WORKING PAPERS
RESEARCH DEPARTMENT

WORKING PAPER NO. 11-28
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RESEARCH DEPARTMENT, FEDERAL RESERVE BANK OF PHILADELPHIA
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We would like to thank Steve Williamson, Rody Manuelli, Gaetano Antinolfi, Chris Waller, David Andolfatto, Albert Marcet, Yinting Li, the seminar participants at the St. Louis Fed, and the participants of the SWIM 2009 and the 2010 Summer Workshop on Money, Banking, Payments, and Finance at the Chicago Fed.

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Abstract

We investigate the extent to which monetary policy can enhance the functioning of the private credit system. Specifically, we characterize the optimal return on money in the presence of credit arrangements. There is a dual role for credit: It allows buyers to trade without fiat money and also permits them to borrow against future income. However, not all traders have access to credit. As a result, there is a social role for fiat money because it allows agents to self-insure against the risk of not being able to use credit in some transactions. We consider a (nonlinear) monetary mechanism that is designed to enhance the credit system. An active monetary policy is sufficient for relaxing credit constraints. Finally, we characterize the optimal monetary policy and show that it necessarily entails a positive inflation rate, which is required to induce cooperation in the credit system.

JEL classification: E4, E5.

Keywords: Fiat money; private credit; costly recordkeeping; imperfect public information; optimal monetary policy.

1. INTRODUCTION

In most modern economies, we have observed the rise of credit arrangements as a means of payment in the past two decades. Consequently, the increased number of transactions involving some kind of credit instrument has fundamentally changed the way in which economists approach monetary economies, leading to a growing literature that investigates the implications of the use of both fiat money and credit as a means of payment. Some papers in this literature include: Shi (1996); Kocherlakota and Wallace (1998); Cavalcanti and Wallace (1999a, 1999b); Aiyagari and Williamson (2000); Corbae and Ritter (2004); Berentsen, Camera, and Waller (2007); Mills (2007); Telyukova and Wright (2008); and Sanches and Williamson (2010).

Some economists have even argued that fiat money is likely to be displaced by other forms of money in the near future. Woodford (2000) provides a useful survey of the debate and discusses some of the implications for monetary policy. The main reason to believe that fiat money is still useful in transactions is twofold. First, it is costly to implement any kind of credit arrangement. For instance, the use of credit requires the construction of a record of transactions as well as the creation of a legal framework to enforce the repayment of liabilities, devoting a significant amount of resources to these activities. Second, the conduct of monetary policy is likely to affect the agents' incentives to use the private credit system, even in a world in which the use of credit as a means of payment is predominant.

In this paper, we take the view that fiat money complements the use of credit in transactions and investigate the extent to which monetary policy can enhance the functioning of the credit system. Specifically, we want to characterize the optimal return on money in a world in which private credit and fiat money coexist as alternative means of payment.

We construct a model in which traders have access to a costly technology that allows them to publicly report their transactions. As a result, these agents may be willing to voluntarily report (at a cost) their trades, together with their identities, to others if such an action allows them to have access to credit. Because agents cannot commit to their future promises and trading opportunities arrive at random, a centralized settlement system along the lines of Cavalcanti, Erosa, and Temzelides (1999) is sufficient to enforce the repayment of private liabilities. The record-keeping technology to which agents have access in our model is similar to the one in Monnet and Roberds (2008), Nosal and Rocheteau (2011), and Li (2011). In particular, a subset of sellers have the ability to verify the identity of their trading partners and publicly report their transactions. If a buyer trades with one of these sellers, then he can choose whether he wants to have his trade reported to others at a cost. If the buyer chooses to report his trade, then he can induce the seller to produce more goods than what his money balances allow him to purchase because he can also promise to make a repayment at a future date.

Alternative credit arrangements have been studied in the literature. In particular, search-theoretic models such as Shi (1996) and Corbae and Ritter (2004) have addressed the issue of the coexistence of fiat money and credit. These authors focus on decentralized credit arrangements that rely either on long-term relationships [Corbae and Ritter (2004)] or on the pledge of collateral [Shi (1996)] to induce cooperation in the credit system. As opposed to these authors, we analyze a centralized credit system that also has the advantage of allowing a central bank to implement certain kinds of policies. We want to focus on the implications of these policies for the functioning of the credit system.

As a benchmark, we characterize the equilibrium allocations in a pure credit economy. Because not all sellers are able to publicly report their trades (due to a technological restriction), buyers face consumption risk when trading bilaterally. Also, because of limited commitment, there can be equilibria in which buyers are credit-constrained, in which case they consume less than the unconstrained efficient quantity. Then, we show that the introduction of fiat money allows buyers to self-insure against the risk of being matched with a seller who is unable to offer him credit. The introduction of fiat money eliminates consumption risk, but the quantity consumed is below the unconstrained efficient quantity.

We conclude that there may be a role for policy intervention. It is now critical to define the way in which the government or monetary authority can intervene in the economy. In particular, we impose two constraints on the implementation of policies. First, any intervention must result in a net transfer of assets to private agents in order to respect incentive compatibility. This is in line with Andolfatto (2010), who imposes voluntary trade as a restriction on the set of incentive-feasible policies to rule out lump-sum tax as an instrument.[1] In addition to voluntary trade, it makes sense in our environment to impose a second constraint on the government's ability to interact with private agents: Any transfer of assets to private agents must be conditional on the available public information. We motivate this constraint on the government policies as essentially a technological restriction that limits the extent to which the government can make transfers to private agents to alter the rate of return on money. We believe this is a relevant constraint in the context of a credit economy in which it is costly to report private trades. Furthermore by considering this extra constraint, we are imposing that the government is not endowed with extra costless informational resources while still preserving the underlying frictions faced by private agents.

Andolfatto (2010) has shown that a nonlinear monetary mechanism involving the payment of interest on money holdings can implement the unconstrained efficient allocation in a pure monetary economy even if lump-sum taxes cannot be used as an instrument. By making interest payments to money holders, it is possible to equalize the real rate of return on money with the rate of time preference (the Friedman rule). Andolfatto's mechanism allows the monetary authority to control, in the long run, both the real return on money and the inflation rate. His results critically depend on the monetary authority's ability to evenly raise the real return on money across *all* money holders. We restrict attention to the same monetary mechanism and add the restriction that interest payments can be made only to those who report their trades. As a result, the monetary authority is unable to uniformly raise the real return on money across all money holders so that we preserve a social role for credit arrangements.

In the presence of the implementation restrictions we have described, we show that an active monetary policy is sufficient for relaxing the credit constraints. By effectively raising the real return on money for the subgroup of agents who currently have the opportunity of trading within the credit/payment system, the monetary authority guarantees that the

[1] This approach is also in line with recent research on optimal taxation such as Golosov, Kocherlakota, Tsyvinski (2003) and Kocherlakota (2005).

unconstrained efficient quantity is traded in each bilateral match in which buyers are able to use credit. Because a deflationary policy is not incentive-feasible, the monetary authority is unable to (indirectly) raise the real return on money for those who currently do not have an opportunity of using the credit/payment system. As a result, it cannot completely eliminate consumption risk so that the unconstrained efficient allocation is not incentive-feasible.

We then characterize the optimal monetary policy. We show that equalizing the real rate of return on money with the rate of time preference (Friedman rule) is incentive-feasible only for those who currently have an opportunity of using the credit/payment system. Moreover, this is only a necessary condition for optimality. Another property of the optimal monetary policy is to have a positive inflation rate. In particular, the monetary authority should optimally choose the *minimum* inflation rate that ensures that buyers who use credit have the incentive to fully repay their debts.

Why is it necessary to have a positive inflation rate? Introducing fiat money into a credit economy gives agents an alternative (decentralized) mechanism to trade. As we have seen, fiat money allows agents to self-insure against the risk of not being able to use credit in some transactions, and their ability to self-insure crucially depends on the real return on money. It is very difficult to induce buyers to repay their liabilities when the alternative to credit is trading with fiat money in an environment in which the inflation rate is virtually zero (especially if buyers are sufficiently patient). For this reason, a slightly positive inflation rate is required to support cooperation in the credit system.

The rest of the paper is organized as follows. In section 2, we describe the model. In section 3, we characterize equilibria for the pure credit economy. In section 4, we introduce fiat money and study its implications for the credit system. In sections 5 and 6, we characterize the optimal monetary policy. In section 7, we discuss the properties of the optimal policy and the nature of our monetary mechanism. Section 8 concludes.

