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THE EMERGENCE AND FUTURE OF CENTRAL
COUNTERPARTIES**

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The Emergence and Future of Central Counterparties[*]

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Abstract

We explain why central counterparties (CCPs) emerged historically. With standardized contracts, it is optimal to insure counterparty risk by clearing those contracts through a CCP that uses novation and mutualization. As netting is not essential for these services, it does not explain why CCPs exist. In over-the-counter markets, as contracts are customized and not fungible, a CCP cannot fully guarantee contract performance. Still, a CCP can help: As bargaining leads to an inefficient allocation of default risk relative to the gains from customization, a transfer scheme is needed. A CCP can implement it by offering partial insurance for customized contracts.

Keywords: Counterparty Risk, Novation, Mutualization, Over-the-counter Markets, Customized Financial Contracts

JEL Classification: G2, G13, D53, D82

1   Introduction

A central counterparty (CCP) is an institution designed to insure counterparty risk. After financial trades are executed, the clearing process reconciles the terms of the trades to make them legally binding. This, however, need not prevent default. Adding a CCP to this clearing process offers two additional services to deal with the costs of a possible default: (i) novation and (ii) mutualization of losses. Novation refers to the legal act of replacing the original contract between the buyer and the seller with a contract between the buyer and the CCP and another one between the seller and the CCP. By doing so, the CCP erases the original obligations between the buyer and the seller and becomes the sole counterparty to both the original buyer and seller. As a consequence, if the CCP is able to fulfill the contract, it eliminates the idiosyncratic risk borne by a trader that his particular counterparty defaults. Still, this does not mean that a CCP eliminates default risk altogether. Rather, the CCP needs to make sure that it has enough resources to cover this risk using proper risk management tools, such as collateral in the form of margins. Should margins not offer enough resources, it can ask its members to cover its losses; in other words, it mutualizes its losses.

When financial trades are taking place on centralized trading platforms like the New York Stock Exchange or the Chicago Mercantile Exchange, clearing takes place through a CCP. Other markets do not have formal clearing arrangements: These are often described as over-the-counter markets (OTC) where trades do not take place on a formal/organized trading platform but are instead bilaterally negotiated. Even before the current financial crisis, many observers criticized the organization of these markets, in particular for their lack of transparency and counterparty risk management. As trades are bilateral, there is no information about who is trading which securities and at what price, with the consequence that risk exposures are difficult to manage. As a consequence, policies have been adopted to impose mandatory clearing through a CCP for a large fraction of standard OTC derivatives to improve transparency and stability in these markets.[1]

We present a framework that explains why CCP-clearing – by which we mean the process of novation and mutualization – has emerged as an efficient part of the market structure on centralized trading platforms. Then we analyze whether it can and should be introduced for OTC markets as well.[2] To do so, we distinguish between standardized and customized

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[1]In the US, the Frank-Dodd bill requires clearing for all sufficiently standardized derivatives. Other proposal point to a minimum fraction of volume and/or value of the OTC market to be formally cleared (see for example BIS (2007) or IMF (2010)).

[2]We do not look at two other services – multilateral netting (see Duffie and Zhu (2009) and the provision

financial contracts in a highly stylized way along two dimensions: fungibility and trading protocol. Standardized contracts are traded competitively on a centralized market with an organized trading protocol. As a consequence, they can be forced to be cleared through a CCP.[3] Such contracts are *fungible*: The contract can be easily replaced by the same contract at any given point in time through a market trade. Once contracts, however, are customized to the needs of their counterparties, they are less fungible and can escape any mandatory clearing. Since these contracts are customized, there is no formal market where they can be traded or replaced. Also, their specific terms are bilaterally negotiated directly between the counterparties. The degree of customization is thus endogenous, making it extremely difficult to mandate CCP clearing as part of the trading arrangement.[4] Hence, CCP clearing of OTC transactions needs to recognize the limited fungibility of contracts and the need to provide incentives for their formal clearing.[5]

When financial contracts are standardized, we find that an efficient allocation is implemented when agents trade on a centralized exchange, where a mutualized CCP novates trade. Novation diversifies counterparty risk, thus reducing the need for collateral, while mutualization insures against the cost of default. Therefore, the CCP can guarantee the full terms of trade. This explains the prevalence of such clearing arrangements on formal exchanges. To the contrary, when contracts are customized, we find that an efficient allocation cannot be implemented for two reasons. First, the scope for insurance through mutualization is limited due to a lack of fungibility. Second, the trading protocol (in our case bilateral bargaining) introduces the inefficiency that – relative to the gains from customization – some counterparties take on too much default risk, while others take on too little. A revenue-neutral transfer scheme could improve the allocation by skewing default risk toward trades with larger gains from customization. But the scheme still needs to give incentives to traders to reveal their

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of trade information (see Koeppl, Monnet and Temzelides (2009) and Acharya and Bisin (2009)) – that are often associated with CCP clearing but need not be performed by a CCP. Leitner (2009) develops a model of a data warehouse that could be taken up by a CCP.

[3]For example, once the traders agree to buy or sell, the clearing process *automatically* sends the contract to the CCP for novation. As such, the agreement to buy or sell includes the agreement to CCP-clearing.

[4]Commonly, a distinction is being made between OTC transactions in standardized and customized contracts. One can argue, however, that any OTC transaction is intrinsically customized as it involves a bilateral trading environment rather than a centralized exchange. Trading on a centralized exchange is usually combined with a specific clearing arrangement offered by a clearinghouse. In general, counterparties can always choose to sufficiently customize a transaction – including the clearing arrangement itself – thereby effectively preventing mandatory clearing by a CCP for sufficiently standardized assets.

[5]Kroszner (1999) gives an historical account of how standardization of financial contracts was crucial for the formation of centralized markets with formal clearing arrangements. As assets became standardized, they could easily be cleared through a CCP and traders worried less or not at all about counterparty risk. As a consequence, they could accept anonymous counterparties, which expanded their set of possible trades. The account, however, is silent on the benefits and effects of customized contracts.

trades in customized contracts. Interestingly, a CCP can provide such incentives in the form of gains from diversifying counterparty risk (i.e., novation), thereby improving the allocation to a second best. Hence, there is room for (limited) CCP clearing of OTC contracts even if they are customized, albeit for a fundamentally different reason than on a centralized trading platform.

This yields three important insights for reorganizing OTC markets that go beyond issues related to systemic risk, standardization, and netting.[6] First, mandating formal clearing for standardized contracts might not be effective to increase transparency: Traders need incentives to also submit for formal clearing OTC transactions that are easily customized. CCP clearing can provide incentives for formal clearing as long as it offers enough direct benefits. Second, these gains are linked to the benefits of novation, which can be offered by a CCP even for customized contracts, and are not related to netting.[7] Third, trading of customized financial contracts can lead to a misallocation of risk. Introducing a CCP for OTC trades can improve the allocation by influencing the terms of trade of customized contracts.

We design a model that allows us to study the economics of clearing in general and that is inspired by the history of the Chicago Mercantile Exchange operating as a futures exchange as described by Kroszner (1999). Risk-averse farmers have to decide on how much wheat to grow before they know the demand for wheat by, say, bakers. The demand for wheat by bakers is uncertain, however, due to an aggregate demand shock. Therefore, farmers' income – and, hence, their consumption – is uncertain. To insure against this income risk, farmers can trade futures contracts; i.e., they can trade promises to deliver some wheat in the future at a given price.

However, futures contracts offer only limited insurance when there is a risk of default. In our model, each farmer has to deal with a single baker that can go bust. If a farmer contracted with a bankrupt baker, he does not need to deliver his wheat and can still sell it on the spot market, but at the spot price. As a consequence, a farmer who is trading futures still faces two types of risk: first, the risk that his counterparty goes bust, and second, the associated

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[6]Formal multilateral netting institutionalizes the tear up and compression of redundant trades that are often used to offset exposures between counterparties in these markets. The potential for such netting increases with more standardized contracts. It can reduce systemic risk and achieve savings in collateral costs but need not be offered within a CCP structure. The failure of Lehman Brothers makes this point clear, as an emergency round of compressions was quite effectively carried out without having a CCP structure in place. However, the reestablishment of positions was more difficult, as this involved an allocation of losses associated with the failure of Lehman as a counterparty (see IMF (2010)).

[7]Multilateral netting becomes less effective as the degree of customization rises and can even lead to opportunity costs that outweigh its benefits (see Duffie and Zhu (2009)). Furthermore, if netting reduces systemic risk, this benefit is not directly enjoyed by the counterparties to the trade.

price risk of having to sell his wheat in the spot market.

We then study three risk management tools adding them in succession: collateral, novation, and mutualization of losses. We model collateral as an asset, gold, that bakers can produce on demand. Posting collateral is costly as it includes a deadweight loss, but it can be seized if a baker goes bust. The deadweight loss reduces the attractiveness of a futures contract for bakers and thus the price of a futures contract. Hence, insurance against default through collateral is costly, and as a consequence, farmers do not fully insure against default.

With novation, a third party – called CCP – becomes the seller of wheat to *all* bakers. Hence, while some bakers still go bust, the CCP does not face idiosyncratic default risk as its position is completely diversified. Therefore, novation eliminates counterparty risk for farmers by pooling idiosyncratic counterparty risk. As this is costless, the optimal collateral policy is to require no collateral.[8] While novation eliminates counterparty risk, it does not eliminate the price risk for farmers. When bakers go bust, the CCP still has to sell the wheat on the spot market at the equilibrium price. Therefore, its revenue also depends on the spot price, which in turn depends on the aggregate demand for wheat; in other words, the CCP incurs replacement cost risk that is state-dependent. Because the promised payment to farmers depends on the CCP revenue, farmers still face the original price risk.

Mutualization of losses, however, eliminates the remaining price risk. Adopting a "survivor-pays-rule," the CCP can impose an additional payment from bakers who did not go bust when wheat was cheap. To make it worthwhile for bakers, the CCP must compensate bakers with a transfer when wheat is expensive. In this way, the CCP can make its revenue independent of the aggregate demand shock. In our framework, farmers are then fully insured against the aggregate uncertainty and are guaranteed to receive the exact value of their futures contract. Our key result here is that a CCP can lower collateral requirements because it can reduce risk exposures for market participants more efficiently. This is not due to netting, but due to diversifying risk (novation) and insuring against it (mutualization).

In the second part of the paper, we address the question of the role for CCP clearing on OTC markets. Such markets are characterized by customized contracts that are traded bilaterally and are not fungible in the sense that it is very hard to replace them (extreme replacement cost risk). We also assume that the gains from customized contracts differ across trades. To capture these features, we introduce the possibility for farmers to produce special or "exotic" types of wheat. Bakers now would like to consume both, exotic and plain wheat. Exotic wheat can only be traded bilaterally as it has to be produced by a farmer to meet the specific

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[8]This certainly seems extreme but is due to default being exogenous. Were defaults endogenous (i.e., strategic), it would still be optimal to require some collateral as an incentive device.

needs of a baker. Importantly, this implies that exotic wheat is not fungible, as only the individual baker for which the wheat has been produced values it. Hence, if the baker for which the exotic wheat has been produced goes bust, the wheat cannot be sold in the spot market. Finally, the gain from customization varies across trades in the sense that the gains from consuming exotic wheat differs across bakers.

Farmers have to decide between producing either plain wheat (and selling it via a futures contract) or exotic wheat (and selling it OTC forward). If they trade OTC, they are matched with a baker. For farmers, the cost of trading OTC is that exotic wheat has no value on the spot market in case a baker defaults. Hence, farmers will require costly collateral to insure against default. The benefit of trading OTC is that the farmer makes a take-it-or-leave-it offer[9] to the baker, and therefore, can extract all the baker's surplus. This means that farmers will trade OTC with bakers only if the OTC contract generates enough surplus; i.e., if a baker's valuation of their exotic wheat is high enough. This gives us a threshold valuation below which farmers prefer to trade on the futures market and that determines the relative size of the futures and OTC market.

There are several sources of inefficiencies on the OTC market. First, farmers again face the risk that their counterparty defaults. A CCP can again alleviate this inefficiency through novation exactly as in the futures market – by pooling default risk, but across all exotic wheat. Fungibility, however, restricts now what the CCP can offer. In our stylized setup, it can only pay out the expected payment from the trade, as any default on a contract implies an irrecoverable total loss (i.e., the replacement cost risk is extreme). In other words, the CCP cannot offer insurance against the aggregate cost of default.[10] Thus, farmers receive always less than the negotiated price when the trade is settled by the CCP, independent of whether their original counterparty defaults or not. Nonetheless, novation makes OTC trading in exotic wheat more attractive, leading to an increase in its size relative to the futures market.

Second, the size of each OTC trade need not reflect the valuation of a baker for his particular exotic wheat. Indeed, in our setup, bilaterally negotiated trades lead to a fixed amount of wheat produced across trades of exotic wheat. Therefore, given a fixed probability of default, the default exposure is then also constant across trades. This is inefficient from a social point of view, as the size of an OTC trade should increase with the gains from customization for the baker. In other words, it is socially efficient when default risk is skewed toward trades

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[9]Our results are robust to changes in bargaining power.

[10]More generally, the less fungible the traded contracts are, the higher the replacement cost risk and, hence, the lower the scope to insure against it via mutualization.

with high gains from producing exotic wheat. A revenue-neutral transfer scheme can achieve this by taxing the surplus from matches with high gains and subsidizing matches with low gains. The intuition for this result is straightforward. A tax or subsidy directly changes the surplus in the match – and indirectly the bargaining power. Since farmers have market power to price the contract, they will try to maintain their profit margin by producing more or less wheat. Somewhat counterintuitively, a tax (subsidy) makes it harder (easier) to extract surplus and gives farmers incentives to produce more (less) exotic wheat. The negotiated payment will decrease, however, as it is given by the marginal benefit from the contract for the buyer. This result holds even when the gains from trading exotic wheat cannot be directly observed by the CCP. The reason is that our bargaining assumptions lead to a distribution of surplus where matches with low gains from trading exotic wheat cannot mimic matches with higher gains.[11]

A CCP is well placed to implement the transfer scheme. A farmer matched with a baker needs to have an incentive to reveal its trade but potentially faces a transfer payment. With novation, the CCP can still induce revelation as it offers savings in collateral costs. This makes novation an important prerequisite for influencing the allocation of risk in OTC markets. CCP clearing of OTC trades then contrasts sharply with mutualization of losses through a CCP in the futures market. The former redistributes default risk by *indirectly* influencing the terms of trades to correct an inefficient distribution of default risk across different trades.[12] To the contrary, the "survivor-pays-rule" is state-dependent and does not influence the terms of trades at all. It is an insurance transfer that allocates default losses optimally across different states.

