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WHY DO MARKETS FREEZE?

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Preliminary. Comments welcome

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Abstract

Consider the sale of mortgages by a loan originator to a buyer. As widely noted, such a transaction is subject to a severe adverse selection problem: the originator has a natural information advantage and will attempt to sell only the worst mortgages. However, a second important feature of this transaction has received much less attention: both the seller and the buyer may have existing inventories of mortgages similar to those being sold. We analyze how the presence of such inventories affects trade. We use our model to discuss implications for regulatory intervention in illiquid markets.

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1 Introduction

Consider the sale of mortgages by a loan originator to a buyer. As widely noted, such a transaction is subject to a severe adverse selection problem: the originator has a natural information advantage and will attempt to sell only the worst mortgages.[1] However, a second important feature of this transaction has received much less attention: both the seller and the buyer may have existing inventories of mortgages similar to those being sold. These inventories affect trade if the buyer and/or seller care about market perceptions of their value.

One direct way in which inventories affect decisions is if market participants are subject to some form of market-value accounting.[2] Any effect of this type is necessarily specific to a particular accounting regime. In this paper, we instead consider an arguably more fundamental effect. When market participants trade, information is released to the market, and market participants update their estimates of asset values. This effect occurs — *independent of the accounting regime* — whenever traders care about market perceptions of the total value of their portfolios. In particular, traders may be financed by short-term debt and so need to ensure that the market valuation of their assets exceeds that of their liabilities.

As in the standard case of trade under asymmetric information, in equilibrium the seller only sells the worst assets, and so trade drives the market valuation of inventories downward. One possible equilibrium response is for the parties to abandon trade — the "freeze" of the title. In this case, no new information about the value of the mortgages emerges. In contrast, if the gains from trade are bounded away from zero, trade never completely breaks down in the standard asymmetric information model

[1] See, for example, Ashcraft and Schuermann (2008).

[2] See, for example, Adrian and Shin (forthcoming), Allen and Carletti (2008), Heaton, Lucas, and McDonald (forthcoming), Plantin, Sapra, and Shin (2008), and Milbradt (2008).

without inventories.

Perhaps surprisingly, however, in other circumstances inventories can have just the opposite effect and *increase* the amount of trade. The reason is that an increase in the probability of equilibrium trade makes the negative information associated with trade *less negative.*

We analyze the conditions under which inventories decrease the amount of trade and, likewise, when they increase the amount of trade. For example, we show that the buyer's inventory can increase trade when he is moderately leveraged, but if the buyer becomes too leveraged, trade is reduced and the market breaks down. We also extend the result to a dynamic setting, showing how changes in leverage can affect prices and volume through time, even when there is no change in fundamentals.

We use our model to discuss implications for regulatory intervention in illiquid markets. On the buyer side, our analysis highlights the potential role of a large investor unencumbered by existing inventories (the government, for example). On the seller side, our analysis suggests limitations to the standard prescription that sellers should retain a stake in the assets they sell. It also suggests that regulatory interventions to buy up assets may need to be large enough to buy *all* the seller's assets.

The paper proceeds as follows. In Section 2, we present the model, which we solve in Section 3. We focus on three cases: A benchmark case, where neither the seller nor the buyer has inventories of similar assets; the case in which only the seller has inventories; and the case in which only the buyer has inventories. In Section 4, we discuss some policy implications, and in Section 5 we extend the model to a dynamic framework. We conclude in Section 6. The appendix contains proofs.

1.1 Some related literature

Several contemporaneous papers also explain why a market may freeze. In Diamond and Rajan (2009) trading freezes because a seller, who may have a liquidity shock that will force him into bankruptcy, prefers to "gamble": If he sells now, he receives a low price that reflects a potential fire sale. If he waits, he can sell the asset at a higher price, but only if he survives. In their model information is symmetric, and in terms of efficiency, it does not matter who holds the asset. Acharya, Gale, and Yorulmazer (2009) explain why short-term borrowing may freeze even when there is no credit risk. In their model the price of the asset falls unless some good news arrives, but when the debt needs to be rolled over very often, the probability of receiving good news is very low, and borrowing freezes. Easley and O'Hara (2008) show that a market may freeze when traders have incomplete preferences over portfolios, and when a trader is assumed to move away from the status quo only if he is better off for every possible belief in the set of beliefs that represent his preferences. Thompson (2009) explores a mechanism by which asymmetric information can lead to delays in borrowing or lending. Allen, Carletti, and Gale (forthcoming) use the term market freeze to describe a situation where interbank trading does not occur because there is excess liquidity and the banks facing the largest demand for liquidity can cover it themselves; in their setting a market freeze is efficient.

Milbradt (2008) studies the (dynamic) trading behavior of a financial institution that is subject to a leverage constraint. In his paper the leverage constraint is based on the price of the last trade (marking to market), so an institution may have the incentive not to trade if the price is low. In our paper the capital constraint is based on Bayes' rule, and the market may break down even if the true value (fundamental price) is high. Also, in his setting the price is exogenous (and becomes public only if the institution trades), whereas in our setting the price is endogenous.

2 The model

There is a risk neutral buyer and a risk neutral seller. The value of an asset is v to the seller and $v+\Delta$ to the buyer, where $\Delta \in (0, 1/2)$ denotes the gains from trade. It is assumed (and it's common knowledge) that v is drawn from a uniform distribution on $[0, 1]$. The seller knows v. Everyone else is uncertain about the value of v. The assumption $\Delta > 0$ implies that trade is always efficient. The assumption $\Delta < 1/2$ ensures that the bid price is always less than one.

We assume the following trading process: The buyer makes a take-it-or-leave-it offer to buy q units of the asset at a price (bid) per unit b.[3] The seller can either accept or reject. If the seller accepts, the seller's profit is $\pi_s = q(b - v)$, and the buyer's profit is $\pi_b = q(v + \Delta - b)$. If the seller rejects, each ends up with a zero profit. The outcome of the bargaining game is publicly observable and is denoted by $\psi = (q, b, \phi)$, where ϕ denotes whether trade occurred ($\phi = 1$) or not ($\phi = 0$).

In addition to the asset discussed above, both the buyer and seller have existing inventories of similar assets; cash; and outstanding debt liabilities. On the asset side the buyer ($i = b$) and the seller ($i = s$) both have cash (Z_i), inventories of x_i units of the traded assets (for which the gains from trade is Δ) and $M_i - x_i$ units of another asset, for which the gain from trade is zero. The values for the two assets are correlated, and for simplicity, we assume perfect correlation; i.e., the value of the second asset is v to both the buyer and the seller. On the liability side, each has total liabilities L_i. Without loss, we assume $x_b = 0$ (it does not matter how buyer's total

[3] From Samuelson (1984), this is the buyer's most preferred trading mechanism in the benchmark case in which the buyer and seller do not care about the market valuation of inventories.

inventory M_b is split between the two perfectly correlated assets), and simply write $x_s = x$.

Both the buyer and the seller are subject to capital constraints, which require that they have enough assets relative to net liabilities. The seller's capital constraint is

$$\alpha_s h(\psi)(M_s - \phi q) + \phi bq + Z_s \geq L_s, \tag{1}$$

where α_s is a constant satisfying $\alpha_s \in (0, 1]$; $h(\psi)$ is the "value" of each unit based on the outcome of the bargaining between the buyer and the seller; $M_s - \phi q$ is the seller's total inventory of assets net of trade; $\phi bq + Z_s$ is the seller's cash, net of trade; and L_s is the seller's liabilities. The value $h(\psi)$ is derived using Bayes' rule, as explained in the next section. If the buyer offers to buy nothing, then $h(\psi) = \frac{1}{2}$, as one can learn nothing about v.

