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Abstract

We study optimal monetary policy in a model in which fiat money and private debt coexist as a means of payment. The credit system is endogenous and allows buyers to relax their cash constraints. However, it is costly for agents to publicly report their trades, which is necessary for the enforcement of private liabilities. If it is too costly for the government to obtain information regarding private transactions, then it relies on the public information generated by the private credit system. If not all private transactions are publicly reported, the government has imperfect public information to implement monetary policy. In this case, we show that there is no incentive-feasible policy that can implement the socially efficient allocation. Finally, we characterize the optimal policy for an economy with a low record-keeping cost and a large number of public transactions, which results in a positive long-run inflation rate.

JEL classification: E4, E5.

Keywords: Fiat money; private credit; costly record-keeping; imperfect public information; optimal monetary policy.

1. INTRODUCTION

Recent work on the microfoundations of monetary economics has emphasized anonymous trade as an essential ingredient for making fiat money socially valuable; see Kocherlakota (1998). However, anonymity makes the implementation of monetary policy difficult because any transfer scheme to anonymous agents is challenging to carrying out. Thus, it is crucial to study the design of optimal monetary policy in an environment in which anonymity also constrains the set of feasible choices for the government. This aspect of monetary policy implementation has not been fully exploited by the literature and motivates our work.[1]

We consider an environment in which the government lacks the technology to verify agents' identity and observe their private trades. As a result, it necessarily relies on the public information voluntarily created by the private sector to implement monetary policy. Thus, all government taxes or transfers are constrained to being conditional on the available public information.[2] Our analysis builds on Lagos and Wright (2005) and Rocheteau and Wright (2005). However, we relax the assumption that all trades are necessarily anonymous. In our environment, the private sector has access to a costly technology that allows agents to publicly report their trades. As a result, private agents may be willing to voluntarily report (at a cost) their trades, together with their identities, to others if such an action allows them to have access to credit. Because agents cannot commit to their future promises, it is necessary to have public information to enforce the repayment of private liabilities through societal punishments.[3] Thus, credit arrangements may compete with fiat money as a means of payment when agents voluntarily report their trades.

The record-keeping technology to which private agents have access in our model is similar to the one in Monnet and Roberds (2008); Nosal and Rocheteau (2009); and Li (2011). In

[1] An exception is a recent paper by Deviatov and Wallace (2010). They study incentive-feasible policies in a model with indivisible money. In our paper, we use a model with divisible money.

[2] In reality, it is very likely that the government has imperfect information regarding private trades when designing and implementing policies.

[3] See Kocherlakota (1998); Kocherlakota and Wallace (1998); Cavalcanti and Wallace (1999); and Araujo and Camargo (2009).

particular, a subset of sellers have the ability to verify the identity of their trading partners and report their trades to other agents in the economy. If a buyer trades with one of these sellers, then he can choose whether he wants to have his trade reported to others at a cost. If the buyer chooses to report his trade, then he can induce the seller to produce more goods than what his money balances allow him to purchase because he can also promise to make a repayment at a future date.

In our environment, not only do public transactions allow credit arrangements within the private sector, but they also permit the government to effectively alter the rate of return on money. As in Andolfatto (2010), we restrict attention to policies that respect voluntary trade, such as interest payments on money holdings, so that money becomes an interest-bearing liability for the government. Given that not all bilateral trades are publicly observable (due to technological restrictions), not all agents are able to receive interest payments from the government on their money holdings. In other words, injections of fiat money are asymmetric.

We characterize the optimal policy rule with constant relative risk aversion (CRRA) preferences for an economy with a sufficiently low record-keeping cost and with a large measure of sellers with the ability of making their trades publicly observable. We show that there is a threshold value for the money growth rate below which the credit system is not used in equilibrium because there is no incentive-compatible repayment amount that can be promised to the sellers. Above the threshold value for the money growth rate, the monetary authority is able to induce agents to report their trades by offering interest payments to those agents who make their trades publicly observable. As a result, the private credit system is operative and produces public information on which the monetary authority can condition its injections of money. However, the socially efficient allocation cannot be implemented by any incentive-feasible policy if not all sellers in the economy have the ability to publicly report their trades.

Finally, we show that the government's optimal policy is unable to eliminate consumption risk: A buyer who has access to credit consumes more than a buyer who is paired with a seller who is unable to make her trade publicly observable (in which case credit is not available). Because not all trades are publicly observable and, as a result, injections of fiat money are asymmetric, the monetary authority cannot increase the rate of return on money so that those who trade exclusively with currency can purchase the socially efficient quantity. Essentially, monetary policy is unable to induce agents to carry more money into the goods market because it cannot eliminate the opportunity cost of holding money over time. Finally, we show that the optimal policy results in a positive long-run inflation rate and a positive nominal interest rate.

2. THE MODEL

2.1. Agents

There is a continuum of infinitely lived buyers and sellers. Each buyer is indexed by $i \in [0,1]$ and each seller is indexed by $j \in [0,1]$. Time is discrete and each period is divided into two subperiods: day and night. Within each subperiod, there is a unique perishable consumption good that is produced and consumed. In the day subperiod, a seller does not want to consume but can produce one unit of the consumption good with one unit of labor. Instead the buyer wants to consume but is unable to produce. At night, both types want to consume and are able to produce one unit of the consumption good with one unit of labor. Neither a buyer nor a seller can commit to his or her promises.

A buyer has preferences given by:

$$u(q_i) + c_i - n_i, \tag{1}$$

where $q_i \in \mathbb{R}_+$ is consumption in the day subperiod, c_i is consumption in the night subperiod, and $n_i \in \mathbb{R}$ is production in the night subperiod. Assume that $u : \mathbb{R}_+ \to \mathbb{R}_+$ is strictly concave, increasing, and continuously differentiable, with $u(0) = 0$ and $u'(0) = \infty$.

A seller has the following utility function over goods and effort:

$$-q_j + c_j - n_j, \tag{2}$$

where $q_j \in \mathbb{R}_+$ is production in the day subperiod, c_j is consumption in the night subperiod, and $n_j \in \mathbb{R}$ is production in the night subperiod. Buyers and sellers have a common discount factor between periods, which we denote by $\beta \in (0, 1)$.

2.2. Markets

Agents trade in two sequential markets every period. In the day market, agents are randomly and bilaterally matched so that each buyer meets a seller. In the night market, agents interact in a centralized location where the terms of trade are given by competitive pricing.