2. THE MODEL

2.1. Agents

Our framework builds on Lagos and Wright (2005) and Rocheteau and Wright (2005). There is a continuum of infinitely lived buyers and sellers. Each buyer is indexed by $i \in [0,1]$ and each seller is indexed by $j \in [0,1]$. Time is discrete, and each period is divided into two subperiods: day and night. Within each subperiod, there is a unique perishable consumption good that is produced and consumed. In the day subperiod, a seller does not want to consume but can produce one unit of the consumption good with one unit of effort; a buyer wants to consume but is unable to produce. In the night subperiod, both types want to consume and are able to produce one unit of the consumption good with one unit of effort. Each agent is endowed with N units of effort. Neither a buyer nor a seller can commit to his or her promises.

A buyer has preferences given by:

$$u(q_i) + c_i - n_i, \tag{1}$$

where $q_i \in \mathbb{R}_+$ is his consumption in the day subperiod, c_i is his consumption in the night subperiod, and $n_i \in \mathbb{R}$ is his production in the night subperiod. Assume that $u : \mathbb{R}_+ \to \mathbb{R}_+$ is strictly concave, increasing, and continuously differentiable, with $u(0) = 0$ and $u'(0) = \infty$.

A seller has the following utility function over goods and effort:

$$-q_j + c_j - n_j, \tag{2}$$

where $q_j \in \mathbb{R}_+$ is her production in the day subperiod, c_j is her consumption in the night subperiod, and $n_j \in \mathbb{R}$ is her production in the night subperiod. Buyers and sellers have a common discount factor between periods, which we denote by $\beta \in (0,1)$.

2.2. Markets

Agents trade in two sequential markets in every period. In the day market, agents are randomly and bilaterally matched in such a way that each buyer meets a seller. In the night market, agents interact in a centralized location where the terms of trade are given by competitive pricing. As in Lagos and Wright (2005), we refer to the first market as the decentralized market and to the second market as the centralized market.

2.3. Recordkeeping

There are two types of sellers: connected and unconnected. A connected seller has access to a record-keeping technology that allows her to verify the identity of her trading partner and record her transaction during the day subperiod. Once a transaction is recorded, it can be reported to other agents. The use of this technology costs $\kappa > 0$ units of the good for the seller in the day subperiod. An unconnected seller does not have access to a record-keeping technology and, consequently, is unable to make her interaction with her trading partner publicly observable. There is a measure $\delta \in [0, 1]$ of connected sellers and a measure $1 - \delta$ of unconnected sellers.

Notice that a connected seller is willing to extend credit to her trading partner in the decentralized market provided that society can enforce any repayment in the centralized location. One possible financial arrangement is to have a seller producing for the buyer with whom she is paired in the day subperiod in exchange for a repayment in the night subperiod. However, a buyer cannot commit to his promise of making a repayment in the night subperiod. To enforce the repayment of private liabilities, there must be some form of societal punishment on defaulters. Otherwise, a seller would not be willing to produce for a buyer in the decentralized market unless she received something tangible and valuable in exchange (such as fiat money). This must be the case for an unconnected seller who is unable to make her trades publicly observable. On the other hand, a connected seller has the ability to make her transaction with a buyer publicly observable. If a buyer does not repay his loan, other agents in the economy will observe his defection. If there exists a mechanism that enforces any repayment in the centralized location, a connected seller is willing to extend credit to her trading partners in the decentralized market. In the next section, we provide more details on the exact punishment that society can impose on defaulters.

3. A PURE CREDIT ECONOMY

In this section, we discuss the existence and uniqueness of equilibrium in a pure credit economy. We believe that the pure credit economy is a useful benchmark that will help us understand the role of government policies. To enforce the repayment of private liabilities, there is a clearinghouse that collects all reports from connected sellers during the day subperiod. The clearinghouse is also responsible for receiving repayments from buyers and making payments to sellers in the centralized location. Any transaction that is reported to the clearinghouse becomes publicly observable. Because a buyer cannot commit to his promises, the clearinghouse needs to impose some kind of punishment on him if he fails to make a repayment. It is not possible for the clearinghouse to directly punish a buyer who has defaulted on his loan. However, the clearinghouse can indirectly punish a defaulter by refusing to make a payment (in the centralized location) to any seller who trades with him. Notice that the identity of a defaulter is publicly observable. As a result, a connected seller will not be willing to extend credit to a defaulter. To extend credit to him, the seller would have to report their transaction, together with their identities, to the clearinghouse, which would then refuse to make a transfer to the seller even if it received a repayment from the buyer in the centralized location. Thus, a buyer who reneges on her liability loses access to credit.

A buyer can choose whether to have his transaction with a connected seller reported to the clearinghouse. If the buyer chooses not to report his trade, then credit will not be available for him: The seller cannot enforce the repayment of a private liability in the centralized location and, as a result, is not willing to extend credit. Therefore, no trade takes place.

To determine the terms of trade in the decentralized market, we assume that the buyer makes a take-it-or-leave-it offer to the seller, who either accepts or rejects it.[2] As a result, the buyer extracts all surplus from trade when proposing the terms of trade. This partic-

[2] Here we implicitly assume that it is infinitely costly for the buyer to make a second offer to the seller if the first one has been rejected.

ular bargaining protocol simplifies the analysis without compromising the generality of our results. In Lagos and Wright (2005), there is an inefficiency arising from the generalized Nash bargaining solution: If the seller has some bargaining power, the socially efficient allocation cannot be implemented as a monetary equilibrium.[3] To concentrate on the informational frictions that we emphasize in this paper, we simplify the analysis by ruling out any potential bargaining inefficient.

3.1. Bilateral Trade with an Unconnected Seller

Consider a match between a buyer and an unconnected seller. In this case, there is no possibility of making their trade publicly observable: The unconnected seller is unable to report their trade to the clearinghouse because she lacks the technology to record their transaction. As a result, such a seller is not willing to extend credit to the buyer; therefore, no trade takes place.

3.2. Bilateral Trade with a Connected Seller

Consider now a match between a buyer and a connected seller. Suppose initially that the buyer chooses to have his trade reported. Let q denote the quantity of the consumption good that the seller produces for the buyer in the day subperiod, and let l denote the repayment amount that the buyer promises to make in the night subperiod. Thus, the terms of trade (q, l) are determined as follows:

$$\max_{(q,l)\in\mathbb{R}_+^2} \left[u\left(q\right) - l\right],$$

subject to the seller's individual rationality constraint,

$$-q - \kappa + l \geq 0,$$

and the buyer's individual rationality constraint,

$$-l + v \geq \hat{v}. \tag{3}$$

[3] Gomis-Porqueras and Peralta-Alva (2010) show that, in order to obtain the first-best allocation, fiscal policies need to be active.

Here v denotes the buyer's expected discounted utility at the end of the night subperiod, and $\hat{v}$ denotes his expected discounted utility upon default (given the punishment that the clearinghouse will impose on him). Condition (3) simply says that the repayment amount l must be such that the buyer weakly prefers to repay his loan to renege on his debt and be punished for taking this action. The solution to this problem is as follows. If the buyer's individual rationality constraint binds (3), then $l = v - \hat{v}$ and $q = v - \hat{v} - \kappa$; otherwise, $l = q^* + \kappa$ and $q = q^*$. Let $\Lambda^r(v, \hat{v})$ denote the buyer's payoff of trading with a connected seller. Thus, we have

$$\Lambda^r(v, \hat{v}) = u(v - \hat{v} - \kappa) - v + \hat{v},$$

if $v < q^* + \kappa + \hat{v}$; and

$$\Lambda^r(v, \hat{v}) = u(q^*) - q^* - \kappa,$$

if $v \geq q^* + \kappa + \hat{v}$. Before concluding the trade, the seller sends the report $\{(i, j), (q, l)\}$ to the clearinghouse, where $i \in [0, 1]$ is the buyer's identity, $j \in [0, 1]$ is the seller's identity, and (q, L) are the terms of trade.

If the buyer chooses not to have his trade reported, then no trade takes place for the same reason as in the case of a meeting with an unconnected seller.