Similar to the seller's, the buyer's capital constraint is

$$\alpha_b h(\psi)(M_b + \phi q) - \phi bq + Z_b \geq L_b, \tag{2}$$

where $\alpha_b \in (0, 1]$.

The seller's utility depends on profits and on whether the capital constraint holds, as follows:

$$U_s = \begin{cases} \phi\pi_s & \text{if } \alpha_s h(\psi)(M_s - \phi q) + \phi bq \geq L_s - Z_s \\ \phi\pi_s - B_s & \text{otherwise.} \end{cases} \tag{3}$$

The buyer's utility is obtained in a similar way:

$$U_b = \begin{cases} \phi\pi_b & \text{if } \alpha_b h(\psi)(M_b + \phi q) - \phi bq \geq L_b - Z_b \\ \phi\pi_b - B_b & \text{otherwise.} \end{cases} \tag{4}$$

We focus on the case in which the capital constraints are satisfied before trading begins. Thus, absent trade, i.e., if the buyer offers to buy nothing, the buyer and seller each obtains a utility of zero. We also assume that B_s and B_b are sufficiently high that the buyer's and seller's first priority is to satisfy their capital constraints. Specifically, we assume that $B_s > x$ and $B_b > x(1+\Delta)$.[4]

Finally, we assume that

Assumption 1 $x < \frac{\alpha_b}{2-\alpha_b} M_b$

This assumption ensures that if $q > 0$, increasing b loosens the buyer's capital constraint. (See subsection 3.3.)

3 Trade, volume, and prices

We focus on three cases: (1) The benchmark case, in which neither the buyer nor the seller cares about the value of their inventories; (2) The case in which only the seller cares about the value of his inventories;[5] (3) The case in which only the buyer cares about the value of his inventories.[6]

3.1 Benchmark case: neither buyer nor seller cares about value of inventories

In the benchmark case, the buyer offers a pair (q, b) to maximize his expected profits subject to $q \leq x$. Since the seller accepts if and only if $v \leq b$, the buyer's expected profit is

$$\pi(q, b) = q \Pr(v \leq b) E(v + \Delta - b | v \leq b). \quad (5)$$

[4]Since $\pi_s \leq x$ and $\pi_b \leq x(1+\Delta)$, the assumption ensures that the disutility (B_i) from violating the capital constraint is always larger than the potential profit.

[5]For instance, the buyer has enough cash relative to his liabilities and the money needed to purchase the asset. A sufficient condition is $Z_b - x \geq L_b$.

[6]For instance, $Z_s \geq L_s$, so the seller's capital constraint is always satisfied.

Since v is uniform on [0,1], we obtain that

$$\pi(q,b) = q\pi(b), \tag{6}$$

where $\pi(b) \equiv b(\Delta - \frac{1}{2}b)$.

The seller accepts the offer with probability b. This is the first term in $\pi(b)$. Conditional on the seller accepting the offer, the expected value of the asset is $\frac{1}{2}b$. Thus, the buyer ends up with an asset whose expected value to him is $\Delta + \frac{1}{2}b$. Since he pays b, the buyer's expected net profit is $\Delta + \frac{1}{2}b - b$. This is the second term in $\pi(b)$.

The buyer's profit-maximizing bid is to buy everything, $q = x$, for a price $b = \Delta$. The probability of trade (b) increases when the gain from trade is higher. The gains from trade are split equally between the buyer and seller; the seller obtains rents because of his private information, and the buyer obtains rents because he is the one making the offer.

Proposition 1 *In the benchmark case, the buyer offers to buy x units at a price per unit Δ. The seller accepts this offer if and only if $v \leq \Delta$.*

3.2 Only seller cares about the value of his inventory

In this subsection we consider the case in which the seller cares about the value of his inventory but the buyer does not. A necessary condition for this case is that the seller's liabilities exceed his cash, $L_s > Z_s$. Since $\pi_s \leq q \leq x < B_s$, and since we assume that the capital constraint is initially satisfied, the seller accepts an offer (q, b) with $q > 0$ if and only if the following two conditions hold: (i) $v \leq b$ and (ii) conditional on accepting the offer, the capital constraint is not violated.

When v is uniform on [0,1], we obtain from Bayes' rule that if the seller accepts an offer with $q > 0$, then $h(\psi) = \frac{1}{2}b$; if the seller rejects, $h(\psi) = \frac{1}{2}(1+b)$. As noted earlier, if $q = 0$, then $h(\psi) = \frac{1}{2}$

Thus, the capital constraint becomes

$$\frac{1}{2}\alpha_s b(M_s - q) + bq \geq L_s - Z_s. \tag{7}$$

This reduces to

$$b \geq \frac{\delta_s}{1 + \beta_s q}, \tag{8}$$

where $\delta_s \equiv \frac{L_s - Z_s}{\frac{1}{2}\alpha_s M_s}$ and $\beta_s \equiv \frac{2-\alpha_s}{\alpha_s M_s}$. Note that δ_s is a measure of the seller's initial leverage; it measures net liabilities relative to assets. For simplicity, we omit the subscript s for the rest of this subsection.

Holding the price b fixed, it is easier to satisfy the seller's capital constraint when q is higher. The reason is that the seller always profits from a trade, so replacing assets with cash adds value to the capital constraint.[7] Consequently, if the buyer finds it worthwhile to bid at all, he bids for the entire quantity available, i.e., $q = x$: bidding for a lower quantity not only lowers the buyer's profits, but it also makes it harder to satisfy the seller's capital constraint and have him accept the offer.

Lemma 1 *The buyer buys everything or nothing: An offer (q, b) with $q \in (0, x)$ that satisfies equation (8) is strictly dominated (from the buyer's perspective) either by (x, b) or by $(0, b)$.*

The buyer's problem reduces to choosing b to maximize his profits $\pi(x, b)$, such that $b \geq \frac{\delta}{1+\beta x}$ so that the seller's capital constraint is satisfied. Since the buyer loses

[7]Formally, since $\alpha_s \in (0, 1]$, it follows that $\beta_s > 0$, and so the the right-hand side in equation (8) is decreasing in q.

money (i.e., $\pi(x, b) < 0$) from bids $b > 2\Delta$, trade is impossible if $\frac{\delta}{1+\beta x} > 2\Delta$. If instead $\frac{\delta}{1+\beta x} \leq 2\Delta$, the buyer bids as close to his benchmark bid of Δ as possible, i.e., $b = \max(\Delta, \frac{\delta}{1+\beta x})$.

Proposition 2 *When only the seller cares about the value of his inventory, trade can occur if and only if $\delta \leq 2\Delta(1 + \beta x)$. In this case, the buyer offers to buy x units at a price per unit* $\max(\Delta, \frac{\delta}{1+\beta x})$*, and the seller accepts if and only if $v \leq b$.*

Figure 1 shows the optimal bid (which equals the probability of trade) as a function of the seller's initial leverage. An increase in leverage first increases trade but then leads to a collapse. Intuitively, when the seller's initial leverage is low, the probability of trade is the same as in the benchmark case because the seller has enough slack to satisfy his capital constraint even though trade reduces the perceived value of his existing assets. When leverage increases, so that the seller has less slack, the buyer must increase his bid to ensure that the seller's capital constraint holds. Finally, if leverage is too high, the market breaks down because any bid that is high enough to satisfy the seller's capital constraint yields negative expected profits to the buyer.

The result that leverage can increase the probability of trade depends on the fact that the buyer has some bargaining power, so that without capital constraints the buyer makes positive profits. This allows him to bid higher and still make positive profits (but less than in the benchmark case) when the capital constraint is binding. We focus on an extreme case in which the buyer has all of the bargaining power, but the nature of the result remains even if the buyer has only some of the bargaining power.