2.3. Recordkeeping

There are two types of sellers: connected and unconnected. A connected seller has access to a record-keeping technology that allows her to verify the identity of her trading partner and record her transaction in the day subperiod. Once a transaction is recorded, it can be reported to other agents in the economy. The use of this technology costs $\kappa > 0$ units of the consumption good for the seller in the day subperiod. An unconnected seller does not have access to a record-keeping technology and, therefore, is unable to make her interaction with her trading partner publicly observable. There is a measure $\delta \in [0, 1]$ of connected sellers and a measure $1 - \delta$ of unconnected sellers.

Notice that a connected seller is willing to extend credit to her trading partner in the decentralized market provided that society can enforce any repayment in the centralized location. One possible financial arrangement is to have a seller producing for the buyer with whom she is paired in the day subperiod in exchange for a repayment in the night subperiod. However, a buyer cannot commit to his promise of making a repayment in the night subperiod. To enforce the repayment of private liabilities, there must be some form of societal punishment on defaulters. Otherwise, a seller would not be willing to produce for a buyer in the decentralized market unless she received something tangible and valuable in exchange (such as fiat money). This must be the case for an unconnected seller who is unable to make her trades publicly observable. On the other hand, a connected seller has the ability to make her transaction with a buyer publicly observable. If a buyer does not repay his loan, other agents in the economy will observe his defection. If there exists a mechanism that enforces any repayment in the centralized location, a connected seller is willing to extend credit to her trading partners in the decentralized market. In the next section, we provide more details on the exact punishment that society can impose on defaulters.

3. MONETARY ECONOMY

As a benchmark, we describe in this section an equilibrium without intervention so that agents are endowed with a fixed stock of money. To enforce the repayment of private liabilities, there is a clearinghouse that collects all reports from connected sellers in the day subperiod. The clearinghouse is also responsible for receiving repayments from buyers and making payments to sellers in the centralized location. Any transaction that is reported to the clearinghouse becomes publicly observable. Because a buyer cannot commit to his promises, the clearinghouse needs to impose some kind of punishment on him if he fails to make a repayment. It is not possible for the clearinghouse to directly punish a buyer who has defaulted on his loan. However, the clearinghouse can indirectly punish a defaulter by refusing to make a payment (in the centralized location) to any seller who trades with him. Notice that the identity of a defaulter is publicly observable. As a result, a connected seller will not be willing to extend credit to a defaulter. To extend credit to him, the seller would have to report their transaction, together with their identities, to the clearinghouse, which would then refuse to make a transfer to the seller even if it received a repayment from the buyer in the centralized location. Thus, a buyer who reneges on her liability loses access to credit: A connected seller may be willing to produce for him in exchange for fiat money but is not willing to extend credit.

Each buyer is endowed with $\bar{M}$ units of fiat money. A buyer can choose whether to have his transaction with a connected seller reported to the clearinghouse. If the buyer chooses not to report his trade, then credit will not be available to him: The seller cannot enforce the repayment of a private liability in the centralized location and, as a result, is not willing to extend credit. In this case, the seller is willing to produce for the buyer only if she receives fiat money in exchange as the unconnected seller does not have access to a record-keeping technology. Hence, credit is incentive-feasible only in a bilateral meeting between a buyer and a connected seller.

As in Lagos and Wright (2005), there is a Walrasian market in the centralized location in which agents can trade goods for fiat money at a competitive price. Trades in this market are always anonymous. To determine the terms of trade in the decentralized market, we assume that the buyer makes a take-it-or-leave-it offer to the seller, who either accepts or rejects it.[4] As a result, the buyer extracts all surplus from trade when proposing the terms of trade. This particular bargaining protocol simplifies the analysis without compromising the generality of our results. In Lagos and Wright (2005) there is an inefficiency arising from the generalized Nash bargaining solution: If the seller has some bargaining power, the first-best allocation cannot be implemented as a monetary equilibrium.[5] To concentrate on the informational frictions that we emphasize in this paper, we simplify the analysis by ruling out any potential bargaining inefficient.

3.1. Bilateral Trade with an Unconnected Seller

Consider the bargaining problem between a buyer and an unconnected seller. Let ϕ_t denote the value of money in the centralized location at date t, and suppose that the buyer has M units of money. Taking ϕ_t and κ as given, the buyer's problem is:

$$\max_{(q,D)\in\mathbb{R}_+^2} \left[u\left(q\right) - \phi_t D\right],$$

subject to the seller's individual rationality constraint,

$$-q + \phi_t D \geq 0, \tag{3}$$

[4] Here we implicitly assume that it is infinitely costly for the buyer to make a second offer to the seller if the first one has been rejected.

[5] Gomis-Porqueras and Peralta-Alva (2010) show that in order to obtain the first-best fiscal policies need to be active.

and the buyer's cash constraint,

$$D \leq M. \tag{4}$$

Here the seller produces q units of the consumption good for the buyer in exchange for D units of money. The solution to this problem is: $q = q^*$ and $D = q^*/\phi_t$ if $M \geq q^*/\phi_t$; or $q = \phi_t M$ and $D = M$ if $M < q^*/\phi_t$. Finally, the buyer's payoff of trading with an unconnected seller is given by

$$\Lambda_t^a(M) = \begin{cases} u(\phi_t M) \text{ if } M < q^*/\phi_t, \\ u(q^*) \text{ if } M \geq q^*/\phi_t. \end{cases} \tag{5}$$

3.2. Bilateral Trade with a Connected Seller

Now consider the bargaining problem between a buyer and a connected seller. First, we describe the solution to the bargaining problem, assuming that the buyer wants to have his trade reported. Let v denote the buyer's expected discounted utility after making a repayment in night subperiod. Similarly, let $\hat{v}$ represent the buyer's expected discounted utility following default in the night subperiod. Taking ϕ_t, v, $\hat{v}$, κ as given, the buyer's problem is as follows:

$$\max_{(q,D,L)\in\mathbb{R}_+^3} \left[u(q) - \phi_t L - \phi_t D\right],$$

subject to the seller's individual rationality constraint,

$$-q - \kappa + \phi_t L + \phi_t D \geq 0, \tag{6}$$

the buyer's cash constraint (4), and the buyer's individual rationality constraint,

$$-\phi_t L + v \geq \hat{v}. \tag{7}$$

Here the seller produces q units of the consumption good for the buyer in exchange for D units of money and a promise of repayment of L units of money in the centralized location. The buyer produces $\phi_t L$ units of the good in exchange for L units of money in the Walrasian market and makes a transfer to the clearinghouse, which in turn makes a payment L to the seller with whom he was paired in the day subperiod. There is no cost of operating the clearinghouse other than the cost of reporting a transaction. A seller needs to pay $\kappa > 0$ to report her transaction with her trading partner in the decentralized market, which makes it harder to satisfy her individual rationality constraint. Because (6) holds with equality at the optimum, the buyer ends up paying for the record-keeping cost. The benefit of having his trade reported is that the buyer can consume more than what his money holdings permit him to purchase because he can promise to make a repayment to the seller in the centralized location through the clearinghouse. Although credit is costly for the buyer, it allows him to relax his cash constraint.

