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PRIVATE LIQUIDITY AND BANKING REGULATION

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Abstract

We show that the regulation of bank lending practices is necessary for the optimal provision of private liquidity. In an environment in which bankers cannot commit to repay their creditors, we show that neither an unregulated banking system nor narrow banking can provide the socially efficient amount of liquidity. If the bankers provided such an amount, then they would prefer to default on their liabilities. We show that a regulation that increases the value of the banking sector's assets (e.g., by limiting competition in bank lending) will mitigate the commitment problem. If the value of the bank charter is made sufficiently large, then it is possible to implement an efficient allocation. Thus, the creation of a valuable bank charter is necessary for efficiency.

Keywords: Private liquidity creation; banking regulation; limited commitment.

JEL classification: E40, E42, G21, G28.

1. INTRODUCTION

The institutions composing the banking system do many things, but one of their main functions is to create liquidity. Among many forms of liquidity creation, banks issue tradeable securities that can be used to facilitate payments and settlement. This is private money. For example, Gorton (1999) highlights the free banking era as a period in American monetary history in which privately issued monies circulated as competing media of exchange. More contemporarily, it has been argued by many observers of the recent financial crisis that repurchase agreements are the private monies of our time (e.g., see Gorton and Metrick, 2010 and the explanations therein). Therefore, a primary concern of monetary economists should be to know whether, putting stability issues aside, a private banking system is creating enough (or too much) of this kind of liquidity to allow society to achieve an efficient allocation of resources. In other words, can a private banking system provide the efficient amount of liquid assets? And if so, what are the characteristics of such a system? Should we leave the job to the invisible hand or should we regulate the banking system? Can narrow banking – whereby the business of lending is separated from the business of deposit-taking – provide the efficient amount of liquidity?

To investigate these questions, we construct a general equilibrium model in which bankers intermediate funds from households to firms and issue liabilities that can be used as a medium of exchange. The frictions explaining the essential role of banks are traders' anonymity and lack of commitment. In our framework, households acquire bank liabilities to use them as a medium of exchange, and the banks use the proceeds to make loans to firms. If the price of the bank liabilities is too high, then the households will not be able to trade the efficient amount and capital accumulation will be suboptimal. This is the mechanism through which the price of liquid assets affects the equilibrium allocation.

What prevents the efficient provision of liquidity here? The answer is simple: Bankers cannot commit to repay creditors, and the threat of terminating their bank charter may not be strong enough to induce them to always redeem their liabilities.[1] To ensure that bankers

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[1]This is very much in the spirit of the hypotheses made in Gu, Mattesini, Monnet, and Wright (forthcom-

do not overborrow and strategically default on their liabilities, we consider a mechanism that imposes individual debt limits on each banker, as in Alvarez and Jermann (2000).[2] These individual debt limits constrain the banker's portfolio choices and discipline private liquidity creation. While these limits guarantee the solvency of each banker, they also constrain the amount of liabilities that each banker can issue.

Our contribution to the literature is to show how the degree of competition in bank lending affects the creation of private liquidity by intermediaries. To determine the bank charter value, we initially consider a situation of perfect competition in bank lending. We show that, in this case, any equilibrium is necessarily inefficient. As banks compete on the asset side of their balance sheet, the return on their assets is relatively low, which reduces the value of the bank charter. As a consequence, the return they are willing to pay on their liabilities cannot be too high, as otherwise they would renege on their promises. This implies that liquid assets are expensive and that bankers are unwilling to supply the socially efficient amount of liquidity.

We then turn to the analysis of a regulated banking system. The role of the regulator is to mitigate the commitment problem by increasing the bank charter value. The way to achieve this goal is to regulate the market for bank loans in order to increase the return on the bankers' assets. We show that if the regulator creates a sufficiently large value for the bank charter, then the bankers are willing to pay a higher return on their liabilities. In this case, they can supply the socially efficient amount of liquidity. Our main contribution is to characterize the way in which the regulator should intervene. In particular, it is necessary to increase the charter value by raising the return on the banking sector's assets, given that it is socially desirable to induce the bankers to pay a high return on their liabilities. Therefore, narrow banking will not be able to provide an efficient amount of liquid assets, as there is no clear way to increase the return on the assets of a bank that keeps 100% in reserves. Also, any regulation that aims to increase the bank charter value by lowering the

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ing), Hart and Zingales (2011), Gertler and Karadi (2011), Boissay (2011), Cavalcanti and Wallace (1999a, 1999b), and Cavalcanti, Erosa and Temzelides (1999).

[2]Their analysis builds on work by Kehoe and Levine (1993) and Kocherlakota (1996).

return paid on bank liabilities will fail to achieve efficiency. Therefore, our theory says that, absent stability issues, Regulation Q in the U.S. was a bad idea.[3]

As financial intermediaries, bankers play a crucial role in allocating funds from savers to finance capital formation; see Gorton and Winton (2003) for a survey. An important aspect of our analysis is to combine the role of bankers as liquidity providers with their role as financial intermediaries. There are several theories explaining why combining intermediation and liquidity provision under the same roof is a good idea. Andolfatto and Nosal (2009) show in the context of a costly state verification model that it is efficient to combine these two activities within the same institution whenever the monitoring cost is sufficiently high. In a similar spirit, Sun (2007) shows that combining liquidity provision and intermediation can be desirable. Kashyap, Rajan, and Stein (2002) have also argued that providing liquidity on both sides of their balance sheets (e.g., through lines of credit) may give banks a competitive advantage. Finally, Williamson (1999) argues that private money creation by intermediaries is optimal because it allows them to undertake productive investment opportunities in states of the world in which they do not have their own funds available.

Empirical work on bank liquidity creation is scant, and the Berger and Bouwman (2009) paper is, to the best of our knowledge, the only one that measures the amount of liquidity created by the banking system. The authors construct a measure of liquidity creation by comparing how liquid the entries on both sides of a bank's balance sheet are. According to this measure, a bank creates more liquidity the more its liabilities are liquid relative to its assets. Among other interesting things, they find that banks that create more liquidity are valued more highly by investors, as measured by the market-to-book and the price-earnings ratios.

Our paper is also related to the large literature on the optimal creation of private liquidity. However, in this literature, the effects of competition in bank lending are usually excluded

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[3]Regulation Q prohibited the payment of interests on deposits and imposed interest rate ceilings on many other types of bank deposits. It was introduced in the U.S. in 1933 and was repealed as part of the Dodd-Frank Act in 2011.

from the analysis. There are two strands in this literature. The first strand focuses on the role of liquidity as a means of payment. Cavalcanti, Erosa, and Temzelides (1999) and Cavalcanti and Wallace (1999a, 1999b) study private liquidity creation in the context of a random matching model. Azariadis, Bullard, and Smith (2001) study private and public provision of liquidity using an overlapping generations model; Kiyotaki and Moore (2001, 2002) propose a theory of inside money based on the possibility of collateralization of part of a debtor's assets; and Monnet (2006) studies the characteristics of the agent that is most able to issue money. The second strand focuses on the role of liquidity as a means of funding investment opportunities. For example, Holmstrom and Tirole (1998, 2011) show that a moral hazard problem may limit the ability of firms to refinance their ongoing projects when there is aggregate uncertainty. They argue that this inefficiency can be resolved by the government issuing bonds to firms.

Other authors have focused exclusively on the study of competition in bank lending without explicitly accounting for the role of bankers as liquidity providers. These include Yanelle (1997) and Winton (1995, 1997). We believe that combining the role of bankers as liquidity providers with their role as financial intermediaries is important for the study of optimal liquidity provision in a competitive environment.

Perhaps the paper closest to ours is Hart and Zingales (2011), who show that an unregulated private banking system creates too much liquidity. They present an environment similar to Gu, Mattesini, Monnet, and Wright (forthcoming), to which our paper also bears a resemblance, where a lack of double coincidence of wants, a lack of commitment, and a limited pledgeability of collateral give rise to an essential role for a medium of exchange. A bank acts as a safe-keeping institution for the collateral and issues receipts that can circulate as a means of payment because the bank is able to commit to pay the bearer of a receipt on demand. Hart and Zingales uncover an interesting externality: A bank that issues more money to its customers increases the price level for all other customers as well. As a result, too much collateral is stored, and banks create too much liquidity. We depart from their analysis in a fundamental way: While they assume that banks can commit to pay back the bearer of the receipts they issued, we assume they cannot. This suffices to overturn their

result: We show that a poorly regulated banking system creates too little liquidity.

To be clear, we are not concerned in this paper with the stability of the banking sector. This is clearly an important issue that also relates to liquidity. In particular, the business of liquidity transformation and the risks it entails have been highlighted most forcefully in the seminal paper by Diamond and Dybvig (1983). Their notion of liquidity is one of immediacy: Bank deposits are useful because they can be redeemed on demand when depositors have an urgency to consume. So the banking system is fragile whenever the bank cannot fulfill the demand for immediate redemption. This is the well-known problem of a bank being illiquid but solvent. However, Jacklin (1987) considers a solution to banks' inherent fragility, namely that banks issue tradeable securities. If depositors have an urge to consume, they can sell these securities instead of running to the bank. This notion of liquidity (namely the ease with which bank liabilities can be traded) is clearly related to ours.

The paper is structured as follows. In Section 2, we present the basic framework, and we discuss the role of its main ingredients in Section 3. In Section 4, we formulate and solve the planner's problem. In Section 5, we characterize equilibrium allocations in the case of an unregulated banking system. In Section 6, we discuss the role of a regulator, where we formulate the regulator's intervention and characterize the equilibrium allocations in the case of a regulated banking system. Section 7 concludes.

2. MODEL

Time $t = 0, 1, 2, ...$ is discrete, and the horizon is infinite. Each period is divided into two subperiods: day and night. There are three physical commodities: a daytime good, a nighttime good, and a capital good. The capital good can be perfectly stored from the day to the night subperiod. It depreciates completely if stored until the following date or if used in the production process. The daytime good can be either immediately consumed or stored to be consumed in the following day subperiod. The storage technology returns $\beta^{-1} > 1$ units of the daytime good at date $t + 1$ for each unit of the daytime good invested at date

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t. Finally, the nighttime good cannot be stored and must be immediately consumed.

There are four types of agents: buyers, sellers, entrepreneurs, and bankers. There is a $[0, 1]$ continuum of each type. Buyers, sellers, and bankers are infinitely lived. Entrepreneurs live for two periods only. At each date t, entrepreneurs are born in the day subperiod and live until the night subperiod of date $t + 1$.