Figure 1 also illustrates that increasing x (the maximum amount that can be sold) has two effects: It increases the region where the probability of trade is the same as in the benchmark case (left region), and it reduces the region in which the market breaks down (right region). Intuitively, when x increases, the non-traded asset becomes a smaller fraction of the seller's balance sheet and it does not matter that its perceived value falls. Increasing Δ has a similar effect because when the gains from trade are higher, the optimal benchmark bid is higher, and it is also easier to satisfy the capital constraint without losing money. Figures 1a and 1b show the probability of trade as a function of Δ and x.

3.3 Only buyer cares about the value of his inventory

In this subsection we consider the case in which the buyer cares about the value of his inventory but the seller does not. A sufficient condition for this case is that the buyer's liabilities exceed his cash, $L_b > Z_b$.

We focus on the case in which $h(\psi)$ is the expected value of $v + \gamma\Delta$, where γ is a constant satisfying $\gamma \in [0, 1]$. At the extreme of $\gamma = 0$, the buyer's capital constraint is based on the value of the asset to the seller, which could also be the value of the asset to a potential lender. At the opposite extreme of $\gamma = 1$, the buyer's capital constraint is based on the value of the asset to the buyer. As before, the expected value of v is based on Bayes' rule. If the seller accepts an offer, $h(\psi) = \frac{1}{2}b + \gamma\Delta$. If the seller rejects an offer, $h(\psi) = \frac{1}{2}(1 + b) + \gamma\Delta$.

The buyer's capital constraint becomes

$$\alpha_b(\frac{1}{2}b + \gamma\Delta)(M_b + q) - bq \geq L_b - Z_b. \tag{9}$$

This reduces to

$$b - \beta_b bq + cq \geq 2\delta_b, \tag{10}$$

where $\delta_b \equiv \frac{L_b - z_b}{\alpha_b M_b} - \gamma\Delta$, $\beta_b \equiv \frac{2-\alpha_b}{\alpha_b M_b}$, and $c \equiv \frac{2\gamma\Delta}{M_b}$. Note that $\frac{\delta_b + \gamma\Delta}{1/2+\gamma\Delta}$ is the buyer's initial leverage (measured as net liabilities over value of assets), and we sometimes refer to δ_b is "leverage." We drop the index b throughout the subsection.

Assumption 1, which reduces to $\beta x < 1$, ensures that increasing the bid always loosens the capital constraint. Increasing b increases the perceived value of existing inventories, which helps loosen the capital constraint, but it also increases the amount the buyer pays for the new asset, which goes against the capital constraint. When the amount of inventories is large relative to the amount for sale, as in Assumption 1, the first effect dominates.

The buyer chooses a bid $(q, b) \in [0, x] \times (0, \infty)$ to maximize $q\pi(b)$. In cases of indifference, we assume the buyer makes the bid associated with the highest quantity q, thereby maximizing social welfare.

Start with the special case $\alpha = \gamma = 1$. Since we must have $b \leq 2\Delta$ (to ensure the buyer does not lose money), the coefficient for q in the capital constraint is nonnegative, and as in Lemma 1, it is optimal to offer either $q = 0$ or $q = x$. Intuitively, when $\alpha = \gamma = 1$, the capital constraint captures the full asset value to the buyer, and since the buyer makes nonnegative profits, buying more assets adds value to the capital constraint. Using a logic similar to that in the seller's case, the capital constraint reduces to $b \geq b(q)$, where

$$b(q) \equiv \frac{2\delta - cq}{1 - \beta q}.$$

Trade can happen only if $b(x) \leq 2\Delta$, which reduces to $\delta \leq \Delta$. If trade happens, the buyer chooses $b = \max(\Delta, b(x))$.

For other parameter values, the buyer may offer to buy less than the maximum amount. This is because buying more units increases profits, but it also tightens the capital constraint.

Proposition 3 *Consider the case in which only the buyer cares about the value of his inventory. If $\alpha = \gamma = 1$, trade can happen if and only if $\delta \leq \Delta$, and if trade* *happens, the buyer offers to buy x units at a price per unit* $\max(\Delta, b(x))$*. Otherwise, trade can happen if and only if* $\delta < \Delta$*, and there exists a number* $\widetilde{\delta} \in (0, \Delta)$*, such that if* $\delta \in [0, \widetilde{\delta}]$*, the buyer offers to buy x units at a price per unit* $\max(\Delta, b(x))$*; and if* $\delta \in (\widetilde{\delta}, \Delta)$ *the buyer offers to buy* $q < x$ *at a price per unit* $b(q)$*, where q is the unique solution to* $\max_{q \in [0,x]} q\pi(b(q))$*. In addition, up to the point at which the market breaks down, the amount offered is continuous and weakly decreasing in δ, and the bid price is continuous and weakly increasing in δ.*

If the buyer's initial "leverage" is low, the benchmark solution is achieved. Otherwise, the buyer can increase the bid price and/or reduce the quantity to ensure that his capital constraint is satisfied. Increasing the bid is effective when the buyer's inventory (M) is high, so leverage is low. In this case, it is optimal to choose $q = x$. However, when the total quantity for sale, x, is high relative to M, it is optimal for the buyer to increase the bid as well as to reduce the quantity; thus, $q < x$. Increasing the bid without reducing the quantity may not be enough to satisfy the capital constraint because the buyer pays more than the asset's borrowing capacity. Even if it is enough, it is suboptimal because bidding a high price for a large amount x substantially reduces profits. By reducing the quantity, the buyer can reduce the price and also satisfy the capital constraint. Finally, if leverage is too high, so that the buyer has only a little slack in his capital constraint, the market freezes because even if the buyer buys just a small quantity, he must bid a high enough price to ensure that the perceived value of his existing assets does not fall too much. But then the buyer expects to make a negative profit.

4 Policy implications

One of our model's main implications is that socially efficient trade can completely break down — "freeze" — when either the buyer or the seller of an asset is both highly leveraged and holds significant inventories of similar assets. This implication is consistent with the ongoing financial market turmoil of the last couple of years. Our analysis has implications for government attempts to defrost markets and for regulatory proposals aimed at improving market functioning.

4.1 Defrosting frozen markets

Consider the case in which only the buyer cares about inventory values and in which trade has completely broken down, i.e., $\Delta \leq \delta_b$ (see Proposition 3).[8] One option open to a government is to offer to buy the seller's asset.

Formally, suppose that the government's valuation of the asset is $v + \Delta_g$, where $\Delta_g < \Delta$, and as before v is private information to the seller. In line with commonly voiced concerns, a general problem with voluntary government purchase schemes is that sellers part with only their worst assets: if the government offers to pay b, the seller only sells if $v \leq b$ (the same as with a private buyer) and makes an expected profit of $\frac{b^2}{2}$.

A central question raised by government purchase schemes is whether they can succeed without taxpayer subsidies (in expectation). Our model has two implications in this respect.

First, observe that because of the rent the seller makes from his informational advantage, a subsidy-free purchase scheme is possible *only if* the asset is worth more

[8] In the special case $\alpha = \gamma = 1$, the market breaks down if $\Delta < \delta_b$, and we can assume below that $\Delta_g \leq \Delta$.

to the government than to the seller, i.e., $\Delta_g > 0$.

Second, even if this demanding condition is satisfied, a subsidy-free purchase scheme imposes a cost on the original potential buyer. Recall that this buyer does not buy the asset himself because doing so violates his capital constraint. However, the same is true when the government buys the asset at unsubsidized terms. To see this, observe that if the purchase is unsubsidized, the market value of the asset after trade is $\frac{b}{2} \leq \Delta_g$. Since $\Delta_g < \Delta \leq \delta_b$, it follows that the buyer's capital constraint is violated after the government purchases the asset (i.e., $b < 2\delta_b$). Moreover, note that a similar issue arises if the government subsidizes a second private buyer to purchase the asset.