The unconstrained solution to the bargaining problem is $\phi_t L + \phi_t D - \kappa = q^*$. This means that, if $\phi_t M - \kappa < q^*$, we must have $D = M$ and $L > 0$ at the optimum. Thus, we can rewrite the buyer's problem as

$$\Lambda_t^r(M) = \max_{L \geq 0} \left[u\left(\phi_t L + \phi_t M - \kappa\right) - \phi_t L \right], \tag{8}$$

subject to (7). The solution to this problem is: $L = (q^* - \phi_t M + \kappa)/\phi_t$ if $q^* - \phi_t M + \kappa \leq v - \hat{v}$ or $L = (v - \hat{v})/\phi_t$ if $q^* - \phi_t M + \kappa > v - \hat{v}$. Finally, the seller sends the report $\{(i, j), (q, D, L)\}$ to the clearinghouse, where $i \in [0, 1]$ is the buyer's identity, $j \in [0, 1]$ is the seller's identity, and (q, D, L) are the terms of trade.

Suppose now that the buyer chooses not to have his trade reported to the clearinghouse. In this case, any trade proposed by the buyer that involves a positive repayment amount will be rejected by the seller. Thus, the buyer's problem is the same as the one he faces when meeting with an unconnected seller in the decentralized market.

3.3. Buyer's Bellman Equation

In a stationary monetary equilibrium, each buyer anticipates that the value of money in the centralized location will be constant over time: $\phi_t = \phi > 0$ for all $t \geq 0$. The buyer's problem in the centralized location can be expressed in terms of the following Bellman equation:

$$v = \max_{M \in \mathbb{R}_+} \left\{ -\phi M + \beta \left[\delta \max\left\{ \Lambda_{t+1}^r(M), \Lambda_{t+1}^a(M) \right\} + (1 - \delta) \Lambda_{t+1}^a(M) + v \right] \right\}, \tag{9}$$

with $\Lambda^r_{t+1}(M)$ given by (8) and $\Lambda^a_{t+1}(M)$ given by (5). Here M is the amount of money that the buyer acquires in the Walrasian market at date t and takes with him into the decentralized market at date $t+1$. Let M^* denote his optimal choice of money holdings in the Walrasian market.

Now we make the same change of variables as in Sanches and Williamson (2010), which will prove to be useful for describing an equilibrium allocation. Let y denote the buyer's daytime consumption if he reports his trade to the clearinghouse and let x denote his daytime consumption if he does not report. Suppose that, at the optimum, we have that:

$$-\phi M^* + \beta\left[\delta\Lambda^r_{t+1}(M^*) + (1-\delta)\Lambda^a_{t+1}(M^*)\right] \geq -\phi M^* + \beta\Lambda^a_{t+1}(M^*).$$

In this case, the buyer chooses to report his trade with a connected seller to other agents in order to have access to credit. We can rewrite equation (9) in terms of x and y as follows:

$$(1-\beta)v = -(1-\beta)x + \beta\delta\left[u(y) - y - \kappa\right] + \beta(1-\delta)\left[u(x) - x\right],$$

with the buyer's individual rationality constraint (7) given by

$$\beta\delta u(y) - (1-\beta+\delta\beta)(y+\kappa) + \beta(1-\delta)\left[u(x) - x\right] \geq (1-\beta)\hat{v}.$$

When making his portfolio decision in the centralized location at date t, a buyer finds it optimal to carry some currency into the decentralized market at date $t+1$ because he may be matched with an unconnected seller (with probability $1-\delta$). In this case, trade takes place only if he has money to pay for his purchase. Even in a trade with a connected seller that is reported to the clearinghouse, the buyer may use both credit and fiat money to pay for the amount y that the seller produces for him.

Suppose now that, at the optimum, we have

$$-\phi M^* + \beta\left[\delta\Lambda^r_{t+1}(M^*) + (1-\delta)\Lambda^a_{t+1}(M^*)\right] < -\phi M^* + \beta\Lambda^a_{t+1}(M^*).$$

In this case, the cost associated with credit exceeds the benefit of trading with credit. As a result, the buyer never reports his trade when he is paired with a connected seller. Thus, he always uses fiat money to pay for his purchase in the decentralized market. We can then rewrite equation (9) as follows:

$$(1-\beta)\,v = -x + \beta u\,(x)\,,$$

with the value of x given by

$$u'\,(\hat{x}) = \beta^{-1}. \tag{10}$$

For fiat money and private debt to coexist as means of payment, the following must hold in equilibrium:

$$-\,(1-\beta)\,x + \beta\delta\,[u\,(y) - y - \kappa] + \beta\,(1-\delta)\,[u\,(x) - x] \geq -\hat{x} + \beta u\,(\hat{x})\,. \tag{11}$$

Otherwise, an equilibrium is one in which all trade in the decentralized market is carried out with only fiat money.

