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CREDIT RISK ANALYSIS OF CREDIT CARD PORTFOLIOS UNDER ECONOMIC STRESS CONDITIONS

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Abstract

We develop an empirical framework for the credit risk analysis of a generic portfolio of revolving credit accounts and apply it to analyze a representative panel data set of credit card accounts from a credit bureau. These data cover the period of the most recent deep recession and provide the opportunity to analyze the performance of such a portfolio under significant economic stress conditions. We consider a traditional framework for the analysis of credit risk where the probability of default (PD), loss given default (LGD), and exposure at default (EAD) are explicitly considered. The unsecure and revolving nature of credit card lending is naturally modeled in this framework. Our results indicate that unemployment, and in particular the level and change in unemployment, plays a significant role in the probability of transition across delinquency states in general and the probability of default in particular. The effect is heterogeneous and proportionally has a more significant impact for high credit score and for high-utilization accounts. Our results also indicate that unemployment and a downturn in economic conditions play a quantitatively small, or even irrelevant, role in the changes in account balance associated with changes in an account's delinquency status, and in the exposure at default specifically. The impact of a downturn in economic conditions and, in particular, changes in unemployment on the recovery rate and loss given default is found to be large. These findings are of particular relevance for the analysis of credit risk regulatory capital under the IRB approach within the Basel II capital accord.

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[2] This paper presents preliminary findings and is being distributed to economists and other interested readers solely to stimulate discussion and elicit comments. The views expressed in the paper are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of New York, Federal Reserve Bank of Philadelphia or the Federal Reserve System. Any errors or omissions are the responsibility of the authors. This paper is available free of charge at www.philadelphiafed.org/research-and-data/publications/working-papers/.

1. Introduction

There are relatively few publicly released studies analyzing the impact of the great recession on the fundamental credit risk components of retail portfolios outside of the spectrum of residential mortgage exposures. There are also relatively few publicly released studies that look at the determinants of credit card loss beyond the probability of default. On the other hand, banks are accustomed to using statistical models of credit loss as a credit risk management tool. Credit loss models also provide key inputs to determining the allowance for loan and lease losses (ALLL) and economic and regulatory capital. This type of analysis of credit card portfolio risk and portfolio losses is particularly relevant at this time, in the aftermath of the recent financial crisis and the worst recession experienced by the credit card industry since its inception.

In this paper we analyze credit bureau data from a panel data set containing account-level credit card information from a 5 percent random sample of individuals with a credit file in the credit bureau database. The data set is maintained by the Retail Risk Analysis unit at the Federal Reserve Bank of Philadelphia. The data set includes information on credit card account characteristics, such as account age, line and utilization, and the individual's credit score, as well as current and past delinquency and account balance information. We apply these data to a traditional framework for analyzing portfolio credit risk. This framework takes into account three components of risk: the risk of default or probability of default (PD), the exposure at default (EAD), and the loss given default (LGD), which represents the percentage of the exposure that is lost at default. Gross expected credit loss is defined as the product of the first two components. Net expected credit loss (EL) is defined as the product of the three components. It is normal industry practice to consider the analysis of each one of these components of loss separately. This practice has also been solidified by the implementation of this framework as part of the process of analysis of regulatory capital in the Basel II rule. In this study, we focus on the analysis of gross credit loss and the quantification of PD and EAD parameters in particular using credit bureau data. We also perform exploratory analysis of LGD, the third component of net credit loss, using recovery and charge-off data reported by select U.S. BHCs comprising the largest credit card issuers in the U.S. in FR Y-9C regulatory reports. The LGD analysis, however, lacks the level of detail and sophistication employed for PD and EAD analyses because useful account-level information on recoveries is not available in the credit bureau data.

Next, we identify a few relevant papers related to our analysis. To begin, Gross and Souleles (2002) analyze credit card delinquency and personal bankruptcy, focusing on the 1990s, using panel data on credit card accounts. The empirical model employed in their analysis is of particular interest because this modeling framework has been adopted by other subsequent papers in this literature. They observe account delinquency status over time. The outcome of interest, or the dependent variable, in their default model is a dichotomous variable that takes a value of one in a particular month if the credit card account defaults in that month and zero otherwise. An account is considered in default if it is seriously delinquent, which is defined as three monthly billing cycles. Using this variable, they model the delinquency behavior over time of credit card accounts using multi-period (i.e., dynamic) probit and logit models, which can also be referred to as discrete time duration models. In this model, the probability of default is a function of origination cohort, account age, control economic variables, and control variables that measure the account's inherent risk. The effect of account age is modeled using a flexible polynomial specification. A relevant result of their analysis is the observation of a significant increase in the propensity to default between 1995 and 1997. Their analysis also suggests that the probability of delinquency increases from the time an account is booked until about its two-year tenure and then declines. Important predictors of default are low credit score, large balances and purchases, or smaller payments. The authors conclude that the relation between default and economic fundamentals appears to have substantially changed over the period of study in ways that are not explained by their control variables.

Agarwal and Liu (2003) examine credit card delinquency and bankruptcy behavior using the same econometric framework of Gross and Souleles (2002). The authors use time dummies rather than a polynomial specification to control for seasoning effects on delinquency. They note that previous empirical studies did not consistently find a significant effect of macroeconomic factors on bankruptcy. They also note that Gross and Souleles (2002) did not find a significant impact of unemployment on credit card default. The authors hypothesize that the lack of a significant effect of unemployment may be the result of a lack of sufficient variation in the data either due to inadequate data or a lack of sufficient variation in the unemployment variable during the period of analysis. Using data from a large sample of credit card accounts over an extended time frame that includes periods of economic expansion and recession, the authors provide conclusive evidence of a significant impact of unemployment on credit card delinquency, with other results broadly consistent with the findings in Gross and Souleles (2002).

The model considered in our analysis of account delinquency transitions is similar in many respects to the one considered in Gross and Souleles (2002) and Agarwal and Liu (2003) with some caveats. As in those studies, the probability of default in our model is a function of origination cohort, account age, economic variables, and control variables that measure the account's inherent risk. Rather than focusing only on the default outcome, we propose a multiple state model that considers the current, delinquent, and default states and separately considers current accounts with medium-low or high utilization rates. The econometric framework considered is a multi-period-multinomial logit specification rather than a multi-period logit specification. Due to data limitations, our model is estimated over time intervals of six months. Also, we estimate separate models for current and delinquent accounts at the time of observation because our analysis indicates that these two groups of accounts historically have performed very differently.

In addition to the analysis of delinquency transitions, our empirical framework also includes the analysis of account balance changes associated with changes in delinquency status. It is common industry practice to estimate changes in account balances for accounts that transition to default. This type of analysis is also required by the Basel II rule as an intermediate step in the process of computing regulatory capital for credit risk. There is a limited relevant academic literature in this area of inquiry. Agarwal, Ambrose, and Liu (2006) studied the utilization of home equity lines at and after origination and found that borrowers with a higher expectation of future deterioration in credit quality originate credit lines to preserve financial flexibility. They also found a statistical relationship between a drop in FICO score and an increase in credit line utilization. Using a sample of Spanish corporate credit lines, Jimenez, Lopez, and Saurina (2009) found that firms that default on their credit lines have significantly higher line utilization rates and these rates increase as the default approaches.

Qi (2009) estimated EAD models for a sample of credit card accounts from a credit bureau over the period 1998–2008. She focused her attention on the estimation of the incremental accumulated dollar balance of an account at default, usually referred to as the loan equivalent exposure (LEQ)[3] separately for current and delinquent accounts and found that utilization rate (utilization > 95 percent), account age, and account balance are significant drivers of LEQ. Credit score and credit limit were found to have a heterogeneous impact on LEQ for current and delinquent accounts — credit score for current accounts and credit limit for delinquent accounts were found to be important determinants of LEQ. The author also found that LEQ declined significantly after implementation of the Bankruptcy Abuse Prevention and Consumer Protection Act (2005) and was higher in periods when overall default rates were high. The author noted that this positive association between LEQ and default rates, which could be interpreted as evidence confirming a positive macroeconomic impact on EAD during economic downturns, was characteristic of the 2002-2003 recession and the relationship between LEQ and defaults changed over time. The author pointed out that the increase in utilization as borrowers approach default can be the result of either an increase in account balance or a decrease in the line originated by the lender. However, her findings suggest that lenders rarely cut back credit limits, although they make it less easy to draw funds as borrowers become more severely delinquent. In our empirical analysis, we avoid this confounding effect by focusing our attention on changes in balance rather than changes in utilization.[4]

Last, we turn our attention to LGD, the third component of net credit loss. Although LGD is an important determinant of credit losses, research on LGD pertinent to retail credit is particularly limited. This is not surprising given the lack of adequate data available to model LGD for retail portfolios. A recent study on LGD was done by Bellotti and Crook (2009). They developed a spate of LGD models using account-level data on major UK retail credit cards and concluded from their findings that ordinary least squares models with macroeconomic variables are best for forecasting LGD, at both the account and the portfolio level. Their findings suggest that higher unemployment is associated with lower recovery rates. In our analysis of LGD, we use a simple

[3] The loan equivalent exposure of an account in period *t* that defaults in *t+12* and can be defined as the incremental accumulated balance on the account between *t* and *t+12* expressed as a percentage of undrawn balances at *t*.
[4] The exposure at default as a percentage of the outstanding balance at the reference time is usually referred to as the credit conversion factor (CCF).

regression model and data on recovery and charge-offs at the portfolio level from regulatory bank filings to assess the impact of macro factors on LGD at banks.

In the next section we present the data and conduct a descriptive statistical analysis. In section 3 we present the empirical methodology. In section 4 we present results and in section 5 we present conclusions. Tables and figures are presented in a separate section at the end of the paper.

2. Data and Descriptive Analysis

We have access to a panel data set containing trade line credit card information from a 5 percent random sample of individuals with a credit file in the credit bureau database. The data set is maintained by the Retail Risk Analysis unit at the Federal Reserve Bank of Philadelphia. The data set includes up to 10 active credit card accounts per individual. For the purpose of data selection, current accounts with zero balance and no activity within the last six months are excluded from the data set. At each observation point, delinquent accounts and accounts active, or with non-zero balance, over the last six months are included in the data set. It is highly unlikely for an individual to have 10 or more active credit card accounts. However, in that case, the 10 most recently opened accounts are retained. For our analysis, we employ a panel with information on credit card accounts from the end of 2005 to the second quarter of 2010. Thus, our data do not include the period around the passage of the recent bankruptcy reform legislation. We observe account snapshot information in six-month windows, in particular for the months of June and December within each relevant year. Given the enormous size of the original data set, in our analysis we employ a 10 percent random subsample from the sample described above, or a 0.5 percent sample of the overall credit bureau sample.

Table 1 lists the account level variables available in our sample as well as any derived variable transformations employed in the empirical analysis. Our data include information on account characteristics, such as account age, line and borrower’s credit score, as well as current and past delinquency status and account balance information. This information is combined with unemployment rate information at the state level, which represents the primary policy variable of analysis. We have divided the range of variation of relevant variables into segments as reflected in the table. This segmentation, when applied to the empirical analysis, allows us to estimate the potential non-linear impact of particular variables without having to rely on specific functional form assumptions.

Table 2a provides information on average values and proportions for relevant variables for specific variable ranges. For accounts with positive utilization, we observe that average utilization was around 72 percent prior to the recession and increased continuously from 72.14 percent in the fourth quarter of 2007 to about 76 percent in the fourth quarter of 2009. The table also provides descriptive information on the distribution of accounts by credit score bands. About 60 percent of accounts are concentrated in the highest credit score band over time. We also observe an increase in the proportion of accounts in the lowest credit score band around the period of economic stress conditions; this pattern is the result of account migration to the lowest band as a result of an increase in the delinquency rate around this time. The table also provides information on delinquency status over time. Between 95 percent and 97 percent of accounts remain current over a six-month time interval from the observation point during the time interval considered. Six-month default rates were higher around the time of the recession, with rates of around 3 percent to 3.4 percent. Re-performing accounts, that is, accounts that have experienced a delinquency (at least 30 days delinquent) over the last two years, represent between 6 percent and 7 percent of the accounts in our sample. Finally, average account age is between six and seven years, and we observe an increase in average account age during the period of economic downturn, which is the result of a lower rate of new account originations during this period.

Additionally, we performed an analysis of LGD, the third component of net credit loss, using recovery and charge-off data reported by select U.S. BHCs in FR Y-9C regulatory reports. BHCs included in the analysis are JPMC, Bank of America, Citi, Wells Fargo, USB, Capital One, and American Express. Overall, these banks accounted for over 80 percent of U.S. credit card receivables at the end of Q3 2010.[5] It is well-known that loss recoveries associated with credit card portfolios are traditionally low, often less than 10 percent of losses and, in some instances, about 20 percent of losses. Recovery rates calculated from FY Y9-C quarterly data submissions are comparable.[6] Table 2b shows average recovery rates for select U.S. BHCs calculated over the

[5] Source: BHC FR Y-9C submissions.
[6] PD and EAD models are developed for consumer credit cards using Equifax credit bureau data. Reported FR Y-9C data on recoveries and charge-offs may include recoveries and charge-offs on small business card portfolios in addition to consumer credit cards.

period Q1 2006 to Q4 2011, and recovery rates over nine quarters spanning the most recent recession, Q1 2008 to Q1 2010. The calculated quarterly recovery rates are percentage quarterly charge-offs recovered by a bank. The average recovery rate across all the BHCs considered in this analysis is estimated to be around 15 percent over the five-year period, and 10 percent over the nine recession quarters. For all BHCs, the recovery rate over the recession quarters was significantly lower. A time-series view of BHC recovery rates is also shown in Chart 1a. It is quite apparent from both Table 2b and Chart 1a that the recovery rate at all banks declined over the period of the most recent recession.

