

#### WORKING PAPER NO. 11-26 A QUANTITATIVE ANALYSIS OF THE U.S. HOUSING AND MORTGAGE MARKETS AND THE FORECLOSURE CRISIS

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Abstract

We construct a quantitative equilibrium model of the housing sector that accounts for the

homeownership rate, the average foreclosure rate, and the distribution of home-equity ratios

across homeowners prior to the recent boom and bust in the housing market. We analyze

the key mechanisms that account for these facts, including the preferential tax treatment

of housing and inflation. We then use the model to gain a deeper understanding of the

recent housing and mortgage crisis by studying the consequence of an unanticipated increase

in the supply of housing (overbuilding shock). We show that the model can account for

the observed decline in house prices and much of the increase in the foreclosure rate if

two additional forces are taken into account: (i) the lengthening of the time to complete a

foreclosure (during which a defaulter can stay rent-free in his house) and (ii) the tightening

of credit constraints in the market for new mortgages.

**Key Words:** Leverage, Foreclosures, Mortgage Crisis

**JEL:** E21 E32 E44 G21 H24

#### 1 Introduction

This study is motivated by the recent collapse in house prices and rising defaults on mortgages in the United States. It has two goals: First, to construct a quantitative model that can account for key long-run features of the US housing market prior to the boom-bust years and, second, to use the model to gain a quantitative understanding of the foreclosure crisis.

The elements of the model environment are as follows. The economy is endowed with an exogenously given stock of housing space. There is a continuum of infinitely lived individuals subject to uninsurable idiosyncratic shocks to earnings. People can buy consumption goods, save in the form of a risk-free savings account, and purchase or rent their housing space. If an individual chooses to purchase his housing space, he can offer his housing space as collateral and borrow funds from a mortgage market. The mortgage market is competitive, with every borrower freely choosing his down payment and being charged an interest rate that exactly reflects his objective probability of default. Because the preferential tax treatment of housing is known to be important for the homeownership decision, the model takes into account the progressivity of the federal tax code, the fact that the implicit rental income from owner-occupancy is not counted as taxable income and that the mortgage interest payments on owner-occupied homes as well as taxes paid on such properties are deductible from taxable income.

Our findings can be summarized as follows. First, we show that a calibrated version of this model can account for the average homeownership rate, the average foreclosure rate, and the distribution of home equity across homeowners prior to the boom-bust years. The tax treatment of housing plays an important role in bringing the model close to reality: Without the exclusion of implicit rental income from taxable income, the model would predict a much lower homeownership rate. And without the mortgage interest deduction, the model would predict much higher home equity ratios. The model accounts for the observed dispersed distribution of home equity ratios because homeowners steadily accumulate home equity as

they pay down their obligations and because inflation steadily increases the nominal value of housing but leaves the nominal value of the corresponding mortgage debt unchanged.

Our second goal is to use the model to evaluate the following narrative of the foreclosure crisis. The trigger for the crisis (or its proximate cause) is an oversupply of housing. In the long run, we expect house prices to fall modestly to absorb this excess supply. But, in the short run, market frictions (such as the transactions costs of buying and selling homes and the difficulty of finding renters for homes meant for owner occupancy) made the demand for housing price inelastic and price (as well as rents) fell more than in the steady state. These drops induced foreclosures because some homeowners with negative home equity found it in their interests to renege on their debt and take advantage of temporarily depressed rents and house prices. The above-normal foreclosures, in turn, disrupted the flow of credit to homebuyers, which further reduced housing demand and further lowered prices and raised foreclosures.

With regard to this goal our findings are as follows.

- In our model, the factor most responsible for the drop in house prices is the disruption in the flow of credit to homebuyers. All else remaining the same, this force accounts for 71% of the observed drop in house prices between 2006 Q2 and 2009 Q2. The oversupply of housing, combined with other features of the model, accounts for the remaining portion.
- The disruption in the flow of credit to new homebuyers and the oversupply of housing alone account for only 50% of the rise in foreclosures since the start of the crisis and slightly *more* than the observed drop in house prices. Even though almost a fifth of the population of homeowners have negative home equity following the price drop, the costs of default motivate most of them to keep their houses. Thus, it appears that some other inducement to default is needed to more fully account for the rise in foreclosures. We consider the fact that the sheer volume of mortgage defaults clogged the judicial process

and allowed delinquent debtors to stay in their homes without making any mortgage or rent payments for substantially longer periods of time than normal. Incorporating this force in the model accounts for 83% of the rise in foreclosures and reduces slightly the downward pressure on house prices. The latter occurs because even though there are more foreclosures, the fraction of foreclosed properties that is offered on the market is lower since defaulters get to stay in the foreclosed homes longer.

• There is feedback between the jump in foreclosures and the drop in house prices but, surprisingly, not much: If foreclosures are prevented altogether, the model still accounts for 84% of the observed drop in house prices. The reason is that the majority of foreclosed properties are not supplied to the market in the period in which the default happens because of foreclosure processing delays.

In addition to these findings, our model permits us to explore the implications of certain types of policy actions for the financial crisis. We find that unexpected changes in the inflation rate matters: For instance, an unexpected decline in inflation leads to a bigger drop in house prices and a higher foreclosure rate. We also find that the oversupply of housing would have had mild effects if the tax code discouraged leverage by not allowing mortgage interest payments to be tax deductible.

## 2 Contributions to the Literature

We build on a growing quantitative-theoretic literature addressing various aspects of the housing sector. In terms of modeling the housing sector, we follow Gervais (2002) in conceiving of the housing market as a market for homogeneous housing *space*, as opposed to houses. We also follow Gervais in giving prominence to the preferential tax treatment of housing for understanding housing market outcomes.<sup>1</sup> We go beyond Gervais (and a host of

<sup>&</sup>lt;sup>1</sup>Gervais (2002) analyzed the distortions resulting from the special tax treatment of housing, namely, the failure to tax the implicit rental income from owner-occupancy and the tax deductibility of mortgage interest

other studies) in allowing for the possibility of default on mortgages.<sup>2</sup> In terms of modeling the mortgage market, we follow Chatterjee et al. (2007) in assuming that each loan is competitively priced to reflect the objective probability of default on the loan (individualized or risk-based pricing). This approach is also taken in Jeske, Krueger and Mitman (2010) and Guler (2008).<sup>3</sup> We go beyond Jeske, Krueger and Mitman in modeling mortgages as long-term contracts wherein the obligation of the borrower to the lender diminishes over time and the borrower steadily accumulates equity in the house.<sup>4</sup> We also advance the literature on consumer and sovereign default by extending the long-maturity unsecured debt framework developed in Chatterjee and Eyigungor (2011) to an environment in which long-maturity debt is issued against collateral whose value may fluctuate over time.

There are two quantitative-theoretic studies that account for certain long-run features of the US housing and mortgage markets with the goal of gaining a better understanding of the foreclosure crisis. Garriga and Schlagenhauf (2009) account for the fraction of different types of mortgages, noting that subprime mortgages display a higher foreclosure rate than prime mortgages, and analyze the impact on mortgage defaults of an unanticipated 15 percent decrease in house prices resulting from a decline in construction cost.<sup>5</sup> Corbae and Quintin

income, in the context of a multi-generational overlapping generations model.

<sup>&</sup>lt;sup>2</sup>Nakajima (2010) uses the same structure as Gervais to study the optimal capital income tax rate when residential capital is a significant portion of tangible capital and residential capital is treated preferentially in the tax code. Diaz and Luengo-Prado (2010) employ an infinite horizon framework to study the joint distribution of capital and housing stocks across the population. Chambers, Garriga and Schlagenhauf (2009a) use a life-cycle structure similar to that of Gervais to study the role of demographics and mortgage innovation in the evolution of the homeownership rate since WWII. Chambers, Garriga and Schalgenhauf (2009b) examine the aggregate consequences of different mortgage contracts. Rios-Rull and Sanchez-Marcos (2008), following on the earlier work of Ortalo-Magne and Rady (2006), model the housing sector as composed of different types of housing and study the migration of households from one type of housing to another. The role of housing investment in business cycles has been analyzed by Davis and Heathcote (2005) and Iacoviello and Pavan (2009), among others. All of these papers abstract from the possibility of default on mortgages.

<sup>&</sup>lt;sup>3</sup>Jeske, Krueger and Mitman (2010) quantify the macroeconomic effects of the subsidy provided by the implicit federal guarantee of GSE debt in the context of an infinite-horizon economy. Guler (2008) examines the impact of better information on household default risk on loan-to-value ratios and interest rates in the mortgage market in the context of a life-cycle model. Both studies allow for the possibility of default on mortgages.

<sup>&</sup>lt;sup>4</sup>Jeske, Krueger and Mitman model mortgages as one-period contracts that are "refinanced" each period. <sup>5</sup>In their model, the price of housing space is determined by the marginal cost of new construction, which is taken as technologically given.

(2010) focus on the role of non-traditional, low down payment mortgage contracts originated during the heyday of the housing boom. Given an unanticipated and exogenous 25 percent decline in house prices, they seek to quantify the contribution of these non-traditional mortgages to the observed rise in foreclosures.