Buyers and sellers want to consume and are able to produce in the day subperiod. Specifically, they produce the daytime good using a divisible technology that returns one unit of the good for each unit of effort they put in. In the night subperiod, buyers are consumers and sellers are producers. Only sellers are endowed with the technology to produce the nighttime good. Such a technology requires k units of the capital good and n units of effort to produce $F(k, n)$ units of the nighttime good. Assume that $F : \mathbb{R}_+^2 \to \mathbb{R}_+$ is twice continuously differentiable, increasing in both arguments, and strictly concave, with $F(0, n) = 0$ for all $n \geq 0$ and $F(k, 0) = 0$ for all $k \geq 0$.

Entrepreneurs specialize in the production of the capital good. Each entrepreneur is endowed with a nontradable, indivisible investment project at birth. Each project requires the investment of *exactly e* units of the daytime good at date t to produce $\gamma \hat{k}$ units of the capital good at the beginning of date $t + 1$, where $\hat{k} > 0$ is a constant. Entrepreneurs are heterogeneous with respect to their productivity levels γ. Specifically, the function $G(\gamma)$ describes the distribution of the productivity levels γ across the population of entrepreneurs. Suppose that $\gamma \in [0, \bar{\gamma}]$ and that there exists a density function $g(\gamma)$.

This means that the daytime good can be either immediately consumed or used in other activities. If it is not consumed at date t, it can be converted into the capital good at date $t + 1$ or can be stored as an inventory.

Bankers are endowed with a technology that allows them to make their actions publicly observable at no cost. This means that if a banker decides to make his actions publicly observable, other agents can keep track of his balance sheet and income statement at each date. Bankers are also endowed with a technology to monitor and enforce contracts at no cost.

We now explicitly describe preferences. Let $x_t^b \in \mathbb{R}$ denote a buyer's daytime net con-

sumption ($x_t^b < 0$ means that the buyer is a net producer), and let $q_t^b \in \mathbb{R}_+$ denote his nighttime consumption. His preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left[x_t^b + u\left(q_t^b\right) \right],$$

where $\beta \in (0,1)$. The function $u : \mathbb{R}_+ \to \mathbb{R}$ is twice continuously differentiable, increasing, and strictly concave, with $u(0) = 0$ and $u'(0) = \infty$. Let $x_t^s \in \mathbb{R}$ denote a seller's daytime net consumption, and let $n_t^s \in \mathbb{R}_+$ denote his nighttime effort level. His preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left[x_t^s - c\left(n_t^s\right) \right],$$

where $c : \mathbb{R}_+ \to \mathbb{R}_+$ is twice continuously differentiable, increasing, and convex. Let $x_t \in \mathbb{R}_+$ denote a banker's daytime consumption. Each banker has preferences given by

$$\sum_{t=0}^{\infty} \beta^t x_t.$$

Finally, an entrepreneur born at date t wants to consume only at date $t+1$. Specifically, each entrepreneur born at date t derives utility x_{t+1}^e if his daytime consumption at date $t+1$ is $x_{t+1}^e \in \mathbb{R}_+$.

Throughout the paper, unless otherwise stated, we assume that all types of agents lack any commitment. We also assume that a buyer cannot carry the capital good to a seller's location. Sellers, who actually need the capital good to produce the consumption good at night, have their previously acquired capital physically fixed in place at production sites. An immediate implication of this assumption is that capital cannot be used as a means of payment in the night market. See Aruoba, Waller, and Wright (2011). Finally, we assume that the property titles on the storage technology are nontradable. Thus, the storage technology corresponds to the concept of illiquid capital in Lagos and Rocheteau (2008).

In the day subperiod, there is a perfectly competitive (Walrasian) market in which agents trade the consumption good and the capital good. In the night subperiod, only buyers and sellers trade. Following the literature, we refer to this night market as the decentralized market. For simplicity, we will use competitive pricing to determine the terms of trade

in this market. Still, a medium of exchange remains essential as long as we maintain the (intertemporal) double coincidence problem and anonymity; see Rocheteau and Wright (2005) for a discussion.

3. DISCUSSION OF THE MODEL

In this section, we explain how the pieces of the model fit together. To generate a demand for liquidity, we build on Lagos and Wright (2005).[4] In the decentralized night market, the absence of commitment and recordkeeping implies that a buyer and a seller can trade only if a medium of exchange is made available. The bankers are well positioned to issue such an instrument because they can make their actions publicly observable. Specifically, they are able to issue securities that can be used as a medium of exchange as long as people believe that they will be willing to redeem them at least in some states of the world at a future date.

Note that the bankers also lack commitment. So we need to have some sort of punishment for default to guarantee that they make good on their promises, a necessary condition for their private liabilities to circulate. As in Cavalcanti, Erosa, and Temzelides (1999) and Cavalcanti and Wallace (1999a, 1999b), we assume the existence of a mechanism that guarantees that the bankers who renege on their promises be punished. Precisely, we assume that a banker who defaults on his liabilities can no longer have his actions publicly observable. Moreover, any assets he holds when he defaults will be seized. This means that a defaulter will loose his "bank charter" (the ability to issue circulating liabilities).

In this respect, the availability of public knowledge of the banker's actions is crucial for allowing people to identify the states of the world in which the banker will be willing to repay his creditors. In the decentralized night market, a seller does not trust a buyer's IOU because he knows that the latter cannot be punished in case of default. But a seller may accept a banker's IOU because the banker can be punished if he fails to redeem his IOUs.

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[4]An alternative tractable framework that also creates a role for a medium of exchange is the large household model in Shi (1997).

Thus, there will be some states of the world in which the banker will be willing to redeem his notes at par, and everybody knows in which states this will happen. Figure 1 shows how a banker's note will circulate in the economy.

Our framework also emphasizes the role of bankers as financial intermediaries. The entrepreneurs need external funds to finance their investment projects. Because entrepreneurs lack commitment, buyers and sellers are not willing to directly lend to entrepreneurs, so a market for one-period consumption loans will not be possible. Because the banker possesses the technology that allows him to monitor and enforce contracts, he will be able to collect a repayment from an entrepreneur. Thus, at each date the bankers borrow from savers and lend to entrepreneurs. The profit they make from these activities finances their own consumption stream. Figure 2 shows the sequence of events in the interactions between a banker and an entrepreneur.

Note that we can interpret our model as a two-sector economy in which capital accumulation and production in one sector (nighttime good) require external finance and banking arrangements, whereas capital accumulation and production in the other sector (daytime good) are frictionless. The bankers in our framework combine two important characteristics of real-world bankers: intermediation of funds and liquidity creation. As intermediaries, bankers play a crucial role in the allocation of funds to finance capital formation in some sectors of the economy. As money issuers, they provide liquid assets that facilitate exchange between households and firms.

4. EFFICIENT ALLOCATIONS

In this section, we formulate and solve the problem of a planner who has access to the technology that allows him to monitor agents and enforce all transfers at zero cost. We assume that the planner treats entrepreneurs of the same generation equally. Thus, he will assign the same consumption level to each member of a given generation. Also, we assume throughout the paper that the planner treats buyers and sellers as being members of the same household. To be precise, a household consists of one buyer and one seller, so the

relevant welfare measure is the sum of the utility of one buyer and one seller (i.e., the utility of a household). Given these assumptions, an efficient allocation is obtained in the usual way: Given some minimum utility level U_t^e assigned to each entrepreneur of generation t, for all generations, and some minimum utility level U assigned to each banker at date $t = 0$, an efficient allocation maximizes the discounted lifetime utility of a household subject to the participation and resource constraints.

It should be clear that the planner will fund only the entrepreneurs who are sufficiently productive. This means that each entrepreneur whose productivity level γ is greater than or equal to a specific marginal type $\gamma_t^p \in [0, \bar{\gamma}]$ will receive e units of the daytime good to undertake his project at date t, whereas the types $\gamma \in [0, \gamma_t^p)$ will not operate their projects. We refer to the type γ_t^p as the date-t marginal entrepreneur. Thus, the planner's problem consists of choosing an allocation

$$\left\{ x_t^b, x_t^s, x_t, x_t^e, q_t, n_t, i_t, k_{t+1}, \gamma_t^p \right\}_{t=0}^{\infty}$$

to maximize the expected discounted utility of the representative household

$$\sum_{t=0}^{\infty} \beta^t \left[x_t^b + u(q_t) \right] + \sum_{t=0}^{\infty} \beta^t \left[x_t^s - c(n_t) \right], \tag{1}$$

subject to the daytime resource constraint

$$x_t^b + x_t^s + x_t^e + x_t + i_t = 0, \tag{2}$$

the nighttime resource constraint

$$q_t = F(k_t, n_t), \tag{3}$$

the restrictions imposed by the investment technology

$$k_{t+1} = \hat{k} \int_{\gamma_t^p}^{\bar{\gamma}} \gamma g(\gamma) \, d\gamma, \tag{4}$$

$$i_t = e \left[1 - G(\gamma_t^p) \right], \tag{5}$$

the entrepreneurs' participation constraints

$$x_t^e \geq U_{t-1}^e, \tag{6}$$

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and the bankers' participation constraint

$$\sum_{t=0}^{\infty} \beta^t x_t \geq U, \tag{7}$$

taking the initial capital stock k_0 and the required utility levels $\{U_{t-1}^e\}_{t=0}^{\infty}$ and U as given. Notice that any Pareto optimal allocation solves the problem described above for a particular choice of required utility levels $\{U_{t-1}^e\}_{t=0}^{\infty}$ and U, and that any solution to the problem above is a Pareto optimal allocation.

Let $k\left(\gamma_t^p\right) \equiv \hat{k} \int_{\gamma_t^p}^{\bar{\gamma}} \gamma g\left(\gamma\right) d\gamma$ denote the aggregate capital stock available at the beginning of date $t + 1$ as a function of the date-t marginal entrepreneur γ_t^p. Since (6) and (7) hold with equality at the optimum, we can rewrite the planner's problem as follows:

$$\max_{\{\gamma_t^p, n_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[eG\left(\gamma_t^p\right) + u\left[F\left(k\left(\gamma_{t-1}^p\right), n_t\right)\right] - c\left(n_t\right)\right],$$

taking the initial capital stock $k_0 = k\left(\gamma_{-1}^p\right) > 0$ as given. The first-order conditions are given by

$$\beta u'\left[F\left(k\left(\gamma_t^p\right), n_{t+1}\right)\right] F_k\left(k\left(\gamma_t^p\right), n_{t+1}\right) \hat{k}\gamma_t^p = e, \tag{8}$$

$$u'\left[F\left(k\left(\gamma_{t-1}^p\right), n_t\right)\right] F_n\left(k\left(\gamma_{t-1}^p\right), n_t\right) = c'\left(n_t\right), \tag{9}$$

for all $t \geq 0$. To marginally increase each buyer's consumption within a household at date $t+1$ without changing the effort level that each seller within a household exerts at date $t+1$, the planner needs to give up e units of the daytime good at date t at the margin to increase the amount of the capital good available for production at date $t + 1$. The left-hand side in (8) gives the marginal benefit of an extra unit of capital at date $t + 1$, whereas the right-hand side gives the marginal resource cost at date t. Similarly, to marginally increase each buyer's consumption within a household at date t given a predetermined stock of capital at the beginning of date t, the planner needs to instruct each seller within a household to exert more effort in the night subperiod. Condition (9) guarantees that the marginal disutility of effort equals the marginal benefit of consuming an extra unit of the nighttime good.