2.1. Descriptive Analysis of Credit Card Market Trends

Charts 1b and 1c show the evolution of total dollar outstanding and net dollar charge-offs on credit card loans held by some of the largest financial institutions in the U.S. Total U.S. consumer revolving debt fell to $801 billion at the end of 2010, down from $866 billion at the end of 2009.[7] The net charge-off rate peaked at the time of the financial crisis, propelled by a decline in dollars outstanding and an increase in the net dollars charge-off rate.

Charts 2a to 2c provide a time-series view of new credit card originations. From Chart 2a we observe that the number of new originations and the aggregated credit card balances of new originations have decreased consistently since early 2008. Chart 2b depicts credit card originations by credit score bands; we observe a significant increase in the proportion of high credit score accounts among new originations starting with 2008. Similarly, Chart 2c depicts credit card originations by credit line bands; we observe a significant reduction in the proportion of high credit line accounts among new originations starting with 2008. Because new originations are likely to be intrinsically risky, other things being equal, we interpret this trend as a risk mitigation strategy on the part of banks in times of economic stress.

Chart 3 depicts six-months-ahead default rates by origination vintage for 2004 to 2009. The vertical axis depicts default rates and the horizontal axis depicts time in six-month intervals. We observe 12 time periods, of six-month intervals, for the 2004 vintage, 10 time periods for the 2005 vintage, and up to two time periods for the 2009 vintage. In order to allow for easy comparisons of the evolution of the default rate across vintages from the time of origination, the

[7] Data are from bank call reports and other regulatory filings.

data are organized so that the first time interval observed in the data for each vintage is aligned with the first observation point in the graph. Overall, this simple descriptive analysis indicates that the 2007 vintage exhibits the highest default rates, while the 2004 vintage exhibits the lowest default rates. These results should be interpreted with caution. The performance of a particular vintage over time is a function of a combination of economic conditions and account- and borrower-specific factors. The identification of these confounding effects is one of the objectives of the econometric analysis conducted in the next section.

Table 3 provides descriptive information, at different sample periods, of credit card delinquency status six months ahead by present delinquency status. The data indicate a trend of increasing delinquency and default rates across the board, starting with the second part of 2007 and continuing to the end of 2009, with a clear improvement in the first six months of 2010. Not surprisingly, the patterns of delinquency and defaults six months into the future are correlated with the original delinquency state. As expected, delinquent accounts are likely to transition into default and higher utilization across delinquency status implies higher risk. Current accounts with zero utilization rates are likely to transition to high utilization rates within a six-month period, at a rate of 35 percent to 48 percent. Default rates six months into the future for this type of account vary from 0.92 percent to 1.65 percent with a peak default rate observed in the last six months of 2009. Current accounts with low utilization are not noticeably more risky than current accounts with zero balances. Current accounts with high utilization are significantly more likely to default than current accounts with lower utilization rates — between four and eight times more likely — and also significantly more likely to be delinquent; the difference in default rate has increased significantly during the period of the last economic downturn. This indicates a clear association between the level of utilization and default risk. Also, accounts with low utilization/high utilization at the present time are likely to remain in the same state six months into the future. For accounts delinquent at time *t*, default rates range from 39 percent to 54 percent, with a peak default rate observed during the last six months of 2009. Looking at the last section of the table, we observe that the average default rate in our sample over a six-month period varies from 2 percent to 3.4 percent, or equivalently, the annualized default rate for our portfolio varies from 4 percent to 6.8 percent.

Table 4 provides descriptive information for different sample periods of credit card delinquency status six months into the future conditional on present credit score across four credit score segments. The first section of the table reports frequency distributions of credit card accounts across credit score segments. In particular, we observe a high concentration of about 80 percent of accounts at the two highest credit score segments and this distribution remains relatively stable over time. The second section of the table provides descriptive information on delinquency status by line size. As expected, credit score is a significant predictor of future delinquency. Default rates six months into the future are significantly lower for higher credit score segments across all different delinquency states, are usually increasing with utilization, and are the highest for delinquent accounts. Average default rates at the lowest credit score segment are relatively similar for current low and high-utilization accounts and range from 14 percent to 21 percent for current low-utilization accounts and from 16 percent to 23 percent for current high-utilization accounts, while default rates range from 47 percent to 62 percent for delinquent accounts. Average default rates at the highest credit score segment range from 0.07 percent to 0.14 percent for current low-utilization accounts, from 0.34 percent to 1.27 percent for current high-utilization accounts, and from 3.83 percent to 10.1 percent for delinquent accounts. Proportionally, the difference in default rates between current low-utilization and high-utilization accounts are relatively small for accounts in the lowest credit score band and increase for higher credit score bands, with very large differences observed for accounts in the highest credit score band.

For accounts delinquent at time t we also report in Table 4 default rates by credit score segment for a six-month lag of the credit score. This strategy allows us to determine the importance of account migration across credit score bands for delinquent accounts. In particular, we observe significantly higher default rates for all the credit score bands except for the lowest one when employing the lag of the credit score. This is consistent with our intuition; a current account in a high credit score band is likely to migrate to a lower credit score when the account becomes delinquent and before it defaults. Taking into account that the credit score rank orders accounts according to risk, accounts that are more likely to default will also be more likely to migrate to lower credit score bands. Accounts that migrate to the lowest credit score band are likely to perform very poorly, similar to other delinquent accounts already in the lowest credit score band. This explains the important differences observed when we consider the lag of the credit score instead of the credit score at time t.

3. Empirical Methodology

We consider a traditional framework for the credit risk analysis of a credit card portfolio. This framework takes into account three components of risk: the risk of default or probability of default (PD), the exposure at default (EAD), and the loss given default (LGD), which represents the percentage of the exposure that is lost at default. Expected loss can be defined as the product of these three components,

$$EL = PD \cdot LGD \cdot EAD$$

It is normal industry practice to consider the analysis of each one of these components of loss separately. This practice has also been solidified by the implementation of this traditional framework as part of the process of analysis of regulatory capital in the Basel II rule. Next, we describe the econometric methodology considered in the analysis of gross credit loss and on the PD and EAD parameters in particular. The lack of useful information on recoveries in our data set prevents us from also conducting a detailed account-level analysis of LGD, the third component of net credit loss.[8] Instead for the analysis of LGD, we resort to more aggregated, publicly available information. The empirical methodology is described in the next three subsections.

3.1. The Probability of Default and the Process of Delinquency Transitions

We assume that a credit card account can be in one of several current or delinquent states at each particular point in time. We model delinquency as a process of transition across states over time with default representing an absorbing state. At each point in time, delinquency status is a function of account characteristics, customer characteristics, economic environment, and past delinquency history up to the present time. In particular, assume that at time *t* a credit card account can be in one of K possible delinquency states $s_t \in \{d_0, \ldots, d_K\}$. For a particular credit card account n, denote the relevant risk drivers at time *t*, including delinquency history up to time *t*, as $R_n(t)$. For accounts active at time *t*, we assume a suitable multinomial logit probability

[8] Loss recoveries associated with credit card portfolios have been traditionally low, often less than 10 percent of losses and, in very rare instances, above 20 percent of losses.

specification for the transition from the present state at time t to any alternative state six months into the future, at time *t+1*, with transition probabilities defined as follows:

$$\mathrm{P}\left(\mathrm{s_{t+1}} = \mathrm{d_k} | \mathrm{s_t} = \mathrm{d_j}, \mathrm{R_t}\right) = \frac{\exp\left(\varphi_{jk}(\mathrm{R_t})\right)}{1 + \sum_{i=1,\dots,K} \exp\left(\varphi_{ji}(\mathrm{R_t})\right)}, \text{ for } k = 0, \dots 0K$$

or $\Pr\left(\mathrm{d_k} | \mathrm{d_j}, \mathrm{R_t}\right)$ for simplicity. In particular, we consider the following convenient specification:

$$\varphi_{ji}(\mathrm{R_t}) = \lambda\left(age(t), \beta_{ji}^{\lambda}\right) + \delta\left(\mathrm{R_t}, \beta_{ji}^{\delta}\right), \text{ for } k = 1, \dots 1K$$

$$and\ \varphi_{j0}(\mathrm{R_t}) = 0$$

where $\lambda\left(age(t), \beta_{ji}^{\lambda}\right)$ represents a baseline hazard of account age with a semi-parametric specification (in the spirit of Han and Hausman, 1990; Meyer, 1990; McCall, 1996; and Deng, Quigley and Van Order, 2000). The factor $\delta\left(R(t), \beta_{ji}^{\delta}\right)$ captures the effect of risk drivers and the account's delinquency history, and in our empirical framework, $\delta\left(R(t), \beta_{ji}^{\delta}\right)$ will be a linear specification for simplicity and convenience of interpretation. The coefficients $\left(\beta_{ji}^{\lambda}, \beta_{ji}^{\delta}\right)$ are specific to the origination and destination delinquency states. The condition $\varphi_{j0}(R_t) = 0$ is consistent with the standard multinomial logit specification (Green, 2002). Within this framework, the contribution to the sample likelihood of account n with account history $\{(d_{nt}, R_{nt}), t = 1, \dots, T\}$ is,

$$\prod_{t=1,\dots,T} \mathrm{P}(\mathrm{d_{nt+1}} | \mathrm{d_{nt}}, \mathrm{R_{nt}})$$

and we obtain the following expression for the likelihood function for a sample of N accounts,

$$\prod_{n=1,\dots,N} \prod_{t=1,\dots,T} \mathrm{P}(\mathrm{d_{nt+1}} | \mathrm{d_{nt}}, \mathrm{R_{nt}}).$$

Re-arranging terms we obtain an equivalent expression of the form

$$\left\{\prod_{t=1,\dots,T}\prod_{n_0=1,1,N_{0t}} \mathrm{P}\big(\mathrm{d}_{n_0\mathrm{t}+1}|\mathrm{s_t}=0,\mathrm{R}_{n_0\mathrm{t}}\big)\right\}\cdot\ldots\cdot\left\{\prod_{t=1,\dots,T}\prod_{n_K=1,1,N_{Kt}} \mathrm{P}\big(\mathrm{d}_{n_K\mathrm{t}+1}|\mathrm{s_t}=\mathrm{K},\mathrm{R}_{n_K\mathrm{t}}\big)\right\}$$

where each component describes, for each *k=0,…,K*, the likelihood function for the transition from state $s_t = k$ to any other state s_{t+1} within a multinomial logit specification. Thus, this expression indicates that, as long as there are no common unobserved elements across the different components, or unobserved heterogeneity, the MLE associated with this specification will be equivalent to considering *K+1* panel multinomial logit specifications, with each one of these specifications independent from each other.[9]

Multi-period-multinomial logit specifications can be interpreted as a particular type of discrete time duration model.[10] Shumway (2001) makes this point theoretically. In particular, proposition 1 in this paper indicates that "a multi-period logit model is equivalent to a discrete-time hazard model [under certain distributional assumptions]." Sueyoshi (1995) also makes a similar point. Shumway's result for the multi-period-multinomial logit has been applied in particular by Agarwal, Ambrose, and Chomsisengphet (2005) in a study of auto loans.

Our model specification incorporates all the basic ingredients that have been employed by many authors in the relevant literature. Like the papers by Gross and Souleles (2002), Agarwal and Liu (2003), Agarwal, Ambrose, and Chomsisengphet (2005), and others, our modeling approach considers the logit specification. The advantages of this specification are the ease of interpretation, as illustrated elsewhere in this paper, and its ideal numerical properties.[11] Also, like the models of Agarwal and Liu (2003) and others, our model uses time dummies to control for seasoning effects. Several econometric studies indicate that the use of a flexible specification to account for time dependency, and time dummies in particular, goes a long way toward minimizing the impact of spurious unobserved heterogeneity, which is necessarily present in any econometric model. Early proponents of this approach include Han and Hausman (1990), Meyer

[9] The presence of unobserved heterogeneity would bring to bear additional computational challenges (Heckman and Singer, 1994, Baker and Melino, 2000, Canals-Cerda and Gurmu, 2007) and conceptual challenges (Heckman, 1981; Cameron and Heckman, 2001; Bearse, P., J. Canals-Cerda, and P. Rilstone, 2007).
[10] Literature surveys of duration models include Kiefer (1988), Canals-Cerda and Stern (2002) and Van Den Berg (2009).
[11] More precisely, models in the logit family have the property of global concavity of the likelihood function, which guarantees convergence of the maximum likelihood estimator to the optimum (Amemiya, 1985).

(1990), and McCall (1996). In particular, the model developed in McCall (1996) to analyze unemployment was subsequently applied in an influential study of mortgage prepayment and default by Deng, Quigley, and Van Order (2000).

3.2. Exposure at Default and the Balance Ratio

In the four-state transition model discussed above, each non-defaulted account at *t* (current with low utilization, current with high utilization, and delinquent) can transition into one of the four possible delinquency states in $t+6$ with default included as the terminal state. As a result, there are 12 transition states, and the projected exposure of accounts corresponding to each transition has to be determined. Typically, econometric models are used to estimate the amount of exposure of defaulting accounts. These models are usually referred to as models of exposure at default (EAD). Both transitions from current to default and transitions from delinquent to default are important from the perspective of credit risk management and loss projection. On the one hand, current accounts have a low risk of default and traditionally constitute the lion's share of a credit card's portfolio but can contribute the largest balance increases at default. On the other hand, delinquent accounts in a well-managed portfolio are likely to contribute only relatively modest future balance increases but typically have a high probability of default.

