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MONEY TALKS

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Money Talks*

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Abstract

We study credible information transmission by a benevolent short-lived central bank. We consider two possibilities: direct revelation through an announcement, versus indirect transmission through monetary policy. These two ways of transmitting information have very different consequences. Since the objectives of the central bank and those of individual investors are not always aligned, private investors might rationally ignore announcements by the central bank. In contrast, information transmission through changes in the interest rate creates a distortion, thus lending an amount of credibility. This induces the private investors to rationally take into account information revealed through monetary policy.

JEL CLASSIFICATION: D80; E40; E52

KEYWORDS: Information; Interest rates; Monetary policy

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1 Introduction

How can a central bank effectively communicate its information about economic fundamentals to the private sector? What is the role of monetary policy as a tool for information revelation in the presence of aggregate risk and of potential coordination problems faced by investors? Why do central banks typically follow policies that lead to positive average levels of inflation? These questions are highly topical and have a long history in economics. However, decisive answers still elude us. This paper studies monetary policy as a tool for credible information transmission by the central bank.¹

We build a model that contains the following three ingredients: (1) money plays a role in facilitating trade, (2) there is aggregate risk about fundamentals, and (3) investment decisions are subject to a coordination problem, an effect emphasized as early as in Keynes (1936), implying that individually optimal decisions might not maximize aggregate welfare. Investors in our economy have expectations about economic fundamentals that affect investment returns. A benevolent central bank (CB) has the ability to print money and to provide loans. In addition, the CB has its own information about the true state of the economic fundamentals. We demonstrate that this information cannot always be credibly transmitted to the private sector. In other words, a simple announcement might not be enough, as the CB may prefer to communicate false information if this led to more socially desirable behavior by investors. We then show that credible information revelation is possible through

¹There is a large literature on optimal monetary policy in the presence of information frictions. See, for example, Weiss (1980) and Barro and Gordon (1983) for two related early models. Cukierman and Meltzer (1986) emphasize the role of ambiguity for central bank policies. Kydland and Prescott (1977) introduce the famous dynamic inconsistency problem. Backus and Driffill (1985) introduce uncertainty about the central bank's type (see also King, Lu and Pasten, 2008). Our approach differs in several ways from these papers. Perhaps the most important one is that we concentrate on the role of monetary policy as a *credible* information transmission mechanism in a model that does not rely on reputation building by a long-lived central bank (see also Phelps, 1983, for the importance of central bank credibility in managing expectations). For a recent model that studies credible information transmission by the central bank, but without explicitly modelling money or monetary policy, see Moscarini (2007). Ellingsen and Söderström (2001) study the effects of monetary policy on the yield curve and document that when monetary policy reveals information about economic fundamentals, interest rates of all maturities move in the same direction as the policy innovation. In this paper, we provide an explanation of why information is revealed through monetary policy instead of announcements. Our paper is also related to Amador and Weill (2007), who study the effects of releasing public information in a Lucas (1972) environment. They show that releasing public information about aggregate fundamentals can lead to greater uncertainty.

monetary policy. In order to gain credibility, however, it is *necessary* that such a policy create a distortion by affecting average inflation. The study of optimal monetary policy in our framework concerns the optimal balancing of the resulting benefits from information revelation against the costs associated with the monetary distortion. More generally, monetary policy in our model can be thought of as the “translation” of the central bank’s information, expressed by the corresponding value of the chosen interest rate. Absent the need for information transmission, the benevolent central bank would always set the nominal interest rate to zero. Thus, positive nominal interest rates serve as a way for the central bank to *credibly* transmit its “message” to the private sector. Since interest rates affect economic fundamentals, this information transmission gains the necessary element of credibility that would be missing in a pure announcement.²

One episode that motivates our analysis concerns the events that took place in Sweden during the period 2005-2007. These events strongly suggest the need for a central bank to supplement its words with deeds. During that period, the Swedish Central bank (the Riksbank) was carefully monitoring the growth of housing prices in Sweden. Concerns about the rapid rise in these prices were repeatedly expressed publicly by the Riksbank. In 2005, concerns about house prices appear in six out of seven press releases that follow monetary policy decisions, as well as in the Financial Stability Report.³ However, the rise in housing prices continued throughout 2005, reaching 10% in the third quarter of 2005 compared to the same quarter in the previous year.⁴ According to the minutes of the Executive Board’s monetary policy meeting of December 2005, it was suggested that raising the repo rate by 25 basis points “would also function as a *signal* that could subdue house price trends and household indebtedness.” From January 2006 onward, the Riksbank started gradually raising interest rates. In the corresponding announcements of monetary policy decisions, this rise was complemented by stressing the concern over housing price developments. According to

²Needless to say, monetary policy in the actual economy serves several purposes. In order to concentrate on monetary policy’s role as a credible communication device, we will abstract from effects related to liquidity provision. For a recent paper that studies the credibility of central bank policies in a different context, see Ennis and Keister (2007).

³The Riksbank changed the interest rate only once in 2005: a 0.50 percentage point cut in June.

⁴Moreover, the average annual growth rate of housing prices was about 7.5% between 1996 and 2005. Household borrowing showed a similar average rate of increase in this time period.

the minutes of December 2006, while the rate of price increases and borrowing remained high, some slowdown had already taken place.⁵ Our reading of these events is that the interest rate increases added credibility to simple announcements by the Riksbank, thus finally changing the behavior of the private sector. This was a task that repeated previous announcements alone had failed to accomplish.