Relative to these studies, our paper advances our understanding of the crisis in three ways. First, as stated already, our goal is to understand the decline in house prices as well as the rise in foreclosures. In contrast to both papers, rents and house prices in our model are determined by the equality of supply and demand. This feature helps us gain insights into the causes underlying the drop in house prices that would not be available if prices are taken as exogenous or technologically determined. For instance, we find that foreclosures partly contribute to the decline in house prices and that an increase in the cost of new mortgages can lead to large declines in house prices. Second, while we do not focus on the role of mortgage innovations per se, we do attempt to match the home equity distribution across households. 6 Since negative home equity is a necessary condition for default, the precrisis home equity distribution, along with the magnitude of the price drop during the crisis, determine the pool of people who might wish to default and is, thus, a key determinant of actual foreclosures. Finally, while negative home equity is a necessary condition for default, it is not sufficient: Whether a negative home equity borrower defaults or not depends on the benefits of homeownership. In contrast to these two studies, we locate these benefits in the preferential tax treatment of housing, which can be measured relatively accurately.<sup>7</sup> In addition, our model allows exploration of the effects of tax policy and inflation (as well as the interaction between them) on housing and mortgage markets outcomes in both the long run and during the crisis.

<sup>&</sup>lt;sup>6</sup>Garriga and Schlagenhauf and Corbae and Quintin target the steady state default rates on different types of mortgages. Their calibration strategy implies some home equity distribution in the background but this distribution is not made explicit and compared to the data.

<sup>&</sup>lt;sup>7</sup>Corbae and Quintin assume that there is an "ownership premium" in preferences while Garriga and Schlagenhauf assume that rental space depreciates faster than owner-occupied space.

## 3 Environment

Time is discrete and indexed by t = 0, 1, 2, ... The economy has an exogenously given stock of rental and owner-occupied housing space  $H_R$  and  $H_O$ .

#### 3.1 People

There is a fixed continuum of individuals. Individuals derive utility from the consumption of a homogeneous consumption good and the service flow from housing space. Let c(t) denote consumption of the homogeneous good in period t, and let h(t) denote the consumption of housing space in period t. Then an individual values the consumption stream  $c = \{c(0), c(1), c(2), \ldots, \}$  and  $h = \{h(0), h(1), h(2), \ldots, \}$  according to:

$$U(c,h) = \sum_{t=0}^{\infty} \beta^t u(c(t), h(t)), \quad 0 < \beta < 1,$$
(1)

where

$$u(c(t), h(t)) = (c(t)^{1-\theta}h(t)^{\theta})^{1-\gamma}/(1-\gamma).$$
(2)

We assume that people must either own their housing space or rent it.

Each individual independently draws an earnings level w from a finite-state Markov process with non-negative positive support  $W \subset R_+$ . The probability that w(t+1) = w' given w(t) = w is F(w', w).

### 3.2 Market Arrangement

The homogeneous consumption/endowment good is the numeraire good. Period t relative prices are expressed in period t consumption goods. In order to properly account for the effect of inflation, we will denote the nominal price of the period t consumption good by  $\Pi(t)$ 

so that the gross inflation rate between period t and t+1 is  $1+\pi(t+1)=\Pi(t+1)/\Pi(t)$ . The path  $\pi(t)$  is taken as exogenously given. Also, in what follows, nominal prices are denoted in upper case while relative prices are denoted in lower case. Thus, the price of period t housing space in terms of the consumption good is denoted p(t) and its nominal price (=  $\Pi(t)p(t)$ ) is denoted P(t). There are four markets in this economy.

- 1. There is a market for owner-occupied housing, with the price per unit of housing space in period t denoted p(t). Owner-occupied housing depreciates randomly with the rate being either high or low:  $\delta_j \in (0,1)$  with probability  $\xi_j$ , j = H, L,  $\delta_H > \delta_L$ . For computational tractability we assume that owner-occupied housing space comes in discrete sizes given by elements of a finite set K.
- 2. There is a market for rental housing, with the rent per unit of housing space in period t denoted z(t). Rental housing space depreciates non-stochastically at the rate  $\Delta \in (0,1)$
- 3. There is a market for risk-free deposits that offers households a constant and exogenously given real interest rate  $\bar{r}$  on deposits with taxable interest income and an exogenously given real interest rate  $\tilde{r}$  on deposits with interest earnings that are not currently taxable. For computational tractability we assume that deposits of both kinds come in discrete sizes given by elements of a finite set A. In addition, there is a global capital market in which the business sector can borrow or lend unrestricted amounts at a constant and exogenously given real interest rate r.
- 4. Finally, there is a market for mortgages where individuals can borrow in nominal terms by offering their house as collateral. When a person takes out a mortgage, he agrees to make a sequence of geometrically declining nominal payments  $\{X, \mu X, \mu^2 X, \ldots\}$  starting the following period. In case of default, the lender gets ownership of the

housing space offered as collateral.<sup>8</sup> In case of sale, the lender receives

$$X\mu^{\tau}\left(\frac{1}{1+i(t+1)}+\frac{\mu}{(1+i(t+1))(1+i(t+2))}+\ldots,\right),$$

where  $\tau$  is the time elapsed since the mortgage was issued and  $1+i(t)=(1+r)(1+\pi(t))$ .  $^9$   $\mu<1$  implies that the value of the mortgagee's obligation declines over time. Because a mortgagee can default in the future, the market price q of a new unit mortgage must take this possibility into account. As we will see below, the decision to default on the mortgage in any period depends on the mortgagee's earnings in that period, his assets in that period, the real value of his debt payment in that period, the size of his house, the depreciation shock, the anticipated path of future house prices, inflation and nominal interest rates. Thus, the market value of a unit mortgage taken out in the current period depends on the mortgagee's current earnings w (because it helps predict future earnings), his post-purchase savings a', the initial nominal payment amount X' to be paid next period, the real value of which in the next period is  $x' = X'/\Pi(t+1)$ , the amount of housing pledged as collateral k' and the time period t (which captures the future paths of all prices and interest rates). The price of a mortgage per real units of goods promised next period is q(w,a',x',k',t) and the value of the mortgage is  $q(w,a',x',k',t) \cdot x'$ .

#### 3.3 Taxes

The amount of taxes to be paid by an individual in nominal terms is modeled after the US tax code. If we let Q(t) denote the present discounted value of the nominal stream  $\{1, \mu, \mu^2, \ldots\}$  starting next period, then the value of debt bought back by the borrower is

<sup>&</sup>lt;sup>8</sup>We assume that when the lender gets ownership of the house, the borrower's obligation to the lender is extinguished and there is no recourse for the lender.

<sup>&</sup>lt;sup>9</sup>This is just the present value of the remaining promised sequence of nominal payments discounted at the nominal risk-free rate facing the business sector

given by  $(X - \mu X)Q(t)$ .<sup>10</sup> Consequently, the portion of X that is the interest payment is  $[1 - (1 - \mu)Q(t)]X$ . Then, the individual's nominal taxable income I is given by:

$$I = \max\{0, W + \omega \bar{i}(t)A - \max\{[1 - (1 - \mu)Q(t)]X + \rho P(t)k', S\}\}$$
(3)

where  $1 + \bar{i}(t) = (1 + \bar{r})(1 + \pi(t))$ , S is the standard deduction,  $\rho P(t)k'$  is property tax paid on housing space owned at the end of period t and  $1 - \omega$  is the fraction of asset returns on which taxes are deferred. For this portion of asset returns we assume that individuals receive an after-tax real return of  $\tilde{r}$  which will (typically) be higher than the after-tax real return on the non-deferred portion.<sup>11</sup> Thus, an individual's currently taxable income consists of wage plus the  $\omega$  portion of interest earnings less the greater of the standard deduction S or the sum of the interest payment on the mortgage and property taxes. The individual's nominal federal tax liability is then given by  $G = \int_0^I T(Y) dY$ , where  $T(\cdot)$  is the marginal tax rate and is weakly increasing in taxable income. In real terms, an individual's overall tax liability is given by:

$$g(w, a, x, k', t) = +\rho p(t)k' + \int_0^{\max\{0, w + \omega \bar{i}(t)a/\pi(t) - \max[1 - (1 - \mu)q(t)/(1 + \pi(t+1))]x + \rho p(t)k', s]\}} \tau(y) \, dy,$$
(4)

where  $\tau(\cdot)$  is the marginal tax rate when income is measured in terms of the current period consumption good (we assume that nominal tax brackets move up with inflation one-for-one) and q(t) is the discounted value of the real stream  $\{1, \mu/(1+\pi(t+2)), \mu^2/(1+\pi(t+2))(1+\pi(t+3)), \ldots\}$  starting next period using the real interest rate r.<sup>12</sup>

 $<sup>^{10}</sup>$ If the borrower does not buy any debt back in the current period only, the present discounted value of his obligations will be XQ(t); when he buys back debt, the present discounted value of his obligations is  $\mu XQ(t)$ . Thus, the value of debt bought back in the current period is  $(1 - \mu)XQ(t)$ .

<sup>&</sup>lt;sup>11</sup>The deferred portion includes the return on assets in retirement accounts and some portion of the return on all assets that accrue as capital gains (taxes on capital gains are paid only when the individual realizes the gain by selling the asset). Since we omit life-cycle features, we cannot explicitly model the deferred payment of taxes on asset returns. The value of the after-tax real return on the deferred portion is discussed in the calibration section.

<sup>&</sup>lt;sup>12</sup>Note that  $Q(t) = q(t)/(1 + \pi(t+1))$ .

