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FORECLOSURES AND HOUSE PRICE DYNAMICS:
A QUANTITATIVE ANALYSIS OF THE MORTGAGE CRISIS
AND THE FORECLOSURE PREVENTION POLICY

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Abstract

We construct a quantitative equilibrium model of the housing market in which an unanticipated increase in the supply of housing triggers default mortgages via its effect on house prices. The decline in house prices creates an incentive to increase the consumption of housing space, but leverage makes it costly for homeowners to sell their homes and buy bigger ones (they must absorb large capital losses). Instead, leveraged households find it advantageous to default and rent housing space. Since renters demand less housing space than homeowners, foreclosures are a negative force affecting house prices. We explore the possible effects of the government's foreclosure prevention policy in our model. We find that the policy can temporarily reduce foreclosures and shore up house prices.

Key Words: Leverage, Foreclosures, Mortgage Crisis, Loan Modifications, Foreclosure Prevention Policy

1 Introduction

The goal of this project is to construct a quantitative equilibrium model of the housing market in which a shock to fundamentals triggers default via its effect on house prices. Our analysis is motivated by the recent experience of declining house prices and rising defaults on mortgages in the United States (and elsewhere) and the US government's attempt to stabilize the housing market by instituting a program of loan modifications to prevent foreclosures. We use our quantitative model to explore the possible impacts of the foreclosure prevention policy.

The key elements of our model environment are as follows. We imagine a city with an exogenously given but potentially time-varying stock of housing. The city is populated by a continuum of infinitely lived residents subject to uninsured idiosyncratic shocks to earnings. Residents can buy consumption goods, save in the form of a risk-free savings account with an exogenously given and constant interest rate, and purchase or rent their housing space. If a resident chooses to purchase housing space, he or she can borrow funds from a mortgage market to do so. Residents must pay tax on earnings and interest income. As in the US federal tax code, interest payments on mortgages can be deducted from taxable income and implicit rental income from owner-occupancy is excluded from taxable income.

We model the mortgage market as competitive, with every borrower being charged an interest rate that exactly reflects the borrower's objective probability of default. A key determinant of this default risk is the level of down payment made on the loan - a level that we assume is freely chosen by the borrower. The endogeneity of the down payment will imply that when the risk of default is perceived to be low, the down payment chosen will be low as well and homeowners will be highly leveraged.

We locate the proximate cause of the mortgage crisis in "overbuilding": an increase in the supply of housing that fails to be matched by an increase in demand at the going price. In equilibrium, the price of housing must fall to absorb the increase in supply. Our model shows how leverage and the tax benefits of homeownership can conspire to turn a simple adjustment of demand and supply into a crisis. Leverage implies that when the price of housing falls, selling one's home imposes a capital loss. Instead of selling their homes and buying a bigger one, the capital loss of a sale induces homeowners to give up their homes in foreclosure and rent for a while. But renting does not have the same tax benefits as homeownership, so the erstwhile homeowners tend to rent less space than they previously owned. In this fashion, foreclosures and renting become forces contributing to *less* housing demand, not more. Consequently, foreclosures end up exacerbating the downward adjustment of house prices.

With regard to foreclosure prevention policy, we find that the policy being pursued by the government can

be effective in decreasing foreclosures and thereby shoring up house prices. However, our analysis also reveals that the expiration of the policy may be followed by another crash in housing prices and another spike in foreclosures. The reason is simply that, by delaying foreclosures, the policy delays adjustment to the higher supply of housing. Thus, once the policy expires, the adjustment starts anew and home prices fall again.

2 Literature Review

There is a rapidly growing literature on the financial crisis and the mortgage crisis. Much of this literature is empirical. To the best of our knowledge, ours is the first quantitative-theoretic investigation of the interaction between the jump in foreclosures (the mortgage crisis) on the one hand and the crash in house prices (the housing crisis) on the other. We are also the first to analyze the implications of the foreclosure prevention policy on house prices in a fully articulated equilibrium model of the housing and mortgage markets.

We build on a small but growing quantitative-theoretic literature on the housing sector. We have in mind studies such as Gervais (2002), Nakajima (2005), Jeske and Krueger (2005), Guler (2008), Ríos-Rull and Sánchez-Marcos (2008), Corbae and Quintin (2009), Iacoviello and Pavan (2009), and Chambers, Garriga, and Schlagenhaut (2009).

In terms of modeling the mortgage market, we follow Jeske and Krueger (2005) and Guler (2008) in assuming that each loan is competitively priced to reflect the objective probability of default on the loan (individualized or risk-based pricing) and in assuming that a borrower controls the objective probability of default by her choice of down payment.

In terms of modeling the housing market, we follow Gervais (2002) and Chambers, Garriga, and Schlagenhaut (2009) in conceiving of the housing market as a market for homogeneous housing *space*, as opposed to houses. This conceptualization is attractive in that there is *one* housing market price – the price per square foot of housing space – to be determined each period.

The role of aggregate shocks in a model of mortgage lending and housing has been analyzed in Ríos-Rull and Sánchez-Marcos (2008). Their analysis is explicitly stochastic, and they employ a forecasting equation (for future house prices) of the type developed in Collusi (2006) to get around the high dimensionality of the state vector. In contrast, our analysis is not explicitly stochastic – we analyze the impact of an unanticipated shock to housing supply – but we work out the perfect foresight equilibrium path following the shock. This approach is congenial for us because we can easily handle the dynamics introduced by the existence of a terminal date for the foreclosure prevention policy.

3 Environment

We will study the housing equilibrium in a representative city. Time is discrete and indexed by $t = 0, 1, 2, \dots$. The city has a fluctuating stock of housing space $H(t) > 0$.

3.1 People

There is a fixed continuum of individuals. Individuals derive utility from the consumption of a homogeneous consumption good and the service flow from housing space. We permit the service flow from owner-occupied housing space to be potentially higher than the service flow from rented housing space. Let c_t denote consumption of the homogeneous good in period t , let h_t denote the consumption of housing space in period t , and let e_t be an indicator variable that is 1 if the person owns the housing space h_t and 0 otherwise. Then an individual values the consumption stream $c = \{c_0, c_1, c_2, \dots\}$, $h = \{h_0, h_1, h_2, \dots\}$ and $e = \{e_0, e_1, e_2, \dots\}$ according to:

$$U(c, h, e) = \sum_{t=0}^{\infty} \beta^t [u(c_t, h_t) + \nu \cdot e_t], 0 < \beta < 1, \nu \geq 0. \quad (1)$$

We assume that

$$u(c_t, h_t, 0) = [c_t^{1-\theta} h_t^\theta]^{1-\gamma} / (1-\gamma). \quad (2)$$

Note that housing space used by a resident must be either fully owned or fully rented.

Each resident independently draws an earnings level w from a finite-state Markov process with non-negative positive support $W \subset R_+$. The probability that $w_{t+1} = w'$ given $w_t = w$ is $F(w', w)$.

3.2 Market Arrangement

The homogeneous consumption/endowment good is the *numeraire* good. There are four markets in this economy.

1. There is a market for owner-occupied housing in which the price per unit of housing space in period t is $P(t)$. An individual who buys k' units of housing pays a purchase price of $P(t) \cdot k'$. We assume that owner-occupied housing comes in discrete sizes given by the finite set K .
2. Second, there is a market for rental housing in which the rent per unit of housing space in period t is $R(t)$. An individual who rents h units of housing space in period t pays $R(t) \cdot h$ as rent.