We model balance changes for account transitions using a "balance ratio," or BR, approach. The balance ratio for a particular account at time *t* is defined as the ratio of the account balance in period $t+6$ to the account balance in period *t*. The econometric approach to estimation of changes in the BR considers a log-linear model specification with the log of the balance ratio as the dependent variable,

$$Log(Balance\ Ratio)_{jit} = \varphi_{ji}\left(\mathrm{R_{jit}}, \mathrm{t}\right) + \varepsilon_{jit}$$

where *i* and *j* represent the account's state at period *t* and $t+1$, respectively, and $\varphi_{ji}\left(\mathrm{R_{jit}}, \mathrm{t}\right) = \alpha_{ji} + \beta_{ji} R_{jit}$, represents a general linear specification considered in our empirical framework where R_{jit} represents the independent variables, or risk drivers, from the set of potential variables defined in Table 1, which also includes interactions across risk drivers for some empirical specifications, and ε_{jit} represents other account- and time-specific idiosyncratic factors.

3.3.Loss Given Default

Unlike data used to estimate PD and EAD models, our data do not include detailed account-level information on borrower characteristics or portfolio characteristics to estimate LGD. Due to data limitations, we estimate LGD at the portfolio level using bank reported data on charge-offs and recoveries in regulatory reports. In fact, our LGD model is a simple model in which the recovery rate is modeled as a simple autoregressive process.

Suppose $\mathrm{RR_{it}}$ is the recovery rate of the i-th bank at time t. Then $\mathrm{RR_{it}}$ is modeled as,

$$RR_{it} = \mu_i + \beta RR_{it-1} + \gamma M_t + \varepsilon_{it}$$

where μ_i *i*s the mean recovery rate of the i-th BHC and ε_{it} is the white noise error term. Our model implies that banks' recovery rate is a stationary stochastic process that is mean reverting and deviations of the recovery rate from the mean in any given period are explained by the recovery rate in the recent past and variables exogenous to the process, here a variable that captures the macroeconomic trend. Note that LGD can be determined once the recovery rate is estimated using the relationship $LGD_{it} = 1 - RR_{it}$. During practical implementation, application of this LGD factor to gross portfolio losses would yield net portfolio losses.

4. Analysis of Empirical Results

In this section, we apply the theoretical econometric framework described in the previous section to analyze credit risk in a generic credit card portfolio. Estimation results for models of delinquency transition, results for balance ratio and exposure at default models, and results for the recovery rate and LGD are presented in the following subsections.

4.1.Probability of Default and the Process of Delinquency Transitions

Each model specification analyzed considers the probability of transition from the present state at time t to any one of several possible states six months into the future. The most simple model specification specifies a framework where accounts transition among three possible delinquency states (current, delinquent, and default). We also considered an extension of this model in which the current state is further segmented into two distinct states according to the level of line utilization, current accounts with high utilization and other current accounts. In this paper we present results for this second, more comprehensive model specification because it contributes some additional insights to the analysis of credit risk and to our understanding of the impact of macro variables on the delinquency transition process.[12]

Model estimation results are presented in Tables 5a to 5c and are discussed in this section. We experimented with a variety of model specifications before selecting the final ones reported in the tables. It is worth noticing that we observed a high correlation between delinquency projections across different sensible model specifications. Model risk drivers include line utilization, re-performing status, a fourth-quarter dummy, vintage, account age, credit score, and two policy variables representing change in unemployment and the unemployment level. The unemployment variables are lagged three and six months, respectively, with respect to the time at which the delinquency outcome is reported. Also, the change in unemployment variable represents a one-year change in unemployment. It is worth noting that there is a high correlation between both unemployment variables; one should take this fact into consideration when interpreting the associated model parameters separately.

Parameter estimates of non-linear models are inherently difficult to interpret; to facilitate this task, parameter estimates are reported as odds ratios.[13] Thus, a parameter estimate above or below one, respectively, represents an increase or a decrease in the odds of a particular outcome as a result of an increase in the value of the associated explanatory variable. A convenient feature of the multinomial logit model is that the odds ratio coefficients are invariant across values of the

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[12] Readers interested in the results from the simpler model specification can request this information from the authors.

[13] Observe that the odds of a particular outcome k relative to the base outcome is defined by the expression

$$\frac{exp(\beta_k x)}{1+\sum_{i=1,\ldots,K} exp(\beta_i x)} : \frac{1}{1+\sum_{i=1,\ldots,K} exp(\beta_i x)} = exp(\beta_k x)$$

and can be denoted as $Odds_k(x)$. We can consider the odds ratio of increasing a certain characteristic x_i by a unit as equal to

$$\frac{Odds_k(x_{-i}, x_i + 1)}{Odds_k(x)} = \frac{exp(\propto + \beta_{-i} x_{-i} + \beta_i x_i + \beta_i)}{exp(\propto + \beta x)} = exp(\beta_i),$$

where, $x = (x_{-i}, x_i)$, $\beta x = \beta_{-i} x_{-i} + \beta_i x_i$ and the odds ratio is independent of characteristics and equal to $exp(\beta_i)$. The odds ratio approach accepts a simple interpretation: if a certain variable has no material impact on the odds of a certain outcome, we would expect the associated odds ratio to be about one. Reported t-values are relevant for the null hypothesis of an odds ratio equal to one. If a certain variable has a positive impact, the odds ratio will increase above one, and if it has a negative impact, the odds ratio will decrease below 1. As an example, if a one-unit increase in a particular variable doubles the odds of a sale, then the odds ratio will be equal to two.

explanatory variables, much like the coefficients in a linear regression model. Given the huge size of the sample employed in our paper, most parameters with associated odds ratios that deviate even slightly from one will be significant.

Table 5a presents parameter estimates for the population of accounts current at time t with low/medium utilization rates, Table 5b presents parameter estimates for the population of accounts current at time t with high utilization rates, and Table 5c presents parameter estimates for the population of accounts delinquent at time t. We present results for two model specifications; the more complex one includes interactions between unemployment and credit score. We also considered simpler model specifications that did not include vintage dummies or interactions between unemployment and score; the inclusion of vintage effects did not seem to have a significant impact on other parameter values and, in particular, the parameters associated with account age

Looking at Table 5a, we observe that the highest credit line group is associated with an increase in the odds of transition to the default state and a decrease in the odds of transition to other states, when compared with the odds of remaining in the current and low/medium utilization state.
When compared with a low utilization rate, a medium utilization rate is associated with an increase in the odds of transition to the delinquent state, and it is also associated with a decrease in the odds of transition to other states, when compared with the odds of remaining in the current low/medium utilization state. Not surprisingly, re-performing accounts are at a high risk of transition to the delinquent and default states, as indicated by the associated odds ratios of 3.6 and 2.2, respectively. With regard to vintage effects, the 2007 vintage, in particular, is associated with an increase in the odds of transition to the delinquent and default states.

Another important variable to consider is account age. This variable enters the model in the form of age dummies, which allows for a great deal of flexibility. Taking the group of accounts with age less than one year as the control group, we observe that the odds of transition to a current and high-utilization state as well as the odds of transition to the delinquent or default states decrease with account age. This result indicates that new accounts are more risky than more seasoned accounts, after controlling for other drivers of risk. This finding is broadly consistent with previous studies (Gross and Souleles, 2002). This result is particularly relevant for assessing the latent risk of account origination strategies by financial institutions. Not surprisingly, credit score is an important determinant of the probability of transition to the delinquent and default states. For an account current at the present time, the odds ratio for the transition to the delinquent or default states are 0.59 and 0.24, respectively, for an account in the second lowest credit score group. These values imply a twofold increase in the odds of delinquency and a fourfold increase in the odds of default for accounts in the lower credit score segment when compared with accounts in the second lowest credit score segment. As expected, the results are even more pronounced when we compare accounts in the lowest credit score segment with accounts in the two highest credit score segments. In particular, for the second highest credit score segments, the odds ratios are 0.20 and 0.05, and for the highest credit score segment, the odds ratios are 0.04 and 0.004, respectively.

The model specification also includes lag-unemployment and lag-unemployment-change as risk drivers, and in the more complex model, this variable is interacted with credit score. The results indicate that both measures of unemployment have a positive association with the likelihood of transition to the delinquent and defaulted states. Interestingly, the interaction between credit score and unemployment indicates that different credit score groups respond differently to an increase in unemployment. Because of the high correlation between the unemployment and change in unemployment variables, it is not very useful to interpret the parameters associated with these variables separately. In the more complex model, the impact of unemployment, for example, is captured by a baseline parameter that affects all credit score groups and an interaction parameter that represents the incremental impact of unemployment in the specific group with respect to the base credit score group, with the baseline group defined as the group with the lowest credit score. The overall conclusion is that an increase in unemployment seems to have proportionally a smaller impact in the lower credit score groups. In particular, our estimates indicate that lower credit score groups have a much higher propensity to default under any kind of economic conditions, but in relative terms, unemployment has a smaller impact on lower credit score groups, as indicated by a smaller change in the odds ratio as a result of an increase in unemployment.

Table 5b presents parameter estimates for the population of accounts current and with high utilization at time t. We observe significant differences in parameter estimates when compared with those in Table 5a. In particular, we observe that both the highest and the second highest credit line groups are associated with a significant increase in the odds of remaining in the current-high-utilization state and an even larger increase in the odds of transition to the delinquent and default states, and this second effect is particularly important. Specifically, the odds ratios associated with accounts with the highest lines are 1.86 and 2.40 for the transition to the delinquent or default states, respectively. We also observe significant differences in the impact of unemployment on the transition across states. Specifically, for this group of accounts we find that unemployment has a much larger positive impact on the odds of remaining in the current-high-utilization state and an even larger impact on the odds of transition to the delinquent and default states.

Table 5c presents parameter estimates for the population of accounts delinquent at time t. The results are broadly consistent with our expectations and, for the most part, are consistent with the results already discussed for current accounts. Accounts in the highest line range are more likely to transition to default. High utilization is an important predictor of transition to default. With regard to the vintage effects, the 2006 and, to a larger extent, the 2007 vintage are associated with an increase in the odds of transition to default. Another important variable to consider is account age. The odds of remaining in the delinquency state seem to be most affected by account age for those accounts that are at least five years old. However, the odds of transition to default decrease significantly with account age. As expected, accounts with low credit scores are significantly more likely to transition to the default state and less likely to remain in the delinquent state. We should point out that for accounts delinquent at t, the score variable considered has a six-month lag. Our analysis indicates that the lagged score variable is less likely to be affected by the current delinquency and is more informative. Both measures of unemployment have a positive association with the likelihood of transition to default, while an increase in unemployment reduces the odds of remaining in the delinquent state. In the more complex model, the interaction between credit score and unemployment indicates that in relative terms, lower credit score groups have a lower propensity to transition to the default states, as indicated by a smaller change in the odds ratio, as a result of an increase in unemployment.

Table 5d presents estimation results for models with state of residence as a fixed effect. The model specifications considered mimic those in Tables 5a to 5c. T-values are not included in order to accommodate various model specifications in a single table. Overall, the results are consistent with those in Tables 5a and 5b and will not be discussed.

Chart 4 graphically compares in-sample and out-of-sample model projections with realized default rates for model specification 2 in Tables 5a to 5c. Out-of-sample model projections are obtained by re-estimating the model with a sample that excludes the last year of data. Thus, the out-of-sample results are for the last sample year. Overall, results do not seem to exhibit a significant or systematic bias. The average bias in the in-sample models is around 3 percent, with the highest bias around 5 percent across models. The average bias for the out-of-sample models is between 3 percent and 5 percent, with the highest bias between 9 percent and 15 percent. The out-of-sample models seem to exhibit a downward bias across model specifications for the last time period considered. Because the economic downturn period in our data is concentrated in the last two years, it should not be surprising that excluding one of these years from the estimation will have an impact on the overall results. We view this as a cautionary tale for risk management, since it suggests that model loss projections may not be sufficiently conservative if the data do not include a sufficiently representative stress period.

4.2. Exposure at Default and the Balance Ratio

Because of the unsecured and revolving nature of credit card lending, the credit risk analysis of a credit card portfolio must take into account the potential impact of changes in account balances on portfolio risk and on potential losses. In particular, for the purpose of calculating expected projected losses, we need to determine the dollars outstanding of accounts expected to default, in conjunction with the risk of default. An account's dollars outstanding, including accrued interest and fees, at the time of default is referred to as exposure at default (EAD). The challenge in estimating EAD for unsecured credit card lines is the determination of the *incremental* additional draws on accounts that are current or delinquent up to the time of default. In this paper, we employ a methodology that considers the estimate of dollar balances at the time of the future default as a percentage of the current account balance, or the balance ratio (BR). In particular, we use BR models to determine the dollar exposure of accounts $t+6$ months into the future expressed as a percentage of the account balance in period t corresponding to the different transition states of accounts being considered.

In the four-state transition model considered in this paper, each non-defaulted account at t (current with low utilization, current with high utilization, and delinquent) can transition into one of the four possible delinquency states in *t+6* (current with low utilization, current with high utilization, delinquent, and default). As a result, we consider 12 transition states, and the exposure of accounts has to be determined for each one of these transitions. From a modeling perspective, we estimate dollar exposures of current and delinquent accounts separately for the defined terminal states. In order to keep the presentation of results within a manageable limit, our analysis of parameter estimates focuses mostly on the discussion of results where default is the terminal state. We do this, keeping in mind that models of balance changes after transition from a current or delinquent state to default are the most pertinent ones in the context of exposure at default.

Table 6 provides descriptive information on average values and distribution of dollar balances of defaulted and delinquent accounts expressed as a percentage of the current account balance. We observe that for the delinquent to default population, on average, balances increase by about 20 percent between delinquency and default. For current accounts, the percentage increase in account balances, on average, on the path to delinquency or default is much higher. It is over 40 percent for accounts that turn delinquent and over 50 percent for accounts that default. These findings seem reasonable given that on average we expect changes in account balances resulting from transitions from delinquent to default to be moderate due to the limited credit available to a typical delinquent account. A further breakdown of current accounts by the level of utilization shows that it is the low-/medium-utilization accounts that contribute largely to the balance increase at delinquency or default. For low-/medium-utilization current accounts, balances increase almost twofold on the path to delinquency or default. For current high-utilization accounts, on the other hand, balance at the time of delinquency (or default) is higher by about 18 percent (or 30 percent). Chart 5 presents balance ratio kernel density estimates for several relevant transitions.