#### 3.4 Financial Intermediaries

Financial intermediaries take in deposits, sell mortgages, and own the housing space rented by people. All intermediaries can borrow or lend funds in a world credit market at a given risk-free interest rate r > 0. We will assume that there is one representative risk-neutral intermediary that takes all prices as given.

#### 4 Decision Problems

#### 4.1 People

For a homeowner, the state variables are  $w, a, x, k, \delta_h, t$ ; for a renter the state variables are w, a, t and whether the renter is excluded from the mortgage market because of a prior default. Denote the value function of a homeowner by  $V_O(w, a, x, k, \delta_h, t)$ , that of a renter who is not excluded from the mortgage market by  $V_R(w, a, t)$  and that of a renter who is excluded by  $V_R^D(w, a, t)$ .

Consider first the decision problem of an individual who does not own housing and who is not excluded from the mortgage market due to prior default. If the individual chooses to purchase, he solves:

$$M_1(w, a, t) = \max_{c \ge 0, k' \in K, x' \ge 0, a' \ge 0} \{ u(c, k') + \beta E_{w', \delta' \mid w} V_O(w', a', x', k', \delta', t + 1) \}$$

$$c = w - g(w, a, x = 0, k', t) + a(1 + \omega r + (1 - \omega)\tilde{r}) - a' - p(t)[1 + \chi_B]k' + q(w, a', x', k', t) \cdot x',$$

where  $\chi_B$  is the percentage transactions cost of purchasing a house. Observe that payment on the chosen mortgage begins in the next period so x = 0 in the tax calculation function g(w, a, x, k', t). If the individual is excluded from the mortgage market due to a prior default but chooses to purchase a house, he solves:

$$M_1^D(w, a, t) = \max_{c \ge 0, k' \in K, a' \ge 0} \{ u(c, k') + \beta E_{w', \delta'|w} V_O(w', a', x' = 0, k', \delta', t + 1) \}$$

$$c = w - g(w, a, x = 0, k', t) + a(1 + \omega r + (1 - \omega)\tilde{r}) - a' - p(t)[1 + \chi_B]k'.$$

We assume that if an excluded individual purchases a house then he is no longer excluded from the mortgage market (the default flag is removed).<sup>13</sup>

If the individual is not excluded from the mortgage market and chooses to rent, he solves

$$M_0(w, a, t) = \max_{c \ge 0, h \ge 0, a' \ge 0} \{ u(c, h) + \beta E_{w'|w} V_R(w', a', t+1) \}$$

$$c = w - q(w, a, x = 0, k' = 0, t) + a(1 + \omega r + (1 - \omega)\tilde{r}) - a' - z(t)h.$$

and if he is excluded and chooses to rent he solves

$$M_0^D(w, a, t) = \max_{c \ge 0, h \ge 0, a' \ge 0} \{ u(c, h) + \beta E_{w'|w} \lambda V_R^D(\cdot, t+1) + (1 - \lambda) V_R(\cdot, t+1) \}$$

$$c = w - g(w, a, x = 0, k' = 0, t) + a(1 + \omega r + (1 - \omega)\tilde{r}) - a' - z(t)h,$$

where  $\lambda$  is the probability that the individual remains excluded. Then  $V_R(w,a,t)$  and  $V_R^D(w,a,t)$  are given by  $\max\{M_1(w,a,t),M_0(w,a,t)\}$  and  $\max\{M_1^D(w,a,t),M_0^D(w,a,t)\}$ , respectively. We denote the decision rules of a non-excluded renter by  $c_R(a,w,t)$ ,  $h_R(a,w,t)$  and  $k_R'(a,w,t)$ , and those of an excluded renter by  $c_R^D(a,w,t)$ ,  $h_R^D(a,w,t)$  and  $k_R'^D(a,w,t)$ . Here it is understood that  $h_R(a,w,t)$  and  $k_R'(a,w,t)$  cannot both be simultaneously positive (similarly for  $h_R^D(a,w,t)$  and  $k_R'^D(a,w,t)$ ).

A homeowner may keep the current house, sell it, or default on the mortgage (if he has one).

<sup>&</sup>lt;sup>13</sup>This assumption is also without much loss of generality because given the substantial transactions costs of purchasing and selling a home, individuals purchase homes for a long duration of time. By the time they need to make another purchase, an excluded individual's exclusion flag would typically be gone. Thus, excluded individuals who purchase a home will behave as if they do not have a default flag.

If he chooses to keep house, he solves:

$$K_0(w, a, x, k, \delta, t) = \max_{c \ge 0, a' \ge 0} \left\{ u(c, k) + \beta E_{w', \delta' | w} V_O(w', a', x\mu/(1 + \pi(t+1)), k, \delta', t+1) \right\}$$

$$c = w - g(w, a, x, k' = k, t) + a(1 + \omega r + (1 - \omega)\tilde{r}) - a' - x - \delta k$$

If he chooses to sell, he solves:

$$K_1(w, a, x, k, \delta, t) = \max_{c \ge 0, h \ge 0, a' \ge 0} \left\{ u(c, h) + \beta E_{w', |w} V_R(w', a', t + 1) \right\}$$

$$c = w - g(w, a, x, k' = 0, t) + a(1 + \omega r + (1 - \omega)\tilde{r}) - a' - x +$$

$$p(t)[1 - \chi_S]k - q(t)\mu x/(1 + \pi') - z(t)h - \delta k,$$

where  $\chi_S$  is the percentage cost of selling a house and  $\pi'$  is the inflation rate between t and t+1. Observe that selling the house requires the individual to pay his current mortgage payments, buy back the promised sequence of future mortgage payments at the nominal risk-free interest rate and move out (i.e., rent housing space in the period of the sale). The arguments of the tax function reflect these assumptions: the current period mortgage interest payment  $x - (1 - \mu)q(t)x/(1 + \pi(t+1))$  is deducted from taxes but since the seller does not consume the services of the house he does not pay property taxes. Observe also that a seller (as well as a keeper) must make good on the depreciation on the house. If the homeowner has a mortgage, he may also choose to default. In this case, he solves:

$$K_D(w, a, x, k, t) = \max_{c \ge 0, h \ge 0, a' \ge 0} \left\{ u(c, h) + \beta E_{w'|w}[(1 - \lambda)V_R(\cdot, t + 1) + \lambda V_R^D(\cdot, t + 1)] \right\}$$
  
$$c = w - g(w, a, x = 0, k' = 0, t) + a(1 + \omega r + (1 - \omega)\tilde{r}) - a' - z(t)h.$$

Foreclosure results in the individual losing the house as well as the mortgage and in his being excluded from the mortgage market for some random length of time. Importantly, a defaulter does not cover the depreciation cost and, therefore,  $\delta$  does not appear as a state

variable in  $K_D$ . Finally,

$$V_O(w, a, x, k, \delta, t) = \max \{K_0(w, a, x, k, \delta, t), K_1(w, a, x, k, \delta, t), K_D(w, a, x, k, t)\}.$$

We denote the decision rules of a homeowner by  $c_O(a, w, x, k, \delta, t)$ ,  $h_O(a, w, x, k, \delta, t)$  and  $k'_O(a, w, x, k, \delta, t)$ . Again, it is understood that  $h_O(a, w, x, k, \delta, t)$  and  $k'_O(a, w, x, k, \delta, t)$  cannot both be simultaneously positive.

#### 4.2 Financial Intermediaries

The (representative) financial intermediary rents out the rental housing stock, accepts deposits and buys mortgages. The rental housing stock has no other use so the intermediary simply supplies whatever it owns at the price z(t). Letting  $p_R(t)$  denote the price of a unit of rental space in period t, the intermediary receives  $z(t) - \rho p_R(t)$  each period. Since the intermediary can always buy or sell rental properties,  $p_R(t)$  satisfies the recursion:

$$p_R(t) = z(t) - \rho p_R(t) + [(1 - \Delta)p_R(t+1)]/(1+r). \tag{5}$$

With regard to deposits and mortgages, denote the intermediary's expected net rate return (profits) on a deposit of size a' by  $\nu(a',t)$  and the net expected return on a mortgage with characteristics w, a', x', k' and t by  $\nu(w,a',x',k',t)$ . Correspondingly, let m(a',t) and m(w,a',x',k',t) denote the measure of such contracts acquired by the financial intermediary. Then the intermediary's decision problem with regard to these financial contracts is:

$$\max_{\{m(a',t),\,m(w,a'x',k',t)\}} \left\{ \int \nu(a',t) m(da',t) + \int \nu(w,a',x',k',t) m(dw,da',dx',dk',t) \right\}$$

For this problem to have a solution, the net expected returns on each type of asset must be non-positive. For deposits this requirement reduces to

$$(1+\bar{r}) \ge (1+r). \tag{6}$$

For mortgages, the expression for net return is more involved. When the intermediary acquires a mortgage it gives up  $q(w, a', x', k'; t) \cdot x'$  in goods. Next period, if the household defaults, the intermediary receives  $p(t+1)[1-\chi_D]k'$  where  $\chi_D$  is the cost of foreclosure to the intermediary; if the household sells the property, the intermediary receives  $x' + q(t+1)\mu x'/(1+\pi'')$ ; and if it neither defaults nor sells, the intermediary receives x' plus the value of the continuing mortgage, which is given by  $q(w', a'', \mu x'/(1+\pi''), k'; t+1)\mu x'/(1+\pi'')$ , where  $\pi''$  is the inflation rate between periods t+1 and t+2. Then, the requirement that the expected net return from a mortgage  $\nu(w, a', x', k', t)$  reduces to:

$$q(w, a', x', k', t) x' \ge (1+r)^{-1} \times$$

$$E_{w', \delta'|w} \{ d(w', a', x', k', \delta', t+1) p(t+1) [1 - \chi_D] k' +$$

$$s(w', a', x', k', \delta', t+1) [x' + q(t+1)\mu x'/(1+\pi'')] +$$

$$(1 - d(\cdot, t+1)) (1 - s(\cdot, t+1)) [x' + q(w', a'', \mu x'/(1+\pi''), k', t+1)\mu x'/(1+\pi'')] \},$$
(7)

where  $d(\cdot)$  and  $s(\cdot)$  are the default and selling decision rules for homeowners that take on the value 1 if the homeowner defaults or sells, respectively, and zero otherwise.