3.3. Buyer's Bellman Equation

The buyer's problem can be formulated in terms of the following Bellman equation:

$$v = \beta\left[\delta \max\left\{\Lambda^r(v, \hat{v}), 0\right\} + v\right].$$

Notice that a buyer trades in the decentralized market only with probability δ. Notice also that a buyer always has the option of not having his trade reported, in which case no trade takes place. Thus, a trade happens if and only if the buyer's payoff $\Lambda^r(v, \hat{v})$ is greater than zero.

3.4. The Value of Defection

As we have seen, the punishment imposed by the clearinghouse results in autarky for a buyer who reneges on his private liabilities. As we have mentioned, a buyer who defaults on his liabilities will no longer be able to trade in the decentralized market because connected sellers will refuse to extend him credit. Recall that the clearinghouse indirectly punishes defaulters by refusing to send payments to connected sellers who have traded with defaulters. This means that the clearinghouse refuses to settle any public transaction involving a defaulter. Thus, we must have $\hat{v} = 0$.

3.5. Stationary Equilibrium

Now we are able to define an equilibrium for the pure credit economy. Throughout the paper, we restrict attention to stationary equilibria. In particular, we can define an equilibrium simply in terms of the value v (the buyer's expected discounted utility). Once we have determined the equilibrium value for v, we can easily recover the loan amount q as well as the repayment amount l observed in each transaction between a buyer and a connected seller.

Definition 1 *A stationary equilibrium of the pure credit economy is a value v^* satisfying*

$$(1-\beta)\, v^* = \beta\delta\Lambda^r\left(v^*, 0\right),$$

together with the condition $v^* \geq \kappa$.

There can be two types of equilibria. Suppose that $\beta\delta u\left(q^*\right) \geq \left(1-\beta+\delta\beta\right)\left(q^*+\kappa\right)$. Then, the equilibrium value for v is given by

$$v^* = \frac{\beta\delta\left[u\left(q^*\right) - q^* - \kappa\right]}{1-\beta}.$$

In this case, the surplus-maximizing quantity q^* is traded in each bilateral match between a buyer and a connected seller. The repayment amount in the centralized location is given by $q^* + \kappa$. The buyer's individual rationality constraint does not bind so that he obtains enough credit to get the surplus-maximizing quantity q^* in the decentralized market.

Suppose now that $\beta\delta u(q^*) < (1-\beta+\delta\beta)(q^*+\kappa)$. Then, the equilibrium value for v is given by

$$v^* = \frac{\beta\delta u(v^*-\kappa)}{1-\beta+\delta\beta}.$$

In this case, the quantity $v^*-\kappa < q^*$ is traded in each bilateral match between a buyer and a connected seller. The repayment amount is now given by v^*. The buyer's individual rationality constraint binds so that he is credit-constrained.

3.6. Discussion

In the pure credit economy, trade only takes place in matches involving connected sellers. Because agents cannot commit to their future promises, credit is infeasible in transactions that cannot be publicly reported. This technological restriction introduces consumption risk for the buyer because with probability $1-\delta$ he gets zero consumption. Notice that the extent to which consumption risk is a problem depends on the available record-keeping technology. For instance, suppose that δ is very close to one. In this case, society has the ability to record almost all transactions, which mitigates the consumption risk that buyers face in decentralized exchange. On the other hand, if δ is close to zero, the consumption risk they face is severe. Notice that a lower value for δ makes the case $\beta\delta u(q^*) < (1-\beta+\delta\beta)(q^*+\kappa)$ more likely to arise, which means that not only is credit unavailable in a (relatively large) fraction $1-\delta$ of matches, but it is also restricted in the trades involving connected sellers.

The pure credit economy makes it clear why fiat money can be socially useful. Fiat money allows two agents to trade even though no public information about their transactions is created. In particular, the only way to induce a seller to produce for a buyer is to have the latter giving something tangible and valuable in exchange, in which case settlement is immediate. As a result, the introduction of fiat money enlarges the set of feasible trades, mitigating consumption risk in the decentralized market. We turn to the analysis of an economy in which fiat money and private debt coexist as a means of payment in the next section.

4. INTRODUCING FIAT MONEY

Suppose now that each buyer is initially endowed with $\bar{M}$ units of money. Suppose also that there is no change in the initial stock of money; that is, there is no government intervention. As in the previous section, a buyer can choose whether to have his transaction with a connected seller reported to the clearinghouse. If the buyer chooses not to report his trade, then credit will not be available for him. In this case, the seller is willing to produce for the buyer only if she receives fiat money in exchange. As in the pure credit economy, a credit transaction is incentive-feasible only in a bilateral meeting between a buyer and a connected seller. As in Lagos and Wright (2005), there is a Walrasian market in the centralized location in which agents can trade goods for fiat money at a competitive price. Trades in this market are always anonymous.

4.1. Bilateral Trade with an Unconnected Seller

Consider the bargaining problem between a buyer and an unconnected seller. Let ϕ_t denote the value of money in the centralized location at date t, and suppose that the buyer has M units of money. Let (q, D) denote the terms of trade; that is, q denotes the amount of goods that the seller produces for the buyer in exchange for D units of money. Taking ϕ_t as given, the buyer's problem is:

$$\max_{(q,D)\in\mathbb{R}_+^2} \left[u\left(q\right) - \phi_t D\right],$$

subject to the seller's individual rationality constraint,

$$-q + \phi_t D \geq 0, \tag{4}$$

and the buyer's cash constraint,

$$D \leq M. \tag{5}$$

The solution to this problem is as follows: $q = q^*$ and $D = q^*/\phi_t$ if $M \geq q^*/\phi_t$; or $q = \phi_t M$ and $D = M$ if $M < q^*/\phi_t$. Finally, the buyer's payoff of trading with an unconnected seller is given by

$$\Lambda_t^a(M) = \begin{cases} u(\phi_t M) \text{ if } M < q^*/\phi_t; \\ u(q^*) \text{ if } M \geq q^*/\phi_t. \end{cases} \tag{6}$$

4.2. Bilateral Trade with a Connected Seller

Now consider the bargaining problem between a buyer and a connected seller. First, we describe the solution to the bargaining problem, assuming that the buyer wants to have his trade reported. As in the previous section, let v denote the buyer's expected discounted utility at the end of the night subperiod, and let $\hat{v}$ denote his expected discounted utility upon default. The terms of trade are given by (q, L, D), where q is the quantity produced by the seller, D is the monetary transfer from the buyer to the seller in the decentralized market, and L is the monetary repayment that the buyer promises to make in the centralized location. Taking ϕ_t, v, $\hat{v}$ as given, the buyer's problem is as follows:

$$\max_{(q,D,L)\in\mathbb{R}_+^3} \left[u(q) - \phi_t L - \phi_t D\right],$$

subject to the seller's individual rationality constraint,

$$-q - \kappa + \phi_t L + \phi_t D \geq 0, \tag{7}$$

the buyer's cash constraint (5), and the buyer's individual rationality constraint,

$$-\phi_t L + v \geq \hat{v}. \tag{8}$$

Here the seller produces q units of the consumption good for the buyer in exchange for D units of money and a promise of repayment of L units of money in the centralized location. In the night subperiod, the buyer produces $\phi_t L$ units of the good and exchange them in the Walrasian market for L units of money. Finally, he makes a transfer to the clearinghouse, which in turn makes a payment L to the seller with whom he was paired in the day subperiod. There is no cost of operating the clearinghouse other than the cost of reporting a transaction. A seller needs to pay $\kappa > 0$ to report her transaction with her trading partner in the decentralized market, which makes it harder to satisfy her individual rationality constraint. Because (7) holds with equality at the optimum, the buyer ends up paying for the record-keeping cost. The benefit of having his trade reported is that the buyer can consume more than what his money holdings permit him to purchase because he can promise to make a repayment to the seller in the centralized location through the clearinghouse. Although credit is costly for the buyer, it allows him to relax his cash constraint.

The unconstrained solution to the bargaining problem is $\phi_t L + \phi_t D - \kappa = q^*$. This means that, if $\phi_t M - \kappa < q^*$, then we must have $D = M$ and $L > 0$ at the optimum. Thus, we can rewrite the buyer's problem as

$$\Lambda_t^r (M, v, \hat{v}) = \max_{L \geq 0} \left[u \left(\phi_t L + \phi_t M - \kappa \right) - \phi_t L \right], \tag{9}$$

subject to (8). The solution to this problem is: $L = (q^* - \phi_t M + \kappa) / \phi_t$ if $q^* - \phi_t M + \kappa \leq v - \hat{v}$; or $L = (v - \hat{v}) / \phi_t$ if $q^* - \phi_t M + \kappa > v - \hat{v}$. Finally, the seller sends the report $\{(i, j), (q, D, L)\}$ to the clearinghouse, where $i \in [0, 1]$ is the buyer's identity, $j \in [0, 1]$ is the seller's identity, and (q, D, L) are the terms of trade.