The parameter estimates of the BR models for the delinquent and current accounts that end up in default are presented in Table 7. Recognizing that we observe individual accounts bi-annually, and that current accounts first have to go through the delinquency state to reach default, regression results for current accounts that enter delinquency are also reported. We do this to lend some insight into the evolution of balances for current accounts that might ultimately default. Specifically, parameter estimates from the regression of current accounts that transition to delinquency and default are presented in the first two columns of the table numbered (1) and (2), respectively. Also, regression results for highly utilized (utilization > 80 percent) current accounts that default are shown in column (3), and regression results for the delinquent accounts that default are shown in column (4). In order to ascertain the sensitivity of the BR to the timing of balance builds, we also estimated balance changes of accounts over a one-year period. The regression results are shown in Panel B. As can be seen, these regression results are not qualitatively different from the ones reported in Panel A. Hence, our discussion of the parameter estimates focuses mostly on the regression results in Panel A.

We focus first on the *t*-th period *delinquent accounts that default* in period *t+6*. (See column (4) of Table 7.) Our results suggest that several account characteristics (credit line, utilization, vintage, and account age) and borrower characteristics (credit score) are important determinants of EAD. However, in this case economic factors do not play a pivotal role in determining dollar exposure of defaulting accounts.

For obvious reasons, the incremental dollar amounts that borrowers can add to balances on their path to default is a function of credit line and utilization. In fact, the potential to draw additional balances is expected to be greater for accounts that have a high credit line and have used relatively little of their credit lines prior to default. As expected, we see that dollar exposure at default expressed as a percentage of dollar balances prior to default (or BR) is highest for accounts that have not used much of their credit line (utilization < 35 percent). Relative to these low-utilization accounts, the BR of accounts with medium utilization (35 percent < utilization < 80 percent) and high utilization (utilization > 80 percent) are 8 percent and 10 percent lower, respectively. We also see that the BR of accounts decreases monotonically with credit line; accounts with credit lines of less than $1,500 have the highest BR, and compared with the low credit line accounts, the BR of accounts with credit lines between $1,500 and $7,500 is 8.6 percent lower and the BR for accounts with credit lines over $7,500 is 11.3 percent lower. These results have important implications in regard to balance drawdowns. Specifically, this indicates that two delinquent accounts with the same level of utilization but with different credit lines will behave differently in terms of balance drawdowns as they approach default; the low line account will draw a greater percentage of the undrawn credit line at default. It is also observed that the BR of accounts varies with account age and vintage. More recent vintages are seen to have a lower BR, with the BR for the 2009 vintage being the lowest. The 2009 vintage BR is 3.5 percent lower than the BR of the 2005 or prior vintage. The BR of accounts decreases monotonically with age, and mature accounts are seen to have a lower BR relative to accounts age one year or less. Among borrower characteristics, lagged credit score is a significant determinant of the BR. Our results suggest that borrowers in the mid credit score range (560 < credit score < 700) have a higher BR compared to borrowers with credit scores of < 560. Interestingly, we see no observed statistical difference in the balance ratios of the lowest and the highest quality borrowers. Overall, our findings indicate that account characteristics, particularly credit line and utilization, are the most important determinants of the BR of delinquent to default accounts.

Next, we focus our attention on the impact of the macroeconomic variable, here the unemployment rate. It is expected that in bad economic times, particularly in periods of high unemployment, borrowers under financial stress would likely use unsecured, unused credit lines even more, adding to EAD. We observe that both unemployment in levels and change in unemployment have a significant positive impact on the BR. However, the magnitude of the macro impact on the BR, hence EAD, is very small. This indicates that economic conditions do not play a significant role in determining dollar exposure of defaulting accounts.

Results for the *current to delinquent and the current to default* population shown in columns (1) – (3) are directionally similar to the results discussed above. Dollar exposure at default of current accounts expressed as a percentage of current account balance in period *t* is decreasing with utilization, credit line, and account age. As with the delinquent to default population, newer vintages and younger accounts are seen to have a lower balance ratio. The vintage effect is seen to be stronger for the current to delinquent population, though, especially for the newer vintages, but its impact is very much muted for the current to default population. Furthermore, compared to delinquent accounts, the marginal impact of utilization, credit line, and age on the BR for current accounts is much stronger. Interestingly, current accounts that have experienced delinquency at least once in the last 24 months have a lower balance ratio, a result that could be driven by successful implementation of risk management strategies at banks that are typically aimed at curbing EAD of riskier accounts (maybe through a credit line reduction). However, the effect is not maintained in the current to default transition. Parameter estimates on lagged credit scores for the current to delinquent population suggest that better quality borrowers have lower BRs, and for the defaulting population, borrowers with credit scores > 700 have a lower BR. Once again, as we have seen for the delinquent to default accounts, macroeconomic variables are seen to have very little impact on the BR of current accounts that become delinquent or default. The macro impact is stronger, though, than what is seen for delinquent accounts.

In addition to the model specifications reported in this paper, we also considered a variety of alternative model specifications that are not reported because they rendered the same conclusions. In particular, we considered model specifications that include time-quarter dummies in order to ascertain potential systematic deviations in balance changes at specific calendar dates, after controlling for observable individual risk factors. The overall conclusion from this exercise is that there are no relevant differences in account balance changes for accounts that transition to default, with the largest measured difference being a 2.5 percent increase in the second quarter of 2008 and an average increase of 1.4 percent during the period of the last recession, in 2008 and 2009.

4.3. Recovery and Loss Given Default

Under the Basel II capital accord, banks have to calculate the expected loss, or expected recovery, for their credit portfolios under depressed economic conditions, and as part of those depressed conditions, LGD has to be estimated. In order to quantify the magnitude of the impact of macro factors on bank recovery rates, we estimated the RR model discussed in Section 3.3 using the unemployment rate and the change in the unemployment rate as the macroeconomic variables of interest, both lagged one quarter. The regression results corresponding to four different RR model specifications are shown in Table 8.[14] Recovery rates are expected to be lower when the economy is under stress. This is because economic downturns, which are typically associated with higher unemployment, reduce borrowers' ability to repay debt. And under such circumstances, loan collection efforts are expected to yield minimal borrower payouts. Our findings are broadly consistent with such views. All three model results associate

[14] Coefficients corresponding to firm-specific dummies are not reported in Table 8.

lower recoveries with a higher unemployment rate and changes in the unemployment rate. The results also indicate that a rapid increase in unemployment has a more severe impact on recoveries; the recovery rate falls by less than one basis point when unemployment increases by one percentage point, but a one-percentage-point increase in the *change* in unemployment reduces the recovery rate by about 4 percent.

5. Conclusions

Using panel data on credit card account performance and account characteristics from a credit bureau, we develop an empirical framework for the analysis of credit risk and loss projections for a generic credit card portfolio. This study substantiates results from the existing literature and highlights some new findings. We are not aware of other publicly released studies in this area that undertake a systematic analysis of credit risk. Prior studies have focused their attention on a single aspect of risk, i.e., the probability of default (Gross and Souleles, 2002, Agarwal and Liu, 2003) or the exposure of default (Qi, 2009) or loss given default (Bellotti and Crook, 2009). While these studies have contributed new insights into our understanding of risk in credit card portfolios, they were not intended to provide a comprehensive view of portfolio credit risk. This is also the first publicly available study of credit risk in credit card portfolios that employs data from the most severe downturn experienced in this field of consumer finance. In this respect, our analysis benefits from a significant variation in policy variables, risk exposure, and performance outcomes observed in our data.

Our results indicate that unemployment, or, in particular, the level and change in unemployment, plays a significant role in the probability of transition across delinquency states in general and the probability of default in particular. In addition, our analysis indicates that unemployment plays a statistically significant role in the increase in account utilization, i.e., an increase in the balance to line ratio, but the effect is comparatively not as large as the effect on the odds of transition to the delinquent and default states. Furthermore, our analysis indicates that the impact of unemployment is heterogeneous across accounts with different credit score and utilization levels. In particular, our estimates indicate that lower credit score groups have a much higher propensity to default, but in relative terms, unemployment has a smaller impact on lower credit score groups, as indicated by a smaller change in the associated odds ratio as a result of an increase in unemployment. Also, the impact of unemployment on the risk of future delinquency and default of current accounts is particularly large for high utilization accounts.

Our results also indicate that unemployment, and, in particular, the level and change in unemployment, plays a quantitatively small or irrelevant role in the changes in account balance associated with changes in an account's delinquency status. In particular, we find that unemployment has, at most, a very small and inconsequential effect on the change in balance associated with accounts that default. We also considered model specifications that include time-quarter dummies in order to ascertain potential systematic deviations in balance changes at specific calendar dates, after controlling for observable individual risk factors. The overall conclusion from these models is that there are no relevant differences in account balance changes for accounts that transition to default, with the largest measured difference being a 2.5 percent increase in the second quarter of 2008 and an average increase of 1.4 percent during the period of the last recession, in 2008 and 2009. Because exposure at default (EAD) is one of the three key retail risk parameters that has to be quantified within the Basel II IRB framework, and because of the emphasis placed on accounting for conditions during an economic downturn within the risk parameterization process, our findings are particularly relevant for the analysis of credit risk regulatory capital.

Last, our findings also indicate that loss given default is severely affected by macro conditions, and this finding should be accounted for within a loss forecasting framework. The availability of detailed account level data could also potentially benefit LGD modeling. The approach developed in this paper can also be applied to other types of revolving retail portfolios, for example, home equity lines of credit. From a practical perspective, our approach can be viewed as a useful tool for risk management and a flexible and informative framework for the analysis of portfolio loss.

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6. Tables and Figures.

Table 1. – Relevant Variable Definitions

Balance	Current and Past Due Balance
Line	Credit Limit
Line 1500-7500	Dummy, credit line from 1500 to 7500
Line 7500-25000	Dummy, credit line from 7500 to 25000
Utilization	Percentage of the line that is being utilized
Low Utilization	Utilization below 35%
Medium Utilization	Utilization between 35% and 80%
High Utilization	Utilization above 80%
Payment Status	
Current	Less than 30 days past due
Cur. Zero Bal.	Current with zero balance
Cur. Low Util.	Current with low utilization
Cur. Medium Util.	Current with medium utilization
Cur. High Util.	Current with high utilization
Delinquent	30 days past due up to 89 days past due
Default	90 days or more past due
Credit score	
Credit Score 1	Score up to 560
Credit Score 2	Score 561 to 620
Credit Score 3	Score 621 to 700
Credit Score 4	Score above 700
Re-performing	Current at this time, was delinquent within last 24 months
Fourth Quarter	Dummy for fourth quarter
Account age	Account age from origination date
Account Age1	Account age is less than one year
Account Age2 to 5	Account age is 2, 3,…,5 years, respectively
Account Age6	Account age is 6 years or more
Unemployment	Unemployment 6 months before time of delinquency outcome
Change in unemployment	One year change in unemp. three months before del. outcome.

Table 2a.- Mean Values and Proportions for Relevant Variables

	Y05Q4	Y06Q2	Y06Q4	Y07Q2	Y07Q4	Y08Q2	Y08Q4	Y09Q2	Y09Q4
Balance	2068	2102	2098	2137	2168	2187	2221	2186	2162
Line	10502	10723	10851	10961	11112	11083	10983	10768	10613
0 – 1500	33.63	33.18	34.77	33.97	33.75	33.96	34.58	37.71	38.25
1500 – 7500	35.69	36.00	34.91	35.20	34.69	34.77	35.05	35.40	35.91
7500 - 25000	30.68	30.83	30.35	30.83	31.56	31.27	30.37	26.89	25.84
Utilization	72.65	72.00	71.99	71.81	72.14	72.73	74.65	75.88	76.08
Status									
Current	96.82	96.26	96.43	95.62	95.81	95.44	95.51	95.19	95.95
Cur. Low U.	55.01	49.1	53.23	50.67	54.13	53.4	50.52	48.18	50.68
Cur. Med U.	12.04	11.82	12.21	11.42	11.7	10.84	10.77	11.45	12.16
Cur. High U.	29.77	35.34	30.99	33.53	29.98	31.2	34.22	35.56	33.11
Delinquent	1.17	1.31	1.28	1.42	1.33	1.49	1.28	1.41	1.18
Default	2.01	2.43	2.3	2.96	2.86	3.06	3.21	3.4	2.87
Credit score									
Credit Score 1	9.59	9.49	10.47	10.70	11.78	11.26	11.96	12.33	12.78
Credit Score 2	9.03	8.79	9.04	9.00	8.79	8.35	8.29	8.12	8.02
Credit Score 3	20.86	20.56	20.68	20.28	19.65	18.95	18.50	18.50	18.62
Credit Score 4	60.52	61.16	59.81	60.02	59.78	61.44	61.25	61.05	60.59
Cur. Zero Ba.	725	724	726	725	728	729	732	734	737
Cur. Low Util	629	629	626	623	619	621	623	627	632
Cur. High Util	629	629	626	623	619	621	623	627	632
Delinquent	500	499	499	492	481	482	482	486	490
Cur. or Del.	743	744	741	743	745	748	750	750	750
Re-performing	6.79	6.93	6.46	6.88	6.56	6.90	6.48	6.76	6.12
Account age	6.72	6.66	6.59	6.55	6.52	6.58	6.69	7.02	7.34

Table 2b: Average Recovery Rate

BHC	Q1 2006-Q4 2011	Q1 2008-Q1 2010
JP Morgan Chase	12.6%	10.0%
Bank of America	9.4%	5.7%
Citi	14.5%	11.1%
Wells Fargo	13.0%	8.3%
Capital One	27.4%	21.0%
USB	10.6%	7.8%
American Express*	15.4%	9.3%

*Amex became a BHC in 2008 and regulatory reporting started in 2009.

