

# WORKING PAPER NO. 12-15 INSURING STUDENT LOANS AGAINST THE FINANCIAL RISK OF FAILING TO COMPLETE COLLEGE

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# Insuring Student Loans Against the Financial Risk of Failing to Complete College\*

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#### Abstract

Participants in student loan programs must repay loans in full regardless of whether they complete college. But many students who take out a loan do not earn a degree (the dropout rate among college students is between 33 to 50 percent). We examine whether insurance, in the form of loan forgiveness in the event of failure to complete college, can be offered, taking into account moral hazard and adverse selection. To do so, we develop a model that accounts for college enrollment and graduation rates among recent US high school graduates. In our model students may fail to earn a degree because they either fail college or choose to leave voluntarily. We find that if loan forgiveness is offered only when a student fails college, average welfare increases by 2.40 percent (in consumption equivalent units) without much effect on either enrollment or graduation rates. If loan forgiveness is offered against both failure and voluntarily departure, welfare increases by 2.15 percent and both enrollment and graduation are higher.

Keywords: College Risk; Government Student Loans; Optimal Insurance

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#### 1 Introduction

Many students who enroll in college fail to earn a college degree. Using the 1990 Panel Study of Income Dynamics (PSID), Restuccia and Urrutia (2004) document that 50 percent of people who enroll do not complete college. Using the National Center for Education Statistics (NCES) data and surveys, we find that 37 percent and 35 percent of students enrolled in 1989-90 and 1995-96, respectively, do not possess a degree and are not enrolled in college five years after their initial enrollment. At the same time, more than 10 million students took out \$95 billion worth of college loans in 2008. While the use of student loans is widespread, the high dropout rate from college suggests that there is considerable financial risk to the student of taking out a college loan: many students who borrow to pay for college fail to earn a college degree. Indeed, this particular risk associated with college loans is evident in the Survey of Consumer Finances (SCF). For the five surveys conducted between 1992 and 2004, the percentage of non-students with a student loan who report not having either a 2- or 4-year college degree is 47 percent, on average. Furthermore, non-students with loans but without a degree have a significantly higher education and consumer debt burden. The ratio of median education debt to median income among non-students with student loans 10 or more years after first taking out the loan was, on average, 0.15 for students without degrees and 0.10 for degree holders.\frac{1}{2}

The financial risk of attempting college but being unable to complete it may discourage some people from enrolling or continuing on in college. While a borrower can choose from a menu of fairly sophisticated repayment options (standard, graduated, income-contingent and extended repayment) under the current system, under each of these payment options the borrower is required to repay the entire loan and associated interest expenses regardless of whether he or she completes college.<sup>2</sup> Thus, even though prospective students may not be financially constrained, a mechanism to share the financial risk of paying for college but failing to earn a degree – the financial risk of failing to complete college – might improve the welfare of college students.<sup>3</sup> It might also encourage more people to attempt and complete college.

<sup>&</sup>lt;sup>1</sup>There is also the risk that the return to a college degree may turn out to be lower than expected (post-college earnings risk). While the latter source of risk is important and has garnered a lot of attention ((Cunha, Heckman, and Navarro, 2005; Heathcote, Storesletten, and Violante, 2008)), the risk of failing to complete college has important adverse consequences for earnings as well because there is a large and growing college degree premium.

<sup>&</sup>lt;sup>2</sup>In a recent survey, Gross, Cekic, Hossler, and Hillman (2009) document that college success as well as the background characteristics of the borrower (in particular college preparedness) play a big role in predicting default. However, student loans in the US cannot be eliminated (discharged) through bankruptcy filing. A borrower who defaults on her loan must reorganize and enter a repayment plan to rehabilitate her defaulted loan and pay the full amount, including collection fees. The majority of borrowers do so soon after default occurs (Volkwein, Szelest, Cabrera, and Napierski-Prancl (1998)). The borrower is permitted to discharge part of her loan only if a repayment effort over 25 years, which is contingent on her income, does not fully cover all obligations. For details on bankruptcy rules and default consequences for student loans, see Ionescu (2011)).

<sup>&</sup>lt;sup>3</sup>Recent research in the education literature provides support for the fact that financial constraints during college-going years are not crucial for college enrollment (Carneiro and Heckman (2002), Cameron and Taber (2001)). Rather, it is student characteristics, such as learning ability, that determine the decision to enroll. Given the generosity of the student loan program, funds are readily available and eligible high school graduates invest in college if they perceive the returns to a college education to be high enough (Ionescu (2009)).

It seems administratively feasible for the student loan program to offer such insurance in the form of a full or partial loan forgiveness for a student who attempts college but is unable to earn a degree. This is the first paper to study the possibility of offering such insurance and to analyze its effects on college investment. Under the current student loan program, a person with low earnings can use the income-contingent repayment plan to reduce the burden of repayment, and recent policy proposals are directed toward making repayments easier for low earners. Nevertheless, the student is still required to repay his or her loan in full. In contrast, the focus of this paper is on loan forgiveness for students who are unable to earn a degree.

Given that such insurance is not currently offered, the goal of this paper is to explore the possible reasons why this might be so. It is possible that moral hazard and adverse selection may prevent such insurance from being offered, or, alternatively, there may not be much value in offering this insurance. To carry out this exploration, we build a simple model consistent with recent college enrollment and graduation facts and use it to determine how much insurance – in the form of loan forgiveness – can be offered against the risk of being unable to complete college. In our model, a student may not complete college either because she leaves college voluntarily or she fails college.

We conduct our investigation under two important constraints. First, we require that the insurance scheme not distribute resources from people with a high probability of completion to people with a low probability of completion (and vice versa). Formally, this requires that the insurance program be self-financing with respect to each person who chooses to participate. The current programs enforce this self-financing constraint regardless of whether the program participant actually graduates from college. We will permit students who do not graduate to pay less than graduates, but each participant will pay the full cost of college in expectation. Second, we require that the insurance program take explicit account of the costs imposed by moral hazard and adverse selection. In our context, moral hazard means that the provision of insurance may induce students to reduce effort in college and therefore elevate the probability of the event against which insurance is being offered. Adverse selection means that the provision of insurance may induce students who would otherwise leave college voluntarily to stay enrolled without putting in effort so as to collect on the insurance (bad risks pooling with good risks). In this case, insurance does not cause any change in the college effort decision of students but induces them to simply substitute failure for leaving.

In the theoretical sections of the paper, we develop a simple model of a student's enrollment and college effort decisions. The model postulates the necessary heterogeneity in student characteristics in order to be consistent with the diversity of enrollment and effort decisions we see in reality and the importance generally assigned to heterogeneity of ability and self-selection into college attendance and completion by researchers (see, for instance, Venti and Wise (1983)). The heterogeneity is in a student's utility cost of putting effort into college and his or her outside option, neither of which is directly observable to loan administrators. Loan

administrators also cannot directly observe a student's effort decision. This asymmetric information leads to the possibility of moral hazard and adverse selection and increases the cost of providing insurance. These costs are modeled and the constrained optimization problem that delivers the optimal insurance program is developed.

In the quantitative section, we calibrate the model to US data on college enrollment, leaving and graduation rates as well as the average college costs of program participants, distinguishing between students of different scholastic ability levels as measured by SAT scores. We then compute the optimal insurance that can be offered to each ability group. We consider two cases: first, we consider the case where loan forgiveness is offered only when a student fails college and, second, where loan forgiveness is offered for both failure as well as voluntary departure. We find that it is optimal to offer full loan forgiveness for both cases for all ability groups. This is so in spite of the fact that offering full loan forgiveness encourages some level of moral hazard as well as adverse selection. However, the cost imposed by these insurance frictions is low enough that it is optimal to tolerate this opportunistic behavior.<sup>4</sup> The welfare gain (in consumption equivalent units) from offering full loan forgiveness against college failure is about 2.40 percent, on average. However, despite the increase in welfare, full failure insurance does not affect enrollment or completion rates much at all. Although students who are already enrolled in college are significantly positively affected, the marginal student affected by failure insurance is relatively rare in the population. When full loan forgiveness is offered against both failure and voluntary departure, the welfare gain is somewhat lower - 2.15 percent - but both enrollment and graduations rates are higher. The positive effects on college investment decisions result from the fact that the latter form of insurance increases the option value of college and encourages more students to enroll and graduate.

There is a rich literature on higher education, with important contributions focusing on college enrollment and completion. Studies that take a quantitative-theoretical approach have given a prominent role to the risk of college failure. These include studies by Akyol and Athreya (2005); Caucutt and Kumar (2003); Garriga and Keightley (2007); Ionescu (2011) and Restuccia and Urrutia (2004). But these studies do not generally consider the possibility of providing insurance against the risk. One exception is Ionescu (2011), who studied the effects of alternative bankruptcy regimes for student loans. She shows that individuals with relatively low ability and low initial human capital levels are affected to a greater degree by the risk of failure and the option to discharge one's debt under a liquidation regime helps alleviate some of this risk. This general conclusion that the lack of insurance can sometimes be the limiting factor for schooling decisions is consistent with the structural estimates in Johnson (2011).<sup>5</sup> Also, with the exception of Garriga and Keightley (2007),

<sup>&</sup>lt;sup>4</sup>Insurance frictions do not completely shut down insurance because both types of opportunistic behavior are costly. Moral hazard is costly because one stands to lose the college premium, which is considerable. Adverse selection is costly because shirkers (students who enroll in college but do not put in effort) lose the wages they could have earned elsewhere.