Suppose now that the buyer chooses not to have his trade reported to the clearinghouse. In this case, any trade proposed by the buyer involving a positive repayment amount will be rejected by the seller. Thus, the buyer's problem is the same as the one he faces when meeting with an unconnected seller in the decentralized market.

4.3. Buyer's Bellman Equation

Throughout the paper, we restrict attention to stationary monetary equilibria in which aggregate real money balances are constant over time. In this kind of equilibrium, each buyer anticipates that the value of money in the centralized location will be constant over time: $\phi_t = \phi > 0$ for all $t \geq 0$. Thus, the buyer's problem in the centralized location can be expressed in terms of the following Bellman equation:

$$v = \max_{M \in \mathbb{R}_+} \left\{ -\phi M + \beta \left[\delta \max \left\{ \Lambda_{t+1}^r (M, v, \hat{v}), \Lambda_{t+1}^a (M) \right\} + (1 - \delta) \Lambda_{t+1}^a (M) + v \right] \right\}, \tag{10}$$

with $\Lambda^r_{t+1}(M, v, \hat{v})$ given by (9) and $\Lambda^a_{t+1}(M)$ given by (6). Here M is the amount of money that the buyer acquires in the Walrasian market at date t and takes with him into the decentralized market at date $t+1$. Let M^* denote his optimal choice of money holdings in the Walrasian market.

Now we make the same change of variables as in Sanches and Williamson (2010), which will prove to be useful for describing an equilibrium allocation. Let y denote the buyer's daytime consumption if he reports his trade to the clearinghouse, and let x denote his daytime consumption if he does not report the trade. Suppose that, at the optimum, we have

$$-\phi M^* + \beta\left[\delta\Lambda^r_{t+1}(M^*, v, \hat{v}) + (1-\delta)\Lambda^a_{t+1}(M^*)\right] \geq -\phi M^* + \beta\Lambda^a_{t+1}(M^*).$$

In this case, the buyer chooses to report his trade with a connected seller to other agents in order to have access to credit. We can rewrite equation (10) in terms of x and y as follows:

$$(1-\beta)v = -(1-\beta)x + \beta\delta[u(y) - y - \kappa] + \beta(1-\delta)[u(x) - x],$$

with the buyer's individual rationality constraint (8) given by

$$\beta\delta u(y) - (1-\beta+\delta\beta)(y+\kappa) + \beta(1-\delta)[u(x) - x] \geq (1-\beta)\hat{v}.$$

When making his portfolio decision in the centralized location at date t, a buyer finds it optimal to carry some currency into the decentralized market at date $t+1$ because he may be matched with an unconnected seller (with probability $1-\delta$). In this case, trade takes place only if he has money to pay for his purchases. Even in a trade with a connected seller that is reported to the clearinghouse, the buyer may use both credit and fiat money to pay for the amount y that the seller produces for him.

Suppose now that, at the optimum, we have

$$-\phi M^* + \beta\left[\delta\Lambda^r_{t+1}(M^*, v, \hat{v}) + (1-\delta)\Lambda^a_{t+1}(M^*)\right] < -\phi M^* + \beta\Lambda^a_{t+1}(M^*).$$

In this case, the cost associated with credit exceeds the benefit of trading with credit. As a result, the buyer never reports his trade when he is paired with a connected seller. Thus, he always uses fiat money to pay for his purchases in the decentralized market. We can then rewrite equation (10) as follows:

$$(1-\beta)\,v = -x + \beta u\,(x)\,,$$

with the value of x given by

$$u'\,(\hat{x}) = \beta^{-1}. \tag{11}$$

For fiat money and private debt to coexist as a means of payment, the following must hold in equilibrium:

$$-\,(1-\beta)\,x + \beta\delta\,[u\,(y) - y - \kappa] + \beta\,(1-\delta)\,[u\,(x) - x] \geq -\hat{x} + \beta u\,(\hat{x})\,. \tag{12}$$

Otherwise, an equilibrium is one in which all trades in the decentralized market are carried out with fiat money only.

4.4. The Value of Defection

If a buyer fails to make a repayment in the centralized location, he will be able to use only fiat currency to pay for his future purchases. When a buyer fails to make a repayment, the clearinghouse makes his defection publicly observable. This means that, if a defaulter wants to have any of his future trades reported, his identity will be revealed to his trading partner. The latter (a connected seller) knows that the clearinghouse will refuse to make her a transfer in the centralized location even if a repayment is actually collected from the buyer. Because the proposed trade would involve a positive repayment amount (otherwise, the buyer would prefer not to report the trade), she would get a negative payoff if she carried out the proposed trade. As a result, the seller will not accept the terms proposed by the buyer. Taking this into account, a defaulter who is paired with a connected seller chooses not to have his trade reported to the clearinghouse because this option would involve a cost without any additional benefit. Thus, the value of defection $\hat{v}$ satisfies the following Bellman equation:

$$\hat{v} = \max_{\hat{M}\in\mathbb{R}_+} \left\{-\phi\hat{M} + \beta\left[u\left(\phi\hat{M}\right) + \hat{v}\right]\right\}. \tag{13}$$

Let z denote the buyer's consumption following defection. Then, we can rewrite (13) as follows:

$$(1-\beta)\,\hat{v} = -z + \beta u\,(z)\,.$$

A defaulter produces and sells z units of the consumption good in the Walrasian market in order to acquire enough money balances at date t to purchase z units of the good in the decentralized market at date $t+1$. We have that $z = \hat{x}$, with $\hat{x}$ satisfying (11).

4.5. Stationary Monetary Equilibrium

The distribution of money holdings at the end of the night subperiod is such that every buyer holds the same amount of money and sellers carry no money into the decentralized market. This result is a direct consequence of the quasilinearity with respect to effort and of the fact that agents have periodic access to centralized trade; see Lagos and Wright (2005) and Rocheteau and Wright (2005). Hence, we can characterize an equilibrium allocation in terms of the daytime consumption y of a buyer who has his trade reported and the daytime consumption x of a buyer who does not have his trade reported, together with the daytime consumption z that a buyer would get had he defaulted on his liabilities.

Definition 2 *A stationary monetary equilibrium with credit is a triple (x, y, z), with $z = \hat{x}$, satisfying the nonnegativity of the repayment amount*

$$y - x + \kappa \geq 0, \tag{14}$$

the first-order condition for the optimal choice of money balances,

$$\delta u'\,(y) + (1-\delta)\,u'\,(x) = \beta^{-1}, \tag{15}$$

and the buyer's individual rationality constraint

$$\beta\delta u\,(y) - (1-\beta+\delta\beta)\,(y+\kappa) + \beta\,(1-\delta)\,[u\,(x) - x] \geq -\hat{x} + \beta u\,(\hat{x})\,, \tag{16}$$

with $y = q^*$ if (16) does not bind.

Notice that (14) and (16) imply that (12) holds. An equilibrium in which fiat money and private debt are used as a means of payment has to be one in which a buyer who is paired with a connected seller finds it optimal to have his trade reported, despite the costs associated with this choice. Equations (15) and (16) characterize the consumption plans x and y. Note that if we set $y = q^*$, we obtain x from (15). If (14) and (16) are satisfied, then the socially efficient quantity q^* is traded in each bilateral trade between a buyer and a connected seller. Otherwise, (15) and (16) holding with equality determine the values of x and y. If there exists no (x, y, z) satisfying (14)-(16), together with $z = \hat{x}$, then an equilibrium is one in which all trades in the decentralized market are carried out with fiat money: Buyers never report their trades to the clearinghouse and, as a result, credit disappears. In this equilibrium, the quantity $\hat{x}$ is traded in every meeting in the decentralized market. In the night subperiod, each buyer then produces $\hat{x}$ and each seller consumes $\hat{x}$.