3. Third, there is a market for risk-free deposits. The interest rate on deposits is $r > 0$. An individual who saves a units receives $a(1 + r)$ next period.
4. Finally, there is a market for mortgages where individuals can borrow to fully or partially fund the purchase of a house. For tractability we assume that all mortgages are perpetuities, in the sense that each mortgagee pays some agreed to amount x each period unless the mortgagee defaults or the mortgage is paid off. In the case of default, the financial intermediary gets ownership of h . In the case where the mortgage is paid off, the lender receives the present value of the promised payment x discounted at the risk-free rate r . Because of the possibility of default, the (unit) price q of a mortgage depends on the amount of housing pledged as collateral h , the payment amount x promised in perpetuity, the individual's post-purchase savings y , his current earnings w , and the time period t . An individual who takes out a mortgage in period t with perpetuity payment x receives $q(x, h, y, w, t) \cdot x$ in the current period.

3.3 Government Sector

There is also a government sector that levies taxes on income. For simplicity, we assume that government consumption of goods does not have to provide any benefits to households. The amount of taxes g to be paid by an individual is modeled after the US tax code. An individual's tax liability depends on the individual's taxable income, which is calculated as the sum of earnings and interest income minus the greater of (i) mortgage payment x or (ii) the standard deduction s . That is:

$$g(w, a, x) = \tau(w + ra - \max[x, s]) \cdot (w + ra - \max[x, s]). \quad (3)$$

Here the tax rate, or bracket, $\tau(\cdot)$ is weakly increasing in taxable income.

3.4 Financial Intermediaries

Financial intermediaries take in deposits, sell mortgages, and own the housing space rented by people. All intermediaries can borrow or lend funds in a world credit market at a given risk-free interest rate $\bar{r} > 0$. We will assume that there is one representative risk-neutral intermediary that takes all prices as given.

4 Decision Problems

4.1 People

The state variables for individuals are k, x, a, w and whether the individual is excluded from the mortgage market or not. Consider first the decision problem of an individual who does not own housing and who is not excluded from the mortgage market due to prior default. For this person $k = 0$ and $x = 0$ and the person may choose to rent or she may choose to buy. Since the person is not excluded from the mortgage market she can borrow to purchase a house. In this case, if the individual chooses to purchase, she solves:

$$M_1(0, 0, a, w; t) = \max_{c \geq 0, k' \in K, x \geq 0, y \geq 0} \{u(c, k') + \nu + \beta E_{w'|w} V(k', x, y, w', t + 1)\}$$

$$c = w - g(w, a, x) + a(1 + r) - y - P(t)[1 + \chi_B]k' + q(x, k', y, w, t) \cdot x$$

$$q(x, k', y, w, t) \cdot x \leq P(t) \cdot k'$$

If the individual is excluded from the mortgage market due to a prior default but chooses to purchase a house, she solves:

$$M_1^X(0, 0, a, w; S) = \max_{c \geq 0, k' \in K, y \geq 0} \{U(c, k') + \nu + \beta E_{w'|w} \lambda V(k', 0, y, w', t + 1) + (1 - \lambda)X(k', 0, y, w', t + 1)\}$$

$$c = w - g(w, a, x) + a(1 + r) - y - P(t)[1 + \chi_B]k'$$

If the individual is not excluded from the mortgage market and chooses to rent, she solves

$$M_0(0, 0, a, w; t) = \max_{c \geq 0, y \geq 0, h \geq 0} \{U(c, h) + \beta E_{w'|w} V(0, 0, y, w', t + 1)\}$$

$$c = w - g(w, a, 0) + a(1 + r) - y - R(t) \cdot h$$

If the individual is excluded from the mortgage market and chooses to rent, she solves

$$M_0^X(0, 0, a, w; S) = \max_{c \geq 0, y \geq 0, h \geq 0} \{U(c, h) + \beta E_{w'|w} \lambda V(0, 0, y, w', t + 1) + (1 - \lambda)X(0, 0, y, w', t + 1)\}$$

$$c = w - g(w, a, 0) + a(1 + r) - y - R(S) \cdot h$$

Then

$$V(0, 0, a, w; t) = \max \{M_1(0, 0, a, w; t), M_0(0, 0, a, w; t)\}$$

And

$$X(0, 0, a, w; t) = \max \{M_1^X(0, 0, a, w; t), M_0^X(0, 0, a, w; t)\}$$

Consider next the decision problem of an individual who owns a house and has an outstanding mortgage. The household may choose to keep the current house, sell it, or default on the mortgage. If she chooses to keep the house, she solves:

$$K_1(k, x, a, w; t) = \max_{c \geq 0, y \geq 0} \{U(c, k) + \nu + \beta E_{w'|w} V(k, x, y, w', t + 1)\}$$

$$c = w - g(w, a, x) + a(1 + r) - y - x$$

If she chooses to sell, she solves:

$$K_0(k, x, a, w; t) = \max_{c \geq 0, h \geq 0, y \geq 0} \{U(c, h) + \beta E_{w'|w} V(0, 0, y, w', t + 1)\}$$

$$c = w - g(w, a, 0) + a(1 + r) - y + P(t)[1 - \chi_S]k - (1 + 1/\bar{r}) \cdot x - R(t) \cdot h$$

Here χ_S is the cost of selling a house and is some percentage of the sale price of the house $P(t)$. Selling the house requires the individual to buy back the perpetuity x at the risk-free interest rate.¹ If the household chooses to default on the mortgage, she solves:

$$K_D(k, x, a, w; S) = \max_{c \geq 0, h \geq 0, y \geq 0} \{U(c, h) + \beta E_{w'|w} [\lambda V(0, 0, y, w', t + 1) + (1 - \lambda)X(0, 0, a', w', t + 1)]\}$$

$$c = w - g(w, a, 0) + a(1 + r) - y - \chi_D - R(t) \cdot h$$

Foreclosure results in the individual losing the house as well as the mortgage and being excluded from the mortgage market for some random length of time.

An individual who is a homeowner but excluded from the mortgage market can either keep her house or sell it. If she chooses to keep her house, she solves:

$$K_1^X(k, 0, a, w; t) = \max_{c \geq 0, a' \geq 0} \{U(c, k) + \nu + \beta E_{w'|w} \lambda V(k, 0, y, w', t + 1) + (1 - \lambda)X(k, 0, y, w', t + 1)\}$$

$$c = w - g(w, a, x) + a(1 + r) - y$$

If she chooses to sell, she solves:

$$K_0^X(k, 0, a, w; t) = \max_{c \geq 0, h \geq 0, y \geq 0} \{U(c, h) + \beta E_{w'|w} \lambda V(0, 0, y, w', t + 1) + (1 - \lambda)X(0, 0, a', w', t + 1)\}$$

$$c = w - g(w, a, 0) + a(1 + r) - y + P(t)[1 - \chi_S]k - R(t) \cdot h$$

Then for $k > 0$ and $x \geq 0$

$$V(k, x, a, w, t) = \max \{K_0(k, x, a, w; t), K_1(k, x, a, w; t), K_D(k, x, a, w; t)\}$$

¹Ideally we should require the individual to buy back the perpetuity at $\tilde{q} \cdot x$ where \tilde{q} is the price the individual paid when he issued the perpetuity. But implementing this would require expanding the state vector.