<sup>&</sup>lt;sup>5</sup>Although insurance against college failure risk is not the focus of their paper, Akyol and Athreya (2005) observe that the

none of these studies recognize that students may choose to drop out.

Our paper is also related to recent research that analyzes student loans in the US with a focus on the importance of borrowing constraints for college investment. Ionescu (2009) quantifies the effects of repayment flexibility (such as to lock-in interest rates or to switch repayment plans) and the relaxation of eligibility requirements for student loans on college enrollment and default rates. Lochner and Monge (2011) study the interaction between borrowing constraints, default, and investment in human capital in an environment based on the US Guaranteed Student Loan Program and private markets where constraints arise endogenously from limited repayment incentives. In the same spirit, Andolfatto and Gervais (2006) show that the endogeneity of credit constraints is important when linking government human capital policies with other transfer programs. These studies, however, abstract from modeling the risk of dropping out from college and insurance arrangements.

However, the empirical research on college behavior calls for a careful modeling of college dropout behavior. Manski and Wise (1983) argue that college students learn over time about what college means and, given this learning, some choose to drop out. In the same vein, Stange (2012) and Stinebrickner and Stinebrickner (2012a) draw attention to what they label the *option value of schooling* – meaning that students who enroll in college have the option to not continue if they learn that college is not for them. Both papers argue that there is considerable option value to schooling. In addition, they suggest that college preparedness is more important than college aspiration for college completion. In related research, Arcidiacono (2004) emphasizes the importance of learning about individual performance in the college environment as well as the characteristics of majors for the decisions of changing major, changing college, or entering the labor force. Also in a recent paper, Stinebrickner and Stinebrickner (2012b) provide evidence on the relative importance of the most prominent alternative explanations for dropout behavior and find that learning about ability plays a particularly important role in this decision.

Our paper is related to studies that focus on merit-based policies. Our insurance arrangement can be interpreted as being merit based: as we show later in the paper, the insurance premium is lower for higher ability types and the amount of insurance offered is higher as well. However, unlike merit-based aid, our insurance arrangement has no aid or grant component – it is self-financed with respect to each individual who participates, in expectation. Caucutt and Kumar (2003) analyze various types of college subsidies and conclude that merit-based aid that uses any available signal on ability increases educational efficiency with little decrease in welfare. Gallipoli, Meghir, and Violante (2008) examine the partial and general equilibrium effects of wealth-based and merit-based tuition subsidies on the distribution of education and earnings. In related work, Redmon and Tamura (2007) use a Mincer model of human capital with ability differences to heavy subsidization of higher education directly mitigates the risk of college failure by reducing the college premium.

characterize the optimal length of schooling by ability class and the importance of school district composition for growth and distribution.

#### 2 Facts

In this section, we report the basic facts that motivate the specific model of college enrollment and completion developed in this paper. The first fact is that students vary with respect to their preparation for college, which in turn affects their probability of success. We use SAT scores as an indicator for college preparation. Table 1 gives the distribution of students who took the SAT in 1999.

Table 1: Distribution of SAT scores

SAT scores	0 - 699	700 - 900	901 - 1100	1101 - 1250	1251 - 1600
Fraction	0.079	0.224	0.342	0.205	0.15

As shown in Table 2, there is considerable diversity of behavior within these observably different groups of students. We use the National Education Longitudinal Study (NELS:88) to collect information on the college enrollment choices of students who were high school seniors in 1992. We consider a student to be enrolled in college if he or she enrolled without any delay after high school and was enrolled in either a 2-year or a 4-year college in October 1992.<sup>6</sup> Notice that enrollment rates are generally high and increase with SAT scores. For the lowest SAT group, about 80 percent of students enroll in college and this percentage increases to 95.3 percent for the highest SAT group.<sup>7</sup>

For completion rates, we use the Beginning Post-secondary Student Longitudinal Survey (BPS 1995/96), which collects data on the intensity of college attendance and completion status of post-secondary education programs for students who enrolled in 1995. As we did for the enrollment rate data, we consider only students who enroll without delay in either a 2- or a 4-year college following high school graduation. Because we do not have part-time enrollment in the model, we consider students who enroll exclusively full-time in their first academic year and enroll full-time in their first and last months of enrollment in future academic years.<sup>8</sup> The survey records the fraction of students (for each ability group) who, in 2001, report having earned a bachelor's degree. This is the degree completion rate reported in Table 2. The completion rates are increasing

<sup>&</sup>lt;sup>6</sup>In this paper we focus on students with SAT scores above 700. According to the BPS data, 56 percent of students with scores below 700 enrolled for less than two years of college or enrolled in two-year colleges and dropped out, 45 percent delayed their enrollment in college and 55 percent did not enroll full-time in the first semester when they enrolled in college.

 $<sup>^{7}</sup>$ We did not want the college performance of students with very low and very high SAT scores to overly affect the performance of their respective groups (the 700-900 group and the 1250-1600 group). We employed a 5 percent Winsorization with respect to SAT scores to reduce the sensitivity of group performance to outliers.

<sup>&</sup>lt;sup>8</sup>Since students can enroll full-time but drop out shortly thereafter, "exclusively full-time enrollment in the first academic year" simply means that the student is enrolled full-time for the months he or she is actually enrolled. For later academic years, we weaken the full-time requirement to apply to only the first and last months of enrollment. This allows students to go part-time for short stretches of time.

Table 2: Enrollment, leaving, completion and graduation rates

SAT scores	700 - 900	901 - 1100	1101 - 1250	1251 - 1600
Enrollment rates	0.795	0.894	0.943	0.953
Leaving rates	0.042	0.019	0.007	0.004
Completion rates	0.602	0.718	0.827	0.871
Graduation rates	0.479	0.642	0.780	0.830

in SAT score but are significantly lower than the corresponding enrollment rates. For the lowest SAT group, the completion rate is 60 percent and it rises to 87 percent for the highest SAT group.

Among the group of students who do not complete college (i.e., do not report having earned a bachelor's degree), there are some who leave shortly after enrolling. These are students who report having last enrolled in the academic year 1995-96. We refer to this group as *leavers*. The percentage of leavers in the lowest SAT group is 4.2 percent and declines to 0.4 percent for students in the highest SAT group.

The final row reports the college graduation rates defined as the fraction of students in each ability group who earn a college degree. It is simply the product of enrollment and completion rates. As is evident, graduation rates are increasing in SAT scores as well.

# 3 Environment

There are two periods, indexed by t=1,2. The first period is the only period in which people make decisions. In period 1, a prospective student makes a one-time decision to enroll in college or not. If she does not enroll, she can work in a low-paid job with disutility of effort  $\theta \geq 0$  and draw lifetime earnings  $y \geq 0$  from a distribution H(y). The disutility of effort  $\theta$  is drawn from the distribution  $F(\theta)$ . At the time of the enrollment decision, the student knows  $\theta$ .

If the individual chooses to enroll in college, she learns the disutility of putting in effort in college,  $\gamma \geq 0$ . The student draws  $\gamma$  from the distribution  $G(\gamma)$ . After she learns  $\gamma$ , the student decides whether to continue on in college or not. If she chooses to leave, she incurs the disutility  $\theta$  from working in the low-paid job and draws her lifetime earnings y from the distribution H(y). She also incurs some partial college expenses  $\phi x$ , where  $0 < \phi < 1$ . If the student continues in college, she incurs the full college cost of x. A continuing student must choose between putting in effort or not. If she chooses to shirk, she will fail with probability 1, but she will not incur effort costs of any kind in period 1 and will start life in period 2 with a lifetime earnings draw y from the distribution H(y) and a debt of x. If she chooses to put in effort, she will complete period 1 with probability  $\pi \in (0,1)$ . If she completes successfully, she begins period 2 as a college graduate

<sup>&</sup>lt;sup>9</sup>Stinebrickner and Stinebrickner (2012a) note that dropouts that occur due to learning happen within the first two years of college.

and debt of x (no interest accumulates on the debt as long as the student continues in college) and draws y from an earnings distribution C(y). If she fails to complete college, she starts period 2 with debt x and as a person with some college credits but no degree. She draws her lifetime earnings y from distribution S(y).