With the introduction of fiat money, a buyer who now finds himself matched with an unconnected seller in the decentralized market can use his money holdings to trade. Consequently, fiat money can be used to self-insure against the risk of being matched with an unconnected seller. However, there is a limit to the extent to which buyers can use fiat money to self-insure. In particular, notice that, in an equilibrium in which fiat money and private debt coexist, we have $x \leq y$, which means that a buyer who reports his trade is able to consume more than a buyer who cannot obtain credit from his trading partner. From a buyer's standpoint, there still exists consumption risk. A buyer who trades with an unconnected seller faces a cash constraint that can eventually bind. On the other hand, a buyer who trades with a connected seller can promise to make a repayment in the centralized location in order to consume more than what his money holdings permit him to purchase. So long as the repayment amount is individually rational, a buyer who trades with a connected seller will be able to consume more in the day market.

Although the possibility of trading with credit on the decentralized market may seem attractive for buyers, the following result shows that there can be no equilibrium in which money and private debt coexist.

Proposition 3 *With a constant money supply, the unique stationary monetary equilibrium is a pure monetary equilibrium in which the quantity $\hat{x}$ is traded in the decentralized market.*

Let us carefully explain the previous result, which may seem surprising initially. To prove Proposition 2, we essentially showed that the buyer's individual rationality constraint cannot be satisfied in equilibrium so that there is no incentive-feasible repayment amount. Why is it so difficult to satisfy (16)? Notice that on the right-hand side of (16) we no longer have the payoff of autarky for the buyer as we had in the pure credit economy. In fact, the buyer's outside option consists of exclusively using money to trade in an environment in which the value of money is constant over time. On the left-hand side, we have the buyer's disutility of repaying the loan amount $y - x + \kappa$ plus the value of continuing to trade using credit. With a stable value of money (zero inflation), agents are able to self-insure at a low cost, effectively raising the value of default. As a result, each buyer has an additional incentive to default on his liabilities. Because sellers anticipate this, they extend no credit in the decentralized market.

With the introduction of fiat money, we have essentially provided households with an alternative mechanism to trade on the decentralized market. As a result, this has critically affected the functioning of the credit system. In particular, the credit system is not used because now it is more difficult to induce buyers to cooperate.[4] As we will show later, it is necessary to implement a positive inflation rate to induce buyers to fully repay their liabilities when they use credit.

Even though the introduction of fiat money eliminates consumption risk, buyers consume less than the unconstrained efficient quantity q^*. In the next section, we study how policy intervention can affect the functioning of the private credit system.

[4] Berentsen, Camera, and Waller (2007) obtain a similar result.

5. WELFARE-IMPROVING POLICIES WITH IMPERFECT PUBLIC INFORMATION

In this section, we discuss the possibility of government intervention. We want to investigate the extent to which a policy intervention can improve upon the laissez-faire economy by altering the real rate of return on money. To be in line with the recent literature on optimal policy, we want to clearly define what is actually incentive-feasible for the government. In particular, we focus on two constraints.

First, any intervention must result in a net transfer of fiat money to private agents in order to respect voluntary trade. This is in line with Andolfatto (2010), who imposes voluntary trade as a restriction on the set of incentive-feasible policies to rule out lump-sum tax as an instrument. In addition to voluntary trade, it makes sense in our environment to impose a second constraint on the government's ability to intervene in the economy: We assume that any transfer of assets to private agents must be conditional on the available public information. We motivate this constraint on the government policies as essentially a technological restriction that limits the extent to which the government can make transfers to private agents to alter the rate of return on money. We believe this is a relevant constraint in the context of a credit economy in which it is costly for agents to report their trades.

As a result, any information the government has about the economy is necessarily reported by agents so that the government needs to interact with the clearinghouse to implement any policy: The transfer of assets can be made only to those who have reported their trades and identities to the clearinghouse. Recall that the clearinghouse keeps a public record of reports and identities.

We consider the same monetary mechanism as Andolfatto (2010): An agent who holds M units of money in the day subperiod and has his trade reported to the clearinghouse can transform his balances into $RM - T$ units of money. Here R is the gross nominal interest rate announced by the government and T represents a redemption fee.

Notice that this kind of intervention creates an additional incentive for agents to report their trades: Not only do public transactions allow credit arrangements within the private sector, but they also permit an agent to receive a net transfer of fiat money. The government can intervene in the economy only if the clearinghouse is used to settle private transactions. The government can induce agents to report their trades by paying interest on money holdings; that is, the benefit of reporting a trade is not only to have access to credit but also to be able to receive an interest payment. However, if it is very expensive to report a transaction, then a buyer who is paired with a connected seller may prefer not to use credit nor to receive an interest payment; as a result, the clearinghouse is not used. In this case, all trade is carried out with fiat currency. Thus, all transactions in the economy are anonymous and the government cannot intervene.

5.1. Government's Budget Constraint

The government's budget constraint is given by

$$\delta T_t + \bar{M}_t - \bar{M}_{t-1} = (R_t - 1)\, \delta \bar{M}_{t-1}.$$

We have anticipated that, in a monetary equilibrium, all buyers who have access to credit choose to use it by reporting their trades to the clearinghouse. We have also anticipated that, at the beginning of the day subperiod at date t, each buyer holds $\bar{M}_{t-1}$ units of money and that sellers carry no money into the decentralized market. We will show later that, in a monetary equilibrium, this will be the endogenous distribution of money holdings across agents at the beginning of the day subperiod. As a result, aggregate interest payments are given by $(R-1)\,\delta\bar{M}_{t-1}$, with an aggregate revenue from the redemption fee equal to δT_t.

We restrict attention to monetary policy rules for which the money supply grows at a constant gross rate $\mu > 0$ ($\bar{M}_t = \mu \bar{M}_{t-1}$ for all $t \geq 0$) and the gross interest rate is constant over time ($R_t = R \geq 1$ for all $t \geq 0$). As in the previous section, let ϕ_t denote the value of money in the centralized location at date t. Thus, we can rewrite the government's budget constraint in real terms as follows:

$$\phi_t T_t = \frac{\phi_t \bar{M}_t}{\delta} \left[\frac{(R-1)\,\delta + 1}{\mu} - 1 \right]. \tag{17}$$

5.2. Bilateral Trade with an Unconnected Seller

The terms of trade are exactly the same as in section 4.1.

5.3. Bilateral Trade with a Connected Seller

Here we describe the bargaining problem between a buyer and a connected seller. Suppose initially that the buyer wants to have his trade reported. Define $\varphi_t(D) = \max\{D, R_t D - T_t\}$. Taking ϕ_t, T_t, R_t, v, $\hat{v}$ as given, the buyer solves the following problem:

$$\max_{(q,L,D)\in\mathbb{R}_+^3} \left[u(q) - \phi_t L - \phi_t D\right],$$

subject to the seller's individual rationality constraint,

$$-q - \kappa + \phi_t L + \phi_t \varphi_t(D) \geq 0,$$

the buyer's cash constraint (5), and the buyer's individual rationality constraint (8). As in the previous section, v denotes the buyer's expected discounted utility at the end of the night subperiod, and $\hat{v}$ denotes his expected discounted utility upon default. If the buyer chooses not to trade with the seller in the day subperiod, he will not be able to receive interest payments because there will be no transaction to be reported to the clearinghouse. For this reason, the buyer's surplus from trade is given by $u(q) - \phi_t L - \phi_t D$.

Suppose that $T_t(R_t - 1)^{-1} < (q^* + \kappa + \phi_t T_t)/\phi_t R_t$. The buyer's surplus from trade as a function of D and L is

$$S_t^r(D, L) = u(\phi_t L + \phi_t D - \kappa) - \phi_t L - \phi_t D,$$

if $0 < D < T_t(R_t - 1)^{-1}$, or

$$S_t^r(D, L) = u(\phi_t L + \phi_t(R_t D - T_t) - \kappa) - \phi_t L - \phi_t D,$$

if $D \geq T_t(R_t - 1)^{-1}$.

Now if $\phi_t \max\{R_t M - T_t, M\} - \kappa < q^*$, then we must have $D = M$ and $L > 0$ at the optimum. Then, we can rewrite the buyer's problem as:

$$\max_{L\in\mathbb{R}_+} S_t^r(M, L),$$

subject to (8). If the constraint (8) binds, we have that $L = (v - \hat{v}) / \phi_t$. Otherwise, the repayment amount is given by

$$L = (q^* + \kappa - \phi_t M) / \phi_t \text{ if } M < T_t (R_t - 1)^{-1}$$

or

$$L = [q^* + \kappa - \phi_t (R_t M - T_t)] / \phi_t \text{ if } T_t (R_t - 1)^{-1} \leq M < (q^* + \kappa + \phi_t T_t) / \phi_t R_t.$$

Again, let $\Lambda_t^r (M, v, \hat{v})$ denote the buyer's payoff from trading with a connected seller when the trade is reported to the clearinghouse. Finally, the seller sends the report $\{(i, j), (q, D, L)\}$ to the clearinghouse.

