the number of simulations. The orthogonality conditions are then given by

$$g_{N,S}(\phi) = m_N - m_S(\phi) \tag{21}$$

Following Hansen (1982), the parameters of our model can be estimated using the following moments estimator:

$$\hat{\phi}_{N,S} \;=\; \arg\min_{\phi \in \Phi} \;\; g_{N,S}(\phi)' \, W_N \, g_{N,S}(\phi) \tag{22}$$

for some positive semi-definite matrix W_N that converges in probability to the asymptotic covariance matrix of the vector of moments, W_0. [27]

[26]The rental rates are \$22.34 and \$19.87 per square ft. per year in the CBD and oCBD, respectively. These rents were obtained from industry data collected by Grubb and Ellis in 2008. Wages in our model are equal to \$48,661, which corresponds to the average yearly income in the financial/service sectors, based on census business patterns data.

[27]Since we can make the simulation error arbitrarily small, we suppress the dependence of our estimator on S. The estimator $\hat{\phi}_N$ is a consistent estimator of ϕ_0 and the asymptotic covariance matrix of the estimator is given in Newey and McFadden (1994). It is straightforward to correct for the sampling error induced into the estimation procedure by the simulations. However, if the

5 Estimation Results

In this section we discuss the results of our empirical analysis. As described in Section 4.2, a subset of parameters can be estimated without solving the entire model. Table 5 reports the parameter estimates and the estimated standard errors. The second column of this table presents parameters estimated using the sample of service-sector data, while the other two columns refer to parameters estimated using only data from the finance and professional/scientific/technical services sectors, respectively.[28] The advantage of focusing on the subset of parameters that can be estimated without solving the entire model is that we can estimate these parameters for any subset of industries without worrying about their possible interaction in the land markets.

Overall, we find that the results are qualitatively similar across different sectors. Nevertheless, there are some interesting differences. Agglomeration effects are stronger (weaker) in the financial (professional) sector than the full sample. Firms in the finance industry operate with smaller labor and land shares than other service sector firms. Entering firms have higher productivity. The opposite result holds for firms in the professional sector.

Table 6 presents parameter estimates for the entire vector of parameters. This requires us to impose the full set of orthogonality conditions implied by the model. The second column refers to a version of the model without exogenous relocation ($\lambda_1 = \lambda_2 = 0$), while the third column allows for small amounts of exogenous relocation between the two communities ($\lambda_1 = 0.01, \lambda_2 = 0.001$).

Consider our preferred specification of the model, the one with small exogenous relocation shocks. The estimate of the labor share parameter is 0.468. The estimate

number of simulations is large, these errors will be negligible. For a discussion, see Gourieroux and Monfort (1993).

[28]The parameters estimated in Table 5 are derived from exactly identified versions of the model and they provide a perfect fit to the moments.

Table 5: Parameter Estimates: Partial Solution

Parameter	Service Sector	Finance Sector	Professional Services
	NAICS 51–62	NAICS 52	NAICS 54
α	0.469	0.328	0.582
	(0.022)	(0.020)	(0.017)
γ	0.086	0.039	0.079
	(0.008)	(0.007)	(0.005)
θ	0.625	1.249	0.303
	(0.098)	(0.490)	(0.072)
μ_e	13.75	18.47	10.89
	(0.098)	(2.946)	(0.550)
σ_{e1}	0.255	0.460	0.104
	(0.033)	(0.073)	(0.025)
σ_{e2}	0.155	0.259	0.091
	(0.018)	(0.041)	(0.019)
ρ	0.967	0.979	0.973
	(0.002)	(0.004)	(0.004)

Notes: This table reports the values of the parameters that can be estimated without solving the entire model, as described in Section 4.