The payoff of students at the start of period 1 is as follows:

1. An individual who does (N)ot enroll gets

$$V^{N}(\theta) = -\theta + \int U(y)dH(y).$$

2. An individual who enrolls, but (L)eaves

$$V^{L}(x,\theta) = -\theta + \int U(y - \phi x) dH(y).$$

3. An individual who enrolls, continues and (S)hirks gets

$$V^{S}(x,\theta) = -\beta\theta + \beta \int U(y-x)dH(y).$$

4. A student who continues and puts in (E)ffort gets

$$V^{E}(\gamma, \pi, x) = -\gamma + \beta \left[ \pi \int U(y - x) dC(y) + (1 - \pi) \int U(y - x) dS(y) \right].$$

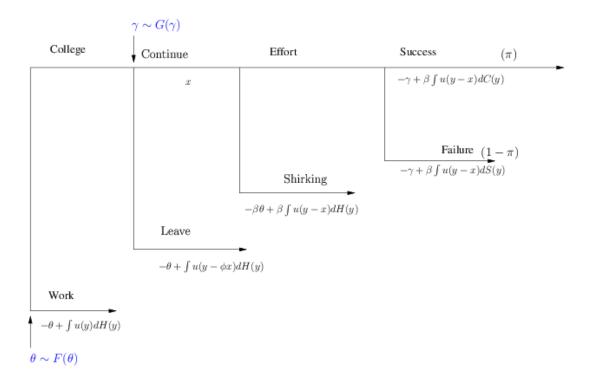
The structure of payoffs is generally self-explanatory but there are some aspects that are worth remarking on. First, we incorporate leavers in the model because we do see students leaving college voluntarily. And we incorporate shirkers because the provision of failure insurance may induce shirking.<sup>10</sup> Second, leaving or shirking forces the individual to work in the low-paid job. In contrast, if the student fails despite putting in effort, she does not incur the disutility  $\theta$  because exerting effort in college leads to some college credit and better job opportunities. Also, note that shirkers do not draw from the same earnings distribution, S (some college) as students who put in effort and fail. We assume that if the student never puts in any effort in college, it will be evident to the employer and so shirkers will draw from the H distribution.

Figure 1 summarizes the timing and the payoff from the various actions.

We make the following set of assumptions on the primitives.

<sup>&</sup>lt;sup>10</sup>We do not believe that shirking is an important issue under the current system but, as noted in the introduction, the provision of insurance may make shirking more important. In particular, if the insurance program forgave all college expenses for failures, then some leavers (who were not planning on completing college anyway) might be induced to stay on campus, not put in effort, fail, and collect on the insurance (in other words, the insurance scheme may induce some leavers to consume leisure at the expense of the good risks).

Figure 1: Timing of decisions



**Assumption 1:**  $U(c): R \to R^{++}$  with  $U'(\cdot) > 0$  and  $U''(\cdot) < 0$ .

**Assumption 2:**  $\beta \int U(y-x)dC(y) > \int U(y)dH(y)$  (college degree is profitable financial investment).

**Assumption 3:**  $\int z(y)dC(y) > \int z(y)dS(y) > \int z(y)dH(y)$  for any z(y) strictly increasing in y (the distribution C FOSD the distribution S and S FOSD the distribution H).

# 4 College Behavior Under the Current System

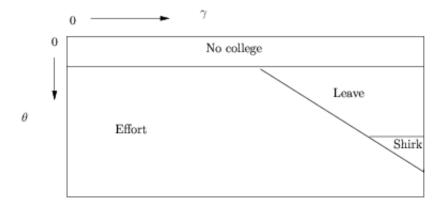
We begin by studying the choice problem. Denote by  $W(\theta, \pi, x)$  the optimal expected lifetime utility of a person prior to making her enrollment decision. At this point, the person knows  $\theta$ ,  $\pi$  and x but not  $\gamma$ . Then,

$$W(\theta,\pi,x) = \max\{V^N(\theta), \int_{\gamma} \max\{\max\{V^S(\theta,x),V^L(\theta,x)\}, V^E(\gamma,\pi,x)\}G(d\gamma)\}$$

In what follows, we will first analyze the choice between  $V^S$  and  $V^L$ . Proposition 4.1 shows that some students would rather spend time in college shirking than leaving so as to delay incurring the disutility  $\theta$  (students who choose to do this are using the student loan program to borrow and consume leisure).

**Proposition 4.1.** There exists a cut-off  $\theta_S(x) > 0$  such that, conditional on not putting in effort in college,

Figure 2: Choices in college



students leave for  $\theta < \theta_S(x)$  and shirk for  $\theta \geq \theta_S(x)$ .

Next, we analyze the decision to put in effort in college or not, which depends on  $\gamma$ ,  $\theta$  and  $\pi$ . Given  $\theta$  and  $\pi$ , if  $\gamma$  is sufficiently high then the student will not put in effort in college. This threshold is higher for students with higher probability of success. In addition, this threshold increases in the disutility  $\theta$ : for any probability of success  $\pi$ , students tolerate a higher effort cost of college if their outside option is worse.

**Proposition 4.2.** There exists a cut-off  $\gamma(\theta, \pi, x) \geq 0$  such that students put in effort for  $\gamma < \gamma(\theta, \pi, x)$  and either leaver or shirk for  $\gamma \geq \gamma(\theta, \pi, x)$ . Furthermore  $\gamma(\theta, \pi, x)$  is increasing in  $\pi$  and  $\theta$ .

Finally, there is also a cut-off value for  $\theta$  that determines who enrolls in college: those with  $\theta$  higher than this cut-off do. The cut-off value is lower for students with higher probability of success, given their relatively higher expected returns to college investment. Hence, students with a high probability of success are more likely to enroll in college.

**Proposition 4.3.** There exists a cut-off  $\theta_N(\pi, x) \geq 0$  such that for  $\theta > \theta_N(\pi, x)$  the student enrolls in college. Furthermore,  $\theta_N(\pi, x)$  is decreasing in  $\pi$ .

Our model of college enrollment and college completion is consistent with the observed diversity in student behavior. First, it predicts that not every student will enroll in college. Second, among those who enroll, some will put in effort, some will shirk and some will leave voluntarily. In terms of outcomes, some students will fail to earn a degree (those who shirk and those who put in effort but fail) and some will complete college successfully. Figure 2 sums up this diversity of behavior as determined by the two types of costs,  $\theta$  and  $\gamma$ .

# 5 Mapping the Model to the Data

We will assume that the utility function is given by  $c^{1-\sigma}/(1-\sigma)$ . Then there are five distributions and five parameters in the model. Among the distributions are  $F(\theta)$  and  $G(\gamma)$  and the three distributions of lifetime earnings of workers with different levels of education, namely, H(y), S(y), and C(y). Among the parameters are two preference-related parameters,  $\beta$  and  $\sigma$ , and three college-related parameters, x,  $\phi$ , and  $\pi$ . In mapping the model to the data, we distinguish students based on scholastic ability, namely, the four ability groupings noted in Table 2. Other than the two preference parameters, which are assumed to be the same for all students, we allow all parameters and distributions to vary with scholastic ability.

#### 5.1 Preference Parameters

We set  $\sigma = 2$  and assume that the annual discount factor is 0.97, both conventional values in quantitative macroeconomics. Since college takes 4 years to complete, the latter value pins  $\beta$  in the model to 0.885.

#### 5.2 The Distribution of Lifetime Earnings

To determine the expected utility from lifetime earnings, we simulate earnings processes specific to each education-ability group. Specifically, we assume that the log earnings at age a of a person of ability i and education (group) k is  $e_a^{k\,i} = \mu_a^{k\,i} + u_a$ , with  $u_a = u_{a-1} + \epsilon_a$ . Here,  $\mu_a^{k\,i}$  is the mean log earnings of all individuals in this group and  $\epsilon$  is a mean zero individual-specific normal random shock with variance  $\sigma^2$ . Thus, the individual's log earnings have a systematic ability-age-education component and an (individual-specific) random-walk component. The individual's present discounted value of lifetime earnings – the quantity we denote as y in the theory – is the random variable  $\sum_{a=A}^T q^{s-a_k} \exp\left(\mu_a^{k\,i} + u_{A-1} + \sum_{a=A}^s \epsilon_a\right)$ , where q = 0.97 is the annual discount factor, A is the age at which the individual begins his working life and T is the individual's retirement age.

To make this expression numerically operational we set  $u_{A-1}$  (the earnings shock prior to the start of the individual's working life) to 0 and, following the estimate reported in Storesletten, Telmer, and Yaron (2004), assume that  $\epsilon$  are drawn from a mean zero (normal) distribution with variance 0.017. We assume T for all individuals is 59 years. For individuals who do not attempt college, we assume that A = 20. Among those individuals who attempt college but do not earn a degree we distinguish between those who leave voluntarily and those who fail. For the former we assume A = 20 and for the latter we assume that A = 24. For college graduates, A = 24.