Suppose now that the buyer chooses not to have his trade reported to the clearinghouse. Then, the buyer's problem is the same as the one he faces when meeting with an unconnected seller.

5.4. Buyer's Bellman Equation

Each buyer takes the value of money $\{\phi_t\}_{t=0}^{\infty}$ and the monetary policy variables $\{\bar{M}_t, T_t, R_t\}_{t=0}^{\infty}$ as given when making his individual decisions. The buyer's problem can be formulated in terms of the following Bellman equation:

$$v = \max_{M \in \mathbb{R}_+} \left\{ -\phi_t M + \beta \left[\delta \max \left\{ \Lambda_{t+1}^r (M, v, \hat{v}), \Lambda_{t+1}^a (M) \right\} + (1 - \delta) \Lambda_{t+1}^a (M) + v \right] \right\}. \quad (18)$$

Given v and $\hat{v}$, let M^* denote the solution to the maximization problem on the right-hand side of (18). Conjecture that, in a monetary equilibrium, agents find it optimal to exercise the option of receiving interest payments from the government. As in the previous section, let y denote the buyer's daytime consumption if he reports his trade to the clearinghouse, and let x denote his daytime consumption if he does not report the trade. Suppose that at the optimum we have

$$-\phi_t M^* + \beta \left[\delta \Lambda_{t+1}^r (M^*, v, \hat{v}) + (1 - \delta) \Lambda_{t+1}^a (M^*) \right] \geq -\phi_t M^* + \beta \Lambda_{t+1}^a (M^*).$$

In this case, the buyer finds it optimal to have his trade reported. Thus, we can rewrite (18) as

$$(1-\beta)\, v = -\mu\,(1-\beta)\, x + \beta\delta\,[u\,(y) - y - \kappa] + \beta\,(1-\delta)\,[u\,(x) - x]\,,$$

with the buyer's individual rationality constraint given by

$$\beta\delta u\,(y) - (1-\beta+\delta\beta)\,(y+\kappa) + \beta\,(1-\delta)\,[u\,(x) - x] \geq (1-\beta)\,(1-\mu)\left(\delta^{-1}-1\right) x + (1-\beta)\,\hat{v}.$$

We have that $y = q^*$ if the buyer's individual rationality constraint does not bind so that the socially efficient quantity will be traded in each bilateral meeting that is reported to the clearinghouse.

Suppose now that at the optimum we have

$$-\phi_t M^* + \beta\left[\delta\Lambda^r_{t+1}\,(M^*, v, \hat{v}) + (1-\delta)\,\Lambda^a_{t+1}\,(M^*)\right] < -\phi_t M^* + \beta\Lambda^a_{t+1}\,(M^*)\,.$$

In this case, the buyer finds it optimal not to have his trade reported: He prefers to exclusively trade with money regardless of the type of his trading partner in the next decentralized market. Then, we can rewrite (18) as

$$(1-\beta)\, v = -\mu\tilde{x} + \beta u\,(\tilde{x})\,,$$

with $\tilde{x}$ given by

$$u'\,(\tilde{x}) = \frac{\mu}{\beta}. \tag{19}$$

In this case, a buyer who is paired with a connected seller chooses not to report his trade so that he exclusively uses fiat money to pay for his purchases in the decentralized market.

Finally, for fiat money and private debt to coexist as means of payment, the following condition must hold in equilibrium:

$$-\,(1-\beta)\,\mu x + \beta\delta\,[u\,(y) - y - \kappa] + \beta\,(1-\delta)\,[u\,(x) - x] \geq -\mu\tilde{x} + \beta u\,(\tilde{x})\,. \tag{20}$$

Otherwise, an equilibrium is one in which all trades in the decentralized market are carried out with fiat money. Condition (20) says that, when choosing how much money to carry into the decentralized market, a buyer prefers to have his trade reported when matched with a connected seller over not having it reported and trading with fiat money only.

5.5. The Value of Defection

As in the previous section, if a buyer fails to make a repayment in the centralized location, then he will be able to use only fiat money to pay for his future purchases. Here we also assume that the government refuses to make interest payments to a defaulter.[5] Thus, the value of defection $\hat{v}$ satisfies the following Bellman equation:

$$\hat{v} = \max_{\hat{M} \in \mathbb{R}_+} \left\{ -\phi_t \hat{M} + \beta \left[u\left(\phi_{t+1} \hat{M} \right) + \hat{v} \right] \right\}. \tag{21}$$

As in the previous section, let z denote the buyer's consumption following defection. Then, we can rewrite (21) as

$$(1-\beta)\,\hat{v} = -\mu z + \beta u\,(z)\,,$$

Upon defaulting on his private liability, a buyer produces and sells μz units of the consumption good in the Walrasian market to acquire money balances at date t. Then, he takes these proceeds into the decentralized market at date $t+1$ to purchase z units of the good. Notice that $z = \tilde{x}$.

5.6. Stationary Monetary Equilibrium

In a stationary monetary equilibrium, the distribution of money holdings across agents at the beginning of the day subperiod at date t is such that each buyer holds $\bar{M}_{t-1}$ units of money and sellers have no money. The distribution of money holdings at the beginning of the night subperiod at date t is such that each seller holds $\bar{M}_t$ units of money and buyers have no money. An agent finds it optimal to receive interest payments if and only if $(R-1)\,x - T \geq 0$. Using (17), this condition holds if and only if:

$$\mu \geq 1 \tag{22}$$

[5] This assumption ensures that a defaulter will be able to trade on the decentralized market with fiat money only. Not only does a defaulter lose access to credit, but he also is not able to receive interest payments from the government. One important implication is that a higher inflation rate punishes defaulters and induces cooperation in the credit market.

and

$$R \geq 1. \tag{23}$$

For a monetary equilibrium to exist, we also need to have that:

$$-\phi_t + \beta R \phi_{t+1} \leq 0, \tag{24}$$

at each date t. Otherwise, agents would demand an infinite amount of money in the Walrasian market, and, as a consequence, a monetary equilibrium would not exist. In a stationary monetary equilibrium, we can rewrite (24) as follows:

$$\frac{R}{\mu} \leq \frac{1}{\beta}. \tag{25}$$

A stationary monetary equilibrium in which aggregate real money balances are constant over time necessarily satisfies (25). Notice that (25) simply says that the real return on money cannot exceed the agents' rate of time preference.

Finally, a government's policy (μ, R) needs to satisfy (22), (23), and (25) to be incentive-feasible.

Definition 4 *For any given incentive-feasible policy (μ, R), a stationary monetary equilibrium with credit is a triple (x, y, z), with $z = \tilde{x}$, satisfying the nonnegativity of the repayment amount,*

$$y + \kappa - x\left(1 + \frac{\mu - 1}{\delta}\right) \geq 0, \tag{26}$$

the first-order condition for the optimal choice of money balances,

$$\delta R u'(y) + (1 - \delta) u'(x) = \frac{\mu}{\beta}, \tag{27}$$

and the buyer's individual rationality constraint,

$$\begin{aligned} & \beta\delta u(y) - (1 - \beta + \delta\beta)(y + \kappa) + \beta(1 - \delta)[u(x) - x] \\ \geq \; & (1 - \beta)(1 - \mu)\left(\delta^{-1} - 1\right)x - \mu\tilde{x} + \beta u(\tilde{x}), \end{aligned} \tag{28}$$

with $y = q^*$ if (28) does not bind.

Notice that condition (28) is simply the buyer's individual rationality constraint stated in terms of the quantities x and y. Notice also that (26) and (28) imply that (20) holds. If an equilibrium with credit exists, then we can have either an unconstrained equilibrium (in which case the buyer gets $y = q^*$ from a connected seller) or a constrained equilibrium (in which case the buyer gets $y < q^*$ from a connected seller). If a stationary monetary equilibrium with credit does not exist, then the unique stationary equilibrium is a pure monetary equilibrium in which the quantity $\hat{x}$ is traded in each bilateral meeting in the decentralized market. In this case, there can be no intervention because no public information is created (all trades are anonymous) so that the money supply remains constant over time.