3.4. The Value of Defection

If a buyer fails to make a repayment in the centralized location, he will be able to use only fiat currency to pay for his future purchases. When a buyer fails to make a repayment, the clearinghouse makes his defection publicly observable. This means that, if a defaulter wants to have any of his future trades reported, his identity will be revealed to his trading partner. The latter (a connected seller) knows that the clearinghouse will refuse to make a transfer to her in the centralized location even if a repayment is actually collected from the buyer. Because the proposed trade would involve a positive repayment amount (otherwise, the buyer would prefer not to report the trade), she would get a negative payoff if she carried out the proposed trade. As a result, the seller will not accept the terms proposed by the buyer. Taking this into account, a defaulter who is paired with a connected seller chooses not to have his trade reported to the clearinghouse because this option would involve a cost without any additional benefit. Thus, the value of defection $\hat{v}$ satisfies the following Bellman equation:

$$\hat{v} = \max_{\hat{M}\in\mathbb{R}_+} \left\{-\phi\hat{M} + \beta\left[u\left(\phi\hat{M}\right) + \hat{v}\right]\right\}. \tag{12}$$

Let z denote the buyer's consumption following defection. Then, we can rewrite (12) as follows:

$$(1-\beta)\,\hat{v} = -z + \beta u\,(z)\,.$$

A defaulter produces and sells z units of the consumption good in the Walrasian market in order to acquire enough money balances at date t to purchase z units of the good in the decentralized market at date $t+1$. We have that $z = \hat{x}$, with $\hat{x}$ satisfying (10).

3.5. Stationary Monetary Equilibrium

Throughout the paper, we restrict attention to stationary monetary equilibria for which aggregate real money balances are constant over time. With constant money supply, this means that the value of money in the centralized location is constant over time. The distribution of money holdings at the end of the night subperiod is such that every buyer holds the same amount of money and that sellers carry no money into the decentralized market. This result is a direct consequence of the quasilinearity with respect to labor supply and that agents have periodic access to centralized trade; see Lagos and Wright (2005) and Rocheteau and Wright (2005). Hence, we can characterize an equilibrium allocation in terms of the daytime consumption of a buyer who has his trade reported to the clearinghouse and the daytime consumption of a buyer who does not have his trade reported, together with the daytime consumption that a buyer would get had he defaulted on his private liability.

Definition 1 *A stationary monetary equilibrium with credit is a triple* (x, y, z)*, with* $z = \hat{x}$*, satisfying the nonnegativity of the repayment amount*

$$y - x + \kappa \geq 0, \tag{13}$$

the first-order condition for the optimal choice of money balances,

$$\delta u'\,(y) + (1-\delta)\,u'\,(x) = \beta^{-1}, \tag{14}$$

and the buyer's individual rationality constraint

$$\beta\delta u\,(y) - (1-\beta+\delta\beta)\,(y+\kappa) + \beta\,(1-\delta)\,[u\,(x) - x] \geq -\hat{x} + \beta u\,(\hat{x})\,, \tag{15}$$

with $y = q^*$ if (15) does not bind.

Notice that (13) and (15) imply that (11) holds. An equilibrium in which fiat money and private debt are used as a means of payment has to be one in which a buyer who is paired with a connected seller finds it optimal to have his trade reported to the clearinghouse, despite the cost associated with this choice. Equations (14) and (15) characterize the consumption plans x and y. Note that if we set $y = q^*$ and obtain x from (14). If (13) and (15) are satisfied, the socially efficient quantity q^* is traded in each bilateral trade between a buyer and a connected seller. Otherwise, (14) and (15) holding with equality determine the values of x and y. Similarly, we need to verify whether (13) is satisfied. If there exists no (x, y, z) satisfying (13)-(15), together with $z = \hat{x}$, then an equilibrium is one in which all trade in the decentralized market is carried out with fiat money (Buyers never report their trades to the clearinghouse and, as a result, credit disappears.). In this equilibrium, the quantity $\hat{x}$ is traded in every meeting in the decentralized market. In the night subperiod, each buyer then produces $\hat{x}$ and each seller consumes $\hat{x}$.

In an equilibrium in which fiat money and private debt coexist, we have that $x \leq y$, which means that a buyer who reports his trade to the clearinghouse is able to consume more than a buyer who cannot obtain credit from his trading partner. From a buyer's standpoint, there is consumption risk. A buyer who trades with an unconnected seller faces a cash constraint that can eventually bind. On the other hand, a buyer who trades with a connected seller can promise to make a repayment in the centralized location in order to consume more than what his money holdings permit him to purchase in the decentralized market. So long as the repayment amount is individually rational, a buyer who trades with a connected seller will be able to consume more in the day subperiod.

Although the possibility of trading with credit in the decentralized market seems attractive for buyers, the following result shows that there can be no equilibrium in which money and private debt coexist.

Proposition 2 *With a constant money supply, the unique stationary monetary equilibrium is a pure monetary equilibrium in which the quantity $\hat{x}$ is traded in the decentralized market.*

Proof. Notice that for any pair (x, y),

$$\begin{aligned} & \beta\delta u(y) - (1 - \beta + \delta\beta)(y + \kappa) + \beta(1 - \delta)[u(x) - x] \\ < \; & \beta\delta u(y) - (1 - \beta + \delta\beta) y + \beta(1 - \delta)[u(x) - x] \\ \leq \; & u(\beta\delta y + \beta(1 - \delta) x) - (1 - \beta + \delta\beta) y - \beta(1 - \delta) x \\ \leq \; & -\hat{x} + \beta u(\hat{x}). \end{aligned}$$

In the second step, we have used the fact that the utility function $u : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is strictly concave. Hence, the buyer's individual rationality constraint (15) cannot be satisfied for any (x, y). This implies that there can be no credit in equilibrium. As a result, the unique stationary monetary equilibrium is one in which $\hat{x}$ is traded in each bilateral meeting in the decentralized market. **Q.E.D.** ■

The previous proposition showed that the buyer's individual rationality constraint cannot be satisfied in equilibrium. This means that there is no incentive-feasible repayment amount. Any buyer would prefer to renege on his liability in the centralized location and use fiat currency to pay for all of his future purchases in the decentralized market. Hence, the credit system is not used in equilibrium. In the subsequent section, we study the possibility of government intervention. There, we will show that there exists a critical value for the money growth rate above which the credit system becomes operative. Moreover, we will show that the critical value for the money growth rate is above zero.

This result is also observed in Berentsen, Camera, and Waller (2007). Like our model, the credit system in their model is endogenous, and the repayment of debt has to be incentive compatible. Our intuition for the previous result is similar to theirs. A buyer who traded with a connected seller and promised to make a repayment in the centralized location has to produce more goods in the Walrasian market to settle his liability relative to a buyer who traded with an unconnected seller. Notice that both the buyer and the seller rebalance their currency portfolio in the same way since there are no wealth effects (due to quasilinear preferences). With zero inflation, agents are able to self-insure at a low cost, which means that having access to costly credit is of no value. A similar result also arises in Aiyagari and

Williamson (2000), although in their environment the informational structure is significantly different.