The calibration of  $\mu_a^{k\,i}$  is a challenge because the data needed to accurately discern the independent effects

Table 3: Influence of scholastic ability on average earnings by education groups

SAT Scores	700-900	901-1100	1101-1250	1251 -
High School Graduate (H)	0.99	1.10	1.10	N/A
High School plus Some College (S)	0.99	1.08	0.94	0.95
College Graduate (C)	0.94	1.02	1.06	1.17

of ability – as measured by SAT scores – and education do not exist. To get a rough measure of how scholastic ability influences earnings, we use the National Education Longitudinal Study of 1988 (NELS:88) data set to group students into our three education groups. Within each of these groups, we distinguish students by their ability. For the resulting 12 ability-education pairs, we compute the mean earnings for students who are five years out from the year they acquired their highest degree and are employed full-time. We then use these mean earnings to compute the mean earnings of each ability-education group relative to the mean earnings of its education group. Table 3 reports our findings. The results show that holding the education group constant, people in higher ability groups typically earn more, on average, than people in lower ability groups. The only exceptions to this are the mean earnings of the (1101-1250, S) and (1251-,S) groups, which turn out to be lower than the average earnings of the S group.

Next, we compute mean age-earnings profiles for each education group from the CPS for 1969-2002 using a synthetic cohort approach. Specifically, we distinguish between the three education groups in our model, namely, those with 12 years of schooling (H), those with at least 12 years but less than 16 years of completed schooling (S) and those with at least 16 years of completed schooling (C). We compute mean real earnings (real values are calculated using the CPI for 1999 as the base) of individuals of type (a, k) by averaging over the earnings of household heads between the ages of a-2 and a+2 in education group k for the appropriate year.<sup>13</sup>

Finally, to obtain  $\mu_a^{k\,i}$  we apply the ability factors in Table 3 to the mean age-earnings profile for each education group. Thus, for instance, we set the age-earnings profile of the lowest ability C group to be 0.94 of

<sup>&</sup>lt;sup>11</sup>Since earnings differentials due to ability are likely to manifest themselves gradually over time, using earnings information from some years out is preferable. We note also that we use the NELS:88 data set because the B&B data set (which reports earnings for more years) covers only college graduates, while the BPS data set covers both high school and college graduates but reports earnings only upon graduation.

 $<sup>^{12}</sup>$ We cannot include the (1251-,H) group because the sample of high ability students who have only a high school diploma is very small in this data set.

<sup>&</sup>lt;sup>13</sup>To be precise, we use the 1969 CPS data to calculate the mean earnings of 20-year-old high school graduates, of 24-year-olds with some college, and of 24-year-old college graduates; the 1970 CPS data to compute the mean earnings of 21-year-old high school graduates, 25-year-olds with some college, and of 25-year-old college graduates; and so on. Our estimates of average real earnings imply that the present value of life-cycle earnings for the H group is 0.66 million, for the S group it is 0.74 million and for the C group it is 0.99 million. Our estimates imply a college premium of 49.6 percent and a premium for acquiring a Bachelor degree over some college of 34.9 percent. Although our calibration procedure is rough, the implied premia are consistent with micro-studies. Willis (1986) and Card (2001) find that the increase in lifetime earnings from each additional year in college is between 8 and 13 percent, and using CPS 1991 data, Jaeger and Page (1996) find that the marginal effect of acquiring a bachelor's degree over completing some college is 33 percent. Restuccia and Urrutia (2004) use a 10 percent rate of return, which corresponds to a lifetime college premium of about 1.5.

the mean age-earnings profile of the C group in the CPS data. In doing these adjustments, we are implicitly assuming that the distribution of ability among different age-education groups in the CPS is comparable to the distribution of ability in each education group in Table 3.

### 5.3 College Costs and Completion Probabilities

The cost for college was \$20,706 per year for private universities and \$8,275 per year for public universities in 1999. Among the students who borrowed for their education, 67 percent went to public and 33 percent to private universities. The enrollment-weighted total college costs are \$49,508 in 1999 dollars (College Board (2001)). We consider heterogeneous costs of college: Using the same enrollment-weighted procedure, we estimate college costs across ability groups using data from the Princeton Review on college rankings in terms of average SAT scores of accepted students and data from USA Today on college costs (tuition and room and board). We estimate college costs for the 4 groups of ability levels to be: \$35,200, \$37,000, \$56,400, and \$73,400 (in 1999 dollars). Thus, we find that high-ability students enroll in more expensive colleges (more selective colleges tend to be more expensive).<sup>14</sup>

Table 4: College costs and probabilities of success

SAT scores	700 - 900	901 - 1100	1101 - 1250	$\geq 1251$
$x_i$ : College cost (in millions)	0.0352	0.0370	0.0564	0.0734
$\pi_i$ : Probability of success in %	79.52	89.35	94.36	95.42

To calibrate  $\pi_i$  we use the Beginning Post-secondary Student Longitudinal Survey (BPS 1995/96), which collects data on the intensity of college attendance and completion status of post-secondary education programs for students who enrolled in 1995. We consider only students who enroll without delay in either 2- or 4-year colleges following high school graduation. Because we do not have part-time enrollment in the model, we consider students who enroll exclusively full-time in their first academic year and enroll full-time in their first and last months of enrollment in future academic years.<sup>15</sup> The survey records the fraction of students (for each ability group) who, in 2001, report having earned a bachelor's degree. This is the degree completion rate and for our universe of students comes out to be  $(c_i, i=1,2,3,4) = (0.602, 0.718, 0.827, 0.871)$ . These rates do not identify  $\pi_i$  because the universe includes students who drop out shortly after enrolling and therefore

<sup>&</sup>lt;sup>14</sup>There is an ongoing debate about the cost that students actually pay and the sticker price of college. In the baseline economy we consider the full college cost. However, some surveys show that the net cost (full college cost net of grants and financial aid) is 77 percent at public colleges and 90 percent at private colleges (College Board (2009)). Given enrollment weights at public and private colleges, the net college cost is therefore 81 percent of the full college cost. If this lower estimate of college costs is used, the quantitative results presented in this paper would change somewhat. In particular, the aggregate welfare gain number reported later in the paper would be scaled down by a factor of about 0.8.

<sup>&</sup>lt;sup>15</sup>Since students can enroll full-time but drop out shortly thereafter, "exclusively full-time enrollment in the first academic year" simply means that the student is enrolled full-time for the months he or she is actually enrolled. For later academic years, we weaken the full-time requirement to apply to only the first and last months of enrollment. This allows students to go part-time for short stretches of time.

never earn a degree as well as students who may not be putting in effort in college (shirking). With regard to the first group, we locate students who, in 2001, report not having earned a bachelor's degree and who report having last enrolled in the academic year 1995-96. We refer to this group as leavers and their fraction (in our universe of students) comes out to be  $(l_i i=1,2,3,4) = (0.042,0.019,0.007,0.004)$ . The complement set is our empirical analog of students who are still enrolled in college after learning  $\gamma$ . To identify  $\pi$ , we assume that all these students put in effort in college. The assumption reflects our prior that large-scale shirking in college is implausible. Therefore, we obtain  $(\pi_i i=1,2,3,4) = ((0.602/(1-0.042),0.718/(1-0.019),0.827/(1-0.007),0.871/(1-0.004)) = (0.6284,0.7319,0.8328,0.8745)$ . Observe that  $\pi$  is increasing in SAT scores, which justifies our initial thought that SAT scores are an observable proxy for  $\pi$ . Table 4 displays the values of  $x_i$  and  $\pi_i$ .

# 5.4 Distributions of Disutility from Effort

The calibration of the distributions  $F_i(\theta)$  and  $G_i(\gamma)$  is achieved via moment matching. The moments we target are enrollment and leaving rates for the four ability groups. In addition, we require that the shirking rate for each education group be zero. We insist on zero shirking rates because our identification of  $\pi_i$  assumed that shirking in the population is zero.

We assume that  $F_i(\theta)$  has a Normal distribution with mean  $\mu_{\theta_i}$  with a common standard deviation  $\sigma_{\theta}$  and the  $G_i(\gamma)$  have an Exponential distribution with  $\mu_{\gamma_i}$ . These distributional assumptions imply that there are 9 parameters to be constrained by 12 moments. The problem reduces to finding the vector of parameters  $\alpha = (\mu_{\theta_{i=1,2,3,4}}, \sigma_{\theta}, \mu_{\gamma_{i=1,2,3,4}})$  that solves

$$\min_{\alpha} \left( \sum_{i=1}^{4} w_i ((e_i - e_i(\alpha))^2 + v_i (l_i - l_i(\alpha))^2 + z_i (0 - s_i(\alpha))^2 \right),$$

where  $e_i(\alpha)$ ,  $l_i(\alpha)$  and  $s_i(\alpha)$  are the enrollment, leaving and shirking rates in the model and  $w_i$ ,  $v_i$  and  $z_i$  are weights assigned to the respective deviations.

The minimization is accomplished via two sets of simulation. In the first set of simulations, the model analogs of  $\int U(y)dH(y)$ ,  $\int U(y-\phi x)dH(y)$ ,  $\int U(y-x)dH(y)$ ,  $\int U(y-x)dS(y)$  and  $\int U(y-x)dC(y)$  are computed. For example,  $\int U(y-x)dC(y)$  for a student of ability i is computed by drawing 10,000 sets of  $\epsilon_{24}, \epsilon_{25}, \ldots, \epsilon_{59}$  from N(0, 0.17). For each set, the expression  $B + (\sum_{a=24}^{59} (0.97)^{a-24} \exp(\mu_a^{C\,i} + \sum_{s=24}^{a} \epsilon_s) - x_i)^{1-\sigma}/(1-\sigma)$  is evaluated. A positive constant B is added to the utility function so as to render all util-

 $<sup>^{16}{\</sup>rm These}$  statistics also reflect a 5 percent Winsorization.