Chart 1a: BHC Recovery Rates Over Time

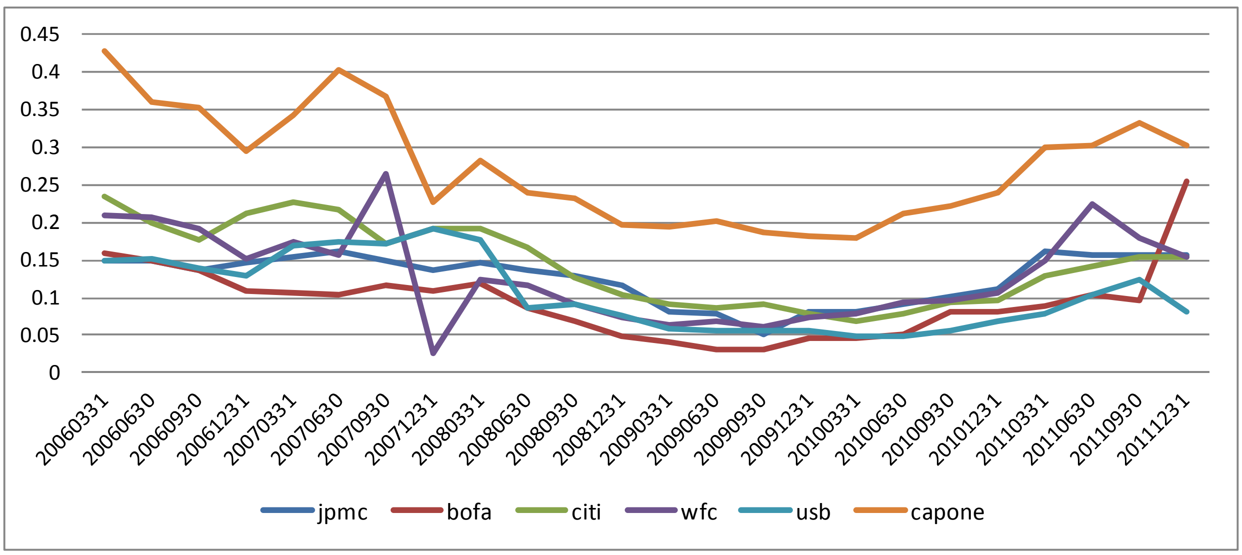
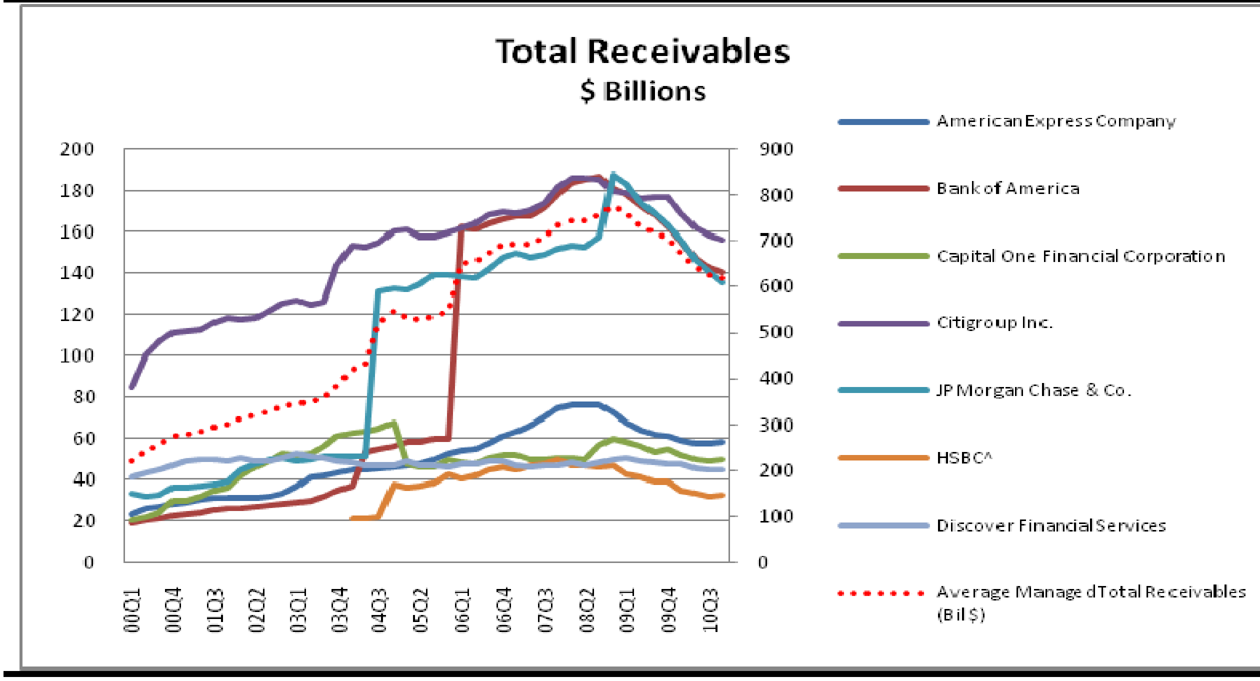
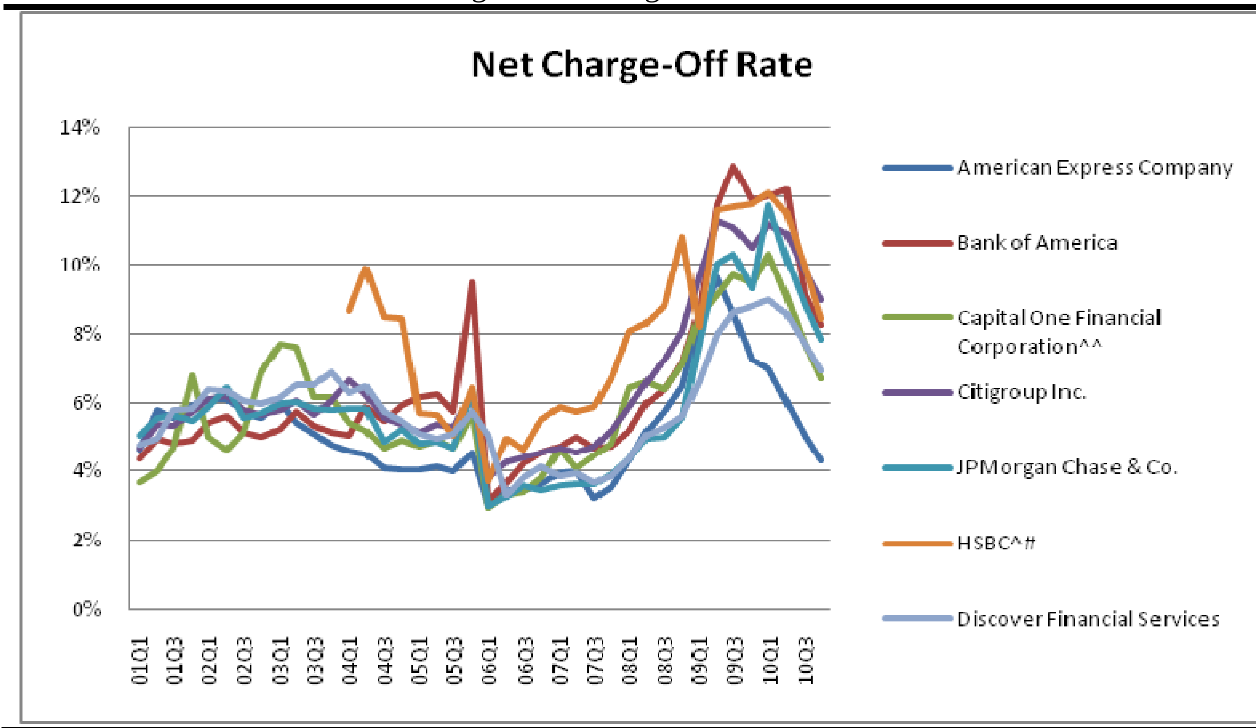


Chart 1b. - Evolution of Total $ Outstanding at Large Financial Institutions in the US



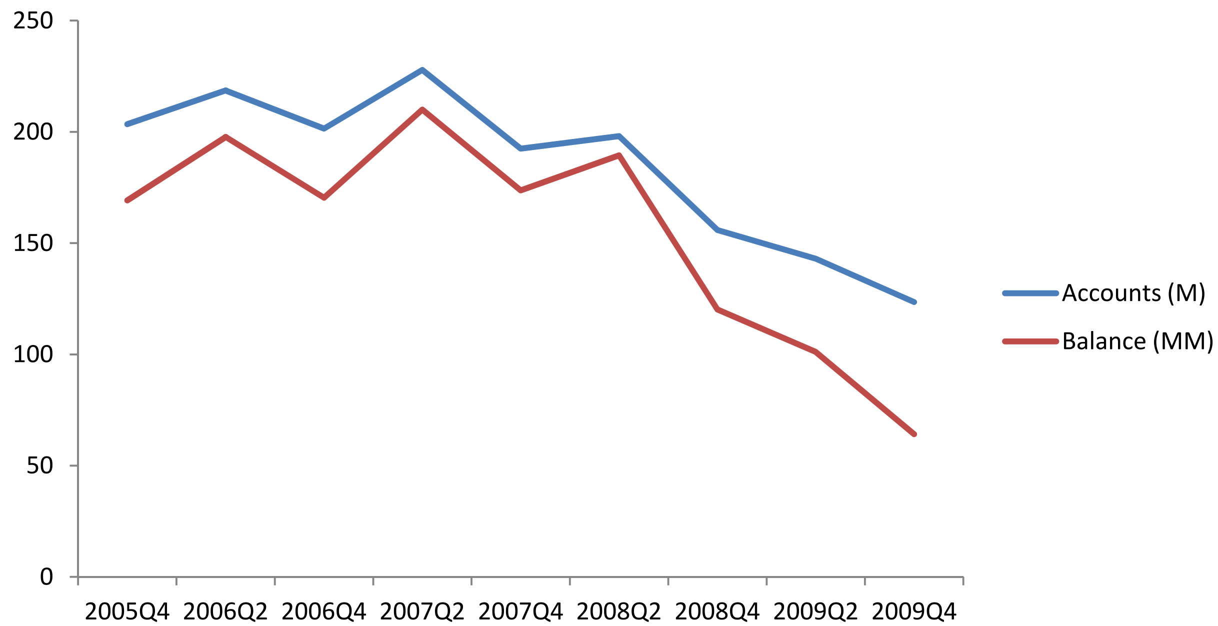
^ Data for HSBC is end of period receivables.

Chart 1c. - Evolution of Net $ Charge-offs at Large Financial Institutions in the US



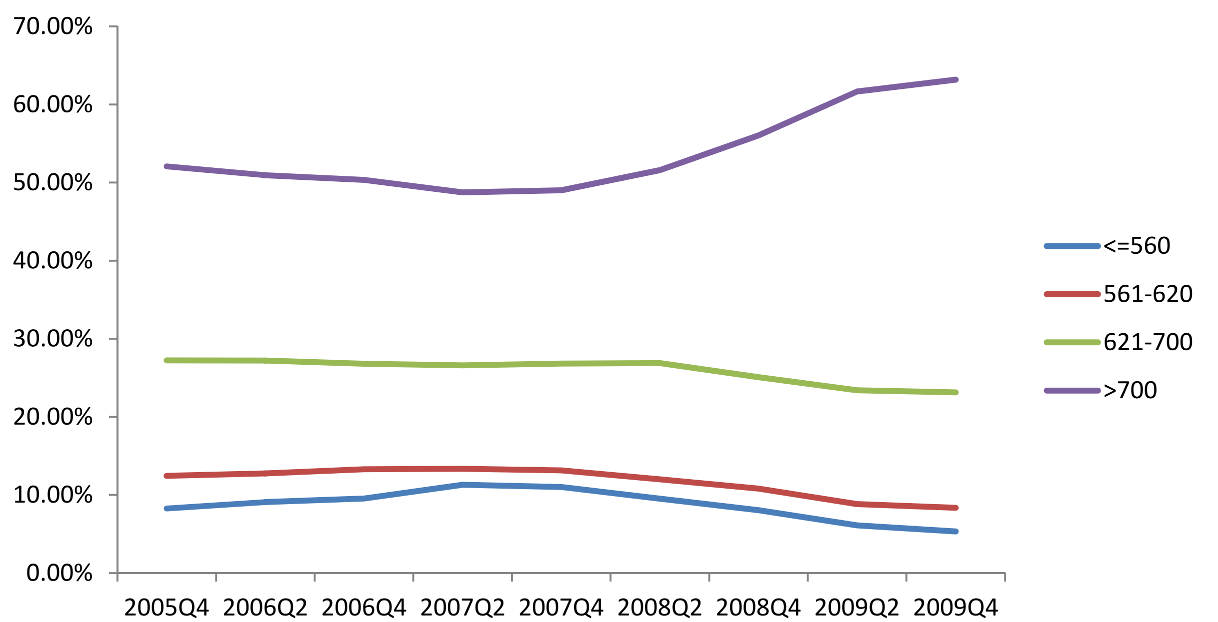
^# Calculated using a weighted average of credit quality figures for HSBC Finance Corp and HSBC USA Inc.