Consequently, if either the asset is worth less to the government than to the seller, $\Delta_g \leq 0$, or if the government wishes to avoid hurting the original buyer, then a taxpayer subsidy is required to defrost the frozen market.

4.2 Should regulation mandate some retention of the asset by the seller?

A commonly voiced regulatory proposal is that sellers of assets subject to asymmetric information problems, such as loan originators, should be required to retain some stake in the assets they sell.[9] Our analysis identifies a potential cost to this proposal, namely, that under some circumstances it leads to a market breakdown. To see this, reinterpret the parameter x in our model as stemming from a regulation mandating that the seller retain a fraction $\frac{M_s - x}{M_s}$ of the asset he is selling. From Proposition 2, whenever x is sufficiently low, trade is impossible, because the seller cares too

[9] See, for example, the recent New Democrat Coalition Principles for Financial Regulatory Reform, February 2009.

much about the market's perception of the value of the assets he is forced to retain. Moreover, notice that this case arises more easily when the seller is highly leveraged (measured by δ_s). And, of course, in addition to the possibility of total market breakdown, restricting the amount the seller can trade reduces the expected volume of trade.

The goal that would-be regulators appear to have in mind with this proposal is to reduce moral hazard on the part of asset sellers — for example, to discourage loan originators from making bad loans and/or shirking on monitoring later on. Our analysis does not speak to this issue, and it seems likely that the proposal would have its intended effect in this regard. Our point here is instead to draw attention to a potentially significant cost of this proposal, namely, that it can lead to the breakdown of socially efficient trade.

5 Dynamic model

So far, we have taken traders' leverage, and hence the tightness of their capital constraints, as given. In practice, both emerge endogenously from prior decisions. Accordingly, in this section we extend the single-period model from Subsection 3.3 to a dynamic framework. We start by formulating the problem of a buyer who can trade sequentially with $n > 1$ potential sellers. Then we focus on the case $n = 2$ and fully characterize it. One of the main results is that changes in the buyer's leverage can affect prices and volume, even when there is no change in asset fundamentals.

5.1 n sellers

There are n types of assets, n potential sellers, and one buyer. Seller i sells asset i, and it is assumed that he can sell at most x_i units. The per unit value of asset i is v_i to the seller and $v_i + \Delta_i$ to the buyer, where $v_1, \ldots, v_n$ are iid with a uniform distribution on [0,1]. As before, it is assumed that $\Delta_i \in (0, 1/2)$ for every i.

The buyer trades sequentially with the n sellers. There are at most n rounds, and in each round, the buyer trades with a different seller. (We sometimes refer to rounds as periods.) At the beginning of round i, the buyer makes a take-it-or-leave-it-offer (q_i, b_i) to seller i, who can either accept or reject the offer. When $q_i = 0$, we say that the buyer did not make an offer; and without loss, we assume $b_i = 0$ in this case. If $i < n$, with probability λ, there are no more rounds (so the buyer cannot make additional offers); with probability $1-\lambda$, the buyer moves on to the next round, where he makes an offer to seller $i+1$. Thus, the total number of rounds is $\min(n, N)$, where N has a geometric distribution with a success probability λ. We sometimes denote $m \equiv 1 - \lambda$.

Before trading begins, the buyer has inventories of M_i units of asset i, $i = 1, \ldots, n$, and L (dollars) in net liabilities. (Z is normalized to be zero.) The buyer's capital constraint is

$$\sum_{i=1}^{n} [\alpha_i h(\psi_i)(M_i + \phi_i q_i) - \phi_i b_i q_i] \geq L, \tag{11}$$

where $\phi_i = 1$ if seller i accepts the offer, $\phi_i = 0$ if seller i rejects, and $h(\psi_i) = \frac{1}{2}b + \gamma_i \Delta_i$ is the perceived value of asset i given the trading outcome $\psi_i = (q_i, b_i, \phi_i)$. (Notation is consistent with that in subsection 3.3.) Note that the perceived value of asset i does not depend on the outcome of trade with seller $j \neq i$ because asset values are independent of each other.

If the capital constraint is satisfied, the buyer ends up with $\sum_{i=1}^{\min(n,N)} \pi(q_i, b_i)$.

Otherwise, he ends up with $\sum_{i=1}^{\min(n,N)} \pi(q_i, b_i) - B$. The buyer's problem is to choose a sequence of offers $(q_i, b_i)_{i=1,\ldots,n}$ to maximize his expected utility. Each offer can depend on the history of trades, but because we assumed that sellers are independent, the relevant information can be summarized by the perceived value of the buyer's assets before he makes an offer. As before, in cases of indifference, we assume the buyer makes the bid associated with the highest quantity q, thereby maximizing social welfare.

We make the following parameter assumptions:

Assumption 2 $\alpha_i = \gamma_i = 1$

Assumption 3 $x_i < \min(M_i, (\frac{1-2\Delta_i}{2\Delta_i})M_i)$ *for every* i*.*

Assumption 2 implies that it is optimal to offer $q_i = 0$ or $q_i = x$. (See Proposition 3 and Lemma 2 below.) This assumption reduces the problem's dimension.

Assumption 3 implies that if an offer is accepted, the value of the buyer's assets falls, and that it falls by less when the buyer offers a higher price (see Lemma 3). (Observe that since $\alpha_i = 1$, $x_i < M_i$ and is consistent with Assumption 1 in the single-period case.)

As before, it is assumed that initially the capital constraint is satisfied (that is, $\sum_{i=1}^{n}(\frac{1}{2} + \Delta_i)M_i \geq L$), and that B is large enough so that satisfying the capital constraint is first priority. We also make some parameter assumptions (details below) that imply that the solution to the problem above (where the capital constraint must hold only when trading ends) is identical to the solution to a problem in which the capital constraint must hold after each round.

We solve the problem using dynamic programming. Denote by $V_i(k)$ the highest utility that the buyer can obtain if he can trade only with sellers $i, i+1, \ldots, n$, and if the value of his assets (the left-hand side in the capital constraint) is k. The end condition is

$$W(k) = \begin{cases} 0 & \text{if } k \geq L \\ -B & \text{otherwise.} \end{cases} \tag{12}$$

The value function is

$$V_i(k) = \max_{(q,b)} \pi(q, b) + (1 - \lambda) E_{v_i}[V_{i+1}(k'(k, q, b))] + \lambda E_{v_i}[W(k'(k, q, b))], \tag{13}$$

where $E_{v_i}[\cdot]$ denotes expectation with respect to the random variable v_i, and k' is a random variable (a function of v_i) defined as follows: If $q = 0$, then $k' = k$. Otherwise,

$$k'(k, q, b) = \begin{cases} k + (\Delta_i - \frac{1}{2}b)q - \frac{1}{2}(1 - b)M_i & \text{if } v_i \leq b \\ k + \frac{1}{2}bM_i & \text{if } v_i > b. \end{cases} \tag{14}$$

(We regularly omit the arguments k, q, b.) The first line in (14) is the value of the buyer's assets in the next period if the seller accepts the offer. The buyer obtains q units of the asset, each unit worth $\Delta_i + \frac{1}{2}b$ to him, on average, but he also pays b per unit, so the net change is $q(\Delta_i + \frac{1}{2}b - b)$. Adding new units of the asset also reduces the value of existing inventories from $(\Delta_i + \frac{1}{2})M_i$ to $(\Delta_i + \frac{1}{2}b)M_i$, with a net change of $-\frac{1}{2}(1 - b)M_i$. The second line is the value of the buyer's assets if the seller rejects the offer. A rejected offer increases the value of existing inventories from $(\Delta_i + \frac{1}{2})M_i$ to $(\Delta_i + \frac{1}{2} + \frac{1}{2}b)M_i$, with a net change of $\frac{1}{2}bM_i$.