<sup>&</sup>lt;sup>17</sup>This identification implies that students who are in good standing but do not complete college must have performed poorly later in college. Lack of transcript information on student GPAs or information on the number of credits earned by those who do not complete college prevents us from verifying this implication. We note, however, that information on self-reported grades available in the BPS do not show much difference between completers and non-completers.

ity values positive. The required integral is simply the average over all 10,000 evaluations. Similarly,  $\int U(y-\phi x)dH(y)$  for an ability i student is computed by averaging over  $B+(\sum_{a=20}^{59}(0.97)^{a-20}\exp\left(\mu_a^{H\,i}+\sum_{s=20}^a\epsilon_s\right)-\phi x_i)^{1-\sigma}/(1-\sigma)$ , etc. In the second set of simulations, the expected utility from various choices is computed. We begin with a large number of individuals in each ability group (10,000). For a given  $\alpha$  vector, we draw a  $\theta$  and  $\gamma$  for each student in each ability group from the appropriate distributions. For each student we compute the model analogs of  $W(\theta)$ ,  $V^N(\theta)$ ,  $V^S(\theta)$  and  $V^E(\theta,\gamma)$  and determine his optimal choice. For the moment matching exercise, only the second set of simulations is repeated for different  $\alpha$  vectors.

Table 5: Enrollment and leaving rates: model and data

SAT scores	700 - 900	901 - 1100	1101 - 1250	$\geq 1251$
Enrollment rates: Data	79.50	89.40	94.30	95.30
Enrollment rates: Model	79.52	89.35	94.36	95.42
Leaving rates: Data	4.20	1.90	0.70	0.40
Leaving rates: Model	4.21	1.91	0.72	0.37
Shirking rates: Model	0	0	0	0

Table 5 gives the outcome of this moment matching exercise for enrollment and leaving rates, with the additional requirement that shirking rates be zero. As is evident, the match between data and model moments is quite good. Although all parameters affect all model moments,  $\mu_i$  is the key determinant of enrollment rates  $(e_i(\alpha))$  and  $\gamma_i$  is the key determinant of leaving rates  $(l_i(\alpha))$ . Thus, these parameters are essentially identified by the corresponding data moments. The  $\sigma_\theta$  parameter, the common variance of the  $\theta$  distributions, is an important determinant of shirking, since it is the high  $\theta$ -high  $\gamma$  students who have the greatest incentive to shirk. Thus,  $\sigma_\theta$  is constrained by the requirement that the shirking rates be zero for each ability group.<sup>18</sup>

We find the distributions  $F_1(\theta) \sim (0.4796, 0.787)$ ,  $F_2(\theta) \sim (0.8155, 0.787)$ ,  $F_3(\theta) \sim (1.07, 0.787)$ , and  $F_4(\theta) \sim (1.082, 0.787)$  and  $G_1(\gamma) \sim (0.1705)$ ,  $G_2(\gamma) \sim (0.1172)$ ,  $G_3(\gamma) \sim (0.0989)$ , and  $G_4(\gamma) \sim (0.0855)$ . Note that the means of the  $\gamma$  distributions decline with ability. This is consistent with our interpretation of  $\gamma$  as the utility cost associated with school work. High-ability students seem to bear fewer costs (i.e., find the college effort more enjoyable) than low-ability students. This result is consistent with Arcidiacono (2005) who argues that higher ability individuals may find college less difficult and therefore may be more likely to attend college. At the same time, the means of the  $\theta$  distributions increase with ability. This implies that high-ability individuals find working in the low-paid job less enjoyable than low-ability individuals. Finally, the average enrollment time for dropouts in our model is 3.8 years, which is in line with the data. According

<sup>&</sup>lt;sup>18</sup>The constraint does not pin down a unique value for  $\sigma_{\theta}$ . To determine  $\sigma_{\theta}$ , we first performed the minimization exercise assuming that the same  $\theta$  distribution applies to all four ability groups. This more tightly parameterized model produced a value of  $\sigma_{\theta} = 0.787$ . The full minimization (where the  $\mu_{\theta}$  is allowed to vary with the ability groups) was performed with the value of  $\sigma_{\theta}$  fixed at 0.787. The results are not sensitive to varying  $\sigma_{\theta}$  within a ±10% range of 0.787.

to the BPS 1996 data, the average completion time is 4.13 years and the average enrollment time for dropouts in college is 3.5 years. These facts are documented in Ionescu (2011) and are also consistent with evidence in Bound, Lovenheim, and Turner (2009).

# 6 Insuring College Failure Risk

### 6.1 Theory

Can the student loan program gainfully offer insurance against the risk of failing college (college failure risk)? As noted in the introduction, we wish to answer this question, recognizing that the student loan program cannot redistribute resources from students with a high probability of success (high ability) to students with a low probability of success (low ability) and recognizing that insurance against college failure may encourage shirking (and therefore failure) and hence raise the cost of such insurance.

To proceed, let f be the indemnity collected by a student who fails college and let s be the insurance premium paid by students who succeed in college.<sup>19</sup> Then, the payoffs from different actions are as follows:

1. An individual who does (N)ot enroll gets

$$V^N(\theta) = -\theta + \int U(y) dH(y).$$

2. An individual who enrolls, but (L)eaves gets

$$V^{L}(x,\theta) = -\theta + \int U(y - \phi x) dH(y).$$

3. An individual who enrolls, continues and (S)hirks gets

$$V^{S}(x, \theta, f) = -\beta \theta + \beta \int U(y - x + f) dH(y).$$

4. A student who continues and puts in (E)ffort gets

$$V^E(\gamma,\pi,x,s,f) = -\gamma + \beta \left[ \pi \int U(y-x-s) dC(y) + (1-\pi) \int U(y-x+f) dS(y) \right].$$

<sup>&</sup>lt;sup>19</sup>Two comments about the theory presented here are worth making. First, we are implicitly assuming that once a student fails college, he or she never attempts college again. If we were to relax this assumption, the insurance arrangement would need to specify that once a student avails herself of insurance, she cannot re-enroll in college without re-paying the indemnity with interest. Second, we are assuming that once a student has accumulated college credits above some pre-specified threshold, she cannot collect on the insurance even if she does not complete college. Some such rule must be in place in order to prevent students from abusing the insurance system by accumulating almost enough credits to earn a degree but not quite enough to actually earn it.

The lifetime utility of a student is then

$$W(\theta,\pi,x,f,s) = \max\{V^N(\theta), \int_{\gamma} \max\{\max\{V^L(\theta,x),V^S(\theta,x,f)\}, V^E(x,\pi,f,s)\}dG(\gamma)$$

As before, these payoffs define cut-offs for  $\theta$  and  $\gamma$  with regard to the leaving/shirking, effort/no effort and enroll/not enroll decisions. Denote these cut-offs by  $\theta_S(x, f)$ ,  $\gamma(\theta, x, \pi, s, f)$  and  $\theta_N(x, \pi, s, f)$  (the existence of these cut-offs follows from the same logic as in Propositions 4.1-4.3). To express the constraint that the insurance offered be self-financing, it is helpful to think of the premium s as being made up of two parts. One part is the "base" premium that covers losses when there is no shirking and is given by  $b(f) = \pi/(1-\pi)f$ . The other part is the additional premium needed to cover the losses imposed by shirkers. Denote this by  $\tau(f)$ . Then, the feasibility constraint can be written

$$\tau(f) \cdot \int_{\theta} 1_{\{\theta \ge \theta_{N}(x,\pi,f,b(f)+\tau(f))\}} \left[ \int_{\gamma(\theta,\pi,x,f,b(f)+\tau(f))}^{\gamma(\theta,\pi,x,f,b(f)+\tau(f))} dG(\gamma) \right] dF(\theta) \cdot \pi$$

$$= f \cdot \int_{\theta} 1_{\{\theta \ge \theta_{S}(x,f)\}} \left[ \int_{\gamma(\theta,\pi,x,f,b(f)+\tau(f))}^{\gamma(\theta,\pi,x,f,b(f)+\tau(f))} dG(\gamma) \right] dF(\theta). \tag{1}$$

The term multiplying  $\tau(f)$  on the l.h.s. of (1) is the measure of enrolled students who put in effort and succeed. Each of them pays the additional premium  $\tau(f)$ . The term multiplying f on the r.h.s. of (1) is the measure of enrolled students who shirk. Each of them collects f from the insurance scheme. For feasibility, the two sides must balance. Since this constraint holds separately for each  $(\pi, x)$  combination, the insurance scheme is self-financing with respect to the pool of students who belong in each  $(\pi, x)$  bin. We say that an insurance level f is feasible if there exists  $\tau(f)$  such that (1) is satisfied.