A stationary solution to the planner's problem involves $\gamma_t^p = \gamma^*$ and $n_t = n^*$ for all $t \geq 0$, with γ^* and n^* satisfying

$$\beta u'\left[F\left(k\left(\gamma^*\right), n^*\right)\right] F_k\left(k\left(\gamma^*\right), n^*\right) \hat{k}\gamma^* = e, \tag{10}$$

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$$u' \left[F \left(k \left(\gamma^* \right), n^* \right) \right] F_n \left(k \left(\gamma^* \right), n^* \right) = c' \left(n^* \right). \tag{11}$$

We also need the initial capital stock to be equal to $k \left(\gamma^* \right)$. In the Appendix, we show the existence and uniqueness of a stationary solution to the planner's problem for at least some specifications of preferences and technologies.

5. UNREGULATED BANK LENDING

In this section, we describe the equilibrium outcome of an economy without intervention in bank lending practices. In this economy, the supply of liquid assets is completely endogenous: The bankers issue private debt that can be used as a medium of exchange so that the aggregate supply of liquid assets depends entirely on the banking sector's willingness to expand its balance sheet.

To finance his investments at date t, the banker raises funds by selling one-period securities to buyers. Then, he uses these funds to make loans to entrepreneurs or to acquire property titles on the storage technology, or both. At date $t + 1$, he collects the proceeds from his investments and repays his creditors, consuming or reinvesting the remaining profits. Specifically, a note issued by a banker at date t gives him one unit of the daytime good and is a promise to repay one unit of the daytime good at date $t + 1$ to the note holder. Each banker has a technology that allows him to create perfectly divisible notes at zero cost. Notes issued by one banker are perfectly distinguishable from those issued by any other banker so that counterfeiting is not a problem. Note that people are willing to hold these notes for two reasons: They may offer them a return greater than that paid on any other asset, and they can be used as a means of payment in the decentralized night market.

Throughout the paper, we restrict attention to symmetric equilibria in which all notes trade at the same price. This means that the notes issued by any pair of bankers are perfect substitutes (as long as people believe both bankers will be willing to redeem them at par). Let ϕ_t denote the common price of a newly issued note in terms of the date-t daytime good so that $1/\phi_t$ gives the real return for anyone who holds a note from date t to date $t + 1$. Every agent in the economy takes the sequence of prices $\{\phi_t\}_{t=0}^{\infty}$ as given when making his

individual decisions.

The goal of this section is to characterize equilibrium allocations in the absence of intervention in bank lending. Because the bankers cannot commit to repay creditors, we need to assume the existence of a regulator whose exclusive role will be to punish bankers who default on their liabilities. Specifically, each banker's trading history enters the public record if and only if the regulator grants him a "bank charter." This means that a bank charter is necessary for the banker's ability to issue notes, and the regulator can punish any banker who defaults on his liabilities by revoking his bank charter and garnishing his assets. In this section, we assume that the *only* role of the regulator is to grant a bank charter to *every* banker at date $t = 0$ and to revoke it if necessary.

5.1. Market for Bank Loans

In the market for bank loans, the bankers make one-period bilateral loans to entrepreneurs on a competitive basis. Therefore, the return on any bank loan must be equal to the return to storage so that the entrepreneurs will capture all surplus from intermediation. Let $r_t(\gamma)$ denote the interest rate offered to a type-γ entrepreneur, which is the interest rate that will prevail in the submarket for bank loans to type-γ entrepreneurs. Then, in the absence of intervention, we must have

$$1 + r_t(\gamma) = \beta^{-1}$$

for any type γ who is funded. Given the interest rate $r_t(\gamma)$, only those entrepreneurs who are sufficiently productive will be able to obtain a loan. A type-γ entrepreneur has a profitable project if and only if $\rho_{t+1}\hat{k}\gamma - e\beta^{-1} \geq 0$, where ρ_t denotes the price of one unit of the capital good in terms of the date-t daytime good. Note that $\rho_{t+1}\hat{k}\gamma$ gives the value of a type-γ entrepreneur's project in terms of the date-$(t+1)$ daytime good, whereas $e\beta^{-1}$ gives the payoff, also in terms of the date-$(t+1)$ daytime good, that the lender would obtain if he invested in the storage technology. In other words, a type-γ entrepreneur has a profitable project if and only if the surplus from intermediation is positive.

Given the relative price of capital ρ_{t+1}, any type-γ entrepreneur for whom

$$\rho_{t+1}\hat{k}\gamma \geq e\beta^{-1} \tag{12}$$

will be funded at date t. Thus, given ρ_{t+1}, we can define the date-t marginal entrepreneur γ_t^m as the type satisfying

$$\gamma_t^m = \frac{e}{\beta\rho_{t+1}\hat{k}}. \tag{13}$$

This means that any entrepreneur indexed by $\gamma \in [0, \gamma_t^m]$ will not be funded, whereas the types $\gamma \in [\gamma_t^m, \bar{\gamma}]$ will be able to obtain funds.

Given the choice of the date-t marginal entrepreneur, the aggregate loan amount at date t is given by

$$\ell_t = e\left[1 - G\left(\gamma_t^m\right)\right].$$

Taking ρ_{t+1} as given, if the bankers are able to raise enough funds, they will devote the amount ℓ_t to fund entrepreneurs' projects at date t, and the aggregate amount of the capital good available at date $t+1$ will be given by

$$k_{t+1} = \hat{k}\int_{\gamma_t^m}^{\bar{\gamma}} \gamma g\left(\gamma\right) d\gamma \equiv k\left(\gamma_t^m\right). \tag{14}$$

5.2. Buyer's Problem

Let $w_t^b\left(a\right)$ denote the value function for a buyer with a portfolio of a notes at the beginning of the day market, and let $v_t^b\left(k, a\right)$ denote the value function for a buyer with a portfolio of k units of capital and a notes at the beginning of the night market. The Bellman equation for a buyer in the day subperiod is given by

$$w_t^b\left(a\right) = \max_{(x,k',a')\in\mathbb{R}\times\mathbb{R}_+^2} \left[x + v_t^b\left(k', a'\right)\right],$$

subject to the budget constraint

$$x + \rho_t k' + \phi_t a' = a.$$

Here k' denotes the amount of capital that the buyer accumulates at the end of the day market, and a' denotes his choice of note holdings at the end of the day market. Because of

quasi-linear preferences, the value $w_t^b(a)$ is an affine function of the form $w_t^b(a) = a + w_t^b(0)$, with the intercept $w_t^b(0)$ given by

$$w_t^b(0) = \max_{(k',a') \in \mathbb{R}_+^2} \left[-\rho_t k' - \phi_t a' + v_t^b(k', a') \right]. \tag{15}$$

Let p_{t+1} denote the price of one unit of the date-t nighttime good in terms of the date-$(t+1)$ daytime good. The Bellman equation for a buyer with a portfolio of k' units of capital and a' notes in the night market is given by

$$v_t^b(k', a') = \max_{q \in \mathbb{R}_+} \left[u(q) + \beta w_{t+1}^b(a' - p_{t+1}q) \right], \tag{16}$$

subject to the liquidity constraint

$$p_{t+1}q \le a'. \tag{17}$$

Using the fact that $w_t^b(a)$ is an affine function, we can rewrite the Bellman equation (16) as follows:

$$v_t^b(k', a') = \max_{q \in \mathbb{R}_+} [u(q) - \beta p_{t+1}q] + \beta a' + \beta w_{t+1}^b(0).$$

First, notice that there is no benefit of accumulating capital (capital cannot be used as a medium of exchange and fully depreciates from one period to the next). Therefore, the buyer optimally chooses $k' = 0$.

The liquidity constraint (17) may either bind or not, depending on the buyer's note holdings. In particular, notice that

$$\frac{\partial v_t^b}{\partial a}(k', a') = \begin{cases} \frac{1}{p_{t+1}} u'\left(\frac{a'}{p_{t+1}}\right) & \text{if } a' < p_{t+1}\hat{q}(p_{t+1}); \\ \\ \beta & \text{if } a' > p_{t+1}\hat{q}(p_{t+1}); \end{cases}$$

where $\hat{q}(p_{t+1}) = (u')^{-1}(\beta p_{t+1})$. If the liquidity constraint does not bind, then the marginal utility of an extra note equals β, which is simply the discounted value of the payoff of one unit of the daytime good at date $t+1$. If the liquidity constraint binds, then the marginal utility of an extra note is greater than β. In this case, the notes offer a liquidity premium. Since the buyer can always use the storage technology, he will hold notes if and only if he obtains a liquidity premium or the return on notes is greater than the return to storage.

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The first-order condition for the optimal choice of note holdings on the right-hand side of (15) is given by

$$-\phi_t + \frac{\partial v_t^b}{\partial a}\left(k', a'\right) \leq 0,$$

with equality if $a' > 0$. If $\phi_t > \beta$, then the optimal choice of note holdings will be given by

$$u'\left(\frac{a'}{p_{t+1}}\right) = \phi_t p_{t+1}, \tag{18}$$

so that notes offer a liquidity premium. Because of quasi-linear preferences, all buyers choose to hold the same quantity of notes at the end of the day market. Thus, condition (18) gives the aggregate demand for notes as a function of the relative price of the nighttime good p_{t+1} and the price of notes ϕ_t.