And for $k > 0$

$$X(k, 0, a, w, t) = \max \{K_0^X(k, 0, a, w; t), K_1^X(k, 0, a, w; t)\}$$

For non-excluded households, let $c(k, x, a, w, t)$, $k'(k, x, a, w, t)$, $x'(k, x, a, w, t)$, $h(k, x, a, w, t)$, $a(k, x, a, w, t)$ and $d(k, x, a, w, t)$ denote the optimal decision rules for consumption, size of owned housing, size of mortgage payments, housing services, financial assets and default ($d = 1$ indicates default and $d = 0$ indicates no default). Also, let $s(k, x, a, w, t) = 1$ indicate the case where the individual sells the house, i.e., $s(k, x, a, w, t) = 1$ if and only if $k > 0$, $d(k, x, a, w, t) = 0$, and $k'(k, x, a, w, t) = 0$. And let $c^X(k, 0, a, w, t)$, $k'^X(k, 0, a, w, t)$, $h^X(k, 0, a, w, t)$, $a^X(k, x, a, w, t)$ and $s^X(k, 0, a, w, t)$ be the decisions of households that are excluded from the mortgage market.

4.2 Financial Intermediaries

The representative financial intermediary accepts deposits, buys mortgages, and buys and rents housing. We will assume that the intermediary chooses how much to engage in these activities on the basis of the expected return in each of these activities. Denote the intermediary's expected net rate return (profits) on a deposit of size a by $\pi(a)$, the net expected return on a mortgage with characteristics k' , x' , y , w and t by $\pi(k', x', y, w, t)$ and the net expected return on the purchase of h units of housing in period t by $\pi(h, t)$. Correspondingly, let $m(a)$, $m(k', x', y, w, t)$ and $m(h, t)$ denote the measure (more precisely, the density or pdf) of such contracts acquired by the financial intermediary. Then, the financial intermediary's decision problem is:

$$\begin{aligned} \Pi(t) = & \max_{\{m(a), m(k', x', y, w, t), m(h, t)\}} \\ & \left\{ \int \pi(a)m(da) + \int \pi(k', x', y, w, t)m(dk', dx', dy, dw, t) + \int \pi(h, t)m(dh, t) \right\} \end{aligned}$$

For there to be a (bounded) solution to this problem the net expected returns on each type of asset must be non-positive. For deposits this requires $\pi(a) = a - [a(1+r)]/(1+\bar{r}) \leq 0$ for all a . This requirement reduces to

$$(1+r) \geq (1+\bar{r}). \quad (4)$$

For housing, this requires that $\pi(h, t) = R(t) \cdot h + [P(t+1) \cdot (1-\delta)h]/(1+\bar{r}) - P(t)h \leq 0$ for all h (financial intermediaries do not pay any cost for selling houses), where δ is the depreciation on rental properties. This requirement reduces to

$$P(t) \geq R(t) + [P(t+1)(1-\delta)/(1+\bar{r})] \quad (5)$$

For mortgages, the expression for net return is more involved. When the intermediary acquires a mortgage it gives up $q(k', x, y, w, t) \cdot x$ in goods. Next period, if the individual defaults, the intermediary receives $P(t+1)[1 - \chi_{LD}]k'$ where χ_{LD} is the cost of foreclosure to the intermediary; if the individual sells the property, the intermediary receives $x \cdot (1 + 1/\bar{r})$; and if she neither defaults nor sells, the intermediary receives x plus the value of the continuing mortgage, which is then given by $q(k', x, a(k', x, y, w', t+1), w', t+1) \cdot x$. Recalling the definition of $d(k, x, a, w, t)$ and $s(k, x, a, w, t)$, the requirement that the expected net return from a mortgage $\pi(k', x', y, w, t)$ be non-positive becomes

$$\begin{aligned}
q(k', x', y, w, t) x' &\geq (1+r)^{-1} \times \\
E_{w'|w} \{ &d(k', x', y, w', t+1) P(t+1) [1 - \chi_{LD}] k' + s(k', x', y, w', t+1) [x' (1 + 1/\bar{r})] + \\
(1 - d(k', x', y, w', t+1)) &(1 - s(k', x', y, w', t+1)) q(k', x', a(k', x', y, w', t+1), w', t+1) \cdot x' \}
\end{aligned} \tag{6}$$

5 Equilibrium

An equilibrium consists of a sequence of rental pricing function $\{R^*(t)\}$, a sequence of housing price functions $\{P^*(t)\}$, a deposit interest rate r^* , a sequence of mortgage price functions $\{q^*(k, x, a, w, t)\}$, a sequence of distributions $\mu^*(k, x, a, w, t)$ of people over the state space, and a sequence of decision rules for excluded and non-excluded individuals such that:

1. The decision rules are optimal given r^* , $R^*(t)$, $P^*(t)$, $q^*(t)$.
2. The net returns (4)-(6) are zero.
3. Demand for housing equals supply, that is, $\int h^*(k, x, a, w, t) \mu^*(dk, dx, da, dw; t) = H(t)$
4. The sequence of distributions $\mu^*(k, x, a, w, t)$ is implied by the decision rules.

6 Parameter Selection and Calibration

Turning first to the Markov process for earnings, we assume that log earnings follow an AR1 process:

$$\ln(w_t) = \rho \ln(w_{t-1}) + \epsilon_t \tag{7}$$

Several studies have estimated this process for the US using PSID earnings data. Estimates of ρ and the standard deviation of ϵ (σ_ϵ) vary across studies. We follow Jeske and Krueger (2005) and set $\rho = 0.95$ and $\sigma_\epsilon = 0.129$. This AR1 process is then approximated by a 5-state Markov chain.

Setting aside the parameters of the income tax schedule, our model economy has 11 other parameters. These include 4 preference parameters ($\beta, \theta, \gamma, \nu$), 3 parameters related to housing transactions (χ_S, χ_B, δ), 3 related to costs of foreclosures ($\lambda, \chi_D, \chi_{LD}$), and 1 related to financial markets (r).

For some parameters we use information from microstudies to restrict their values. Among preference parameters, we set $\theta = 0.24$ based on the findings in Davis and Ortalo-Magne (2009) that across US cities with quite different real rental prices of housing, expenditure on rent as percentage of household income is remarkably close to 0.24. Among the housing transactions parameters, we appeal to Gruber and Martin (2003) to set $\chi_B = 0.025$ and $\chi_S = 0.075$, and among the foreclosure cost parameters, we set $\chi_{LD} = 0.22$ on the basis of the findings reported in Pennington-Cross (2006). The real interest rate is set at 4 percent.

The remaining 6 parameters are determined by computing the steady state of the model economy for a given housing stock and setting these parameters so as to match certain data targets. In this version of the paper, we have set both ν and χ_D to 0. The remaining 4 parameters are chosen to come as close as possible to the (i) the average homeownership rate over the period 1993-2003, (ii) average fraction of mortgages going into foreclosure over the period 1993-2003, (iii) the ratio of mean housing wealth to mean financial wealth in 1998 (the midpoint of the period 1993-2003), and (iv) the ratio of the median household net worth to median household income in 1998. We chose to target averages over the period 1993-2003 so as not to distort our parameter choices by booms and busts in the residential real estate markets. These targets are displayed in Table 1.

