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Exogenous vs. Endogenous Separation*

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Abstract

This paper assesses how various approaches to modeling the separation margin affect the ability of the Mortensen-Pissarides job matching model to explain key facts about the aggregate labor market. Allowing for realistic time variation in the separation rate, whether exogenous or endogenous, greatly increases the unemployment variability generated by the model. Specifications with exogenous separation rates, whether constant or time-varying, fail to produce realistic volatility and productivity responsiveness of the separation rate and worker flows. Specifications with endogenous separation rates, on the other hand, succeed along these dimensions. In addition, the endogenous separation model with on-the-job search yields a realistic Beveridge curve correlation and performs well in accounting for the productivity responsiveness of market tightness. While adopting the Hagedorn-Manovskii calibration approach improves the behavior of the job finding rate, the volume of job-to-job transitions in the on-the-job search specification becomes essentially zero.

JEL codes: E24, J63

Keywords: Separation rate, job finding rate, unemployment

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1 Introduction

In its complete form, the Mortensen-Pissarides job matching model (henceforth MP model) endogenously determines both the match creation and separation margins.¹ While researchers agree that match creation is appropriately viewed as endogenous, there is little consensus as to the proper treatment of the separation margin. In this paper, we assess how these various approaches to modeling the separation margin affect the ability of the MP model to explain key facts about unemployment, transition rates, worker flows and other variables. A discrete-time version of Pissarides’s (2000) specification is calibrated at weekly frequency. Match separation is parameterized in four ways: (i) constant separation rate; (ii) exogenous separation rate following an AR(1) process; (iii) endogenous separation rate without on-the-job search (OJS); and (iv) endogenous separation rate with OJS. For the two specifications with endogenous separation, match-specific productivity factors follow a persistent stochastic process. The dynamic properties of the model are examined after solving the model by a nonlinear method that parameterizes match surplus and market tightness (i.e., the vacancy-unemployment ratio) and iterates backward to exploit stability of the backward dynamics.

In calibrating the model, the values of the workers’ unemployment benefits and bargaining weight, as well as the elasticity parameter of the matching function, are set to standard values proposed in the literature (e.g., Shimer (2005) and Mortensen and Nagypál (2007a)). The calibration of the vacancy posting cost draws on survey evidence from Barron and Bishop (1985) and Barron et al. (1997). Other parameters are chosen to match the mean monthly job finding and separation rates calculated by Fujita and Ramey (2006), who consider data from the Current Population Survey (CPS) over the 1976-2005 period. In addition, each of the three specifications with time-varying separation rates is calibrated to match the standard deviation and the persistence of the separation rate observed in the Fujita-Ramey data. For the OJS specification, the cost of OJS is calibrated by matching the average job-to-job transition rate measured by Moscarini and Thompson (2007) using CPS data.

Statistics calculated from simulated data for the four specifications are compared to corresponding statistics obtained from the Fujita-Ramey data. The results show, first of all, that the model with constant separation rate fares poorly in accounting for the volatility of key labor market variables. It does not, of course, explain the substantial variability of the separation rate observed in the data, nor does it generate anywhere near the empirical volatility of unemployment, a point stressed by Shimer (2005) and Costain and Reiter (2008). In addition, the cyclical behavior of gross worker flows in this model is clearly counterfactual: in the data, both unemployment-to-employment (UE) and employment-to-unemployment (EU) worker flows are countercyclical, whereas they are both procyclical in the model. This counterfactual implication arises due to the omission of the cyclical variations of the separation rate.

On the other hand, the three specifications with time-varying separation rates, which are

¹Throughout this paper, the terms “separation” and “job finding” denote movements of workers between employed and unemployed status.

calibrated to match the volatility of the empirical separation rate, each generate substantially greater volatility of unemployment and worker flows. In the model with OJS, for example, the standard deviation of unemployment equals 60 percent of its empirical value. Moreover, the three specifications match closely the standard deviations of UE and EU flows. Introducing realistic variability at the separation margin thus substantially improves the performance of the MP model in accounting for unemployment and worker flow variability.

In the data, the separation rate and the two worker flow variables exhibit substantial negative correlations with productivity. Both versions of the MP model with exogenous separation fail to replicate this pattern. The two versions with endogenous separation, however, exhibit realistic responsiveness of these variables to productivity. Thus endogeneity of the separation rate appears central to explaining the cyclical properties of the separation rate and worker flows.

The two endogenous separation specifications differ in their ability to account for the Beveridge curve relationship, wherein unemployment and vacancies display strong negative correlations. In the absence of OJS, the model produces a counterfactually positive unemployment-vacancy correlation, due to the “echo” effect that higher unemployment during downturns makes it easier to find workers, stimulating the vacancy posting. With OJS, however, downturns also imply a fall in the number of employed searchers, militating against the rise in unemployment. The unemployment-vacancy correlation becomes strongly negative in this case, matching closely the empirical value. Endogenous separation is therefore consistent with the Beveridge curve relationship when OJS is added to the model. Moreover, this specification captures the negative correlation between the job finding and separation rates seen in the data.

In summary, the endogenous separation specification with OJS implies empirically reasonable volatility and productivity responsiveness of unemployment, the separation rate and worker flows, along with realistic Beveridge curve and transition rate correlations. Each of the remaining three specifications fails decisively along one or more of these dimensions. This provides strong support for the OJS model as the most valid specification.

The results also show, however, that the MP model under the standard calibration does not produce realistic volatility of the job finding rate, irrespective of how the separation margin is modeled. The empirical standard deviation of the job finding rate is nearly five times the simulated value for each of the four specifications, and the comparison is similar for the productivity elasticity. This failure to generate realistic behavior at the job finding margin, which lies at the heart of the Hall-Shimer critique of the MP model, is thus not resolved by introducing realistic behavior at the separation margin.

The three specifications without OJS also deliver insufficient productivity responsiveness of market tightness. In the OJS specification, however, this variable is much more responsive to productivity: the productivity elasticity of market tightness in the simulated data amounts to roughly 50 of the empirical value. In the OJS specification, a substantial variation in the unemployment rate, and vacancies that are negatively correlated with unemployment, produce the relatively large responsiveness to productivity of market tightness.

Hagedorn and Manovskii (2008) propose an alternative calibration strategy, drawing

on empirical information on wages and profits, that raises the volatility of unemployment, market tightness and other variables in the constant separation rate model. To investigate the robustness of the current findings to this alternative, the constant separation rate and OJS specifications are suitably recalibrated. In line with Hagedorn and Manovskii’s findings, this procedure yields much more realistic volatility of unemployment, the job finding rate, vacancies and market tightness. It does not, however, remedy the key failings of the constant separation rate model: in particular, the separation rate and worker flows continue to display unrealistic variability and productivity comovement. Moreover, in the OJS specification the volume of job-to-job transitions becomes essentially zero. This is because the worker’s bargaining weight is very low under the alternative calibration, making OJS unattractive in nearly all circumstances.

Numerous previous papers have evaluated the properties of various versions of the MP model. First, Shimer (2005), Hall (2005), Hagedorn and Manovskii (2008), Fujita and Ramey (2007), Hornstein et al. (2005), Mortensen and Nagypál (2007a), Yashiv (2006) and others specify that matches break up at a rate that is exogenous and constant, affected by neither incentives nor cyclical factors. A subset of this group, including Mortensen and Nagypál (2007a), Shimer (2005) and Yashiv (2006), considers the possibility that separation rates vary over time in a random manner following an AR(1) process. On the other hand, papers such as Mortensen and Pissarides (1994), Cole and Rogerson (1999), Den Haan et al. (2000), Mortensen and Nagypál (2007b) and others allow match dissolution to be responsive to incentives facing the worker and firm. Mortensen (1994), Pissarides (1994), Krause and Lubik (2006), Nagypál (2005), Tasci (2006) and others allow for separation directly to new jobs. Mortensen (1994) calibrates and simulates endogenous separation versions of the standard MP model in continuous time, and stresses the model’s ability to explain facts about job creation and destruction in manufacturing. He also allows for OJS and shows that the model delivers countercyclical worker flows and a negative Beveridge correlation, consistent with the results obtained in this paper. Krause and Lubik (2006) and Tasci (2006) offer modifications of the MP model, also incorporating OJS.² Both of these papers show that their models yield significantly greater unemployment volatility than does the standard constant separation rate specification, and they obtain negative Beveridge correlations.³ Finally, a number of papers have embedded the MP model into dynamic stochastic general equilibrium frameworks (e.g, Merz (1995), Andolfatto (1996), Cooley and Quadrini (1999), Den Haan et al. (2000), Walsh (2005), and Nason and Slotsve (2005)). This body of work focusses chiefly on dynamic propagation of aggregate technology and monetary shocks.