4. WELFARE-IMPROVING POLICIES WITH IMPERFECT PUBLIC INFORMATION

In this section, we consider the possibility of government intervention. Any intervention must result in a net transfer of fiat money to private agents in order to respect the constraint that all trades have to be voluntary, as in Andolfatto (2010). Moreover, any transfer must be conditional on the public information created by the private sector. We implicitly assume that the government lacks the technology to verify agents' identity and observe their trades. Notice that this kind of intervention creates an additional incentive for agents to report their trades: Not only do public transactions allow credit arrangements within the private sector, but they also permit an agent to receive a net transfer of fiat money.

Any information the government has about the economy is necessarily reported by agents. As a result, the government needs to interact with the clearinghouse to implement any policy because the interest payment can be made only to those who have reported their trades and identities to the clearinghouse. Recall that the clearinghouse keeps a public record of reports and identities. Specifically, an agent who holds M units of money in the decentralized market can transform his balance into $RM - T$ units of money. Here R is the gross nominal interest rate announced by the government and T represents a redemption fee.

The government can intervene in the economy only if the clearinghouse is used to settle private transactions. The government can induce agents to report their trades by paying interest on money holdings; that is, the benefit of reporting a trade is not only to have access to credit but also to be able to receive an interest payment. However, if it is very expensive to report a transaction, then a buyer who is paired with a connected seller may prefer not to use credit nor to receive an interest payment; as a result, the clearinghouse is not used. In this case, all trade is carried out with fiat currency. Thus, all transactions in the economy are anonymous and the government cannot intervene.

4.1. Bilateral Trade with an Unconnected Seller

An unconnected seller is unable to report her trade in the decentralized market to the clearinghouse. Thus, she will not be willing to extend credit to the buyer because no repayment amount can be enforced in the centralized location. Moreover, neither the buyer nor the seller is able to receive an interest payment because they are unable to contact the clearinghouse in the day subperiod. As a result, the bargaining problem between a buyer and an unconnected seller is exactly the same as the one described in section 3. Thus, the buyer's payoff of trading with an unconnected seller, denoted by $\Lambda_t^n(M)$, is given by (5).

4.2. Bilateral Trade with a Connected Seller

Here we describe the bargaining problem between a buyer and a connected seller. Suppose first that the buyer wants to have his trade reported. Define $\varphi_t(D) = \max\{D, R_t D - T_t\}$. Taking ϕ_t, T_t, R, κ, v, $\hat{v}$ as given, the buyer solves the following problem:

$$\max_{(q,L,D)\in\mathbb{R}_+^3} \left[u(q) - \phi_t L - \phi_t D\right],$$

subject to the seller's individual rationality constraint,

$$-q - \kappa + \phi_t L + \phi_t \varphi_t(D) \geq 0,$$

the buyer's cash constraint (4), and the buyer's individual rationality constraint (7). If the buyer chooses not to trade with the seller in the day subperiod, he will not be able to receive an interest payment because there will be no transaction to be reported to the clearinghouse. For this reason, the buyer's surplus from trade is given by $u(q) - \phi_t L - \phi_t D$.

Suppose that $T_t(R_t - 1)^{-1} < (q^* + \kappa + \phi_t T_t)/\phi_t R_t$. Now if $\phi_t \max\{R_t M - T_t, M\} - \kappa < q^*$, then we must have $D = M$ and $L > 0$ at the optimum. The buyer's surplus from trade as a function of D and L is

$$S_t^r(D, L) = u(\phi_t L + \phi_t D - \kappa) - \phi_t L - \phi_t D,$$

if $0 < D < T_t (R_t - 1)^{-1}$, or

$$S_t^r (D, L) = u (\phi_t L + \phi_t (R_t D - T_t) - \kappa) - \phi_t L - \phi_t D,$$

if $D \geq T_t (R_t - 1)^{-1}$. Then, we can rewrite the buyer's problem as:

$$\max_{L \in \mathbb{R}_+} S_t^r (M, L),$$

subject to (7). If the constraint (7) binds, we have that $L = (v - \hat{v}) / \phi_t$. Otherwise, the repayment amount is given by

$$L = (q^* + \kappa - \phi_t M) / \phi_t \text{ if } M < T_t (R_t - 1)^{-1}$$

or

$$L = [q^* + \kappa - \phi_t (R_t M - T_t)] / \phi_t \text{ if } T_t (R_t - 1)^{-1} \leq M < (q^* + \kappa + \phi_t T_t) / \phi_t R_t.$$

Again, let $\Lambda_t^r (M)$ denote the buyer's payoff of trading with a connected seller (as a function of his money holdings) when the trade is reported to the clearinghouse. Finally, the seller sends the report $\{(i, j), (q, D, L)\}$ to the clearinghouse.

Suppose now that the buyer chooses not to have his trade reported to the clearinghouse. Then, the buyer's problem is the same as the one he faces when meeting with an unconnected seller.

4.3. Government's Budget Constraint

The government's budget constraint is given by

$$\delta T_t + \bar{M}_t - \bar{M}_{t-1} = (R_t - 1) \delta \bar{M}_{t-1}.$$

We have anticipated that, in a monetary equilibrium, all buyers who have access to credit choose to use it by reporting their trades to the clearinghouse. We have also anticipated that, at the beginning of the day subperiod at date t, each buyer holds $\bar{M}_{t-1}$ units of money and that sellers carry no money into the decentralized market. We will show later that, in a monetary equilibrium, this will be the endogenous distribution of money holdings across agents at the beginning of the day subperiod. As a result, aggregate interest payment is $(R-1)\,\delta\bar{M}_{t-1}$, with an aggregate revenue from the redemption fee equal to δT_t.