## 5 Equilibrium

An equilibrium consists of a stock of rental housing  $H_R$ , a stock of owner-occupied housing  $H_O$ , initial distributions of excluded and non-excluded renters over individual states  $\mu_R(w, a, 0)$  and  $\mu_R^D(w, a, 0)$ , an initial distribution of homeowners  $\mu_O(w, a, x, k, \delta, 0)$ , a sequence of rents  $\{z^*(t)\}$ , a sequence of rental housing prices  $\{p_R^*(t)\}$ , a sequence of owner-

occupied housing prices  $\{p^*(t)\}$ , deposit interest rate  $r^*$  and  $\tilde{r}^*$ , a sequence of mortgage price functions  $\{q^*(w, a', x', k', t)\}$ , a sequence of decision rules and a sequence of distributions  $\mu_R^*(w, a, t), \mu_R^{*D}(w, a, t)$  and  $\mu_O^*(w, a, x, k, \delta, t), t \geq 1$  such that:

- 1. The decision rules are optimal given  $r^*$ ,  $\tilde{r}^*$ ,  $z^*(t)$ ,  $p^*(t)$ ,  $q^*(t)$ .
- 2. The sequence  $z^*(t)$  is strictly positive and  $\{p_R^*(t)\}$  satisfies (5).
- 3. The net returns (6)-(7) are zero.
- 4. Demand for rental housing equals supply for all t

$$\int h_R^*(w, a, t) \mu_R^*(dw, da, t) + \int h_R^{D*}(w, a, t) \mu_R^{D*}(dw, da, t) + \int h_O^*(a, w, x, k, \delta, t) \mu_O^*(da, dw, dx, dk, d\delta, t) = H_R.$$

5. Demand for owner-occupied housing equals supply for all t

$$\int k_R'^*(w, a, t) \mu_R^*(dw, da, t) + \int k_R'^{D*}(w, a, t) \mu_R^{D*}(dw, da, t) + \int k_O'^*(a, w, x, k, \delta, t) \mu_O^*(da, dw, dx, dk, d\delta, t) = H_O.$$

6. The sequence of distributions  $\mu_R^*(w, a, t)$ ,  $\mu_R^{*D}(w, a, t)$  and  $\mu_O^*(w, a, x, k, \delta, t)$ ,  $t \geq 1$  are implied by the sequence of decision rules and initial distributions  $\mu_R(w, a, 0)$ ,  $\mu_R^D(w, a, 0)$  and  $\mu_O(w, a, x, k, \delta, 0)$ .

## 6 Parameter Selection and Calibration

Turning first to the Markov process for earnings, we assume that log earnings follow an AR1 process:

$$\ln(w_t) = \rho \ln(w_{t-1}) + \epsilon_t \tag{8}$$

Several studies have estimated log earnings processes for the US using PSID earnings data.<sup>14</sup> Estimates of  $\psi$  and the standard deviation of  $\epsilon$  ( $\sigma_{\epsilon}$ ) vary across studies. We follow Storesletten, Telmer and Yaron (2004a,b) in setting  $\sigma_{\epsilon} = 0.129$ .and  $\rho = 0.97$  (which is near the lower bound of  $\rho$  estimates reported by the authors). This AR1 process is then approximated by a 17-state Markov chain.

Setting aside the parameters of the income tax schedule, our model economy has 18 other parameters. These include 3 preference parameters  $(\beta, \theta, \gamma)$ , 8 parameters related to housing transactions  $(\chi_S, \chi_B, \psi, \Delta, \{\delta_j, \xi_j\}, j = H, L)$ , 1 related to the mortgage contract  $(\mu)$ , 2 related to the costs of foreclosures  $(\lambda, \chi_D)$ , and 3 related to the asset market  $(\omega, \tilde{r}, r)$  and, finally, the steady-state inflation rate  $(\pi)$ .

Of the preference parameters,  $\gamma$  is set to 2, which is a standard value in macro studies and the value of  $\theta$  is set to 0.20, based on the reported share of rents in expenditures for renters.<sup>15</sup>

Of the housing transactions parameters, Gruber and Martin (2003) find (from the Survey of Consumer Expenditures) that transaction costs account for around 7 percent of the house value; we split this into 6 percent selling cost and 1 percent buying cost, which fixes  $\chi_S$  and  $\chi_B$ , respectively. The average property tax rate in the US in 2007 was 1.38 percent, so  $\psi$  was set to 0.0138.<sup>16</sup> The value of the depreciation rate for rental housing space,  $\Delta$ , was set to 1.66 percent, which is in line with the estimate reported in Shilling, Sirmans and Dombrow (1991) for rental properties built more than 10 years ago.<sup>17</sup> The value of  $\lambda$  was set to 0.5, which implies that exclusion from the mortgage market upon default lasts 2 years, on

<sup>&</sup>lt;sup>14</sup>These processes are typically modeled as the sum of a fixed random effect, an AR1 process and a purely transitory shock. For reasons of tractability, we ignore the fixed random effect and the purely transitory shock.

<sup>&</sup>lt;sup>15</sup>According to Consumer Expenditure Tables (http://www.bls.gov/cex/csxshare.htm#1989) the share of rents in total expenditures varied from 19.7 percent in 1989 to 21.6 percent in 1999.

 $<sup>^{16}{\</sup>rm This}$  was reported in http://www.nytimes.com/2007/04/10/business/11leonhardt-avgproptaxrates.html.

<sup>&</sup>lt;sup>17</sup>The authors report that the average depreciation rate for owner-occupied properties is roughly between 1.93 percent in Year 1 and 1.06 percent in Year 10, while that for rental properties ranges from 2.54 percent in Year 1 to 1.66 percent in Year 10. The findings are robust to alternative specifications of the hedonic price model

average.<sup>18</sup> The loss in value of a house that goes into default is set to 15 percent, which fixes  $\chi_D$  to 0.15.<sup>19</sup> We also assumed that the high depreciation shock for homeowners is equal to  $\chi_D$ , so  $\delta_H = 0.15$ .<sup>20</sup>

We set the average inflation rate to 2.5 percent, which sets  $\pi = 0.025$ .

Turning to the asset market parameters, we set the real pre-tax return on financial assets to 4 percent, which fixes r to 0.04. Regarding the tax treatment of interest earnings, we recognize that only a portion of the nominal returns on financial assets is taxed at the relevant individual income tax rate; the remaining portion is taxed at a (potentially) lower rate because some of the return on assets is in the form of capital gains (which are typically taxed at a lower rate) and both capital gains and dividends and interests on assets that are in retirement accounts are not taxed until the the individual reaches retirement. We assume that the portion that is taxed at the relevant income tax rate is 40 percent, which sets  $\omega$  to 0.40.<sup>21</sup>. We assume that the returns on this latter portion are taxed at a flat rate of 20 percent after a period of 10 years. Given an inflation rate of 2.5 percent and a real return of 4 percent, this is equivalent to annual after-tax earnings from long-term investments of 2.973 percent, which fixes  $\tilde{r} = 0.0297$ .<sup>22</sup>

These values are summarized in Table 1.

<sup>&</sup>lt;sup>18</sup>We chose a relatively short exclusion period because lenders may well lend to households with a fore-closure in their credit history as long as the household is willing to put down enough down payment on the mortgage.

<sup>&</sup>lt;sup>19</sup>Shilling, Benjamin and Sirmans (1990) estimate that the price per square foot of foreclosed properties is about 10 percent less than that of non-distressed properties. In addition, there are other costs borne by lenders that further lower the net realized value from foreclosure.

<sup>&</sup>lt;sup>20</sup>The motivation for this assumption is simply that in the steady state, the high depreciation shock leads to a default when the home-equity ratio is low enough. Thus, we assume the same loss in home value as in a foreclosure.

<sup>&</sup>lt;sup>21</sup>In 2009, the fraction of household financial assets in retirement accounts was 35 percent (Investment Company Institute (2009)). Of the remaining 65 percent, we assume that 70 percent is allocated to equity. The return on equity due to capital gains has been about 58 percent (Ibbotson and Chen (2003, Figure 1)). Thus, the portion of return on financial assets that are taxed at a lower rate is  $0.35 + (0.65)(0.70)(0.58) \approx 0.60$ .