We present the model in Section 2, and we analyze optimal trading and clearing arrangements for standard assets in Section 3. The results on customized assets and OTC trades are presented in Section 4, that also includes the case with private information. We conclude in Section 5.

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[11]Interestingly, with private information, the CCP must be able to decline clearing of certain trades in order to implement the transfer scheme.

[12]The transfer scheme falls short of achieving full efficiency that would require a direct change in the distribution of bargaining power for OTC transactions. This could be done via intermediating OTC trades, which goes beyond the clearing of trades.

2 The Environment

We describe now a general equilibrium model with incomplete markets. The fact that markets are incomplete is important as agents will not be able to perfectly insure against two sources of risk. The first risk arises from an aggregate demand shock leading to price fluctuations, while the second one is related to default on trades, in the sense that some counterparty will be unable to make good on its promises. This allows us to analyze how a particular asset structure in combination with clearing arrangements can complete the market.

2.1 Model

The economy has two periods and there are two goods: Gold that is storable and wheat. There is a measure 1 of farmers that can produce wheat. Producing wheat, however, takes time: any amount needs to be produced in $t = 1$ for consumption in $t = 2$. There are many possible states in $t = 2$, which we denote by θ. For simplicity, we represent a farmer's preferences by the following expected utility,

$$U(x(\theta), q) = -q + E_\theta [\log x(\theta)],$$

where $x(\theta)$ is the amount of gold consumed at date 2 in state θ and $q \geq 0$ is the amount of wheat produced . We take expectations only over second period utility as no uncertainty will be resolved at $t = 1$.

There is also a measure $1/(1 - \delta) > 1$ of bakers in the economy that can produce gold in both periods for consumption in $t = 2$. To introduce the idea of default, we assume that bakers die with probability δ at the end of $t = 1$, so that there is only a measure 1 of bakers alive in $t = 2$. A baker's death is a random event that does not depend on the action of the baker. Again, we assume that a baker's expected utility takes the following simple form

$$V(y(\theta, \theta_i), x_1, x_2(\theta, \theta_i)) = -\mu x_1 + (1 - \delta) E_{\theta_i, \theta} [\theta_i \log (y(\theta, \theta_i)) - x_2(\theta, \theta_i)],$$

where the parameter θ_i is a preference shock for the baker realized in $t = 2$, y is the amount of wheat consumed in $t = 2$, $x_1 \geq 0$ is the amount of gold produced in the first period, and x_2 is the net production of gold in the second period.[13] To capture the idea that it is in general costly to pledge collateral, we assume that producing gold is more costly in the first period, i.e., $\mu > 1$. Finally, we model the preference shock θ_i with the aggregate component

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[13]If $x_2 < 0$, a baker consumes gold in $t = 2$.

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θ and an idiosyncratic component ε_i according to

$$\theta_i = \theta + \varepsilon_i,$$

where θ is drawn from a distribution F with mean 1 and ε_i is iid drawn from a distribution G with mean 0. Therefore, θ denotes the aggregate state in $t = 2$.

2.2 Trading Frictions

We now discuss the role of some of our assumptions and how they help us in getting to the idea of a CCP. First, farmers need to produce wheat in the first period, before the realization of the aggregate demand shock θ. Given their preferences, they will want to insure against this shock by locking in the price. Full prepayment, however, is costly, but one can trade futures contracts, in which bakers promise to pay gold against delivery of wheat in the second period. With full commitment such a promise is credible.

Second, bakers can die, which implies the possibility for default on the futures contract. Thus, whenever farmers trade with a specific baker in the first period, they face a default exposure. While being insured against the demand shock, a farmer who writes a futures contract now faces a default risk that effectively limits this insurance.

Third, an individual baker's utility for wheat is realized only in the second period. So, at the time a baker purchases a futures contract, he does not yet know his exact preference for wheat. This creates the need for bakers to trade among themselves, since there are gains from trade between bakers with high and low idiosyncratic shocks. This gives us a rationale to introduce a spot market in period 2.

To summarize, our environment formalizes the fundamental frictions that will allow us to endogenize the need for futures contracts and proper clearing arrangements. Spot trading of wheat can allocate wheat across surviving bakers efficiently. A futures contract between an individual farmer and baker can partially insure against the aggregate demand shock but exposes farmers to counterparty default. Collateral in form of prepayment by gold that has been produced early can insure against such default but is costly. This provides a rationale for *clearing arrangements* that can provide cheaper and better insurance against default risk. In the next section, we look at different market mechanisms and how close they fare in achieving a benchmark, which is the first-best allocation without trading frictions.

2.3   The First-Best Allocation

As a benchmark for our environment, we consider a social welfare function that gives equal weight to all farmers and bakers. A (symmetric) first best is described by an allocation of gold (x^*, x_1^*, x_2^*) and an allocation of wheat (q^*, y^*) across farmers and bakers, measurable with respect to the realized shocks. Given any realization of θ_i across bakers, it will be sufficient to only use the aggregate shock θ to define the allocation of gold across farmers; therefore we denote it by $x^*(\theta)$. Similarly, the optimal allocation for bakers will depend only on the aggregate shock and a baker's idiosyncratic shock; hence we simply denote it by $x_2^*(\theta, \theta_i)$ and $y^*(\theta, \theta_i)$ respectively. All other allocations cannot depend on the realization of the shocks in period $t = 2$. A first-best allocation solves the following problem

$$\max \quad \log(x(\theta)) - q + \int \theta_i \log(y(\theta, \theta_i)) - x_2(\theta, \theta_i) - \frac{1}{1-\delta}\mu x_1 dF \times G$$

subject to

$$\int y(\theta, \theta_i)dG \leq q \text{ for all } \theta \tag{1}$$

$$x(\theta) \leq \int x_1 + x_2(\theta, \theta_i)dG \text{ for all } \theta. \tag{2}$$

The objective function takes into account that some of the bakers will die after the first period. The constraints (1) and (2) are resource constraints for the consumption of wheat and gold for any given realization of the aggregate shock.

Consumption of gold takes place only in the second period, and it is inefficient to produce gold early on, as $\mu > 1$ and $\delta > 0$. Hence, the efficient allocation has no early production of gold, $x_1^* = 0$. Also, the strict concavity of the farmers' utility function from their consumption of gold implies that it is optimal to insure farmers perfectly against the aggregate demand shock. Similarly, our assumption on preferences implies that bakers' consumption is proportional to his realized preference for wheat. It is straightforward to verify that the optimal allocation satisfies $x^*(\theta) = 1$, $q^* = 1$ and $y^*(\theta, \theta_i) = \theta_i/\theta$ with $x_2^*(\theta, \theta_i)$ being indeterminate.

3   Efficient Trading and Clearing Arrangements

3.1   Spot Market

We assume first that there is only a perfectly competitive market in $t = 2$, where bakers can purchase wheat from farmers against gold. We call this market the *spot market*, because in

this market any trade is immediately settled with gold and each unit of wheat costs $p(\theta)$ units of gold. Wheat is produced in the first period so that the quantity of wheat q available later on is fixed.

The problem of a baker entering the spot market with wealth equal to ω units of gold is then

$$\tilde{V}(\omega) = \max_{y,x_2} \theta_i \log(y) - x_2 \tag{3}$$
$$\text{subject to}$$
$$p(\theta)y \leq x_2 + \omega.$$

Since the budget constraint will hold with equality, we can rewrite this problem as

$$\tilde{V}(\omega) = \omega + \max_y \theta_i \log(y) - p(\theta)y.$$

Hence, the demand of bakers is independent of initial wealth ω. Since gold is more expensive to produce in $t = 1$, bakers will not produce any gold early on. This implies that we can set $\omega = 0$.

We now turn to the problem of farmers in the spot market. Farmers have q units of wheat to sell. Since farmers do not value wheat in the second period, they will sell all wheat against gold. For each unit, they get $p(\theta)$ so that they can consume $p(\theta)q$ units of gold. Farmers then choose their initial investment in wheat to maximize their expected payoff, taking as given the spot price of wheat, $p(\theta)$ in $t = 2$. Note that a farmer is small; therefore his individual production decision does not affect the equilibrium spot price. He thus solves

$$\max_q -q + \int \log(p(\theta)q) \, dF(\theta) \tag{4}$$

with solution $q = 1$ for each farmer.

It will be convenient later on to define a spot market equilibrium in the following way. For an aggregate supply of wheat Q, a *spot market equilibrium* in $t = 2$ is given by a price schedule $\{p(\theta)\}_\theta$ and an allocation for bakers $(y(\theta, \theta_i), x_2(\theta, \theta_i))$ for each state θ, such that (i) taking $p(\theta)$ as given, bakers choose $y(\theta, \theta_i)$ and $x_2(\theta, \theta_i)$ to solve (3) with $\omega = 0$ and (ii) the market for wheat clears for all θ, $\int y(\theta, \theta_i) dG(\varepsilon_i) = Q$.

The solution to the baker's problem is given by

$$y(\theta_i) = \theta_i/p(\theta), \tag{5}$$

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with $x_2(\theta, \theta_i) = p(\theta)y(\theta_i)$. Using market clearing and the fact that in equilibrium $Q = q = 1$, we obtain the equilibrium price for all θ,

$$p(\theta) = \theta/Q = \theta, \tag{6}$$

Note that our assumptions on preferences imply that both the equilibrium quantity produced by farmers $q = 1$ and the equilibrium spot price $p(\theta) = \theta$ will remain unchanged when we vary the market structure later on.

A spot market for wheat in $t = 2$ suffices to allocate wheat optimally across bakers. Their individual consumption of wheat and the expected payment are first best, since we have $\int py(\theta_i)dG(\varepsilon_i) = 1$. Even though the bakers' aggregate consumption stays constant, the equilibrium spot price moves one for one with the aggregate demand shock θ. This implies immediately that farmers' consumption of gold depends on θ. Hence, they are not insured against aggregate demand fluctuations. Replacing the equilibrium allocation, ex-ante welfare is given by

$$W^s = \int \log(\theta)\, dF(\theta) - 1 + \int \theta_i \log\left(\frac{\theta_i}{\theta}\right) - \theta_i dF \times G. \tag{7}$$

3.2 Futures Market

On top of the spot market in $t = 2$, we now add a *futures market* in the first period where agents trade futures contracts. A futures contract is basically a promise to deliver one unit of wheat at $t = 2$ in exchange for gold.

We make the assumption that a farmer can trade only with a single baker. More precisely, we postulate that bakers can acquire wheat either by trading in the futures market in $t = 1$ or by waiting and trading in the spot market in $t = 2$. Therefore, there will be trade on the futures market only if bakers are at least as well off as trading on the spot market. If this is the case, we will assume that a random measure 1 of bakers is selected to participate in the futures market. In a symmetric equilibrium, all farmers will supply the same quantity of futures contracts, and bakers will demand the same quantity. Therefore, we can assume that one farmer is trading with a single baker. Bakers who are not selected to trade in the futures market will buy wheat in the spot market.

This implies that a farmer is exposed to *counterparty risk*, as any single baker dies with probability $\delta > 0$. In this case, we say there is default.[14] To insure against the risk of

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[14]We choose to have an exogenous default rate because endogenous default introduces intricacies that would blur the main message of this paper. The case with endogenous default is available from the authors

Figure 1: Market Structure – Spot and Futures

default, farmers who are trading a futures contract may want to require bakers to post some gold as collateral $t = 1$. We let k denote the required collateral per unit of wheat traded forward.

A futures contract is a pair (p_f, k), with the understanding that one contract entitles the owner to one unit of wheat at a price p_f with the requirement to pledge k units of collateral. There is then no difference between pledging collateral and prepaying so that one can think of it as settling an obligation by netting collateral with the final payment. In case of default, farmers are not required to honor their obligation from the futures contract to deliver wheat so that they can still sell it on the spot market. Figure 1 depicts the market structure with collateral.

If a baker purchases q_b units of a futures contract (p_f, k), he has a claim to q_b units of wheat in $t = 2$, subject to paying the futures price p_f minus the collateral k he has already pledged. Since he can also sell these units on the spot market at price $p(\theta)$, his net wealth is given by $\omega = (p(\theta) + k - p_f) q_b$. Using the fact that his demand on the spot market $y(\theta_i)$ is independent of his wealth position (see equation (5)), a baker will choose the number of contracts q_b to maximize his expected revenue, or

$$\tilde{V}^f (p_f, k) = \max_{q_b} -\mu k q_b + (1 - \delta) \int (p(\theta) + k - p_f) q_b dF(\theta), \tag{8}$$

where the first term expresses the additional costs of securing the trade with collateral when purchasing wheat forward.

Farmers can either sell their wheat in the futures market or in the spot market. However, we

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upon request.

can show that they prefer to lock in the price by selling their entire production in the futures market. If farmers sell q futures contracts, q is also the amount of wheat available in $t = 2$. If their counterparty does not default, the revenue from selling q units of futures contracts is given by $p_f q$. To the contrary, a farmer who faces default keeps the posted collateral kq and sells his wheat on the spot market. Hence, he obtains a state-dependent revenue equal to $(p(\theta) + k) q$, where $p(\theta)$ is the equilibrium spot price. The farmer's problem in period 1 is then given by

$$\max_{q} -q + (1 - \delta) \log (p_f q) + \delta \int \log ((p(\theta) + k) q) \, dF(\theta). \tag{9}$$

Given a spot market price schedule $\{p(\theta)\}_\theta$, a *futures market equilibrium* is a futures contract (p_f, k) and supply and demand of such contracts, such that (i) the demand q_b solves (8), (ii) the supply q solves (9), (iii) the market clears $q_b = q$, and (iv) the price schedule $\{p(\theta)\}_\theta$ is a spot equilibrium price schedule given q.

To solve for the futures contract price p_f, bakers must be indifferent between trading spot and trading futures. This leads to the following no-arbitrage pricing condition

$$p_f = \int p(\theta) dF(\theta) - \left(\frac{\mu}{1 - \delta} - 1 \right) k. \tag{10}$$

Arbitrage pricing implies that bakers need to be fully compensated for the cost of posting collateral, which is composed of the direct cost $\mu > 1$ and the indirect cost that collateral is lost for the baker if he dies ($\delta > 0$). Hence, farmers bear all the cost of collateral, as the futures price declines for any k. Moreover, given the equilibrium price (10), bakers are indifferent between pledging a high amount of collateral and a low price, or inversely.

