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ON THE INHERENT INSTABILITY OF
PRIVATE MONEY**

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Abstract

A fundamental question in monetary economics is whether private agents can provide a stable monetary framework in the absence of government intervention. I show that a purely private monetary system is inherently unstable due to the properties of endogenously determined limits on private money creation. In particular, there is a continuum of equilibria characterized by a self-fulfilling collapse of the exchange value of private liabilities that is accompanied by a persistent reduction in the demand for these liabilities. I interpret this class of equilibrium allocations as self-fulfilling banking crises. However, it is possible to formulate a government intervention that ensures the local determinacy of equilibrium so that the value of privately issued liabilities does not collapse.

Keywords: private money, self-fulfilling crises, central banking

JEL classifications: E42, E58, G21

1. INTRODUCTION

A fundamental question in monetary economics is the following: Can we rely on private agents to provide a stable monetary framework? Some economists have argued that many forms of government intervention in the monetary system can be a source of instability and that private markets are capable of providing a sound monetary framework. Others have argued that government control over the monetary system is necessary for achieving stability. In particular, there has been much emphasis on two polar views. Friedman (1959) has argued that the government should be the sole issuer of currency because private creation of government money substitutes will necessarily lead to excessive volatility in the supply of money and, consequently, an unstable monetary system. At the other extreme, we have the argument made in Hayek (1976) that private agents through private markets can effectively achieve desirable outcomes, even in the field of money and banking. According to this view, there is no reason to believe that any form of government intervention is necessary for the establishment of a stable monetary system.¹

Recent developments in monetary theory have provided a rigorous theoretical framework to evaluate the benefits of alternative monetary arrangements. In this paper, I study the properties of a purely private monetary system and investigate whether it is possible to achieve a stable monetary framework in the absence of government intervention. My main result shows that a purely private monetary system is inherently unstable due to the properties of endogenously determined limits on private money creation. Precisely, there exist multiple equilibrium allocations that are characterized by a self-fulfilling collapse of the value of privately issued liabilities that circulate as a medium of exchange. Because of this intrinsic instability, I characterize a government intervention that stabilizes the value of privately issued liabilities. Thus, my results indicate that a purely private monetary system requires a specific form of government intervention to ensure monetary stability (and I will show that the required intervention is small).

¹See also King (1983) and Friedman and Schwartz (1986) for a critical examination of some proposals for monetary reform.

In my formal analysis, I construct a model in which privately issued liabilities circulate as a means of payment. As in Cavalcanti and Wallace (1999a, 1999b), I assume that a subset of private agents, referred to as bankers, will have their actions publicly monitored so that it is possible to construct a monetary system in which the liabilities issued by bankers, referred to as bank notes, circulate as a means of payment. The acceptability of privately issued notes is endogenously determined so that each trader's decision to stop accepting notes, if he believes the issuer will not fulfill his promises, is sufficient to discipline note creation by private agents. And this is the key to understanding my analysis.

The intuition for the main result is as follows. When a seller decides whether to accept a privately issued note in exchange for something he owns or is able to produce, he worries about whether the private agent (a banker) who has issued the note is willing to fulfill her promise of paying noteholders on demand so that either the seller can redeem the note himself or the seller can use the note to acquire something he wants from someone else (in which case the person accepting the note as a means of payment will have the same concerns as the original seller). This means that the seller has to consider the issuer's incentives to fulfill her promise of converting privately issued notes into a specified amount of a commodity or outside money.

The willingness of the issuer to convert her notes depends on the profitability of the note-issuing business. If the present value of the flow of income derived from the note-issuing business is sufficiently large, then the issuer is less inclined to renege on her promises, given that the decision to renege on her promises will lead people not to trust her in future transactions (in which case she will no longer be able to issue notes that are widely accepted as a means of payment). Thus, to determine the willingness of an issuer to keep her promise, the seller must form beliefs regarding the flow of income derived from the note-issuing business, which in turn depends on beliefs regarding the exchange value of privately issued notes in future periods. My main result shows that there exist multiple beliefs that are consistent with an equilibrium outcome, including a class of beliefs that is characterized by a self-fulfilling collapse of the exchange value of privately issued circulating liabilities (i.e., a belief that privately issued notes will be redeemed at increasing discounts

over time). This class of equilibrium allocations can be surely interpreted as a self-fulfilling banking crisis.

In view of these difficulties, I formulate a government intervention that guarantees the local determinacy of equilibrium. This intervention requires the granting of privileges to a *small* fraction of bankers, who in turn will issue and retire notes according to a government policy. These privileges are such that each banker is willing to voluntarily implement the monetary policy prescribed by the monetary authority so there is no need for the establishment of a government monopoly. By tying the equilibrium value of privately issued notes to a path of government policy, there exists an intervention that provides a condition for determining the initial choice of the equilibrium value of private notes in such a way that the value of these notes remains stable over time. Finally, I argue that the local determinacy of equilibrium is sufficient to ensure the stability of the monetary system.

2. RELATED LITERATURE

The literature on inside money is vast.² To emphasize my contribution to this literature, it is helpful to explain how my analysis relates to two particular papers. Cavalcanti, Erosa, and Temzelides (1999), using a standard random-matching model, have shown that a private monetary system can be stable in the sense that it is possible to show the existence of a stationary equilibrium. They introduce a mechanism for note exchange that makes the creation of private money possible. Such a mechanism imposes the condition that any banker who fails to redeem his notes on demand will lose his note-issuing privileges. As a result, it is possible to obtain cooperation owing to the threat of exclusion from the business of issuing notes, disciplining the amount of notes issued by any individual banker. In particular, they have shown that a version of the law of reflux holds, guaranteeing that

²Some prominent papers analyzing the issue include Champ, Smith, and Williamson (1996), Cavalcanti, Erosa, and Temzelides (1999), Cavalcanti and Wallace (1999a, 1999b), Kahn and Roberds (1999), Smith and Weber (1999), Williamson (1999), Azariadis, Bullard, and Smith (2001), Li (2001, 2006), Martin and Schreft (2006), Berentsen (2006), Mills (2007), He, Huang, and Wright (2008), and Gu, Mattesini, Monnet, and Wright (forthcoming), among many others.

bankers do not overissue notes. Their notion of stability is restricted to the existence of a stationary equilibrium with private note creation. My goal is precisely to characterize the complete set of equilibrium allocations (including nonstationary allocations), and I show that the analysis of nonstationary equilibria matters for the conclusions regarding the stability of a private monetary system.

Azariadis, Bullard, and Smith (2001) have characterized the dynamic properties of a purely private monetary system and a hybrid system in which privately and publicly issued notes coexist. The authors construct an overlapping generations model in which trade is imperfectly coordinated due to spatial separation. As a result, privately issued liabilities can circulate as a medium of exchange. In contrast to their analysis, my framework emphasizes the properties of endogenously determined limits on private money creation. This emphasis results in very different conclusions. In particular, I show that a purely private monetary system can result in very large fluctuations that, in most cases, drive the economy to autarky, as a result of a self-fulfilling collapse of the banking system.

The rest of the paper is structured as follows. In Section 3, I present the basic framework. Section 4 provides a discussion of the main elements of the model. In Section 5, I characterize equilibrium allocations in the case of a private monetary system. In Section 6, I characterize a monetary intervention that ensures the local determinacy of equilibrium. In Section 7, I discuss how my analysis relates to some proposals for monetary reform. Section 8 concludes.

3. MODEL

Time $t = 0, 1, 2, \dots$ is discrete, and the horizon is infinite. Each period is divided into two subperiods. There are two physical commodities: good 1 and good 2. Good 1 can be produced only in the first subperiod, and good 2 can be produced only in the second subperiod. If not immediately consumed, good 1 will perish completely. Good 1 can also be used as an input in a production process. In particular, there exists a safe investment technology that returns $\beta^{-1} > 1$ units of good 1 at date $t + 1$ for each unit of good 1 invested at date t . Good 2 cannot be stored and completely depreciates if not immediately

consumed.