We restrict attention to monetary policy rules for which the money supply grows at a constant gross rate $\mu > 0$ ($\bar{M}_t = \mu\bar{M}_{t-1}$ for all $t \geq 0$) and the gross interest rate is constant over time ($R_t = R \geq 1$ for all $t \geq 0$).[6] Thus, we can rewrite the government's budget constraint in real terms as follows:

$$\phi_t T_t = \frac{\phi_t \bar{M}_t}{\delta}\left[\frac{(R-1)\,\delta+1}{\mu} - 1\right]. \tag{16}$$

4.4. Buyer's Bellman Equation

Each buyer takes the value of money $\{\phi_t\}_{t=0}^{\infty}$ and the monetary policy variables $\{\bar{M}_t, T_t, R_t\}_{t=0}^{\infty}$ as given when making his individual decisions. The buyer's problem can be formulated in terms of the following Bellman equation:

$$v = \max_{M\in\mathbb{R}_+}\left\{-\phi_t M + \beta\left[\delta \max\left\{\Lambda_{t+1}^{r}(M), \Lambda_{t+1}^{a}(M)\right\} + (1-\delta)\,\Lambda_{t+1}^{a}(M) + v\right]\right\}. \tag{17}$$

Let M^* denote the solution to the maximization problem on the right-hand side of (17). Conjecture that, in a monetary equilibrium, agents find it optimal to exercise the option of receiving interest payment from the government. Suppose that at the optimum we have

$$-\phi_t M^* + \beta\left[\delta\Lambda_{t+1}^{r}(M^*) + (1-\delta)\,\Lambda_{t+1}^{a}(M^*)\right] \geq -\phi_t M^* + \beta\Lambda_{t+1}^{a}(M^*).$$

Thus, we can rewrite (17) as

$$(1-\beta)\,v = -\mu\,(1-\beta)\,x + \beta\delta\,[u(y) - y - \kappa] + \beta\,(1-\delta)\,[u(x) - x],$$

with the buyer's individual rationality constraint given by

$$\beta\delta u(y) - (1-\beta+\delta\beta)(y+\kappa) + \beta\,(1-\delta)\,[u(x)-x] \geq (1-\beta)(1-\mu)\left(\delta^{-1}-1\right)x + (1-\beta)\,\hat{v}.$$

We have that $y = q^*$ if the buyer's individual rationality constraint does not bind so that the socially efficient quantity will be traded in each bilateral meeting that is reported to the clearinghouse.

[6] We show later that stationary equilibria with $\mu < 1$ are not incentive compatible because agents would prefer not to exercise the option of receiving interest payment from the government.

Suppose now that at the optimum we have

$$-\phi_t M^* + \beta \left[\delta \Lambda^r_{t+1}(M^*) + (1-\delta)\Lambda^a_{t+1}(M^*)\right] < -\phi_t M^* + \beta \Lambda^a_{t+1}(M^*).$$

Then, we can rewrite (17) as

$$(1-\beta) v = -\mu \tilde{x} + \beta u(\tilde{x}),$$

with $\tilde{x}$ given by

$$u'(\tilde{x}) = \frac{\mu}{\beta}. \tag{18}$$

In this case, a buyer who is paired with a connected seller chooses not to report his trade so that he exclusively uses fiat money to pay for his purchases in the decentralized market.

Finally, for fiat money and private debt to coexist as means of payment, the following condition must hold in equilibrium:

$$-(1-\beta)\mu x + \beta\delta[u(y) - y - \kappa] + \beta(1-\delta)[u(x) - x] \geq -\mu\tilde{x} + \beta u(\tilde{x}). \tag{19}$$

Otherwise, an equilibrium is one in which all trade in the decentralized market is carried out with fiat money.

4.5. The Value of Defection

We assume that the government refuses to make interest payment to a defaulter.[7] Thus, the value of defection $\hat{v}$ satisfies the following Bellman equation:

$$\hat{v} = \max_{\hat{M} \in \mathbb{R}_+} \left\{ -\phi_t \hat{M} + \beta \left[u\left(\phi_{t+1}\hat{M}\right) + \hat{v} \right] \right\}. \tag{20}$$

Let z denote the buyer's consumption following defection. Then, we can rewrite (20) as

$$(1-\beta)\hat{v} = -\mu z + \beta u(z),$$

[7] This assumption ensures that a defaulter will be able to trade in the decentralized market only with fiat money. Not only a defaulter loses access to credit, but he is not able to receive interest payment from the government. One important implication is that a higher inflation rate punishes defaulters and induces cooperation in the credit market.

Upon defaulting on a private liability, a buyer produces and sells μz units of the consumption good in the Walrasian market to acquire money balances at date t. Then, he takes these proceeds into the decentralized market at date $t+1$ to purchase z units of the good. We have that $z = \tilde{x}$.

4.6. Stationary Monetary Equilibrium

In a stationary monetary equilibrium, the distribution of money holdings across agents at the beginning of the day subperiod at date t is such that each buyer holds $\bar{M}_{t-1}$ units of money and sellers have no money. The distribution of money holdings at the beginning of the night subperiod at date t is such that each seller holds $\bar{M}_t$ units of money and buyers have no money. A buyer finds it optimal to receive interest payment if and only if $(R-1)\,x - T \geq 0$. Using (16), this condition holds if and only if:

$$\mu \geq 1 \tag{21}$$

and

$$R \geq 1. \tag{22}$$

For a monetary equilibrium to exist, we also need to have that:

$$-\phi_t + \beta R \phi_{t+1} \leq 0, \tag{23}$$

at each date t. Otherwise, agents will demand an infinite amount of money in the Walrasian market and a monetary equilibrium would not exist. In a stationary monetary equilibrium, we can rewrite (23) as follows:

$$\frac{R}{\mu} \leq \frac{1}{\beta}. \tag{24}$$

A stationary monetary equilibrium in which aggregate real money balances are constant over time necessarily satisfies (24). As a result, a government's policy (μ, R) needs to satisfy (21), (22), and (24) to be incentive-feasible.

Definition 3 *For any given incentive-feasible policy* (μ, R)*, a stationary monetary equilibrium with credit is a triple* (x, y, z)*, with* $z = \tilde{x}$*, satisfying the nonnegativity of the repayment* *amount,*

$$y + \kappa - x\left(1 + \frac{\mu - 1}{\delta}\right) \geq 0, \tag{25}$$

the first-order condition for the optimal choice of money balances,

$$\delta R u'(y) + (1 - \delta) u'(x) = \frac{\mu}{\beta}, \tag{26}$$

and the buyer's individual rationality constraint,

$$\begin{aligned} & \beta\delta u(y) - (1 - \beta + \delta\beta)(y + \kappa) + \beta(1 - \delta)[u(x) - x] \\ \geq \quad & (1 - \beta)(1 - \mu)\left(\delta^{-1} - 1\right) x - \mu\tilde{x} + \beta u(\tilde{x}), \end{aligned} \tag{27}$$

with $y = q^*$ if (27) does not bind.