Chart 2a: Account Originations by Number of Accounts and by Balance



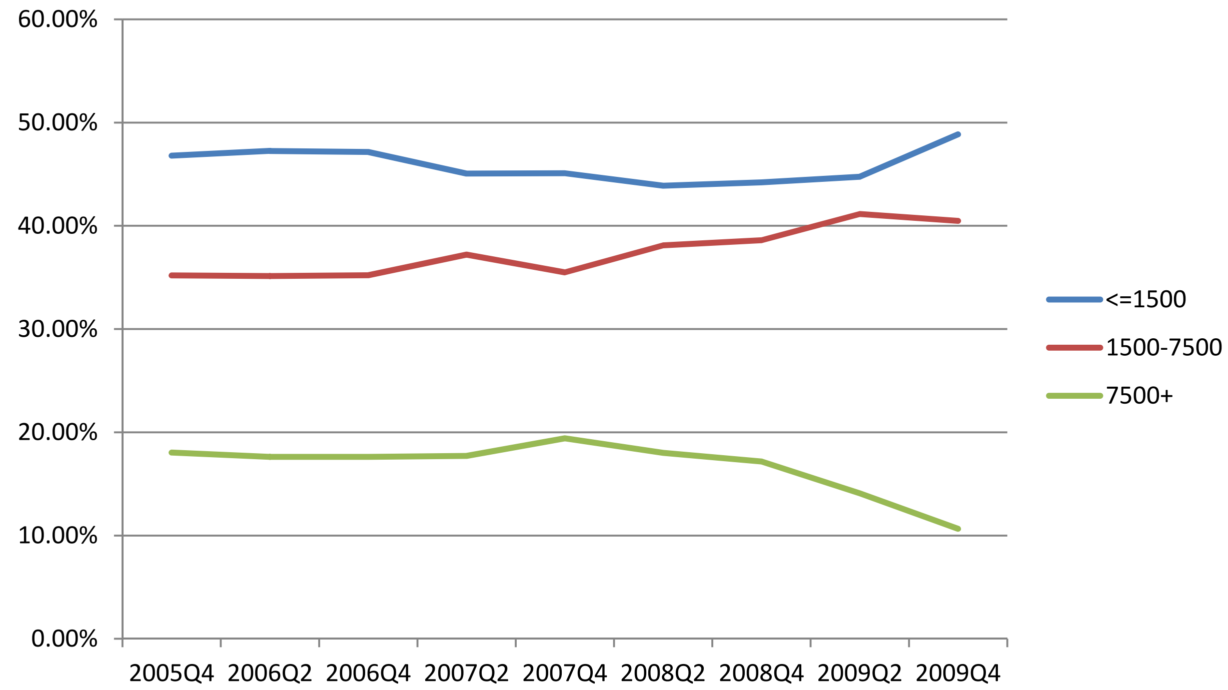
Note: Credit bureau data.

Chart 2b: Account Originations by Credit Score Band



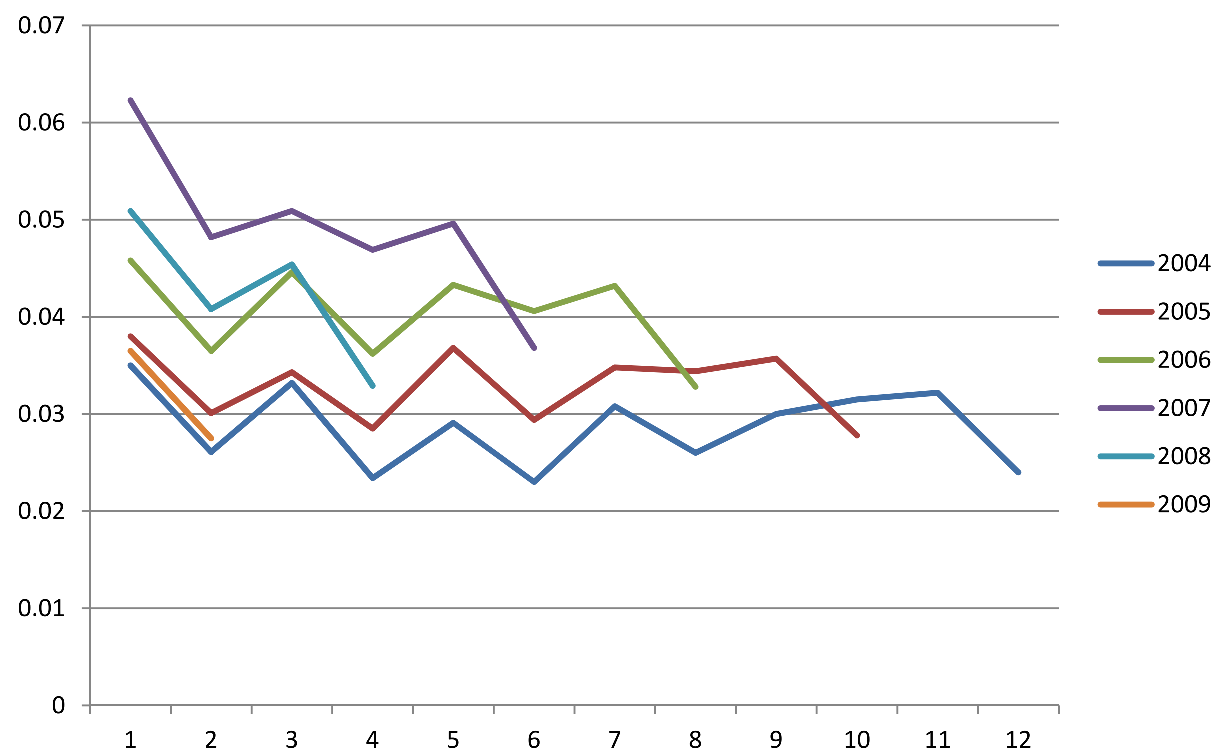
Note: Credit bureau data.

Chart 2c: Account Originations by Line



Note: Credit bureau data

Chart 3.- Six Months Ahead Default Rates by Origination Vintage



Note: Credit bureau data

Table 3: Frequency Tables of Delinquency Status at t+6 by Status at t

	Y05Q4	Y06Q2	Y06Q4	Y07Q2	Y07Q4	Y08Q2	Y08Q4	Y09Q2	Y09Q4
Current and Zero Bal. at t									
Cur. Low Util	61.97	50.11	59.96	58.07	63.87	63.32	55.34	51.46	57.78
Cur. High Util	37.05	48.56	38.91	40.56	34.98	35.33	43.29	46.74	40.78
Delinquent	0.16	0.17	0.18	0.19	0.18	0.18	0.13	0.15	0.13
Default	0.82	1.16	0.95	1.18	0.97	1.17	1.24	1.65	1.31
Default ($ w.)	0.45	0.49	0.40	0.61	0.68	0.60	0.98	0.59	0.58
Current and Low Util at t									
Cur. Low Util	83.81	82.05	82.25	79.18	83.14	82.34	83.23	83.43	82.19
Cur. High Util	14.52	15.92	15.98	18.59	14.96	15.36	14.67	14.31	16
Delinquent	0.82	0.97	0.86	1.01	0.86	1.09	0.88	0.98	0.76
Default	0.85	1.06	0.91	1.23	1.04	1.21	1.21	1.28	1.05
Default ($ w.)	0.67	0.76	0.67	0.85	0.88	1.00	1.25	1.23	1.14
Current and High Util at t									
Cur. Low Util	32.07	27.8	29.71	24.93	25.26	20.22	22.03	19.44	24.05
Cur. High Util	59.66	62.18	61.06	62.75	63.54	66.34	66.04	67.69	66.39
Delinquent	3.51	3.89	3.76	4.54	4.19	5.27	4.17	4.74	3.48
Default	4.76	6.13	5.48	7.78	7	8.17	7.76	8.13	6.09
Default ($ w.)	3.85	4.42	4.03	5.22	5.35	6.3	6.93	7.16	5.66
Delinquent at t									
Cur. Low Util	25.36	20.33	24.28	17.49	18.74	15.83	16.74	14.61	17.65
Cur. High Util	23.43	19.88	22.62	22.37	22.87	20.52	22.81	22.25	24.12
Delinquent	12.68	16.95	12.54	12.15	10	10.3	9.04	9.16	8.91
Default	38.53	42.84	40.57	48	48.38	53.34	51.41	53.97	49.33
Default ($ w.)	43.01	45.72	44.13	49.26	51.45	57.9	57.87	60.96	58.62
Current or Delinquent at t									
Cur. Low Util	67.05	60.93	65.44	62.09	65.83	64.24	61.29	59.62	62.84
Cur. High Util	29.77	35.34	30.99	33.53	29.98	31.2	34.22	35.56	33.11
Delinquent	1.17	1.31	1.28	1.42	1.33	1.49	1.28	1.41	1.18
Default	2.01	2.43	2.3	2.96	2.86	3.06	3.21	3.4	2.87
Curr. L. U. ($ w.)	52.48	52.35	52.44	52.21	50.47	47.03	44.88	44.38	45.45
Curr. H. U. ($ w.)	42.26	41.93	42.2	41.54	43.12	45.37	46.74	47.07	47.26
Delinq. ($ w.)	2.43	2.63	2.41	2.78	2.58	3.23	2.99	3.11	2.46
Default ($ w.)	2.83	3.09	2.95	3.47	3.83	4.37	5.39	5.43	4.83
#Cur. Low Util	1702	1534	1580	1533	1575	1683	1554	1482	1439
#Cur. High Util	756	890	748	828	718	817	868	884	758
#Delinquent	30	33	31	35	32	39	32	35	27
#Default	51	61	56	73	69	80	81	85	66
Overall	2539	2518	2415	2469	2394	2619	2535	2486	2290

($ w.) represents weighted by account balance. # represents accounts in thousands.

Table 4: Delinquency Status at t+6 by delinquency Status at t and Credit Score Band or Line

	Y05Q4	Y06Q2	Y06Q4	Y07Q2	Y07Q4	Y08Q2	Y08Q4	Y09Q2	Y09Q4
Default Rate at t+ 6 by Status at t and Risk Score									
Risk Score 1									
Current at t:									
Low Utilization	14.51	18.65	15.09	19.53	16.08	19.09	18.7	21.2	17.03
High Utilization	15.74	19.68	17.19	23.13	19.44	22.45	20.84	22	17.41
Delinquent at t	47.14	52.21	49.58	57.56	56.62	62.07	58.91	61.6	56.18
Delinquent at t*	47.37	52.46	48.12	57.48	56.75	61.45	59.12	61.35	55.19
Risk Score 2									
Current at t:									
Low Utilization	3.39	4.49	3.59	4.67	4.03	4.89	4.96	5.95	4.71
High Utilization	4.27	5.69	4.89	6.82	5.97	7.09	6.94	8.06	5.89
Delinquent at t	15.5	17.3	16.9	17.95	18.59	22.74	24.5	27.57	26.21
Delinquent at t*	39.87	43.36	42.53	48.91	47.99	54.23	51.68	54.59	49.92
Risk Score 3									
Current at t:									
Low Utilization	0.62	0.86	0.72	0.89	0.88	1.09	1.3	1.45	1.2
High Utilization	1.52	1.96	1.63	2.36	2.21	3	3.4	3.81	2.97
Delinquent at t	6.24	7	7.11	7.72	8.32	9.24	11.17	13.09	13.26
Delinquent at t*	29.07	31.95	31.77	35.97	38.69	44.23	44.98	46.89	45.93
Risk Score 4									
Current at t:									
Low Utilization	0.07	0.08	0.08	0.09	0.08	0.09	0.14	0.14	0.13
High Utilization	0.34	0.39	0.4	0.46	0.54	0.79	1.11	1.27	1.16
Delinquent at t	3.83	8.07	6.37	4.95	7.71	10.06	6.86	10.39	6
Delinquent at t*	19.7	20.93	22.46	26.56	30.9	32.38	32.38	37.28	33.04
Delinquency Status at t+6 for Non-Defaulted Accounts by Line.									
Line up to 1500									
Curr. L/M Util	58.78	55.67	56.63	49.08	51.61	49.42	52.92	51.72	54.33
Curr. High Util	32.87	34.02	33.83	38.18	36.4	37.46	35.53	36.56	36.71
Delinquent	2.99	3.58	3.38	3.99	3.64	4.19	3.35	3.66	2.79
Default	5.35	6.73	6.16	8.74	8.36	8.93	8.19	8.05	6.17
Line 1500-7500									
Curr. L/M Util	68.23	65.65	66.55	64.01	65.69	63.47	63.47	63.85	63
Curr. High Util	27.96	29.92	29.47	31.13	29.64	30.84	30.83	30.3	32.3
Delinquent	1.67	1.94	1.66	1.95	1.76	2.28	1.87	2.02	1.53
Default	2.14	2.49	2.32	2.92	2.9	3.41	3.83	3.82	3.16
Line 7500-25000									
Curr. L/M Util	82.56	80.78	80.68	78.29	81.06	79.66	77.54	77.9	78.98
Curr. High Util	16.02	17.59	17.8	19.95	17.11	18.07	19.78	19.29	18.62
Delinquent	0.66	0.76	0.66	0.76	0.71	0.95	0.93	0.98	0.77
Default	0.76	0.87	0.86	1.01	1.12	1.32	1.75	1.82	1.63

(*) indicates use of 6-month lag of credit score

Table 5.a: Parameter Estimates for 4 States Model [Curr./Curr High Util./Del./Def.] – Account Status at Time t Current & Low/Medium Utilization