We can now formulate the general optimal insurance problem. Let  $\Phi$  be the set of  $0 \le f \le x$  of insurance levels that are feasible. Then the optimal f solves

$$\sup_{f \in \Phi} \int_{\theta} \left[ \int_{\gamma} W(\theta, \gamma, \pi, x, f, b(f) + \tau(f)) dG(\gamma) \right] dF(\theta).$$

Note that  $\Phi$  is non-empty since the "insurance" scheme f = 0 is trivially feasible as  $\tau(0) = 0$  will satisfy (1). Furthermore, the fact that all payoffs are bounded above by  $\int U(y)dC(y)$  implies that the supremum must exist. Even if no f actually attains the supremum, insurance levels exist that come arbitrarily close to attaining it.

In this formulation, the insurance frictions of moral hazard and adverse selection manifest themselves in the cost term  $\tau(f)$ . To develop some intuition for this term, consider first the nature of optimal insurance when loan administrators can observe effort. In this case shirkers can be excluded from insurance and, hence, the  $\tau(f)$  can be set to zero and s is simply  $\pi/(1-\pi)f$ . Ignoring the  $-\gamma$  term, the expected utility from putting in effort in college is then given by

$$\pi \cdot \int U(y-x-[(1-\pi)/\pi]f)dC(y) + (1-\pi) \cdot \int U(y-x+f)dS(y).$$

Maximizing the above expression with respect to f yields the following first-order condition:

$$\int U'(y - x - [(1 - \pi)/\pi]f)dC(y) = \int U'(y - x + f)dS(y).$$

The value of f that attains the maximum is the one that equalizes the expected marginal utility of consumption following failure and success. Denote this value of  $\bar{f}$ . Because there is a college premium in earnings (meaning that the distribution C(y) first-order stochastic dominates the distribution S(y)), the value of  $\bar{f}$  will typically exceed the cost of college x as, in fact, is true for all ability levels in our calibrated economy. The implication is that when effort is observable, it is optimal to set f = x, i.e., it is optimal to offer full loan forgiveness in case of failure (full failure insurance).<sup>20</sup>

When effort is unobservable, however, loan administrators cannot exclude shirkers from taking advantage of failure insurance. Furthermore, if loan administrators offer full failure insurance they end up swelling the ranks of potential shirkers and, possibly, of actual shirkers. Since this goes to the heart of the adverse selection friction, it is worthwhile to explain this point in some detail.

Observe that when failure insurance is absent, the payoff from leaving and shirking is given by  $V^L(\theta, x) = -\theta + \int U(y - \phi x) dH(y)$  and  $V^S(\theta, x) = \beta[-\theta + \int U(y - x) dH(y)]$ , respectively. This implies that students with  $\theta > (1 - \beta)^{-1} \left[ \int U(y - \phi x) dH(y) - \beta \int U(y - x) dH(y) \right]$  would find shirking to be the best outside option relative to putting in effort in college. If full failure insurance is offered, the r.h.s. of the inequality drops to  $(1-\beta)^{-1} \left[ \int U(y - \phi x) dH(y) - \beta \int U(y) dH(y) \right]$  (the payoff from leaving is unchanged but the payoff from shirking goes up). Thus the cutoff  $\theta$  will decline and the mass of potential shirkers will increase.<sup>21</sup> An increase in the mass of potential shirkers does not mean that actual shirking will increase because full failure insurance also increases the value of putting in effort in college. We can show, however, that the utility gain

 $<sup>^{20}</sup>$ Furthermore, since insurance does not affect the payoff from leaving or shirking, it raises  $V^E$  without altering  $V^L$  or  $V^S$  and therefore leads to a higher  $\gamma$  cut-off. This means that fewer students leave college after learning their  $\gamma$ . Furthermore, a higher  $V^E$  raises the ex-ante utility from enrolling in college, which raises  $\theta_N$ . This means that more students enroll in college. Thus, both enrollment and completion rates as well as welfare are positively affected by full failure insurance.

 $<sup>^{21}</sup>$ If the term in square brackets remains positive, the cutoff  $\theta$  will decline but remain positive. However, it is also possible that the term is actually negative, in which case the cutoff  $\theta$  will be negative and all students with positive  $\theta$  would view shirking as a better option than leaving.

from putting effort in college when there is full failure insurance versus when there is no insurance (denoted  $\Delta V^E$ ) is, in fact, smaller than the utility gain from shirking when there is full insurance versus when there is no insurance (denoted  $\Delta V^S$ ). To see this, observe that

$$\Delta V^{E} = \pi \int [U(y - x - s) - U(y - x)] dC(y) + (1 - \pi) \int [U(y) - U(y - x)] dS(y)$$

and

$$\Delta V^S = \pi \int [U(y) - U(y-x)] dH(y) + (1-\pi) \int [U(y) - U(y-x)] dH(y).$$

The term multiplying  $\pi$  in  $\Delta V^E$  is less than the term multiplying  $\pi$  in  $\Delta V^S$  and, since the function U(y)-U(y-x) is decreasing in y (from strict concavity of the utility function) and the distribution S(y) FOSD the distribution H(y), the term multiplying  $(1-\pi)$  in  $\Delta V^E$  is also less than the term multiplying  $(1-\pi)$  in  $\Delta V^S$ . Since shirking is the best outside option of all enrolled students, it is now entirely possible that some students who chose to leave when no insurance was offered will shirk and some students who chose to put in effort in college when no insurance was offered will shirk as well.<sup>22</sup> Thus, both adverse selection and moral hazard will contribute to raising the failure rate above  $1-\pi$  and make such insurance impossible to offer at an actuarially fair price. In other words, the feasibility constraint (1) would be violated for  $\tau(\bar{f})=0$ . To offer such insurance, loan administrators would have to consider raising  $\tau$  above zero. However, it is not certain that a  $\tau$  for which feasibility is restored will exist. The problem is that as  $\tau$  is raised,  $V^E$  will decline, which will induce more students to shift from putting in effort in college to shirking. If this feedback from higher insurance costs to the measure of shirkers is strong enough, there may not exist a  $\tau$  for which full failure insurance is feasible. Furthermore, even if there exists a  $\tau(\bar{f})$  that restores the feasibility of  $\bar{f}$ , it may impose too high a cost on successful students to be the optimal insurance policy.

These considerations give some indication of the nature of the tradeoff involved in the general optimal insurance problem. Basically, failure insurance comes at the cost of some level of opportunistic behavior (shirking) and this cost has to be borne by program participants in the form of  $\tau(f)$  – the additional

$$\gamma^{'}(e) = \pi^{'}(e)\beta [\int U(y-x-b(f)-\tau(f))dC(y) - \int U(y-x+f)dS(y)].$$

If  $\gamma''(e) > 0$  and  $\pi''(e) < 0$  (as seems plausible) then increasing f, holding fixed  $b(f) + \tau(f)$ , would lower the value of the (integral) term within  $[\cdot]$  on the r.h.s. and lower e. Thus insurance would lower the probability of success for someone who puts effort in college. However, if the effort choice in the absence of insurance is at a corner, i.e.,

$$\gamma'(e_{\max}) < \pi'(e_{\max})\beta[\int U(y-x)dC(y) - \int U(y-x)dS(y)],$$

where  $e_{\text{max}}$  is the maximum amount of effort possible, then e need not fall even if insurance lowers the value of the integral term. By taking effort to be a binary decision, we are in effect assuming that we are in this corner situation.

 $<sup>^{22}</sup>$ If effort was a continuous variable, the first-order condition for an interior choice of e would be

Table 6: Optimal failure insurance

SAT scores	700 - 900	901 - 1100	1101 - 1250	1251 - 1600
$f^*$	\$35,200	\$37,000	\$56,400	\$73,400
$f^*$ as percent of $x$	100	100	100	100
$s^*$ as percent of $x$	59.1	36.6	20.1	14.4
$\tau^*$ as percent of $x$	0.0528	0.00581	0.00217	0.00194
Percent shirking	0.0317	0.0042	0.0018	0.0017

premium collected in excess of the actuarially fair level. The higher the level of insurance offered the greater these costs are likely to be  $(\tau(f))$  is increasing in f). The optimal level of failure insurance will balance the benefits of providing insurance against these costs.

#### 6.2 Quantitative Findings

In this section we report the quantitative results regarding the optimal level of insurance for each of the four ability groups in our economy.

The first task is to determine the set of feasible insurance levels for each ability group. We divide  $[0, x_i]$  into a fine grid and for each grid point attempt to find a  $\tau_i$  that satisfies (1) by iterating on  $\tau_i$ . For iteration k, we set  $\tau_i^k$  to the value that satisfies (1), given the decision rules corresponding to  $\tau_i^{k-1}$ . We start the iterations with  $\tau_i^0 = 0$ . If this iterative process converges, we classify the particular grid point as feasible; otherwise we classify it as infeasible. We find that for each ability group, all values in  $[0, x_i]$  are feasible. Even though shirking occurs when failure insurance is offered, the cost of offering this insurance is small and the  $\tau_i$  sequence easily converges for all values (grid points) in  $[0, x_i]$ .