Table 6: Parameter Estimates: Full Solution

	Model Version	
Parameter	No Relocation Shock	Relocation Shock
α	0.482 (0.015)	0.495 (0.015)
γ	0.091 (0.006)	0.096 (0.006)
θ	0.656 (0.025)	0.667 (0.026)
μ_e	13.77 (0.099)	13.75 (0.113)
σ_{e1}	0.247 (0.013)	0.237 (0.012)
σ_{e2}	0.142 (0.013)	0.131 (0.013)
ρ	0.967 (0.002)	0.967 (0.002)
μ_ε	0.452 (0.061)	0.453 (0.071)
σ_ε	0.188 (0.030)	0.174 (0.037)
c_{f1}	32,908 (9,153)	21,011 (22,048)
c_{f2}	96,753 (18,026)	86,257 (21,194)
c_r	1,356,598 (661,830)	1,436,026 (724,396)
ξ	0.752 (0.037)	0.753 (0.049)
A_1/A_2	1.464 (0.131)	1.494 (0.186)

Notes: The second column presents parameter estimates for the version of the model with $\lambda_1 = \lambda_2 = 0$. The third column presents estimates for the case $\lambda_1 = 0.01$ and $\lambda_2 = 0.001$.

of the land share parameter is 0.086. These estimates are broadly consistent with those reported in the literature.[29] The estimate of the agglomeration externality is 0.667. The restriction $\theta > \gamma$ is necessary to obtain an equilibrium sorting pattern in which high productivity firms prefer locations with high agglomeration externalities. Productivity shocks are highly correlated across time. The point estimate of 0.967 is also consistent with previous estimates in the literature (Hopenhayn & Rogerson, 1993). We find that the distribution of productivity of entrants in the CBD has a larger standard deviation than the distribution of entrants outside the CBD.

Entry costs are larger inside than outside the CBD, \$398,570 versus \$71,759. Fixed costs of operation are \$21,011 inside and \$86,257 outside the CBD. Relocation costs are large, approximately \$1.4 million. Only the largest firms relocate in the version of the model without exogenous relocation shocks. For that reason we also add the exogenous relocation shocks to the model. This accounts for the fact that in the data we observe smaller firms relocating. Adding exogenous relocation to the model does not affect the estimates of the other structural parameters.

The rental rate for office space is 12 percent higher in the CBD than the rate outside the CBD. The rental rate price ratio along with the estimate of the externality parameter, θ, implies that firms located in the CBD receive an 8 percent productivity gain over firms located outside the CBD. This gain is due to the local agglomeration externality.

The productivity gain associated with being located in the CBD relative to the outside location is of the same order of magnitude as the productivity effects of agglomeration estimated by the rest of the literature. The exact magnitude of this effect and the associated interpretation of the size of the parameter θ are closely tied to many elements that are specific to our model, such as the exact specification of

[29]Estimates about the land share are reported in Deckle and Eaton (1993), Adsera (2000), and Caselli and Coleman (2001).

agglomeration effects and the geographic units of analysis. For example, Ciccone and Hall (1996) and Davis et al. (2011) express agglomeration effects as a function of output rather than employment density. The former paper takes a county as the unit of analysis, while the latter uses a metropolitan area. Unlike in these papers, our measure of physical space is not land area but office space. Finally, differen from other papers, we focus our analysis on the service sector.

Table 7: Fit: Moments Used in Estimation

Moment	Sample	Model Version No Relocation Shock	Model Version Relocation Shock
Labor share	0.4686	0.4822	0.4954
Land share	0.0863	0.0910	0.0955
Mean log employment entrants (CBD)	1.0158	1.0405	1.0602
Mean log employment entrants (oCBD)	0.8032	0.8012	0.7998
Variance log employment entrants (CBD)	0.7372	0.8033	0.8548
Variance log employment entrants (oCBD)	0.2730	0.2670	0.2626
Autocorrelation of log employment	0.9673	0.9673	0.9672
Mean log employment incumbents (CBD)	1.6528	1.6334	1.6201
Mean log employment incumbents (oCBD)	1.2948	1.3001	1.3027
Variance log employment incumbents	1.2394	1.2356	1.2316
Mean log employment exiting firms	0.8172	0.8051	0.8020
Ratio of entrants to incumbents	0.5243	0.5237	0.5236
Ratio of incumbents in CBD vs oCBD	0.1712	0.1680	0.1662
Ratio of entrants in CBD vs oCBD	0.0922	0.0970	0.0995
Fraction relocating from CBD to oCBD	0.0022	0.0026	0.0030

We also evaluate the within-sample fit of our model. Table 7 reports the empirical moments observed in the data and the corresponding moments predicted by our model. Overall, we find that the within-sample fit of our model is good, which is not surprising since we are only using a small number of over-identifying restrictions in