Notice that (25) and (27) imply that (19) holds. If an equilibrium with credit exists, we can have either an unconstrained equilibrium (in which case the buyer gets $y = q^*$ from a connected seller) or a constrained equilibrium (in which case the buyer gets $y < q^*$ from a connected seller). If a stationary monetary equilibrium with credit does not exist, then the unique stationary equilibrium is a pure monetary equilibrium in which the quantity $\hat{x}$ is traded in each bilateral meeting in the decentralized market. In this case, there can be no intervention because no public information is created (all trades are anonymous) so that the money supply remains constant over time.

4.7. Optimal Monetary Policy

In this subsection, we characterize the optimal policy rule subject to the implementation constraint that all trade has to be voluntary and that all monetary transfers to private agents have to be conditional on the public information available. The social welfare associated with an equilibrium with credit (x, y, z) is given by:

$$\delta[u(y) - y] + (1 - \delta)[u(x) - x] - \delta\kappa, \tag{28}$$

and the social welfare associated with an equilibrium without credit is

$$u(\hat{x}) - \hat{x}. \tag{29}$$

Without a credit system, the only allocation that can be implemented (other than autarky) is one in which each buyer gets $\hat{x}$ from a seller in the decentralized market and produces $\hat{x}$ in the centralized location. All trades are anonymous so that the private sector does not create any public information on which the government can condition its transfers.

Notice that $x = y = q^*$ maximizes the social welfare associated with an equilibrium with credit. If we can implement the socially efficient quantity q^*, then the maximum welfare level is given by

$$u(q^*) - q^* - \delta\kappa.$$

We say that a society has low record-keeping cost if the following holds:

$$u(q^*) - q^* - \delta\kappa > u(\hat{x}) - \hat{x}. \tag{30}$$

In this case, the equilibrium with credit dominates the pure monetary equilibrium without intervention, provided that the socially efficient quantity q^* can be implemented by an incentive-feasible policy. Thus, our first step is to verify whether the socially efficient quantity can indeed be implemented.

Lemma 4 *$x \leq \tilde{x}$ in an unconstrained monetary stationary equilibrium.*

Proof. Condition (24) implies

$$\left(\frac{\mu}{\beta} - R\delta\right)\frac{1}{1-\delta} \geq \frac{\mu}{\beta}.$$

Because the utility function $u : \mathbb{R}_+ \to \mathbb{R}_+$ is strictly concave, we conclude that $x \leq \tilde{x}$ as claimed. **Q.E.D.** ■

The previous result shows that we must have $x = \tilde{x} = q^*$ in order to implement the socially efficient allocation. However, we show next that there is no incentive-feasible policy that can implement $x = \tilde{x} = q^*$ as an unconstrained equilibrium.

Proposition 5 *The socially efficient allocation cannot be implemented as a stationary monetary equilibrium.*

Proof. From (21) and (24), we have that

$$\left(\frac{\mu}{\beta} - R\delta\right)\frac{1}{1-\delta} \geq \frac{1}{\beta} > 1, \tag{31}$$

which means that the maximum consumption amount (in an unconstrained equilibrium) that a buyer who is paired with an unconnected seller can get is $\hat{x} < q^*$. Since the socially efficient allocation can be implemented only as an unconstrained equilibrium, (31) implies that such an allocation is infeasible. **Q.E.D.** ■

It is not possible for the government to eliminate the consumption risk that a buyer faces in the decentralized market (in an unconstrained equilibrium). With probability δ, a buyer consumes q^* because his trade is reported to the clearinghouse, and with probability $1-\delta$, he consumes less than q^* because his money holdings are insufficient to purchase the quantity q^* from his trading partner. The government cannot induce him to carry more money balances into the decentralized market so that the efficient quantity cannot be traded in a bilateral meeting between a buyer and an unconnected seller.

We now characterize the optimal monetary policy rule under the assumption that the buyer has CRRA preferences.

Assumption 1 Suppose $u(q) = (1-\sigma)^{-1}\left[(q+b)^{1-\sigma} - b^{1-\sigma}\right]$, with the coefficient of relative risk aversion satisfying $\sigma < 1$ and with $b \in (0,1)$.

Next we show that the optimal policy will be given by $(\mu, R) = \left(\bar{\mu}, \beta^{-1}\bar{\mu}\right)$ for some money growth factor $\bar{\mu} > 1$, provided that the measure of connected sellers in the economy is sufficiently large. The money growth factor $\bar{\mu}$ is such that the buyer's individual rationality constraint holds with equality in an unconstrained equilibrium. In this case, the buyer is indifferent between defaulting on his private liability and making a repayment to the clearinghouse. The next proposition shows how to uniquely determine $\bar{\mu}$.

Proposition 6 *For κ small and $\delta \geq \sigma$, there exists a unique $\bar{\mu} > 1$ such that, with $x = \tilde{x}$ and $y = q^*$, (25) holds as a strict inequality and (27) holds with equality. As a result,*

$(x, y, z) = (\tilde{x}, q^*, \tilde{x})$ *is an unconstrained stationary monetary equilibrium in which the repayment amount is strictly positive.*

Proof. Define the function $\varphi(\mu)$ by

$$\begin{aligned}\varphi(\mu) &= \beta\delta u(q^*) - (1-\beta+\delta\beta)(q^*+\kappa) + \beta(1-\delta)[u(\tilde{x}) - \tilde{x}] \\ &\quad - [\beta u(\tilde{x}) - \mu\tilde{x}] - (1-\beta)\left(\delta^{-1} - 1\right)(1-\mu)\tilde{x}.\end{aligned}$$

Under Assumption 1, we have that $\varphi'(\mu) > 0$ for all μ. Notice that $\varphi(\beta) < 0$. Also, we have that

$$\varphi(1) = \beta\delta u(q^*) - (1-\beta+\delta\beta)(q^*+\kappa) - [\beta\delta u(\hat{x}) - (1-\beta+\delta\beta)\hat{x}] < 0.$$

Finally, notice that $\varphi(\mu) > 0$ for μ sufficiently large. To verify this claim, observe that

$$\beta\delta u(q^*) - (1-\beta+\delta\beta)(q^*+\kappa) > 0$$

for δ sufficiently close to one. Also, we have that, for any $\mu \geq 1$,

$$\varphi(\mu) > \beta\delta u(q^*) - (1-\beta+\delta\beta)(q^*+\kappa) - \beta\delta[u(\tilde{x}) - \tilde{x}] + (1-\beta)\left(\delta^{-1}-1\right)(\mu-1)\tilde{x}.$$

Because both $\tilde{x} \to 0$ and $\mu\tilde{x} \to 0$ as $\mu \to \infty$, we must have $\varphi(\mu) > 0$ for some μ sufficiently large. As a result, there exists a unique $\bar{\mu} > 1$ such that (27) holds with equality at $x = \tilde{x}$ and $y = q^*$.