Denote the argument inside the maximization problem in (13) by $V_i(k, q, b)$; that is, $V_i(k) = \max_{(q,b)} V_i(k, q, b)$. Denote the optimal solution by $(q_i(k), b_i(k))_{i=1,\ldots,n}$. The initial value of the buyer's assets is denoted by k_0.

Lemma 2 *1. $V_i(k)$ increases (weakly) in k.*

2. If $q_i(k) > 0$, then $\pi(q_i, b_i) \geq 0$ (or equivalently, $b_i \leq 2\Delta_i$).

3. Either $q_i(k) = 0$ or $q_i(k) = x_i$ (i.e., with each seller the buyer offers to buy everything or nothing.)

Note that an offer with $q = 0$ is equivalent to the offer $(q, b) = (x, 0)$. Thus, from now on we can assume without loss that all offers have $q = x$.

Denote $k_a \equiv k + (\Delta_i - \frac{1}{2}b)q - \frac{1}{2}(1-b)M_i$. This is the value of the buyer's assets given that an offer is accepted. (Equation (14)).

Lemma 3 *(i) If an offer is accepted, the value of the buyer's assets falls (i.e., $k_a < k$). (ii) The value of assets falls by less when the buyer offers a higher price (i.e., $\frac{\partial k_a}{\partial b} > 0$).*

The next lemma says that the capital constraint must hold at each point in time (i.e., not only after the last round, but after any round).

Lemma 4 *If $k \geq L$, then either $q_i(k) = 0$ (i.e., no offer is made) or $q_i(k) > 0$ and $b_i(k) \geq \frac{2(L-k)+M_i-2\Delta_i x_i}{M_i - x_i}$. (In other words, b is chosen so that $k_a \geq L$.)*

5.2 Two sellers

From now on we focus on the case $n = 2$, assuming that $M_i = M$, $\Delta_i = \Delta$, $x_i = x$ for $i \in \{1, 2\}$. Since the parameters in each round are the same, it is suboptimal to delay offers. In other words, if it is suboptimal to make an offer in the first round, it is also suboptimal to make an offer in the second round.

Denote the bidding strategy by $(b; b_a, b_r)$, where b denotes the offer to the first seller, and b_a, b_r denote the offer to the second seller given that the first seller accepted or rejected, respectively. (From Lemma 2, all offers have $q_i = x$.) Denote the optimal bids by $(b^*; b_a^*, b_r^*)$. It turns out that *whenever* the buyer's first-period bid is rejected, his capital constraint in the second period is sufficiently slack that he can make his unconstrained optimal bid of Δ:

Lemma 5 *If $b^* > 0$, then $b_r^* = \Delta$.*

Using Lemma 5, the problem reduces to finding b and b_a. From equation (14) and Lemma 3, if the buyer offers $b > 0$ and the offer is accepted, the value of his assets becomes

$$k_a \equiv k_0 + [(\Delta - \frac{1}{2}b)x - \frac{1}{2}(1-b)M] < k_0.$$

If the buyer then offers $b_a > 0$, the value of his assets becomes

$$k_{aa} \equiv k_a + [(\Delta - \frac{1}{2}b)x - \frac{1}{2}(1-b)M] < k_a.$$

Thus, if $b, b_a > 0$, the relevant capital constraint is

$$k_{aa} \geq L, \tag{15}$$

and if $b > 0$ but $b_a > 0$, the relevant constraint is

$$k_a \geq L. \tag{16}$$

Since $k_0 = 2M(1/2 + \Delta)$, equation (15) reduces to

$$b + b_a \geq H, \tag{17}$$

and equation (16) reduces to

$$b \geq H - \sigma, \tag{18}$$

where

$$H \equiv \frac{L - 2\Delta(M + x)}{\frac{1}{2}(M - x)}. \tag{19}$$

and $\sigma \equiv \frac{\frac{1}{2}M - \Delta x}{\frac{1}{2}(M-x)} > 2\Delta$. The last inequality follows since $\Delta < 1/2$. Observe that, everything else equal, an increase in H corresponds to an increase in L, the buyer's net liabilities.

The buyer's expected utility reduces to $xV(b, b_a)$, where

$$V(b, b_a) = \pi(b) + mb\pi(b_a) + m(1-b)\pi(\Delta). \tag{20}$$

Since $b, b_a \leq 2\Delta$ (Lemma 2), trade can happen if and only if $2\Delta \geq H - \sigma$, which reduces to $H \leq 2\Delta + \sigma$. If $H \leq 2\Delta$, the benchmark solution is achieved (i.e., $b = b_a = \Delta$). The rest of this section deals with the case $H \in (2\Delta, 2\Delta + \sigma]$. We assume, without loss, that if the buyer is indifferent between bidding once or bidding twice, he bids twice.

Proposition 4 *There exists some $H_2 > 2\Delta$ such that whenever $H \in (2\Delta, H_2)$, the buyer always bids in both periods. Moreover, he bids more than benchmark Δ in the first period, and even more in the second period (i.e., $b_a^* > b^* > \Delta$).*

Proposition 4 captures one aspect of dynamic behavior. If initial leverage is relatively moderate, the buyer has enough slack in his capital constraint to make two rounds of offers. But unless leverage is very low, the buyer still needs to consider his capital constraint, and this leads him to bid more than the benchmark in both periods. If his first bid is accepted, his capital constraint is tightened, forcing him to bid even more in the second period. In other words, the price at which trade occurs rises over time (conditioning on trade occurring in both periods).

If instead initial leverage is high, the buyer has insufficient slack to have two bids accepted:

Proposition 5 *There exists some $H_1 < 4\Delta$ such that whenever $H \in (H_1, 2\Delta + \sigma)$, trade occurs in at most one period. Within this range, if H exceeds (is less than) $\Delta + \sigma$ the first period bid is above (below) the benchmark bid Δ.*

By Proposition 5, once leverage is high, there must be a period in which trade does not occur. In particular, if the buyer's first period offer is accepted, his capital constraint is too tight to make a bid in the second period; i.e., the market freezes.

Proposition 5 also sheds light on the price path leading up to this market freeze. When leverage is very high, the bid before the market freeze is also very high (more than benchmark). In contrast, when leverage is only moderately high, the bid before the market freeze is low. The reason for this last result is that, knowing he has just one trade left, the buyer is tempted to submit a low bid in the first period: if the bid is accepted, he has high profits, and if it is rejected, he still gets to make the benchmark bid Δ in period 2. However, he cannot bid *too little* for fear of violating his capital constraint.

6 Summary

We analyze how existing stocks of assets — inventories — affect the volume and terms of trade. Inventories can either increase or decrease the volume of trade, depending on the leverage of the trading parties. As leverage increases from low to moderate to high levels, prices and the probability of trade first increase, and then markets break down completely.

We also show the following dynamic effect in a two-stage model: The buyer bids aggressively in the first stage and purchases assets, which increases his inventory and leverage. In the second stage, the buyer either bids even more aggressively, paying higher prices for assets, generating more volume and further increasing his inventory and leverage; or else he does not bid at all, i.e., trade breaks down. For some initial levels of leverage, the buyer may reduce his bid in the first round and not bid at all in the second round. This sequence of events matches the broad outlines of market developments in recent years.