5.3. Seller's Problem

Let $w_t^s(a)$ denote the value function for a seller with a portfolio of a notes at the beginning of the day market, and let $v_t^s(k, a)$ denote the value function for a seller with a portfolio of k units of capital and a notes at the beginning of the night market. The Bellman equation for a seller in the day market is given by

$$w_t^s(a) = \max_{(x, k', a') \in \mathbb{R} \times \mathbb{R}_+^2} \left[x + v_t^s\left(k', a'\right)\right],$$

subject to the budget constraint

$$x + \rho_t k' + \phi_t a' = a.$$

Here k' denotes the amount of capital that the seller accumulates at the end of the day market, and a' denotes his choice of note holdings at the end of the day market. Similarly, the value $w_t^s(a)$ is an affine function, $w_t^s(a) = a + w_t^s(0)$, with the intercept $w_t^s(0)$ given by

$$w_t^s(0) = \max_{(k', a') \in \mathbb{R}_+^2} \left[-\rho_t k' - \phi_t a' + v_t^s\left(k', a'\right)\right]. \tag{19}$$

The Bellman equation for a seller with a portfolio of k' units of capital and a' notes in the night market is given by

$$v_t^s\left(k', a'\right) = \max_{n \in \mathbb{R}_+} \left[-c(n) + \beta w_{t+1}^s\left(p_{t+1} F\left(k', n\right) + a'\right)\right]. \tag{20}$$

Using the fact that $w_t^s(a)$ is an affine function, we can rewrite the right-hand side of (20) as follows:

$$\max_{n \in \mathbb{R}_+} \left[-c(n) + \beta p_{t+1} F(k', n) \right] + \beta a' + \beta w_{t+1}^s(0).$$

The first-order condition for the optimal choice of nighttime effort is given by

$$c'(n) = \beta p_{t+1} F_n(k', n). \tag{21}$$

Because $(\partial v_t^s / \partial k)(k', a') = \beta p_{t+1} F_k(k', n)$, the first-order condition for the optimal choice of capital on the right-hand side of (19) is given by

$$\rho_t = \beta p_{t+1} F_k(k', n). \tag{22}$$

Thus, conditions (21) and (22) determine the demand for capital and the nighttime effort decision as a function of the relative price of the nighttime good p_{t+1} and the relative price of capital ρ_t. Combining (21) with (22), we obtain the following condition:

$$\frac{\rho_t}{c'(n)} = \frac{F_k(k', n)}{F_n(k', n)}. \tag{23}$$

Finally, the first-order condition for the optimal choice of note holdings is given by

$$-\phi_t + \beta \leq 0,$$

with equality if $a' > 0$. This means that the seller does not hold notes if $\phi_t > \beta$.

5.4. Banker's Problem

Now we describe the decision problem of a banker. Let $w_t(b_{t-1}, i_{t-1})$ denote the value function for a banker with debt b_{t-1} and assets i_{t-1} at the beginning of date t. The banker's assets at the beginning of date t consist of loans to entrepreneurs made at date $t-1$ and titles on the storage technology acquired at date $t-1$, whereas the banker's debt refers to the amount of notes issued at date $t-1$. As we have seen, the marginal return on the banker's assets is given by β^{-1} in the absence of intervention, whether he invests in the

entrepreneurs' projects or in the storage technology. Thus, the banker's decision problem can be formulated as follows:

$$w_t\left(b_{t-1}, i_{t-1}\right) = \max_{(x_t, i_t, b_t) \in \mathbb{R}^3_+} \left[x_t + \beta w_{t+1}\left(b_t, i_t\right)\right] \qquad (24)$$

subject to the daytime budget constraint

$$i_t + x_t + b_{t-1} = \beta^{-1} i_{t-1} + \phi_t b_t$$

and the debt limit

$$b_t \leq \bar{B}_t.$$

Here i_t denotes the amount of resources (units of the daytime good) that the banker decides to invest at date t. In other words, i_t gives the banker's assets at the beginning of date $t + 1$. When making his investment decisions at each date, the banker takes as given the sequence of debt limits $\{\bar{B}_t\}_{t=0}^{\infty}$, the marginal return on his assets β^{-1}, and the sequence of prices $\{\phi_t\}_{t=0}^{\infty}$.

If $\phi_t > \beta$, then the banker finds it optimal to borrow up to his debt limit, i.e., he will choose $b_t = \bar{B}_t$. Because the return paid on his notes (his cost of funds) is lower than the return on his assets, he makes a positive profit by borrowing and investing the proceeds in the storage technology. Note also that, because the return on his assets equals his rate of time preference, he is indifferent between immediately consuming and reinvesting the proceeds from his previous profits (his retained earnings). Therefore, a solution to the banker's optimization problem is $i_t = \phi_t \bar{B}_t$, which means that the banker invests all funds he has borrowed at date t but does not use his own funds. Thus, the balance sheet of a typical banker will have no equity, only debt. In this case, the banker's consumption at date t is simply given by

$$x_t = \bar{B}_{t-1}\left(\beta^{-1}\phi_{t-1} - 1\right).$$

We refer to the bank charter *value* as the lifetime utility associated with a particular choice of the return on the banker's assets, the sequence of debt limits, and the sequence of prices for the banker's liabilities. At each date t, the bank charter value is given by

$$w_t\left(\bar{B}_{t-1}, \phi_{t-1}\bar{B}_{t-1}\right) = \sum_{\tau=t}^{\infty} \beta^{\tau-t} \bar{B}_{\tau-1}\left(\beta^{-1}\phi_{\tau-1} - 1\right).$$

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Perfect competition in the market for bank loans implies that the return on the banker's assets is the smallest possible, β^{-1} at each date, which lowers the bank charter value. As we will see, the introduction of banking regulation will play a crucial role in increasing the return on the banker's assets, thus raising the bank charter value.

5.5. Aggregate Note Holdings

Let a_t denote the date-t aggregate note holdings. For any price $\phi_t > \beta$, the liquidity constraint (17) is binding, in which case the value of the notes in circulation must equal the value of the aggregate production in the night market,

$$a_t = p_{t+1} F\left(k\left(\gamma_{t-1}^m\right), n_t\right). \tag{25}$$

This is the equivalent of the equation for the quantity theory of money.[5] Note that the aggregate production depends on the current capital stock and the effort level that each seller is willing to exert in order to produce the nighttime good. Combining (18) with (25), we obtain

$$u'\left[F\left(k\left(\gamma_{t-1}^m\right), n_t\right)\right] = \phi_t p_{t+1}.$$

Using (21) to substitute for p_{t+1}, we get the following equilibrium condition:

$$u'\left[F\left(k\left(\gamma_{t-1}^m\right), n_t\right)\right] = \frac{\phi_t}{\beta} \frac{c'\left(n_t\right)}{F_n\left(k\left(\gamma_{t-1}^m\right), n_t\right)}. \tag{26}$$

This condition determines the equilibrium nighttime effort decision, given the predetermined capital stock. The price of notes ϕ_t influences this decision in the following way: A lower price for the bankers' notes increases the return on these notes and the buyer's expenditure decision, raising the relative price p_{t+1} and inducing each seller to exert more effort.

As we have seen, the choice of the date-t marginal entrepreneur is given by (13). Using (23) to substitute for ρ_{t+1}, we obtain the following equilibrium condition:

$$\beta u'\left[F\left(k\left(\gamma_t^m\right), n_{t+1}\right)\right] F_k\left(k\left(\gamma_t^m\right), n_{t+1}\right) \hat{k}\gamma_t^m = e\frac{\phi_{t+1}}{\beta}. \tag{27}$$

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[5]The difference from the quantity theory is that both sides of (25) are a function of ϕ_t, as we will show below, whereas in the quantity theory the money supply is fixed by the government.

This condition determines the equilibrium capital accumulation decision at date t given the nighttime effort decision at date $t+1$. Notice that a lower anticipated value for ϕ_{t+1} results in a larger capital stock at date $t+1$, holding n_{t+1} constant.

We can use (26) and (27) to implicitly define the functions $\gamma_{t-1}^m = \gamma^m(\phi_t)$ and $n_t = n(\phi_t)$. Using these functions, we can define the aggregate production of the nighttime good by $q(\phi_t) = F[k(\gamma^m(\phi_t)), n(\phi_t)]$. Then, the aggregate note holdings as a function of the price ϕ_t are given by

$$a(\phi_t) = \frac{u'[q(\phi_t)]\, q(\phi_t)}{\phi_t}. \tag{28}$$

5.6. Equilibrium

To define an equilibrium, we need to specify the sequence of debt limits $\{\bar{B}_t\}_{t=0}^{\infty}$ in such a way that the bankers are willing to supply the amount of notes other agents demand and are willing to fully repay their creditors (note holders). We take two steps to define a sequence of debt limits satisfying these two conditions. First, for any given sequence of prices $\{\phi_t\}_{t=0}^{\infty}$, we set

$$\bar{B}_t = a(\phi_t) \tag{29}$$

at each date t. This condition guarantees that each banker is willing to supply the amount of notes in (28) at the price ϕ_t. Then, given this choice for the individual debt limits, we need to verify whether a particular choice for the price sequence $\{\phi_t\}_{t=0}^{\infty}$ implies that each banker does not want to renege on his liabilities at any date. As we have seen, a banker who reneges on his liabilities will lose his bank charter, in which case he will no longer be able to issue notes. Moreover, he will have his assets seized to repay creditors. Thus, a particular price sequence $\{\phi_t\}_{t=0}^{\infty}$ is consistent with the solvency of each banker if and only if

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} a(\phi_{\tau-1})\left(\beta^{-1}\phi_{\tau-1} - 1\right) \geq a(\phi_{t-1})\left(\beta^{-1}\phi_{t-1} - 1\right) + \phi_t a(\phi_t)$$

holds at each date t. As in Alvarez and Jermann (2000), these solvency constraints allow the banker to borrow as much as possible without inducing him to default on his liabilities. The left-hand side gives the beginning-of-period continuation value. The right-hand side

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gives the current payoff the banker gets if he decides *not* to invest the resources he has borrowed at date t. In this case, he can increase his current consumption by the amount $a(\phi_t)\phi_t$, but he will permanently lose his note-issuing privileges at date $t+1$. We can rewrite the solvency constraints above as follows:

$$-\phi_t a(\phi_t) + \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} a(\phi_{\tau-1}) \left(\beta^{-1}\phi_{\tau-1} - 1\right) \geq 0. \tag{30}$$

Before we formally define an equilibrium, note that, in the market for bank loans, only the submarkets indexed by $\gamma \geq \gamma^m(\phi_{t+1})$ will be open in equilibrium for any given price ϕ_{t+1}. And the interest rate $r_t(\gamma)$ that will prevail in each one of these submarkets is given by

$$1 + r_t(\gamma) = \beta^{-1} \tag{31}$$

for each type $\gamma \geq \gamma^m(\phi_{t+1})$. Finally, throughout the paper, we will restrict attention to non-autarkic equilibria for which the price sequence $\{\phi_t\}_{t=0}^{\infty}$ is bounded.