	Current & High Utilization at t+6				Delinquent at t+6				Default at t+6			
Model Number:	(1)		(2)		(1)		(2)		(1)		(2)	
	Coef.	t-val.	Coef.	t-val.	Coef.	t-val.	Coef.	t-val.	Coef.	t-val.	Coef.	t-val.
Line 1500-7500	0.774	-195.9	0.774	-196.1	0.968	-4.39	0.970	-4.18	0.967	-5.83	0.970	-5.29
Line 7500-25000	0.580	-374.4	0.581	-373.7	0.940	-6.07	0.944	-5.62	1.087	9.56	1.100	10.89
Util2	0.739	-195.4	0.738	-195.8	2.337	126.92	2.332	126.55	0.682	-75.06	0.679	-75.97
Util3												
Re-performing	1.324	99.45	1.321	98.62	3.635	163.79	3.627	163.45	2.255	140.33	2.240	139.22
Fourth Quarter	0.859	-140.2	0.859	-140.2	0.829	-29.84	0.829	-29.81	0.793	-48.8	0.794	-48.61
Vintage 2006	0.993	-3.54	0.995	-2.26	1.021	1.81	1.024	2.06	1.051	5.88	1.059	6.69
Vintage 2007	0.981	-8.05	0.987	-5.34	1.063	4.66	1.068	4.97	1.089	8.81	1.100	9.79
Vintage 2008	0.934	-22.38	0.945	-18.42	1.010	0.6	1.014	0.79	1.010	0.75	1.014	1.07
Vintage 2009	0.968	-7.42	0.980	-4.77	1.029	1.13	1.022	0.84	1.105	5.25	1.087	4.40
Acc. Age2	0.806	-106.7	0.809	-104.5	0.700	-32.13	0.701	-32.03	0.815	-24.63	0.817	-24.34
Acc. Age3	0.710	-145.3	0.715	-142.0	0.626	-35.66	0.628	-35.38	0.773	-26.52	0.777	-25.88
Acc. Age4	0.715	-125.7	0.721	-122.4	0.614	-32.01	0.616	-31.69	0.786	-21.32	0.792	-20.60
Acc. Age5	0.662	-139.5	0.667	-136.7	0.601	-30.26	0.603	-30.01	0.748	-22.98	0.753	-22.39
Acc. Age6	0.594	-225.2	0.598	-221.3	0.609	-37.18	0.611	-36.87	0.663	-39.48	0.668	-38.69
Credit score												
Score2	1.237	57.18	1.152	11.99	0.595	-57.8	0.544	-21.07	0.241	-235.9	0.209	-82.96
Score3	1.078	22.58	0.922	-7.82	0.260	-149.1	0.203	-58.56	0.049	-436.7	0.033	-164.8
Score4	0.845	-51.26	0.653	-43.12	0.051	-256.3	0.037	-104.9	0.004	-525.6	0.002	-206.1
Policy Variables												
Chg. Unemp.	1.014	30.91	1.006	2.37	1.032	11.37	1.031	5.78	1.059	28.69	1.041	13.99
Chg. Unemp. X Score2			1.015	4.90			1.007	0.89			1.016	3.32
Chg. Unemp. X Score3			1.003	1.21			1.004	0.56			1.052	9.68
Chg. Unemp. X Score4			1.009	3.67			0.982	-2.35			1.035	4.66
Unemp.	1.009	26.22	0.975	-14.55	1.004	2	0.972	-7.63	1.030	21.08	1.001	0.55
Unemp. X Score2			1.008	3.71			1.013	2.42			1.019	5.54
Unemp. X Score3			1.024	12.81			1.040	7.99			1.051	13.64
Unemp. X Score4			1.040	22.42			1.056	9.95			1.084	16.31
LLF	-116452											

Note: t-values not reported

Table 5.b: Parameter Estimates for 4 States Model [Curr./Curr High Util./Del./Def.] – Account Status at Time t Current & High Utilization

	Current & High Utilization at t+6				Delinquent at t+6				Default at t+6			
Model Number:	(1)		(2)		(1)		(2)		(1)		(2)	
	Coef.	t-val.	Coef.	t-val.	Coef.	t-val.	Coef.	t-val.	Coef.	t-val.	Coef.	t-val.
Line 1500-7500	1.384	106.65	1.386	106.98	1.427	52.43	1.431	52.79	1.506	70.59	1.513	71.37
Line 7500-25000	1.327	73.03	1.331	73.70	1.848	61.95	1.861	62.59	2.396	102.57	2.424	103.84
Util2												
Util3												
Re-performing	0.656	-95.93	0.657	-95.78	1.732	74.72	1.735	74.91	1.214	30.08	1.216	30.38
Fourth Quarter	0.852	-61.21	0.852	-61.21	0.724	-55.46	0.724	-55.31	0.718	-67.22	0.719	-66.89
Vintage 2006	1.081	17.15	1.090	18.81	1.071	6.91	1.086	8.26	1.134	15.19	1.151	16.92
Vintage 2007	1.109	19.73	1.129	23.02	1.147	12.15	1.181	14.70	1.213	20.54	1.253	23.97
Vintage 2008	0.997	-0.38	1.026	3.75	0.994	-0.41	1.039	2.50	0.993	-0.58	1.046	3.57
Vintage 2009	0.818	-17.93	0.836	-15.90	0.827	-7.12	0.859	-5.71	0.842	-8.18	0.883	-5.94
Acc. Age2	1.258	51.76	1.267	53.33	0.973	-2.84	0.984	-1.67	0.842	-22.02	0.854	-20.23
Acc. Age3	1.326	54.73	1.347	57.53	0.881	-11.09	0.902	-9.00	0.767	-28	0.790	-24.84
Acc. Age4	1.403	56.72	1.429	59.64	0.880	-9.38	0.907	-7.18	0.773	-22.62	0.801	-19.36
Acc. Age5	1.473	58.02	1.500	60.56	0.887	-7.78	0.913	-5.87	0.784	-18.61	0.813	-15.81
Acc. Age6	1.689	98.9	1.720	101.93	0.926	-6.18	0.953	-3.83	0.773	-24.43	0.801	-20.92
Credit score												
Score2	0.790	-54.51	0.723	-24.14	0.425	-115.1	0.361	-43.66	0.217	-235.5	0.167	-88.74
Score3	0.554	-140.2	0.431	-68.63	0.163	-213.9	0.110	-90.47	0.062	-356.6	0.032	-153.7
Score4	0.395	-197.4	0.278	-97.34	0.036	-213.6	0.017	-90.73	0.013	-289.1	0.004	-127.3
Policy Variables												
Chg. Unemp.	1.071	62.15	1.036	13.44	1.107	40.74	1.062	15.00	1.133	59.7	1.087	25.99
Chg. Unemp. X Score2			1.024	6.94			1.028	4.55			1.039	7.43
Chg. Unemp. X Score3			1.044	13.65			1.077	12.05			1.090	15.30
Chg. Unemp. X Score4			1.043	12.51			1.096	8.05			1.099	8.79
Unemp.	1.020	26.66	0.990	-5.41	1.018	10.04	0.979	-7.48	1.036	24.71	0.990	-4.35
Unemp. X Score2			1.011	4.41			1.022	5.11			1.035	9.48
Unemp. X Score3			1.035	16.01			1.052	11.76			1.089	22.38
Unemp. X Score4			1.052	22.00			1.108	13.44			1.176	23.18
LLF		-30754										

Table 5.c: Parameter Estimates for 4 States Model [Curr./Curr High Util./Del./Def.] – Account Status at Time t Delinquent

	Current & High Utilization at t+6				Delinquent at t+6				Default at t+6			
Model Number:	(1)		(2)		(1)		(2)		(1)		(2)	
	Coef.	t-val.	Coef.	t-val.	Coef.	t-val.	Coef.	t-val.	Coef.	t-val.	Coef.	t-val.
Line 1500-7500	1.406	27.00	1.407	27.05	1.465	25.58	1.467	25.65	1.352	25.86	1.352	25.86
Line 7500-25000	1.462	20.91	1.466	21.07	1.591	20.91	1.595	21.03	1.737	32.18	1.740	32.21
Util2	0.629	-23.63	0.629	-23.67	1.425	15.40	1.428	15.48	1.596	26.57	1.587	26.21
Util3	3.996	91.01	3.989	90.79	3.567	62.71	3.577	62.76	7.162	128.12	7.101	127.37
Fourth Quarter	0.868	-12.75	0.868	-12.76	0.697	-27.60	0.697	-27.60	0.752	-28.34	0.752	-28.25
Vintage 2006	1.202	9.11	1.206	9.27	1.069	2.74	1.068	2.69	1.225	11.26	1.235	11.70
Vintage 2007	1.190	7.46	1.197	7.70	1.111	3.72	1.108	3.62	1.281	11.96	1.300	12.63
Vintage 2008	1.077	2.38	1.082	2.52	0.968	-0.83	0.963	-0.95	1.101	3.50	1.113	3.89
Vintage 2009	1.036	0.65	1.018	0.32	0.792	-3.01	0.778	-3.24	0.925	-1.64	0.902	-2.18
Acc. Age2	1.023	1.08	1.022	1.02	1.071	2.65	1.067	2.49	0.664	-22.54	0.663	-22.58
Acc. Age3	1.041	1.68	1.042	1.74	1.107	3.49	1.102	3.33	0.578	-26.22	0.580	-25.96
Acc. Age4	1.081	2.87	1.084	2.96	1.084	2.43	1.078	2.28	0.558	-24.24	0.562	-23.89
Acc. Age5	1.096	3.09	1.099	3.19	1.158	4.11	1.154	3.99	0.531	-23.87	0.535	-23.52
Acc. Age6	1.081	3.16	1.083	3.22	1.308	8.97	1.301	8.81	0.438	-37.60	0.441	-37.25
Credit score												
L. Score2	0.979	-1.48	0.932	-1.58	0.692	-22.67	0.682	-7.46	0.648	-34.10	0.583	-13.58
L. Score3	0.898	-7.47	0.753	-6.50	0.449	-45.76	0.423	-16.16	0.442	-62.00	0.306	-29.88
L. Score4	0.806	-11.30	0.605	-9.40	0.256	-50.54	0.233	-18.41	0.254	-71.78	0.135	-36.59
Policy Variables												
Chg. Unemp.	1.039	8.66	1.038	4.95	0.960	-7.52	0.970	-3.77	1.090	21.05	1.082	12.14
Chg. Unemp. X L. Score2			1.012	1.02			0.996	-0.33			1.012	1.21
Chg. Unemp. X L. Score3			1.009	0.84			0.991	-0.64			1.022	2.15
Chg. Unemp. X L. Score4			0.972	-2.09			0.896	-5.37			1.009	0.68
Unemp.	1.026	8.23	1.008	1.48	1.024	6.37	1.011	1.98	1.045	15.24	1.015	3.28
Unemp. X L. Score2			1.006	0.80			1.003	0.30			1.015	2.13
Unemp. X L. Score3			1.028	3.62			1.012	1.21			1.057	7.90
Unemp. X L. Score4			1.053	5.60			1.033	2.32			1.097	10.02
LLF	-421021											

Table 5.d: Parameter Estimates for 4 States Model with Fixed Effects

Account Status at t:	Current			Current & High Util.			Delinquent		
	Curr. +	Del.	Def.	Curr. +	Del.	Def.	Curr. +	Del.	Def.
Line 1500-7500	0.774	0.968	0.971	1.387	1.431	1.513	1.407	1.462	1.355
Line 7500-25000	0.580	0.942	1.098	1.333	1.861	2.425	1.464	1.593	1.742
Util2	0.738	2.334	0.679				0.628	1.431	1.581
Util3							3.980	3.596	7.069
Re-performing	1.321	3.619	2.238	0.656	1.731	1.219			
Fourth Quarter	0.858	0.830	0.792	0.853	0.726	0.720	0.867	0.702	0.747
Vintage 2006	0.995	1.039	1.061	1.081	1.096	1.162	1.200	1.084	1.234
Vintage 2007	0.989	1.096	1.105	1.113	1.201	1.277	1.186	1.140	1.294
Vintage 2008	0.941	1.062	**1.018**	**1.002**	1.072	1.081	**1.063**	**1.026**	1.101
Vintage 2009	0.958	1.096	1.074	0.815	0.907	0.924	**0.986**	**0.863**	0.871
Acc. Age2	0.806	0.710	0.815	1.260	**0.994**	0.861	**1.016**	1.084	0.659
Acc. Age3	0.712	0.644	0.776	1.332	0.919	0.804	**1.032**	1.137	0.574
Acc. Age4	0.717	0.635	0.791	1.409	0.929	0.820	1.072	1.120	0.556
Acc. Age5	0.664	0.623	0.753	1.477	0.936	0.833	1.085	1.201	0.529
Acc. Age6	0.596	0.633	0.670	1.690	**0.978**	0.822	1.069	1.351	0.436
Credit score									
Score2	1.148	0.545	0.208	0.724	0.362	0.167	0.932	0.691	0.579
Score3	0.916	0.204	0.033	0.430	0.110	0.032	0.753	0.431	0.306
Score4	0.652	0.037	0.002	0.277	0.017	0.004	0.607	0.236	0.137
Policy Variables									
Chg. Unemp.	**0.996**	1.030	1.029	1.043	1.068	1.086	1.034	**0.986**	1.066
Chg. Unemp. X Score2	1.015	**1.008**	1.016	1.024	1.029	1.039	**1.012**	**0.997**	**1.012**
Chg. Unemp. X Score3	**1.003**	**1.007**	1.053	1.042	1.079	1.090	**1.009**	**0.993**	**1.020**
Chg. Unemp. X Score4	1.010	0.984	1.038	1.041	1.096	1.099	0.971	0.897	**1.007**
Unemp.	0.982	0.962	1.007	0.991	0.969	0.984	1.014	**0.989**	1.027
Unemp. X Score2	1.009	1.012	1.020	1.010	1.021	1.035	**1.007**	**1.001**	1.017
Unemp. X Score3	1.025	1.038	1.052	1.035	1.051	1.090	1.029	**1.009**	1.057
Unemp. X Score4	1.040	1.057	1.084	1.053	1.108	1.177	1.053	1.033	1.097
LLF		-116328			-3071054			-419894	

Geographic states included as fixed effects. Coefficients that are insignificant at the standard significant levels are highlighted.