Table 1

Statistic	Target	Source
homeowners/total households	0.664	Orzechowski and Sapielli (2003) Table A, p. 5
foreclosure initiation/outstanding mortgages	0.0056	Mortgage Bankers Assoc., Guler (2008)
mean housing wealth/mean household net worth	0.337	O. & S. Table A, p. 5
median hh net worth/median hh income	1.28	O. & S. Table C, p. 8 & Table H-8 (Cen.B.)

The homeownership rate may be computed as the fraction of households that are homeowners or as the fraction of housing units that are owner-occupied. The number reported in the table is the former calculation using the Survey of Consumer Finances for 1998 and reported in Orzechowski and Sapielli (2003). The latter calculation, done for 1993-2003 using Census Bureau data, yields 0.662. The foreclosure initiation rate comes from the Mortgage Bankers Association and is the average number of new foreclosures of conventional fixed

rate mortgages initiated each year as a fraction of outstanding conventional fixed rate mortgages for the years 1998-2003 (the data for earlier years are not readily available). The figure is comparable to the one reported in Guler (2008). The ratio of mean housing wealth to mean household net worth is from the 1998 SCF and reported in Orzechowski and Sapielli. These authors also report the median household net worth for the same survey as \$49,932 and the Census Bureau reports the median household income for 1998 as \$38,885. The table reports the ratio of these two figures.

To complete the calibration of the model, we need to specify the tax schedule $\tau(\cdot)$ and the standard deduction s . The tax schedule is chosen to match the tax table for 1998. In our model, people are viewed as individuals (this seems consistent with the earnings data). But we will take individuals to be married. Hence, the tax table we use is the tax table for married filing separately. According to the Census Bureau, male median income of year-round full-time workers age 25 and older in 1998 was \$37,906 and that of females was \$27,956. We use the average of these two numbers, which is \$32,931, as the median income of an individual filing for taxes. Normalizing the tax brackets for 1998 by this estimate of median income, we obtain the following tax schedule $\tau(\cdot)$:

Table 2

Tax Brackets	Tax Rate
0 - 0.64	0.15
0.64 - 1.55	0.28
1.55 - 2.37	0.31
2.37-4.23	0.36
4.23 -	0.396.

And normalizing the standard deduction for a married person filing separately by median income gives $s = 0.1116$.²

Table 3 displays the steady-state model statistics and the parameter values that generate them. With the exception of the net worth-to-income ratio, which is too high in the model relative to the data, the steady-state statistics come reasonably close to their target values.³ The implied preference parameters, β and γ

²Our tax schedule overstates the taxes paid by low-income people because we ignore the earned income tax credit. However, what is important for our study is the tax benefit of owner-occupied housing and this benefit is *not* affected by the earned income tax credit. This is because the credit is calculated on a person's adjusted gross income and, therefore, does not depend on whether the household rents or owns.

³A lower β would bring the net worth-to-income ratio closer to the target (people will save less), but it also raises the probability of foreclosure above its target value because the punishment for foreclosure (exclusion from the mortgage market) becomes less effective.

are not implausible.⁴ The value of λ implies that following a foreclosure a person is allowed back into the mortgage market in 3.33 years, on average. The value of δ , which is the depreciation of rental properties, is rather small.⁵

Table 3

Statistic	Target	Model	Parameters
homeowners/total households	0.664	0.645	$\gamma = 2$
foreclosure initiation/outstanding mortgages	0.0056	0.0049	$\lambda = 0.3$
mean housing wealth/mean household net worth	0.337	0.314	$\delta = 0.002$
median hh net worth/median hh income	1.28	1.48	$\beta = 0.93$

It is useful to discuss some of the more salient features of the steady state. First, one may ask, why do people in our model have an incentive to purchase homes? In our calibration, we have assumed that there is no direct benefit to homeownership ($\nu = 0$) and the “rental externality” (the fact that rental properties depreciate but owner-occupied units do not) is negligible ($\delta = .002$). Furthermore, in computing the solution to our model, we assume that households that rent can choose their h freely, while those that purchase their homes must choose from a finite set of house sizes, i.e., a from a finite set of k (and, therefore, h) values. Thus, homeownership affords no direct utility benefit, involves somewhat less flexibility (in terms of house sizes) than renting, and involves the payment of significant transactions costs in its acquisition. Nevertheless, the majority of people choose to purchase their homes. This is because of the tax benefit of homeownership: the implicit rental income from ownership is not counted as part of income and therefore not taxed. This exemption means that people – especially those in the higher tax brackets – have a strong incentive to purchase their homes. The deductibility of mortgage interest payments merely encourages these households to *borrow* to finance the purchase of their homes as opposed to paying for the purchase from accumulated assets.

Second, one may ask, why are there foreclosures in the steady state at all? In the steady state the value of a house does not change over time, and if the house was purchased with some down payment, the owner can recover his or her home equity by selling the house. So, why default? Two conditions must be satisfied. First, it must be the case that the owner’s home equity is less than the transactions costs of selling the house; otherwise selling will always dominate default. Formally, the condition is

$$[1 - \chi_S]p^*k - x(1 + 1/r) < 0 \tag{8}$$

⁴The discount factor β may seem too low relative to other quantitative macro models. But the possibility of default makes borrowing expensive, and to get people to borrow in spite of high interest rates requires the discount factor to be low. Low discount factors also pervade models of unsecured consumer credit as well as models of sovereign debt and default.

⁵The value needs to be small in order to make renting sufficiently attractive relative to owning.

where p^* is the steady-state price of a unit of housing space. In the steady state, 36 percent of homeowners satisfy this condition. But this condition is not sufficient. It must also be the case that the punishment from default is low. Recall that the punishment for default is exclusion from the mortgage market for several years. This punishment is ineffective for an individual whose current and future income is expected to be low, as such a person does not gain from homeownership and would prefer to rent anyway. Therefore, for default to happen the homeowner must have low home equity *and* low income. Since homeowners tend to be relatively high earners, the probability of default on a mortgage is pretty low because earnings are a persistent process and it is unlikely that a homeowner would get an income shock bad enough to make him want to default.

The third salient feature of our model is that homeowners choose their down payment and this choice is regulated by the interest rate on the resulting loan. As noted above, a low down payment is a pre-condition for default, and therefore, the implicit interest rate on such loans will reflect the probability of default. A borrower can lower the cost of the mortgage by lowering his or her promised payments x without changing the value of k , which is equivalent to putting down a higher down payment since the down payment is simply $p^*k - q(k, x, y, w)x$ and the term $q(k, x, y, w)x$ is generally increasing in x . In the steady state, the average ratio of $q(k, x, y, w)x$ to p^*k (the loan-to-price ratio) is 0.9127. So, the average down payment is little under 10 percent. How does this compare to the data? Between 1993-2003, according to data from the Mortgage Bankers Association, the average loan-to-price ratio on all *new* mortgages was 0.7748. Note, however, that in actuality households take out subsequent loans against their home equity as well (something that does not happen in our model). Taking these loans into account will likely reduce the discrepancy between model predictions and data.