Turning to the supply of futures contract, the solution to (9) is again given by $q = 1$. Since only a measure 1 of bakers can participate in the futures market, market clearing implies immediately that $q_b = q = 1$ *independent of the collateral policy*.[15] Hence, the aggregate supply of wheat has not changed relative to the equilibrium with only a spot market. All wheat is still allocated among bakers through spot trades in $t = 2$, yielding again the

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[15]Since bakers are indifferent between trading in futures or only in the spot market, in equilibrium some bakers will not engage in futures transactions. Furthermore, our preference structure makes the amount sold on the futures market independent of the price since the log function implies that the substitution and revenue effects cancel each other. With a more general utility function, the amount produced on the futures market would be higher than if production was only traded spot, as a futures contract offers partial insurance.

equilibrium spot price schedule $p(\theta) = \theta$. Thus, in equilibrium, we obtain

$$p_f = 1 + (1 - \mu(\delta))k, \tag{11}$$

where we have defined the effective cost of collateral for bakers by $\mu(\delta) = \mu/(1 - \delta) > 1$. As bakers are fully reimbursed for the collateral cost, welfare is given by

$$W^f = (1 - \delta) \log \left(1 + (1 - \mu(\delta)) k\right) + \delta \int \log \left(\theta + k\right) dF(\theta) - 1 + \int \theta_i \log \left(\frac{\theta_i}{\theta}\right) - \theta_i dF \times G, \tag{12}$$

where the first term is the farmers' utility from a performing futures contract (one where the baker is still alive) and the second from seizing collateral and selling wheat in the spot market when a counterparty defaults.

The equilibrium is parametrized by the amount of collateral k. To find the optimal collateral policy, we assume that bakers compete. Therefore, the optimal collateral policy maximizes farmers' welfare (9) given (11). Using collateral to get insurance is costly. Still, farmers prefer using collateralized futures contract than only trading on the spot market as long as the collateral cost is low or the default probability is high. The proof of this result is relegated to the Appendix.

Proposition 1. *Spot and futures markets with collateral (Pareto) dominate a spot market alone. The equilibrium price on the futures market equals the expected spot price minus collateral costs, i.e., $p_f^k = 1 + (1 - \mu(\delta))k \leq 1$.*

It is never optimal for farmers to fully insure against default through collateral. The optimal collateral policy is $k^* = 0$, if and only if $\mu \geq \underline{\mu}(\delta) > 1$, where $\partial \underline{\mu}/\partial \delta > 0$ for all $\delta > 0$.

A futures market partially insures farmers against the aggregate price risk of selling wheat on the spot market. The insurance is imperfect, as farmers face the risk that their counterparty defaults with probability $\delta > 0$. Then, farmers have to sell their wheat on the spot market assuming risk in their consumption of gold. This gives rise to two sources of inefficiency for farmers. First, default reintroduces aggregate price risk in the futures contract. And second, farmers suffer from a lack of diversification, as they can only trade with a specific counterparty. One way to limit these risks is to require collateral.

Somewhat surprisingly, farmers never fully collateralize their trades. But the intuition is simple. In case of default, farmers can still sell their production spot in period 2. If farmers were to fully collateralize – i.e., require full prepayment $(k = p_f)$ – they would enjoy too

much consumption in default states at the expense of lower consumption in nondefault states. Therefore, they prefer to undercollateralize their exposures. This implies that costly collateral is not a perfect substitute for insurance. Hence, we look next at better clearing arrangements offered through CCP clearing.

3.3   Central Counterparty Clearing

We now introduce the notion of a CCP. This is a third party such as a clearing agent or a clearinghouse that *clears* all trades.[16] While the CCP takes the terms of trades as given, it can affect them indirectly by modifying the trading environment, such as setting additional collateral requirements.

3.3.1   Novation

Novation is a mechanism whereby the CCP becomes the buyer to every seller and the seller to every buyer. More precisely, the original futures contract between a farmer and a baker is superseded by two contracts: One between the farmer and the CCP and one between the CCP and the baker. This means that farmers and bakers are now facing only the CCP in the second period when settling futures contracts. Without loss of generality, we assume that only the CCP can set and administer collateral requirements. Naturally, these requirements will change the price of a futures contract, which we now denote by p_f^n.

With novation, given q futures contracts (p_f^n, k) have been signed in equilibrium, the revenue of the CCP – and hence its payout to the farmers – in $t = 2$ is

$$R^n(\theta) = kq + \left(p_f^n - k\right)(1 - \delta)q + p(\theta)\delta q. \tag{13}$$

In this period, the CCP receives all payments from bakers and all wheat from farmers and uses these proceeds to fulfill the obligations from the futures contracts.[17] It can also seize the

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[16]In its most primitive function, it could simply be a collateral storage facility. A collateral storage facility might be necessary if farmers cannot commit to make the necessary investment in wheat when they sell wheat forward against collateral. The third party then holds the collateral in escrow until the quantity of wheat promised forward is released to the baker. Hence, with a two-sided strategic default problem, a neutral third party that stores collateral is essential for managing default risk, as has been pointed out in a companion paper (see Koeppl and Monnet, 2008). See also Mattesini, Monnet and Wright (2009). Rather than making this notion precise in this framework, we abstract from this issue and assume that farmers can perfectly commit.

[17]Novation is not a guarantee. In order for it to be a guarantee, we would have to require that the CCP satisfies a solvency constraint. In other words, the CCP would guarantee to settle all trades at the price p_f^n

collateral of all bakers that default on their futures trade. The first term in (13) is all of the collateral that the CCP collects from bakers in the first period, where we have used market clearing $q_b = q$. The second term is the overall gold payments – net of collateral postings – made by nondefaulting bakers for settling their futures contracts. In exchange, the CCP delivers a total of $(1 - \delta)q$ units of wheat. Finally, the CCP can sell the remaining amount of wheat δq on the spot market at price $p(\theta)$, which is the final term. The CCP's revenue is thus state-dependent, as the spot price of wheat varies with the aggregate demand shock.

An equilibrium with novation is a futures market equilibrium where instead of (9), farmers solve

$$\max_q -q + E\log\left(R^n(\theta)\right).$$

In the symmetric equilibrium,[18] we have that $q_b = q = 1$ and, hence, each farmer receives a payment equal to

$$R^n(\theta) = p_f^n + \delta\left(p(\theta) + k - p_f^n\right). \tag{14}$$

By no-arbitrage, the equilibrium futures price needs to make bakers indifferent between trading in the futures markets or only on the spot market. Hence, the equilibrium price is unaffected by novation and it is given by

$$p_f^n = 1 + (1 - \mu(\delta))\,k. \tag{15}$$

Replacing this in the revenue equation (14), we obtain that farmer's ex-ante utility is given by

$$\log\left((1 - \delta)(1 + (1 - \mu(\delta))k) + \delta(\theta + k)\right) - 1. \tag{16}$$

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at which farmers sold wheat forward. This would pin down a collateral requirement, again influencing the price. Our approach is more general as it also allows for a partial guarantee.

[18]The revenue is expressed for a symmetric equilibrium where all farmers produce the same quantity q. Given all other farmers produce q, it is not beneficial for a farmer to produce something else other than q when the terms of the futures contract are (p_f^n, k). To see this, suppose farmer $i \in [0,1]$ produces q_i. Then the revenue of the CCP is

$$R^n(\theta) = k \int q_i di + \left(p_f^n - k\right)(1 - \delta)\int q_i di + p(\theta)\delta\int q_i di.$$

Consider the rule for the CCP to divide its revenue pro rata among farmers so that farmer i gets

$$R(q_i) = \frac{q_i}{\int q_i di} R^n(\theta).$$

Given this payment from the CCP, a farmer's production choice is independent of all other farmer's choices. Indeed, a farmer would solve

$$\max_{q_i} -q_i + \int \log\left(\frac{q_i}{\int q_i di} R^n(\theta)\right) dF$$

with the solution again being $q_i = 1$ for all i.

Given collateral k, risk-averse farmers obtain the average revenue across all futures trades. Novation thus acts as a substitute for diversification ex-ante. If farmers cannot perfectly diversify counterparty risk upfront, they can do so with a CCP that averages this exposure. Since the bakers' payoff is independent of the collateral posted, the optimal collateral policy maximizes a farmer's expected payoff. From equation (16), it is straightforward to see that average consumption for farmers decreases with k.

Proposition 2. *Novation perfectly diversifies counterparty risk. The optimal collateral policy with novation is then $k^* = 0$.*

The intuition for this result is straightforward. Collateral is costly to produce, and these costs have to be borne entirely by farmers. Hence, collateral is a costly insurance device against counterparty risk. Novation, to the contrary, simply pools all the counterparty risk of individual farmers, thereby perfectly diversifying it. Requiring collateral would just lower the revenue in all states without providing any additional insurance neither against idiosyncratic default risk, nor against the aggregate price risk.[19]

Novation is not equivalent to guaranteeing farmers the futures price p_f^n, as the revenue of the CCP fluctuates with the price risk θ. Still, novation benefits farmers by providing savings on collateral – implying a higher futures price ($p_f^n > p_f$) – and perfect diversification of counterparty risk, even though there is always some default in the aggregate. However, novation alone cannot deliver the first-best allocation. The CCP's revenue depends on the spot price, as it needs to sell the wheat from defaulted trades. There is thus still some room to insure farmers against the aggregate price risk.

3.3.2 Novation and Mutualization

We now introduce *mutualization of losses*. When losses are mutualized, surviving bakers pay (or receive) an additional fee $\phi(\theta)$ to the CCP, which can depend on the aggregate state of the economy. We denote by p_f^m the price of the futures contract with mutualization and again allow the CCP to request collateral k for any unit of wheat traded on the futures market. An equilibrium with novation and mutualization is defined as an equilibrium with

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[19]With endogenous default, it is optimal to impose some collateral requirement. The reason is that collateral acts as an incentive device to lower default. The derivation is available upon request from the authors. Still, novation leads to a reduction in the optimal collateral requirement.

novation where $\phi(\theta) \neq 0$ for some θ. Bakers choose to trade futures contracts according to

$$\max_{q_b} -\mu k q_b + (1-\delta)\int \left(p(\theta) + k - p_f^m - \phi(\theta)\right) q_b dF(\theta), \quad (17)$$

where the state-dependent fee schedule $\phi(\theta)$ now influences the wealth of bakers. The no-arbitrage condition then gives

$$p_f^m = 1 + (1 - \mu(\delta))k - \int \phi(\theta)dF(\theta). \quad (18)$$

where we have used the spot price $p(\theta) = \theta$. The futures price reflects the expected costs of the mutualization scheme $\phi(\theta)$ for bakers. Using the fact that the total production of wheat is sold on the futures market and that farmers will choose the same production level $q = 1$,[20] the revenue of the CCP is given by

$$R^m(\theta) = (1-\delta)p_f^m + \delta\left(p(\theta) + k\right) + (1-\delta)\phi(\theta). \quad (19)$$

The CCP's revenue is composed of three terms. The first two terms are due to the CCP novating futures contracts. As in the previous section, the CCP receives the payment p_f^m from nondefaulting bakers. Also, the CCP can sell the wheat that was due to be transferred to defaulting bakers on the spot market for the price $p(\theta)$, while still keeping the collateral they pledged. Finally, the third term is the additional payment that bakers who have not defaulted make to the CCP.

We now construct a fee schedule $\phi(\theta)$ such that (i) $R^m(\theta) = 1$, (ii) $k = 0$, and (iii) $\int \phi(\theta)dF(\theta) = 0$. Given such a fee schedule, the CCP can fully insure farmers – at the expected fair price of $p_f^m = 1$ for producing $q = 1$ – while not relying on costly collateral at all to safeguard against default.[21] As the mutualization fee averages to zero, bakers are indifferent in period 1 in participating in a futures market operated by a mutualized CCP

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[20] The argument for symmetric production is similar to the one for novation.

[21] Indeed, if $k > 0$, one can never ensure a constant payment across states θ of at least 1. Integrating the revenue equation with respect to the aggregate shock yields

$$\int R^m(\theta)dF(\theta) = 1 + k(1-\mu) < 1.$$

Hence, for some state the aggregate revenue must be less than 1. More generally, one can design a fee structure with constant revenue for any given collateral level and show that given this fee structure, it is optimal to impose no collateral requirement.

and one that is not. Setting $R^m(\theta) = 1$ and $k = 0$, we obtain from the revenue equation (19)

$$\phi(\theta) = \frac{\delta(1 - \theta)}{1 - \delta}. \tag{20}$$

Given this fee schedule, we obtain that the futures price is again the expected spot price

$$p_f^m = \int p(\theta) dF(\theta) = 1. \tag{21}$$

Since there is no expected transfers between bakers and farmers, the futures price does not adjust at all and equals the fixed payment $R^m(\theta) = 1$ from the CCP to a farmer for selling wheat forward. The transfer schedule implies that for $\theta < 1$ bakers that have not defaulted must pay more than the agreed futures price, while they pay less whenever $\theta > 1$. Hence, mutualization guarantees a fixed payment that enables bakers to perfectly insure farmers against the aggregate price risk.

Proposition 3. *Novation and mutualization of losses implement the efficient allocation through trading on futures and spot markets.*

To summarize, futures markets can allow for a better allocation of price risk. However, allocating this risk is imperfect whenever there is also counterparty risk: default reintroduces price risk into futures contracts. Clearing arrangements help deal with this counterparty risk. Novation improves the allocation by diversifying counterparty risk but does not insure against it. Mutualization of losses provides such insurance. Importantly, these clearing arrangements, which indirectly influence the terms of trades, do not directly alter the incentives to trade.

4 Over-the-Counter Markets

While a CCP can achieve the first-best allocation when products are standardized and traded on a centralized exchange, how would it fare when products are specialized and traded over the counter? To answer this question, we now introduce OTC trading, as originating from the demand for differentiated products that cannot be traded centrally. Hence, the trading environment is linked to the type of product being traded and cannot be changed. In the model, aside from plain wheat, each baker can now also demand a special type of wheat (called *exotic*) which only he can consume. We use exotic wheat as a metaphor for those financial contracts that are designed to fulfill the specific needs of the contract's holder.

4.1 The Model

The environment is the same as before except for that there are now two types of wheat: Plain and exotic. Within the type of exotic wheat, there are as many varieties of wheat as there are bakers: Each baker likes to consume only his variety of exotic wheat. Farmers need to specialize in their production. They can produce either plain wheat or some variety of exotic wheat, but not both. We assume that farmers are as good in producing plain wheat as exotic wheat, so that their preferences are represented by

$$U(x(\theta), q, s) = -q - s + E_\theta[\log x(\theta)]$$

where s is the amount of any exotic wheat they produce for a specific baker. Since farmers have to specialize, we require that $s_i s_j = 0$ for any $i \neq j$ and $qs_i = 0$ for any i where s_i is the production for a specific baker.