<sup>&</sup>lt;sup>22</sup>The nominal gross after-tax return on a dollar invested in the long-term asset is  $[(1.025 \times 1.04)^{10} - 1](1 - 0.20) + 1 = 1.6890$  and the real return is  $1.6890/(1.025)^{10} = 1.3194$ , which implies an after-tax real rate of return of  $1.3194^{1/10} = 1.02973$ . Hence  $\tilde{r} = 0.0297$ .

Table 1

Parameter	Value	Description	
λ	0.5	probability of re-entry after default	
$\bar{r}$	0.04	risk-free real interest rate	
ρ	0.97	autocorrelation of earnings	
σ	0.129	sd of innovation to earnings shock	
Δ	0.0166	depreciation of rentals each year	
$\delta_H$	0.15	high depreciation rate for homeowners	
θ	0.20	exponent to housing consumption	
γ	2.0	risk-aversion coefficient	
$\pi$	0.025	steady-state inflation	
$\chi_B$	0.01	cost of buying	
$\chi_S$	0.06	cost of selling	
$\chi_D$	0.15	foreclosure cost	
ψ	0.0138	property tax rate	
$\tilde{r}$	0.02973	real after-tax annual return on long-term investment	
$\omega$	0.40	portion of asset return that is currently taxable	

We need to specify the tax schedule  $\tau(\cdot)$  and the standard deduction s. The tax schedule is chosen to match the tax table for 1998. In our model, people are viewed as individuals (this seems consistent with the earnings data). But we will take individuals to be married. Hence, the tax table we use is the tax table for married, filing separately. According to the Census Bureau, median income of year-round full-time male workers age 25 and older in 1998 was \$37,906 and that of females was \$27,956. We use the average of these two numbers, which is \$32,931, as the median income of an individual filing for taxes. Normalizing the tax brackets for 1998 by this estimate of median income, we obtain the following tax schedule  $\tau(\cdot)$ :

Table 2			
Tax Brackets	Tax Rate		
0 - 0.64	0.15		
0.64 - 1.55	0.28		
1.55 - 2.37	0.31		
2.37-4.23	0.36		
4.23 -	0.396.		

And normalizing the standard deduction for a married person filing separately by median income gives  $s=0.1116.^{23}$ 

The remaining 4 parameters  $(\beta, \delta_L, \xi_H, \mu)$  are determined by computing the steady state of the model so as to match the ratio of mean financial assets to mean income, the homeownership rate, the steady-state default rate and the fraction of homeowners with a home equity ratio less than or equal to 25 percent.

We solve for the steady state under the additional assumption that  $p^* = p_R^*$ . The motivation for this assumption is that if new housing space of both types was being produced each period and the marginal cost of producing both types of space was the same, the price of a unit of owner-occupied space will be the same as the price of a unit of rental space. Then,

$$p^* = z^* - \rho p^* + [p^*(1 - \Delta)/(1 + r)]. \tag{9}$$

The right-hand side of (9) is the present discounted value of the flow from one unit of rental housing, if rental housing space sells for the same price as owner-occupied housing space. Thus, even though we do not model the construction sector explicitly, we assume that the long-run equilibrium is one in which (9) is satisfied. Furthermore, we normalize  $z^*$  to 1, which then determines  $p^*$ . The implied demand for rental and owner-occupied housing space is then

 $<sup>^{23}</sup>$ Our tax schedule overstates the taxes paid by low-income people because we ignore the earned income tax credit. However, what is important for our study is the tax benefit of owner-occupied housing and this benefit is *not* affected by the earned income tax credit. This is because the credit is calculated on a person's adjusted gross income and, therefore, does not depend on whether the household rents or owns.

taken to be equal to the available stocks of these spaces. Thus steady-state stocks of rental and owner-occupied housing space is effectively endogenous. The same procedure is followed in all steady-state comparisons.

In computing the ratio of financial wealth to income we used the 1998 Survey of Consumer Finances and ignored the top 3 percent of the wealthiest households (by net worth), since it is well-known that this class of models cannot match the upper tail of the wealth distribution.<sup>24</sup> The homeownership rate we target is the 10-year average ending in 2003 (we stopped at 2003 so as not to distort our parameter choices by booms and busts in the residential real estate markets). The foreclosure rate is the average number of foreclosures started each year as a percentage of all mortgages for 1991-1998. The facts about the home equity ratio distribution were obtained from the 1998 Survey of Consumer Finances.<sup>25</sup>

The results of the matching exercise are displayed in Table 3. The model matches the target statistics almost exactly. The parameter values that achieve this match are listed in the final column. Although these statistics are jointly targeted, the parameter listed in each row is the parameter that is the key for determining the corresponding statistic. The table also lists statistics that were not targeted but are relevant for understanding model mechanics (these will be discussed in the next section).

<sup>&</sup>lt;sup>24</sup>Financial wealth is defined as financial assets - credit card balance - margin loans, loans against pensions, loans against life insurance - other lines of credit not secured by equity in home - educational installment loans.

<sup>&</sup>lt;sup>25</sup>The home equity ratio is defined as the value of the home minus housing debt to the value of the home.

Table 3

Targeted Statistics	Data	Model	Parameter Values	
Avg financial asset/avg income	1.52	1.51	Discount factor, $\beta$	0.9555
Homeownership rate	0.664	0.664	Low depr rate for homeowners, $\delta_L$	0.0049
Steady-state foreclosure rate	0.0135	0.0135	Prob of high depr rate, $\xi_H$	0.078
Frac of homeowners with $\leq 25\%$ equity	0.19	0.18	Prop of maturing mortgages, $(1 - \mu)$	0.015
Non-targeted Statistics				
Avg inc of homeowners/avg inc of renters	2.02	1.94		
Avg housing wealth/avg income	1.28	1.48		
Average home equity ratio	0.64	0.68		
Frac of homeowners with < 0% equity	3.03	4.07		
Frac of homeowners with $< 10\%$ equity	7.83	10.15		
Frac of homeowners with < 20% equity	13.73	15.53		
Frac of homeowners with < 30% equity	22.95	20.49		

## 7 Analysis of the Steady State

In this section, we analyze the nature of the steady-state equilibrium and the key forces at work in our model.

## 7.1 Why Do Households Purchase Homes?

In our model, homeownership yields no direct utility benefit, involves less flexibility (in terms of house sizes) than renting, involves only slightly less (expected) depreciation than rental properties and imposes significant transactions costs in its acquisition. Nevertheless, the majority of people choose to purchase their homes. This is because of a key tax benefit of homeownership: the implicit rental income from ownership is not counted as part of income and therefore not taxed. This exemption means that people – especially those in the

higher tax brackets – have a strong incentive to purchase their homes.<sup>26</sup> The deductibility of mortgage interest payments encourages these households to *borrow* to finance the purchase of their homes as opposed to paying for the purchase from accumulated assets. Both effects operate more strongly for richer households because their tax rate is higher and the mortgage interest payment deductions are more likely to exceed standard deductions as they buy bigger homes. Given this, homeowners are concentrated among richer households. In our model, the average income of homeowners is 1.97 times the average income of renters, which compares very favorably with the data, where it is 2.02.

Our model is also consistent with the observation that owner-occupants consume more housing space, on average, than renters. In our equilibrium the per capita housing space of renters is 54 percent of the per capita housing space of owner-occupants. There are two reasons for this: First, high earners choose to buy houses, which makes the housing space of owner-occupants larger than that of renters. In addition, the tax benefit of owner-occupancy makes owner-occupants consume more housing than renters. To establish this point, Table 4 shows how the steady state is altered if these tax benefits were to be eliminated. If the mortgage deduction is eliminated, the homeownership rate declines from 0.664 to 0.605. Average equity rises to more than 95.9 percent, since there is no benefit to taking on leverage. In addition, average housing consumption in the whole economy (including homeowners and renters) declines by 5.8 percent. If, in addition to getting rid of the mortgage tax deduction, the implicit rental income from owner-occupancy is taxed in exactly the same fashion as returns from financial assets, the incentive to own homes goes away entirely. Furthermore, average housing consumption declines by 13.4 percent relative to the baseline model.

<sup>&</sup>lt;sup>26</sup>Let's say the household is deciding between saving in a risk-free asset or saving in a home. When the household saves in a risk-free asset, it pays taxes on the nominal interest return. If the household saves by buying a house, the return to that saving comes as (implicit) rental income and appreciation in the value of the house, both of which are not taxed. So there is an additional tax benefit to homeownership.

Table 4

Statistic	Baseline	No Mtg Ded	No Mtg Ded &Taxes on Impl Rental Inc
Homeownership rate	0.664	0.605	0
Avg Home Equity	0.675	0.959	0
Avg Housing Cons	1.0	0.942	0.866

# 7.2 Why Do People Not Hold An Even Higher Portion of Their Wealth in Housing?

In the steady state, the benefit to having home equity occurs because the implicit return in rental income from owning the house is not taxed while if that equity was transferred to other assets, the returns would be taxed. But the higher implicit return on housing must be balanced against the fact that the higher return must be spent on housing consumption. Thus a homeowner's investment in home equity is bounded by the utility flow from housing services. In our calibration, the exponent to housing services in the Cobb-Douglas utility function is 0.20 (which implies fairly sharply diminishing marginal utility from housing services) and results in average home equity to average income ratio of 1.48. This is somewhat higher than what we find in the data.