Relative to the existing literature, this paper focuses on the implications of the various approaches to modeling the separation margin. In doing so, we use the textbook MP models

²Krause and Lubik (2006) specify a constant rate of separation into unemployment and introduce permanent productivity differences across jobs in order to elicit OJS. Tasci (2006) posits that each match undergoes an initial phase of learning about productivity whose outcome can induce endogenous separation.

³Menzio and Shi (2011) analyze worker flows and transition rates using a matching model that features directed search across labor submarkets, together with complete commitment of wage contracts. Their findings with respect to unemployment volatility and the Beveridge curve conform with those of Krause and Lubik (2006) and Tasci (2006).

exposed by Pissarides (2000) that differ only with respect to how the separation margin is modeled. This approach allows us transparently to examine the role of the separation margin in a unified framework. Furthermore, our evaluation of the model is based on the dynamic stochastic equilibrium of the model solved via a nonlinear method, rather than based on the steady-state comparative statics.⁴ Note also that we allow for persistence in the match-specific productivity factor, in contrast to the i.i.d. assumption sometimes adopted in the literature.⁵ As we will discuss in the text, persistence of match-specific productivity is not an innocuous consideration with respect to the cyclicity of the aggregate separation rate. Because our focus is on the behavior of the separation margin and its interaction with other aggregate variables, it is important to allow for the persistence in the match-specific factor.

The paper proceeds as follows. Section 2 introduces the four specifications of the MP model and constructs theoretical measures that correspond to the empirical data series. The calibration procedure and numerical solution method are discussed in Section 3, and results are presented in Section 4. In Section 5, the dynamic interrelationships of labor market variables are considered. Section 6 investigates the implications of the Hagedorn-Manovskii calibration approach, and Section 7 concludes.

2 MP Model

There is a unit mass of atomistic workers and an infinite mass of atomistic firms. Time periods are weekly. In any week t , a worker may be either matched with a firm or unemployed, while a firm may be matched with a worker, unmatched and posting a vacancy, or inactive. Unemployed workers receive a flow benefit of b per week, representing the total value of leisure, home production and unemployment insurance payments. Firms that post vacancies pay a posting cost of c per week. Let u_t and v_t denote the number of unemployed workers and posted vacancies, respectively, in week t . In the case of no OJS, the number of new matches formed in week t is determined by a matching function $m(u_t, v_t)$, having a Cobb-Douglas form:

$$m(u_t, v_t) = Au_t^\alpha v_t^{1-\alpha}.$$

Thus, an unemployed worker's probability of obtaining a match in week t , denoted by $f(\theta_t)$, equals $A\theta_t^{1-\alpha}$, where the variable $\theta_t = v_t/u_t$ indicates market tightness. The job filling rate for a vacancy, denoted by $q(\theta_t)$, equals $A\theta_t^{-\alpha}$. The value of v_t in each week is determined by free entry.

A worker-firm match can produce an output level of $z_t x$ during week t , where z_t and x are aggregate and match-specific productivity factors, respectively. The aggregate factor is determined according to the following process:

$$\ln z_t = \rho_z \ln z_{t-1} + \varepsilon_t^z, \tag{1}$$

⁴The latter approach has been used in some recent papers; see, for example, Mortensen and Nagypál (2007a,b) and Pissarides (2009).

⁵The papers that adopt the i.i.d. assumption include Den Haan et al. (2000) and Krause and Lubik (2007).

where ε_t^z is an i.i.d. normal disturbance with mean zero and standard deviation σ_z . Determination of x is discussed below.

Before engaging in production in week t , the worker and firm negotiate a contract that divides match surplus according to the Nash bargaining solution, where π gives the worker's bargaining weight and the disagreement point is severance of the match. Let $S_t(x)$ indicate the value of match surplus in week t for given x , and let U_t and V_t be the values received by an unemployed worker and a vacancy-posting firm, respectively. The worker and firm will agree to continue the match if $S_t(x) > 0$, while they will separate if separation is jointly optimal, in which case $S_t(x) = 0$. As the outcome of bargaining, the worker and firm receive payoffs of $\pi S_t(x) + U_t$ and $(1 - \pi)S_t(x) + V_t$, respectively. Let x^h denote the value of the match-specific productivity in a new match. The unemployment value satisfies:

$$U_t = b + \beta \mathbb{E}_t \left[f(\theta_t) \pi S_{t+1}(x^h) + U_{t+1} \right], \quad (2)$$

where \mathbb{E}_t represents the expectation operator with respect to the aggregate states in t and β is the discount factor. The continuation value of a vacant job is written as:

$$V_t = -c + \beta \mathbb{E}_t \left[q(\theta_t) (1 - \pi) S_{t+1}(x^h) + V_{t+1} \right]. \quad (3)$$

In free entry equilibrium, $V_t = 0$ for all t , implying that:

$$\beta q(\theta_t) (1 - \pi) \mathbb{E}_t S_{t+1}(x^h) = c. \quad (4)$$

This condition determines θ_t in every period.

2.1 Exogenous Separation

In the exogenous separation version of the MP model, $x = x^h$ is assumed to hold at all times and for all matches. At the end of each week, matches face a risk of exogenous separation. Let s_t denote the probability that any existing match separates at the end of week t . In the constant separation version of the model, s_t is held constant at a fixed value s for all t . When s_t is assumed to fluctuate exogenously, it is governed by:

$$\ln s_t = \rho_s \ln s_{t-1} + (1 - \rho_s) \ln s + \varepsilon_t^s, \quad (5)$$

where ε_t^s is i.i.d. normal with mean zero and standard deviation σ_s .

Let $M_t(x)$ denote the value of a match in week t when the match-specific factor is x . Since the worker and firm seek to maximize match value as part of Nash bargaining, $M_t(x^h)$ must satisfy the following Bellman equation:

$$M_t(x^h) = \max \left\{ M_t^c(x), U_t + V_t \right\},$$

where $M_t^c(x)$ represents the value of the match when the continuation is chosen, and is

written as:

$$M_t^c(x^h) = z_t x^h + \beta \mathbb{E}_t \left[(1 - s_t) M_{t+1}(x^h) + s_t (U_{t+1} + V_{t+1}) \right].$$

Match surplus then can be expressed as:

$$\begin{aligned} S_t(x^h) &= M_t(x^h) - U_t - V_t \\ &= \max \left\{ S_t^c(x^h), 0 \right\}, \end{aligned} \quad (6)$$

where $S_t^c(x)$ represents match surplus after the continuation is chosen. Substituting for U_t from (2) and setting $V_t = 0$ for all t , we can express this term as follows:

$$S_t^c(x^h) = z_t x^h - b + \beta (1 - s_t - f(\theta)\pi) \mathbb{E}_t S_{t+1}(x^h).$$

Equations (4) and (6) determine the equilibrium paths of θ_t and $S_t(x^h)$ for given realizations of the z_t and s_t processes.

2.2 Endogenous Separation

In the endogenous separation version (without OJS), s_t is held constant at the value s , whereas x follows a Markov process. All new matches start at $x = x^h$, but the value of x may switch in subsequent weeks. At the end of each week t , a switch occurs with probability λ . In the latter event, the value of x for week $t + 1$ is drawn randomly according to the c.d.f. $G(x)$, taken to be truncated lognormal with parameters μ_x and σ_x for $x < x^h$, and $G(x^h) = 1$. With probability $1 - \lambda$, x maintains its week t value into week $t + 1$. When OJS is not allowed, match value satisfies:

$$M_t(x) = \max \left\{ M_t^c(x), U_t + V_t \right\}$$

where $M_t^c(x)$ again represents the value of the match when the continuation of the match is chosen, and is expressed as follows:

$$M_t^c(x) = z_t x + \beta \mathbb{E}_t \left[(1 - s) \left(\lambda \int_0^{x^h} M_{t+1}(y) dG(y) + (1 - \lambda) M_{t+1}(x) \right) + s (U_{t+1} + V_{t+1}) \right].$$

The Bellman equation for match surplus is

$$S_t(x) = \max \left\{ S_t^c(x), 0 \right\}, \quad (7)$$

where $S_t^c(x)$ represents the value of match surplus after continuation of the match is chosen:

$$S_t^c(x) = z_t x - b + \beta \mathbb{E}_t \left[(1 - s) \left(\lambda \int_0^{x^h} S_{t+1}(y) dG(y) + (1 - \lambda) S_{t+1}(x) \right) - f(\theta_t)\pi S_{t+1}(x^h) \right].$$

Equations (4) and (7) determine the equilibrium paths of θ_t and $S_t(x)$ for given realizations of the z_t process.