The preferences of baker i over general and exotic wheat are given by

$$V(y(\theta, \theta_i), s_i, x_1, x_2(\theta, \theta_i)) = -\mu x_1 + (1-\delta)E_{\theta_i, \theta}[\theta_i \log(y(\theta, \theta_i)) + \sigma v(s_i) - x_2(\theta, \theta_i)],$$

where v is concave, $v(0) = 0$, $v'(0) = \infty$ and θ_i is again a preference shock in $t = 2$. We assume that $\sigma \in [\underline{\sigma}, \overline{\sigma}]$ is distributed across bakers according to some distribution H. It is a fixed, observable ex-ante characteristic of a baker and expresses how much a baker likes his type of exotic wheat relative to general wheat. It thus describes the gains from customization. We analyze the case in which σ is private information in Section 4.7 below.

An important feature of our setup is that bakers still value plain wheat, even if they consume exotic wheat. More precisely, while bakers can do without exotic wheat, i.e., $s_i = 0$ is possible, they cannot do without plain wheat. Therefore, all bakers will want to obtain some plain wheat – either on the futures market in the first period or in the spot market in second period – while some bakers may not consume their exotic wheat.

To formalize bilateral OTC trading in exotic wheat, we employ a model of a one-sided search in which farmers and bakers bargain over trading exotic wheat.[22] The sequence of events is as follows. Each farmer is randomly matched with exactly one baker. We assume for simplicity that the farmers make a take-it-or-leave-it offer to the baker that specifies a price p_i, a contract size s_i, and a collateral requirement k_i.[23] If the baker accepts the offer, the

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[22]Trading exotic wheat in a centralized market with Walrasian pricing is impossible, as only one baker likes a type of exotic wheat.

[23]In the Appendix, we briefly discuss our results with more general Nash bargaining.

farmer produces the exotic wheat for the baker, which is delivered in $t = 2$. If the baker rejects it, the farmer moves on to trade on the competitive futures market, where a CCP operates with novation and mutualization in place, as analyzed in the previous section.[24] There is still a spot market for plain wheat in $t = 2$, where all bakers can purchase such wheat against gold.

Once again, let us stress that a baker likes only his type of exotic wheat. Since exotic wheat is special to only one baker, it is not worth anything to any other baker. Hence, there is no value for this wheat on the spot market. In other words, it is *not fungible*. This introduces extreme price risk into producing exotic wheat, as it cannot be sold after default on the market.[25]

4.2   OTC Trading and Equilibrium

As we established earlier, equation (5) implies that a baker's demand for plain wheat in the spot market depends only on the spot price $p(\theta)$ and is not influenced by trading exotic wheat. Furthermore, since bakers who access the futures market do not consume exotic wheat, their problem on the futures market is as described previously. Therefore, futures contracts continue to be priced by no-arbitrage so that a baker is indifferent between trading on the spot market or in the futures market. Also, since a CCP is operating under novation and mutualization on the futures market, we know that $k = 0$. Therefore, given the equilibrium futures contract $(p_f, 0)$, a baker accepts an OTC offer (s_i, p_i, k_i), if and only if

$$-\mu k_i + (1 - \delta) \left[\sigma v(s_i) - (p_i - k_i) + \int \tilde{V}(0) dF \times G \right] \geq (1 - \delta) \int \tilde{V}(0) dF \times dG, \quad (22)$$

If the baker accepts the offer, a baker needs to pledge collateral k_i in $t = 1$. If he is still alive in the second period, the baker obtains s_i units of exotic wheat but incurs the cost of paying the remaining $p_i - k_i$ units of gold. Finally, the baker can acquire plain wheat on the spot market with expected value being $E\tilde{V}(0)$. If he rejects the offer, the baker can either trade on the futures market in which case he gets an expected payoff $\tilde{V}^f$, or on the spot market. So, no arbitrage pricing yields $\tilde{V}^f = (1 - \delta) \int \tilde{V}(0) dF \times dG$.

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[24]If there is a measure n of farmers in the futures market, a measure n of bakers is randomly selected to participate in the futures market – among those bakers are those who were not matched with a farmer and those who rejected an offer. This is feasible as there are always more bakers than farmers that do not trade in exotic wheat.

[25]This is an extreme assumption that could be weakened by allowing exotic wheat to be sold as general wheat in the spot market at a discount $\lambda \in (0, 1)$.

The equilibrium contract is then given by the farmer's take-it-or-leave-it offer which solves

$$\max_{(s_i, p_i, k_i)} -s_i + (1 - \delta) \log(p_i) + \delta \log(k_i)$$

$$\text{subject to}$$

$$-\mu k_i + (1 - \delta) \left[\sigma v(s_i) - (p_i - k_i) \right] \geq 0.$$

Here, with probability δ, the baker defaults leaving the farmer with only his collateral, as exotic wheat is worthless and the farmer had to specialize his production. The first-order conditions yield

$$v'(s_i) = v(s_i) \tag{23}$$

$$k_i = \frac{\delta}{1 - \delta} \frac{p_i}{\mu(\delta) - 1} \tag{24}$$

$$p_i = (1 - \delta) \sigma v'(s_i) \tag{25}$$

where the last equation follows from the participation constraint of bakers.

Let $\bar{s}$ be the solution to equation (23). It is independent of σ, the surplus generated from producing exotic wheat.[26] Farmers use the price to extract all of the baker's expected surplus from consuming exotic wheat. To ensure participation of farmers in the OTC market, their payoff from trading OTC must be higher than their payoff from selling a futures contract for plain wheat. If there is a fraction n of farmers producing plain wheat, this participation constraint is given by

$$1 - \bar{s} + \log\left((1 - \delta)\sigma v'(\bar{s})\right) + \delta \log\left(\frac{\delta}{(1 - \delta)(\mu(\delta) - 1)}\right) \geq \log\left(\int \frac{R^m(\theta)}{n} dF(\theta)\right) \tag{26}$$

where $R^m(\theta)/n$ is defined by the CCP's revenue as given in equation (19). Since only a fraction n of farmers is active on the futures market, we have that aggregate production is nq, and since the production of plain wheat is $q = 1$, the aggregate supply of wheat is n. The spot price is then given by $p(\theta) = \theta/n$, and arbitrage pricing yields a futures price equal to $p_f = 1/n$. This implies again that the CCP using novation and mutualization on the futures market can ensure $R^m(\theta) = 1$, and we obtain that farmers make an offer to produce exotic wheat if and only if the baker's valuation σ satisfies

$$1 - \bar{s} + \log\left((1 - \delta)\sigma v'(\bar{s})\right) + \delta \log\left(\frac{\delta}{(1 - \delta)(\mu(\delta) - 1)}\right) \geq \log\left(\frac{1}{n}\right). \tag{27}$$

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[26]This is an implication of the distribution of bargaining power and log-utility. As discussed in the Appendix, it does not influence our results.

Therefore, there is a threshold $\hat{\sigma}(n)$ below which farmers prefer to produce plain wheat on the futures market. This implies that the number of OTC trades is equal to the number of bakers with σ above the threshold $\hat{\sigma}(n)$ that are matched with a farmer. Hence, we have that

$$n = 1 - \min\left\{\frac{1 - H(\hat{\sigma}(n))}{(1-\delta)}, 1\right\} \tag{28}$$

where $\hat{\sigma}(n)$ satisfies equation (27) with equality.

Finding an equilibrium amounts to finding a fixed point n^* of equation (28). When $n = 0$, there is no production of plain wheat, and the spot price goes to infinity. Hence, in equilibrium there always must be some production of plain wheat. To have OTC trades in equilibrium, we simply need to require that a farmer being matched with the highest σ always prefers to produce exotic wheat.[27]

Proposition 4. *An equilibrium with OTC trades exists if and only if*

$$1 - \bar{s} + \log\left((1-\delta)\bar{\sigma}v'(\bar{s})\right) + \delta\log\left(\frac{\delta}{(1-\delta)(\mu(\delta)-1)}\right) > 0.$$

Figure 2 summarizes the payoff in the equilibrium allocation for farmers. Below the equilibrium threshold $\hat{\sigma}$, farmers produce plain wheat and sell it in the futures market to obtain the payoff $u\left(p_f\left(\hat{\sigma}\right)\right)$. All other farmers produce the same quantity $s(\sigma) = \bar{s}$ of exotic wheat but extract increasingly more surplus as prices increase with σ. The farmers' pay-offs from OTC trades are thus increasing in σ. Note that farmers on the OTC market choose to bear the default risk associated with their counterparty. Again, this suggests that there can be gains from employing CCP clearing.

4.3   Efficient Allocations on the OTC Market

In order to evaluate the gains from CCP clearing on the OTC market, we next establish two different benchmarks that represent constrained efficient allocations. In the first benchmark, a planner is constrained by feasibility and participation constraints only. In the second benchmark, the planner is in addition constrained by the fact that farmers should receive all the surplus from the match.

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[27]If we had assumed a futures market without CCP clearing, the value of the outside option for farmers to produce plain wheat would be lower. In equilibrium, there would then be more trades in the OTC market.
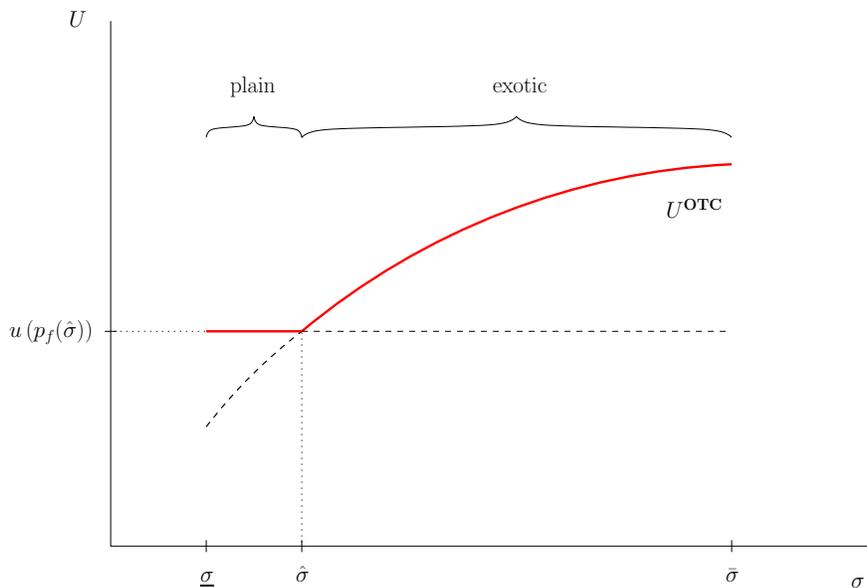
Figure 2: Payoff for Farmers – Equilibrium with OTC Market

4.3.1 Efficient Allocations

We take the size of the OTC market as exogenously given, in the sense that there are n farmers in the futures market and $1-n$ farmers that produce in the OTC market.[28] We give only an informal discussion of the (constrained) efficient allocation and relegate the analysis to the Appendix.

The social planning problem takes the matches – parametrized by σ – as given. The planner can direct farmers to produce a quantity $s(\sigma)$ of exotic wheat and bakers to pay $x_2(\sigma)$ units of gold in those matches. Hence, the planner is restricted in that only one farmer can produce for one baker: If the farmer produces $s(\sigma)$ units of exotic wheat, then this is exactly the consumption of the baker with characteristic σ. The planner, however, can redistribute the aggregate gold payment $\int_{\hat{\sigma}}^{\bar{\sigma}} x_2(\sigma) dH$ across farmers. Assuming that the planner in charge of

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[28]Qualitatively, the properties of the efficient allocation for the OTC market are independent of its size. So we abstract from solving the optimal market size for OTC trading. However, this is an important issue, as introducing CCP clearing for OTC trades will modify the size of the OTC market. An analysis of allocating trades across markets is available from the authors upon request.

allocating exotic wheat is utilitarian, his problem is described by[29]

$$\max_{s(\sigma),x_2(\sigma),x(\sigma)} \int_{\hat{\sigma}}^{\bar{\sigma}} (1-\delta)\left[\sigma v(s(\sigma)) - x_2(\sigma)\right] + \log(x(\sigma)) - s(\sigma)dH(\sigma)$$

subject to

$$\int_{\hat{\sigma}}^{\bar{\sigma}} x(\sigma)dH(\sigma) = \int_{\hat{\sigma}}^{\bar{\sigma}} (1-\delta)x_2(\sigma)dH(\sigma)$$

$$(1-\delta)\left[\sigma v(s(\sigma)) - x_2(\sigma)\right] \geq \bar{v} \text{ for all } \sigma \in [\hat{\sigma}, \bar{\sigma}]$$

$$\log(x(\sigma)) - s(\sigma) \geq \bar{u} \text{ for all } \sigma \in [\hat{\sigma}, \bar{\sigma}]$$

$$s(\sigma), x_2(\sigma), x(\sigma) \geq 0 \text{ for all } \sigma \in [\hat{\sigma}, \bar{\sigma}].$$

The first constraint is a resource constraint on allocating gold across farmers. The other constraints are participation constraints on farmers and bakers, and non-negativity restrictions.

One can show that – independent of the participation constraints and similar to the OTC equilibrium – it is always optimal to "price" the OTC contract as

$$(1-\delta)\sigma v'(s(\sigma)) = x(\sigma), \tag{29}$$

for a given contract size $s(\sigma)$. Hence, farmers receive consumption according to their marginal contribution to the match surplus, i.e. the expected marginal benefit from exotic wheat consumption. If participation does not constrain the planner, the first-best allocation equates marginal benefits and costs of producing gold and exotic wheat, thus yielding a constant consumption across all matches, $x(\sigma) = 1$. This implies that, while the production of exotic wheat is increasing in the surplus σ of the match, farmers suffer more production costs as σ increases but consume the same so that their utility is decreasing in σ.

However, with this allocation, farmers who are in a high surplus match (i.e., large σ) have more incentives to trade on the futures market; their consumption is fixed, but they have to produce a lot of exotic wheat, possibly shrinking their utility below $\bar{u}$, the utility they would obtain on the futures market.[30] Figure 3 shows the farmer's payoff for the efficient allocation when farmers have the outside option of trading on the futures market. For $\sigma \geq \sigma^*$, farmers have an incentive to trade on the futures market, and this outside option drives a wedge into the production of exotic wheat. Farmers need to be rewarded for higher production of exotic wheat with higher consumption. Given the "pricing" formula (29), the production of

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[29]We set $x_1(\sigma) = 0$ so that no baker produces gold in the first period, as this would be inefficient.