## 7.3 Why Is There Default in the Steady State?

For there to be default on a mortgage, the selling option must be inferior to default – which can happen only if the value of the homeowner's obligation to the lender exceeds the sale price of the house less the transaction costs of selling. Thus, negative home equity is a necessary condition for default. It is, however, not sufficient and there are two reasons for this.

First, even if selling imposes a capital loss, this loss has to be weighed against the costs of default. This cost stems from the loss of access to mortgage markets for some (random)

length of time. If the defaulter does not have the personal wealth to purchase a house, the loss of access to mortgage markets implies loss of the tax benefits of homeownership. These tax benefits rise with earnings, since high earners face a higher tax rate and are in a better position to take advantage of the mortgage interest deductions. Therefore, if the capital loss from selling is small enough and the tax benefits of homeownership are valuable enough, the selling option may dominate the default option.

Second, even if home equity is negative and the default option dominates the selling option, the individual may choose not to default because the option of keeping the house and paying the mortgage may dominate the default option. For default to dominate the keeping option, utility from keeping must be relatively low. This will tend to be the case if, since taking out the mortgage, the individual's income has changed sufficiently to make the size of his house and/or the size of his mortgage suboptimal relative to his current resources. In particular, if he has experienced bad income shocks his house and/or mortgage may be too large relative his earnings and, thus, the keeping option may be low relative to the default option. Also, since the cost of default tends to decline with earnings, the default option is relatively higher for such a borrower.

In any event, a precondition for default is negative home equity. In the steady state, with a constant price of housing, there are only two ways in which an individual can end up with negative home equity. One is for the individual to knowingly borrow more than the value of the house less selling costs. This channel is permitted in our model but it is rarely active: In our calibration, borrowers prefer to increase their down payment rather than pay the default premium on a negative home equity loan. The other way is for there to be an *idiosyncratic* loss in the value of the house offered as collateral. The random depreciation shocks,  $\delta$ , allow for this possibility. When the high depreciation shock hits, a homeowner with relatively low home equity may end up with negative home equity.

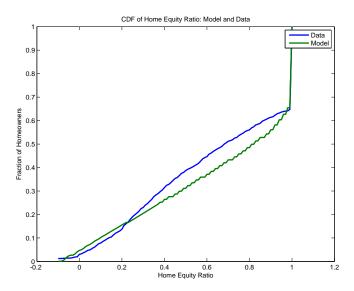
But since home equity is something that a borrower can control via his down payment, the mere existence of the adverse depreciation shock is not sufficient to generate default. The additional assumption we need is that following such a shock, the further costs of foreclosure to the financial intermediary is negligible: i.e.,  $\chi_D = \xi_H$ . Given that the lender is risk neutral and the depreciation shocks are independent across homeowners, it is then efficient for lenders to insure risk-averse borrowers against the risk of a bad depreciation shock: The lender expects that if the depreciation shock happens early on in the contract (when home equity is still low), the homeowner will choose to avoid the depreciation cost by passing the house back to the lender. In return, the lender charges the homeowner a higher interest payment spread through the years. If there is a further cost of foreclosure, i.e.,  $\chi_D > \xi_H$ , lenders will ask for higher premia and the cost of insurance will rise beyond what is actuarially fair (from the perspective of the homeowner). The higher cost will motivate individuals to supply a larger down payment and thereby lower the probability of default on the loan (or eliminate it altogether).

## 7.4 What Factors Determine the Home-Equity Distribution in the Model?

The home-equity distribution among new buyers is bi-modal (U-shaped). A significant fraction of people buy houses with very little equity, and these are mostly high wage earners who wish to use the tax deduction of mortgage interest payments since their interest payments are likely to exceed the standard deduction. The middle is relatively empty, with the second big mass coming at the other end with people choosing very high home equity rates. These are middle- to low-income households who wish to buy smaller houses and for whom – had they taken out a big mortgage – the interest payments would have fallen below the standard deduction. When the mortgage payment is below or very close to the standard deduction, it is more beneficial for a person to invest more of his existing savings into the house and get the tax-free implicit rental returns from that savings. Consistent with this reasoning, the average income of people taking out (new) mortgages with less than 50 percent home equity is 1.62 times the average income of households taking out new mortgages with more than 50

percent home equity. Also, the average house size of the former group is about 1.37 times the average house size of the latter group. Aside from these considerations, the decision of how much to borrow to finance the purchase of the house depends also on the household's financial wealth: Some households get a large loan because they wish to become homeowners but do not have the financial wealth for a large down payment.

Turning to the distribution of home equity across the population of all homeowners, two other important factors come into play: Geometric decay and inflation. Because the mortgage contract is nominal, inflation steadily reduces the real value of debt over time and, therefore, steadily increases the real value of home equity. In addition, households steadily pay down their debt at a constant geometric rate. The almost linear form of the CDF of the home equity distribution – both in the model and in the data – results from these two steady forces at work. Figure 1 displays the home-equity distribution in the model and in the data.



#### 7.5 How Does Steady-State Inflation Affect the Housing Market?

Table 5 compares the baseline steady state, which has an inflation rate of 2.5 percent, with the steady state when the inflation rate is 4 percent. There are two effects: First, a higher inflation rate increases the nominal interest rate and, therefore, the tax benefits of the mortgage deduction. The benefits of the mortgage deduction increase with inflation because nominal interest payments are tax-deductible, not real. This encourages households to take on more debt and buy bigger houses. On the other hand, higher inflation erodes the value of debt faster and thus causes households to accumulate home equity at a faster rate. In the experiment, the first effect dominates and average home equity goes down and the homeownership rate goes up along with average housing consumption. Higher inflation also makes saving in housing more attractive as the effective tax rate on financial assets becomes higher when inflation is higher, since those taxes also depend on the nominal return, not the real return.

Table 5

Statistic	SS infl of 1 percent	Baseline	SS infl of 4 percent
Homeownership rate	0.650	0.664	0.664
Average equity	0.72	0.68	0.65
Average housing consumption	0.99	1.0	1.01

## 8 Foreclosure Crisis

#### 8.1 Calibration

In this section, we use the model to quantitatively study the foreclosure crisis story outlined in the introduction. The two key facts we want the model to account for is the rise in the foreclosure rate between 2006 Q2 and 2010 Q2 and the drop in house prices between 2006 Q2 and 2009 Q2. The cumulative fall in house prices over this latter period is about 19

percent and the cumulative foreclosure rate over the former period is 15 percent.<sup>27</sup>

We model the over-supply of housing as a permanent, unanticipated increase in the stock of owner-occupied housing space in period 1. We assume that this additional housing space is initially owned by developers who reside outside our economy. Developers supply the extra space on the market as long as  $p(t) > -\rho p(t) + [p(t+1)(1-E\delta)]/(1+r)$ . The right-hand side of the inequality is the return from keeping a unit of space unsold for one period; if  $p(t) = -\rho p(t) + [p(t+1)(1-E\delta)]/(1+r)$ , the entities are indifferent between holding on to the space or selling it. We compute the perfect foresight transition path to the new steady state adhering to the constraint that

$$[p(t) + \rho p(t) - [p(t+1)(1-E\delta)]/(1+r)] \cdot I(t+1) = 0,$$

where I(t+1) is the stock of unsold homes in the hands of developers at the start of period t+1. Along the equilibrium path, the evolution of this stock follows:

$$\begin{split} I(t+1) &= \max \left\{ 0, H_O^* + I(t) - \int k_R'^*(w, a, t) \mu_R^*(dw, da, t) \right. \\ &- \int k_R'^{D*}(w, a, t) \mu_R^{D*}(dw, da, t) - \int k_O'^*(a, w, x, k, \delta, t) \mu_O^*(da, dw, dx, dk, d\delta, t) \right\}, \end{split}$$

where I(1) is the initial stock of unsold housing in the hands of developers.<sup>28</sup>

We start with an estimate of I(1). McNulty (2009) reports that between 2005 and 2007, the housing stock increased by 3.8 million units but the number of occupied housing units increased by only 1.8 million units. Thus, about 2 million housing units were added that did not find occupants. Since houses typically sit on the market for some time before

<sup>&</sup>lt;sup>27</sup>The drop in house prices is computed from the LoanPerformance house price index excluding distressed sales. According to this series, house prices peaked in 2006Q2 and then fell and temporarily stabilized in 2009 Q2. We use 5 quarter averages centered around these peak and trough quarters to calculate the percentage decline in price. For the foreclosure rate, we simply sum the quarterly foreclosure rate between 2006 Q2 to 2010 Q2. We cumulate up to the 2010 Q2 because foreclosures take time to process and the foreclosures that occurred in 2010 Q2 presumably started around 2009 Q2 (or earlier).