There are three types of infinitely lived agents, referred to as buyers, sellers, and bankers, with a $[0, 1]$ continuum of each type. Each seller wants to consume good 1 but cannot produce it. Each buyer is able to produce this good using a divisible technology that delivers one unit of the good for each unit of effort he exerts. Each buyer wants to consume good 2, but only a seller is able to produce such a good. In particular, each seller is endowed with a divisible technology that requires one unit of effort to produce each unit of good 2. Finally, each banker wants to consume good 1 but cannot produce either good. Each banker has a technology that allows him to create, at zero cost, divisible and durable objects, referred to as notes, that perfectly identify him. Thus, notes issued by one banker are perfectly distinguishable from those issued by any other banker so that counterfeiting will not be a problem. As will become clear, a note issued by a banker will be a promise to pay goods on demand and, for this reason, will be used as a medium of exchange.

There exists a centralized location in the economy. Buyers and sellers visit the centralized location periodically, whereas bankers always stay in this location. Specifically, each buyer and each seller visit the centralized location only in the first subperiod. However, I assume that buyers and sellers do *not* overlap in the centralized location. In particular, all buyers arrive at the centralized location first and leave the centralized location before all sellers arrive. In the second subperiod, each buyer is randomly matched with a seller. Following the literature, I refer to the second market as the decentralized market. See Figure 1 for a sequence of events within each period.

Buyers and sellers are anonymous, and their trading histories are privately observable. The trading history of each banker is publicly observable. I now explicitly describe preferences. Let $y_t \in \mathbb{R}_+$ denote a buyer's production of good 1, and let $q_t \in \mathbb{R}_+$ denote his consumption of good 2. His preferences are represented by

$$\sum_{t=0}^{\infty} \beta^t [-y_t + u(q_t)],$$

where $\beta \in (0, 1)$. The function $u : \mathbb{R}_+ \rightarrow \mathbb{R}$ is twice continuously differentiable, increasing, and strictly concave, with $u'(0) = \infty$. Let $x_t \in \mathbb{R}_+$ denote a seller's consumption of good

1, and let $n_t \in \mathbb{R}_+$ denote his production of good 2. His preferences are represented by

$$\sum_{t=0}^{\infty} \beta^t [x_t - c(n_t)],$$

where $c : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is twice continuously differentiable, increasing, and convex. Let $z_t \in \mathbb{R}_+$ denote a banker's consumption of good 1. Each banker has preferences represented by

$$\sum_{t=0}^{\infty} \beta^t z_t.$$

Note that all types of agents have the same discount factor over periods.

Finally, I assume throughout the paper that the amount invested by any individual in the safe technology is privately observable, i.e., other people do not know how much an individual has invested in the safe technology at each date.

4. DISCUSSION OF THE MODEL

My framework builds on Lagos and Wright (2005).³ However, it departs from the standard Lagos-Wright framework in a fundamental way. I assume that buyers and sellers meet *only* in the decentralized market (random bilateral meetings). Similar to the work of Freeman (1996), I assume that buyers and sellers do not overlap in the centralized location. This assumption implies that bankers will play an essential intermediation role in the economy. As I will show, bankers will be able to provide a medium of exchange in the form of notes redeemable on demand.⁴

In particular, each banker has an opportunity to trade sequentially with buyers and sellers, respectively, in the centralized location and each banker's trading history is publicly observable. For this reason, a banker will be able to issue personal liabilities that will be

³See also Rocheteau and Wright (2005). An alternative tractable framework that also creates an essential role for a medium of exchange is the large household model in Shi (1997).

⁴It is important to note that bankers would play an essential intermediation role even if they were not allowed to issue private money. If government-created fiat money were the only available medium of exchange, then bankers would still be essential to the functioning of the monetary system because they would ensure that the money stock would move from the hands of sellers to the hands of buyers after each round of decentralized exchange.

used as a medium of exchange in the decentralized market, provided that people believe he will be willing to redeem them at par value at a future date. In this case, each seller is willing to accept these privately issued liabilities as a means of payment, so each buyer is willing to use them as a means of payment.

Figure 2 shows how a banker's note will circulate in the economy. Each buyer has an opportunity to acquire notes in the first subperiod (during the first round of interactions). In particular, a buyer is able to acquire notes by producing some amount of good 1 and selling it in the market in exchange for bank notes. In the second subperiod, the buyer is randomly matched with a seller in the decentralized market. To pay for the amount of good 2 the seller is willing to produce for the buyer, the latter transfers his bank notes to the seller. In the following period, the seller has an opportunity to redeem his bank notes in the first subperiod (during the second round of interactions). The redemption of a bank note means that the banker who has issued the note is supposed to convert such a note into a specified amount of good 1.

In this respect, the availability of public knowledge of the banker's trading history, together with the possibility of endogenously punishing any banker who reneges on his promises, is crucial for the circulation of privately issued notes. In the decentralized market, a seller does not trust a buyer's IOU because he knows the latter cannot be (endogenously) punished in case of default. But the same seller may accept a banker's IOU as a means of payment because a banker can be (endogenously) punished if he fails to fulfill his promise of converting his IOUs into goods on demand.

The existence of a centralized location where noteholders can claim the face value of privately issued notes implies that a banker's notes will be periodically presented for redemption. The banker's willingness to pay his noteholders today will depend on the exchange value of his notes in future periods. If future monetary conditions are more favorable for him, then the continuation value of his note-issuing business is higher, so he will be less inclined to renege on his promises. As a result, his ability to raise funds today through the sale of notes increases because his liability holders know that he will have more to lose if he reneges on his promises. This means that the creation of private notes at any given date

will crucially depend on beliefs about future monetary conditions. And this is the key to understanding my results.

Finally, note that goods invested in the safe technology will not be available for use in the decentralized market. Because each buyer is anonymous and lack any commitment, he cannot credibly use claims on the goods invested in this technology as a means of payment in the decentralized market, since he cannot commit to deliver them in the following period. Thus, the investment technology corresponds to the concept of illiquid capital in Lagos and Rocheteau (2008).

5. PRIVATE MONETARY SYSTEM

In this section, I characterize the set of equilibrium allocations in the case of a purely private monetary system. The money supply is completely endogenous: Each banker issues private liabilities, referred to as notes, that can be used as a medium of exchange so that the aggregate money supply depends entirely on his willingness to create these notes. Specifically, a note issued by a banker is a contract that provides him with one unit of good 1 at date t and entitles each note holder to receive π_t units of good 1 at any future date.

There is a Walrasian market in which buyers demand notes and bankers supply notes. This market takes place during the first round of interactions in the first subperiod when all buyers and all bankers meet in the centralized location. During the second round of interactions in the first subperiod, all sellers and all bankers meet in the centralized location. This meeting gives each seller an opportunity to redeem any amount of notes he has previously acquired by trading bilaterally with a buyer in the previous decentralized market. Note that a buyer who has previously acquired notes and, for some reason, has decided not to spend them (not to trade with a seller) can also redeem these notes in the centralized location during the first round of interactions.

Throughout the paper, I restrict attention to symmetric equilibria in which all privately issued notes trade at the same price. This means that, in equilibrium, the notes issued by any pair of bankers will be perfect substitutes, which happens if and only if people believe

both bankers will be willing to redeem them at the expected face value. Each agent in the economy takes the sequence of prices $\{\pi_t\}_{t=0}^{\infty}$ as given when making his individual decisions.