[30]The bakers' outside option is $\bar{v} = 0$, as they do not derive any surplus form trading on the futures market compared with only trading general wheat in the spot market.
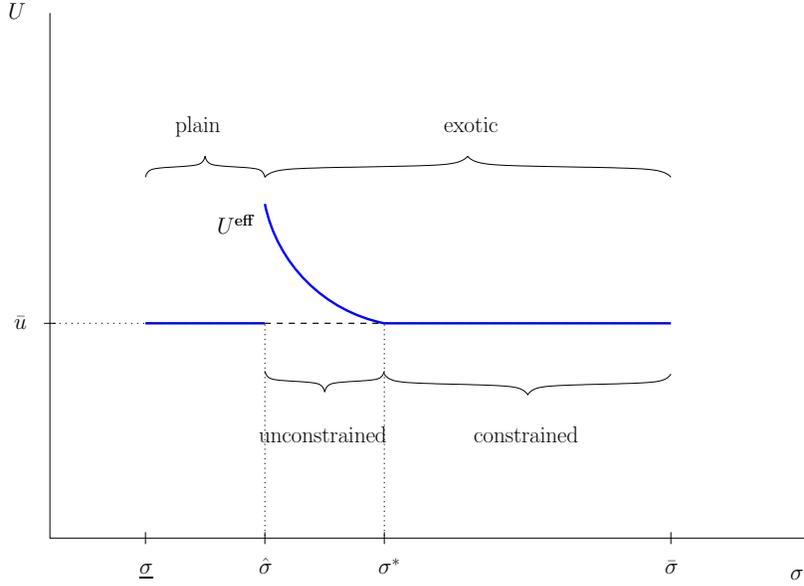
Figure 3: Farmer's Payoff – Efficient Allocation

exotic wheat is now given by

$$\log\left((1-\delta)\sigma v'(s(\sigma))\right) - s(\sigma) = \bar{u} \tag{30}$$

where $\bar{u}$ is the utility a farmer can obtain when trading on the futures market. Hence, it is still efficient to have the production of exotic wheat increasing with σ, but less so as farmers need to be compensated for their production.[31]

4.3.2 Efficient Allocations with Bargaining

We now also impose the bargaining protocol on the planner. We do so by imposing that the planner's allocation must give all the surplus to farmers while bakers receive $\bar{v} = 0$.[32] The planner is therefore constrained to give at least as much as what the farmer would get if he would resort to a bilaterally negotiated contract. The problem of the planner is the same as before, except that bakers have no surplus, $\sigma v(s(\sigma)) = x_2(\sigma)$, and the farmers' outside

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[31]We have that

$$0 < \frac{ds}{d\sigma} = \frac{1}{\sigma}\frac{v'}{v'-v''} - \frac{1}{\sigma}\frac{v'}{v''}.$$

[32]Since farmers extract all surplus when trading bilaterally, it must be the case that the surplus bakers receive is 0. If there were an allocation in the match that would make both the farmer and the baker better off, the farmer could replicate the allocation and make himself better off by extracting all surplus via a take-it-or-leave-it offer from the bakers.
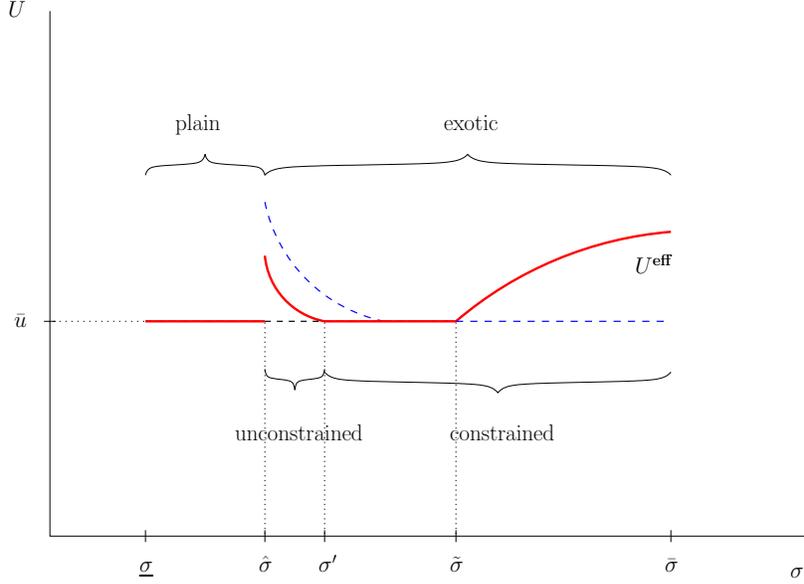
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Figure 4: Payoff for Farmers – Efficient Allocation with Bargaining

option is given by

$$\bar{u} = \max\left\{-\bar{s} + \log\left((1-\delta)\sigma v(\bar{s})\right) + \delta\log\left(\frac{\delta}{\mu - (1-\delta)}\right), -1 + \log\left(\frac{1}{n}\right)\right\}, \quad (31)$$

which compares the options to trade bilaterally *or* on the futures market.

Figure 4 shows an efficient allocation when we allow for the option to trade and clear bilaterally. When σ is low, the planner again achieves an allocation that depends only on the outside option to trade in the futures market, with the production of wheat increasing in this region. But now, the bilateral outside option becomes relevant for high σ. The planner can still induce a contract size that is higher than the one with an OTC equilibrium, as the planner can offer insurance against idiosyncratic default risk without having to resort to collateral. This increases the surplus in matches that can be redistributed to leave farmers with exactly the same utility as with bilateral clearing. Most important, the contract size, however, is then independent of σ for high valuations. This is a direct consequence of the bilateral outside option in equation (31) and our log-linear preference structure.

To summarize, the equilibrium allocation differs from the efficient allocations with bilateral outside options along two dimensions. First, farmers are not insured against the default risk. Second, bargaining leads to a constant contract size across some OTC trades, although it is efficient to have the contract size increase with the surplus from the match as expressed by σ.

This suggests again that there is some room for CCP clearing to improve on the equilibrium allocation. Novation can diversify the default risk, and mutualization can change the incentives to negotiate a particular contract size and payment. An important feature here is that efficient allocations – independent of the assumed outside option – share with the equilibrium allocation that in any match the payment is equal to the expected marginal benefit of exotic wheat (see equation (29)). Hence, in order to achieve efficiency gains, CCP clearing cannot distort bargaining directly. It will have to influence bargaining indirectly by changing the surplus in a match.

4.4   CCP Clearing for OTC Trades

Suppose there is a CCP for clearing exclusively OTC trades.[33] We again assume that the CCP takes the terms of OTC trades (s_i, p_i, k_i) as given but novates the trade, i.e., becomes the counterparty to every trade. Obviously, novation and mutualization will affect the terms of the trade, and we will solve for the equilibrium OTC contract in due course. We first specify what happens when a trade is submitted for clearing.

There are two important differences between clearing OTC contracts and futures contracts for plain wheat. First, exotic wheat is not fungible; therefore, the CCP cannot obtain any additional revenue from selling the exotic wheat on a spot market. Second, trading futures on plain wheat implies automatic CCP clearing, i.e., there is no incentive to clear plain wheat bilaterally by collateralizing the trade with $k > 0$. This is not the case on the OTC market: Consider an OTC trade with terms (s_i, p_i, k_i). As shown above, the contracts differ in the underlying surplus σ and, hence, in their terms. If the CCP were to split its revenue R^n equally across all farmers, as done when clearing futures, some farmers would get a lower utility than others and might have no incentive to submit the trade for clearing through the CCP. This implies that the CCP has to design a payment rule $m(s_i, p_i, k_i)$ so that there are incentives for the farmer and the baker to submit their trade for clearing to the CCP.[34] A payment rule $m(\cdot)$ of the CCP is incentive compatible, if for every OTC trade (s_i, p_i, k_i)

$$-s_i + \log\left(m(s_i, p_i, k_i)\right) \geq -\bar{s} + \log\left((1 - \delta)\sigma v'(\bar{s})\right) + \delta \log\left(\frac{\delta}{(1 - \delta)(\mu(\delta) - 1)}\right) \quad (32)$$

where the right-hand side denotes the farmer's payoff from the optimal OTC contract when

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[33]Koeppl, Monnet, and Temzelides (2009) consider the problem of a CCP operating on two platforms and possibly cross-subsidizing its operations.

[34]The problem of providing incentives to submit OTC trades to formal clearing was first pointed out and modeled by Koeppl, Monnet, and Temzelides (2009).

clearing bilaterally. Note that while the CCP takes the OTC contract as given, its promised payment $m(\cdot)$ and additional collateral requirements will alter the negotiated contract.[35]

Denote the additional collateral requested by the CCP $\tilde{k}_i$. Its revenue is then given by

$$R^{OTC} = (1 - \delta) \int_{\hat{\sigma}}^{\bar{\sigma}} (p_i + \phi_i) dH(\sigma) + \delta \int_{\hat{\sigma}}^{\bar{\sigma}} \left(k_i + \tilde{k}_i \right) dH(\sigma). \tag{33}$$

Since exotic wheat is not fungible, there is extreme counterparty risk, causing the revenue for the CCP to be independent of the aggregate demand shock θ. For an incentive compatible payment rule $m(\cdot)$, all farmers with a match above the cut-off point $\hat{\sigma}$ for OTC trades submit their trade for clearing with the CCP. The CCP then delivers then all exotic wheat s_i to non-defaulting bakers and receives in return the outstanding net payment $(p_i - k_i - \tilde{k}_i)$. The CCP also obtains the collateral pledged for each OTC trade, which is equal to $(k_i + \tilde{k}_i)$. We again have included an additional (positive or negative) payment for bakers ϕ_i, which is now independent of the aggregate demand shock θ, but can depend on the characteristics σ and as such is lump-sum. The exotic wheat delivered to the CCP and owed to defaulting bakers is worthless. The payment rule $m(\cdot)$ is then resource feasible, if and only if

$$\int_{\hat{\sigma}}^{\bar{\sigma}} m(s_i, p_i, k_i) dH(\sigma) = R^{OTC}. \tag{34}$$

4.5   Gains from Novation

We first consider a payment schedule that insures against the counterparty risk associated with a specific contract (s_i, p_i, k_i). Consider the payment schedule

$$m(s_i, p_i, k_i) = (1 - \delta)p_i + \delta \left(k_i + \tilde{k}_i \right). \tag{35}$$

This payment schedule perfectly diversifies the counterparty risk associated with a contract σ_i, as it simply pays out all funds the CCP receives, which are the payments from performing contracts and the collateral seized from contracts in default. Here, we have set $\phi_i = 0$ for all σ. In this sense, novation shares only default risk among farmers. It is immediate that the payment schedule is resource feasible for the CCP. Also, note that novation through the payment schedule m depends only on the contract terms, but not on σ directly.

Given the payment schedule (35), if the trade is cleared through the CCP, farmers will make

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[35]Here we have required that the payment $m(s_i, p_i, k_i)$ fully insures farmers against the risk of default. This does not have to be the case, but as farmers are risk averse, insurance against counterparty risk saves the CCP resources.

a take-it-or-leave-it offer according to

$$\max_{(s_i,p_i,k_i)} -s_i + \log\left(m(s_i, p_i, k_i)\right) \tag{36}$$

subject to

$$-\mu\left(k_i + \tilde{k}_i\right) + (1-\delta)\left[\sigma v(s_i) - \left(p_i - k_i - \tilde{k}_i\right)\right] \geq 0 \tag{37}$$

with first-order conditions given by

$$-1 + \xi(1-\delta)\sigma_i v'(s_i) = 0 \tag{38}$$

$$\frac{1}{(1-\delta)p_i + \delta\left(k_i + \tilde{k}_i\right)} \leq \xi \tag{39}$$

$$\frac{1}{(1-\delta)p_i + \delta\left(k_i + \tilde{k}_i\right)} \leq \xi\frac{(1-\delta)(\mu(\delta)-1)}{\delta}, \tag{40}$$

where ξ is the multiplier on the participation constraint for the baker. Since $(1-\delta)(\mu(\delta)-1) > \delta$, the last constraint will not hold with equality implying $k_i = 0$. It follows then from the binding participation constraint that

$$(1-\delta)\sigma\left(v(s_i) - v'(s_i)\right) = (\mu-1)\tilde{k}_i. \tag{41}$$

Equation (41) gives us the production of exotic wheat s_i as a function of collateral requirements by the CCP. Requiring collateral increases insurance but drives a wedge in the bargaining problem that causes the contract size s_i to increase with σ.[36] When the CCP diversifies counterparty risk through novation, it is thus never optimal for the CCP to provide additional insurance through collateral.

Lemma 5. *If the CCP shares default risk through novation, it is optimal to not require collateral $(\tilde{k}_i = k_i = 0)$.*

Proof. The baker has no benefit from CCP clearing. Hence, we look at the utility gains from collateral for farmers. Taking into account the solution of the bargaining problem, the farmer receives the payment

$$m(s_i, p_i, k_i) = (1-\delta)\sigma v'(s_i(\tilde{k}_i)).$$

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[36]Hence, having collateral requirements increase with the gains from customization can increase the contract size for such contracts. Similarly, a *negative* collateral requirement could subsidize a trade, thereby *lowering* the contract size s_i. However, as collateral is costly ($\mu > 1$), providing incentives through it will be dominated by a tax-transfer scheme.

The value for a farmer from submitting the contract to the CCP is then given by

$$-s_i(\tilde{k}_i) + \log\left((1-\delta)\sigma v'(s_i(\tilde{k}_i))\right).$$

Differentiating with respect to s_i, we obtain

$$\left(-1 + \frac{v''(s_i)}{v'(s_i)}\right)\frac{\partial s_i}{\partial \tilde{k}_i} < 0,$$

since v is strictly concave and the contract size s_i increases with collateral. $\qquad\square$

Novation gives farmers an incentive to submit all OTC trades for clearing to the CCP, as the incentive constraint for clearing (32) is satisfied. The contract size stays unchanged at $\bar{s}$, but novation saves collateral costs and guarantees farmers exactly their expected payment $(1-\delta)p_i$. The negotiated price changes and is now given by

$$p_i = \sigma v'(\bar{s}), \tag{42}$$

which ensures that farmers obtain the payment of a bilateral contract independent of default. Hence, they are again perfectly insured against counterparty risk.

The cut-off point for OTC trades $\hat{\sigma}$ is now given by

$$1 - \bar{s} + \log\left((1-\delta)\hat{\sigma}v'(\bar{s})\right) = \log\left(\frac{1}{n^*}\right) \tag{43}$$

with n^* solving (28) where $\sigma(n) = \hat{\sigma}$. Since the value of an OTC trade increases for any given σ, it must be the case that the cut-off point $\hat{\sigma}$ decreases, and as a consequence, there are more OTC trades in equilibrium. We can thus characterize how the equilibrium on the OTC market changes with novation through a CCP.