Definition 1 *An equilibrium is an array $\left\{\gamma_t^m, n_t, a_t, \bar{B}_t, \phi_t, r_t(\gamma)\right\}_{t=0}^{\infty}$ satisfying (26), (27), (28), (29), (30), and (31) at each date t.*

5.7. Welfare Properties

Now we want to show a very important property of any equilibrium allocation in the absence of intervention (even though we have not shown existence yet). If we compare equations (26) and (27) with the solution to the planner's problem, given by equations (8) and (9), we realize that setting $\phi_t = \beta$ at each date $t \geq 0$ makes the choices of the marginal entrepreneur and the nighttime effort level exactly the same as those in the planner's solution. Thus, $\phi_t = \beta$ for all $t \geq 0$ is a necessary condition for efficiency so that the optimal return on notes at each date should be given by β^{-1}. But condition (30) implies that the banker's solvency constraints are necessarily violated in this case, so we cannot have an equilibrium with $\phi_t = \beta$ for all $t \geq 0$. This means that any allocation that can be implemented in the absence of intervention in the market for bank loans is necessarily *not* Pareto optimal. We summarize these findings in the following proposition.

Proposition 2 *Any equilibrium allocation in the absence of intervention in bank lending practices is inefficient.*

Why are the bankers unwilling to supply the socially efficient amount of liquidity? As we have seen, the return on the banker's assets is the same as the return to storage and the rate of time preference. Because of perfect competition in the market for bank loans, there is no markup over the return to storage. This means that, to implement the optimal return on notes, we must drive the value of the bank charter to zero, which is inconsistent with the solvency constraints. As a result, there exists an upper bound on the return the bankers are willing to offer on their liabilities without inducing them to default. Any return above this bound makes the banker prefer to default on his liabilities. Such a bound exists because the return on the banking sector's assets is relatively low when there is perfect competition in bank lending.

The previous result is extremely useful to define the role of banking regulation because it says that any kind of regulation that seeks to restrict competition on the liability side of banks' balance sheets will result in an inefficient amount of liquidity, regardless of the kind of intervention that is carried out on the asset side. Regulation Q in the U.S. is an example of a regulatory measure aimed at increasing the bank charter value by restricting the return that banks are allowed to pay to their depositors. Our analysis thus predicts that these measures lead to an inefficient amount of bank liquidity creation.

5.8. Existence

To show existence, we will restrict attention to stationary equilibria for which the aggregate amount of notes issued at each date is constant over time. In the Appendix, we discuss the characterization of non-stationary equilibria. In the case of stationary allocations, we have $\phi_t = \phi$, $\gamma_t^m = \gamma^m$, $n_{t+1} = n$, $\bar{B}_t = \bar{B}$, and $a_t = a$ for all $t \geq 0$. We must also have $r_t(\gamma) = \beta^{-1} - 1$ for all t and for each active submarket γ.

Note that we can use (26) and (27) to define the choices of the marginal entrepreneur γ^m and the nighttime effort level n as a function of the price ϕ and then define the aggregate

note holdings in the same way. The following Lemma guarantees that, at any given price ϕ, the bankers will be able to raise enough resources from the sale of notes in order to finance all entrepreneurs whose projects have a positive surplus.

Lemma 3 *For any given $\phi > \beta$, we have $\phi a (\phi) > e\left[1 - G\left(\gamma^m (\phi)\right)\right]$.*

Finally, any stationary equilibrium must also satisfy the solvency constraints (30). In particular, a stationary solution satisfies these constraints if and only if

$$-\phi a (\phi) + \frac{\beta}{1 - \beta} a (\phi) \left(\beta^{-1}\phi - 1\right) \geq 0. \tag{32}$$

Because $a (\phi) > 0$ for any $\phi > \beta$, note that condition (32) holds if and only if

$$\phi \geq 1.$$

This means that the bankers are willing to supply any amount of notes for which the return on these notes is nonpositive. In other words, in the case of perfect competition in bank lending, the bankers need to charge for their liquidity services in order to be individually rational for them to redeem their notes at par. As we have seen, this result has a crucial implication for the welfare properties of equilibrium allocations. In particular, the nonpositive-return-on-notes property arises in the case of stationary allocations and implies that any stationary equilibrium is necessarily inefficient.

The following proposition establishes the existence of multiple equilibria for some specifications of preferences and technologies.

Proposition 4 *Suppose that $u (q) = (1 - \sigma)^{-1} \left(q^{1-\sigma} - 1\right)$, with $0 < \sigma < 1$, $c (n) = n$, and $F (k, n) = k^\alpha n^{1-\alpha}$, with $0 < \alpha < 1$. Suppose also that $g (\gamma) = 1$ for all $0 \leq \gamma \leq 1$ and $g (\gamma) = 0$ otherwise. Then, there exist infinitely many stationary equilibria, each of which is indexed by a price $\phi \geq 1$.*

Under these specifications of preferences and technologies, it is straightforward to show that the aggregate amount of notes $a (\phi)$ is strictly decreasing in ϕ. This means that the lack of intervention in the market for bank loans results in an inefficiently small amount

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of liquidity, in which case the price of notes will be too high to allow society to achieve a Pareto optimal allocation. This result suggests that we can mitigate the commitment problem only by increasing the return on the banker's assets, which can be accomplished through the creation of banking regulation.

Before we discuss banking regulation, it should be noted that the public provision of liquidity will not resolve the problem. Even though the government can provide liquidity to the private sector by issuing the same kind of liabilities as those issued by the private banking system, it cannot perform the same kind of intermediation as that performed by private banks (recall that the bankers are endowed with the technology to monitor and enforce loans). In fact, an intervention in which the government issues too many notes may result in a situation in which the bankers will not be able to raise enough resources to fund all profitable investment projects, which is clearly inefficient.

6. REGULATED BANK LENDING

In the previous section, we have shown that a system in which bank lending practices are left unregulated fails to deliver an efficient allocation of resources. Our results have also suggested that the way to achieve efficiency is by raising the bank charter value, which will allow us to "relax" the bankers' solvency constraints. In this section, we consider the possibility of regulating the market for bank loans in order to increase the return on the bankers' assets. We study banking regulation by considering the existence of a regulatory mechanism that sets the terms of trade in the market for bank loans by means of interest rate controls. So, the goal of the regulator is to find the minimum interest rates $r_t(\gamma)$ that imply a sufficiently high return on the bankers' assets to allow them to supply the socially efficient amount of notes. We start by describing a regulatory mechanism.

6.1. A Regulatory Mechanism

Here we describe the regulatory mechanism (or just mechanism for short) in the market for bank loans. The mechanism announces a return function $R_{t+1}(i_t)$ that promises to

deliver $i_t R_{t+1}(i_t)$ units of the date-$(t+1)$ daytime good if the banker decides to invest i_t units of the date-t daytime good. Then, the mechanism collects all funds raised from the bankers and allocates these resources to fund entrepreneurs' projects and acquire property titles on the storage technology. Because of the possibility of using the storage technology, the participation constraint for the banker is given by

$$R_{t+1}(i_t) \geq \beta^{-1}. \tag{33}$$

We will restrict attention to return functions of the form:

$$R_{t+1}(i_t) = \begin{cases} \beta^{-1} + \mu_t \text{ if } i_t < e\left[1 - G\left(\gamma^m\left(\phi_{t+1}\right)\right)\right], \\ \\ \beta^{-1} \text{ if } i_t \geq e\left[1 - G\left(\gamma^m\left(\phi_{t+1}\right)\right)\right], \end{cases} \tag{34}$$

where $\mu_t \geq 0$ denotes the date-t markup over the return to storage. At each date t, the mechanism chooses a portfolio that devotes the amount $e\left[1 - G\left(\gamma^m\left(\phi_{t+1}\right)\right)\right]$ to fund entrepreneurs indexed by $\gamma \geq \gamma^m\left(\phi_{t+1}\right)$ and invests the remaining resources in the storage technology. In this way, we can guarantee that any entrepreneur whose project has a positive surplus, given the price ϕ_{t+1}, is funded provided that the bankers have enough resources (raised from the sale of notes and their own retained earnings).

The mechanism also requires that the interest rate $r_t(\gamma)$ that the regulator wants to implement in each active submarket $\gamma \in [0, \bar{\gamma}]$ satisfies the type-γ entrepreneur's participation constraint:

$$\rho_{t+1}\hat{k}\gamma - [1 + r_t(\gamma)]e \geq 0. \tag{35}$$

Also, we must have that, given the interest rates $r_t(\gamma)$, the announced return function $R_{t+1}(i_t)$ satisfies

$$\left(\beta^{-1} + \mu_t\right)\left[1 - G\left(\gamma^m\left(\phi_{t+1}\right)\right)\right] \leq \int_{\gamma^m\left(\phi_{t+1}\right)}^{\bar{\gamma}} [1 + r_t(\gamma)]\, g(\gamma)\, d\gamma. \tag{36}$$

The left-hand side gives the amount of resources that the mechanism promised at date t to deliver at date $t+1$, and the right-hand side gives the total repayment received at date $t+1$ from the entrepreneurs who were funded at date t. Thus, condition (36) guarantees that

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the announced return function $R_{t+1}(i_t)$ is feasible. Finally, the participation constraints (33) and (35) imply that the interest rates $r_t(\gamma)$ can neither be too large nor too small:

$$\beta^{-1} \leq 1 + r_t(\gamma) \leq \beta^{-1} \frac{\gamma}{\gamma^m(\phi_{t+1})} \tag{37}$$

for each type $\gamma \geq \gamma^m(\phi_{t+1})$.

Definition 5 *Given a sequence of prices $\{\phi_t\}_{t=0}^{\infty}$, a mechanism consists of a sequence of markups $\{\mu_t\}_{t=0}^{\infty}$ and a sequence of interest rate functions $\{r_t(\gamma)\}_{t=0}^{\infty}$ satisfying (36) and (37) at each date.*

The mechanism specifies a sequence of markups and interest rates as a function of the sequence of prices $\{\phi_t\}_{t=0}^{\infty}$. We can think of this mechanism as a regulated mutual fund in which all bankers invest their resources. The rules of the fund then determine the amount of resources that will be devoted to finance entrepreneurs and to acquire property titles on the storage technology. The mutual fund then "hires" one banker to monitor and enforce all loans the fund has decided to make.[6] The choice of a particular mechanism then determines the marginal return on each unit invested in the fund. Different choices for the interest rates $r_t(\gamma)$ by the regulator will imply different values for the markup μ_t, determining the profitability of the fund.