Let me start by describing the decision problem of a typical buyer. Let $W(a, t)$ denote the value function for a buyer holding $a \in \mathbb{R}_+$ privately issued notes at the beginning of the first subperiod, and let $V(a, t)$ denote the value function for a buyer holding $a \in \mathbb{R}_+$ notes at the beginning of the second subperiod. The Bellman equation for a buyer in the first subperiod is given by

$$W(a, t) = \max_{(y, \hat{a}) \in \mathbb{R}_+^2} [-y + V(\hat{a}, t)]$$

subject to the budget constraint

$$\hat{a} = y + \pi_{t-1}a.$$

Here, $\hat{a} \in \mathbb{R}_+$ denotes his choice of note holdings in the first subperiod, $y \in \mathbb{R}_+$ denotes his production of good 1, and $\pi_{t-1} \in \mathbb{R}_+$ denotes the value of a note issued at the previous date (in terms of good 1). In the case of an interior solution for y , the value $W(a, t)$ can be written as $W(a, t) = \pi_{t-1}a + W(0, t)$, where

$$W(0, t) = \max_{\hat{a} \in \mathbb{R}_+} [-\hat{a} + V(\hat{a}, t)]. \quad (1)$$

In the decentralized market, the buyer makes a take-it-or-leave-it offer to the seller.⁵ The buyer chooses the amount of good 2, denoted by $q \in \mathbb{R}_+$, the seller is supposed to produce and the quantity of notes, denoted by $d \in \mathbb{R}_+$, he is supposed to transfer to the seller. Formally, the terms of trade (q, d) are determined by the solution to the following problem:

$$\max_{(q, d) \in \mathbb{R}_+^2} [u(q) - \beta\pi_t d]$$

subject to the seller's participation constraint

$$-c(q) + \beta\pi_t d \geq 0$$

and the buyer's liquidity constraint

$$d \leq a, \quad (2)$$

⁵In what follows, nothing hinges on this particular choice of the bargaining protocol.

where $a \in \mathbb{R}_+$ is the amount of notes the buyer has taken with him into the decentralized market. The solution to this problem is as follows:

$$q(a, t) = \begin{cases} c^{-1}(\beta\pi_t a) & \text{if } a < (\beta\pi_t)^{-1}c(q^*), \\ q^* & \text{if } a \geq (\beta\pi_t)^{-1}c(q^*), \end{cases}$$

$$d(a, t) = \begin{cases} a & \text{if } a < (\beta\pi_t)^{-1}c(q^*), \\ (\beta\pi_t)^{-1}c(q^*) & \text{if } a \geq (\beta\pi_t)^{-1}c(q^*). \end{cases}$$

The Bellman equation for a buyer holding $a \in \mathbb{R}_+$ notes at the beginning of the decentralized market is given by

$$V(a, t) = u(q(a, t)) + \beta W(a - d(a, t), t + 1). \quad (3)$$

Using the fact that $W(a, t)$ is an affine function in its first argument, I can rewrite (3) as follows:

$$V(a, t) = u(q(a, t)) + \beta\pi_t[a - d(a, t)] + \beta W(0, t + 1).$$

Thus, the first-order condition for the optimal choice of note holdings in the first subperiod is given by

$$-1 + \frac{\partial V}{\partial a}(a, t) \leq 0,$$

with equality if $a > 0$. If $\pi_t < \beta^{-1}$, then the optimal choice of note holdings will be given by

$$\frac{u'(q(a, t))}{c'(q(a, t))} = \frac{1}{\beta\pi_t}, \quad (4)$$

in which case $q(a, t) = c^{-1}(\beta\pi_t a)$. Note that all buyers choose to hold the same quantity of notes in the first subperiod and, consequently, will be able to purchase the same amount of good 2 in the decentralized market. Thus, condition (4) gives the aggregate demand for privately issued notes as a function of the value π_t .

In the description of the buyer's decision problem, I have implicitly assumed that each seller voluntarily accepts privately issued notes in exchange for his output in the second subperiod. A seller's decision to accept a banker's notes as a means of payment depends on the seller's beliefs about the banker's willingness to redeem them at some expected value.

In particular, I have written the seller's participation constraint under the assumption that each banker who has issued notes at the current date is willing to redeem them at the expected value π_t at the following date.

Let me now describe the decision rule for each seller with respect to the acceptance of privately issued notes in bilateral trades. A seller is willing to accept privately issued notes as a means of payment at each date provided that the amount of notes issued by each banker does not exceed an upper bound $\bar{B}_t \in \mathbb{R}_+$ at each date $t \geq 0$. If this upper bound is exceeded at some date t , then each seller chooses not to accept privately issued notes as a means of payment in the decentralized market. This means that a seller's acceptance rule depends on the current bound on note issue and on all future bounds. As I will show later, it is possible to construct a sequence $\{\bar{B}_t\}_{t=0}^{\infty}$ such that the seller's acceptance rule is individually rational.

Now I describe the decision problem of a typical banker. Let $J(n, s, t)$ denote the value function for a banker who had issued $n \in \mathbb{R}_+$ notes at the previous date and who had invested $s \in \mathbb{R}_+$ units of good 1 in the safe technology at the previous date. The banker's decision problem can be formulated as follows:

$$J(n, s, t) = \max_{(z, \hat{n}, \hat{s}) \in \mathbb{R}_+^3} [z + \beta J(\hat{n}, \hat{s}, t + 1)], \quad (5)$$

subject to the budget constraint

$$\hat{s} + z + \pi_{t-1}n = \beta^{-1}s + \hat{n}$$

and the upper bound on the number of notes that can be issued at each date

$$\hat{n} \leq \bar{B}_t.$$

Here, $\hat{s} \in \mathbb{R}_+$ denotes the amount of good 1 the banker decides to invest in the safe technology at the current date, $z \in \mathbb{R}_+$ denotes his current consumption, and $\hat{n} \in \mathbb{R}_+$ denotes the number of notes he decides to issue at the current date. The constraint $\hat{n} \leq \bar{B}_t$ incorporates the seller's acceptance rule into the banker's decision problem. Thus, when making his decisions at each date, each banker takes as given the sequence of values $\{\pi_t\}_{t=0}^{\infty}$ as well as the sequence of individual limits on note issue $\{\bar{B}_t\}_{t=0}^{\infty}$.

If $\pi_t < \beta^{-1}$, then each banker finds it optimal to issue as many notes as possible (i.e., he chooses $\hat{n} = \bar{B}_t$). Because the rate of return paid on his notes (his cost of funds) is lower than the rate of return on the safe technology, he makes a positive profit by issuing notes and investing the proceeds in the safe technology. Also, note that because the return on the safe technology equals his rate of time preference, he is indifferent between immediately consuming and reinvesting the proceeds from his previously accumulated earnings. Therefore, an optimal investment decision is given by $\hat{s} = \bar{B}_t$, which can be interpreted as the decision to voluntarily hold in reserve all proceeds from the sale of notes in the current period. This means that each banker is willing to issue notes that are fully secured by a riskless asset. In this case, the banker's optimal consumption is given simply by

$$z = \bar{B}_{t-1} (\beta^{-1} - \pi_{t-1}).$$

Thus, the banker's lifetime discounted utility is given by

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} \bar{B}_{\tau-1} (\beta^{-1} - \pi_{\tau-1}).$$

This means that his lifetime utility at any point in time depends on the sequence of individual limits on note issue and the return paid on privately issued notes today and in future periods.

5.1. Equilibrium

For any value $\pi_t < \beta^{-1}$, the liquidity constraint (2) binds. Let q_t denote the amount of good 2 traded in each bilateral meeting at date t . Thus, we can rewrite (4) as follows:

$$\frac{u'(q_t)}{c'(q_t)} = \frac{1}{\pi_t \beta}. \quad (6)$$

This condition determines the production of good 2 in each bilateral meeting as a function of the face value of notes. Thus, I can use (6) to implicitly define $q_t = q(\pi_t)$, with $q'(\pi_t) > 0$ for any $\pi_t > 0$. Thus, a higher rate of return on privately issued notes results in a larger amount produced and traded in each bilateral meeting. The aggregate note holdings as a

function of the value π_t are given by

$$a(\pi_t) = \frac{c(q(\pi_t))}{\beta\pi_t}. \quad (7)$$

Note that the demand for notes can be either increasing or decreasing in the purchasing power of each note depending on the specification of preferences.