Table 8: Fit of Model - Moments Not Targeted in the Estimation

Description	Data	Model with Relocation Shock
Mean log employment (CBD)	1.10	1.37
Mean log employment (oCBD)	0.69	0.97
Median log facility size (CBD)	7.82	7.41
Median log facility size (oCBD)	7.79	7.13
Median age (CBD)	12	9
Median age (oCBD)	10	6
Percent establishments relocating from oCBD to CBD	0.22	0.30
Percent establishments relocating from CBD to oCBD	0.33	0.09
Percent employment relocating from oCBD to CBD	0.48	3.45
Percent employment relocating from CBD to oCBD	0.29	0.29
Median log employment relocators from oCBD to CBD	1.38	4.67
Median log employment relocators from CBD to oCBD	1.25	1.37
Percent establishments exiting economy (3 years)	29.48	34.37
Percent establishments entering economy (3 years)	34.39	34.37
Median log employment exiting firms (CBD)	0.69	1.17
Median log employment exiting firms (oCBD)	0.69	0.58
Mean log employment exiting firms (CBD)	1.14	1.28
Mean log employment exiting firms (oCBD)	0.76	0.75

the estimation.

More interesting is the comparison between sample and predicted moments that are not used in estimation. Table 8 reports a variety of additional interesting statistics. Consider first median log employment, log facility size and age. We find that the model provides a good fit of the data as well. We slightly underestimate the median age, but age (derived from self-reported year-established data) is probably measured with a fair bit of error in the data, which also explains why we do not use these moments in estimation. The remainder of Table 8 focuses on characteristics of firms entering, exiting and relocating. Overall, our model fits the entry and exit distribution of firms reasonably well. It is harder to fit the distribution of relocating firms. As we discussed in detail in Section 2, it is difficult to measure relocation in the data.

6 Policy Analysis

Our analysis has some important policy implications. Relocation costs prevent establishments from moving because the gains for the individual firm are smaller than the moving costs. However, this decision may not be efficient, since firms ignore the external benefits of density and agglomeration to other firms when making locational decisions. Since agglomeration effects are present in both the CBD and the locations outside the CBD, in principle it is not clear whether a relocation subsidy will generate an improvement in aggregate welfare.

We consider a subsidy to relocation to the CBD financed by a proportional wage tax paid only by firms located in the CBD. In other words, this is a policy that could be implemented unilaterally by a central city. Specifically, the relocation cost from outside to inside the CBD is subsidized at the rate s and wages are taxed at the rate τ. Given that labor is elastically supplied at the rate w, the after-tax wage becomes $w\,(1+\tau)$, so that the tax is fully absorbed by firms in the CBD in terms of higher labor costs. A Balanced-budget requirement calls for the revenue from the wage tax to cover the equilibrium amount of subsidies paid out to relocating establishments:

$$\tau\, w \int n_1(\varphi)\mu_1(d\varphi) \;=\; s\, c_r\, \xi\, (1-\lambda_2) \int 1\left\{x_2(\varphi)=1\right\}\mu_2(d\varphi) \;+\; s\, c_r\, \xi\, \lambda_2 \int \mu_2(d\varphi) \tag{23}$$

We adopt as a welfare measure the aggregate surplus generated by our economy (see also Rossi-Hansberg (2004)):

$$\text{Surplus} = \int_0^{L_1^*} \left(r_1^* - A_1 L^{\delta}\right) dL + \int_0^{L_2^*} \left(r_2^* - A_2 L^{\delta}\right) dL. \tag{24}$$

This corresponds to the areas between the equilibrium rents and the inverse land supply functions in both locations.[30]

Table 9 reports the quantitative implications of two policies that differ in the magnitude of the relocation subsidy. Qualitatively, results are similar across these two experiments. The evaluation of the policy is based on the parameter values of the version of the model that allows for exogenous relocation shocks.

A relocation subsidy financed by a tax on wages increases employment relocation from outside to inside the CBD. Rents increase in the CBD and decline outside the CBD. Aggregate employment and facility use increase in the CBD despite a reduction in entry and in the measure of firms in that location. The opposite pattern is observed outside the CBD. Turning to the welfare implications of this policy, we find that the surplus in the CBD increases, while it decreases outside the CBD. Overall welfare, measured by total surplus, increases.