Note that setting $\mu_t = 0$ at each date gives us the competitive solution that we have analyzed in the previous section. We now characterize equilibria for which the markup μ_t is positive at each date so that the bankers will be able to extract some of the surplus from the entrepreneurs. As a result, the average return on the bankers' assets will be higher.

The buyer's and seller's decision problems are the same as in the previous section. Thus, the aggregate note holdings are also given by (28). It remains now to describe the banker's decision problem given a regulated market for bank loans.

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[6]The regulator will prohibit bankers to make loans on their own. This means that bankers can either invest in the fund or in the storage technology.

6.2. Banker's Problem

The banker's decision problem can now be formulated as follows:

$$w_t \left(b_{t-1}, i_{t-1} \right) = \max_{(x_t, i_t, b_t) \in \mathbb{R}^3_+} \left[x_t + \beta w_{t+1} \left(b_t, i_t \right) \right]$$

subject to the daytime budget constraint

$$i_t + x_t + b_{t-1} = R_t \left(i_{t-1} \right) i_{t-1} + \phi_t b_t$$

and the debt limit

$$b_t \leq \bar{B}_t.$$

The return function $R_{t+1} \left(i_t \right)$ is given by (34) with $\mu_t > 0$. The banker takes the announced return functions as given when making his individual decisions, as well as the sequence of debt limits $\{\bar{B}_t\}_{t=0}^{\infty}$ and prices $\{\phi_t\}_{t=0}^{\infty}$.

As before, if $\phi_t > \beta$, then we have $b_t = \bar{B}_t$ at the optimum, so the banker finds it optimal to borrow up to his debt limit. If the banker has enough funds at date t, then the optimal choice for i_t is such that $i_t \geq e \left[1 - G \left(\gamma^m \left(\phi_{t+1} \right) \right) \right]$ because $\mu_t > 0$. If the investment amount i_t is lower than $e \left[1 - G \left(\gamma^m \left(\phi_{t+1} \right) \right) \right]$, then the return to each incremental amount invested at date t is greater than the rate of time preference. In this case, the banker would be better off if he increased his investment at date t. If the investment at date t exceeds $e \left[1 - G \left(\gamma^m \left(\phi_{t+1} \right) \right) \right]$, then the return to each extra unit invested at date t equals the rate of time preference. In this case, the banker is indifferent between immediately consuming and investing one extra unit. This means that $i_t = \phi_t \bar{B}_t$ is part of a solution to the banker's decision problem provided that $\phi_t \bar{B}_t \geq e \left[1 - G \left(\gamma^m \left(\phi_{t+1} \right) \right) \right]$. We will later show that this will be the case in equilibrium.

6.3. Equilibrium

To construct an equilibrium, we follow the same steps as in the previous section. We need to find a sequence of debt limits that guarantees that the bankers are willing to supply the

amount of notes other people demand and are willing to fully repay their creditors at each date. The banker's solvency constraints are now given by

$$-\phi_t a\left(\phi_t\right) + \sum_{\tau=t+1}^{\infty} \beta^{\tau-t}\left[\Pi\left(\phi_{\tau-1}, \phi_\tau, \mu_{\tau-1}\right) - a\left(\phi_{\tau-1}\right)\right] \geq 0 \qquad (38)$$

at each date $t \geq 0$, where the date-t revenue $\Pi\left(\phi_{t-1}, \phi_t, \mu_{t-1}\right)$ is given by

$$\Pi\left(\phi_{t-1}, \phi_t, \mu_{t-1}\right) \equiv \mu_{t-1} e\left[1 - G\left(\gamma^m\left(\phi_t\right)\right)\right] + \beta^{-1}\phi_{t-1} a\left(\phi_{t-1}\right).$$

The solvency constraints (38) are similar to those that we have obtained in the previous section, except that now the banker's date-t revenue has increased by the amount $\mu_{t-1} e\left[1 - G\left(\gamma^m\left(\phi_t\right)\right)\right]$. The definition of an equilibrium is now straightforward.

Definition 6 *An equilibrium is an array* $\left\{\gamma_t^m, n_t, a_t, \bar{B}_t, \phi_t, r_t\left(\gamma\right), \mu_t\right\}_{t=0}^{\infty}$ *satisfying (26), (27), (28), (29), (36), (37), and (38) at each date t.*

6.4. Existence

To show existence, we will restrict attention to stationary equilibria in which the aggregate amount of notes issued at each date is constant over time. In the Appendix, we characterize non-stationary equilibria. First, consider a solution to the banker's decision problem when $\phi_t = \phi$ and $\bar{B}_t = a\left(\phi\right)$ at each date $t \geq 0$. In this case, we have $b_t = a\left(\phi\right)$ and $i_t = \phi a\left(\phi\right)$. Second, note that Lemma 3 guarantees that $\phi\bar{B} = \phi a\left(\phi\right) > e\left[1 - G\left(\gamma^m\left(\phi\right)\right)\right]$ for any $\phi > \beta$. Finally, we need to find the set of stationary prices ϕ for which the solvency constraints hold. Given a stationary markup $\mu > 0$, any price ϕ satisfying

$$-\phi a\left(\phi\right) + \frac{\beta}{1-\beta}\left[\hat{\Pi}\left(\phi, \mu\right) - a\left(\phi\right)\right] \geq 0 \qquad (39)$$

implies that the repayment of creditors is individually rational for each banker. Here the value $\hat{\Pi}\left(\phi, \mu\right)$ is defined by

$$\hat{\Pi}\left(\phi, \mu\right) \equiv \mu e\left[1 - G\left(\gamma^m\left(\phi\right)\right)\right] + \phi a\left(\phi\right)\beta^{-1},$$

which gives the banker's revenue at each date as a function of the price ϕ and the markup μ.

The markup μ must satisfy the following condition:

$$\beta^{-1} < \beta^{-1} + \mu \leq \beta^{-1} \frac{\int_{\gamma^m(\phi)}^{\bar{\gamma}} \gamma g(\gamma) \, d\gamma}{\gamma^m(\phi) [1 - G(\gamma^m(\phi))]} \equiv R(\phi). \tag{40}$$

Here $R(\phi)$ gives the average return on the banking sector's loan portfolio if the mechanism is such that, for each type $\gamma \geq \gamma^m(\phi)$, the interest rate $r(\gamma)$ makes the type-γ entrepreneur's participation constraint hold with equality. This interest rate is given by

$$1 + r(\gamma) = \beta^{-1} \frac{\gamma}{\gamma^m(\phi)}. \tag{41}$$

This means that the average return $\beta^{-1} + \mu$ on the banking sector's loan portfolio can range from the competitive return β^{-1} to the monopolist return $R(\phi)$, depending on the regulatory mechanism.

Indeed, given a particular choice of the interest rates $\{r(\gamma)\}_{\gamma \geq \gamma^m(\phi)}$, the markup μ will be given by

$$\mu = \frac{\int_{\gamma^m(\phi)}^{\bar{\gamma}} [1 + r(\gamma)] g(\gamma) \, d\gamma}{[1 - G(\gamma^m(\phi))]} - \beta^{-1}. \tag{42}$$

Thus, given any price ϕ, a stationary mechanism consists of a stationary markup μ and an interest rate function $\{r(\gamma)\}_{\gamma \geq \gamma^m(\phi)}$ satisfying (42) and

$$\beta^{-1} \leq 1 + r(\gamma) \leq \beta^{-1} \frac{\gamma}{\gamma^m(\phi)} \tag{43}$$

for each $\gamma \geq \gamma^m(\phi)$.

One immediate consequence of the existence of a positive markup is that the average return on the banker's assets is higher than the average return he gets in the case of perfect competition. Specifically, for any given ϕ, the banking sector's revenue exceeds the revenue obtained in the case of perfect competition by the amount $\mu e [1 - G(\gamma^m(\phi))]$. As a consequence, the set of stationary equilibrium prices must be larger than the one we obtain in the case of unregulated bank lending because a higher return on assets essentially relaxes the solvency constraints. The following proposition shows this result formally.

Proposition 7 *Suppose that $u(q) = (1 - \sigma)^{-1} (q^{1-\sigma} - 1)$, with $0 < \sigma < 1$, $c(n) = n$, and $F(k, n) = k^\alpha n^{1-\alpha}$, with $0 < \alpha < 1$. Suppose also that $g(\gamma) = 1$ for all $0 \leq \gamma \leq 1$ and*

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$g(\gamma) = 0$ *otherwise. Then, there is $\bar{\phi} < 1$ such that, for any $\phi \geq \bar{\phi}$, there exist infinitely many stationary equilibria for which $\mu > 0$, each of which is indexed by a price $\phi \geq \bar{\phi}$.*

With a positive markup, it is possible to have an equilibrium in which the return on notes is strictly positive. As should be expected, a positive markup raises the bank charter value, mitigating the commitment problem associated with the note-issuing privileges. As a result, there exist equilibria in which the bankers are willing to pay a strictly positive return on their notes. In other words, there exist equilibria in which the price of liquid assets is lower, and the aggregate supply of these assets is larger than those that we have obtained in the absence of regulation.

6.5. Welfare Properties

Now we turn to the welfare implications of having a regulated market for bank loans. In particular, we want to know whether the regulation of bank lending practices will allow us to implement the optimal return on notes.

Proposition 8 *If β is sufficiently close to one, then an equilibrium with $\phi_t = \beta$ for all $t \geq 0$ exists.*

For any μ sufficiently close to the upper bound, given by the monopolist markup ($\mu = R(\phi) - \beta^{-1}$), it is possible to have an equilibrium in which the return on notes equals the rate of time preference. In this case, we eliminate the opportunity cost of holding liquid assets, maximizing the surplus from trade in the decentralized night market. Because any other allocation that makes at least one entrepreneur better off necessarily makes a banker worse off, we conclude that setting $\phi_t = \beta$ for all $t \geq 0$ is both necessary and sufficient for efficiency.

The regulatory mechanism has a crucial impact on the efficiency of an equilibrium allocation. Without such a regulatory mechanism, bankers compete on the asset side of their balance sheets and can only get a positive charter value if they offer a low return on their liabilities. Therefore, the role of the regulator is to increase the bank charter value by

limiting competition in bank lending and consequently raising the return on assets. This is important because such an intervention will allow bankers to increase the return on their liabilities, thus favouring the provision of liquidity. As we have shown, the bankers are willing to supply the optimal amount of liquidity (i.e., the amount that results in a Pareto optimal allocation) only if the average return on their assets is sufficiently close to the return that a monopolist banker would obtain.