2.3 OJS

The OJS version of the MP model extends the endogenous separation version by allowing matched workers to search at a cost of a . The worker search pool expands to $u_t + \phi_t$, where ϕ_t indicates the number of matched workers who search in week t . Total match formation in week t is now equal to $m(u_t + \phi_t, v_t)$. The expressions for matching probabilities stay the same as before with a suitable redefinition of the market tightness variable; that is, $\theta_t \equiv v_t / (u_t + \phi_t)$.

When a matched searching worker makes a new match in week t , the worker must renounce the option of keeping his old match before bargaining with the new firm at the start of week $t + 1$. As a consequence, the worker receives a payoff of $\pi S_{t+1}(x^h) + U_{t+1}$ from the new match. Since the worker's payoff from the old match cannot exceed this value, it is optimal for the worker always to accept a new match.

In the OJS version of the model, the optimal decision of the match is characterized by:

$$M_t(x) = \max \left\{ M_t^{cs}(x), M_t^{cn}(x), U_t + V_t \right\},$$

where $M_t^{cs}(x)$ and $M_t^{cn}(x)$ represent the value of continuation of the match with and without OJS, respectively. These two terms are expressed as follows:

$$\begin{aligned} M_t^{cs}(x) &= z_t x - a + \beta \mathbb{E}_t \left[f(\theta_t) \left(\pi S_{t+1}(x^h) + U_{t+1} + V_{t+1} \right) \right. \\ &\quad \left. + (1 - f(\theta_t)) \left\{ (1 - s) \left(\lambda \int_0^{x^h} M_{t+1}(y) dG(y) + (1 - \lambda) M_{t+1}(x) \right) + s(U_{t+1} + V_{t+1}) \right\} \right], \\ M_t^{cn}(x) &= z_t x + \beta \mathbb{E}_t \left[(1 - s) \left(\lambda \int_0^{x^h} M_{t+1}(y) dG(y) + (1 - \lambda) M_{t+1}(x) \right) + s(U_{t+1} + V_{t+1}) \right]. \end{aligned}$$

Assuming that the worker's search decision is contractible, the Bellman equation for match surplus is written as:

$$S_t(x) = \max \left\{ S_t^{cs}(x), S_t^{cn}(x), 0 \right\}, \quad (8)$$

where $S_t^{cs}(x)$ and $S_t^{cn}(x)$ represent match surplus with and without OJS. Using the expression for U_t and setting $V_t = 0$, these two terms may be expressed as:

$$S_t^{cs}(x) = z_t x - a - b + \beta(1 - f(\theta_t))(1 - s) \mathbb{E}_t \left[\lambda \int_0^{x^h} S_{t+1}(y) dG(y) + (1 - \lambda) S_{t+1}(x) \right], \quad (9)$$

$$S_t^{cn}(x) = z_t x - b + \beta \mathbb{E}_t \left[(1-s) \left(\lambda \int_0^{x^h} S_{t+1}(y) dG(y) + (1-\lambda) S_{t+1}(x) \right) - f(\theta_t) \pi S_{t+1}(x^h) \right]. \quad (10)$$

Equilibrium θ_t and $S_t(x)$ are determined by (4) and (8) in this case.

2.4 Measurement

Equilibrium worker transition rates and flows are measured as follows. A worker who is unemployed in week t becomes employed in week $t+1$ with probability $f(\theta_t) = A\theta_t^{1-\alpha}$. Thus, for all specifications the measured job finding rate and number of UE flows for week $t+1$ are

$$JFR_{t+1} = A\theta_t^{1-\alpha}, \quad UE_{t+1} = A\theta_t^{1-\alpha}u_t.$$

Moreover, in the exogenous separation version, a worker who is employed in week t becomes unemployed in week $t+1$ with probability s_t , giving the following measured separation rate and the number of EU flows:

$$SR_{t+1} = s_t, \quad EU_{t+1} = s_t(1-u_t).$$

Separation rates and EU flows in the two endogenous separation versions of the model depend on the distribution of x across existing matches. Let $e_t(x)$ denote the number of matches in week t having match-specific factors less than or equal to x ; note that $e_t(x^h)$ gives total employment. Since $S_t(x)$ is strictly increasing in x wherever $S_t(x) > 0$, there exists a value R_t such that $S_t(x) = 0$ if and only if $x \leq R_t$. Thus, separation into unemployment occurs at the start of week $t+1$ whenever $x \leq R_{t+1}$.⁶ In equilibrium, $e_{t+1}(x) = 0$ for $x \leq R_{t+1}$. The employment distribution differs depending on whether or not OJS is allowed.

Endogenous separation without OJS. In the absence of OJS, the employment distribution evolves according to:

$$e_{t+1}(x) = (1-s)\lambda[G(x) - G(R_{t+1})]e_t(x^h) + (1-s)(1-\lambda)[e_t(x) - e_t(R_{t+1})].$$

for $x \in (R_{t+1}, x^h)$. For $x = x^h$, it evolves according to:

$$e_{t+1}(x^h) = (1-s)\lambda[1 - G(R_{t+1})]e_t(x^h) + (1-s)(1-\lambda)[e_t(x^h) - e_t(R_{t+1})] + f(\theta_t)u_t, \quad (11)$$

which gives the evolution of the stock of employment. Next, total EU flows and the separation rate are, respectively, given by:

$$EU_{t+1} = se_t(x^h) + (1-s)\lambda G(R_{t+1})e_t(x^h) + (1-s)(1-\lambda)e_t(R_{t+1}), \quad (12)$$

⁶When $x = R_{t+1}$, the firm and worker could also choose to continue their match, as a matter of indifference. It is slightly more convenient for notational purposes to specify that separation occurs at the R_{t+1} margin.

$$SR_{t+1} = \frac{EU_{t+1}}{e_t(x^h)}. \quad (13)$$

The implied law of motion for unemployment is:

$$u_{t+1} = u_t + EU_{t+1} - UE_{t+1}. \quad (14)$$

Lastly, vacancies are determined simply by:

$$v_t = \theta_t u_t.$$

Endogenous separation with OJS. Allowing for the possibility of OJS somewhat complicates the evolution of labor market variables. First, it can be shown that there exists a value R_t^s such that the match surplus from OJS exceeds the surplus from continuing the match with no OJS if and only if $x < R_t^s$. In other words, OJS is chosen whenever $R_t < R_t^s$ and $x \in (R_t, R_t^s)$. Therefore, for $x \in (R_t, R_t^s)$,

$$\begin{aligned} e_{t+1}(x) = & (1-s)\lambda[G(x) - G(R_{t+1})][e_t(x^h) - e_t(R_t^s) + (1-f(\theta_t))e_t(R_t^s)] \\ & + (1-s)(1-\lambda)(1-f(\theta_t))[e_t(x) - e_t(R_{t+1})], \end{aligned}$$

while, for $x \in [R_t^s, x^h)$,

$$\begin{aligned} e_{t+1}(x) = & (1-s)\lambda[G(x) - G(R_{t+1})][e_t(x^h) - e_t(R_t^s) + (1-f(\theta_t))e_t(R_t^s)] \\ & + (1-s)(1-\lambda)[e_t(x) - e_t(R_t^s) + (1-f(\theta_t))(e_t(R_t^s) - e_t(R_{t+1}))]. \end{aligned}$$

In essence, the presence of OJS alters the evolution of the employment distribution in that OJS makes it possible for those on-the-job searchers to avoid endogenous separation into unemployment (when the search is successful) given that their match-specific factor starts at the highest level x^h in the following period. The law of motion for the total employment stock is written as:

$$\begin{aligned} e_{t+1}(x^h) = & (1-s)\lambda[1 - G(R_{t+1})][e_t(x^h) - e_t(R_t^s) + (1-f(\theta_t))e_t(R_t^s)] \\ & + (1-s)(1-\lambda)[e_t(x^h) - e_t(R_t^s) + (1-f(\theta_t))(e_t(R_t^s) - e_t(R_{t+1}))] \\ & + f(\theta_t)(u_t + e_t(R_t^s)). \end{aligned} \quad (15)$$

Note that (15) differs from the corresponding equation for the version without OJS (equation (11)), even though job-to-job transitions simply reshuffle workers within the employment pool. This property comes from the fact that when on-the-job searchers find a new job, they essentially avoid endogenous separation into unemployment. Accordingly, EU flows are measured differently in the model with OJS, relative to those in the model without OJS:

$$\begin{aligned} EU_{t+1} = & se_t(x^h) + (1-s)\lambda G(R_{t+1})[e_t(x^h) - e_t(R_t^s) + (1-f(\theta_t))e_t(R_t^s)] \\ & + (1-s)(1-\lambda)(1-f(\theta_t))e_t(R_{t+1}). \end{aligned} \quad (16)$$

The expressions for the separation rate and the law of motion for unemployment remain the same as (13) and (14), respectively. Lastly, vacancies are determined by:

$$v_t = \theta_t(u_t + e_t(R_t^s)).$$

3 Simulation

Before examining the quantitative properties of the different versions discussed above, this section discusses the calibration procedure. We then present the method to compute the stochastic dynamic equilibrium of the model and summarize the procedure to evaluate various quantitative aspects of the model.