We interpret the results in the following way. A relocation subsidy benefits firms located outside the CBD because it makes it cheaper for them to relocate to the CBD. The increases in the expected value of entering outside the CBD induces an increase in the measure of entrants and incumbents in that location. However, since the largest establishments from outside the CBD relocate to the CBD, aggregate employment outside the CBD falls. Moreover, aggregate employment falls more than facility use, leading to a reduction in employment density and agglomeration effects outside the CBD. Hence, there is a decline in equilibrium rents and surplus in that location. The opposite mechanism is operative in the CBD: the employment reallocation from outside the CBD increases aggregate employment, employment density, agglomeration effects and equilibrium rents. Entry of new firms in the CBD falls because of the tax

[30]Note that there is no surplus associated with workers because they supply labor in a perfectly elastic manner. Similarly, the industry under consideration operates under aggregate returns to scale (taking entry into account) and does not generate any surplus.

Table 9: Policy Experiments

Subsidy rate (%)	10.00	20.00
Tax rate (%)	0.31	0.75
Employment relocation	6.96	14.56
Rent CBD	0.62	1.51
Rent oCBD	-0.09	-0.20
Measure of entrants in CBD	-11.60	-22.94
Measure of entrants oCBD	2.28	4.84
Aggregate measure of entrants	0.94	2.15
Measure of firms CBD	-8.58	-16.38
Measure of firms oCBD	2.02	4.25
Aggregate measure of firms	0.62	1.47
Aggregate employment CBD	1.57	3.83
Aggregate employment oCBD	-0.26	-0.60
Aggregate employment	0.47	1.17
Aggregate facility CBD	1.25	3.05
Aggregate facility oCBD	-0.18	-0.40
Aggregate facility	0.35	0.88
Surplus in CBD	1.88	4.61
Surplus oCBD	-0.26	-0.60
Total surplus	0.59	1.48

Note: Except for the first two rows, entries are percent changes relative to the benchmark values implied by the model with relocation shocks.

used to finance the subsidy.

Overall, our results indicate that there might be too little relocation of establishments to the CBD and are supportive of policy efforts targeted to increase the extent of agglomeration effects in CBDs.

7 Conclusions

We have developed a new dynamic general equilibrium model to explain firm entry, exit, and relocation decisions in an urban economy with multiple locations. We have characterized the stationary distribution of firms that arises in equilibrium. The parameters of the model can be estimated using a nested fixed point algorithm by matching the observed distribution of firms by location and the one implied by our model. We have implemented the estimator using unique data collected by Dun and Bradstreet for the Pittsburgh metropolitan area. Firms located in the central business district are older and larger than firms located outside the urban core. They use more land and labor in the production process. However, they face higher rental rates for office space, which implies that they operate with a higher employee per land ratio. Our estimates imply that agglomeration externalities increase the productivity of firms by 8 percent. Economic policies that subsidize firm relocations can potentially have large effects on economic growth and firm concentration in central business districts.

We view the findings of this paper as promising for future research. The modeling framework can be extended to analyze investment and innovation decisions. While Dun and Bradstreet does not collect such data, other data sets are available that allow researchers to study these issues.

In preliminary work, we have compiled statistics characterizing firm sorting in

Table 10: Characteristics of Manufacturing Establishments in the Shanghai Metropolitan Area in 2007

	Inner City Shanghai	Outer City Shanghai
Age	13.0	9.3
Average employment	221	178
Fixed assets per employee	206	148
Value added per employee	230	168
Average wage	9202	5220
TFP level (Olley-Pakes)	-0.26	-0.47
TFP growth (2006-07)	0.015	-0.012
Sample size	3,063	20,927
Source: Calculations performed by Jipeng Zhang based on Census data of manufacturing firms from China's Statistical Bureau. The productivity estimates were kindly provided by Brandt, van Biesebroeck, and Zhang (2012)		

Shanghai using data from the Chinese Industrial Survey that are summarized in Table 10.[31] Some empirical patterns observed in Shanghai are surprisingly similar to those described in this paper. For example, manufacturing firms located in the inner city in Shanghai are larger, more productive and invest more than firms located outside the inner city.[32] Now obviously, there are important differences between fairly stationary urban economies such as Pittsburgh and those that are undergoing a large transformation like urban economies in places such as Shanghai.