Table 6 presents the optimal indemnity offered,  $f^*$ , as well as the base premium,  $b^*$ , and the cost,  $\tau^*$ . We find that it is optimal to forgive the entire college loan in the case of failure. This result holds even though full-cost insurance induces shirking for all ability groups. We find that the percentage of shirkers is highest for the lowest ability group and the corresponding  $\tau$  is relatively large, owing to the fact that the probability of success  $\pi$  is lowest and the mean of the G distribution is highest for this group. Both factors combine to make the elasticity of the measure of potential shirkers with respect to the insurance offered the highest for this group. For the other three ability groups, the optimal insurance features small levels of shirking and the associated  $\tau(f)$  are quite small as well.

Table 7 displays the effects of optimal failure insurance. Insurance results in significant welfare improvement for all ability groups. The welfare gains range from 1.4 - 3.1 percent. In the aggregate, the gain in welfare is 2.4 percent. Given that full failure insurance is offered at almost actuarially fair rates (meaning that the optimal  $\tau^*$  is quite small), the gain in welfare reflects the gain from having almost fairly priced

insurance against a bad outcome.

As one would expect from our earlier discussion of the effects of full failure insurance, there is a measurable decline in leaving rates for all ability groups. The sharpest drop occurs for the lowest ability group. For this group, the leaving rates decline by 2.8 percentage points and, as displayed in Table 6, the shirking rate rises by 3.2 percentage points. The additional 0.4 percentage point gain in shirking (beyond the 2.8 percent) is the result of some students switching to shirking from putting in effort in college. There is a small but measurable decline in completion rates as a result. For the other three ability groups, the decline in leaving rates is almost entirely balanced by an increase in shirking rates and there is no change in completion rates.

One surprising result is that the improvement in welfare fails to show up in enrollment rates. This results from the fact that the mass of students around the cut-off  $\theta_N$  is too small for enrollment rates to be appreciably affected. In other words, the marginal student (the one who is indifferent between enrolling in college or not) is pretty rare in the population. This rarity presumably reflects the fact that the current student loan program is successful in drawing in most students who believe that they will choose to put in effort in college, following enrollment.

Finally, given the small change in enrollment and completion rates, failure insurance has almost no effect on the percentage of college graduates in the student population.

900 901- $1\overline{100}$ 1251-SAT scores 700-1101 -1250 Fail Ins Data Fail Ins Data Fail Ins Data Fail Ins Data 2.97 2.693.05 1.35 % Welfare gain 1.38 4.2 1.45 1.90 0.550.70 0.20 0.40% Leaving rates % Completion rates 59.98 60.2 71.83 71.80 82.67 82.70 87.13 87.10 % Enrollment rates 79.53 79.5 89.35 89.40 94.36 94.30 95.42 95.30 % Coll graduation rates 47.70 47.9 64.1864.20 78.01 78.00 83.14 83.00

Table 7: Effects of Optimal failure insurance

# 7 Insuring College Dropout Risk

#### 7.1 Theory

Students in our economy fail to complete college either because they cannot accomplish what is required of them or they choose to leave college voluntarily. So far we have considered the possibility of providing insurance against the first scenario. In this section we consider the possibility of insuring both leavers and failures, the group we call *dropouts*. The motivation for widening the scope of insurance in this way is that insuring leavers is tantamount to providing insurance against a bad (high) draw of  $\gamma$ , which can be

welfare improving in its own right. Second, leavers are the main source of adverse selection when insurance is provided against failure only: erstwhile leavers stay on and shirk and collect on the insurance. Providing leavers with some insurance is likely to attenuate the adverse selection problem.

The payoffs from this insurance arrangement is the same as in the previous section, except for the payoff from leaving which is now given by

$$V^L(\theta, x, f) = -\theta + \int U(y - \phi x + \max\{\phi x, f\}) dH(y).$$

Thus, the insurance program also provides an indemnity to leavers but only up to the maximum of their college costs or f. As before f is any element of [0, x]. The requirement for the feasibility of f now includes (on the r.h.s. of 1) the indemnity collected by students who leave following enrollment, namely

$$\max\{\phi x, f\} \cdot \int_{\theta} 1_{\{\theta < \theta_S(x, f)\}} \left[ \int_{\gamma(\theta, \pi, x, f, b(f) + \tau(f))} dG(\gamma) \right] dF(\theta).$$

SAT scores 700 - 900901 - 11001101 - 12501251 - 1600\$35,200 \$56,400 \$37,000 \$73,400  $f^*$  as percent of x100 100 100 100  $\max\{\phi x, f^*\}$  as percent of x 25 25 25 25 20.1 14.4  $s^*$  as percentage of x 59.1 36.6 $\tau^*$  as percent of  $\overline{x}$ 0.0112 0.02880.0151 0.01 0.02790.00394 0.001720.0016 Percent shirking

Table 8: Optimal dropout insurance

#### 7.2 Quantitative Findings

Table 8 reports the optimal insurance and premia collected. The optimal insurance,  $f^*$ , is full college cost for all ability groups as in the case where only failure is insured and leavers receive the full cost of their college expenses. This arrangement reduces shirking, although each successful graduate pays the additional premium  $\tau(f)$  to fund the students who leave.

Table 9 reports the effects of dropout insurance (DI) relative to failure insurance (FI) and to the data. In and of itself, dropout insurance should increase welfare relative to failure insurance, since both adverse draws of  $\gamma$  and failure are being insured. However, the insurance is more costly because leavers have to be compensated for their college costs as well. Overall, there is a welfare gain relative to the no-insurance case but there is a decline in welfare relative to the failure insurance case.

Dropout insurance increases leaving rates significantly, relative to the failure insurance case as well as the data. As observed in Table 8, the increase in leaving comes partially at the expense of shirking. This is easy to understand: When only failure insurance is offered,  $\theta_S$  (the cut-off value of  $\theta$  above which a student finds shirking better than leaving) is given by  $(1-\beta)^{-1} \left[ \int U(y-\phi x) dH(y) - \beta \int U(y) dH(y) \right]$ . With dropout insurance, the cost  $\phi x$  is also covered, so that the first term within the square brackets goes up. Indeed,  $\theta_S$  is now given by  $\int U(y) dH(y)$ . Thus, the pool of potential shirkers shrinks with dropout insurance.

But dropout insurance also comes at the expense of putting in effort in college. When only full failure insurance is offered, students who choose between putting in effort (with payoff  $-\gamma + \beta EV^E$ ) and leaving (with payoff  $-\theta + \int U(y)dH(y)$ ) may now find the option to leave more attractive. For this reason, dropout insurance leads to a measurable drop in completion rates for all ability groups relative to the failure insurance case as well as the data.

Enrollment rates rise with dropout insurance. This is in sharp contrast to the case of full failure insurance where enrollment rates barely budged. The reason is that dropout insurance lowers the costs to students of learning their  $\gamma$  by trying out college. Students who expect to leave college (after enrolling) with a relatively high probability will find their decision to enroll largely unaffected by the provision of failure insurance, since the insurance is being offered conditional on a decision (putting in effort in college) that they are unlikely to take. But with dropout insurance, these students enroll since the insurance is being offered for an event they think is more likely. In other words, the provision of dropout insurance increases the option value of enrolling in college. Thus, our results confirm the importance of the option value of schooling in students' enrollment decisions that both Stange (2012) and Stinebrickner and Stinebrickner (2012a) stress.

The rise in enrollment rates is large enough to make the percent of college graduates in the population higher with dropout insurance. Thus, the negative impact of lower completion rates is more than offset by the positive effect of higher enrollment rates.

In the aggregate, dropout insurance induces an increase in enrollment rates to 95.44 percent compared to enrollment rates of 89.62 percent in the case of failure insurance. Out of everyone who enrolls, 6.42 percent decide to leave compared to 1.00 percent who decide to leave in the case where failure insurance is offered. There are 0.75 percent of shirkers on average compared to 0.78 percent in the case of failure insurance. The average completion rate is lower: 70.61 percent compared to 74.49 percent in the case of failure insurance. Overall the percentage of high school graduates who acquire a college degree is 67.4 percent compared to 66.75 percent with failure insurance. The gain in aggregate welfare is 2.15 percent.

Overall, the optimal provision of dropout insurance increases welfare for all ability groups. It is still the case, however, that from a pure welfare point of view dropout insurance is less beneficial than failure insurance. The potential benefits of insuring against a bad  $\gamma$  draw is lower than the cost imposed on successful students from this additional insurance. On the other hand, dropout insurance significantly increases enrollment rates and (slightly) boosts the fraction of the student population that ultimately ends up earning a college degree.<sup>23</sup> Thus, the "positive" effects of dropout insurance are more in line with what policymakers seem to desire: a more highly educated workforce.