Now I have to ensure that each seller's acceptance rule is indeed individually rational. Thus, I need to specify the sequence of individual limits on note issue $\{\bar{B}_t\}_{t=0}^\infty$ in such a way that each banker is willing to supply the amount of notes other people demand and is willing to fulfill his promise of converting notes into goods. I take two steps to define individual limits on note issue satisfying these two conditions. First, for any given sequence $\{\pi_t\}_{t=0}^\infty$, I set

$$\bar{B}_t = a(\pi_t) \quad (8)$$

at each date t . This condition guarantees that, given π_t , each banker is willing to supply the amount of notes in (7) so that the market for privately issued notes will clear at each date. Then, I need to verify whether a particular choice for the sequence $\{\pi_t\}_{t=0}^\infty$ implies that each banker does not want to renege on his promises. A particular sequence $\{\pi_t\}_{t=0}^\infty$ is consistent with convertibility if and only if

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} a(\pi_{\tau-1}) (\beta^{-1} - \pi_{\tau-1}) \geq a(\pi_{t-1}) (\beta^{-1} - \pi_{t-1}) + a(\pi_t)$$

holds at each date t . As in Kehoe and Levine (1993) and Alvarez and Jermann (2000), these constraints allow each banker to issue as many notes as possible without inducing him to opportunistically renege on his promises. The left-hand side gives the banker's beginning-of-period lifetime utility. The right-hand side gives the short-term payoff the banker gets if he decides *not* to hold in reserve the proceeds from the sale of notes at date t . In this case, he can increase his current consumption by the amount $a(\pi_t)$, but he will inevitably renege on his promises at date $t+1$, resulting in the autarkic payoff from date $t+1$ onward. Recall that the amount invested by any individual in the productive technology is unobservable to other people.

As previously mentioned, a seller's acceptance rule specifies that he chooses not to accept privately issued notes in the decentralized market if the amount of notes issued by a banker exceeds the upper bound $\bar{B}_t = a(\pi_t)$ at some date. Thus, I can interpret the banker's decision to renege on his promises as the dissolution of his note-issuing business, given that nobody will be willing to produce to acquire his notes in future periods if he defaults on his obligations today.

If I restrict attention to individual limits on note issue that are not too tight, then I can rewrite the convertibility constraints as follows:

$$-a(\pi_t) + \beta J_{t+1} = 0, \quad (9)$$

where J_t denotes the banker's lifetime utility at the beginning of date t ,

$$J_t = \sum_{\tau=t}^{\infty} \beta^{\tau-t} a(\pi_{\tau-1}) (\beta^{-1} - \pi_{\tau-1}).$$

I can also rewrite (5) as follows:

$$J_t = a(\pi_{t-1}) (\beta^{-1} - \pi_{t-1}) + \beta J_{t+1}. \quad (10)$$

Note that the term $a(\pi_{t-1}) (\beta^{-1} - \pi_{t-1})$ gives the banker's current profit at date t . Specifically, at date $t-1$, the banker received the amount $a(\pi_{t-1})$ in exchange for his notes and invested this amount in the safe technology, obtaining the revenue $\beta^{-1} a(\pi_{t-1})$ at date t . Because each note issued at date $t-1$ is a promise to pay π_{t-1} units of good 1 at date t , his current disbursement is given by $\pi_{t-1} a(\pi_{t-1})$. Thus, his profit will be given by the difference between the revenue $\beta^{-1} a(\pi_{t-1})$ and the disbursement $\pi_{t-1} a(\pi_{t-1})$. As I have shown, he will immediately consume any profit he makes.

Combining (9) with (10), I obtain the following equilibrium law of motion for the exchange value of notes:

$$a(\pi_t) = \pi_{t-1} a(\pi_{t-1}). \quad (11)$$

The formal definition of a perfect-foresight equilibrium is now straightforward.

Definition 1 *A monetary equilibrium is a sequence $\{\pi_t\}_{t=0}^{\infty}$ satisfying $0 \leq \pi_t \leq \beta^{-1}$ and (11) at each date t .*

Note that (9) indicates that the supply of notes today depends on the exchange value of notes in future periods. If future monetary conditions are more favorable for each banker, then the value of his note-issuing privileges is higher, making it more costly for him to renege on his promises. In this case, the supply of notes today is higher. If future monetary conditions are less favorable for each banker, then he will be more inclined to renege on his promises. In this case, the supply of notes today is lower.

For the rest of the paper, I focus on the case in which the demand for notes is strictly increasing in the expected value of notes.⁶ As we will see, this is precisely the case in which we will observe self-fulfilling collapses, the main topic of this paper.

Assumption 1 Assume $a'(\pi) > 0$ for all $\pi > 0$.

Given this assumption, I can formally establish the existence and uniqueness of a stationary monetary equilibrium.

Proposition 2 $\pi_t = 1$ for all $t \geq 0$ is the unique stationary monetary equilibrium.

Proof. It is straightforward to verify that the constant sequence $\pi_t = 1$ for all $t \geq 0$ satisfies (11). The uniqueness of this *interior* solution immediately follows from the fact that $a'(\pi) > 0$ for any $\pi > 0$. In this case, the amount of good 2 produced by the seller in each bilateral meeting is given by

$$\frac{u'(\hat{q})}{c'(\hat{q})} = \frac{1}{\beta}.$$

Q.E.D. ■

In this equilibrium, the exchange value of notes remains constant over time. People do not expect monetary conditions to change over time, so the amount of notes issued at each date, as well as their expected return, remains constant over time. In particular, people expect the value of each note to be one in all future periods and know that, as long as the amount raised from the sale of notes equals $a(1)$ for each banker at each date, such a banker

⁶This will be the case if, for instance, we assume $c(q) = q$ and $u(q) = (1 - \sigma)^{-1} q^{1-\sigma}$, with $0 < \sigma < 1$.

will be willing to maintain the convertibility of his notes. As a result, no banker will ever renege on his promises along the equilibrium path.

I can interpret this stationary equilibrium as follows. Given that each buyer acquires a note at the cost of one unit of good 1 and that each note is convertible into one unit of the same good, people expect privately issued notes to be redeemed at par value, and this expectation is confirmed in equilibrium. This means that it is possible to construct an equilibrium in which the value of privately issued notes is stable over time so that the equilibrium allocation is stationary. However, as I will show next, it is possible to construct other equilibria in which the exchange value of notes is not constant over time (i.e., people fully anticipate that privately issued notes will not be redeemed at par value). These equilibria exist because other beliefs about the exchange value of notes in future periods will also be consistent with an equilibrium outcome (a self-fulfilling prophecy). In these equilibria, the amount of goods produced and traded in the decentralized market will vary over time. The dynamics will be completely driven by expectations about future monetary conditions.

5.2. Self-Fulfilling Collapses

In this subsection, I characterize equilibria for which people expect monetary conditions to constantly deteriorate over time. I interpret this kind of equilibrium as a self-fulfilling collapse of the banking system characterized by a persistent decline in the amount of notes in circulation driven by expectations that future monetary conditions will persistently deteriorate. As I will show, this kind of equilibrium will have an adverse impact on trading activity. In particular, the quantities produced and traded in the decentralized market will monotonically decline over time.

Note that

$$\pi a'(\pi) + a(\pi) > 0$$

for any $\pi > 0$. As a result, we have

$$\frac{d\pi_t}{d\pi_{t-1}} = \frac{\pi_{t-1} a'(\pi_{t-1}) + a(\pi_{t-1})}{a'(\pi_t)} > 0,$$

which means that (11) defines an implicit mapping $\pi_t = f(\pi_{t-1})$ that is strictly increasing.

In particular, we have

$$\left. \frac{d\pi_t}{d\pi_{t-1}} \right|_{\pi_{t-1}=\pi_t=1} = \frac{a'(1) + a(1)}{a'(1)} > 1,$$

which means that the mapping $\pi_t = f(\pi_{t-1})$ crosses the 45-degree line from below at $(\pi_{t-1}, \pi_t) = (1, 1)$. Note also that, for any initial condition $\pi_0 > 1$, there will be no equilibrium because the condition $\pi_t \leq \beta^{-1}$ will necessarily be violated at some finite date t . See Figure 3.