In an equilibrium in which fiat money and private debt coexist, we have that $x \leq y$, which means that a buyer who reports his trade to the clearinghouse is able to consume more than a buyer who cannot obtain credit from his trading partner. A buyer who trades with an unconnected seller faces a cash constraint that can eventually bind. On the other hand, a buyer who trades with a connected seller can promise to make a repayment in the centralized location in order to consume more than what his money holdings permit him to purchase in the decentralized market. So long as the repayment amount is individually rational, a buyer who trades with a connected seller will be able to consume more in the day subperiod.

6. OPTIMAL MONETARY POLICY

In this section, we characterize the optimal policy rule subject to the implementation constraint that all trade has to be voluntary and that all monetary transfers to private agents have to be conditional on the available public information. The social welfare associated with an equilibrium with credit (x, y, z) is given by

$$\delta \left[u\left(y\right) - y\right] + \left(1 - \delta\right)\left[u\left(x\right) - x\right] - \delta\kappa, \tag{29}$$

and the social welfare associated with an equilibrium without credit is given by

$$u\left(\hat{x}\right) - \hat{x}. \tag{30}$$

Without a credit system, the only allocation that can be implemented (other than autarky) is one in which each buyer gets $\hat{x}$ from a seller in the decentralized market and produces $\hat{x}$ in the centralized location. All trades are anonymous so that the private sector does not create any public information on which the government can condition its transfers.

Notice that $x = y = q^*$ maximizes the social welfare associated with an equilibrium with credit. If we can implement the socially efficient quantity q^*, then the maximum welfare level is given by

$$u(q^*) - q^* - \delta\kappa.$$

We say that a society has a low record-keeping cost if the following holds:

$$u(q^*) - q^* - \delta\kappa > u(\hat{x}) - \hat{x}. \tag{31}$$

In this case, the equilibrium with credit dominates the pure monetary equilibrium without intervention, provided that the socially efficient quantity q^* can be implemented by an incentive-feasible policy. Thus, our first step is to verify whether the socially efficient quantity can indeed be implemented.

Lemma 5 *$x \leq \tilde{x}$ in an unconstrained monetary stationary equilibrium, with $x = \tilde{x}$ if and only if (25) holds with equality.*

The previous result shows that we must have $x = \tilde{x} = q^*$ in order to implement the socially efficient allocation. However, we show next that there is no incentive-feasible policy that can implement $x = \tilde{x} = q^*$ as an unconstrained equilibrium.

Proposition 6 *The unconstrained efficient allocation cannot be implemented as a stationary monetary equilibrium.*

It is not possible for the government to eliminate the consumption risk that a buyer faces in the decentralized market (in an unconstrained equilibrium). With probability δ, a buyer consumes q^* because his trade is reported to the clearinghouse, and with probability

$1-\delta$, he consumes less than q^* because his money holdings are insufficient to purchase the quantity q^* from his trading partner. The government cannot induce him to carry more money balances into the decentralized market so that the efficient quantity cannot be traded in a bilateral meeting between a buyer and an unconnected seller.

We now characterize the optimal monetary policy rule under the assumption that the buyers have CRRA preferences.

Assumption 1 Suppose $u(q) = (1-\sigma)^{-1}\left[(q+b)^{1-\sigma} - b^{1-\sigma}\right]$, with the coefficient of relative risk aversion satisfying $\sigma < 1$ and with $b \in (0,1)$.

The optimal monetary policy requires (25) to hold with equality so that we can guarantee that the buyer's daytime consumption when trading with an unconnected seller is the highest possible. This is the way in which we can implement the Friedman rule without resorting to lump-sum taxes.

Proposition 7 *For κ small and $\delta \geq \sigma$, there exists a unique $\bar{\mu} > 1$ such that, with $x = \tilde{x}$ and $y = q^*$, (26) holds as a strict inequality and (28) holds with equality. As a result, $(x, y, z) = (\tilde{x}, q^*, \tilde{x})$ is an unconstrained stationary monetary equilibrium in which the repayment amount is strictly positive.*

With CRRA preferences, the repayment amount in an unconstrained equilibrium is strictly increasing in the money growth factor μ, which means that the higher the long-run inflation rate (which is also given by μ), the larger the repayment amount is. As we would expect, credit becomes relatively more important in transactions as the inflation rate rises. Notice that the buyer's individual rationality constraint can be satisfied only for values of μ above the threshold value $\bar{\mu}$, which means that a stationary monetary equilibrium in which the credit system is operative exists if and only if the money growth factor is sufficiently large (in particular, above the threshold value $\bar{\mu}$).

Notice that there are two effects on the buyer's incentives to move away from credit as we vary μ. First, a higher return on money (lower μ for any given R) gives agents an additional incentive to default on their liabilities and, consequently, abandon the credit system. Second, a higher return on money lowers the buyer's opportunity cost of holding such an asset, making it more valuable as an alternative to credit. Lenders (sellers) are aware of this effect and, consequently, reduce the loan amounts they are willing to extend.

Finally, we need to verify whether the welfare associated with the allocation $(\tilde{x}, q^*, \tilde{x})$ is greater than the welfare associated with the equilibrium without credit (the pure monetary equilibrium $\hat{x}$). Specifically, we need to verify whether the following holds:

$$\delta \left[u\left(q^*\right) - q^*\right] + (1-\delta)\left[u\left(\tilde{x}\right) - \tilde{x}\right] - \delta\kappa > u\left(\hat{x}\right) - \hat{x}. \tag{32}$$

The left-hand side of (32) gives the expected surplus from trade associated with an equilibrium in which fiat money and credit coexist as a means of payment, whereas the right-hand side of (32) gives the expected surplus associated with the pure monetary economy. Notice that q^* is the surplus-maximizing quantity so that $u\left(q^*\right) - q^* > u\left(\hat{x}\right) - \hat{x}$. However, condition (32) may not hold when the cost κ is relatively large.

Proposition 8 *Suppose that $\delta \geq \sigma$ is sufficiently close to one and κ is sufficiently small such that (32) holds. Let $\bar{\mu} > 1$ be the threshold value for the money growth factor as defined in Proposition 5. Let $\tilde{x}$ be given by*

$$u'\left(\tilde{x}\right) = \frac{\bar{\mu}}{\beta}.$$

Then, the allocation $(\tilde{x}, q^*, \tilde{x})$ achieves the highest welfare level.

This means that only a society with a sufficiently sophisticated record-keeping technology can benefit from public transactions. Not only do public transactions allow credit arrangements within the private sector, but they also permit the government to alter the rate of return on money. If the number of transactions that is reported in equilibrium is large relative to those that are anonymous, then the impact of the policy rule $(\mu, R) = \left(\bar{\mu}, \beta^{-1}\bar{\mu}\right)$ on social welfare is bigger. This means that public trades are socially desirable so long as the cost of reporting private trades is low and the relative fraction of these trades is sufficiently large.

7. DISCUSSION

In the pure credit economy, buyers not only face consumption risk in the decentralized market, but they also may consume less than the surplus-maximizing quantity q^* in a credit trade if they are credit-constrained. Thus, we conclude that, regardless of the available record-keeping technology, there is a social role for fiat money in transactions and for an active monetary policy. Indeed, we have shown that it is possible to implement welfare-improving policies even if we restrict these policies to respect voluntary trade and the constraint that all transfers to private agents must be contingent on the available public information. In particular, it is necessary to have an active monetary policy to guarantee that the surplus-maximizing quantity q^* is traded in each match involving a connected seller (Propositions 3 & 7).

In Andolfatto (2010), any incentive-feasible policy rule for which $R/\mu = \beta^{-1}$ (a version of the Friedman rule) implements the unconstrained efficient allocation in a pure monetary economy. In particular, the monetary authority can implement such an allocation with zero inflation by setting $(\mu, R) = \left(1, \beta^{-1}\right)$. As a result, there is no social role for credit. Andolfatto's results depend crucially on the assumption that the monetary authority is able to evenly raise the real return on money across all money holders. In our analysis, we impose the constraint that the transfers to agents have to be conditional on the available public information. Because the record-keeping technology is not available to all sellers, the monetary authority is unable to directly raise the real return on money for the subgroup of traders who temporarily cannot report their trades. Notice that, in principle, it could indirectly raise the real return on money for this subgroup of agents by means of a deflationary policy. But we have shown that such a policy is not incentive-feasible under the implementation restrictions we have considered.