Table 9: Effects of insuring college dropout risk

SAT scores	700	-	900	901	-	1100	1101	-	1250	1251	-	
	DI	FI	Data	DI	FI	Data	DI	FI	Data	DI	FI	Data
% Welfare gain	2.46	3.05		1.23	1.35		2.85	2.97		2.62	2.69	
% Leaving rates	12.72	1.38	4.2	6.29	1.45	1.90	4.13	0.55	0.70	2.75	0.20	0.40
% Completion rates	53.10	59.98	60.2	68.30	71.83	71.80	79.70	82.67	82.70	84.90	87.13	87.10
% Enrollment rates	91.01	79.53	79.5	94.558	89.35	89.40	98.34	94.36	94.30	98.39	95.42	95.30
% Coll graduation rates	48.32	47.70	47.9	64.57	64.18	64.20	78.37	78.01	78.00	83.53	83.14	83.00

#### 8 Conclusion

A large fraction of students who enroll in college do not earn a degree. Many of these students borrow money to finance their (failed) college education. Our paper examines – theoretically and quantitatively – if the risk of failing to complete college can be insured and quantifies the value and other effects of offering such insurance. We develop a model of student enrollment and effort decisions that is broadly consistent with the diversity of student behavior observed in reality. We develop the notion of optimal insurance against the risk of failing to complete college, taking into account the costs imposed by moral hazard and adverse selection on such insurance. Using the calibrated model, we compute the optimal insurance and quantify the effect of optimal insurance on student behavior as well as on student welfare.

We find that it is optimal to offer full insurance against risk of college failure, meaning that it is optimal to offer full loan forgiveness in the event the student fails college. Although failure insurance encourages some students to "fake failure" and collect on the insurance, the incidence of such behavior turns out to be quite low in our model. Thus, full failure insurance can be offered at almost actuarially fair rates. Consequently, there is a significant improvement in the welfare of students. A somewhat surprising finding is that the provision of full failure insurance, while welfare improving, does not much affect enrollment, completion, or degree attainment rates. The insensitivity of these rates appears to be the result of the fact that most

<sup>&</sup>lt;sup>23</sup>We are abstracting from the adverse effects on the private returns to college education that may stem from policy-induced increases in the numbers of college graduates. Card and Lemieux (2001) as well as Bound, Lovenheim, and Turner (2009) find evidence of congestion effects in higher education: an increase in the number of people seeking higher education tends to be associated with a decline in educational attainment. It should also be kept in mind that because higher education is subsidized by federal and state governments, changes in enrollment and graduation rates induced by insurance will change the level of subsidy received by the higher education sector. The welfare costs of this change in subsidy are ignored in this study.

students who anticipate making a serious effort to earn a college degree already enroll.

We also investigated the consequences of extending insurance to students who enroll in college but leave voluntarily (as opposed to failing). When insurance to offered to both failures and leavers (the group we call dropouts), we find that it is optimal to offer full dropout insurance: students who leave or fail have all their college costs forgiven. We find that both enrollment and degree attainment rates are measurably positively affected by full dropout insurance: More students enroll in college and more students end up earning college degrees. These findings suggest that there is a substantial group of high school graduates who expect to find college so hard that they do not believe they will stay in college long enough to benefit from failure insurance. But these students are motivated to enroll in college if the cost of trying out college and leaving is insured as well. Nevertheless, precisely because dropout insurance increases enrollment rates, it also increases the premium burden on successful students. Thus, while full dropout insurance improves overall college-performance measures, it leads to a smaller welfare gain for the student body as a whole.

In conclusion, we would like to stress that our study focused on a form of insurance that has not been analyzed previously. Most existing studies in the related literature focus on offering insurance against bad outcomes in the labor market. In contrast, we focused on offering insurance against bad outcomes in the education market. At first blush, such insurance schemes seem implausible but our exploration revealed that that is not the case and that there is merit to studying such insurance arrangements more thoroughly.

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# A Appendix

In this section we provide proofs for the propositions presented in the paper.

Proof of Proposition 4.1. Consider the function  $V^L(x,\theta) - V^S(x,\theta) = -\theta(1-\beta) + \int U(y-\phi x)dH(y) - \beta \int U(y-x)dH(y)$ , which is continuous and strictly decreasing in  $\theta \in [0,\infty)$ . We have  $V^L(x,0) - V^S(x,0) = \int U(y-\phi x)dH(y) - \beta \int U(y-x)dH(y) > 0$ . By continuity and strict monotonicity with respect to  $\theta$ , there exists  $\theta_S(x) > 0$  such that  $V^L(x,\theta_0(x)) - V^S(x,\theta_S(x)) = 0$ . For any  $\theta$  below this cut-off, leaving is strictly preferred to shirking and at or above this cut-off, shirking is weakly or strictly preferred to leaving.

Proof of Proposition 4.2. Consider the function  $Z(x,\pi,\gamma,\theta)=V^E(x,\pi,\gamma)-\max[V^L(x,\theta),V^S(x,\theta)]$  which is continuous and strictly decreasing for  $\gamma\in[0,\infty)$ . If  $Z(x,\pi,0,\theta)\leq 0$  then  $\gamma(x,\pi,\theta)=0$ . If  $Z(x,\pi,0,\theta)>0$  then, by continuity and strict monotonicity with respect to  $\gamma$ , there exists a unique  $\gamma(x,\pi,\theta)>0$  such that  $Z(x,\pi,\gamma(x,\pi,\theta))=0$ . For any  $\gamma<\gamma(x,\pi,\theta)$ , putting in effort in college is strictly preferred to either leaving or shirking and for any  $\gamma\geq\gamma(x,\pi,\theta)$  either shirking or leaving is weakly or strictly preferred to putting effort in college. To show that  $\gamma(x,\pi,\theta)$  is increasing in  $\pi$ , it is sufficient to note that by Assumption 3  $V^E$  is increasing in  $\pi$ . To show that it is increasing in  $\theta$ , it is sufficient to note that  $\max\{V^S(x,\theta),V^L(x,\theta)\}$  is decreasing in  $\theta$ .

Proof of Proposition 4.3. Consider the function  $Z(x, \pi, \theta) = \int \max\{V^E(x, \pi, \gamma), V^L(x, \theta), V^S(x, \theta)\}dG(\gamma) - W(\theta)$ . We will show that this function is increasing in  $\theta$ . Observe that

$$Z(x,\pi,\theta) = \int_0^{\gamma(x,\pi,\theta)} V^E(x,\pi,\gamma) dG(\gamma) + \int_{\gamma(x,\pi,\theta)} \max[V^D(x,\theta), V^S(x,\theta)] dG(\gamma) - W(\theta).$$

Let  $\theta$  increase by  $\Delta > 0$ . Consider the effect of this change on  $Z(x, \pi, \theta)$  in 2 parts:

$$Z(x,\pi,\theta+\Delta) - Z(x,\pi,\theta) = [Z(x,\pi,\theta+\Delta) - \bar{Z}(x,\pi,\theta+\Delta)] + [\bar{Z}(x,\pi,\theta+\Delta) - Z(x,\pi,\theta)].$$

where

$$\bar{Z}(x,\pi,\theta+\Delta) = \int_0^{\gamma(x,\pi,\theta)} V^E(x,\pi,\gamma) dG(\gamma) + \int_{\gamma(x,\pi,\theta)} (e=1) \max[V^L(x,\theta+\Delta),V^S(x,\theta+\Delta)] dG(\gamma) - W(\theta+\Delta).$$

Then  $[\bar{Z}(x, \pi, \theta + \Delta) - [Z(x, \pi, \theta)]$  is given by

$$\int_{\gamma(x,\pi,\theta)} \max\{-(\theta+\Delta) + \int u(y-x/4)H(dy), -(\theta+\Delta)\beta + \beta \int u(y-x)H(dy)\}dG(\gamma)\} - \int_{\gamma(x,\pi,\theta)} \max\{-\theta + \int u(y-x/4)H(dy), -\theta\beta + \beta \int u(y-x)H(dy)\}dG(\gamma)\} + \Delta dG(\gamma)$$

Observe that the above change is non-negative because the positive  $\Delta$  term contributes  $\Delta$  while the negative  $\Delta$  term contributes either  $-\Delta G(\gamma(x,\pi,\theta))$  (in the case where  $\theta + \Delta < \theta_0$ ) or  $-\beta \Delta G(\gamma(x,\pi,\theta))$  (in the case where  $\theta + \Delta \geq \theta_0$ ). Furthermore, the term  $[Z(x,\pi,\theta+\Delta)-\bar{Z}(x,\pi,\theta+\Delta)]$  is non-negative by optimality. Hence,  $Z(x,\pi,\theta+\Delta)-Z(x,\pi,\theta)\geq 0$ . Thus  $Z(x,\pi,\theta)$  is increasing in  $\theta$ . Therefore, there must be a cut-off value  $\theta_N(x,\pi)\geq 0$  such that for all  $\theta>\theta_N(x,\pi)$  the student will enroll.

To establish that  $\theta(x,\pi)$  is decreasing in  $\pi$ , it is sufficient to note that  $V^E(x,\pi,\gamma)$  is strictly increasing

in  $\pi$  and, therefore,  $Z(x,\pi,\theta)$  is strictly increasing in  $\pi.$