A third salient feature of our model is that there is a positive transactions volume in the housing market. Individuals buy and sell houses. These transactions occur because household earnings change over time and households may wish to either up-size or down-size their housing space as a result. In our steady state, 0.64 percent of homeowners sell each year and 2.06 percent of renters buy each year. The former figure is an order of magnitude lower than the comparable figure in the data. According to data published by the National Association of Realtors, during 1993-2003 the number of single-family home sales as a percentage of owner-occupied units was 4.27 percent, on average. However, our model contains one motivation for sale – a change in income that makes the existing house size inappropriate (too large or small). In reality, there are other motivations for sale, such as relocation and life events (marriage, divorce, and change in family size). Thus, it would be surprising if the model came close to matching actual transactions volumes.

Finally, our model is consistent with the observation that owner-occupants consume more housing space, on average, than renters. In our equilibrium the per capita housing space of renters is 54 percent of per capita

housing space of owner occupants. There are two reasons for this. First, because high earners choose to buy houses, the housing space of owner- occupants will be greater than that of renters. In addition, the tax benefit of owner-occupancy makes owner-occupants consume more housing than renters; i.e., if an owner-occupant were to rent, he or she would rent *less* space. In the steady steady, if homeowners were to rent, their demand for space would be lower by 12 percent. This fact will play an important role in generating a mortgage crisis in our model.

7 The Mortgage Crisis

We model the mortgage crisis as stemming from an unexpected increase in the stock of housing. In particular, we assume that the economy (city) enters the initial period, period 0, in the steady state corresponding to some housing stock \bar{H} . Then, unexpectedly, the housing stock in period 1 rises to $(1 + \phi)\bar{H}$, $\phi > 0$, and stays at that level forever. We assume that the new supply of housing, $\phi\bar{H}$, shows up as an additional endowment in the hands of the financial intermediary sector/business sector. Our analysis will focus on what happens in steady state as a result of this increase and what happens along the transition to the new steady state.

We conduct our analysis under the assumption that $\phi = 0.05$. This might seem high if our reference point is the whole economy.⁶ But by all accounts the housing bubble affected some parts of the country more than others. Thus, the areas that have seen the most foreclosures and the steepest drop in housing prices presumably experienced more overbuilding than the economy as a whole.

Table 4 displays the steady-state effects of the permanent increase in the supply of housing. The price of housing space declines by 4 percent and there is about a 2-percentage-point increase in the homeowner-ship rate and about a 1-percentage-point decrease in the loan-to-price ratio. There is an increase in the transactions volume (selling and buying) and a decrease in the foreclosure rate.

⁶According to some analysts, long-run demographic and housing trends imply a normal demand for new housing units of about 1.5 million units each year. By this metric, about 2 million “extra” housing units were built during 2001-2006. The average stock of occupied housing units during this period was about 106 million units, which would imply an “oversupply” of less than 2 percent.

Table 4

Variable	Pre-shock SS	Post-shock SS
Price per unit of housing space	1	0.96
Average LTV (%)	91.27	90.31
% of homeowners	64.55	66.39
% of homeowners selling	0.64	0.74
% of homeowners foreclosing	0.49	0.41
% of renters buying	2.06	2.32

These changes accord with intuition. The (consumption good) price of housing space must decline in order for the economy to absorb the additional supply of housing. A lower price of housing, in turn, encourages more buying and selling because transactions costs (as a percentage of earnings) decline with the decline in the price of housing. To understand why the foreclosure rate and the loan-to-price ratio decline consider, again, the necessary condition for default, namely, the condition in (8). Observe that for a mortgage for which this condition holds, a proportionate decline in both p^* and x will cause the difference to become a *smaller* negative number. In other words, the loss in wealth involved in selling the house as opposed to defaulting on it would be smaller. All else remaining the same, this decline in the real cost of selling will lower the probability of default and the cost of the mortgage. Individuals respond to this decline in the cost of borrowing by increasing x (borrowing more) and this accounts for the increase in the loan-to-price ratio. Evidently, however, borrowers (home purchasers) do not increase x to completely neutralize the decrease in the probability of default so that, in equilibrium, there is a small decline in the foreclosure rate as well.

Compared to these long-run (steady-state) changes in the housing market, the on-impact effects (i.e., the effects in the period of the period of the shock) of the increase in housing supply are dramatically different. Table 5 displays the on-impact effects along with the old and new steady states.

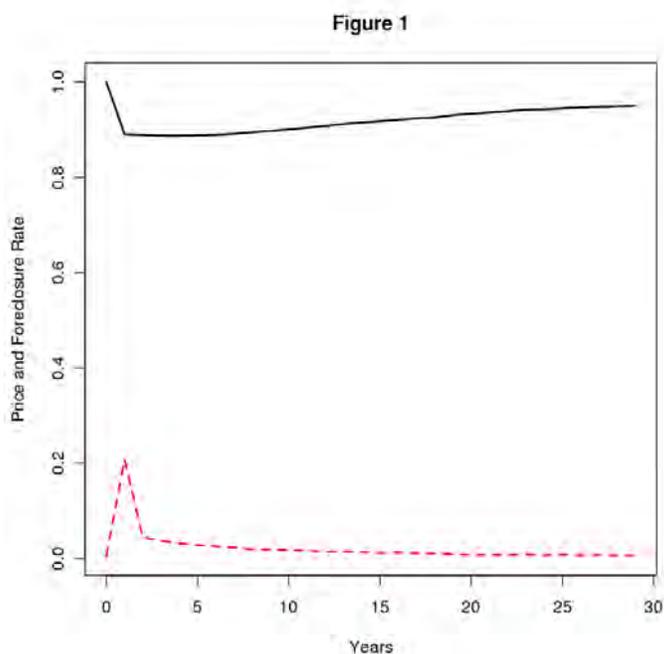
Table 5

Variable	Pre-shock SS	On-Impact Effects	Post-shock SS
Price per unit of housing space	1	0.89	0.96
Average loan-to-price (%)	91.27	90.67	90.31
% of homeowners	64.55	52.01	66.39
% of homeowners selling	0.64	0	0.74
% of homeowners foreclosing	0.49	20.81	0.41
% of renters buying	2.06	1.50	2.32

Observe that on impact (i.e., in period 1) the price of housing declines by 11 percent and the foreclosure rate spikes up to almost 21 percent. Correspondingly, the homeownership rate drops to 52 percent and there is

a complete cessation of selling activity. Renters continue to purchase homes but the rate of buying on their part drops as well. Indeed, the total measure of individuals participating in the housing market, either as sellers or as buyers, drops by 37.18 percent. The only statistic that is not very different from its steady-state value is the loan-to-price ratio, which stays close to 90 percent.

Figure 1 displays what happens to the rate of foreclosures (the red dashed line) and the price of housing after the initial period. The foreclosure rate falls back to around 4 percent and then declines monotonically toward its new steady-state value. The price of housing rises monotonically toward its new steady-state value as well. In both cases, the rise is very gradual and convergence takes many years.



What explains these results? In the long run, both owner-occupants and renters must increase their consumption of housing space so as to absorb the increased supply of housing. Indeed, in the new steady state the per capita consumption of housing space of both renters and owner-occupants goes up by 4 percent. But the adjustment by owner-occupants is slowed down by the transactions costs of selling and buying a house. Thus, in the aftermath of the shock, the burden of adjustment falls disproportionately on the rental market and rents decline below their new steady-state value. The decline in rents causes the price of housing space to decline, since rents and prices must satisfy (5) with equality: if prices do not fall but rents do, the business sector would have an incentive to sell the property it owns, since holding on to it and renting it out would be less remunerative.