7 Appendix

Proof of Lemma 1. Consider an offer (q, b) with $q \in (0, x)$ and $b \geq \frac{\delta}{1+\beta q}$. If $\pi(q, b) > 0$, the buyer can increase his profits by offering (x, b). Since $\beta > 0$, the new offer satisfies (8). If $\pi(q, b) < 0$, the buyer is better off offering $(0, b)$. Finally, if $\pi(q, b) = 0$, the buyer can increase his payoff by offering $(x, b - \varepsilon)$, where $\varepsilon \in (0, b)$. Reducing b increases the buyer's payoff, and if ε is small (and since $\beta > 0$), it is possible to increase q, so that equation (8) is satisfied. **Q.E.D.**

Proof of Proposition 3:

If $\delta \leq \frac{1}{2}(\Delta - \beta\Delta x + cx)$, the capital constraint does not bind and the buyer bids (x, Δ). The remainder of the proof deals with the case $\delta > \frac{1}{2}(\Delta - \beta\Delta x + cx)$.

First, note that the problem has a solution, as follows. For any $q > 0$, the buyer's best choice of b is $\widehat{b}(q) = \min\left\{\max\left\{\Delta, \frac{2\delta - cq}{1-\beta q}\right\}, 2\Delta\right\}$, which is continuous in q. Thus, the buyer's problem can be rewritten as: $\max_{q\in[0,x]} q\pi\left(\widehat{b}(q)\right)$. Since the objective is continuous in q, a solution exists.

Let (q^*, b^*) denote the buyer's optimal offer. Given the assumption $\beta x < 1$, clearly $b^* \geq \Delta$ if $q^* > 0$, since otherwise the buyer could strictly increase profits by increasing b. Moreover, $b^* \leq 2\Delta$ since otherwise the buyer loses money.

Whenever $q^* > 0$, the capital constraint must hold with equality: if instead it is slack, either $q^* < x$, in which case q can be increased, or $q^* = x$ and $b^* > \Delta$, in which case b can be decreased.

The capital constraint reduces to $b \geq b(q)$, where $b(q) = \frac{2\delta - cq}{1-\beta q}$. Observe that $b'(q) = \frac{2\beta\delta - c}{(1-\beta q)^2}$. In addition, $\alpha \in (0, 1]$ implies that $\beta > 0$.

If $\delta \leq \frac{c}{2\beta}$, increasing q helps relax (weakly) the capital constraint. In this case, the optimal quantity q^* cannot lie in $(0, x)$, as in Lemma 1. Thus, either $(q, b) = (x, b(x))$, or $q = 0$.

Moreover, since $\delta \leq \frac{c}{2\beta}$ the bid $(q, b) = (x, b(x))$ generates positive profits, and hence is preferred, as follows. We must show $b(x) \leq 2\Delta$, or equivalently, $\delta \leq \frac{1}{2}cx + \Delta(1 - \beta x)$. So it suffices to show $\frac{c}{2\beta} \leq \frac{1}{2}cx + \Delta(1 - \beta x)$. This last inequality is equivalent to $\gamma \leq \frac{2-\alpha}{\alpha}$, which is indeed satisfied since $\gamma \leq 1$ and $\alpha \leq 1$.

Note that $\frac{c}{2\beta} = \frac{\gamma\alpha}{2-\alpha}\Delta \leq \Delta$, with strict inequality unless $\alpha = \gamma = 1$. Hence $\delta \leq \frac{c}{2\beta}$ only if $\delta \leq \Delta$.

If instead $\delta > \frac{c}{2\beta}$, increasing q makes the capital constraint tighter. In this case, trade can happen if and only if $b(0) < 2\Delta$, which reduces to $\delta < \Delta$. The problem becomes: Find $q \in [0, x]$ that maximizes $f(q) = q\pi(b(q))$. We can add the constraint $b(q) \geq \Delta$, without loss. Thus, we can maximize $f(q)$ over the compact set $[0, x] \cap Q$, where $Q \equiv \{q : b(q) \geq \Delta\}$. Since $f(q)$ is strictly concave over this set, there exists a unique solution. To see why f is concave, note that

$$f'(q) = \pi(b(q)) + q\pi'(b(q))b'(q)$$

$$f''(q) = 2\pi'(b(q))b'(q) + q\pi''(b(q))b'(q) + q\pi'(b(q))b''(q)$$

Since $b(q) \geq \Delta$, we know that $\pi'(b(q)) \leq 0$. In addition, $\pi''(b(q)) < 0$, $b'(q) > 0$, and since $\beta > 0$, it follows that $b''(q) > 0$. Thus, $f''(q) < 0$.

If $f'(x) \geq 0$, the solution is $q^* = x$. Otherwise, we need to rule out the other corner, $f'(q^*) = 0$, $q^* < x$, and it follows from the implicit function theorem that $\frac{\partial q^*}{\partial \delta} < 0$. (Observe that

$$\frac{\partial f'(q)}{\partial \delta} = \pi'(b(q))\frac{\partial b(q)}{\partial \delta} + q\pi''(b(q))\frac{\partial b(q)}{\partial \delta}b'(q) + q\pi'(b(q))\frac{\partial b'(q)}{\partial \delta} < 0,$$

since $\frac{\partial b(q)}{\partial \delta} = \frac{2}{1-\beta q} > 0$, and $\frac{\partial b'(q)}{\partial \delta} = \frac{2\beta}{(1-\beta q)^2} > 0$.)

It remains to show that $\frac{\partial b^*}{\partial \delta} > 0$ when $q < x$. First, consider the case $\beta b^* - c \leq 0$. Fix $\tilde{\delta} > \delta$, and let the associated optimal bid be $\left(\tilde{b}^*, \tilde{q}^*\right)$. From above, $\tilde{q}^* < q^*$. But then $b^* - \tilde{q}^*(\beta b^* - c) - 2\tilde{\delta} < b^* - q^*(\beta b^* - c) - 2\delta = 0$; i.e., the bid $(b^*, \tilde{q}^*)$ violates the capital constraint. So $\tilde{b}^* > b^*$.

Second, consider the case $\beta b^* - c > 0$. The buyer's problem can be written as $\max_b q(b)\pi(b)$, where $q(b) = \frac{b-2\delta}{\beta b - c}$ is the value of q that sets the capital constraint to equality for a given b. To show the result use the implicit function theorem. Specifically, let $g(b) = q(b)\pi(b)$; observe that $q'(b) = \frac{2\beta\delta - c}{(\beta b-c)^2} > 0$, $q''(b) < 0$,

$$g'(b) = q'(b)\pi(b) + q(b)\pi'(b),$$

$$g''(b) = q''(b)\pi(b) + 2q'(b)\pi'(b) + q(b)\pi''(b) < 0,$$

and

$$\frac{\partial g'(b)}{\partial \delta} = \frac{2\beta}{(\beta b - c)^2}\pi(b) - \frac{2}{\beta b - c}\pi'(b) > 0.$$

Q.E.D.

Proof of Lemma 2:

1. The proof is by backward induction. Since k' and $W(k)$ increase in k, it follows that $V_n(k)$ increases in k. Suppose the claim is true for $V_{i+1}(k)$, and consider the round i maximization problem. Fix a value of k, say, $\check{k}$, and let $\left(\check{q}, \check{b}\right)$ be the utility-maximizing choice associated with this $\check{k}$. Consider any $\tilde{k} > \check{k}$. Since k', V_{i+1} and W are weakly increasing in k,

$$\begin{aligned} V_i\left(\tilde{k}\right) &\geq \pi(\check{q}, \check{b}) + (1-\lambda)E_{v_i}[V_{i+1}(k'\left(\tilde{k}, \check{q}, \check{b}\right))] + \lambda E_{v_i}[W(k'\left(\tilde{k}, \check{q}, \check{b}\right))] \\ &\geq \pi(\check{q}, \check{b}) + (1-\lambda)E_{v_i}[V_{i+1}(k'\left(\check{k}, \check{q}, \check{b}\right))] + \lambda E_{v_i}[W(k'\left(\check{k}, \check{q}, \check{b}\right))] = V_i\left(\check{k}\right). \end{aligned}$$

2. The proof is by contradiction. Suppose $b_i > 2\Delta_i$. Then $\pi_i(q_i, b_i) < 0$ (from benchmark case), and we can increase $\pi_i(q_i, b_i)$ by lowering q_i. Lowering q_i not only makes $\pi(q_i, b_i)$ less negative, but it also increases k' with probability b_i (note that if $b_i > 2\Delta_i$, the coefficient of q_i in (14), $\Delta_i - \frac{1}{2}b_i$, is negative) and keeps it unchanged with probability $1 - b_i$. Therefore, $V_{i+1}(k_{i+1})$ and $W(k_{i+1})$ increase (using part 1), and overall, $V_i(k)$ increases in contradiction to the optimality of $q_i(k)$.