In our policy experiment the tax-subsidy scheme implemented by the CBD occurs in the context of a non-strategic environment where the second location is assumed not to try to attract establishments from the CBD. In large U.S. metropolitan areas, there are often many independent communities that compete among each other to attract business using targeted subsidies. It is not obvious that this type of tax and subsidy competition among communities increases economic welfare. Our model

[31] These data are also used in Hsieh and Klenow (2009).

[32] The inner city of Shanghai is defined according to the postal codes that represent its central urban area, i.e., the six-digit postal codes starting with 200.

could be usefully extended to allow for this sort of strategic behavior.

Another fruitful extension would focus on competition among regional economies or metropolitan areas for firms. This type of firm relocation entails different and broader trade-offs from the ones analyzed in this paper, as firms would have to consider issues such as distance from customers and variation in local labor market conditions.

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A Firm Location Choices in a Sample of Large U.S. Cities

To get some additional quantitative insights into firms' sorting behavior, we collected census data for a number of metro areas. We define a business district within the metropolitan area as those zip codes within a city that have a relatively high density of firms, signifying local agglomeration. To make this concept operational, we use an employment density of at least 10,000 employees per square mile. These locations need not be contiguous, as some metropolitan areas exhibit multiple dense business districts.

Table 11: Concentration of Employment in Dense Business Districts

MSA	Total Emp. Outside CBD	Total Emp. in CBD	Avg. Emp. outside CBD	Avg. Emp. in CBD	% Services outside CBD*	% Services in CBD**
Atlanta	1,115,398	229,002	15.79	29.25	45.24%	63.31%
Boston	1,728,075	531,349	15.66	39.01	41.99%	59.90%
Chicago	3,070,387	528,529	15.86	24.47	41.85%	66.50%
Columbus	705,534	63,278	18.69	23.73	42.88%	58.64%
Hartford	499,718	18,783	17.26	26.95	40.31%	61.41%
Houston	1,720,625	286,574	16.38	28.47	42.86%	65.51%
Jacksonville	491,959	24,315	15.24	25.38	43.09%	66.28%
Los Angeles	4,257,269	974,693	15.02	19.39	44.16%	52.39%
Philadelphia	1,921,626	196,428	15.91	27.66	43.99%	55.74%
Phoenix	1,551,921	64,793	18.31	27.78	47.79%	71.01%
Pittsburgh	822,013	157,009	14.58	40.04	39.16%	60.90%
Salt Lake	440,239	53,086	15.22	21.08	45.64%	58.90%
San Antonio	655,740	26,572	17.21	20.49	43.22%	56.59%
Seattle	1,260,335	179,230	14.55	20.33	42.07%	58.97%
St Louis	1,253,959	84,034	16.38	42.57	41.41%	52.43%
Wash. DC	1,930,848	303,770	15.42	21.68	49.96%	60.05%

Source: 2006 Zip Code Business Patterns, U.S. Census

*Percentage of establishments outside the CBD that are in the service industries.

**Percentage of establishments in the CBD that are in the service industries.

Table 11 shows the concentration of employment in dense business districts for a sample of U.S. cities. First, we report statistics using all firms located in the metro area. We find that there is a significant amount of heterogeneity among the cities in our sample. There are some cities such as Phoenix and Hartford where employment is not concentrated in dense business districts. Most larger cities in the U.S., such as Los Angeles, Chicago, Boston, Washington, Philadelphia, and Houston, have a significant fraction of firms located in high density central business districts. This finding is also true for a variety of mid-sized cities such as Pittsburgh and Seattle. Focusing on the differences between firms located inside and outside of the CBD, we find that firms in the CBD are larger than the MSA average. This indicates that they have higher levels of productivity. This finding is common among all cities in our sample. In addition, firms in the service sector are more concentrated in the CBD compared to firms in general, suggesting that service-oriented firms benefit more from local agglomeration than other sectors.

B Computation of Equilibrium

The task of computing an equilibrium can be simplified by exploiting some properties of the parameterization used in our computational model.