The decline in price, in turn, causes the necessary condition for default, condition (8), to hold for a greater number of borrowers. In fact, the condition holds for 99 percent of the homeowners. So, the decline in price sets the precondition for foreclosure. But why do borrowers choose to default? In the steady state the trigger for default is a fall in income, which makes servicing the mortgage payments relatively onerous. But that is not the motivation for default at the start of the crisis. In the period of the shock, borrowers default because they wish to take advantage of the low rental price of housing space: defaulters give up the tax benefits of homeownership in favor of the gain from consuming housing space at a (much) reduced rent. In subsequent periods, the trigger for defaults reverts to bad income shocks.

But exit from homeownership via foreclosure makes achieving equilibrium in the housing market more difficult. The reason was alluded to at the end of the previous section: all else remaining the same, a renter desires *less* housing space than an owner-occupant. Thus, as the ranks of owner-occupants shrink there is a tendency for the demand for housing space to shrink as well. Consequently, rents have to fall further to equate demand with supply. Of course, this in turn causes further declines in price, which induces more foreclosures. The sharp on-impact effects on price and foreclosures is the result of this “circular dynamic.”

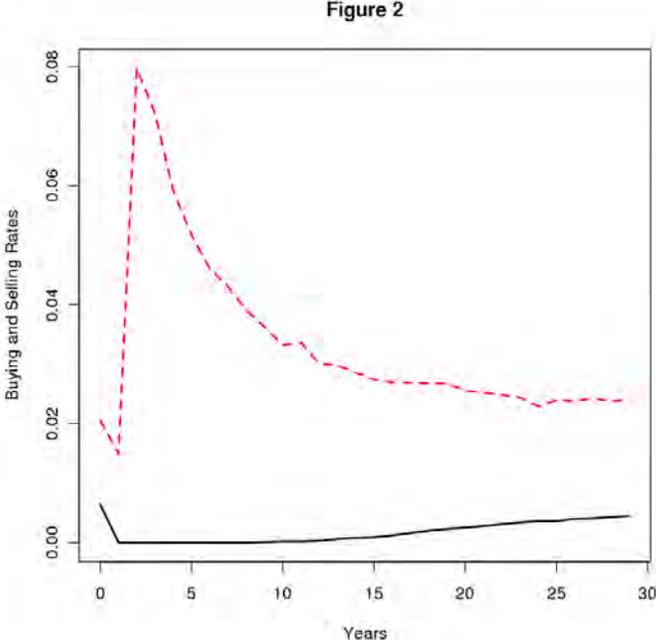
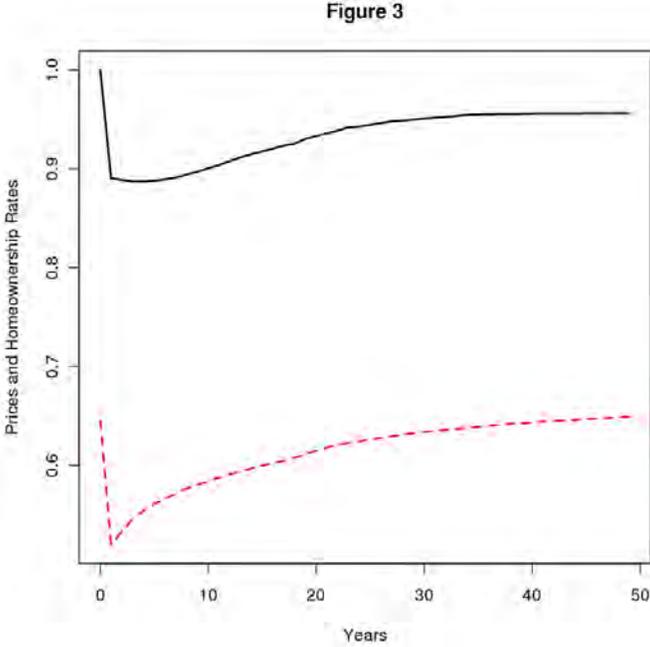


Figure 2 displays what happens to transactions volumes. The decline in price makes it very costly to sell a house because the seller has to absorb the capital loss. As a result, there are no sales in the period of the shock. Indeed, there are no sales for several years following the shock. There is also a decline in the

percentage of renters buying houses in the period of the shock. This happens because in the period of the shock, the ranks of renters are swelled with defaulters and defaulters (or sellers, for that matter) cannot purchase homes rightaway. In the period following the shock, there is a spike in the percentage of renters purchasing homes; this comes from defaulters who are lucky enough to be let back into the housing market. But the percentage of defaulters moving into homeownership declines over time as the measure of defaulters in the renter pool declines over time. Renters who are not defaulters do not normally wish to purchase homes because they are low earners. These people could be tempted to purchase homes in the expectation of capital gains, but the rise in price is too gradual for that to be profitable.

Why is the rise in price so gradual? Following the period of the shock, the spike in new buyers is roughly offset by new defaulters. Thus, the percentage of homeowners in period 2 is only slightly larger than in period 1. Because the foreclosure rate stays elevated for many years into the future, there is a continuing partial offset to the measure of new buyers coming into the housing market. The net result of these dynamics is that the homeownership rate rises back to its new steady state level very gradually (Figure 3). Because homeowners tend to occupy more space than renters (all else the same), the increase in the demand for housing space rises only gradually over time and, therefore, so does the price of housing.



Our assumption about the competitive pricing of loans has a bearing on the speed with which homeownership recovers after the shock. Our pricing equation (6) assumes that each loan is priced *independently* of any

other loan - including loans that were made prior to the crisis and which suffer from elevated rates of default after the shock. But we can also imagine a situation where these “legacy” loans - to borrow the term often used to refer to poorly performing assets in the current crisis - contaminate the pricing of new mortgages. This will happen if new mortgages have to be made by existing financial intermediaries who must use “legacy loans” (not new mortgages) as collateral for borrowed funds. Since “legacy” loans suffer from much higher rates of default, the implicit interest rate at which the financial intermediary can borrow will be much higher following the shock. This will make new mortgages more expensive and reduce the number of renters who move back into homeownership and thereby lead to an even more gradual increase in house price than shown in Figure 3.

8 The Foreclosure Prevention Policy

This section analyzes the impact of the “loan modification” program implemented by the US government since March 2009. The key features of this policy are as follows. To be eligible for a loan modification, the mortgagor must have current income than his income at the time of the origination of the loan (the “hardship” requirement). Furthermore, the ratio of mortgage payments to income must be higher than 31 percent (there are some asset restrictions as well that we ignore). If a mortgage is eligible, the government promises to reimburse investors half the cost of reducing the mortgage payment-to-income ratio from 38 percent to 31 percent. The modification alters only the monthly payments; if the mortgagor were to sell the property, the outstanding loan amount has to be paid. Also, only one modification per mortgage is permitted (i.e., a modified mortgage cannot be modified again), and the modification ends in 6 years after which the mortgage payment is gradually raised back to the original amount. The option to modify an eligible mortgage is available for only 4 years.