3. If $q_i > 0$, then $b_i \leq 2\Delta_i$ (from part 2), and $\pi(q_i, b_i) \geq 0$. Increasing q_i to x_i, and keeping b_i unchanged, increases $V_i(k)$. In particular, $\pi(q_i, b_i)$ increases, as well as k' and $W(k')$. From part 1 it follows that $V_{i+1}(k')$ increases. In addition, the transition probabilities b_i and $1 - b_i$ remain unchanged. **Q.E.D.**

Proof of Lemma 3: The first part follows from the assumption $x_i < (\frac{1-2\Delta_i}{2\Delta_i})M_i$. To see why, note that since $b_i \geq 0$, it follows that $(\Delta_i - \frac{1}{2}b)q \leq \Delta_i x_i$, and from part 2 in Lemma 2 it follows that $\frac{1}{2}(1-b)M_i \geq \frac{1}{2}(1-2\Delta_i)M_i$. Finally, note that the assumption is equivalent to $\Delta_i x_i < \frac{1}{2}(1-2\Delta_i)M_i$. The second part follows from the assumption $q \leq x_i < M_i$. **Q.E.D.**

Proof of Lemma 4: We need to show that it is possible to choose B that is high enough so that the claim is true. The proof is by contradiction. Suppose $b_i(k) < \frac{2(L-k)+M_i-2\Delta_i x_i}{M_i - x_i}$. Let $\bar{b}$ be some number such that $\bar{b} \in (0, \Delta)$. If $b_i \geq \bar{b}$, the loss from choosing $b_i(k)$ compared to choosing $b_i = \frac{2(L-k)+M_i-2\Delta_i x_i}{M_i - x_i}$ is at least $b_i(k)\lambda B \geq \bar{b}\lambda B$. If λB is large enough the potential loss is larger than the expected benefit. If $b_i < \bar{b}$, one can improve $V_i(k)$ by choosing $b_i = \Delta$. The benefit is at least $\pi(\Delta) - \pi(b_i) \geq \frac{1}{2}\Delta^2 - \bar{b}(\Delta - \frac{1}{2}\bar{b})$. The cost is that the offer is less likely to be rejected, so we are less likely to start the next period with higher wealth. An upper bound on the cost is $(1-\lambda)(1-b)[V_{i+1}(k + \frac{1}{2}bM) - V_{i+1}(k)] \leq \frac{1}{2}\Delta^2 \sum_{j=i+1}^{n}(1-\lambda)^j$ (by induction). If λ is large enough and/or $\bar{b}$ is small enough, the cost is less than the benefit. **Q.E.D.**

Proof of Lemma 5:

Since choosing $b_r = \Delta$ maximizes profits in the second round, it is enough to show that the capital constraint is not violated after choosing $b_r = \Delta$. From equation (14), after the first offer is rejected, the value of the buyer's assets is $k_0 + \frac{1}{2}Mb$; if the buyer subsequently offers $b_r = \Delta$, and this offer is accepted, the value of the buyer's assets becomes

$$\begin{aligned} k_{ra} &\equiv k_0 + \frac{1}{2}Mb + (\Delta - \frac{1}{2}\Delta)x - \frac{1}{2}(1-\Delta)M \\ &= k_0 + \frac{1}{2}Mb + \frac{1}{2}\Delta(M-x) + (\Delta x - \frac{1}{2}M). \end{aligned}$$

We also know from Lemma 4 and equation (14), that if $b > 0$, we must have

$$k_0 + \frac{1}{2}Mb - \frac{1}{2}xb + (\Delta x - \frac{1}{2}M) \geq L. \tag{21}$$

Since $M > x$, equation (14) implies that $k_{ra} \geq L$. **Q.E.D**.

Proof of Proposition 4:

Assume $H \in (2\Delta, 2\Delta + \sigma]$. If the buyer wishes to make sure he can bid in the second period even if his first period bid is rejected, his capital constraint binds (since $H > 2\Delta$), and his expected profit is

$$R_2(H) = \max_{b \in [\Delta, 2\Delta]} \pi(b) + mb\pi(H-b) + m(1-b)\pi(\Delta).$$

If instead the buyer follows a strategy of bidding only once, his capital constraint may or may not bind, and his expected profit is

$$R_1(H) = \max_{b \in [\Delta, 2\Delta] \text{ s.t. } b \geq H - \sigma} \pi(b) + m(1-b)\pi(\Delta).$$

Both R_1 and R_2 are continuous. Observe that $R_2(2\Delta) > R_1(2\Delta)$, and $R_2(4\Delta) < R_1(4\Delta)$. Hence, there exists some $H_2 > 2\Delta$ such that $R_2(H) > R_1(H)$ whenever $H < H_2$. For such values of H, the buyer chooses his first-period bid to make sure he can bid in the second period even if his first-period bid is rejected.

To establish the claimed ordering of the bids, we need to show that the b that maximizes $V(b, H-b)$ lies between Δ and $\frac{H}{2}$ (so that $b < H - b$). Note first that $V(b, H-b)$ is a cubic in b, and the coefficient on the cubic term is negative. In the case under consideration, $H > 2\Delta$. Given this, the result follows if

$$\left.\frac{d}{db}V(b, H-b)\right|_{b=\Delta} > 0 > \left.\frac{d}{db}V(b, H-b)\right|_{b=H/2}.$$

Evaluating,

$$\frac{d}{db}V(b, H-b) = \pi'(b) + m\pi(H-b) - mb\pi'(H-b) - m\pi(\Delta).$$

Since π is a quadratic with its extremum at Δ, for any $\tilde{b}$, $\pi\left(\tilde{b}\right) = \pi\left(\Delta\right) + \frac{1}{2}\left(\tilde{b} - \Delta\right)\pi'\left(\tilde{b}\right)$. Given this,

$$\begin{aligned}\left.\frac{d}{db}V(b, H-b)\right|_{b=\Delta} &= m\left(\pi(H-\Delta) - \Delta\pi'(H-\Delta) - \pi(\Delta)\right) \\ &= m\left(\frac{1}{2}\left(H - \Delta - \Delta\right) - \Delta\right)\pi'\left(H-\Delta\right),\end{aligned}$$

which is positive, since $H - \Delta > \Delta$ implies that $\pi'\left(H-\Delta\right) < 0$, and $H < 4\Delta$. Similarly,

$$\begin{aligned}\left.\frac{d}{db}V(b, H-b)\right|_{b=H/2} &= \left(1 - m\frac{H}{2}\right)\pi'\left(\frac{H}{2}\right) + m\left(\pi\left(\frac{H}{2}\right) - \pi\left(\Delta\right)\right) \\ &= \left(1 - m\frac{H}{2} + \frac{m}{2}\left(\frac{H}{2} - \Delta\right)\right)\pi'\left(\frac{H}{2}\right) \\ &= \left(1 - m\frac{H}{4} - \frac{m}{2}\Delta\right)\pi'\left(\frac{H}{2}\right),\end{aligned}$$

which is negative, since $\pi'\left(H/2\right) < 0$, and $1 > m\frac{H}{4} + m\frac{\Delta}{2}$, since $m < 1$ and $H/4 < \Delta < 1/2$. **Q.E.D.**