<sup>&</sup>lt;sup>28</sup>We assume that only developers can carry unsold homes; intermediaries who receive housing space through foreclosures pay the depreciation cost and immediately sell the house.

they find occupants, part of the increase in unoccupied housing units is simply a reflection of "frictional" vacancy. McNulty estimates that the increase in unoccupied units due to frictional vacancies to be about 0.28 million units, which leaves an excess of 1.72 million units. As a percentage of the stock of owner-occupied housing units in 2005, this is about 2.3 percent. In our analysis, we assume that the excess supply of owner-occupied housing is 3 percent. We chose a somewhat higher excess supply in order to compensate for the fact that our model leaves out features that, in the real world, tend to lower the elasticity of housing demand with respect to the price of housing space.<sup>29</sup>

Second, we assume that following the shock of over-supply, there is a disruption in the flow of credit to the mortgage market. We model the disruption as a wedge  $(1 - \Psi(t))$ such that when a household makes a promise to pay the sequence  $\{x', \mu x'/(1+\pi''), \ldots\}$ he obtains  $q(w,a',x',k',t)(1-\Psi(t))x'$  in the current period. We assume that the wedge remains constant for the first 4 periods following the shock, then declines at the rate of 20 percent per year. The wedge increases the cost of new mortgages: The borrower now has to promise more in the future to get the same level of resources as before. Obviously, this lowers the demand for owner-occupied housing space and its price has to drop to motivate renters to become homeowners and absorb the additional supply. We chose the size of the initial wedge so that the model produces a decline in the price of owner-occupied housing of 19 percent in the initial period. Calibrated in this way,  $\Psi = 0.14$  in the initial period, which is roughly equal to a 1-percentage-point increase in the cost of funds beyond the riskfree rate. Hall (2011, Table 2) reports that the spread between AAA corporate bonds and constant-maturity 20-year Treasuries rose 1.08 percentage points during the worst of the crisis and the spread between BAA bonds and Treasuries rose 3.65 percentage points. He interprets the widening spread between essentially default-free debt instruments as reflecting the emergence of a financial friction "wedge." Our calibration of  $\Psi$  is broadly consistent with this evidence.

<sup>&</sup>lt;sup>29</sup>For instance, in our model, a drop in the price of housing space leads to an increase in the measure of small owner-occupied houses. In reality, the measure of different size houses is unlikely to change much when house prices drop.

Finally, we assume that for the first 4 years following the shock, households that choose to foreclose on their homes get to live in their house rent-free with some fixed probability. This assumption captures the fact that the time to foreclosure lengthened during the crisis. In normal times a foreclosure takes about 6 months to complete but during the crisis foreclosures have been taking an additional 7.5 months, on average.<sup>30</sup> The lengthening has resulted from processing delays because of the sheer volume of foreclosures.<sup>31</sup> This extra time for foreclosure is taken into account by setting the probability with which a person declaring foreclosure gets to stay in the house rent-free for an additional year to 0.63.

#### 8.2 Baseline Results

Table 6				
	SS	Post-shock SS	Shock Period	
House Prices	1	0.98	0.81	
Rents	1	0.98	0.88	
Foreclosures (%)	1.35	1.39	12.25	

Table 6 displays the equilibrium outcome regarding house prices, rents and foreclosures for the new steady state and for the initial period (the period of the shock). In the new steady state, the increase in the supply of owner-occupied housing has benign effects: The 3 percent increase in the supply of owner-occupied housing space leads to a roughly 2 percent decline in the price of owner-occupied housing and a 2 percent decline in rents. The excess supply is absorbed through an increase in the average housing space occupied by owners and an increase in the fraction of homeowners. There is only a slight increase in the foreclosure rate.

<sup>&</sup>lt;sup>30</sup>We compared the average days delinquent for foreclosure in August 2010 (468 days) with January 2008 (249) days, which implies a lengthening of around 7 months. The data are from Loan Processing Services (LPS); see <a href="http://www.lpsvcs.com/LPSCorporateInformation/ResourceCenter/PressResources/Pages/MortgageMonitorArchive.aspx">http://www.lpsvcs.com/LPSCorporateInformation/ResourceCenter/PressResources/Pages/MortgageMonitorArchive.aspx</a>.

<sup>&</sup>lt;sup>31</sup>While we model the lengthening of the time to foreclosure as an exogenous event, it is possible that the lengthening is a self-fulfilling outcome wherein a large number of individual borrowers expect processing delays from a high volume of defaults and then default and thus confirm these expectations. See Arellano and Kocherlakota (2008) for a model of sovereign default with this feature.

In the period of the shock, however, the gross and net foreclosure rates rise to 12.25 percent and 10.9 percent, respectively. These predictions are reasonably in line with what actually transpired. The the movement of renters into owner-occupancy reduces demand for rental space in the period of the shock and lowers rents.

The 19 percent decline in house prices along with the selling costs of 6 percent means that homeowners with about 25 percent or less home equity prior to the shock satisfy the necessary condition for default following the shock. In our model, the fraction of such homeowners is 18 percent, which is calibrated to be close to what it is in the data. But whether a person with low or negative home equity actually defaults depends on his best alternative. Given the large decline in prices, the option of selling the house (which would inflict a large capital loss on the homeowner) is typically dominated by the option of keeping the house. Therefore, for a homeowner with negative home equity to default, the default option must be better than the keeping option. Some conditions under which this would be true have been discussed earlier (namely, a change in earnings since the mortgage was taken out that makes the size the house and/or mortgage suboptimal relative to his current resources). The new forces at work is that house prices and rents are temporarily low, and there is a chance that after declaring default the individual gets to live rent-free for one year. These factors raise the value of the default option.

In the periods following the shock, the default remains somewhat elevated for about 4 years and then essentially goes to zero. After the shock, default continues at a somewhat higher rate because there are more individuals with negative or low home equity and if they receive a shock that makes them unhappy with the size of their house or their mortgage, they may wish to default. The fact that they get to live rent-free with some probability over this four-year period is also another reason that foreclosures happen with higher frequency.

In the rest of this section, we quantify the contribution of different factors to the decline in the price of housing and to the rise in foreclosures. The results are summarized in Table 7.

#### 8.3 Do Foreclosures Depress House Prices?

As we have noted, the drop in the price of owner-occupied housing space creates the necessary conditions for the foreclosure crisis by increasing the fraction of homeowners with low or negative home equity. One may also ask if there is feedback from the rise in foreclosures to the decline in the price of owner-occupied space. In particular, if we prevented foreclosures, would house prices fall less?

The answer depends on a defaulter's next best alternative. If the defaulter is choosing between selling and defaulting, preventing him from defaulting would push him to sell. This would increase the supply of space in the owner-occupied market. The reason is that when a person defaults, there is only a 37 percent chance that the person will have to move out of the house and the space would be offered for sale. In contrast, if the homeowner is forced to sell, then the space would be offered for sale with probability 1. If the next best alternative to default is to keep the house, then preventing default will decrease the supply of space. Because of the large drop in price, the next best alternative to default for most individuals is to keep the house. Thus, foreclosures are a depressive force on house prices: if we prevented households from defaulting, we would reduce the supply of owner-occupied housing space offered for sale and, therefore, mitigate the drop in the price of owner-occupied housing space. We find that if foreclosures are eliminated, the drop in price would be 16 percent, as opposed to 19 percent. Thus, without foreclosures the drop in house prices would still be 84% of the observed drop in house prices.

## 8.4 The Role of Lengthened Time to Foreclosure

The fact that the foreclosure process has lengthened considerably during the crisis may have contributed to the crisis itself. We can examine what equilibrium default and price decline would be like if the probability of staying "rent-free" for one year is set to zero. Then, the fraction of mortgages that default in the period of the shock is only 6.9 percent, which is

about half of what is observed. Although the frequency of foreclosure drops, the amount of owner-occupied housing space offered for sale actually *increases* because all foreclosed properties are now offered for sale. The drop in the price of housing, however, is not much affected: the decline is only slightly larger. The reason for this is that p(1) hits the lower bound implied by the possibility of an unsold inventory of homes. In the period of the shock, about 16.67 percent of the new owner-occupied housing space is carried over as unsold homes in the hands of developers. The constraint does not bind in any other period.<sup>32</sup> From this experiment we conclude that if over-supply and mortgage disruptions were the only forces at work and the foreclosure process did not lengthen, we would see only half the foreclosures actually observed and a drop in price only somewhat larger than what is observed.

#### 8.5 The Role of Mortgage Market Disruptions

In the model, as well as in the real world, the foreclosure crisis disrupted the flow of funds into the mortgage market. As noted above, in the model we take this into account by incorporating the factor  $(1 - \Psi)$  (corresponding to a 1 percent additional cost, beyond the risk-free rate of getting a new mortgage). The additional cost is present for 4 model periods and then declines rapidly at a geometric rate. What happens if this additional cost is eliminated? In the period of the shock, the foreclosure rate rises to 9.14 percent, as opposed to 12.25 percent in the baseline model, but the decline in price is a lot less. Now, in the period of the shock, the price of owner-occupied housing declines by only 5.5 percent. The reason is that the renters are more willing to jump in and buy houses when the cost of mortgages is lower and, when they do buy houses, they buy bigger ones. Also, because there are fewer foreclosures, there is less downward pressure on the price of owner-occupied housing space. On all these counts, the decline in house prices is much more moderate.

Although the price decline is moderate, the foreclosure rate is still quite high. The reason

<sup>&</sup>lt;sup>32</sup>In the baseline exercise, the price of owner-occupied housing is always greater than this lower bound in each period, so no housing space meant for owner-occupancy is ever left empty.

for this is that the lower cost of mortgages also makes defaulting attractive: Some defaulters default with an eye to buying a house in the near future when the exclusion period is over. A lower mortgage cost makes this strategy more attractive and thus increases foreclosures.