Chart 4: Default Rates by Del. Status at t (in sample and out of sample projected vs. realized)

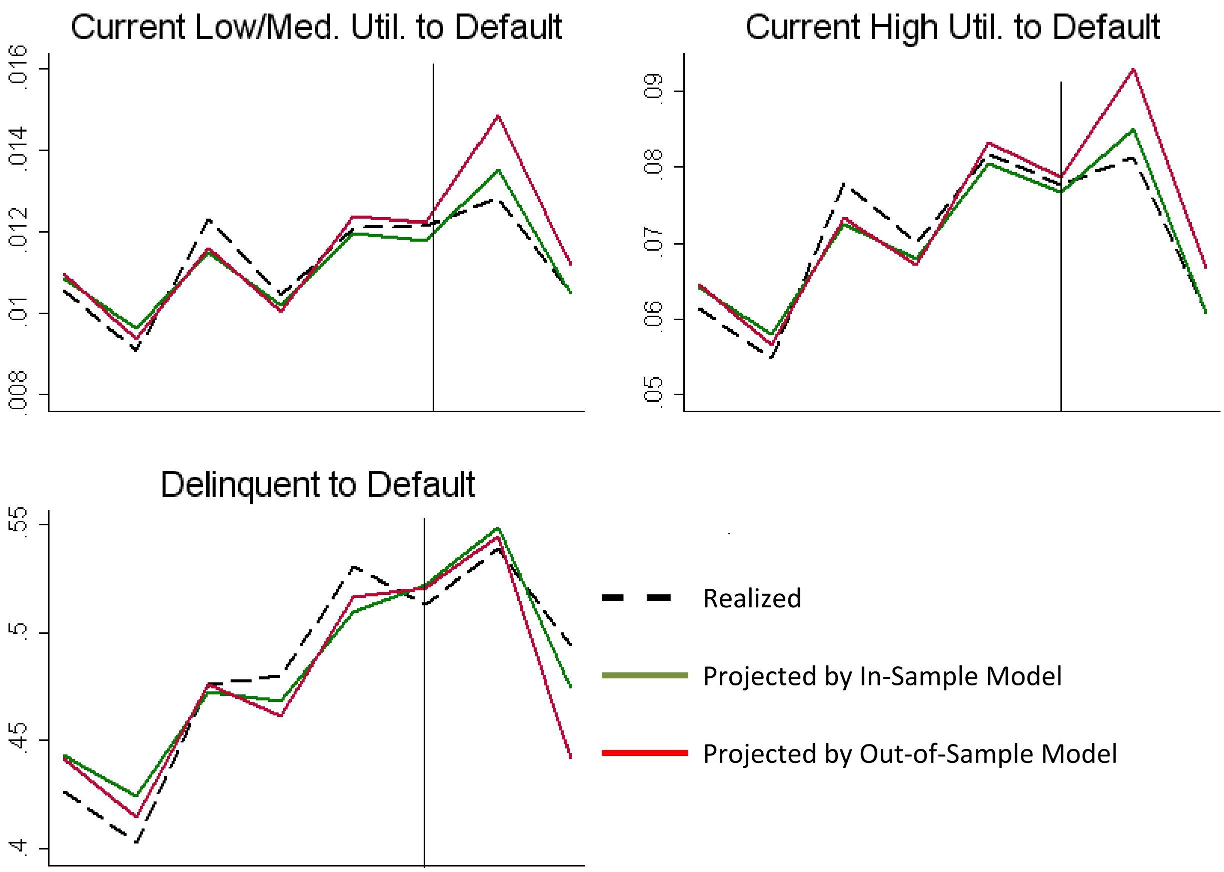


Table 6: Balance Ratio Descriptive Statistics

	Mean	Std.	Centiles 20	40	50	60	80
Curr. Low/Med. Util. to Del.	2.185	2.811	0.971	1.138	1.287	1.486	2.330
Curr. Low/Med. Util. to Def.	1.817	2.061	1.161	1.161	1.161	1.304	2.009
Curr. High Util. to Del.	1.187	0.431	1.009	1.077	1.115	1.166	1.347
Curr. High Util. to Def.	1.307	0.469	1.143	1.161	1.206	1.277	1.527
Curr All to Del.	1.575	1.849	1.000	1.088	1.144	1.226	1.565
Curr. All to Def.	1.468	1.245	1.150	1.161	1.196	1.281	1.607
Del. to Def.	1.214	0.341	1.100	1.161	1.161	1.161	1.309

Chart 5: Kernel Density of Balance Rate for Several Delinquency Transitions

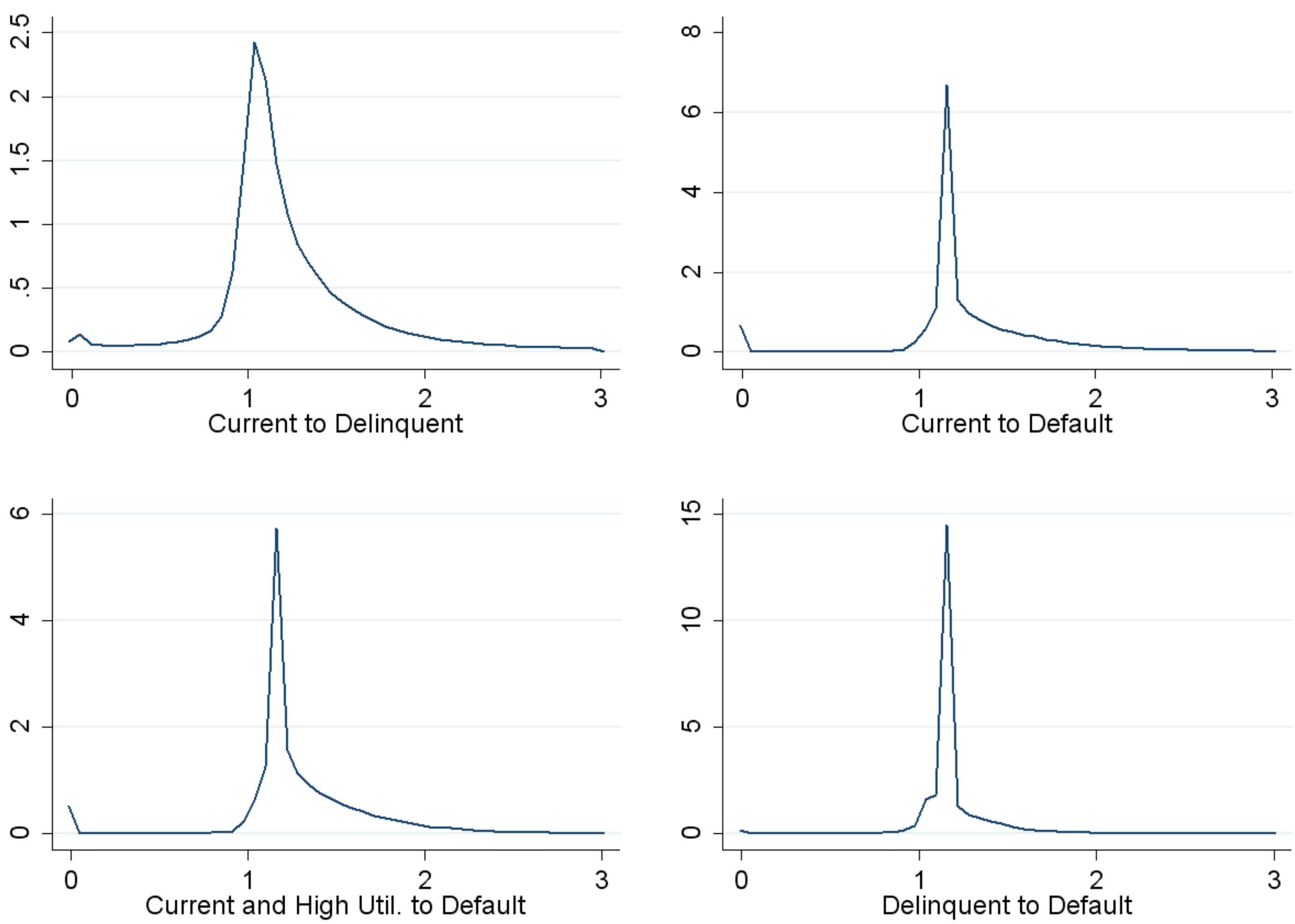


Table 7a: Parameter Estimates for Exposure at Default Models over a 6-month Horizon

	Curr. To Del.		Curr. To Def.		Curr. + to Def.		Del. to Def.	
Models:	(1)		(2)		(3)		(4)	
	Coef.	t-val.	Coef.	t-val.	Coef.	t-val.	Coef.	t-val.
Line 1500-7500	-0.189	-59.54	-0.186	-115.98	-0.175	-139.87	-0.097	-68.26
Line 7500-25000	-0.221	-48.01	-0.228	-93.96	-0.206	-107.98	-0.137	-61.23
Util2	-0.481	-105.31	-0.209	-85.19			-0.08	-24.62
Util3	-0.581	-139.46	-0.292	-135.92			-0.106	-38.35
Re-performing	-0.1	-30.96	-0.001	-0.51	-0.013	-10.16		
Fourth Quarter	-0.037	-13.6	-0.01	-7.93	-0.009	-8.37	-0.008	-7.26
Vintage 2006	-0.007	-1.57	-0.001	-0.28	-0.004	-2.33	-0.013	-6.74
Vintage 2007	-0.012	-2.19	0.002	0.84	-0.003	-1.33	-0.012	-5.52
Vintage 2008	-0.028	-3.95	-0.002	-0.56	-0.01	-3.82	-0.016	-5.37
Vintage 2009	-0.044	-3.65	0.004	0.69	-0.024	-5.34	-0.035	-6.8
Acc. Age2	-0.102	-22.17	-0.085	-39.72	-0.06	-37.02	-0.01	-5.61
Acc. Age3	-0.128	-23.23	-0.108	-41.38	-0.079	-39.73	-0.021	-9.45
Acc. Age4	-0.154	-23.65	-0.119	-37.95	-0.088	-36.07	-0.024	-8.91
Acc. Age5	-0.146	-20.07	-0.122	-34.2	-0.088	-31.64	-0.029	-9.27
Acc. Age6	-0.183	-30.79	-0.142	-48.32	-0.094	-40.93	-0.034	-13
L. Score2	-0.025	-7.1	0.019	11.13	0.005	4.11	0.013	9.31
L. Score3	-0.075	-19.93	0.007	3.79	-0.014	-9.73	0.016	9.71
L. Score4	-0.169	-31.53	-0.02	-6.95	-0.035	-14.27	0.005	1.68
Chg. Unemp.	0.008	7.2	0.005	8.97	0.005	10.73	0.001	1.22
Unemp.	-0.003	-3.82	-0.002	-5.03	-0.002	-5.42	0.001	2.58
Cte.	1.006	139.57	0.729	209.77	0.418	188.96	0.326	90.62
R-sq.		0.1168		0.1433		0.1734		0.0561

Table 7b: Parameter Estimates for Exposure at Default Models over a 12-month Horizon

	Curr. To Del.		Curr. To Def.		Curr. + to Def.		Del. to Def.	
Models:	(1)		(2)		(3)		(4)	
	Coef.	t-val.	Coef.	t-val.	Coef.	t-val.	Coef.	t-val.
	(1f)		(2f)		(3f)		(4f)	
Line 1500-7500	-0.218	-50.65	-0.168	-116	-0.165	-138.23	-0.092	-68.21
Line 7500-25000	-0.26	-42.82	-0.204	-93.29	-0.206	-111.79	-0.132	-61.18
Util2	-0.723	-125.11	-0.27	-131.49			-0.091	-6.58
Util3	-0.879	-163.46	-0.385	-210.19			-0.112	-12.46
Re-performing	-0.157	-34.05	-0.044	-28.6	-0.04	-32.18		
Fourth Quarter	0.01	2.59	-0.009	-7.27	-0.005	-4.61	-0.01	-7.24
Vintage 2006	-0.008	-1.21	-0.006	-2.85	-0.005	-2.98	-0.011	-6.77
Vintage 2007	-0.016	-2.08	-0.006	-2.54	-0.005	-2.4	-0.012	-5.65
Vintage 2008	-0.049	-4.4	-0.007	-2.08	-0.007	-2.56	-0.013	-5.6
Vintage 2009	-0.047	-1.79	0.015	1.94	-0.01	-1.5	-0.041	-6.95
Acc. Age2	-0.103	-16.24	-0.073	-37.62	-0.045	-29.13	-0.011	-5.77
Acc. Age3	-0.133	-17.07	-0.093	-38.09	-0.062	-31.17	-0.019	-9.64
Acc. Age4	-0.164	-17.89	-0.104	-34.88	-0.065	-26.54	-0.025	-9.1
Acc. Age5	-0.167	-16.89	-0.109	-33.13	-0.063	-23.25	-0.024	-9.44
Acc. Age6	-0.225	-27.86	-0.137	-50.91	-0.07	-31.44	-0.042	-13.19
L. Score2	-0.019	-3.7	0.02	12.6	0.006	4.7	0.011	9.31
L. Score3	-0.063	-11.91	0.01	5.68	-0.014	-9.81	0.018	9.68
L. Score4	-0.177	-24.79	-0.017	-6.68	-0.046	-19.22	-0.024	1.67
Chg. Unemp.	0.009	4.97	0.005	8.12	0.003	5.54	0	-0.41
Unemp.	-0.007	-4.94	-0.004	-7.83	-0.001	-2.83	0	-0.24
Cte.	1.317	111.55	0.85	221.44	0.428	151.07	0.37	0.83
R-sq.		0.1605		0.1179		0.1058		0.0351

Table 8: Parameter Estimates for Recovery Rate Model

	(1)		(2)		(3)		(4)	
Models:	Coef.	t-val.	Coef.	t-val.	Coef.	t-val.	Coef.	t-val.
Intercept	0.041	4.260	0.072	3.050	0.066	6.250	0.123	5.200
1 Qtr Lag Recovery Rate	0.725	11.51	0.647	7.790	0.600	9.260	0.445	5.200
1 Qtr Lag Unempl Rate			-0.003	-1.420			-0.005	-2.690
1 Qtr Lag Chg Unempl Rate					-0.036	-4.590	-0.041	-5.190
Num Obs		144		144		144		144