3.1 Calibration

Two specifications of the exogenous separation version are considered: s_t may either be constant at s , or else follow an AR(1) process given by (5) with $\sigma_s > 0$. Combined with the two endogenous separation versions of the model, there are four specifications to calibrate. Parameter choices for the four cases are given in Table 1.

The parameters b , α and π are set to the standard values, as discussed by Mortensen and Nagypál (2007a). First, the flow value of unemployment is set to 0.7, which implies the replacement ratio of roughly 70%, given that that productivity is normalized to unity.⁷ We will later consider the calibration proposed by Hagedorn and Manovskii (2008), in which b is set to a higher value. The elasticity parameter of the matching function α and the bargaining weight of workers π are both set to 0.7. This choice is close to the value (0.72) used in Shimer (2005). Mortensen and Nagypál (2007a) argue that 0.72 is empirically too high and estimate it at 0.45. However, a more recent paper by Brügemann (2008) reconciles the two estimates and proposes estimates between 0.54 and 0.63. In this paper, we use the value originally estimated by Shimer (2005) as a benchmark value and then later examine the robustness of our results when $\alpha = \pi = 0.5$ is used.

Calibration of the vacancy posting cost c draws on survey evidence on employer recruitment behavior. Survey results cited in Barron et al. (1997) point to an average vacancy duration of roughly three weeks. Moreover, Barron and Bishop (1985) show an average of about nine applicants for each vacancy filled, with two hours of work time required to process each application. These figures suggest an average investment of 20 hours per vacancy filled, or 6.7 hours per week the vacancy is posted. This amounts to 17 percent of a 40-hour workweek; thus, it is reasonable to assign this value to c , given that weekly productivity is normalized to unity.

Next, to ensure comparability across different versions of the model, the highest value of idiosyncratic productivity x^h is adjusted to generate mean match productivity of unity in all cases. The cost of searching on the job a in the OJS specification is chosen so that

⁷Wage is close to productivity and thus the replacement ratio is nearly 70%.

Table 1: Parameter Values for Benchmark Calibrations

Parameter	Constant	AR(1)	Endogenous without OJS	Endogenous with OJS
b	0.7	0.7	0.7	0.7
c	0.17	0.17	0.17	0.17
a	—	—	—	0.13
A	0.095	0.095	0.094	0.096
α	0.7	0.7	0.7	0.7
π	0.7	0.7	0.7	0.7
x_h	1	1	1.15	1.1
s	0.005	0.005	0.0034	0.0042
ρ_s	—	0.965	—	—
σ_s	—	0.018	—	—
λ	—	—	0.085	0.085
σ_x	—	—	0.16	0.214
ρ_z	0.9895	0.9895	0.9895	0.9895
σ_z	0.0034	0.0034	0.0034	0.0034
β	0.9992	0.9992	0.9992	0.9992

Notes. b : unemployment payoff; c : vacancy posting cost; a : OJS cost; A : scale parameter of the matching function; α : elasticity parameter of the matching function; π : worker bargaining weight; x^h : highest value of idiosyncratic productivity; s : exogenous separation rate; ρ_s : persistence of the exogenous separation rate process; σ_s : S.D. of the innovation to the exogenous separation rate process; λ : arrival rate of the idiosyncratic shock; σ_x : S.D. of the idiosyncratic shock; ρ_z : persistence of the aggregate productivity process; σ_z : S.D. of the aggregate productivity shock; β : discount factor.

the mean monthly job-to-job transition rate in the simulated data matches the value of 3.2 percent calculated by Moscarini and Thompson (2007) using the CPS data.

The parameters for the aggregate productivity process ρ_z and σ_z are set to the values proposed by Hagedorn and Manovskii (2008).⁸ The value of the weekly discount factor β is consistent with an annual interest rate of four percent.

Selection of the remaining parameters relies on monthly job finding and separation rate data from Fujita and Ramey (2006). These data derive from the CPS for the 1976-2005 period and are adjusted for margin error and time aggregation error. In all cases, the parameters A and s are chosen to ensure that the simulated data generate mean monthly job finding and separation rates of 34 percent and two percent, respectively, consistent with the Fujita-Ramey evidence.

In the AR(1) specification, ρ_s and σ_s are selected to match the standard deviation and first-order autocorrelation of the simulated separation rate series, aggregated to quarterly,

⁸In the earlier version of the paper, we set these two parameters based on the VAR evidence in Fujita and Ramey (2007), in which case these two parameters are set to 0.99 and 0.027, respectively, instead of the values in Table 1. We find that the results are robust with respect to this alternative calibration.

logged and HP filtered (with smoothing parameter 1,600), to the empirical values of these moments in the Fujita-Ramey data. For endogenous separation versions of the model, the standard deviation of idiosyncratic shock σ_x and the arrival rate λ are chosen in a similar manner. That is, σ_x is used to achieve the standard deviation of the cyclical component of the empirical separation rate series. Finally, we adjust λ to match the first-order autocorrelation coefficient of the series. The chosen value 0.085 implies a mean waiting time of three months between switches of the match-specific productivity factor. The persistence of the separation rate is useful in identifying the arrival rate in that more (less) frequent arrival of the shock tends to raise (lower) the persistence. In Section 5, we will discuss the intuition behind this property of the model, when we examine the robustness of the results with respect to a different value of the arrival rate.

3.2 Solution Method

The model consists of the free entry condition (4), the surplus equation (6), (7) or (8), and the driving processes (1) and (5). To solve the model, let the stochastic elements be represented on grids. The method of Tauchen (1986) is used to represent the processes z_t and s_t as Markov chains having state spaces $\{z_1, \dots, z_I\}$ and $\{s_1, \dots, s_K\}$ and transition matrices $\Delta^z = [\delta_{ij}^z]$ and $\Delta^s = [\delta_{kl}^s]$, where $\delta_{ij}^z = \Pr\{z_{t+1} = z_j | z_t = z_i\}$ and $\delta_{kl}^s = \text{Prob}\{s_{t+1} = s_l | s_t = s_k\}$. $G(x)$ is approximated by a discrete distribution with support $\{x_1, \dots, x_M\}$, satisfying $x_1 = 1/M$, $x_m - x_{m-1} = x^h/M$ and $x_M = x^h$. The associated probabilities $\{\gamma_1, \dots, \gamma_M\}$ are $\gamma_m = g(x_m)/M$ for $m = 1, \dots, M-1$, where $g(x)$ is the lognormal density, and $\gamma_M = 1 - \gamma_1 - \dots - \gamma_{M-1}$. Market tightness and match surplus may be represented as:

$$\theta_t = \theta(z_i, s_k), \quad S_t(x_m) = S(z_i, s_k, x_m),$$

where z_i and s_k are the aggregate states prevailing in period t . Equations (4), (6) and (7) take the forms, for $i = 1, \dots, I$, $k = 1, \dots, K$, $m = 1, \dots, M$:

$$\beta A \theta(z_i, s_k)^{-\alpha} (1 - \pi) \sum_{j,l} \delta_{ij}^z \delta_{kl}^s S(z_j, s_l, x^h) = c, \quad (17)$$

$$S(z_i, s_k, x^h) = \max \left\{ z_i x^h - b + \beta(1 - s_k - \beta A \theta(z_i, s_k)^{1-\alpha} \pi) \sum_{j,l} \delta_{ij}^z \delta_{kl}^s S(z_j, s_l, x^h), 0 \right\}, \quad (18)$$

$$\begin{aligned} S(z_i, s_k, x_m) = & \max \left\{ z_i x_m - b + \beta(1 - s_k) \lambda \sum_{j,l,n} \delta_{ij}^z \delta_{kl}^s \gamma_n S(z_j, s_l, x_n) \right. \\ & + \beta(1 - s_k)(1 - \lambda) \sum_{j,l} \delta_{ij}^z \delta_{kl}^s S(z_j, s_l, x_m) \\ & \left. - \beta A \theta(z_i, s_k)^{1-\alpha} \pi \sum_{j,l} \delta_{ij}^z \delta_{kl}^s S(z_j, s_l, x^h), 0 \right\}, \quad (19) \end{aligned}$$

and similarly for (8). Numerical solutions are obtained via backward substitution. For example, let $\theta^T(z_i, s_k)$ and $S^T(z_i, s_k, x^h)$ be the functions obtained after T iterations of (17) and (18). At iteration $T + 1$, these functions are updated to

$$S^{T+1}(z_i, s_k, x^h) = \max \left\{ z_i x^h - b + \beta(1 - s_k - \beta A \theta^T(z_i, s_k)^{1-\alpha} \pi) \sum_{j,l} \delta_{ij}^z \delta_{kl}^s S^T(z_j, s_l, x^h), 0 \right\}$$

$$\theta^{T+1}(z_i, s_k) = \left(\frac{\beta A (1 - \pi)}{c} \sum_{j,l} \delta_{ij}^z \delta_{kl}^s S^{T+1}(z_j, s_l, x^h) \right)^{\frac{1}{\alpha}} .$$