## 8.6 Unexpected Disinflation and Foreclosures

In this subsection, we study the effects of a lower inflation path on house prices and foreclosures. We assume that in the period of the shock, the anticipated inflation rate going forward falls to 1 percent for 5 years and then recovers back to the steady state value of 2.5 percent. To do this we need to be clear about the nature of the mortgage contract. We assume that the contracts written prior to the shock stipulate that upon sale of the house, the present discounted value of the outstanding payment stream is evaluated at the current market interest rate. In the steady state this is equivalent to the present discounted value being calculated at the time the mortgage is written. However, when the shock hits, the nominal interest rate at which the payment stream is evaluated is now (unexpectedly) lower – because anticipated inflation is lower.

The lower inflation path increases the default rate from 12.25 percent to 17.07 percent. With a lower inflation rate, the real value of mortgage debt does not erode as rapidly as in the baseline model. Thus, the value of keeping the house is lower. And the value from selling the house decreases as well because the present discounted value of the outstanding loan to be repaid upon sale is now higher. For both reasons, more households find default a better option. The higher default rate does not have much of an impact on the price of houses. House prices fall by about 20 percent now as opposed to 18 percent in the baseline model. It appears that the supply of homebuyers is fairly elastic at this price: As the price drops slightly, the excess supply of housing stemming from higher default is soaked up by new buyers and existing buyers buying bigger homes.

## 8.7 Mortgage Deduction and the Mortgage Crisis

In this section, we study how the crisis would have fared if there was no mortgage incentive to take on leverage. As noted in the discussion of the steady state of the baseline model, eliminating the mortgage deduction lowered the incentive to own homes and greatly lowered the incentive to take on mortgage debt. Thus average home equity is much higher and the average size of owner-occupied housing is also lower. Also, given the very high home equity there is no default in the steady state. A 3 percent increase in the supply of owner-occupied housing lowers the price by about 10 percent. The drop in price does not create any incentive to default, so there is no increase in foreclosures in the period of the shock. The drop in price in the period of the shock exceeds the steady-state drop for the same reasons as in the baseline model: The costs of selling and buying homes make the demand for owner-occupied housing space insensitive to a change in the price in the short run. Since there are no foreclosures, it makes sense to also analyze what would happen if the mortgage cost is eliminated as well. In this case, the drop in price is about 5.5 percent.

Table 7

Experiment	House Price Decline in %	Foreclosure Rate in %
Baseline	19	12.25
No Default	16	0
No Free-Rental	19	6.90
No Mortgage Cost	5.5	9.14
No Mortgage Ded	10	0
Unexpectedly Low Inflation	20	17.05

# 9 Conclusion

This paper developed a quantitative model of the US housing and mortgage market. We calibrated the model to be consistent with a small number of long-run facts in these markets.

The model turned out to be consistent with a range of other facts as well. We pointed out that the federal tax code has important implications for these markets. The non-taxability of implicit rental income is key for getting a large number of homeowners. The deductibility of mortgage interest payments in computing taxable income is key for getting people to borrow to purchase their homes. And inflation is important in getting the dispersed distribution of home equity we see in the data.

We used the model to understand the foreclosure crisis. We showed that a modest level of over-supply in the housing market, coupled with a plausible increase in the cost of new mortgages, can account for the steep decline in house prices. Given the decline in house prices, the model can account for much of the observed rise in foreclosures if we also take into account the lengthening of the time to complete a foreclosure.

We used the model to perform counterfactual experiments to get a quantitative understanding of the importance of different factors that go into generating the drop in price and the rise in foreclosures. Three findings are worth stressing. First, we found that the lengthening of the time to foreclosure, which allows homeowners who default on their mortgages to continue to stay in the house "rent-free," is an important factor in the increase in the foreclosure rate but not an important factor in the drop in house prices. Second, the increase in the cost of obtaining new mortgages is an important factor in generating the decline in house prices but not important for generating the increase in foreclosures. We also argued that foreclosures are a depressive force on house prices in that if foreclosures are prevented altogether, the decline in house prices would be less. Finally, we also noted that leverage played a big role in the foreclosure crisis. If the tax code did not encourage leverage by making the interest payment of mortgages tax deductible, home equity would be much higher. In this case, the overbuilding shock would not cause any foreclosures and the price drop would be a lot less.

# 10 Appendix on the Computational Algorithm

We start with a steady state without any aggregate shocks and then perturb the economy with a permanent unanticipated shock to the supply of housing and solve for the perfect foresight transition path to the new steady state.

## 10.1 Main Algorithm

The algorithm is as follows. We assume that the transition from the initial steady state to the new steady state takes 50 periods (years). The solution algorithm will alert us if 50 periods is too short.

### START OF OUTER LOOP

1. Guess a sequence of z(t) and a sequence of p(t) for periods 1 through 51. For t = 1, we normalize z(1) = 1 and set p(1) as the solution to equation 9.

### START OF INNER LOOP

- (a) Guess value functions and mortgage pricing functions for period 1 and period 51. That is, guess  $V_R(w, a, t)$ ,  $V_R^D(w, a, t)$ ,  $V_H(w, a, x, k, t)$ , and q(w, a, x, k, t) for t = 1, 51.
- (b) Solve for decision rules for t = 51 assuming that the value functions and the pricing function for t = 52 are the same as the guessed value and pricing functions for t = 51. This assumption imposes that we are in steady state in period 51. The decision rule for t = 51 implies new value functions and a new pricing function for t = 51. Replace the guessed value and pricing functions by these new value and pricing functions. Recompute the period 51 decision rule. Continue repeating this step until the new value and pricing functions are close to the guessed value and pricing functions.

- (c) Use the converged decision rule for t = 51 and the converged pricing function for period t = 51 to compute the pricing function for period t = 50 (see equation 7). Use this pricing function for t = 50 and the converged value function for t = 51 to compute the value function and decision rules for t = 50.
- (d) Proceed backwards in this way, calculating new value functions, pricing functions and decision rules all the way back to t = 2.
- (e) Solve for decision rules in t = 1 assuming that the value function and pricing function for t = 2 are the same as the guessed value and pricing functions  $V_R(w, a, 1)$ ,  $V_R^D(w, a, 1)$ ,  $V_H(w, a, x, k, 1)$ , and q(w, a, x, k, 1). Again this imposes the assumption that we are in the steady state in period 1 (this is where we use the assumption that the shock that happens in period 2 is unanticipated). Update the price and value functions for t = 1 until they converge just as in step b.

#### END OF INNER LOOP

- 2. Use the converged decision rules for t = 1 to compute the initial steady state distribution of people over the state space. Set the total owner-occupied housing space in period t = 1 to the total demand for owner-occupied housing space implied by the initial steady state distribution and the total supply of rental housing space to the total demand for rental housing space implied by the initial distribution.
- 3. Starting from this initial distribution in t=1, use the decision rules computed for periods  $t=2,3,\ldots,51$  to compute the distribution of households over the state space for periods  $t=2,3,4,\ldots,51$ .
- 4. Use these distributions to compute excess demand for housing space in each of the years. The supply of owner-occupied housing space in periods 2 through 51 is simply (1.03) times the supply of owner-occupied housing space for t = 1 determined in step 2 and the supply of rental space in periods 2 through 51 is the supply of rental space for t = 1 determined in step 2.

- 5. For t = 2, 3, ..., 51, update z(t) and p(t) appropriately (increasing the price slightly if there is an excess demand in that period and decreasing it slightly if there is an excess supply).
- 6. Repeat 1-5 until excess demand in each market in each period is almost zero.

### END OF OUTER LOOP.

Note that if the converged sequence of housing and rental prices does not change very much in the last several periods, that is a good indication that we are close to steady state by period 51.

## 10.2 Computation of Value Functions and Decision Rules

The value functions and decision rules are solved on a grid. The number of grid points for w is 17; for a it is 75; for x it is 80; and for k it is 15 (the depreciation shock  $\delta$  has a two-point distribution). When solving for the decision rules for a' and x', we allow for choices that are off the grid. In particular, we search over  $15 \times 75$  points for a' and  $5 \times 80$  points for x'. For values of a' and x' that are not on the grid we use linear interpolation of the (future) value function (in effect, we assign them randomly to the relevant adjacent grid points).

To calculate the excess demand for owner-occupied and rental properties, we simulate the economy, keeping track of the measure of individuals on each grid point. For the simulations, we assume that if an individual chooses a' or x' off the grid, the individual is sent to the relevant adjacent grids according to the probabilities defined by the interpolation step above.

To ensure the continuity of mortgage pricing function q(w, a, x, k, t) with respect to the current and future prices embedded in the aggregate state variable t, it is sometimes necessary to allow for small independent errors in the decision rule of default. Specifically, we assume that a homeowner chooses to default if  $K_D - \max\{K_0, K_1\} > \kappa$ , where  $\kappa$  is drawn from a mean zero extreme value distribution with a very small variance. The presence of this error

"smooths out" the default response of individuals who are almost indifferent between default and some other option to small changes in housing and rental prices.<sup>33</sup>

Finally, we check the steady-state equilibrium for sensitivity to changes in the number of grid points.

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 $<sup>^{33}\</sup>mathrm{See}$  Chatterjee and Eyigungor (2011) for a discussion of how in difference between two best options can lead to a lack of continuity of price functions like  $q(\cdot)$  in discrete state models and, consequently, to lack of convergence of numerical algorithms employed to compute the solution to discrete state default models.

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