For any initial condition $\pi_0 \in (0, 1)$, there exists a unique equilibrium trajectory that is strictly decreasing. Along this equilibrium path, the individual limits on note issue, given by $\bar{B}_t = a(\pi_t)$, decrease monotonically over time and converge to zero. This means that the equilibrium allocation approaches autarky as $t \rightarrow \infty$. As a result, the exchange value of privately issued notes depreciates over time as their purchasing power persistently declines. As a result, buyers and sellers will be able to trade smaller amounts of goods in the decentralized market. I summarize these findings in the following proposition.

Proposition 3 *For each initial condition $\pi_0 \in (0, 1)$, there exists a unique monetary equilibrium $\{\pi_t\}_{t=0}^{\infty}$ in which the exchange value of notes is strictly decreasing and converges to zero.*

I interpret this kind of equilibrium as a self-fulfilling collapse of the banking system. As I have shown, the determination of equilibrium quantities and prices completely depends on people's beliefs regarding future monetary conditions. Because people believe that the exchange value of private notes will persistently decrease over time (i.e., people fully anticipate that notes will be retired at increasing discounts), the amount of funds devoted to each banker will be lower today. This means that the number of notes in circulation today will be lower. In fact, the number of notes in circulation will monotonically decrease over time, resulting in a decreasing amount of goods produced and traded in the decentralized market. From a buyer's standpoint, his demand for notes will decrease over time because the purchasing power of notes is depreciating over time, allowing him to purchase ever smaller amounts of goods from his trading partners.

The existence of other equilibria with undesirable properties for initial conditions arbitrarily close to $\pi_0 = 1$ implies that such a system is necessarily unstable. These equilibria arise because there is no condition to pin down the initial choice of the exchange value of privately issued notes. As a result, multiple beliefs about the exchange value of privately issued notes in future periods are consistent with an equilibrium outcome. In particular, some of these equilibria will be characterized by a persistent decline in the supply of notes because of the expectation that their exchange value will depreciate over time. In this respect, inside money shares some of the same properties as outside fiat money, namely, indeterminacy of equilibrium. See Woodford (1984) and Lagos and Wright (2003) for a description of the properties of outside money.⁷

6. GOVERNMENT INTERVENTION

The goal of this section is to investigate whether there exists a government intervention that can ensure the stability of a private monetary system of the kind previously described. Suppose the government intervenes by granting a privilege (or subsidy) to a fraction $\alpha \in (0, 1)$ of bankers. If a banker who is eligible to receive a subsidy decides to follow the government's policy with respect to note issue, then each one of them will receive a series of lump-sum payments such that his lifetime utility is at least the same as the lifetime utility of those who do not receive a subsidy. The government will impose a lump-sum tax on each buyer to finance the subsidy payments. As we will see, the size of this lump-sum tax will be arbitrarily small.

Let $\{\bar{D}_t\}_{t=0}^{\infty}$ denote the path of note creation for each banker who accepts the government privilege (i.e., this is the amount of notes that each one of them is supposed to issue at each date). This means that there will be two types of private notes in circulation: those issued by privileged bankers and those issued by nonprivileged bankers. As a result, there exists an exchange rate between these two types of money that is necessarily market-determined.

⁷See also Gu, Mattesini, Monnet, and Wright (forthcoming) for a study of the dynamic properties of pure credit economies.

On the other hand, the government wants people to treat the notes issued by privileged bankers and those issued by other bankers as perfect substitutes, i.e., it wants the exchange rate (or price ratio) to be one at each date.

The government can achieve this goal by conferring the status of legal tender to both types of notes and simultaneously requiring privileged bankers to be willing to exchange, on demand, each note issued by a nonprivileged banker for one of his own notes, and vice versa. This intervention necessarily implies that the legal ratio is one and that an equilibrium in which both monies circulate as a medium of exchange can exist if and only if the market price ratio equals the legal ratio.⁸

Given this form of intervention, the prices of both types of notes must be the same. In this case, the function $a(\pi_t)$ defined in (7) continues to represent the aggregate demand for notes, so the amount of resources devoted to each nonprivileged banker at date t is given by

$$\frac{a(\pi_t) - \alpha \bar{D}_t}{1 - \alpha} = \bar{B}_t.$$

To guarantee that each privileged banker voluntarily follows the government's prescription with respect to the amount of notes to be issued at each date, the subsidy payments $\{s_t\}_{t=0}^{\infty}$ must satisfy

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} [\bar{D}_{\tau-1} (\beta^{-1} - \pi_{\tau-1}) + s_{\tau}] \geq \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[\frac{a(\pi_{\tau-1}) - \alpha \bar{D}_{\tau-1}}{1 - \alpha} \right] (\beta^{-1} - \pi_{\tau-1}) \quad (12)$$

at each date $t \geq 0$. As we will see, the size of these subsidy payments will be very small, and so will be the taxes required to finance them. To guarantee that each seller accepts the notes issued by privileged bankers, the path $\{\bar{D}_t\}_{t=0}^{\infty}$ must be such that the condition

$$-\bar{D}_t + \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} \bar{D}_{\tau-1} (\beta^{-1} - \pi_{\tau-1}) \geq 0 \quad (13)$$

holds at each date $t \geq 0$. This condition ensures that a particular choice of a monetary regime $\{\bar{D}_t\}_{t=0}^{\infty}$ by the government is consistent with the convertibility constraints so that

⁸Indeed, one of the crucial provisions of the banking act passed during the U.S. Civil War was that each national bank was required to accept the notes issued by other national banks at par value.

the notes issued by the privileged bankers and those issued by other bankers will be treated as perfect substitutes. All that matters for the nonbank public is whether privileged and nonprivileged bankers are willing to fulfill their promises to pay note holders on demand (i.e., whether the convertibility constraints are satisfied at each date). If both types are willing to voluntarily convert their notes into goods, then the nonbank public will treat the notes issued by any pair of bankers as perfect substitutes, regardless of any subsidy payment. Now it is straightforward to define an equilibrium in the presence of government intervention.

Definition 4 *A monetary equilibrium is a sequence $\{\pi_t, \bar{D}_t, s_t\}_{t=0}^{\infty}$ satisfying (12), (13), $0 \leq \pi_t \leq \beta^{-1}$,*

$$[a(\pi_{t-1}) - \alpha \bar{D}_{t-1}] \pi_{t-1} = a(\pi_t) - \alpha \bar{D}_t, \quad (14)$$

and

$$a(\pi_t) \geq \alpha \bar{D}_t \quad (15)$$

at each date t .

Note that if the government chooses $\alpha = 0$, then the dynamic system is exactly the same as that derived in the previous section, given by (11). Thus, I have already characterized the full set of equilibrium allocations for the case $\alpha = 0$. In this section, I will consider the possibility of a monetary intervention ($\alpha > 0$) and restrict attention to the local determinacy of equilibrium in the case of monetary regimes $\{\bar{D}_t\}_{t=0}^{\infty}$ that involve small fluctuations in the amount of notes issued by privileged bankers over time. Even though the global determinacy of equilibrium is perhaps the most desirable outcome of a government intervention, I consider the local determinacy of equilibrium a sufficiently satisfactory criterion to judge the effectiveness of a particular government intervention.

It is important to mention that I do not wish to claim that the government intervention proposed in this section completely solves the problem of indeterminacy associated with a purely private monetary system, but I do claim that the local determinacy is a satisfactory criterion to judge the effectiveness of the intervention in promoting monetary

stability. Thus, when I refer to monetary stability I mean precisely the local determinacy of equilibrium under a specific monetary arrangement.