Convergence follows as a consequence of the saddlepoint stability property of the matching model, which makes for stability in the backward dynamics.⁹

3.3 Evaluation procedure

The empirical data series used for purposes of model evaluation are constructed as follows. Job finding and separation rates, and UE and EU flows are quarterly averages of the monthly series from Fujita and Ramey (2006), covering 1976Q1-2005Q4. Employment and the unemployment rate are quarterly averages of the CPS official monthly series covering the same period. The productivity series is obtained by dividing quarterly GDP by the employment series in the CPS. Vacancies are measured as quarterly averages of the monthly composite Help-Wanted index constructed by Barnichon (2010). All quarterly series are logged and HP filtered, with a smoothing parameter of 1,600.

To conform with the empirical series, the weekly data from the model are averaged to quarterly frequency, logged and HP filtered using smoothing parameter 1,600. Each simulated quarterly series consists of 620 observations, of which the last 120 are used to calculate the reported statistics. For each of the four specifications, 1,000 replications are run and averages of the statistics across the replications are presented.

4 Main Results

This section discusses the main results of the paper. Table 2 presents various second moment properties of the four specifications of the MP model as well as those of the observed data. In each panel of the table, we present the standard deviation, the elasticity with respect to labor productivity, the correlation coefficient with labor productivity, and the autocorrelation coefficient for the seven variables listed across the first row of the table. Later, we also consider cross correlations between unemployment and vacancies, i.e., the Beveridge correlations, and between the separation rate and the job finding rate.

⁹In solving the model, $I = K = 13$ and $M = 200$ are chosen. The tolerance for pointwise convergence of $\theta(z_i, s_k)$ and $S(z_i, s_k, x_m)$ is 10^{-8} .

Table 2: Second Moment Properties: Benchmark Calibration

X_t	u_t	JFR_t	SR_t	UE_t	EU_t	v_t	$\frac{v_t}{u_t}$
(a) Data							
σ_X	0.096	0.077	0.058	0.042	0.052	0.126	0.218
$cor(p_t, X_t)$	-0.460	0.369	-0.535	-0.337	-0.521	0.564	0.527
$\frac{cov(p_t, X_t)}{\sigma_p}$	-5.914	3.786	-4.157	-1.879	-3.644	9.524	15.437
$cor(X_t, X_{t-1})$	0.926	0.804	0.631	0.416	0.560	0.920	0.930
(b) Exogenous separation: constant							
σ_X	0.011	0.013	-	0.006	0.001	0.034	0.043
$cor(p_t, X_t)$	-0.884	0.997	-	0.572	0.861	0.988	0.999
$\frac{cov(p_t, X_t)}{\sigma_p}$	-0.722	0.961	-	0.258	0.042	2.491	3.213
$cor(X_t, X_{t-1})$	0.860	0.768	-	0.395	0.860	0.706	0.768
(c) Exogenous separation: AR(1)							
σ_X	0.048	0.013	0.058	0.046	0.056	0.055	0.043
$cor(p_t, X_t)$	-0.203	0.994	-0.004	0.068	0.006	0.594	0.996
$\frac{cov(p_t, X_t)}{\sigma_p}$	-0.724	0.958	-0.002	0.252	0.040	2.479	3.203
$cor(X_t, X_{t-1})$	0.781	0.767	0.620	0.768	0.606	0.746	0.767
(d) Endogenous separation without OJS							
σ_X	0.056	0.013	0.057	0.044	0.054	0.021	0.042
$cor(p_t, X_t)$	-0.926	0.994	-0.909	-0.867	-0.898	-0.446	0.998
$\frac{cov(p_t, X_t)}{\sigma_p}$	-4.142	1.006	-4.134	-3.064	-3.898	-0.773	3.367
$cor(X_t, X_{t-1})$	0.828	0.768	0.613	0.827	0.590	0.652	0.768
(e) Endogenous separation with OJS							
σ_X	0.059	0.014	0.058	0.048	0.055	0.042	0.096
$cor(p_t, X_t)$	-0.888	0.995	-0.908	-0.802	-0.903	0.969	0.998
$\frac{cov(p_t, X_t)}{\sigma_p}$	-3.992	1.024	-4.010	-2.891	-3.766	3.057	7.229
$cor(X_t, X_{t-1})$	0.848	0.768	0.705	0.847	0.687	0.679	0.768

Notes. σ_X : standard deviation of the variable X ; $cor(p_t, X_t)$: correlation between p_t and X_t ; $cov(p_t, X_t)/\sigma_X$: elasticity of labor productivity p_t with respect to X_t ; $cor(X_t, X_{t-1})$: correlation between X_t and X_{t-1} . Data sources: u_t : quarterly average of monthly official BLS unemployment rate; transition rates and worker flows: quarterly average of the monthly series constructed by Fujita and Ramey (2006); v_t : quarterly average of monthly composite Help-Wanted index constructed by Barnichon (2010); p_t : quarterly output per worker constructed by dividing real GDP by CPS employment series; sample period: 1976Q1-2005Q4. All series are logged and HP filtered, with smoothing parameter 1,600. See subsection 2.4 for measurement in the model. Simulated data are quarterly averages of weekly series, logged and HP filtered, with smoothing parameter 1,600. Each replication computes simulated statistics from a sample of 120 quarterly observations. Reported statistics are averages over 1,000 replications.

4.1 Unemployment and Worker Transition Rates

The first three columns of Table 2 compare the empirical moments of unemployment and worker transition rates with the values obtained from the four specifications of the model.

The empirical standard deviation of unemployment, equalling 9.6 percent, is over 6 times greater than the value of roughly 1.5 percent generated by the constant separation rate specification. This conforms to the observation of Costain and Reiter (2008) and Shimer (2005) that the MP model with a constant separation rate produces far too little unemployment volatility.

However, the empirical separation rate is not in fact constant, as it has a standard deviation of 5.8 percent. The other three versions of the MP model, which allow for fluctuations in the separation rate, are calibrated to match the standard deviation of the empirical series. All three specifications yield significantly greater unemployment volatility. The standard deviation of unemployment in the OJS specification, for example, is 5.9 percent, or over 60 percent of its empirical value. Thus, incorporating variability at the separation margin, under any of the three specifications, greatly enhances the ability of the MP model to produce realistic unemployment volatility.

At the same time, all four specifications of the MP model yield highly unrealistic volatility of the job finding rate, with the empirical standard deviation being 5.5 times the simulated value in each specification. Improving the model's performance at the separation margin does not mitigate its problems at the job finding margin.

With respect to contemporaneous correlations with productivity, the constant, endogenous and OJS specifications each produce strong negative comovement between unemployment and productivity, while the AR(1) specification generates little comovement. All four specifications give rise to strong positive productivity comovement for the job finding rate. Actually, the positive correlation in the model is much stronger than the empirical value. This is because the model does not adequately replicate the sluggishness of the labor market, as pointed out by Fujita and Ramey (2007), who introduce a one-time job creation cost in the exogenous separation version of the model to address this problem. The two exogenous separation specifications fail to replicate the negative correlation between productivity and the separation rate that is a robust feature of the data. The two endogenous separation rate specifications succeed in capturing this negative correlation.

Elasticities of the variables with respect to productivity are shown in the next row.¹⁰ The productivity elasticities offer somewhat cleaner measures of comovement, insofar as they reflect the effects of variations in productivity in isolation from other disturbances; see Mortensen and Nagypál (2007a). The elasticities may also be interpreted as rough measures of responsiveness to productivity shocks. For unemployment, the empirical productivity elasticity of -5.9 is roughly eight times greater in magnitude than the elasticities produced by the two exogenous separation specifications. However, when the separation margin is endogenized, whether OJS is allowed or not, the elasticity increases considerably to a level not far from the empirical counterpart.