Lemma 6. *CCP clearing with novation increases surplus for OTC trades and, therefore, increases the size of the OTC market, i.e., $\hat{\sigma}$ and n^* decline. This improves farmers' welfare but lowers bakers' welfare, as the futures price increases.*

Figure 5 shows that CCP clearing shifts the payoff upwards for farmers that have traded OTC beforehand. This will draw additional farmers to the OTC market, and as a consequence, less plain wheat is produced. As this pushes up prices for plain wheat, all farmers gain from
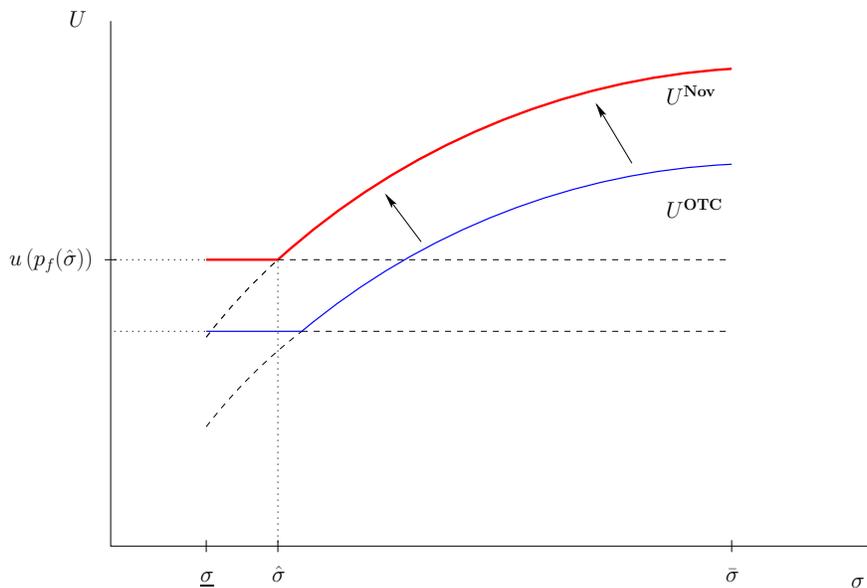
Figure 5: Payoff for Farmers – CCP for OTC Market with Novation

introducing a CCP on the OTC market. However, this comes at a cost to bakers: They are worse off because they have to pay more for plain wheat, and they get no extra surplus from exotic wheat. This creates a possible conflict of interest for introducing CCP clearing on OTC markets, which we do not analyze further.[37]

As is evident from Figure 5, with novation alone, CCP clearing cannot achieve an efficient allocation in the OTC market. The reason is that bargaining leads to socially inefficient contract sizes across σ. It would be optimal to have the contract size increase with σ. This implies that farmers in matches with high σ should produce more wheat than farmers in matches with low σ, at the cost of reducing their payoff.[38] We show next that a redistributive, revenue-neutral transfer scheme that charges additional fees to surviving bakers can accomplish a better allocation.

4.6 Improving the Allocation of Default Risk

Beyond novation, a CCP now also charges additional fixed fees $\phi(\sigma)$ for clearing – which can be positive or negative for bakers that do not default, but *depend on the (observable)*

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[37]While this stark result is somewhat an artifact of farmers extracting all the surplus, it will survive for a sufficiently unequal distribution of bargaining power.

[38]This is akin to a standard production externality where high productivity matches do not take into account how their marginal product compares with the average one in the economy.

characteristic of the match σ. With such fees, the farmer will offer a contract to solve

$$\max_{(s_i, p_i, k_i)} -s_i + \log\left(m(s_i, p_i, k_i)\right)$$

subject to

$$-\mu k_i + (1-\delta)\left[\sigma v(s_i) - p_i - \phi(\sigma) - k_i\right] \geq 0.$$

Suppose the CCP uses the same payment schedule $m(\cdot)$ as with novation (see equation (35)). The fee $\phi(\sigma)$ leaves the structure of the contract the same but influences the total surplus of the OTC trade. As the first-order conditions remain the same, it follows immediately from the participation constraint that the negotiated contract size s_i solves

$$\sigma\left[v(s_i) - v'(s_i)\right] = \phi(\sigma). \tag{44}$$

Like collateral with novation, the fee $\phi(\sigma)$ drives a wedge into the choice of the contract size, but without influencing the pricing of the contract directly. As v is concave, this wedge causes s_i to be an increasing function of $\phi(\sigma)$.

Taking the size of the OTC market with novation as given, the CCP can thus influence the contract size across trades through its fee schedule ϕ. A positive fee will reduce surplus in a match. The farmer would like to maintain the surplus by adjusting his offer to produce more at a lower price. Similarly, a negative fee subsidizes an OTC trade by increasing the surplus. It is now easier for the farmer to extract surplus, and he will produce less at a higher price.

The CCP, however, faces additional restrictions on the fee schedule ϕ. Given the optimal collateral policy $\tilde{k}_i = k_i = 0$,[39] the CCP's revenue is now given by

$$R^{OTC} = (1-\delta)\int_{\hat{\sigma}}^{\bar{\sigma}} (p_i + \phi(\sigma))\,dH(\sigma). \tag{45}$$

Hence, the payment schedule $m(\cdot)$ is resource feasible if and only if the fees ϕ are purely redistributive (or revenue neutral) across OTC trades,

$$\int_{\hat{\sigma}}^{\bar{\sigma}} \phi(\sigma)dH(\sigma) = 0. \tag{46}$$

Furthermore, the CCP needs to induce trades to be submitted for clearing; i.e., the payment schedule has to be incentive feasible according to equation (32). Finally, we also require that farmers do not have an incentive to switch to the futures market because ϕ reduces the

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[39]It is straightforward to verify that the optimal collateral policy with novation is not affected by the mutualization scheme.

surplus of a trade. Hence, we require that the schedule $\phi(\sigma)$ induces a contract size such that

$$1 - s_i(\phi(\sigma)) + \log\left((1-\delta)\sigma v'(s_i(\phi(\sigma)))\right) = \log\left(\frac{1}{n^*}\right), \tag{47}$$

where we have used the payment schedule m and the fact that the CCP takes the size of the OTC market with novation as given. Since these restrictions simply mirror the trading frictions for a planner, there exists a fee schedule ϕ^* that implements the constrained efficient allocation with trading frictions.

Proposition 7. *CCP clearing with novation together with a revenue-neutral transfer scheme achieves the efficient allocation with bargaining $(x^*(\sigma), s^*(\sigma))$ on the OTC market by employing a payment schedule and fee structure*

$$
\begin{aligned}
m^*(s_i, p_i, k_i) &= (1-\delta)p_i \\
\phi^*(\sigma) &= \sigma\left[v(s^*(\sigma)) - v'(s^*(\sigma))\right].
\end{aligned}
$$

The optimal fee structure ϕ^* implies a fixed positive fee for high valuations. As σ declines, the fee structure eventually declines and becomes negative.

Proof. Let $(x^*(\sigma), s^*(\sigma))$ be the constrained efficient allocation where the planner is restricted by bargaining. Given the payment schedule $m(s_i, p_i, k_i) = (1-\delta)p_i$, the solution to the bargaining problem has to satisfy

$$\sigma v'(s_i) = p_i.$$

and the binding participation constraint. Hence, we can use the price p_i and the fee schedule to obtain

$$\sigma\left[v(s^*(\sigma)) - v'(s^*(\sigma))\right] = \phi(\sigma) = \sigma\left[v(s_i) - v'(s_i)\right]$$

for all $\sigma \in [\hat{\sigma}, \bar{\sigma}]$. By concavity of v, we then have $s_i = s^*(\sigma)$ and a payment equal to

$$(1-\delta)p_i = (1-\delta)\sigma v'(s_i^*) = x^*(\sigma)$$

for all $\sigma \in [\hat{\sigma}, \bar{\sigma}]$. By construction, the concavity of v implies then that the solution to the bargaining problem is $(x^*(\sigma), s^*(\sigma))$ for all $\sigma \in [\hat{\sigma}, \bar{\sigma}]$.

Hence, it suffices to show that the resource constraint of the CCP is satisfied by the payment and the fee schedule. Since $k_i = \tilde{k}_i = 0$, we only need to show that

$$\int_{\hat{\sigma}}^{\bar{\sigma}} \phi(\sigma)dH(\sigma) = 0.$$

We have

$$
\begin{aligned}
\int_{\hat{\sigma}}^{\bar{\sigma}} \phi(\sigma) dH(\sigma) &= \int_{\hat{\sigma}}^{\bar{\sigma}} \sigma \left[v(s^*(\sigma)) - v'(s^*(\sigma)) \right] dH(\sigma) \\
&= \int_{\hat{\sigma}}^{\bar{\sigma}} \sigma v(s^*(\sigma)) dH(\sigma) - \int_{\hat{\sigma}}^{\bar{\sigma}} \sigma v'(s^*(\sigma)) dH(\sigma) \\
&= \int_{\hat{\sigma}}^{\bar{\sigma}} \sigma v(s^*(\sigma)) dH(\sigma) - \int_{\hat{\sigma}}^{\bar{\sigma}} \frac{x^*(\sigma)}{1 - \delta} dH(\sigma) \\
&= 0,
\end{aligned}
$$

where the last equality follows from the fact that the efficient allocation with bargaining is resource feasible.

For the shape of ϕ^*, we need only to characterize how the optimal contract size changes with σ. For σ high enough, the optimal contract size is given by

$$
\bar{s} - s^*(\sigma) + \log \left(\frac{v'(s^*(\sigma))}{v'(\bar{s})} \right) = \delta \left(\log \left(\frac{\delta}{\delta + (\mu - 1)} \right) \right) < 0,
$$

which implies that $s^*(\sigma)$ is constant and larger than the equilibrium value with bilateral clearing $\bar{s}$. Hence, $\phi^*(\sigma) > 0$.

For σ such that the futures market is the relevant outside option of farmers, the relationship

$$
-s^*(\sigma) + \log \left((1 - \delta) \sigma v'(s^*(\sigma)) \right) = -1 + \log \left(\frac{1}{n^*} \right)
$$

implies that $s^*(\sigma)$ increases with σ, and so does ϕ^*. Finally, when farmers' utility declines with σ, the fact that $x^*(\sigma) = (1 - \delta) \sigma v'(s^*(\sigma))$ implies that $s^*(\sigma)$ has to increase as well. $\quad\square$

As shown in Figure 6, the optimal fee schedule ϕ^* taxes high surplus matches and subsidizes low surplus ones. Taxing surplus reduces what farmers can extract and, as a consequence, the contract size will increase. A subsidy of course has exactly the opposite effect. Interestingly, the CCP needs to employ novation in order to change contract sizes. Novation offers diversification at no cost, while bilateral clearing would require costly collateral as a substitute for diversifying risk. The CCP is able to use this benefit in order to extract some of the surplus from the match by imposing a positive fee $\phi(\sigma)$. At this fee, matches with high valuations are then made indifferent between clearing bilaterally with collateral and clearing through a CCP, as represented in the figure by the downward shift in farmers' utility. In this sense, only novation makes the transfer system feasible for the CCP.
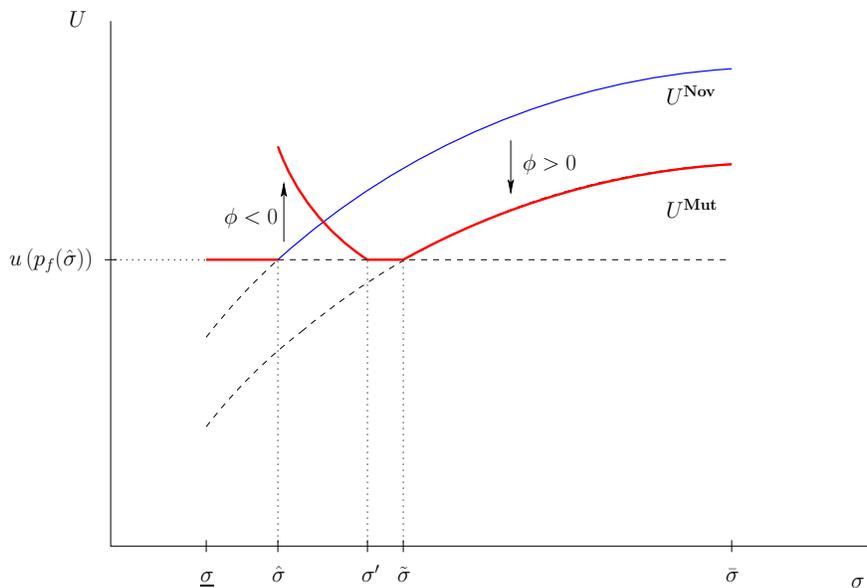
Figure 6: Payoff for Farmers – CCP for OTC Market with Novation and Mutualization

Since the CCP takes the terms of the bilaterally negotiated contract as given, it cannot influence the structure of the contract and achieve a better allocation. Here, the key frictions are that farmers can extract all the surplus from bakers.[40] The transfer scheme can therefore alleviate but not entirely remove the production inefficiency in the OTC market. The CCP could circumvent this friction and negotiate the terms of trade directly with bakers and farmers. It would thus assume the role of a dealer that goes well beyond the mere clearing and settlement of trades. We will discuss this issue further below.

4.7 The Transfer Scheme with Private Information

The CCP's fee schedule is conditional on the valuation of bakers σ. However, it is unlikely that this valuation is publicly observable. It seems natural to assume that it is private information for the parties of the OTC trade. Next we outline the constrained efficient allocation under private information and relegate details to the Appendix.

The planner now needs to provide incentives for the match to reveal the true valuation σ of the OTC trade; in other words, we require that a match cannot achieve a higher payoff by misrepresenting its valuation.[41] We use a direct mechanism where agents in a match

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[40] As we argue in the Appendix, this holds more generally whenever the distribution of bargaining power differs from the weights in the planner's objective function.

[41] This relates our problem to the literature on Mirleesian Taxation, in which a planner taxes labor income

directly report their valuation σ with the planner, imposing an allocation $(x_2(\sigma), s(\sigma))$ that is a function of the reported σ.