Having said that, now consider monetary regimes $\{\bar{D}_t\}_{t=0}^{\infty}$ for which the amount of notes issued by privileged bankers remains bounded, and suppose the government chooses an arbitrarily small fraction of bankers to receive the subsidy. Because the government chooses both the quantity of notes that each privileged banker is supposed to issue at each date and the fraction of privileged bankers, it is able to keep the total amount of these notes, given by $\alpha\bar{D}_t$, within *any* bounded interval. In this case, we can use standard methods to determine the local determinacy of equilibrium; see, for instance, Azariadis (1993) and Woodford (2003). Note that (14) is a nonautonomous, nonlinear difference equation describing the evolution of the variable π_t for any given government policy. Define $\hat{\pi}_t \equiv \pi_t - 1$. Then, a linear approximation to (14) is given by

$$\hat{\pi}_t = b\hat{\pi}_{t-1} + \Delta_t,$$

where

$$b \equiv \frac{a'(1) + a(1)}{a'(1)},$$

$$\Delta_t \equiv \frac{\alpha}{a'(1)} (\bar{D}_t - \bar{D}_{t-1}).$$

Because $b > 1$, this equation can be solved forward to obtain a *unique* bounded solution

$$\hat{\pi}_t = -\frac{1}{b} \sum_{j=0}^{\infty} \left(\frac{1}{b}\right)^j \Delta_{t+1+j}. \quad (16)$$

In other words, there exists a sufficiently small neighborhood around $\pi = 1$ such that the unique equilibrium can be approximated by (16). This means that the equilibrium value of notes today depends on the future path of government policies with respect to the amount of notes issued by privileged bankers.

It remains to verify whether (13) is satisfied. Suppose that we choose $\{\bar{D}_t\}_{t=0}^{\infty}$ to be weakly increasing and bounded, so $0 \leq \bar{D}_t \leq \delta$ for some upper bound $\delta > 0$. Suppose also that α is arbitrarily small so that $\alpha\bar{D}_t$ remains within a small neighborhood of zero. This

implies $\Delta_t \geq 0$ for all $t \geq 1$ and, consequently, $\pi_t \leq 1$ for all $t \geq 0$. In this case, we have

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} \bar{D}_{\tau} (\beta^{-1} - \pi_{\tau}) \geq \frac{D_t}{\beta}$$

at each date $t \geq 0$, which implies that (13) is indeed satisfied. Thus, this particular choice of a monetary regime $\{\bar{D}_t\}_{t=0}^{\infty}$ is consistent with an equilibrium outcome in which the notes issued by privileged bankers and those issued by other bankers are treated as perfect substitutes. Again, I can make α arbitrarily small to justify the local approximation because the government is able to choose the fraction of privileged bankers and still satisfy the convertibility constraints.

Example 5 *Suppose the government instructs privileged bankers to follow the policy $\bar{D}_t = \delta - \sigma(1+t)^{-1}$ at each date $t \geq 0$, where $0 < \sigma < \delta$. In this case, for α sufficiently small, we have $\Delta_t = \alpha\sigma\alpha'(1)^{-1} [t^{-1} - (1+t)^{-1}] > 0$ for all $t \geq 1$, so $\pi_t < 1$ for all $t \geq 0$. Thus, it is possible to keep the equilibrium value of π_t arbitrarily close to one by appropriately choosing a small value for α .*

Because the equilibrium value of notes depends on the future path of government policies with respect to the amount of notes issued by privileged bankers, monetary conditions will remain relatively stable if the government keeps the aggregate amount of these notes within a sufficiently small neighborhood of zero. This means that the equilibrium value of notes will remain within a small neighborhood of one, implying arbitrarily small fluctuations in the aggregate supply of notes. As a result, the quantity of goods produced and traded in the decentralized market will remain within an arbitrarily small neighborhood of \hat{q} . It is convenient to summarize the previous findings in the following proposition.

Proposition 6 *There exists a monetary regime $\{\bar{D}_t\}_{t=0}^{\infty}$ that results in a unique monetary equilibrium in which the face value of notes π_t remains arbitrarily close to one and the aggregate amount of notes issued by privileged bankers $\alpha\bar{D}_t$ remains within an arbitrarily small neighborhood of zero. The unique equilibrium path $\{\pi_t\}_{t=0}^{\infty}$ can be approximated by (16).*

I have characterized a government intervention that ensures the local determinacy of equilibrium. By tying the equilibrium value of privately issued notes to a path of government policies with respect to the amount of notes issued by privileged bankers, the intervention described above provides a condition for determining the initial choice of the equilibrium value of notes. Because the government can choose both the fraction of privileged bankers and the amount of notes each one of them is supposed to issue at each date, a unique equilibrium path is obtained in case the government keeps the aggregate amount of notes issued by privileged bankers within a sufficiently small neighborhood of zero. Finally, it is important to emphasize that the taxes required to finance the subsidy payments to the privileged bankers are arbitrarily small, which means that the government intervention characterized in this section is minimal.

7. DISCUSSION

My analysis certainly relates to some proposals for monetary reform. For instance, Friedman (1959) has proposed a reform of the banking system that requires commercial banks to hold the full value of their demand deposits in the form of safe short-term assets. Also, Wallace (1996) provides a critical examination of a similar proposal to achieve a stable banking system, referred to as the narrow banking proposal. More recently, Gorton (2012) has also argued in favor of the creation of narrow funding banks as a solution to the instability of shadow banks (i.e., financial intermediaries that issue debt backed by securitized bonds).

Friedman (1959) has argued that one of the problems of the national banking system was that, despite the requirement to fully secure national bank notes with safe government bonds, the reserve requirements for demand deposits were less than the full nominal value of these deposits. Thus, an effective solution to the instability associated with abrupt fluctuations in the desired ratios of deposits to currency and of deposits to reserves is to make the 100 percent reserve requirements uniform to all demandable liabilities of the banking system. According to Friedman, this would eliminate the problem.

In my analysis, I have assumed that people do not observe the amount that each banker

has invested in the productive technology at each date. This assumption, combined with a lack of commitment, implies that people have to worry about a banker's decision to suspend convertibility. As I have shown, each banker is willing to voluntarily hold in reserve the full face value of each note issued at a particular date provided that expected prices are such that he will be better off by maintaining the convertibility of his notes, given that he knows that people will stop accepting his notes in private transactions in case he strategically suspends convertibility.

As I have shown, it is possible to achieve a stable exchange value of bank liabilities under this arrangement (i.e., bank liabilities are always retired at par value). However, the fact that the decision to accept bank liabilities in bilateral trades is endogenously determined means that other beliefs are also consistent with an equilibrium outcome. In particular, the nonstationary equilibria characterized above can surely be interpreted as a self-fulfilling collapse of the value of privately issued liabilities. Similar to what happened in a typical banking panic episode, people believe that privately issued bank liabilities will not be redeemed at par value and, consequently, reduce their demand for them. Moreover, people believe that the discount on bank liabilities will increase over time, as in a typical banking panic episode, so the demand for private notes decreases over time. And I have shown these beliefs become self-fulfilling.

Finally, the kind of government intervention I have characterized in this paper has some support in the historical evidence. For instance, the Bank of England, far from behaving as a typical profit-maximizing institution, was heavily influenced and overtly supported by the British government throughout the 18th and 19th centuries, receiving several privileges from Parliament. In exchange for these privileges, the bank gradually accepted its duties as the nation's central bank, issuing and retiring notes to implement specific policies. The First and Second Banks of the United States were also government-sponsored financial institutions that operated to influence money market conditions in order to implement specific policies.

8. CONCLUSION

I have characterized the properties of a monetary system in which private bank notes are fully secured by safe short-term assets. The key frictions in the model are people's inability to commit to their future promises and a lack of common knowledge of trading histories for most individuals. Those who have the ability to make their actions publicly observable are able to issue liabilities that circulate as a medium of exchange and enjoy a higher lifetime utility than that associated with autarky precisely because of their note-issuing privileges. I have referred to these agents as bankers.

In my analysis, individual limits on note issue are endogenously determined, so each banker's ability to issue notes today depends on beliefs about the exchange value of his notes in future periods. As a result, there can be multiple equilibria under a private monetary system. Some of these equilibria have undesirable properties. In most cases, we observe a self-fulfilling collapse of the banking system in which the amount of notes in circulation persistently declines over time as note holders continuously reduce their demand for these notes, adversely affecting trading activity. In particular, consumers will be able to purchase ever smaller quantities of goods because of a persistent depreciation of the exchange value of privately issued notes.