Findings are similar for the separation rate elasticities, where the exogenous separation specification provides highly unrealistic values, while those of the endogenous separation specifications are empirically plausible. Across all four specifications, however, the produc-

¹⁰These productivity elasticities are computed as follows. Let p_t denote productivity in quarter t , and let X_t be any series. Then the productivity elasticity is $Cov(p_t, X_t)/SD(p_t)$.

tivity elasticities of the job finding rate are far too low: the empirical value is 3.8, while the simulated values are always around 1.

In summary, introducing variability at the separation margin greatly magnifies the degree of unemployment volatility generated by the MP model, whether the separation rate is determined exogenously or endogenously. Moreover, when the separation rate is endogenous, the model generates realistic responsiveness of unemployment and the separation rate to productivity shocks, whereas the exogenous separation versions yield little or no responsiveness. For all of the specifications considered, the simulated job finding rate is deficient in both its volatility and its responsiveness to productivity.

4.2 Worker Flows

The fourth and fifth columns consider gross flows of workers between unemployment and employment. As panel (b) of Table 2 indicates, the constant separation rate specification produces almost no volatility in UE and EU flows. This is contrary to the data, where the standard deviations for both flows are roughly half of the standard deviation of unemployment. The three specifications with variable separation rates, in contrast, do a good job of matching the empirical standard deviations of both UE and EU flows. Thus, variability at the separation margin is crucial for producing realistic variability in worker flows.

In terms of correlation with productivity, the constant separation rate specification gives rise to a counterfactual pattern that worker flows exhibit a strong positive correlation with productivity. This contradicts the substantial negative correlation seen in the data. In the constant separation rate model, worker flows are driven principally by procyclical movements in the job finding rate, allowing little scope for explaining their observed countercyclical movements. The AR(1) specification, in turn, yields essentially acyclical movements in worker flows, reflecting the fact that exogenous separation rate shocks are uncorrelated with the productivity process. The two endogenous separation rate specifications, on the other hand, produce strong negative correlations between productivity and worker flows.

Results on productivity elasticity indicate that worker flows are almost entirely unresponsive to productivity in the two exogenous separation rate specifications, whereas they exhibit strong negative responses in the two endogenous separation specifications.

Note that the constant separation rate specification produces procyclical separation (EU) flows because employment is procyclical and thus the number of workers who separate is procyclical. For a symmetric reason, hiring (UE) flows would be countercyclical if the job finding rate were to be treated as constant. The countercyclicity of both EU and UE flows is a salient feature that indicates the importance of the separation margin in understanding the labor market dynamics, as emphasized by Fujita and Ramey (2006) and Fujita (2011).

4.3 Vacancies and Market Tightness

Vacancies and market tightness are considered in the last two columns of Table 2. First, all four specifications produce insufficient volatility of both vacancies and market tightness, consistent with the low volatility of the job finding rate discussed earlier. The standard

deviation of market tightness in the OJS version is significantly greater than in the other specifications, however. Observe that the standard deviation of tightness in the OJS specification is more than twice that of the no-OJS version, even though the differences in the standard deviations of unemployment and vacancies are not large. This comes from the fact that the two variables are strongly negatively correlated in the version with OJS, while in the version without OJS, they are positively correlated. We elaborate on this point in the following paragraph.

Both versions of the exogenous separation model replicate the procyclical movements of vacancies seen in the data, whereas the endogenous separation model without OJS yields countercyclical movements. The latter finding reflects conflicting effects on the incentive to post vacancies. Following a negative productivity shock, the returns to forming a new match are relatively low, reducing vacancy posting incentives. This effect drives vacancies downward in the constant and AR(1) versions of the model. In the endogenous separation version without OJS, however, the separation rate rises in response to the productivity shock, pushing up the number of unemployed workers. This raises the vacancy matching probability and enhances the incentive to post vacancies. On balance, the latter effect dominates, and vacancies become negatively correlated with productivity. The OJS model, however, produces a strong positive correlation between vacancies and productivity, despite the fact that the separation rate is determined endogenously. With OJS, a negative productivity shock induces a fall in the number of employed searchers, which partially offsets the rise in unemployment.¹¹ Thus, endogenous separation is consistent with realistic vacancy comovement once OJS is incorporated. Note finally that all four specifications yield positive productivity comovement for market tightness, consistent with the data.

The empirical productivity elasticity of vacancies far exceeds the elasticities obtained from all four specifications, in line with the performance in terms of the standard deviations. For market tightness, however, the OJS version of the model performs noticeably better than the other three versions, generating productivity elasticity that is roughly one-half of its empirical value.

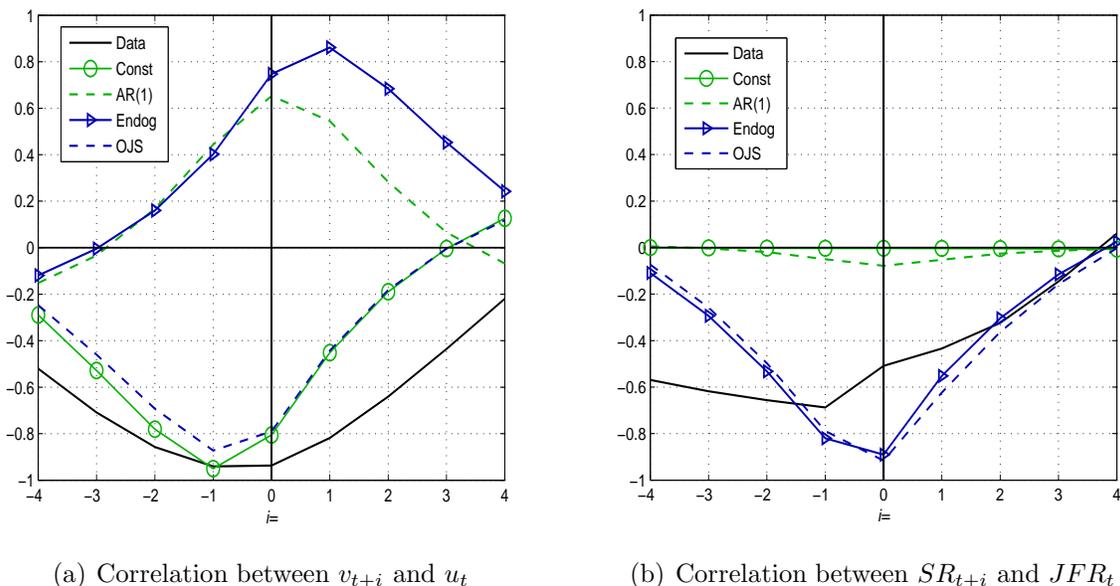
In summary, the OJS version of the model performs the best among all versions considered. It matches all correlation patterns. In particular, the OJS version overcomes the deficiency of the endogenous separation model (without OJS) that vacancies become countercyclical. Even with OJS, however, the MP model fails to match the volatility of vacancies and thereby the job finding rate.

4.4 Cross Correlations

Next, we examine whether the various versions of the MP model can replicate the dynamic relationship between the key labor market variables. Specifically, we consider cross correlations between unemployment and vacancies (i.e., the Beveridge curve) and between the separation rate and the job finding rate.

¹¹The mechanism discussed here explains why endogenizing the separation margin without OJS reduces the volatility of vacancies relative to the constant separation rate specification and introducing OJS restores the level of the volatility.

Figure 1: Cross Correlations



(a) Correlation between v_{t+i} and u_t

(b) Correlation between SR_{t+i} and JFR_t

Panel (a) of Figure 1 presents the Beveridge correlations, where the current-period unemployment rate is associated with future and lagged values of vacancies up to four quarters. First observe that a large value of contemporaneous correlation between unemployment and vacancies observed in the data is reasonably well matched by the value generated by the constant separation rate specification. The AR(1) specification, in contrast, produces a highly counterfactual value of 0.65, and for the endogenous separation specification the value is an even more unrealistic 0.75. In the AR(1) model, a small positive separation rate shock induces a large inflow into unemployment, because the stock of employed workers is relatively large. It becomes easier for workers to find a job, while productivity is unchanged, and thus incentives to post vacancies rise. A related effect operates in the endogenous separation model, where a negative productivity shock drives up unemployment, making workers easier to find and raising the incentive to post vacancies. For the OJS model, the unemployment-vacancy contemporaneous correlation amounts to -0.79 , which is reasonably close to the empirical value. Here, procyclical movements in the number of employed searchers lead to procyclical changes in vacancy posting incentives, giving rise to a realistic Beveridge correlation.

With respect to the lead-lag relationship, the observed data suggest some tendency for vacancies to lead unemployment.¹² Qualitatively, this pattern is captured well in the constant separation rate version and the OJS version of the model. This reflects the mechanics of the model, wherein the search friction produces some lagged response in unemployment after the response in vacancy posting. In the AR(1) version and the endogenous separation version

¹²For this, observe that correlations between lagged values of vacancies and current unemployment tend to be larger (in absolute value) than those between future values of vacancies and current unemployment.