Proof of Proposition 5: The existence of H_1 follows from the proof of Proposition 4. To find the bid in the first round, note that if $b_a = 0$, the optimal b maximizes $V(b, 0)$, subject to $b \geq H - \sigma$. Observe that $V(b, 0)$ is quadratic. The unconstrained solution is $\widetilde{b} = \Delta - \frac{1}{2}m\Delta^2$. If $\widetilde{b} \geq H - \sigma$, then $b^* = \widetilde{b}$. Otherwise, we want to get as close as possible to $\widetilde{b}$ while still satisfying the constraint. Thus, $b^* = \max(\widetilde{b}, H - \sigma)$. **Q.E.D.**

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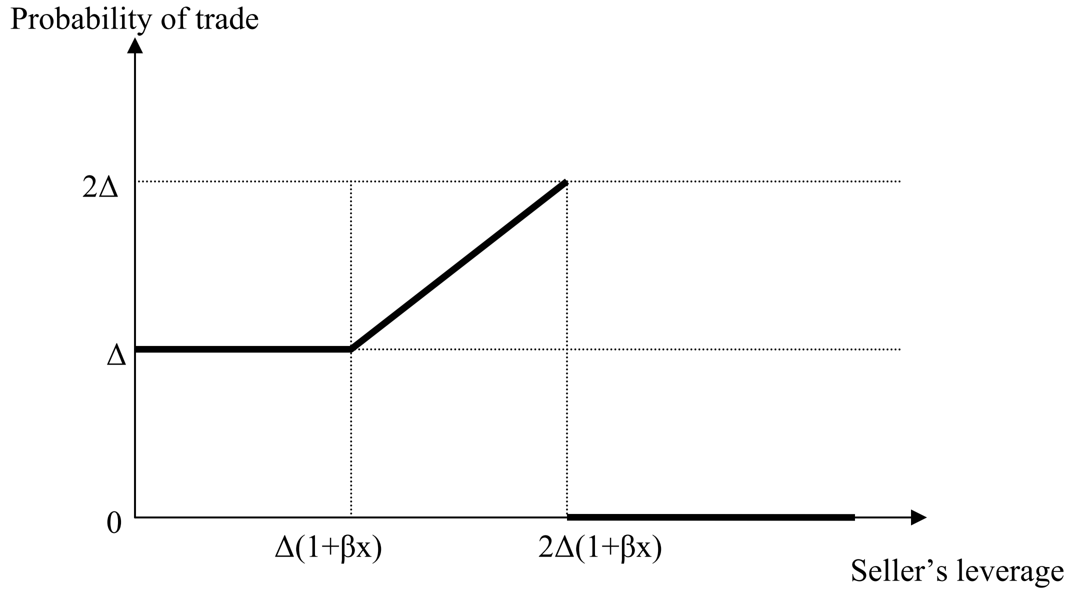

Figure 1: Probability of trade as a function of seller's leverage, when the seller cares about the value of his inventory.

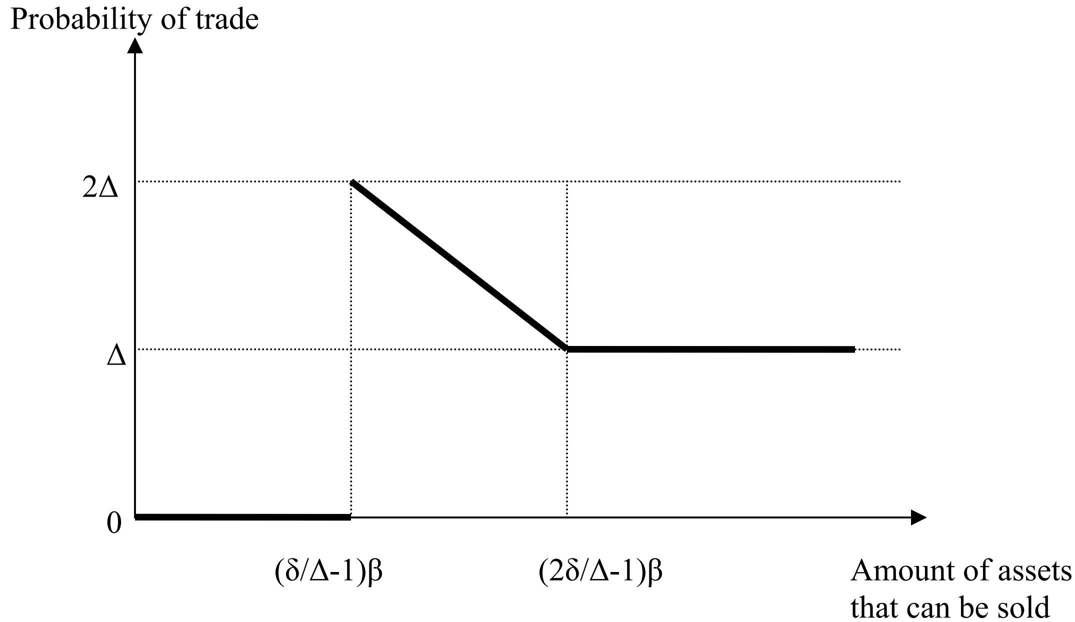

Figure 1a: Probability of trade as a function of the amount of assets that can be sold, when the seller cares about the value of his inventory.

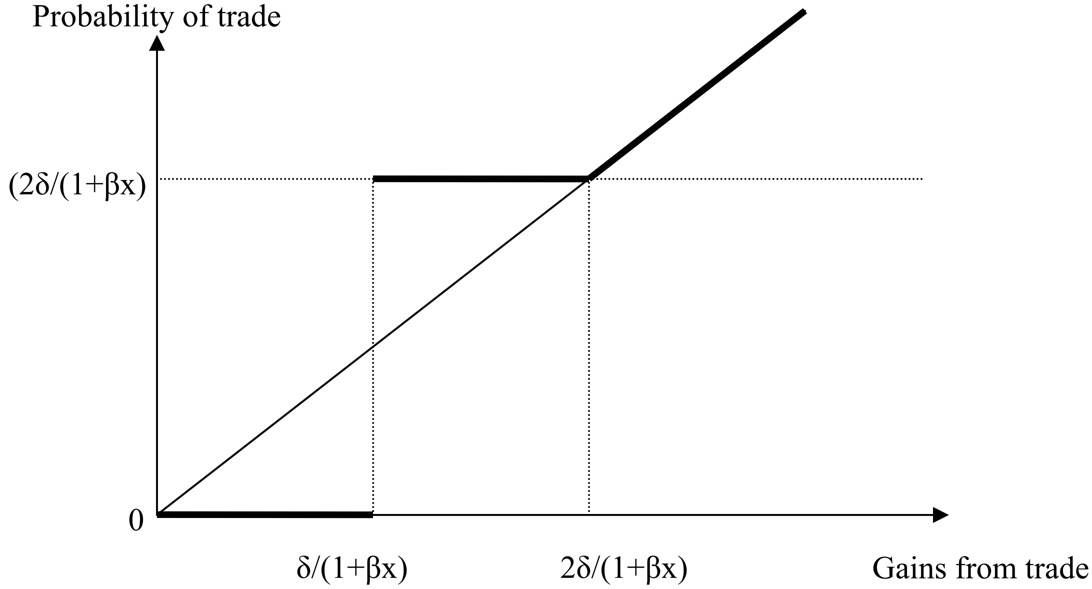

Figure 1b: Probability of trade as a function of the gains from trade, when the seller cares about the value of his inventory.