(1) Externality as a function of rents. From the static first-order condition that determines the ratio of land and labor inputs, we obtain that the labor/land ratio is the same for all firms in the same location j:

$$\frac{n}{l} = \frac{\alpha}{\gamma}\frac{r_j}{w}. \tag{25}$$

Aggregating over all firms in a location, we obtain that:

$$\frac{N_j}{L_j} = \frac{\alpha}{\gamma}\frac{r_j}{w} \tag{26}$$

and using the parameterization of the externality as $e_j = (N_j/L_j)^\theta$ we obtain an expression linking the externality, e_j in each location to that location's rent, r_j

$$e_j = \left(\frac{\alpha}{\gamma}\frac{r_j}{w}\right)^\theta. \tag{27}$$

We can, therefore, solve the Bellman equations as a function of the rents (r_1, r_2) only.

(2) Equilibrium rents. The free-entry conditions, assumed to hold as equality, are then:[33]

$$\int_0^\infty V_1(\varphi)d\nu_1(\varphi) = c_1 \tag{28}$$
$$\int_0^\infty V_2(\varphi)d\nu_2(\varphi) = c_2. \tag{29}$$

These are two equations in two unknowns (r_1, r_2). The non-linearity of the model implies that there may be more than one possible candidate value for equilibria with entry in both locations.

(3) Stationary distributions and equilibrium entry ratio. Next, define the ratio of entrants in the two locations as $m = M_1/M_2$ and the distribution of firms standardized by the mass of entrants in location 2 as:

$$\hat{\mu}_j = \frac{\mu_j}{M_2}. \tag{30}$$

[33] In addition to equilibria with entry in both locations, as assumed here, it is also possible to have equilibria in which entry occurs only in one of the two locations.

The standardized stationary distributions satisfy

$$\int_0^{\varphi'} \hat{\mu}_1(dz) = \xi\,(1-\lambda_1)\int F(\varphi'|z)1\left\{x_1(z)=1\right\}\hat{\mu}_1(dz) \tag{31}$$
$$+\xi\,(1-\lambda_2)\int F(\varphi'|z)1\left\{x_2(z)=1\right\}\hat{\mu}_2(dz)$$
$$+\xi\lambda_2\int F(\varphi'|z)\hat{\mu}_2(dz) + m\int_0^{\varphi'}\nu_1(dz)$$

and

$$\int_0^{\varphi'} \hat{\mu}_2(dz) = \xi\,(1-\lambda_1)\int F(\varphi'|z)1\left\{x_1(z)=2\right\}\hat{\mu}_1(dz) \tag{32}$$
$$+\xi\,(1-\lambda_2)\int F(\varphi'|z)1\left\{x_2(z)=2\right\}\hat{\mu}_2(dz)$$
$$+\xi\lambda_1\int F(\varphi'|z)\hat{\mu}_1(dz) + \int_0^{\varphi'}\nu_2(dz).$$

Given a value for m, forward iteration on these two equations yields the equilibrium standardized stationary distributions $\widehat{\mu}_j$, $j=1,2$. To find the equilibrium value of m, substitute the aggregate demands for land in the two locations into the inverse land supply functions and take their ratios to obtain:

$$\frac{r_1}{r_2} = \frac{A_1}{A_2}\left[\frac{\int l_1(\varphi)\hat{\mu}_1(d\varphi)}{\int l_2(\varphi)\hat{\mu}_2(d\varphi)}\right]^{\delta}. \tag{33}$$

This equation determines the equilibrium value of m.

(4) **Level of entry.** Finally, the mass of entrants in location 2, M_2, is determined by the market clearing condition for land:

$$\left(\frac{r_2}{A_2}\right)^{\frac{1}{\delta}} = M_2\int l_2(\varphi)\hat{\mu}_2(d\varphi), \tag{34}$$

Note that M_2 can be solved for analytically. Given M_2 and m, it is feasible to retrieve

$M_1 = mM_2$.

C Analytical Properties of Equilibrium

To get some additional insights into the properties of our model, it is useful to simplify the structure of the model and shut down the future productivity shocks. We can then characterize the equilibrium of the model almost in closed form.[34] Let us impose the following additional assumptions.