Table 3: Parameter Values for Alternative Calibrations: $\alpha = \pi = 0.5$

	Constant	Endog.	Endog. with OJS		Constant	Endog.	Endog. with OJS
b	0.7	0.7	0.7	s	0.005	0.0019	0.0038
c	0.17	0.17	0.17	λ	—	0.085	0.085
a	—	—	0.08	σ_x	—	0.27	0.326
A	0.068	0.067	0.071	ρ_z	0.9895	0.9895	0.9895
α	0.5	0.5	0.5	σ_z	0.0034	0.0034	0.0034
π	0.5	0.5	0.5	β	0.9992	0.9992	0.9992
x_h	1	1.18	1.125				

without OJS, however, the feedback from the movement of the separation rate into vacancy posting as discussed above erases this feature, generating the tendency that unemployment leads vacancies.

Panel (b) presents cross correlations between the two transition rates, where the current-period job finding rate is associated with future and lagged values of the separation rate. In the data, job finding and separation rates exhibit strong negative correlation contemporaneously. The correlations are essentially zero in the two exogenous separation specifications. The two endogenous separation specifications, on the other hand, produce strong negative contemporaneous correlations, on the order of -0.9 . The latter specifications achieve the correct transition rate comovement chiefly because the two rates themselves respond realistically to the common underlying productivity process.

Turning to the lead-lag relationship, the data imply that the separation rate leads the job finding rate, which is indicated by the fact that larger negative correlations are achieved when lagged values of the separation rate are associated with the current-period job finding rate. While the correlations for the two endogenous separation versions exhibit a slight negative phase shift, they fail to adequately capture the overall dynamic pattern. Of course, all of these correlations are zero in the constant separation rate model.

5 Robustness

This section considers how the results change under alternative calibrations. First, we consider the case in which the worker bargaining power together with the elasticity parameter of the matching function are set at a lower value of 0.5.¹³ The model is re-calibrated to achieve the same moment conditions discussed above. Next, we examine the effect of raising the

¹³As mentioned before, our benchmark calibration of these two parameters is based on Shimer (2005) who estimates the elasticity with respect to unemployment at 0.72. Mortensen and Nagypál (2007a) estimate the elasticity parameter at 0.45 using a different method. Brügemann (2008) then reconciles the difference between these two estimates and proposes a value between 0.54 and 0.63. Our choice of 0.5 is thus a conservative value for checking the robustness of our results.

Table 4: Second Moment Properties: Calibration with $\alpha = \pi = 0.5$

X_t	u_t	JFR_t	SR_t	UE_t	EU_t	v_t	$\frac{v_t}{u_t}$
(a) Constant separation							
σ_X	0.019	0.022	—	0.010	0.001	0.029	0.045
$cor(p_t, X_t)$	-0.884	0.997	—	0.573	0.859	0.953	0.999
$\frac{cov(p_t, X_t)}{\sigma_p}$	-1.244	1.659	—	0.447	0.072	2.083	3.326
$cor(X_t, X_{t-1})$	0.861	0.768	—	0.397	0.861	0.636	0.768
(b) Endogenous separation without OJS							
σ_X	0.064	0.023	0.057	0.043	0.054	0.026	0.045
$cor(p_t, X_t)$	-0.920	0.992	-0.915	-0.815	-0.902	-0.531	0.997
$\frac{cov(p_t, X_t)}{\sigma_p}$	-4.828	1.831	-4.282	-2.904	-4.001	-1.149	3.677
$cor(X_t, X_{t-1})$	0.836	0.768	0.612	0.830	0.584	0.706	0.768
(c) Endogenous separation with OJS							
σ_X	0.067	0.024	0.056	0.045	0.053	0.032	0.094
$cor(p_t, X_t)$	-0.907	0.995	-0.943	-0.770	-0.936	0.963	0.998
$\frac{cov(p_t, X_t)}{\sigma_p}$	-4.624	1.849	-4.036	-2.680	-3.761	2.341	7.130
$cor(X_t, X_{t-1})$	0.853	0.768	0.708	0.843	0.687	0.648	0.768

Notes. See notes to Table 2 for details on data construction and simulation. See panel (a) of Table 2 for empirical moments. Parameter values are presented in Table 3.

arrival rate of the idiosyncratic shock in the endogenous separation versions of the model. In this latter experiment, we change only this parameter while keeping other parameters at the same values as before.¹⁴

5.1 Lower Bargaining Weight and Elasticity of Matching Function

Calibrated parameter values in the constant separation rate version and the two endogenous separation versions are presented in Table 3. The AR(1) version is omitted for brevity. The results are presented in Table 4. The results are very similar to those under the benchmark calibration. That is, the endogenous separation version without OJS performs much better than the constant separation rate version in terms of volatility of the separation rate and unemployment as well as productivity correlation of the separation rate; all versions face the same difficulty of generating enough volatility of vacancies and the job finding rate; and the OJS version fixes the counterfactual behavior of vacancies in the endogenous separation version without OJS and generates a relatively large volatility of market tightness. Evaluations based on cross correlations also yield the same conclusions as in the benchmark calibration.

¹⁴The main purpose of this second experiment is to highlight the effect of this parameter change, particularly on the persistence of the separation rate, rather than to examine the sensitivity of the results. Therefore, we did not recalibrate the model in the second experiment.

Table 5: Parameter Values for Alternative Calibrations: $\lambda = 0.125$

	Endogenous	Endogenous with OJS		Endogenous	Endogenous with OJS
b	0.7	0.7	s	0.0037	0.0043
c	0.17	0.17	λ	0.125	0.125
a	—	0.13	σ_x	0.215	0.271
A	0.093	0.096	ρ_z	0.9895	0.9895
α	0.7	0.7	σ_z	0.0034	0.0034
π	0.7	0.7	β	0.9992	0.9992
x_h	1.23	1.16			

5.2 More Frequent Shock Arrival

Recall that in our benchmark calibration, we used the arrival rate parameter to match the persistence of the separation rate. This moment is useful in identifying the arrival rate because, in the model, more frequent shock arrival tends to *raise* the persistence of the separation rate. To see this point, suppose that the idiosyncratic shock is i.i.d. over time. This property implies that the separation rate is calculated simply as the mass of employment relationships whose new productivity draws come below the threshold level R_t . This feature recurs every period. On the other hand, when the idiosyncratic shock is persistent, increases in the separation rate in the face of a recessionary shock tend to be concentrated in the impact period and then to come down in the following periods, even though underlying aggregate productivity is persistently low. In other words, once the matches that have become unviable due to the negative shock are destroyed on impact, only those that experience a switch of productivity can potentially be destroyed in the ensuing periods.

Tables 5 and 6 present the parameter values and simulation results, respectively when the arrival rate of the idiosyncratic shock is raised from 0.085 to 0.125. This corresponds to changing the mean arrival time of the idiosyncratic shock from 3 months to 2 months. Again, the main results that we have already discussed so far remain the same. The main difference can be observed in the persistence of the separation rate, especially in the version without OJS, in which the first order autocorrelation coefficient increases from 0.61 to 0.67. Accordingly, persistence of EU flows also increases. When OJS is allowed, the effect is relatively minor. This is because the job finding rate directly affects the separation rate, thus reducing the impact of the concentration effect mentioned in the previous paragraph on the behavior of the separation rate (see (12) and (16)).

6 Hagedorn-Manovskii Calibration

Lastly, we look at implications of the calibration proposed by Hagedorn and Manovskii (2008, henceforth HM) in our setup. This is a natural extension given the finding so far that insufficient volatility of the job finding rate remains a weakness of the model regardless of the

Table 6: Second Moment Properties: Calibration with More Frequent Shock Arrival

X_t	u_t	JFR_t	SR_t	UE_t	EU_t	v_t	$\frac{v_t}{u_t}$
(a) Endogenous separation without OJS							
σ_X	0.058	0.013	0.059	0.047	0.056	0.025	0.042
$cor(p_t, X_t)$	-0.900	0.993	-0.922	-0.829	-0.916	-0.393	0.997
$\frac{cov(p_t, X_t)}{\sigma_p}$	-4.232	1.014	-4.372	-3.134	-4.122	-0.837	3.394
$cor(X_t, X_{t-1})$	0.842	0.768	0.674	0.842	0.655	0.695	0.768
(b) Endogenous separation with OJS							
σ_X	0.058	0.014	0.057	0.047	0.053	0.044	0.096
$cor(p_t, X_t)$	-0.869	0.995	-0.894	-0.770	-0.890	0.974	0.997
$\frac{cov(p_t, X_t)}{\sigma_p}$	-3.809	1.020	-3.817	-2.711	-3.585	3.190	7.186
$cor(X_t, X_{t-1})$	0.850	0.768	0.720	0.848	0.703	0.685	0.768

Notes. See notes to Table 2 for details on data construction and simulation. Parameter values are presented in Table 5

way the separation margin is modeled. For brevity, only the constant and OJS specifications are considered. The question is whether the HM calibration can raise the volatility of the model without adversely influencing other desirable features of the OJS version of the model.