We have shown that setting $R/\mu = \beta^{-1}$ (Friedman rule) does not implement the unconstrained efficient allocation (Proposition 6). As we have seen, this policy equalizes the real rate of return on money with the rate of time preference only for the subgroup of traders who are currently able to report their trades. From Proposition 7, a necessary and sufficient condition for optimality is to have both $R/\mu = \beta^{-1}$ and $\mu = \bar{\mu} > 1$. This means that a slightly positive inflation rate is required to support cooperation in the credit system; that is, an inflation rate of at least $\bar{\mu}$ is required to guarantee that the buyer's individual rationality constraint is satisfied so that credit transactions are indeed incentive-feasible.

Why should the monetary authority choose the minimum incentive-feasible inflation rate consistent with an unconstrained monetary equilibrium? As we have seen, any money growth factor above $\bar{\mu}$ results in a strictly lower value for x, reducing the value of the social welfare function. Notice that those who are able to use credit in their current transactions are essentially "protected" against inflation: Even though inflation raises the opportunity cost of holding money, those who have their trades reported receive an offsetting interest payment, completely eliminating the opportunity cost of holding money provided that $R = \beta^{-1}\mu$. On the other hand, those who are currently unable to report their trades endure a cost of holding money. In principle, the monetary authority would want to use inflation as an instrument to induce cooperation in the credit system (a punishment on those who default on their debts). However, there are always some traders in the economy who currently do not have an opportunity to use credit and who would be punished if the inflation rate were too high. For this reason, the monetary authority wants to choose the minimum incentive-feasible inflation rate.

Finally, it is important to discuss the nature of our monetary mechanism. In particular, why do we think it is relevant? Our analysis explicitly assumes that monetary policy is implemented through the payment/credit system. We initially imposed such a restriction to essentially preserve a social role for credit arrangements. Our monetary mechanism is a relevant institutional arrangement if we consider the possibility of using monetary policy to enhance the payment/credit system. In reality, monetary policy implementation is much more complex than what our simple mechanism suggests (specifically, it involves the purchase and sale of government securities in the open market, the creation of liquidity facilities such as the discount window, and recently the payment of interest on reserves). However, our analysis focuses on the interaction between monetary policy and the private credit system, in which case our theoretical mechanism provides a useful characterization.

Even though we restricted the monetary authority to intervene through the credit/payment system, we have shown that it is possible to affect the sellers' incentives to extend credit as well as the buyers' incentives to fully repay their debts.

8. CONCLUSION

We have taken the view that fiat money complements the use of credit as a means of payment and have characterized the way in which monetary policy affects the functioning of the credit system. In our analysis, credit arrangements are constrained by the borrowers' inability to commit to fully repay their debts and by the available record-keeping technology. We have characterized the equilibrium allocations of a pure credit economy and have shown that (due to a technological restriction) there is a social role for fiat money. Then, we have shown that an active monetary policy is sufficient for relaxing the credit constraints. Finally, we have shown that if credit arrangements are essential, then the optimal monetary policy entails a positive inflation rate in order to induce cooperation in the credit system.

One limitation of our analysis is that we explicitly abstracted from other aspects of monetary policy. Specifically, we restricted attention to the design of policies that aim at influencing the effectiveness of credit arrangements. However, our monetary mechanism allowed us to provide a useful characterization of the interactions between optimal monetary policy and credit arrangements.

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APPENDIX

Proof of Proposition 3

Notice that for any pair (x, y),

$$\begin{aligned} & \beta\delta u(y) - (1-\beta+\delta\beta)(y+\kappa) + \beta(1-\delta)[u(x)-x] \\ < \; & \beta\delta u(y) - (1-\beta+\delta\beta)\, y + \beta(1-\delta)[u(x)-x] \\ \leq \; & \beta u(\delta y + (1-\delta)x) - (1-\beta+\delta\beta)\, y - \beta(1-\delta)\, x \\ \leq \; & -\hat{x} + \beta u(\hat{x}). \end{aligned}$$

In the second step, we used the fact that the utility function $u : \mathbb{R}_+ \to \mathbb{R}_+$ is strictly concave. Hence, the buyer's individual rationality constraint (16) cannot be satisfied for any (x, y). This implies that there can be no credit in equilibrium. As a result, the unique stationary monetary equilibrium is one in which $\hat{x}$ is traded in each bilateral meeting in the decentralized market. **Q.E.D.**

Proof of Lemma 5

Condition (25) implies

$$\left(\frac{\mu}{\beta} - R\delta\right)\frac{1}{1-\delta} \geq \frac{\mu}{\beta}.$$

Because the utility function $u : \mathbb{R}_+ \to \mathbb{R}_+$ is strictly concave, we conclude that $x \leq \tilde{x}$ as claimed. In particular, if $R = \beta^{-1}\mu$, then from (27) we have $x = \tilde{x}$. **Q.E.D.**

Proof of Proposition 6

From (22) and (25), we have that

$$\left(\frac{\mu}{\beta} - R\delta\right)\frac{1}{1-\delta} \geq \frac{1}{\beta} > 1, \tag{33}$$

which means that the maximum daytime consumption (in an unconstrained equilibrium) for a buyer who is paired with an unconnected seller is $\tilde{x} < q^*$. Since the socially efficient allocation can be implemented only as an unconstrained equilibrium, (33) implies that such an allocation is infeasible. **Q.E.D.**

Proof of Proposition 7

Define the function $\varphi(\mu)$ by

$$\begin{aligned} \varphi(\mu) &= \beta\delta u(q^*) - (1-\beta+\delta\beta)(q^*+\kappa) + \beta(1-\delta)\left[u(\tilde{x}) - \tilde{x}\right] \\ &\quad - \left[\beta u(\tilde{x}) - \mu\tilde{x}\right] - (1-\beta)\left(\delta^{-1}-1\right)(1-\mu)\,\tilde{x}. \end{aligned}$$

Under Assumption 1, we have that $\varphi'(\mu) > 0$ for all μ. Notice that $\varphi(\beta) < 0$. Also, we have that

$$\varphi(1) = \beta\delta u(q^*) - (1-\beta+\delta\beta)(q^*+\kappa) - \left[\beta\delta u(\hat{x}) - (1-\beta+\delta\beta)\,\hat{x}\right] < 0.$$

Finally, notice that $\varphi(\mu) > 0$ for μ sufficiently large. To verify this claim, observe that

$$\beta\delta u(q^*) - (1-\beta+\delta\beta)(q^*+\kappa) > 0$$

for δ sufficiently close to one. Also, we have that, for any $\mu \geq 1$,

$$\varphi(\mu) > \beta\delta u(q^*) - (1-\beta+\delta\beta)(q^*+\kappa) - \beta\delta\left[u(\tilde{x}) - \tilde{x}\right] + (1-\beta)\left(\delta^{-1}-1\right)(\mu-1)\tilde{x}.$$

Because both $\tilde{x} \to 0$ and $\mu\tilde{x} \to 0$ as $\mu \to \infty$, we must have $\varphi(\mu) > 0$ for some μ sufficiently large. As a result, there exists a unique $\bar{\mu} > 1$ such that (28) holds with equality at $x = \tilde{x}$ and $y = q^*$.

Now we need to verify whether the repayment amount is nonnegative. Define the function $\psi(\mu)$ by

$$\psi(\mu) = 1 + \kappa - \left(\frac{\beta}{\mu}\right)^{\frac{1}{\sigma}}\left[1 + \delta^{-1}(\mu-1)\right],$$

which gives the repayment amount as a function of μ. We have that $\psi'(\mu) > 0$ for all $\mu \geq (1-\delta)/(1-\sigma)$. Notice that $(1-\delta)/(1-\sigma) \leq 1$ if and only if $\delta \geq \sigma$. Because $\psi(1) > 0$, we have that $\psi(\mu)$ is strictly increasing for any $\mu \geq 1$. This means that for the value $\bar{\mu}$ such that $\varphi(\bar{\mu}) = 0$ we also have that $\psi(\bar{\mu}) > 0$. Therefore, we conclude that $(x, y, z) = (\tilde{x}, q^*, \tilde{x})$, with $\tilde{x}$ given by

$$u'(\tilde{x}) = \beta^{-1}\bar{\mu},$$

is an unconstrained stationary monetary equilibrium. **Q.E.D.**

Proof of Proposition 8

Given condition (32), it follows immediately from Proposition 5. **Q.E.D.**