We require again that the planner has to respect the bargaining frictions, so that farmers extract all surplus from trade; in other words, we impose the restriction $x_2(\sigma) = \sigma s(\sigma)$ for all σ on the allocations the planner can propose. This restrictions limits what σ a match can report. If a match with true valuation σ reported σ' instead, it must be the case that

$$\sigma v(s(\sigma')) - x_2(\sigma') \geq 0 = \sigma' v(s(\sigma')) - x_2(\sigma'), \tag{48}$$

since otherwise the baker in the match would receive a negative surplus and, hence, would prefer to trade on the futures markets. Hence, only reports of lower valuations than the true one ($\sigma' \leq \sigma$) are feasible. Taking into account that bakers never receive any surplus, the truth-telling constraints for any valuation σ are thus given by

$$-s(\sigma) + \log\left(x(\sigma)\right) \geq -s(\sigma') + \log\left(x(\sigma')\right) \text{ for all } \sigma' \leq \sigma. \tag{49}$$

This condition implies that in the constrained efficient allocation, the utility for farmers must be weakly increasing in σ. If it were not, farmers would have an incentive to misrepresent the valuation of the match.

Still, it is efficient to have a high production of exotic wheat for high valuation matches. Therefore, the production of exotic wheat should be increasing in σ, but farmers also need to have a (weakly) increasing payoff in σ. Figure 7 exhibits the constrained efficient allocation. The planner guarantees a minimum payoff for farmers, but above a threshold $\tilde{\sigma}$ the bilateral outside option binds so that the payoff is strictly increasing in σ. It is important to realize here that while a farmer in a match with valuation $\sigma < \tilde{\sigma}$ has an incentive to report any $\sigma' > \tilde{\sigma}$, such a report is not feasible as the baker would just object to it.

A CCP can implement this constrained efficient allocation as before. Crucially, we assume that the CCP declines to clear any trade (s_i, p_i) that does not satisfy

$$p_i = \sigma v'(s(\sigma)) \tag{50}$$

for some $\sigma \in [\hat{\sigma}, \bar{\sigma}]$. This restricts the possibilities for OTC trades to misrepresent their valuation when submitting it for CCP clearing.[42] Gains from novation can be achieved with

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with output being observable, but productivity being private information.

[42]This then corresponds to a direct revelation mechanism where traders simply report σ to the CCP. The CCP then levies a mutualization fee $\phi(\sigma)$ and "clears" the corresponding contract $(s(\sigma), p(\sigma))$, where we have taken into account that no collateral will be used.
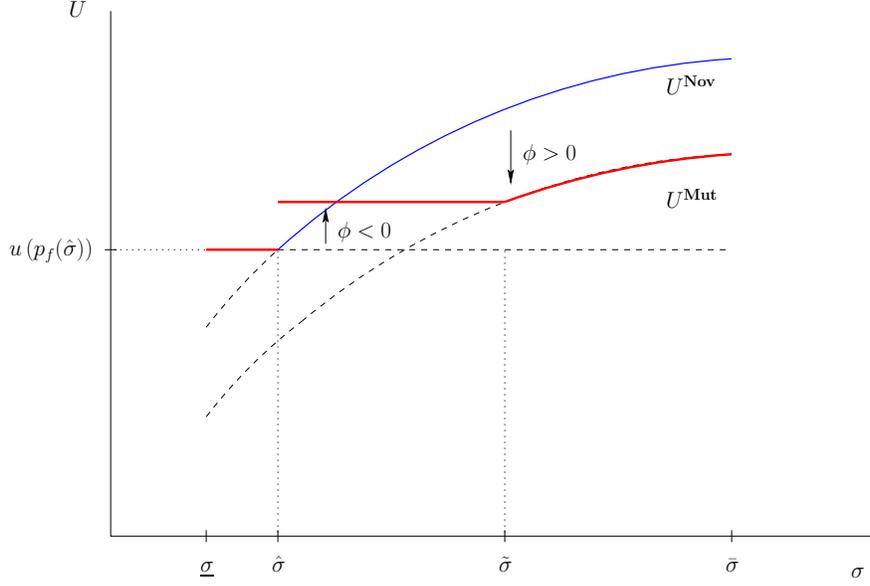
Figure 7: Payoff for Farmers – Mutualization with Private Information

the payment schedule $m = (1 - \delta)p_i$ and are necessary for taxing high valuation matches. It is then feasible for a match with σ to submit a trade corresponding to some σ' if and only if

$$\sigma v(s(\sigma')) - p_i(\sigma') - \phi(\sigma') \geq 0. \tag{51}$$

Since $\sigma' v(s(\sigma')) - p_i(\sigma') - \phi(\sigma') = 0$, only farmers with $\sigma > \sigma'$ can pretend to be σ'. Also, a baker would get a positive payoff whenever a farmer were to lie, since the payment would be $\sigma' v'(s(\sigma')) < \sigma v'(s(\sigma'))$. Hence, the fee schedule $\phi(\sigma)$ satisfies truth-telling if for all σ

$$U(\sigma, \phi(\sigma)) \geq U(\sigma, \phi(\sigma')) \text{ for all } \sigma' \leq \sigma, \tag{52}$$

where $U(\cdot)$ is the utility for a farmer of making announcement σ' given the true valuation is given by σ.

Using the constrained efficient allocation as shown in Figure 7, we obtain that the payment schedule $m = (1 - \delta)p_i$ and the fee schedule

$$\phi(\sigma) = \sigma \left[v(s^*(\sigma)) - v'(s^*(\sigma)) \right]$$

satisfies the truth-telling constraint. The reason is that the bilateral outside option implies again a fixed positive fee for high σ, which gives a strict preference to farmers to reveal their type of match. For low σ, the fee schedule is negative and increases with σ so that farmers

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are indifferent between announcing their true valuation or any lower one. For high σ, the fee schedule is positive and increasing. In this way, a match with a high valuation has less surplus if it is cleared through the CCP. To induce bakers to participate, given the terms of the take-it-or-leave-it offer, requires a higher production of exotic wheat. Hence, the contract size is again increasing with the valuation in a match. The CCP can charge $\phi(\sigma) > 0$ to high σ, as it taxes the additional surplus that originates from the diversification of counterparty risk. This revenue can then be transferred to matches with a lower valuation, with the effect of reducing their production of exotic wheat. Hence, private information does not prevent a redistribution of default risk but limits it.

5   Conclusion

Our paper offers a formal model of a CCP and of clearing more generally. We find that CCP clearing with novation and mutualization of losses is part of an efficient market structure for standardized financial contracts that are centrally traded on a competitive market. A CCP that clears OTC trades has to take into account, however, that fungibility of contracts is limited and that due to their customized nature formal clearing remains a choice for the counterparties.

The discussion about formal clearing of OTC transactions has primarily focused on the benefits offered by netting but has largely overlooked how intricate clearing is for such transactions. A CCP can still offer novation – albeit not in the form of a guarantee – and it is precisely these gains from novation that can give incentives for counterparties to formally clear OTC transactions through a CCP. With such incentives in place, a CCP is then perfectly situated to affect both the level of risk individual counterparties have to bear and the overall allocation of risk in the market.

We have deliberately abstracted from one important question that the introduction of CCP clearing might entail moral hazard. This is clearly pivotal for addressing the optimal collateral structure of a CCP, and we think it deserves particular attention. In this context, it will be necessary to study the optimal scope for CCP clearing in the sense that one creates an institution that is too-big-to-fail and entails potentially an overall increase in risk due to a moral hazard problem.

Finally, some of our assumptions are quite strong but are driven by the desire to derive stark results. One issue is to extend our analysis to cases in which counterparties contemplate default if it is in their interest. Collateral will then play a crucial role as an incentive device.

Also, we have assumed that preferences are represented by log-linear utility. This simplifies the analysis greatly, as there are no wealth effects from introducing insurance and there are no distortions from reallocating risk. It would be interesting to see how our results fare under different preference structures, but we doubt this would affect the main message of what CCP clearing adds to financial markets and how it differs for OTC markets.

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7 Appendix

7.1 Proof of Proposition 1

The bakers' welfare is not affected by the market structure. Since $k = 0$ is feasible, farmers must be better off with a futures market, as they are partially insured against the aggregate shock θ.

The utility of a farmer given collateral k is given by

$$U_f = (1 - \delta) \log \left(1 + (1 - \mu(\delta)) k\right) + \delta \int \log \left(\theta + k\right) dF(\theta).$$

Hence, the optimal level of collateral solves

$$\partial U_f / \partial k \equiv \varphi(k) = \frac{1 - \mu(\delta)}{1 + (1 - \mu(\delta))k}(1 - \delta) + \delta \int \frac{1}{\theta + k} dF(\theta) = 0$$

It is easy to check that utility is strictly concave in k, i.e., $\varphi'(k) < 0$ for all δ. Hence, it is optimal to set $k > 0$, unless $\varphi(0) \le 0$. This is the case for critical values of μ and δ such that

$$\underline{\mu}(\delta) = 1 - \delta + \delta \int \frac{1}{\theta} dF(\theta).$$

Since $1/\theta$ is a strictly convex function and $E(\theta) = 1$, we have $\underline{\mu}(\delta) > 1$ and $\partial \underline{\mu}(\delta)/\partial \delta > 0$ for all $\delta > 0$.

To show that futures trades are never fully collateralized, set $p_f^k = \bar{k} = 1 + (1 - \mu(\delta))\bar{k}$. The first-order condition then yields

$$\frac{1 - \mu(\delta)}{\bar{k}}(1 - \delta) + \delta \int \frac{1}{\theta + \bar{k}} dF(\theta) =$$
$$= \frac{1}{\bar{k}} \left[(1 - \mu(\delta)) + \mu(\delta)\delta + \delta \int \left(\frac{\bar{k}}{\theta + \bar{k}} - 1\right) dF(\theta)\right]$$
$$\frac{1}{\bar{k}} \left[(1 - \mu) + \delta \int \left(\frac{\bar{k}}{\theta + \bar{k}} - 1\right) dF(\theta)\right] < 0,$$

as $\mu > 1$ and $\theta > 0$. Since utility is concave, any $k \ge \bar{k}$ can thus never be optimal, which completes the proof.

7.2 Constrained Efficient Allocations on OTC Markets

7.2.1 First-Best Allocations

Consider all matches in the OTC market and suppose that there are no outside options the planner has to respect. Then, the planner has to satisfy only a resource constraint and his problem solves

$$\max_{(s(\sigma),x(\sigma),x_2(\sigma))} \int_{\hat{\sigma}}^{\bar{\sigma}} \left((1-\delta) \left[\sigma v(s(\sigma)) - x_2(\sigma) \right] + \log\left(x(\sigma)\right) - s(\sigma) \right) dH(\sigma)$$

subject to

$$\int_{\hat{\sigma}}^{\bar{\sigma}} x(\sigma)dH(\sigma) \leq \int_{\hat{\sigma}}^{\bar{\sigma}} (1-\delta)x_2(\sigma)dH(\sigma) \qquad (\lambda_1)$$

$$x_2(\sigma) \geq 0 \qquad (\lambda_2(\sigma))$$

The first-order condition yields

$$(1-\delta)\sigma v'(s(\sigma)) = 1$$
$$1/x(\sigma) = \lambda_1$$
$$\lambda_1 = 1 - \lambda_2(\sigma)$$

Therefore, if $\lambda_2(\sigma) = 0$ for all σ so that all bakers produce, then $\lambda_1 = 1$ and $x(\sigma) = 1$. The first-best allocation is therefore given by

$$(1-\delta)\sigma v'(s(\sigma)) = 1$$
$$x(\sigma) = 1.$$

7.2.2 Efficient Allocations

The constrained planner's problem introduces participation constraints and is described by

$$\max_{s(\sigma),x(\sigma),x_2(\sigma)} \int_{\hat{\sigma}}^{\bar{\sigma}} \left\{ (1-\delta)\left[\sigma v(s(\sigma)) - x_2(\sigma)\right] + \log\left(x(\sigma)\right) - s(\sigma) \right\} dH(\sigma)$$

subject to

$$\int_{\hat{\sigma}}^{\bar{\sigma}} x(\sigma)dH(\sigma) = \int_{\hat{\sigma}}^{\bar{\sigma}} (1-\delta)x_2(\sigma)dH(\sigma) \qquad (\lambda_1)$$

$$x_2(\sigma) \geq 0 \qquad (\lambda_2(\sigma))$$

$$(1-\delta)\left[\sigma v(s(\sigma)) - x_2(\sigma)\right] \geq \bar{v} \qquad (\lambda_3(\sigma))$$

$$\log\left(x(\sigma)\right) - s(\sigma) \geq \bar{u} \qquad (\lambda_4(\sigma))$$

where $\bar{v} = 0$ and $\bar{u}$ are the payoffs from the bakers' and farmers' outside options, respectively. By the Inada condition, $s(\sigma) > 0$ and $x(\sigma) > 0$ for any σ. The first-order conditions are

$$(1-\delta)\,\sigma v'(s(\sigma)) = \frac{1+\lambda_4(\sigma)}{1+\lambda_3(\sigma)}$$

$$\frac{1}{x(\sigma)} = \frac{\lambda_1}{1+\lambda_4(\sigma)}$$

$$\lambda_1 = 1 + \lambda_3(\sigma) - \frac{\lambda_2(\sigma)}{1-\delta}.$$

Since $s(\sigma) > 0$ and $\sigma > 0$ for all σ, we can assume without loss of generality that $x_2(\sigma) > 0$ for all σ. Hence, $\lambda_2(\sigma) = 0$. Therefore, we obtain a single first order necessary condition for all matches,

$$(1-\delta)\,\sigma v'(s(\sigma)) = x(\sigma).$$

The participation constraints give us the optimal allocation. First, notice that $\lambda_3(\sigma) = \lambda_3$ for all σ as $\lambda_2(\sigma) = 0$. Hence, we need to consider only two cases.

Case 1: $\lambda_3 = 0$. In this case, we know $\lambda_1 = 1$. If $\lambda_4(\sigma) = 0$, then the solution is the efficient allocation, i.e. $(x(\sigma), s(\sigma)) = (x^*, s^*(\sigma))$. Since $x(\sigma) = x^* = 1$ is constant and $s^*(\sigma)$ is increasing in σ, the farmer's participation constraint may be binding when σ is large so that $\lambda_4(\sigma) > 0$. In particular, this is the case for all $\sigma > \sigma^*$ such that $\log\left((1-\delta)\,\sigma^* v'(s^*(\sigma))\right) = \bar{u} + s^*(\sigma)$.

Suppose then $\lambda_3 = 0$ and $\lambda_4(\sigma) > 0$. The allocation for the match is

$$
\begin{aligned}
\log\left((1-\delta)\,\sigma v'(s(\sigma))\right) &= \bar{u} + s(\sigma) \\
(1-\delta)\,\sigma v'(s(\sigma)) &= x(\sigma) > 1.
\end{aligned}
$$

Note that farmers consume more than the efficient amount.