Assumption 9

1. *The productivity shock φ is drawn upon entry once and for all from a uniform distribution in $[0,1]$:*

$$\nu_j(\varphi) = 1 \text{ for } \varphi \in [0,1] \text{ and } j = 1,2. \tag{35}$$

2. *There is no capital in the model: $\eta = 0$.*

3. *The externality parameter θ satisfies the following restrictions: $\theta = 1 - \alpha > \gamma$.*

4. *There is no fixed production cost: $f_j = 0$, $j = 1,2$.*

In what follows we characterize the unique equilibrium of this version of the model in which $r_1 > r_2$ and firms move from location 2 to location 1, but not vice versa. Firms that enter in location 1 stay there all the time or exit. All exits are exogenous.

[34] The model cannot be entirely solved in closed form because the equilibrium r_2 has to satisfy a highly non-linear equation. Sufficient conditions on the model's parameters for r_2 to exist and be unique are imposed instead. Conditional on r_2, everything else can be solved for analytically.

First note that under assumptions 2–4 above, the indirect profit functions can be written as:

$$\pi_j(\varphi) = r_j \Delta \varphi^{\psi},\ j = 1, 2, \tag{36}$$

where

$$\Delta = \left[\frac{\alpha}{w}\right]^{\frac{1}{1-\alpha-\gamma}} \left(\frac{1-\alpha-\gamma}{\gamma}\right) > 0, \tag{37}$$

$$\psi = \frac{1}{1-\alpha-\gamma} > 1 \tag{38}$$

are known functions of the parameters of the model. Notice that since there are no fixed costs, all entrants stay in the economy until they are exogenously forced to exit through the shock ξ.

We have the following result.

Proposition 1 *If $r_1 > r_2$, then firms in location $j = 2$ follow a simple cut-off rule. Firms below the threshold φ_r stay in location 2, and firms with shocks larger than φ_r move to location 1. The cut-off φ_r is defined as:*

$$\varphi_r = \left[\frac{(1-\beta\xi)\, c_r}{(r_1 - r_2)\,\Delta}\right]^{\frac{1}{\psi}}. \tag{39}$$

For the model to be meaningful we assume that parameters are such that $\varphi_r < 1$.

Proof:
Notice that since profits are increasing in r and we are assuming that $r_1 > r_2$, firms located in 1 never want to switch location. The value function of firms in location 1 is therefore:

$$V_1(\varphi) = \frac{\pi_1(\varphi)}{1-\beta\xi}. \tag{40}$$

Next consider the decision rule of firms located in 2. Firms with shocks in $(0, \varphi_r)$ stay in 2 forever (as long as they survive the exogenous destruction shock). Firms with high shocks move to 1. The indifference condition for staying vs moving is:

$$\frac{\pi_2(\varphi_r)}{1-\beta\xi} = \pi_2(\varphi_r) + \beta\xi\left(V_1(\varphi_r) - c_r\right). \tag{41}$$

This equation defines the cut-off value φ_r, which can be solved for analytically to obtain the expression in equation (39). The lemma then follows from the result that the benefits of switching to location 1 monotonically increase with φ. Q.E.D.

Next we consider the free-entry conditions and show that these conditions determine the rents in both locations. We have the following result:

Proposition 2 *There is at most one set of rental rates (r_1, r_2) that are consistent with entry in both locations. Conditions on the parameter values guarantee the existence of (r_1, r_2).*

Proof (of uniqueness):
First consider the free-entry condition in location 1, which is given by

$$\int V_1(\varphi)\,\nu(\varphi)\,d\varphi = c_1. \tag{42}$$

Substituting in our optimal decision rule and simplifying, we obtain the equilibrium rent in location 1:

$$r_1 = \frac{c_1(1+\psi)(1-\beta\xi)}{\Delta}. \tag{43}$$

Free entry in location 2 requires:

$$\int V_2(\varphi)\,\nu(\varphi)\,d\varphi = c_2. \tag{44}$$

Replacing the value function in location 2 and taking into account the definition of φ_r in (39) this equation simplifies to:

$$(\varphi_r)^{\psi+1} + \frac{(\psi+1)}{\psi\beta\xi}\left(\beta\xi + \frac{c_2 - c_1}{c_r}\right)(\varphi_r)^{\psi} + \frac{1-\beta\xi}{\psi\beta\xi} = 0. \tag{45}$$

A solution to this equation requires the entry cost differential between the two locations to be sufficiently large:

$$c_1 - c_2 > 2\beta\xi c_r. \tag{46}$$

Under this condition, the equilibrium value of φ_r, if it exists, is unique. In turn, φ_r is monotonically related to r_2 by equation (39). Thus, if the solution φ_r to equation (45) is unique, the equilibrium value of r_2 is also unique. Q.E.D.