In view of these difficulties, I have characterized a government intervention that results in the local determinacy of equilibrium, which is a desirable property of a monetary system. Specifically, it is possible to formulate an intervention that results in a stable value of privately issued bank notes so that trading activity does not contract as a result of a self-fulfilling collapse. Thus, private agents are able to provide a sound monetary framework provided that a monetary authority intervenes in the way described above.

REFERENCES

- [1] F. Alvarez, U. Jermann. "Efficiency, equilibrium, and asset pricing with risk of default," *Econometrica* 68 (2000), pp. 775-797.

- [2] C. Azariadis. *Intertemporal Macroeconomics*, Oxford: Blackwell (1993).
- [3] C. Azariadis, J. Bullard, B. Smith. "Private and public circulating liabilities," *Journal of Economic Theory* 99 (2001), pp. 59-116.
- [4] A. Berentsen. "On the private provision of fiat currency," *European Economic Review* 50 (2006), pp. 1683-1698.
- [5] R. Cavalcanti, A. Erosa, T. Temzelides. "Private money and reserve management in a random-matching model," *Journal of Political Economy* 107 (1999), pp. 929-945.
- [6] R. Cavalcanti, N. Wallace. "A model of private bank-note issue," *Review of Economic Dynamics* 2 (1999a), pp. 104-136.
- [7] R. Cavalcanti, N. Wallace. "Inside and outside money as alternative media of exchange," *Journal of Money, Credit and Banking* 31 (1999b), pp. 443-457.
- [8] B. Champ, B. Smith, S. Williamson. "Currency elasticity and banking panics: Theory and evidence," *Canadian Journal of Economics* 29 (1996), pp. 828-864.
- [9] S. Freeman. "The payments system, liquidity, and rediscounting," *American Economic Review* 86 (1996), pp. 1126-1138.
- [10] M. Friedman. *A Program for Monetary Stability*, New York: Fordham University Press (1959).
- [11] M. Friedman, A. Schwartz. "Has government any role in money?," *Journal of Monetary Economics* 17 (1986), pp. 37-62.
- [12] G. Gorton. *Misunderstanding Financial Crises: Why We Don't See Them Coming*, Oxford University Press (2012).
- [13] C. Gu, F. Mattesini, C. Monnet, R. Wright. "Banking: A new monetarist approach," *Review of Economic Studies*, forthcoming.

- [14] C. Gu, F. Mattesini, C. Monnet, R. Wright. “Endogenous credit cycles,” *Journal of Political Economy*, forthcoming.
- [15] F. Hayek. *Denationalisation of Money*, Hobart paper 70, 2nd ed., The Institute of Economic Affairs, London (1976).
- [16] P. He, L. Huang, R. Wright. “Money, banking, and monetary policy,” *Journal of Monetary Economics* 55 (2008), pp. 1013-1024.
- [17] C. Kahn, W. Roberds. “Demandable debts as a means of payment: Banknotes versus checks,” *Journal of Money, Credit and Banking* 31 (1999), pp. 500-525.
- [18] T. Kehoe, D. Levine. “Debt-constrained asset markets,” *Review of Economic Studies* 60 (1993), pp. 868-888.
- [19] R. King. “On the economics of private money,” *Journal of Monetary Economics* 12 (1983), pp.127-158.
- [20] R. Lagos, G. Rocheteau. “Money and capital as competing media of exchange,” *Journal of Economic Theory* 142 (2008), pp. 247-258.
- [21] R. Lagos, R. Wright. “Dynamics, cycles and sunspot equilibria in ‘genuinely dynamic, fundamentally disaggregative’ models of money,” *Journal of Economic Theory* 109 (2003), pp. 156-171.
- [22] R. Lagos, R. Wright. “A unified framework for monetary theory and policy analysis,” *Journal of Political Economy* 113 (2005), pp. 463-84.
- [23] Y. Li. “A search model of money and circulating private debt with applications to monetary policy,” *International Economic Review* 42 (2001), pp. 925-946.
- [24] Y. Li. “Banks, private money, and government regulation,” *Journal of Monetary Economics* 53 (2006), pp. 2067-2083.
- [25] A. Martin, S. Schreft. “Currency competition: A partial vindication of Hayek,” *Journal of Monetary Economics* 53 (2006), pp. 2085-2111.

- [26] D. Mills. “A model in which outside and inside money are essential,” *Macroeconomic Dynamics* 11 (2007), pp. 347-366.
- [27] G. Rocheteau, R. Wright. “Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium,” *Econometrica* 73 (2005), pp. 175-202.
- [28] S. Shi. “A divisible search model of fiat money,” *Econometrica* 65 (1997), pp. 75-102.
- [29] B. Smith, W. Weber. “Private money creation and the Suffolk banking system,” *Journal of Money, Credit and Banking* 31 (1999), pp. 624-659.
- [30] N. Wallace. “Narrow banking meets the Diamond-Dybvig model,” Federal Reserve Bank of Minneapolis *Quarterly Review* 20 (1996), pp. 3-13.
- [31] S. Williamson. “Private money,” *Journal of Money, Credit and Banking* 31 (1999), pp. 469-491.
- [32] M. Woodford. “Indeterminacy of equilibrium in the overlapping generations model: A survey,” Unpublished Manuscript (1984).
- [33] M. Woodford. *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton: Princeton University Press (2003).