Now we need to verify whether the repayment amount is nonnegative. Define the function $\psi(\mu)$ by

$$\psi(\mu) = 1 + \kappa - \left(\frac{\beta}{\mu}\right)^{\frac{1}{\sigma}}\left[1 + \delta^{-1}(\mu - 1)\right],$$

which gives the repayment amount as a function of μ. We have that $\psi'(\mu) > 0$ for all $\mu \geq (1-\delta)/(1-\sigma)$. Notice that $(1-\delta)/(1-\sigma) \leq 1$ if and only if $\delta \geq \sigma$. Because $\psi(1) > 0$, we have that $\psi(\mu)$ is strictly increasing for any $\mu \geq 1$. This means that for the value $\bar{\mu}$ such that $\varphi(\bar{\mu}) = 0$ we also have that $\psi(\bar{\mu}) > 0$. Therefore, we conclude that $(x, y, z) = (\tilde{x}, q^*, \tilde{x})$, with $\tilde{x}$ given by

$$u'(\tilde{x}) = \beta^{-1}\bar{\mu},$$

is an unconstrained stationary monetary equilibrium. **Q.E.D.** ■

With CRRA preferences, the repayment amount in an unconstrained equilibrium is strictly increasing in the money growth factor μ, which means that the higher the long-run inflation rate (which is also given by μ), the larger the repayment amount is. As we would expect, credit becomes relatively more important in transactions as the inflation rate rises.

Notice that the buyer's individual rationality constraint can only be satisfied for values of μ above the threshold value $\bar{\mu}$, which means that a stationary monetary equilibrium in which the credit system is operative exists if and only if the money growth factor is sufficiently large (in particular, above the threshold value $\bar{\mu}$).

Finally, we need to verify whether the welfare associated with the allocation $(\tilde{x}, q^*, \tilde{x})$ is greater than the welfare associated with the equilibrium without credit (the pure monetary equilibrium $\hat{x}$). Specifically, we need to verify whether the following holds:

$$\delta \left[u\left(q^*\right) - q^*\right] + (1-\delta)\left[u\left(\tilde{x}\right) - \tilde{x}\right] - \delta\kappa > u\left(\hat{x}\right) - \hat{x}. \tag{32}$$

This condition holds if δ is close to one and the record-keeping cost κ is small. This means that only a society with a sufficiently sophisticated record-keeping technology can benefit from public transactions. Not only do public transactions allow credit arrangements within the private sector, but they also permit the government to alter the rate of return on money. If the fraction of transactions that is reported in equilibrium is large relative to those that are anonymous, then the impact of the policy rule $(\mu, R) = \left(\bar{\mu}, \beta^{-1}\bar{\mu}\right)$ on social welfare is bigger. This means that public trades are socially desirable so long as the cost of reporting private trades is low and the relative fraction of these trades is sufficiently large.

5. DISCUSSION

In Andolfatto (2010), the policy rule $(\mu, R) = \left(1, \beta^{-1}\right)$, which is a version of the Friedman rule, implements the socially efficient allocation in a pure monetary economy. In this equilibrium, the price level is constant over time so that the long-run inflation rate is zero.

If we impose the additional constraint that all transfers of fiat money to private agents have to be conditional on the available public information, then we find that, with CRRA preferences, the optimal policy rule is a deviation from $\left(1, \beta^{-1}\right)$. For an economy with δ close to one and with a very low record-keeping cost κ, the optimal policy rule $\left(\bar{\mu}, \beta^{-1}\bar{\mu}\right)$, with $\bar{\mu} > 1$, is pretty close to $\left(1, \beta^{-1}\right)$. As a result, the optimal monetary policy results in a small but strictly positive long-run inflation rate.

As in Andolfatto's analysis, money is an interest-bearing government liability. The key difference of our analysis relative to Andolfatto's is that, because of the additional frictions in the environment, not all agents are able to receive an interest payment on their money holdings. This means that, from an agent's standpoint, money is an interest-bearing asset with probability δ. In our model, there is an endogenous credit system that allows buyers to relax (at a cost) their cash constraints when making their purchases in the goods market. Because agents cannot commit to their promises, the effective functioning of the credit system creates public information regarding private trades to enforce the repayment of private liabilities through societal punishments. Given that the government is unable to contact anonymous agents in the (decentralized) goods market, only those who decide to report their trades are able to receive an interest payment on their money holdings. As a result, asymmetric injections of fiat money arise in our environment due to informational frictions.

6. CONCLUSION

We have constructed a model in which private debt and interest-bearing government liabilities coexist as a means of payment. The credit system is endogenous and allows buyers to relax their cash constraints when trading in the goods market. Such a system is costly for society because private agents have to report (at a cost) their trades to others in the economy, which is necessary for the enforcement of private liabilities. The government is unable to contact anonymous agents in the goods market, which in our model implies that injections of fiat money are asymmetric. As a result, from an agent's standpoint, fiat money is an interest-bearing asset only with a certain probability: Only agents who have their trades publicly reported are able to receive an interest payment from the government. In our analysis, the asymmetric effects of monetary policy arise due to informational frictions.

The Friedman rule is infeasible, and the socially efficient allocation cannot be implemented as an equilibrium. We characterized the optimal monetary policy rule with CRRA preferences for an economy with a large fraction of connected sellers (who can potentially make their trades publicly observable) and with a low record-keeping cost. The optimal policy results in a strictly positive long-run inflation rate and nominal interest rate. This means that the constraint that all transfers of fiat money to private agents have to be conditional on the available public information is relevant for monetary policy implementation. The kinds of informational friction that we have emphasize in this paper are likely to arise in real-world monetary policymaking. These frictions provide a rationale for targeting a strictly positive inflation rate in the long run, which seems to be the norm among central banks of developed economies.

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