It is important to notice that a monopolist banker would not choose an efficient allocation because he would certainly not choose the price of his liabilities to be $\phi_t = \beta$ at each date. We have to keep in mind that we have assumed a perfectly competitive market for the bankers' notes, which is crucial for the efficiency of the system. The fact that a monopolist would obtain a high return on his assets does not mean that he would be willing to offer the socially efficient return on his liabilities. To obtain efficiency, a monopolist banker would have to be regulated as well.

An important corollary that follows immediately is that narrow banking cannot provide the efficient amount of liquidity. Indeed, narrow banking does not offer any means to increase the return on the banking sector's assets.

Even though we have assumed that the regulator directly chooses the interest rates in the market for bank loans, we can also interpret the regulator's intervention as the imposition of entry restrictions for some submarkets, where each submarket is indexed by the entrepreneur's type γ. Restricting entry in some submarkets would increase the degree of concentration there, which would allow the bankers permitted to make loans in these submarkets to capture some of the surplus from intermediation.

7. CONCLUSION

In this paper, we have emphasized two distinct characteristics of banks: liquidity provision and intermediation of funds. We have characterized the interplay between these two activities in the context of a simple general equilibrium model. We have shown that taking this interplay into account is important when studying the welfare properties of equilibrium

allocations in the absence of commitment.

In particular, we have shown that a banking system in which bank lending practices are left unregulated is unable to supply an efficient amount of liquid assets. In the absence of intervention, the return on the bankers' assets will be relatively low because of competition in the market for bank loans, reducing the bank charter value. This makes the option of defaulting on their liabilities relatively more attractive. Thus, the bankers will be willing to offer to pay only a low return on their liabilities, creating a cost for their liability holders (that they are willing to bear because these liabilities provide them with a transaction service). For this reason, any equilibrium allocation in the absence of intervention is necessarily inefficient.

In view of this inefficiency, we have considered the possibility of regulating bank lending practices. In particular, we have characterized an optimal intervention in the market for bank loans. The way to induce bankers to supply an efficient amount of liquidity is to sufficiently raise the bank charter value by increasing the return on their assets. The regulator's goal is to ensure that the bankers get some of the surplus from the borrowers in the market for bank loans. This can be achieved by either imposing direct interest rate controls or restricting entry in some submarkets to increase the degree of concentration there.

So far, we have left aside the role of banks as risk transformers, whereby banks undertake risky investments but issue relatively safe debt, or alternatively whereby banks' assets are information sensitive while they issue information-insensitive liabilities (an idea that dates back to Gorton and Pennacchi, 1990, but has regained some traction recently; see Gorton, 2010). This is clearly an important issue that will impact the optimal provision of liquidity, and we leave it for future work.

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APPENDIX

A.1. Existence of a Unique Stationary Solution to the Planner's Problem

Here we show the existence of a unique stationary solution to the planner's problem for some specifications of preferences and technologies. In particular, we assume that $u(q) = (1-\sigma)^{-1}\left(q^{1-\sigma}-1\right)$, with $0 < \sigma < 1$, $c(n) = n$, and $F(k,n) = k^{\alpha}n^{1-\alpha}$, with $0 < \alpha < 1$. We also assume that $g(\gamma) = 1$ for any $0 \le \gamma \le 1$ and $g(\gamma) = 0$ otherwise. In this case, conditions (10) and (11) become

$$n = \chi \left[\gamma^{-1}\left(1-\gamma^2\right)^{1-\alpha+\alpha\sigma}\right]^{\frac{1}{(1-\alpha)(1-\sigma)}} \equiv Z(\gamma), \tag{44}$$

$$n = \lambda \left(1-\gamma^2\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \equiv H(\gamma), \tag{45}$$

respectively, where the constants χ and λ are defined as

$$\chi \equiv \left(\frac{1}{2}\right)^{\frac{1-\alpha+\alpha\sigma}{(1-\alpha)(1-\sigma)}} \left[\frac{e}{\alpha\beta\hat{k}^{\alpha(1-\sigma)}}\right]^{\frac{1}{(1-\alpha)(1-\sigma)}}, \tag{46}$$

$$\lambda \equiv \left[(1-\alpha)\left(\frac{\hat{k}}{2}\right)^{\alpha(1-\sigma)}\right]^{\frac{1}{\alpha+\sigma(1-\alpha)}}. \tag{47}$$

Notice that $Z'(\gamma) < 0$ for all $\gamma \in (0,1)$. Also, we have that $\lim_{\gamma\to 0} Z(\gamma) = +\infty$ and $\lim_{\gamma\to 1} Z(\gamma) = 0$. This means that the function $Z(\gamma)$ is strictly decreasing in the open interval $(0,1)$. With respect to the function $H(\gamma)$, we have that $H'(\gamma) < 0$ and $H''(\gamma) < 0$ for all $\gamma \in (0,1)$. Also, we have that $\lim_{\gamma\to 0} H(\gamma) = \lambda$ and $\lim_{\gamma\to 1} H(\gamma) = 0$. This means that the function $H(\gamma)$ is strictly decreasing and concave in the open interval $(0,1)$. This means that a unique interior solution exists.

A.2. Proof of Lemma 3

Note that we can rewrite the expression for the aggregate note holdings as follows:

$$\phi a(\phi) = \frac{e\phi F\left[k\left(\gamma^m(\phi)\right), n(\phi)\right]}{\beta^2 \hat{k}\gamma^m(\phi) F_k\left[k\left(\gamma^m(\phi)\right), n(\phi)\right]}.$$

For any price $\phi > \beta$, we have

$$
\begin{aligned}
\frac{e\phi F\left[k\left(\gamma^m\left(\phi\right)\right), n\left(\phi\right)\right]}{\beta^2 \hat{k}\gamma^m\left(\phi\right) F_k\left[k\left(\gamma^m\left(\phi\right)\right), n\left(\phi\right)\right]} \quad &> \quad \frac{e\phi k\left(\gamma^m\left(\phi\right)\right)}{\beta^2 \hat{k}\gamma^m\left(\phi\right)} \\
&> \quad \frac{ek\left(\gamma^m\left(\phi\right)\right)}{\beta \hat{k}\gamma^m\left(\phi\right)} \\
&> \quad \frac{ek\left(\gamma^m\left(\phi\right)\right)}{\hat{k}\gamma^m\left(\phi\right)} \\
&= \quad \frac{e\int_{\gamma^m\left(\phi\right)}^{\bar{\gamma}} \gamma g\left(\gamma\right) d\gamma}{\gamma^m\left(\phi\right)} \\
&> \quad e\left[1 - G\left(\gamma^m\left(\phi\right)\right)\right].
\end{aligned}
$$

Q.E.D.

A.3. Proof of Proposition 4

Suppose that $u\left(q\right) = \left(1-\sigma\right)^{-1}\left(q^{1-\sigma} - 1\right)$, with $0 < \sigma < 1$, $c\left(n\right) = n$, and $F\left(k, n\right) = k^\alpha n^{1-\alpha}$, with $0 < \alpha < 1$. Suppose also that $g\left(\gamma\right) = 1$ for any $0 \le \gamma \le 1$ and $g\left(\gamma\right) = 0$ otherwise. In this case, conditions (26) and (27) become

$$
n = \chi^e\left(\phi\right)\left[\gamma^{-1}\left(1-\gamma^2\right)^{1-\alpha+\alpha\sigma}\right]^{\frac{1}{(1-\alpha)(1-\sigma)}} \equiv Z^e\left(\gamma, \phi\right), \tag{48}
$$

$$
n = \lambda^e\left(\phi\right)\left(1-\gamma^2\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \equiv H^e\left(\gamma, \phi\right), \tag{49}
$$

respectively, where the functions $\chi^e\left(\phi\right)$ and $\lambda^e\left(\phi\right)$ are given by

$$
\chi^e\left(\phi\right) = \left(\frac{1}{2}\right)^{\frac{1-\alpha+\alpha\sigma}{(1-\alpha)(1-\sigma)}}\left[\frac{e\phi}{\alpha\beta^2\hat{k}^{\alpha(1-\sigma)}}\right]^{\frac{1}{(1-\alpha)(1-\sigma)}},
$$

$$
\lambda^e\left(\phi\right) = \left[\left(1-\alpha\right)\frac{\beta}{\phi}\left(\frac{\hat{k}}{2}\right)^{\alpha(1-\sigma)}\right]^{\frac{1}{\alpha+\sigma(1-\alpha)}}.
$$

Notice that $d\chi^e/d\phi > 0$, whereas $d\lambda^e/d\phi < 0$. Also, we have that $\chi^e\left(\beta\right) = \chi$ and $\lambda^e\left(\beta\right) = \lambda$, where χ and λ are given by (46) and (47), respectively. For any fixed $\phi > \beta$, we have that $\partial Z^e/\partial\gamma < 0$ for all $\gamma \in \left(0, 1\right)$, $\lim_{\gamma\to 0} Z^e\left(\gamma, \phi\right) = +\infty$, and $\lim_{\gamma\to 1} Z^e\left(\gamma, \phi\right) = 0$.

For any fixed $\phi > \beta$, we also have that $\partial H^e/\partial\gamma < 0$ and $\partial^2 H^e/\partial\gamma^2 < 0$ for all $\gamma \in (0,1)$, $\lim_{\gamma\to 0} H^e(\gamma,\phi) = \lambda^e(\phi)$, and $\lim_{\gamma\to 1} H^e(\gamma,\phi) = 0$. Thus, for any fixed $\phi > \beta$, a unique interior solution exists. Moreover, the Implicit Function Theorem implies that $d\gamma^m/d\phi > 0$ and $dn/d\phi < 0$.

As we have seen, condition (32) holds if and only if $\phi \geq 1$. Thus, for each price $\phi \in [1,\infty)$, there exists a unique stationary non-autarkic equilibrium for which $\gamma^m = \gamma^m(\phi)$, $n = n(\phi)$, $a = a(\phi)$, where $a(\phi)$ is given by

$$a(\phi) = \phi^{-1}\left(\frac{\hat{k}}{2}\right)^{\alpha(1-\sigma)}\left[1 - \gamma^m(\phi)^2\right]^{\alpha(1-\sigma)} n(\phi)^{(1-\alpha)(1-\sigma)}. \tag{50}$$

Q.E.D.