Next we characterize the equilibrium distribution of firms in each location.

Proposition 3 *For each value of M_2, there exists a unique stationary equilibrium distribution of firms in each location.*

Proof:

Without loss of generality, let us normalize the model so that entry in location 2 is always equal to $M_2 = 1$. This implies a specific choice of A_2. Given this the mass of firms in location 2 is $\mu_2(\varphi)$:

$$\mu_2(\varphi) = \begin{cases} \frac{1}{1-\xi}\varphi_r \text{ if } \varphi \in [0, \varphi_r] \\ 1 - \varphi_r \text{ if } \varphi > \varphi_r \end{cases}. \tag{47}$$

Firms with $\varphi > \varphi_r$ move to 1, and there is a measure $1 - \varphi_r$ of them. Firms in the group $\varphi \in [0, \varphi_r]$ remain in 2 forever subject to surviving the death shock ξ .

Let m denote entry in location 1. The mass of firms in location 1 is:

$$\mu_1(\varphi) = \begin{cases} \frac{m}{1-\xi}\varphi_r \text{ if } \varphi \in [0, \varphi_r] \\ \frac{(\xi+m)}{1-\xi}(1-\varphi_r) \text{ if } \varphi > \varphi_r \end{cases}. \tag{48}$$

Firms in the first group originate in 1 and stay in 1 forever. Firms with $\varphi > \varphi_r$ arrive from 2 sources: (1) firms that entered in 1 and stayed there forever subject to the death shock $m(1-\varphi_r)/(1-\xi)$ and (2) firms that entered in location 2 last period, survived the shock and moved to 1 where they remain forever: $\xi(1-\varphi_r)/(1-\xi)$. Q.E.D.

Finally, we have the following result:

Proposition 4 *There is at most one value of m such that the relative demand for land equals the relative supply of land. Under conditions on the parameters, m is shown to exist.*

Proof:
Given the equilibrium distributions, we can solve for the equilibrium value for entry, denoted by m. Note that given the assumptions the demand for land is:

$$l_j(\varphi) = \left[\frac{\alpha}{w}\right]^{\psi} \varphi^{\psi}. \tag{49}$$

The equilibrium value of m is such that it solves the relative land equilibrium condition which can be written as,

$$\frac{r_1}{r_2} = \frac{A_1}{A_2}\left[\frac{\int \varphi^{\psi}\mu_1(d\varphi)}{\int \varphi^{\psi}\mu_2(d\varphi)}\right]^{\delta} \tag{50}$$

where the left-hand side does not depend on m. The right-hand side is monotonically increasing in m through the mass $\mu_1(\varphi)$. This means that if m exists, it is unique. For $m \to \infty$ the right-hand side of (50) goes to infinity. For $m \to 0$ the right-hand side is strictly positive. To show that it is less than the left-hand side for $m = 0$, A_1 must be sufficiently small. Since the rest of the equilibrium is independent of A_1 one can always choose A_1 small enough in order to guarantee existence. Thus, there exists a unique value of m. Q.E.D.

In what follows we present the equilibrium of the model in a numerical example.

Result 1 *Consider the following parameter values:* $\beta = 0.5$, $\alpha = 0.65$, $\theta = 0.35$, $\xi = 0.9$, $\gamma = 0.01$, $w = 1$, $A_1 = 0.5$, $A_2 = 1.0$, $c_1 = 0.1$, $c_2 = 0.01$, $c_r = 0.01$, $\delta = 0.5$. *Then, the unique equilibrium of the model is characterized by the following:* $\varphi_r = 0.33$, $r_1 = 0.02$, $r_2 = 0.007$, $m = 3.21$.

The analysis of this section shows that there exists a unique equilibrium with entry in both locations. This reinforces the notion that equilibria with entry in both locations are often locally unique.