To map this policy into our model we simplify matters somewhat. First, we assume that the percentages 31 and 38 apply to *earnings* as opposed to income. Since earnings are exogenous in our model this simplification reduces the computational burden significantly. Define $x^m(w) = 0.31w$ and let $x^{\bar{m}}(w) = 0.38w$. Then, on a modified mortgage with original payment x and current earnings w , the intermediary receives

$$x^m(w, x) = x^m(w) + \frac{1}{2} \{ \min\{x, x^{\bar{m}}(w)\} - x^m(w) \} \quad (9)$$

and the mortgagor pays $x^m(w)$. The difference is made up by the government. In the event the mortgagor sells the property, he must repay $x/(1 + 1/r)$ to the intermediary. Thus, there is no loan forgiveness.⁷ We assume that the modification program is offered in the period of the shock only (i.e., in period 1 only) and

⁷The policy permits forgiveness of interest but this is an option, not a requirement for modification

that each agreed-to modification ends in 5 years. Under these rules, the period $t = 1, 2, \dots, 5$ market value of a mortgage with modified payment x^m and original payment x is given by the recursion

$$\begin{aligned}
q_x(k, x^m, y, w, t) x^m &= (1+r)^{-1} \times E_{w'|w} \{ \\
d_x(k, x^m, y, w', t+1) [1 - \chi_D] P(t+1) k &+ s_x(k, x^m, y, w', t+1) [x(1+1/r)] + \\
(1 - d_x(k, x^m, y, w', t+1)) (1 - s_x(k, x^m, y, w', t+1)) &\times \\
[I_{\{t+1 \leq 5\}} (1 + q_x(k, x^m, a_x(k, x^m, y, w', t+1), w', t+1)) x^m &+ I_{\{t+1=6\}} (x^m + q(k, x, a(k, x, y, w', t+1), w', t+1)) x] \}
\end{aligned} \tag{10}$$

where $d_x(k, x^m, y, w, t)$ is the period t default decision rule of a borrower household with a modified mortgage, given that the unmodified mortgage has payment x . Similarly, $s_x()$ and $a_x()$ are the corresponding sale and savings decision rules under modification. Note that the original payment amount x continues to be state variable along with the modified amount x^m . For this reason, the decision rules are indexed by x .

Since $x^m < x$ for an eligible mortgage, the mortgager will always prefer a modification. But for a modification to actually happen, the lender (intermediary) must not be worse off. Thus an eligible mortgage will be modified if and only if

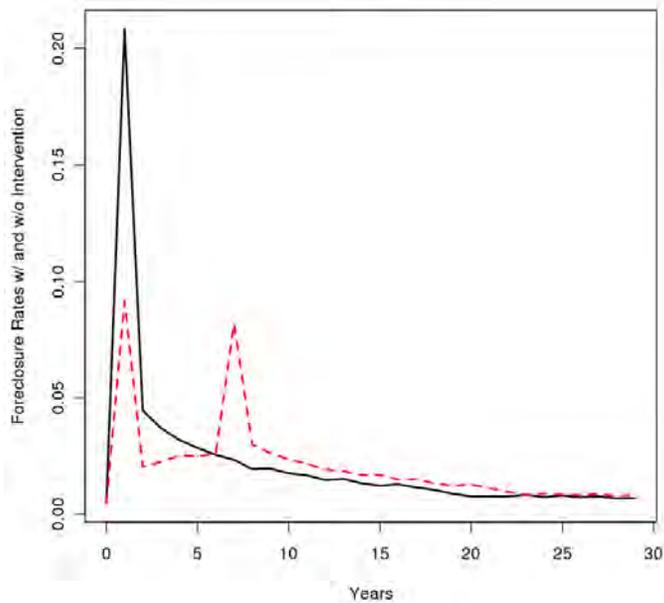
$$q_x(k, x^m, a_x(k, x^m, a, w, t), w, t) x^m > q(k, x, a(k, x, a, w, t), w, t) x. \tag{11}$$

Finally, we assume that borrowers currently in the highest earnings state are the only ones ineligible for a loan modification. These people have the same or higher earnings compared to the period in which they took out the loan and therefore fail the hardship test. Borrowers below the top earnings state may fail the hardship test as well, but we take these borrowers to be eligible for the program. It turns out that enlarging the scope of the program in this way does not affect the results because the mortgages that do ultimately get modified are almost always the ones that pass the hardship test as well.

Figure 4 displays the foreclosure path with and without the policy in place (in all ensuing figures with the exception of Figure 6, the (red) dotted line corresponds to the path under intervention and the (black) solid line the path under no intervention). The policy is quite effective in reducing foreclosures, but only temporarily so. In the period of the shock the foreclosure rate rises to a little over 9 percent, which is less than half of the spike in foreclosures in the absence of the policy.⁸ The foreclosure rate remains below the non-interventionist rate for as long as the modifications remain effective (year 6 in the Figure).

⁸The effectiveness of the foreclosure prevention policy seems consistent with the micro-evidence on the gains from foreclosure prevention, which are typically large (see, for instance, Family Housing Fund (1998)). In practice, the foreclosure prevention program has not been as successful as one would expect. Reasons for its lack of success are unclear but may have something to do with the increase in coordination costs among lenders when the loan is securitized and sold to *many* investors. See Piskorski, Seru and Vig (2009) for a discussion and Foote *et al.* (2009) for a dissenting view.

Figure 4

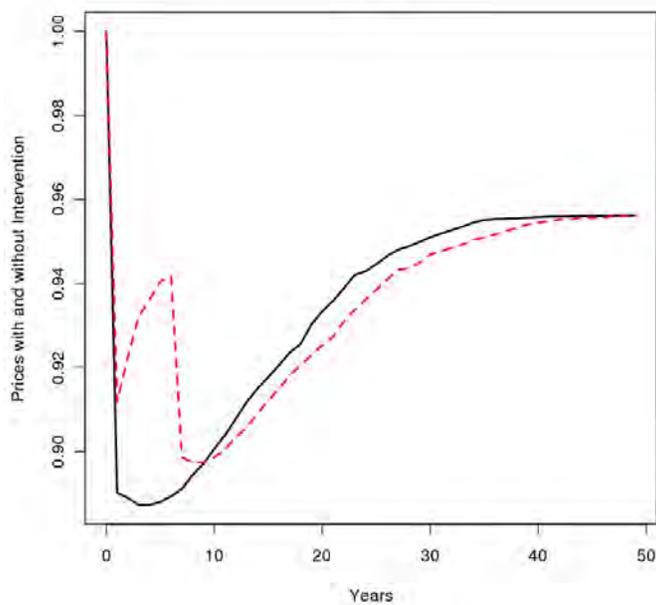


Once the modifications expire, there is a second spike in the foreclosure rate of more than 8 percent, and the foreclosure rate remains higher than the non-interventionist path for quite some time. The rates with and without intervention converge around 25 years in the future.

What explains this pattern? As noted earlier, the main motivation for foreclosures in the period of the shock is to take advantage of the low price of rental housing space. This incentive is blunted if the homeowner can reduce his or her mortgage payment – which is exactly what the loan modification program does. Consequently, as long as the homeowner can continue to live in the house with lowered payments and does not receive an adverse income shock, he or she does not wish to default. This lowers the foreclosure rate for as long as the modification is in place. Once the modification expires, the incentive to take advantage of the low price of housing space re-emerges and the foreclosure rate spikes up.