6.1 Calibration Strategy

HM propose an approach to calibrating the MP model that draws on wage and profit data. In all four specifications of the MP model, the wage rate determined by Nash bargaining may be expressed as:

$$w_t(x) = (1 - \pi)b + \pi(z_t x + \theta_t c),$$

where x is identically equal to x^h in the exogenous separation specifications. HM point out that under standard calibrations, the empirical productivity elasticity of wages is much lower than the elasticity generated by the model. They propose an alternative calibration strategy that aims to match this elasticity, along with the empirical relationship between mean wage and profit levels.

To assess the implications of the HM calibration, this paper follows Hornstein et al. (2005) in varying the calibrated values of b and π in order to match the productivity elasticity of wages and the steady-state wage-productivity ratio to the values 0.5 and 0.97, respectively.

The new calibrations are reported in Table 7. As noted by Hornstein et al. (2005), matching the empirical statistics requires large increases in the b parameter and large decreases in the π parameter. For the constant separation rate model, the A parameter is adjusted to match the mean job finding rate, while for the OJS model the parameters x^h , s and σ_x are also adjusted to normalize mean productivity and match the mean and standard deviation of the separation rate. We fix the shock arrival rate at 0.085 as in the benchmark calibration. Importantly, under the HM calibration the volume of job-to-job transitions is essentially

Table 7: Parameter Values for Hagedorn-Manovskii Calibrations

	Constant	Endogenous with OJS		Constant	Endogenous with OJS
b	0.93	0.934	s	0.005	0.0048
c	0.17	0.17	λ	—	0.085
a	—	0	σ_x	—	0.124
A	0.065	0.061	ρ_z	0.9895	0.9895
α	0.7	0.7	σ_z	0.0034	0.0034
π	0.082	0.063	β	0.9992	0.9992
x_h	1	1.13			

zero, even when the search cost parameter a is set to zero; we cannot match the evidence from Moscarini and Thompson (2007) used in the other calibrations. The model is solved and simulated according to the procedures discussed earlier.

6.2 Results

Results are presented in Table 8. One can immediately see that the HM calibration produces much more realistic volatility of unemployment and the job finding rate for both the constant and OJS specifications. Moreover, the job finding rate becomes highly responsive to productivity. The responsiveness of the separation rate in the OJS model declines considerably, however. This reflects the fact that, following a negative productivity shock, strong downward movement in the job finding rate reduces separation incentives by worsening workers' outside option.

The HM calibration enhances the volatility of UE flows in the constant separation rate model, but it does not appreciably raise the volatility of EU flows, nor does it mitigate the counterfactual procyclicality of worker flows implied by this specification. In the OJS version of the model, worker flows become less responsive to productivity. For UE flows, in particular, strong procyclical movements in the job finding rate serve to neutralize the countercyclical movements in the separation rate, leaving only small responsiveness to productivity. Recall that in the other calibrations, the OJS version successfully matches the countercyclicality of worker flows (see the discussion in subsection 4.2). However, this feature of the model is lost in the HM calibration. The HM calibration greatly improves the performance of both specifications in matching the empirical features of vacancies and market tightness. Finally, the Beveridge and transition rate correlations are essentially unaffected for the constant separation rate version, while they become somewhat smaller in magnitude for the OJS version.¹⁵ Although fluctuations in the number of employed searchers play virtually no role in this case, the correct Beveridge correlation emerges because vacancies become much more responsive to productivity fluctuations.

¹⁵Cross correlations for the HM calibration are similar to those shown in Figure 1; a corresponding figure for the HM case is available upon request.

Table 8: Second Moment Properties: Hagedorn-Manovskii Calibration

X	u_t	JFR_t	SR_t	UE_t	EU_t	v_t	$\frac{v_t}{u_t}$
(a) Data							
σ_X	0.096	0.077	0.058	0.042	0.052	0.126	0.218
$cor(p_t, X_t)$	-0.460	0.369	-0.535	-0.337	-0.521	0.564	0.527
$\frac{cov(p_t, X_t)}{\sigma_p}$	-5.914	3.786	-4.157	-1.879	-3.644	9.524	15.437
$cor(X_t, X_{t-1})$	0.926	0.804	0.631	0.416	0.560	0.920	0.930
(b) Constant separation							
σ_X	0.043	0.051	0.000	0.024	0.003	0.133	0.169
$cor(p_t, X_t)$	-0.870	0.981	-0.003	0.565	0.834	0.970	0.983
$\frac{cov(p_t, X_t)}{\sigma_p}$	-2.742	3.693	0.000	1.022	0.162	9.581	12.321
$cor(X_t, X_{t-1})$	0.861	0.765	0.996	0.405	0.860	0.703	0.765
(c) Endogenous separation with OJS							
σ_X	0.076	0.048	0.057	0.049	0.054	0.111	0.160
$cor(p_t, X_t)$	-0.720	0.969	-0.371	0.075	-0.219	0.935	0.973
$\frac{cov(p_t, X_t)}{\sigma_p}$	-3.970	3.563	-1.779	-0.297	-1.520	7.957	11.907
$cor(X_t, X_{t-1})$	0.830	0.766	0.416	0.605	0.402	0.669	0.766

Notes. See notes to Table 2 for data construction and simulation. Parameter values are presented in Table 7.

6.3 HM Calibration and Incentives for OJS

Incentives for OJS are linked to the size of the worker's bargaining weight. Using (10) and (9), the net gain in match surplus from searching versus not searching may be expressed as:

$$\begin{aligned} \text{Net gain from OJS} &= -a + f(\theta_t)\beta\mathbb{E}_t\pi S_{t+1}(x^h) \\ &\quad - f(\theta_t)(1-s)\beta\mathbb{E}_t\left[\lambda\int_0^{x^h} S_{t+1}(y)dG(y) + (1-\lambda)S_{t+1}(x)\right]. \end{aligned}$$

Observe that the benefit of OJS derives from the prospect of starting a new match at the highest level of surplus, $S_{t+1}(x^h)$. The current worker-firm match obtains only proportion π of this surplus, however. Thus, at very low values of π , such as that associated with the HM calibration, worker-firm matches receive a very small share of the surplus from new matches, so incentives for OJS are low.

7 Conclusion

This paper considers four specifications of the standard MP model that differ in how they treat the separation margin. The specifications are calibrated at weekly frequency and solved using a nonlinear method. Allowing for realistic time variation of the separation rate

greatly increases the volatility of unemployment in the simulated data. In the specification with OJS, for example, the standard deviation of unemployment equals 60 percent of its empirical value. Thus, moving beyond constant separation rates goes a long way toward readdressing the problem of insufficient unemployment volatility in the MP model.

Both of the specifications with exogenous separation rates fail to reproduce the empirical volatility and productivity responsiveness of the separation rate and worker flows. The endogenous separation specifications, in contrast, yield empirically reasonable behavior along these dimensions, and the specification with OJS also generates a realistic Beveridge curve correlation. Furthermore, the endogenous separation specifications imply more realistic dynamic interrelationships in comparison to the exogenous separation ones.

Two broad conclusions emerge from this analysis. First, the endogenous separation specification with OJS dominates both of the exogenous separation specifications along all dimensions considered. From the empirical standpoint, there seems to be no justification for assuming exogenous separation when modeling the separation margin.

Second, the OJS version of the MP model, as articulated in Pissarides (2000), does a remarkable job of matching labor market facts even under the standard calibration, although the model still generates insufficient volatility of the job finding rate and related variables. Adopting the HM calibration largely resolves the latter failings with some costs. In particular, the HM calibration implies virtually no job-to-job transitions in the OJS specification. It also makes the cyclical behavior of worker flows counterfactual. Exploring possible remedies for these issues appears to be an important topic for future research.

Lastly, the inability of the MP model to generate sluggish dynamics suggests that it does not deal adequately with key structural features of the labor market. Fujita and Ramey (2007) argue that fixed costs of vacancy creation may be salient in practice, and they show that introducing these costs into the MP model with constant separation rates leads to substantial improvements in its dynamic performance. Further investigations in this direction might be useful for deepening our understanding of labor market dynamics.

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