Figure 1: Sequence of Events within a Period

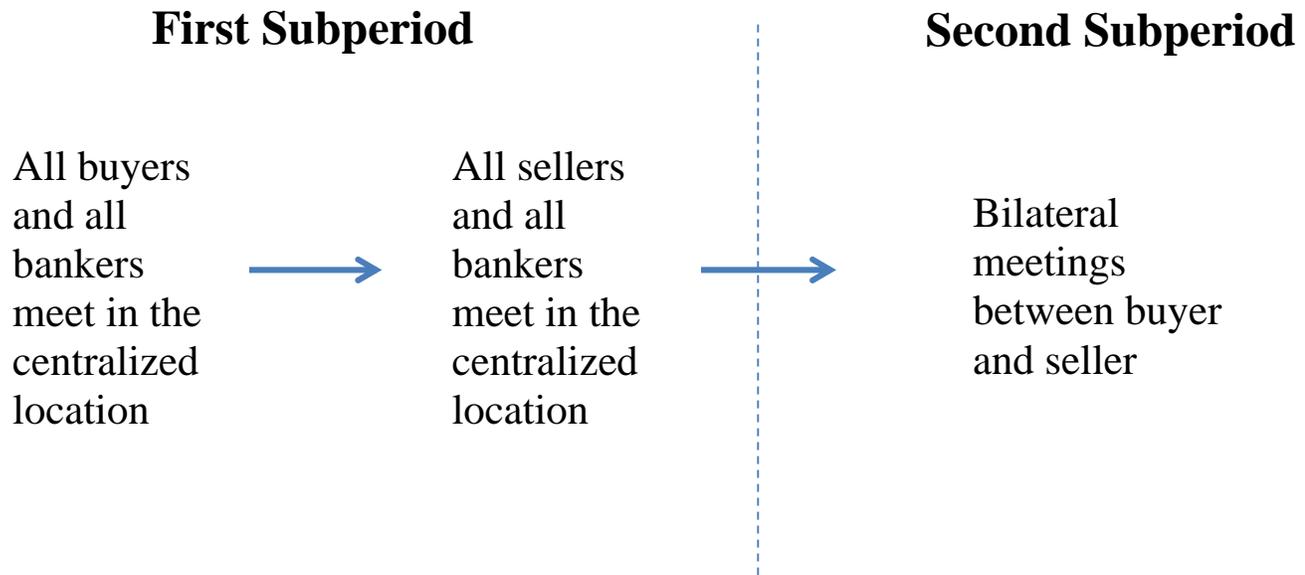


Figure 2: Circulation of Bank Notes

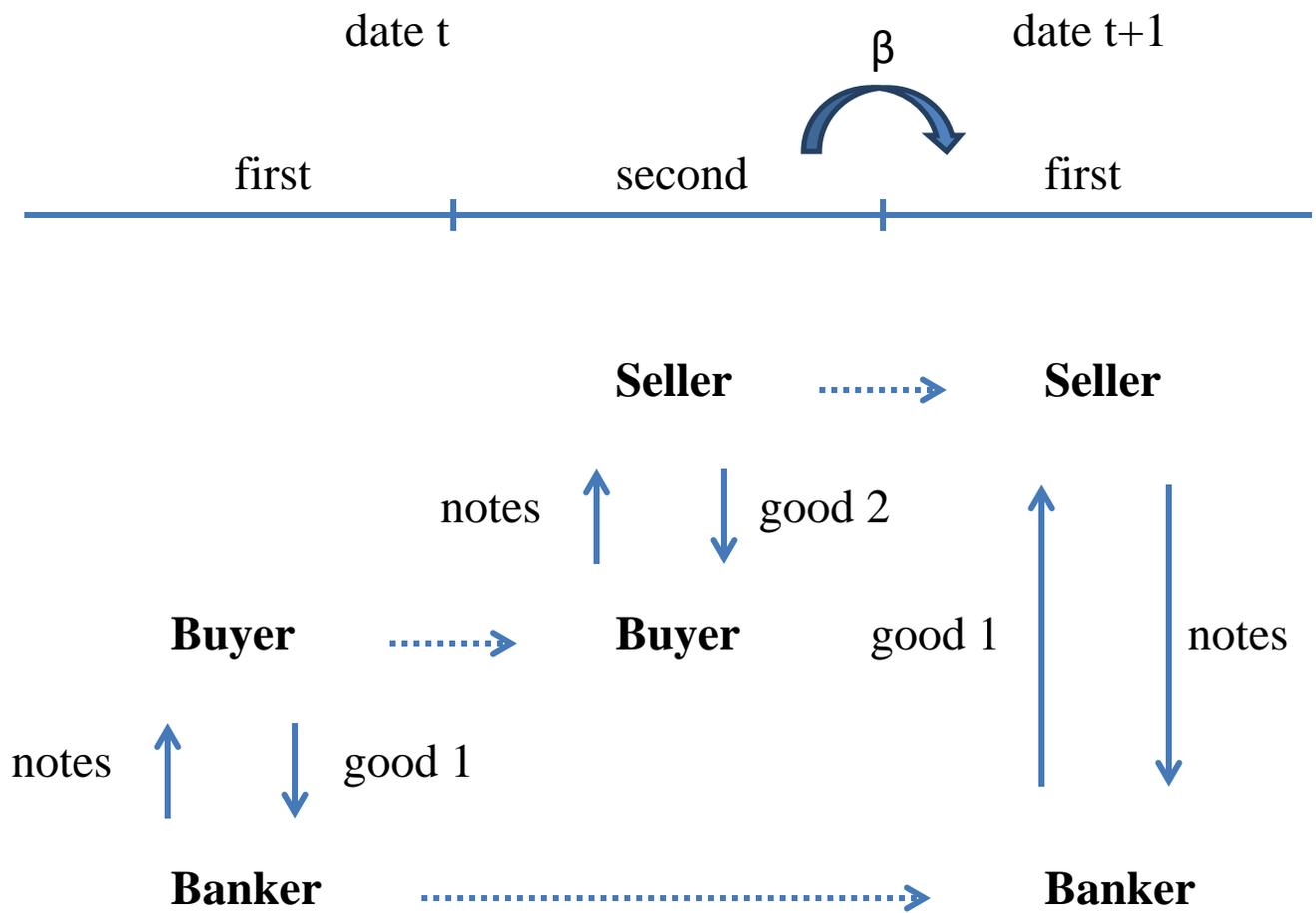
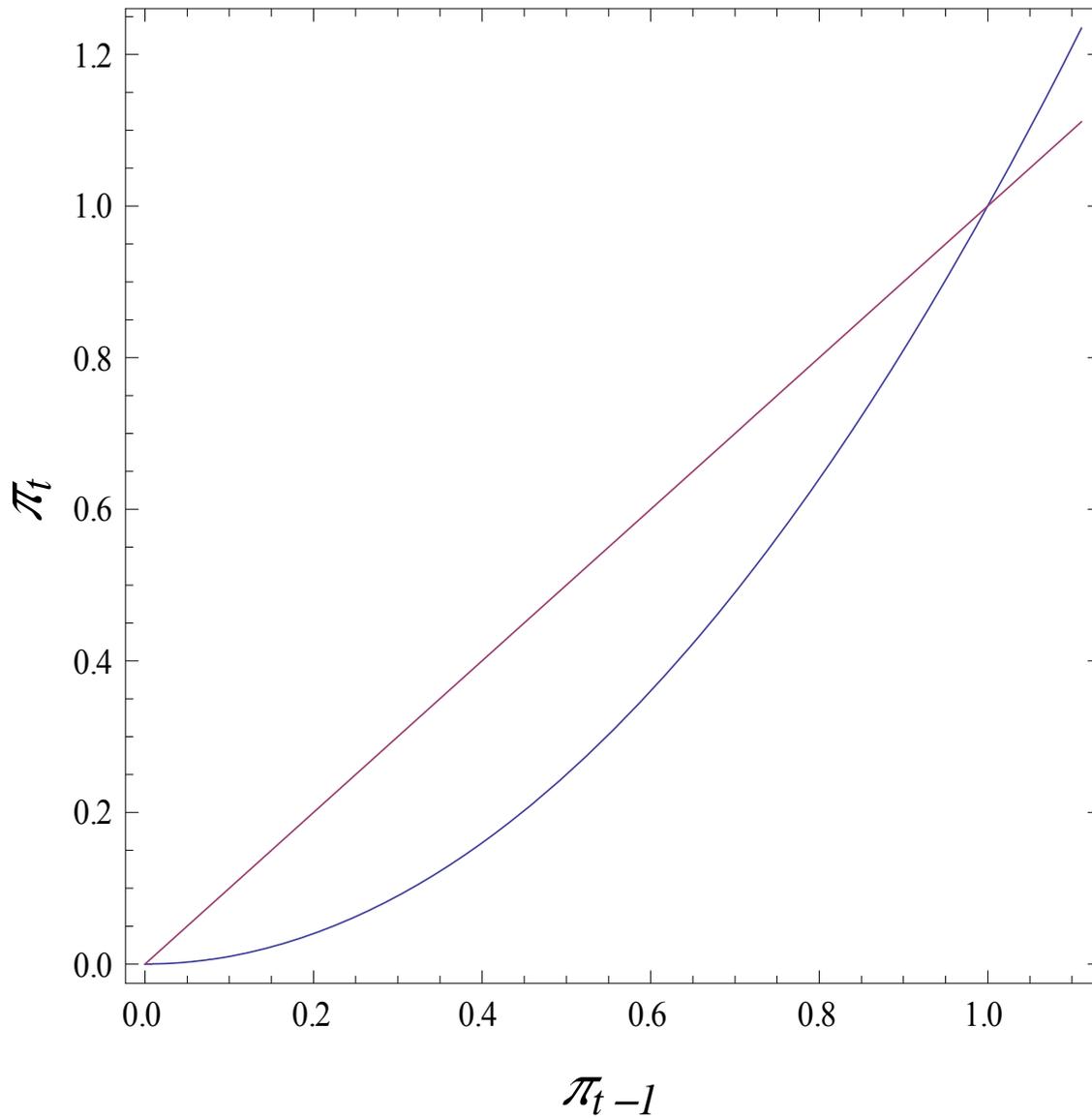


Figure 3: Self-Fulfilling Banking Collapses



$$u(q)=2\sqrt{q} ; c(q)=q ; \text{ and } \beta=0.9$$