The individual payment $x_2(\sigma)$ is indeterminate, but the total aggregate payment needs to be sufficient to cover the consumption of the farmers. Hence, a necessary and sufficient condition for this case is that

$$
\int_{\hat{\sigma}}^{\bar{\sigma}} \sigma v(s(\sigma)) dH(\sigma) \geq \int_{\hat{\sigma}}^{\sigma^*} \frac{x^*}{1-\delta} + \int_{\sigma^*}^{\bar{\sigma}} \sigma v'(s(\sigma)) dH(\sigma).
$$

This is sufficient, since the planner can then set $x_2(\sigma) = \sigma v(s(\sigma)) - \varepsilon$, for ε small enough. This is necessary, since the equilibrium we consider gives a positive surplus to all bakers so that $\sigma v(s(\sigma)) > x_2(\sigma)$ for all σ. When this condition is not satisfied, bakers have no surplus, which is the second case.

To summarize, the constrained efficient allocation is then given by

$$
(x^*, s^*(\sigma))
$$

for all $\sigma < \sigma^*$ and

$$
\begin{aligned}
\log\left((1-\delta)\sigma v'(s(\sigma))\right) &= \bar{u} + s(\sigma) \\
(1-\delta)\,\sigma v'(s(\sigma)) &= x(\sigma)
\end{aligned}
$$

for all $\sigma \geq \sigma^*$.

Case 2: $\lambda_3 > 0$. When the above allocation is not feasible, the constrained efficient allocation must be such that bakers have no surplus, or $x_2(\sigma) = \sigma v(s(\sigma))$. If $\lambda_4(\sigma) = 0$, we get

$$
x(\sigma) = \tilde{x} = \frac{1}{1+\lambda_3} < x^*.
$$

Hence, the payment is constant, but less than the efficient amount. The first-order conditions give us $s(\sigma)$ as the solution to

$$
(1-\delta)\,\sigma v'(s(\sigma)) = \tilde{x}.
$$

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When $x(\sigma)$ is constant, the farmer's participation constraint might bind for σ large enough, as $s(\sigma)$ is increasing in σ. Hence given $\tilde{x}$, there is some cut-off $\tilde{\sigma}$, such that for all $\sigma > \tilde{\sigma}$, the farmer's participation constraint will be violated if $s(\sigma)$ is set such that $(1 - \delta)\,\sigma v'\,(s(\sigma)) = \tilde{x}$. This implies $\lambda_4(\sigma) > 0$ for all $\sigma > \tilde{\sigma}$, and the allocation is given by

$$
\begin{aligned}
\log\left((1-\delta)\sigma v'(s(\sigma))\right) &= \bar{u} + s(\sigma) \\
(1-\delta)\sigma v'(s(\sigma)) &= x(\sigma).
\end{aligned}
$$

To complete the characterization of the constrained optimal solution when $\lambda_3 > 0$, we find $\tilde{x}$ from the resource constraint

$$
\int_{\hat{\sigma}}^{\tilde{\sigma}} \tilde{x}dH(\sigma) + \int_{\tilde{\sigma}}^{\bar{\sigma}} x(\sigma)dH(\sigma) = (1-\delta)\int_{\hat{\sigma}}^{\bar{\sigma}} \sigma v(s(\sigma)))dH(\sigma).
$$

To summarize, when bakers have no surplus, the constrained efficient allocation is described by

$$
\begin{aligned}
\tilde{x} &< x^* \\
(1-\delta)\,\sigma v'(s(\sigma)) &= \tilde{x}
\end{aligned}
$$

for all $\sigma < \tilde{\sigma}$ and

$$
\begin{aligned}
\log\left((1-\delta)\sigma v'(s(\sigma))\right) &= \bar{u} + s(\sigma) \\
(1-\delta)\sigma v'(s(\sigma)) &= x(\sigma)
\end{aligned}
$$

for all $\sigma \geq \tilde{\sigma}$.

7.2.3   Efficient Allocations with Bargaining

The planner now has to respect that farmers make a take-it-or-leave-it offer to bakers and that a match akways has the option to trade bilaterally with collateral. This implies that

$x_2(\sigma) = \sigma v'(s(\sigma))$ in any allocation. The planner's problem is then given by

$$\max_{(s(\sigma),x(\sigma))} \int_{\hat{\sigma}}^{\bar{\sigma}} -s(\sigma) + \log(x(\sigma)) \, dH(\sigma)$$

subject to

$$\int_{\hat{\sigma}}^{\bar{\sigma}} x(\sigma) dH(\sigma) \leq (1-\delta) \int_{\hat{\sigma}}^{\bar{\sigma}} \sigma_i v(s(\sigma)) dH(\sigma) \qquad (\lambda)$$

$$-s(\sigma) + \log(x(\sigma)) \geq \bar{u} \qquad (\lambda(\sigma))$$

The first-order conditions are given by

$$x(\sigma) = \frac{1 + \lambda(\sigma)}{\lambda} \qquad (53)$$

$$(1-\delta)\sigma v'(s(\sigma)) = \frac{1 + \lambda(\sigma)}{\lambda}. \qquad (54)$$

Note that $\bar{u}$ has the following definition when farmers can either trade futures (and a CCP operates on the futures market) or clear bilaterally,

$$\bar{u} = \max\left\{ -\bar{s} + \log\left((1-\delta)\sigma v(\bar{s})\right) + \delta \log\left(\frac{\delta}{\mu - (1-\delta)}\right) ; -1 + \log\left(\frac{1}{n}\right) \right\}.$$

For low σ, it is more profitable for farmers to trade futures than clear bilaterally. The farmers participation constraint reads

$$-s(\sigma) + \log\left((1-\delta)\sigma v'(s(\sigma))\right) \geq -1 + \log\left(\frac{1}{n}\right)$$

Denote by $s'(\sigma)$ the level of production such that this constraint binds. We have that $s'(\sigma)$ is an increasing function of σ with the utility of farmers being constant so that their consumption of gold has to increase over this range of σ. As σ increases further, it becomes more profitable to clear bilaterally than to trade futures. Then, for these levels of $\sigma > \tilde{\sigma}$, and to replace the first-order condition in the farmers' participation constraint, we obtain the following participation constraint for farmers

$$\bar{s} - s(\sigma) + \log\left(\frac{v'(s(\sigma))}{v(\bar{s})}\right) \geq \delta \log\left(\frac{\delta}{\mu - (1-\delta)}\right)$$

so that $s(\sigma) = s'$ is the same for all σ such that this constraint binds. As the right-hand side is negative, by concavity of v we have that $s' > \bar{s}$. The solution is then the same as in the case in which there is no bargaining friction and there is no surplus for any bakers.

To summarize, the constrained efficient allocation is given by

$$(1 - \delta)\sigma v'(s(\sigma)) = x'$$

for all $\sigma < \sigma'$,

$$-s(\sigma) + \log\left((1 - \delta)\sigma v'(s(\sigma))\right) = -1 + \log\left(\frac{1}{n}\right)$$

for all $\sigma' < \sigma < \tilde{\sigma}$ and

$$(1 - \delta)\,\sigma v'(s') = x(\sigma)$$

for all $\sigma \geq \tilde{\sigma}$. The value of x' is again given by the resource constraint.

7.2.4   Efficient Allocations with Bargaining and Private Information

We now consider the planner's problem with bilateral outside options under private information. The planner does not observe σ. The match then makes a report regarding σ to the planner, and the planner chooses an allocation based on the report.

We assume that the planner knows how the bargaining power is distributed. Since bakers will have no surplus, a report must be feasible for the match, which implies that the truth-telling constraint is given by

$$-s(\sigma) + \log\left(x(\sigma)\right) \geq -s(\sigma') + \log\left(x(\sigma')\right) \text{ for all } \sigma \geq \sigma'.$$

Note that this condition requires that farmers' utility is nondecreasing in σ. The planner's problem is then given by

$$\max_{s(\sigma), x(\sigma))} \int_{\hat{\sigma}}^{\bar{\sigma}} -s(\sigma) + \log\left(x(\sigma)\right) dH(\sigma)$$
$$\text{subject to}$$
$$(1 - \delta) \int_{\hat{\sigma}}^{\bar{\sigma}} \sigma v(s(\sigma)) dH(\sigma) \geq \int_{\hat{\sigma}}^{\bar{\sigma}} x(\sigma) dH(\sigma)$$
$$-s(\sigma) + \log\left(x(\sigma)\right) \geq -1 + \log\left(\frac{1}{n}\right)$$
$$-s(\sigma) + \log\left(x(\sigma)\right) \geq \bar{u}(\sigma)$$
$$-s(\sigma) + \log\left(x(\sigma)\right) \geq -s(\sigma') + \log\left(x(\sigma')\right) \text{ for all } \sigma \geq \sigma'$$

where $\bar{u}(\sigma)$ is the outside option of a bilateral OTC trade with collateral.

Suppose first that none of the participation constraints is binding. Note that $x(\sigma)$ is in-

creasing with σ, since any constrained efficient allocation satisfies $(1-\delta)\sigma v'(s(\sigma)) = x(\sigma)$. The constrained optimal solution must then give the same utility $\bar{u}$ to all farmers with the solution being described by

$$
\begin{aligned}
(1-\delta)\sigma v'(s(\sigma)) &= x(\sigma) \\
-s(\sigma) + \log(x(\sigma)) &= \bar{u}.
\end{aligned}
$$

To see this, suppose by way of contradiction that there is a subset Σ with positive measure such that for all $\sigma \in \Sigma$, $-s(\sigma) + \log(x(\sigma)) > \bar{u}$. Without loss of generality, we can assume that for $\sigma \in \Sigma$, $-s(\sigma) + \log(x(\sigma)) = \bar{u}' > \bar{u}$. Hold $s(\sigma)$ constant for all $\sigma \in \Sigma$, define $\tilde{x}(\sigma) = x(\sigma) - \varepsilon(\sigma)$ for all $\sigma \in \Sigma$ such that $-s(\sigma) + \log(\tilde{x}(\sigma)) = \bar{u}' - \varepsilon$ for some $\varepsilon > 0$ sufficiently small. This frees up total resources $\int_\Sigma \varepsilon(\sigma) dH(\sigma)$ that can be distributed to all farmers in matches with $\sigma \notin \Sigma$ so that their utility shifts up uniformly still satisfying truth-telling. Since $x(\sigma)$ is increasing with σ and the utility is concave, the gain in utility for these agents will more than compensate for the loss in utility of farmers with $\sigma \in \Sigma$. Hence, the original allocation was not constrained efficient. A contradiction.

Suppose now that the participation constraints bind for some σ. Since truth-telling requires non-decreasing utility for farmers, it is optimal to have utility for farmers as high as possible, but constant, until the bilateral outside option becomes relevant. Recall that $\bar{u}(\sigma)$ is increasing in σ. Consider then $\bar{u}$ such that $\bar{u} \geq -1 + \log\left(\frac{1}{n}\right)$, but $\bar{u} \geq \bar{u}(\sigma)$ only for $\sigma < \tilde{\sigma}$ for some $\tilde{\sigma}$ and $\bar{u} < \bar{u}(\sigma)$ otherwise. The constrained efficient allocation under private information is then described by

$$
\begin{aligned}
(1-\delta)\sigma v'(s(\sigma)) &= x(\sigma) \\
-s(\sigma) + \log(x(\sigma)) &= \bar{u} \text{ for all } \sigma \leq \tilde{\sigma} \\
-s(\sigma) + \log(x(\sigma)) &= \bar{u}(\sigma) \text{ for all } \sigma > \tilde{\sigma}
\end{aligned}
$$

where $\tilde{\sigma}$ is chosen to satisfy the feasibility constraint

$$
(1-\delta)\int_{\hat{\sigma}}^{\bar{\sigma}} \sigma v(s(\sigma))dH(\sigma) = \int_{\hat{\sigma}}^{\bar{\sigma}} x(\sigma)dH(\sigma).
$$

7.3 Generalized Nash Bargaining

The goal of this section is to show that Nash bargaining also leads to an inefficient contract size in OTC trading. We first solve for the equilibrium allocations on the OTC market where

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the counterparties to an OTC trade have access to novation by a CCP, but where there is no mutualization. Novation here increases surplus in the OTC trade, which is split between the farmer and baker according to the distribution of bargaining power. As before, this does not influence the optimal structure of the contact.

In an OTC trade, the valuation σ is common knowledge for the trading parties. Suppose there is Nash bargaining where η is the relative weight of farmers. Define the surplus of farmers and bakers as S_1 and S_2 respectively. We then have

$$
\begin{aligned}
S_1 &= \log\left((1-\delta)p_i\right) - s(\sigma) - \bar{u} \\
S_2 &= (1-\delta)\left[\sigma v(s(\sigma)) - p_i\right] - \bar{v},
\end{aligned}
$$

where we already have used the payment schedule m under novation. The outside options are participation in the futures market, which offers no surplus for bakers ($\bar{v} = 0$), but positive surplus for farmers. Again, with novation it is not optimal to use collateral and the bargaining problem with no collateral is given by

$$
\max_{(s(\sigma),p_i)} S_1^\eta S_2^{1-\eta}
$$

yielding the following first-order conditions

$$
\begin{aligned}
p_i &= (1-\delta)\sigma v'(s) \\
\frac{\eta S_2}{(1-\eta)S_1} &= (1-\delta)\sigma v'(s).
\end{aligned}
$$

The pricing of the OTC contract is once again independent of the bargaining assumption and equates the price to the expected marginal benefit of the transactions for bakers. Hence, there is no inefficiency in the pricing of the OTC contract.

Rewriting, we obtain

$$
\frac{v(s)}{v'(s)} - 1 = \frac{1-\eta}{\eta}\left[\log\left((1-\delta)\sigma v'(s)\right) - s - \bar{u}\right].
$$

Note that $\bar{u}$ is constant. Hence, for any given $\eta \in (0,1)$, the contract size increases with σ (i.e., $ds/d\sigma > 0$). Again, there is a cut-off point with respect to σ – depending on η – such that only matches with a higher surplus will carry out OTC trades. Also, if the bargaining power shifts toward bakers (i.e., η declines), the contract size s will increase for all σ.

The efficient allocation that respects the outside option to trade on the futures market gives

zero surplus to farmers for sufficiently high σ (see the previous section in this Appendix). If the farmers receive zero surplus in the solution above, we have that $v'(s) = v(s)$ independent of σ. Otherwise, the surplus is positive for farmers (unless all bargaining power rests with bakers, or $\eta = 0$), and hence, there is an inefficient contract size due to bargaining. Also, note that this inefficiency does not disappear if the bargaining power is equally distributed $(\eta = 1/2)$ and, thus, mirrors the weighting in the planner's objective function. This implies that the benefits of mutualization are independent of our bargaining assumption.