A.4. Proof of Proposition 7

Suppose that $u(q) = (1-\sigma)^{-1}\left(q^{1-\sigma} - 1\right)$, with $0 < \sigma < 1$, $c(n) = n$, and $F(k,n) = k^\alpha n^{1-\alpha}$, with $0 < \alpha < 1$. Suppose also that $g(\gamma) = 1$ for any $0 \leq \gamma \leq 1$ and $g(\gamma) = 0$ otherwise. Note that we can rewrite (39) as follows:

$$-\phi a(\phi) + \frac{\beta}{1-\beta}a(\phi)\left(\phi\beta^{-1} - 1\right) + \frac{\beta}{1-\beta}\mu\left[1 - \gamma^m(\phi)\right] \geq 0, \tag{51}$$

where

$$a(\phi) = \phi^{-1}\left(\frac{\hat{k}}{2}\right)^{\alpha(1-\sigma)}\left[1 - \gamma^m(\phi)^2\right]^{\alpha(1-\sigma)} n(\phi)^{(1-\alpha)(1-\sigma)}.$$

We have already shown that

$$-\phi a(\phi) + \frac{\beta}{1-\beta}a(\phi)\left(\phi\beta^{-1} - 1\right) \geq 0$$

if and only if $\phi \geq 1$. This means that there exists $\bar{\phi} < 1$ such that, for any $\phi \geq \bar{\phi}$, there exist $\{r(\gamma)\}_{\gamma\geq\gamma^m(\phi)}$ and $\mu > 0$ satisfying (42), (43), and (51). **Q.E.D.**

A.5. Proof of Proposition 8

Suppose now that $r(\gamma)$ is given by (41) for any given ϕ. Then, (39) can be written as

$$\frac{e}{\beta}\left[\frac{1}{2\gamma^m(\phi)} + \frac{\gamma^m(\phi)}{2} - 1\right] - \left(\frac{1-\phi}{\phi}\right)\left(\frac{\hat{k}}{2}\right)^{\alpha(1-\sigma)}\left[1 - \gamma^m(\phi)^2\right]^{\alpha(1-\sigma)} n(\phi)^{(1-\alpha)(1-\sigma)} \geq 0.$$

Taking the limit as $\phi \to \beta$ from above, the left-hand side of this expression converges to

$$e \left(\frac{1}{2\gamma_\beta^*} + \frac{\gamma_\beta^*}{2} - 1 \right) - (1 - \beta) \left(\frac{\hat{k}}{2} \right)^{\alpha(1-\sigma)} \left[1 - (\gamma_\beta^*)^2 \right]^{\alpha(1-\sigma)} \left(n_\beta^* \right)^{(1-\alpha)(1-\sigma)},$$

where $\left(\gamma_\beta^*, n_\beta^* \right)$ denotes the solution to the planner's problem [i.e., the unique interior solution to the system (44)-(45)] for any given discount factor $\beta < 1$. As $\beta \to 1$ from below, we have that $0 < \lim_{\beta \to 1} \gamma_\beta^* < 1$. This means that there exists $\beta < 1$ sufficiently close to one such that the expression above is strictly positive. Therefore, we have constructed an equilibrium in which $\phi_t = \beta$,

$$1 + r_t(\gamma) = \beta^{-1} \frac{\gamma}{\gamma_\beta^*},$$

for each $\gamma \geq \gamma_\beta^*$, and

$$\mu_t = \frac{\beta^{-1}}{2} \left(\frac{1}{\gamma_\beta^*} - 1 \right).$$

for all $t \geq 0$. **Q.E.D.**

A.6. Non-Stationary Equilibria

In this subsection, we consider the existence of non-stationary equilibria in which the solvency constraints hold with equality at each date. Using the same terminology as Alvarez and Jermann (2000), we characterize equilibria in which the solvency constraints are not too tight. Consider first the case of unregulated bank lending. Let w_t denote the banker's discounted lifetime utility at the beginning of date t. Then, the equations defining the equilibrium dynamics of w_t and ϕ_t are given by

$$w_t = a \left(\phi_{t-1} \right) \left(\beta^{-1} \phi_{t-1} - 1 \right) + \beta w_{t+1} \tag{52}$$

and

$$\phi_t a \left(\phi_t \right) = \beta w_{t+1}. \tag{53}$$

Combining these two conditions, we can define an equilibrium as a sequence of prices $\{\phi_t\}_{t=0}^\infty$ satisfying

$$\phi_t a \left(\phi_t \right) = a \left(\phi_{t-1} \right), \tag{54}$$

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given an initial condition $\phi_{-1} > 0$.

Note that there exists a unique stationary non-autarkic solution: $\phi_{t-1} = \phi_t = 1$. Suppose now that $u(q) = (1 - \sigma)^{-1} (q^{1-\sigma} - 1)$, with $0 < \sigma < 1$, $c(n) = n$, and $F(k, n) = k^{\alpha} n^{1-\alpha}$, with $0 < \alpha < 1$. Suppose also that $g(\gamma) = 1$ for any $0 \leq \gamma \leq 1$ and $g(\gamma) = 0$ otherwise. Using the Implicit Function Theorem, we find that

$$\left. \frac{d\phi_t}{d\phi_{t-1}} \right|_{\phi_{t-1}=\phi_t=1} = \frac{a'(\phi_{t-1})}{\phi_t a'(\phi_t) + a(\phi_t)} > 1,$$

where the function $a(\phi)$ is given by (50). This means that, for any initial value $\phi_{-1} > 1$, the equilibrium price trajectory is strictly increasing and unbounded, so the equilibrium allocation approaches the autarkic allocation as $t \to \infty$. Along this equilibrium path, the debt limits, given by $\bar{B}_t = a(\phi_t)$, shrink over time and converge to zero. This means that liquidity becomes scarcer and more expensive over time, and households are able to trade smaller amounts of goods in the decentralized night market.

Consider now the case of regulated bank lending. Suppose that $r_t(\gamma)$ is given by (41). In this case, the equations defining the equilibrium dynamics of w_t and ϕ_t are given by (53) and

$$w_t = a(\phi_{t-1})(\beta^{-1}\phi_{t-1} - 1) + e\beta^{-1}\frac{[1 - \gamma^m(\phi_t)]^2}{2\gamma^m(\phi_t)} + \beta w_{t+1}. \tag{55}$$

Combining (53) with (55), we can define an equilibrium as a sequence of prices $\{\phi_t\}_{t=0}^{\infty}$ satisfying

$$a(\phi_{t-1}) = e\beta^{-1}\frac{[1 - \gamma^m(\phi_t)]^2}{2\gamma^m(\phi_t)} + \phi_t a(\phi_t), \tag{56}$$

given an initial condition $\phi_{-1} > 0$. Suppose that $u(q) = (1 - \sigma)^{-1}(q^{1-\sigma} - 1)$, with $0 < \sigma < 1$, $c(n) = n$, and $F(k, n) = k^{\alpha} n^{1-\alpha}$, with $0 < \alpha < 1$. Suppose also that $g(\gamma) = 1$ for any $0 \leq \gamma \leq 1$ and $g(\gamma) = 0$ otherwise. Notice that, for β sufficiently close to one, $\phi_{t-1} = \phi_t = \beta$ is the unique non-autarkic stationary solution. Again, for any initial condition $\phi_{-1} > \beta$, the debt limits shrink over time and the price of liquid assets grows unbounded as the economy approaches autarky. In this case, the creation of public liquidity can rule out some of these explosive trajectories for the price of liquid assets, even though it cannot *per se* guarantee efficiency.
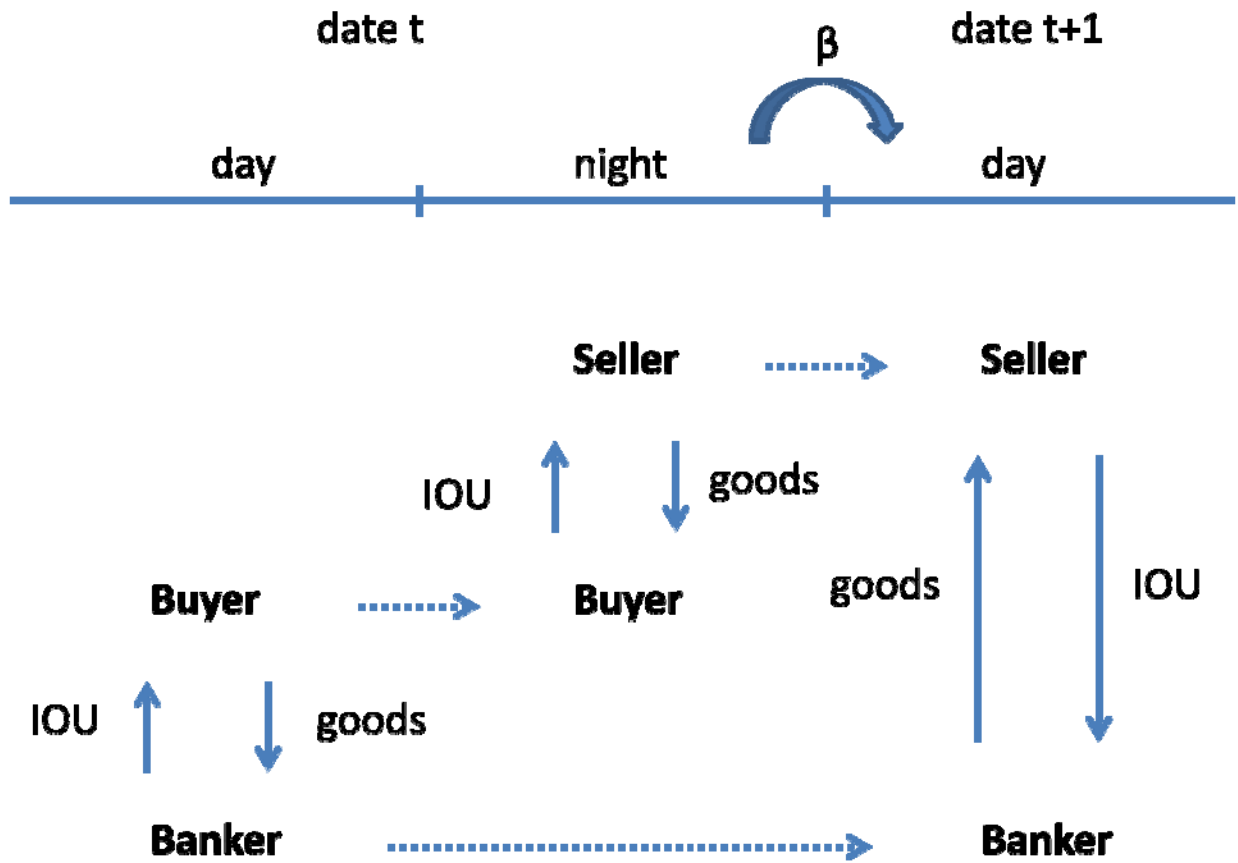
Figure 1 – Banker's IOUs

Figure 2 – Capital Accumulation