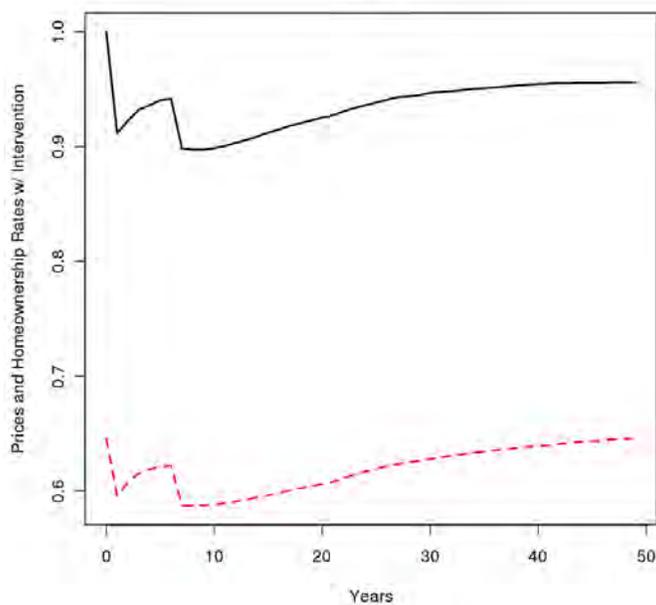
Figure 5 displays the path of prices under the two scenarios. There is less of a decline in prices in the period of the shock and prices actually recover some during periods 2 – 6. Then there is a crash in period 7. Following the second crash, there is a gradual recovery in prices.

Figure 5

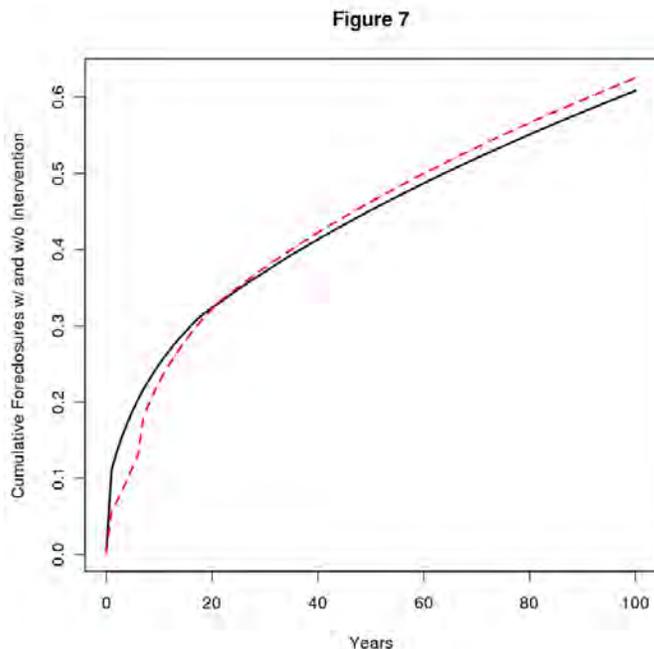


The path of prices basically mirrors the homeownership rate under intervention. This is evident from the comparison in Figure 6. The homeownership rate (the red dotted line) rises between periods 2 – 6 and then declines, and, correspondingly, prices rise and then decline.

Figure 6



In this discussion, we have simplified matters by assuming that the modifications expire at the end of 6 years. In reality, the plan is for the modifications to be taken away gradually. The gradualness may make the second rise in foreclosures less sharp, but it is unlikely to completely eliminate it. The temporary nature of foreclosure relief suggests looking at the effects of policy from a cumulative perspective. Are there fewer total foreclosures as a result of the intervention? Figure 7 shows the cumulative path of foreclosures.



Evidently, the policy reduces total foreclosures for about 20 years out but then leads to slightly higher total foreclosures in the very long run.

9 Conclusions

We developed a quantitative equilibrium model of the housing market in which an unanticipated increase in the supply of housing depresses house prices and triggers default on mortgages. The decline in house prices creates an incentive to increase the consumption of housing space but leverage makes it costly for homeowners to sell their homes and buy bigger ones (they must absorb large capital losses). Instead, leveraged households find it advantageous to default and rent housing space. Since renters demand less housing space than homeowners (homeowners demand more space because of the tax benefits of owner-occupancy), foreclosures depress housing prices even further. Thus, our model features a “circular dynamic”

that contributes to a downward spiral of higher foreclosures and lower house prices.

Because foreclosures contribute to lower house prices, a foreclosure prevention policy can potentially stabilize house prices. To explore this possibility we implement a somewhat simplified version of the government's foreclosure prevention policy in our model. We find that the policy is indeed quite effective in reducing foreclosures and stabilizing house prices in the model. However, we also find that these effects are temporary in that once the program expires in 6 years, foreclosures spike and housing prices crash again.

References

- [1] Chambers, M.S., Garriga C. and Schlagenhaut D., “Accounting for Changes in Homeownership Rate,” *International Economic Review*, February 2009.
- [2] Colussi, A., “Migrants Networks: An Estimable Model of Illegal Mexican Immigration,” Working Paper, University of Western Ontario, January 2006.
- [3] Corbae, D. and Quintin E., “Mortgage Innovation and the Foreclosure Boom,” Working Paper, University of Texas at Austin, June 2009.
- [4] Davis, M.A. and Ortalo-Magne, F., “Household Expenditures, Wages, Rents,” Working Paper, Department of Real Estate and Urban Land Economics, University of Wisconsin-Madison, April 2009.
- [5] Family Housing Fund, “Cost Effectiveness of Mortgage Foreclosure Prevention,” Report, Minneapolis MN, 1998.
- [6] Foote C.L, Gerardi K.S., Goette L., and Willen P.S., “Reducing Foreclosures,” Public Policy Discussion Papers, FRB Boston, 2009.
- [7] Iacoviello, M. and Pavan M., “Housing and Debt over the Life Cycle and over the Business Cycle,” Working Paper, Boston College, June 2009.
- [8] Gervais, M., “Housing Taxation and Capital Accumulation,” *Journal of Monetary Economics*, 49(7), pp. 1461-1489, 2002.
- [9] Gruber, J.W. and Martin, R.F., “Precautionary Savings and the Wealth Distribution with Illiquid Durables,” International Finance Discussion Papers No. 773, Board of Governors of the Federal Reserve System, September 2003.
- [10] Guler, B., “Innovation in Information Technology and the Mortgage Market,” Working Paper, University of Texas at Austin, November 26, 2008.
- [11] Jeske, K. and Krueger D., “Housing and the Macroeconomy: The Role of Implicit Guarantees for the Government-Sponsored Enterprises,” Working Paper No. 2005-15, Federal Reserve Bank of Atlanta, August 2005.
- [12] Nakajima, M., “Rising Earnings Instability, Portfolio Choice and House Prices,” Working Paper, University of Illinois Urbana-Champaign, July 2005.
- [13] Orzechowski S. and Sapielli P., “Net Worth and Asset Ownership of Households: 1998 and 2000,” *Current Population Reports*, US Census Bureau, May 2003.

- [14] Pennington-Cross, A. "The Value of Foreclosed Property," *Journal of Real Estate Research* 28(2), 2006.
- [15] Piskorski T., Seru A., and Vig V., "Securitization and Distressed Loan Renegotiation: Evidence from Subprime Mortgage Crisis," Chicago Booth School of Business Research Paper No. 09-02, 2009.
- [16] Rios-Rull, J-V. and Sanchez-Marcos, V., "Aggregate Shocks and the Volatility of House Prices," Mimeo, September 2008.
- [17] U.S. Treasury, "Home Affordable Modification Program Guidelines," The Treasury Department, March 4 2009.