Throughout the paper, we study the efficient and credible ways for a CB to communicate its information to private investors. Motivated by the Swedish experience, we consider two possibilities. First, the CB makes a direct “announcement.” For example, one could think of a press statement or an interview delivered, say, by the chairman of the CB. Alternatively, the CB may indirectly transmit its information through monetary policy, i.e., through varying the nominal interest rate. Our main finding is that these two ways of transmitting information can have very different consequences. Since the objectives of the benevolent CB and that of an individual investor in the economy are not directly aligned, the CB might have an incentive to misrepresent its information if this would lead to private investment decisions that improve social welfare. The private investors might, in turn, rationally choose to ignore such announcements. In contrast, credible information transmission through changes in the interest rate is not “cheap-talk,” as it reduces welfare through creating a distortion associated with a violation of the Friedman rule. We demonstrate that, provided that the costs associated with inflation are sufficiently high, this adds a necessary amount of credibility that induces the private investors to rationally take into account the information provided by the CB. At the same time, social welfare is higher than in the absence of credible information transmission.

There is an ongoing debate in macroeconomics about whether the CB should react to the developments in financial markets that are considered excessive, and that can lead to suboptimal investment decisions. In particular, the effectiveness of monetary policy in affecting investors’ choices is often questioned. The argument is that to induce a change in investors’ behavior, a corresponding change in the interest rate would have to be very large. This would have a negative impact on the real economy, thus more than outweighing the

⁵See <http://www.riksbank.com/templates/ItemList.aspx?id=27260> for the minutes of the Executive Board’s monetary policy meetings.

benefits of improved resource allocation. Our analysis suggests a new channel through which potentially modest interest rate changes can have significant beneficial effects.

Finally, in the Appendix we formalize the equivalence between the set of feasible monetary policies and the information set of the CB. Whenever this equivalence holds, at the cost of imposing a monetary distortion, the CB can use monetary policy in order to credibly transmit its information to the private sector.

The paper proceeds as follows. Section 2 describes the economic environment and considers the full information benchmark, while Section 3 studies the model under private information. A brief Conclusion follows. The Appendix contains the proofs of the propositions in the text and discusses the equivalence result.

2 The Environment

Our basic model uses the setup developed by Berentsen and Monnet (2008).⁶ Time $t = 0, 1, \dots$, is infinite. The economy consists of a continuum of infinitely lived agents. In addition, there is a benevolent central bank (CB) that has the ability to make public announcements, to print money, and to make loans to the private sector. The CB serves for one term only and is replaced by a new CB at the end of each period.⁷

Each period is divided into three stages: 0, 1, and 2. There are three goods: an *investment good*, k , a *stage-1 good*, q , and a *stage-2 good*, z . Investment takes place in stage 0 of each period. The market for good q opens in stage 1. In stage 2, investment pays off in units of good z . Future periods are discounted at rate $\beta \in (0, 1)$. There is no discounting between stages. We consider each stage in more detail next.

In stage 0 of each period, half of the agents are randomly chosen to be producers (*in-*

⁶They, in turn, build on Lagos and Wright (2005) and on Kiyotaki and Wright (1989). Other choices of a monetary model are also consistent with our findings. Our choice was for a tractable model in which to study information transmission by the central bank and where money is essential. Indeed, our model can be easily converted to a cash-in-advance economy.

⁷This feature allows us to abstract from reputation effects, which, as we mentioned before, have been the focus of study in other papers. In the case of a long-lived CB, truthful information transmission can be assured if (sufficiently patient) investors trigger a severe penalty when they discover that the CB has miscommunicated its information. Our analysis focuses on achieving credible truthful communication in the absence of such reputation effects.

vestors). The remaining $\frac{1}{2}$ proportion of agents become consumers. Each investor i chooses how much of an investment good, k , to produce. The utility cost to investor i from producing $k_i > 0$ units of the investment good is $c(k_i) = \frac{1}{2}(k_i - \gamma K)^2$, where $K = \int k_i di$ is the aggregate production of the investment good and $\gamma \in (0, 1)$. Thus, investors costs are subject to an externality. The cost is zero if $k_i = 0$. All investments mature in stage 2. The return on this investment is uncertain and is given by θ^2 units of good z per unit of k , where θ is a random variable with an improper uniform prior on $(-\infty, +\infty)$ and is *iid* across periods.⁸ Nature draws θ at the start of stage 0.

The random variable θ is our way of introducing aggregate risk about the profitability of investment, a feature that can be traced back at least to Keynes (1936). Some information regarding this risk is available in the economy. More precisely, when the true state is θ , the CB receives a signal $y = \theta + \eta$, while all investors receives a signal $x = \theta + \varepsilon$. These signals are received in stage 0 of each period, prior to the investment decisions, and are *iid* across time.⁹ We assume that the noise terms η and ε are normally distributed with mean zero and respective precisions given by $\alpha = 1/\sigma_\eta^2$ and $\delta = 1/\sigma_\varepsilon^2$. The assumption that investors have some available information about the future profitability of their investments seems natural. In addition, CBs employ large numbers of specialists in order to collect and analyze economic data. The fact that these data is treated with secrecy suggests that CBs consider the information in their possession important and that they care about how and when this information is revealed to the public. Notice that we do not take a stand on whether the CB or the private investors have more accurate information.

In stage 1, investors can produce the stage-1 good, q , at a cost $c(q) = -q$. The remaining agents (consumers) can consume q , deriving utility $u(q)$, where u is increasing, concave, and satisfies the usual Inada conditions. In particular, there is a unique q^* such that $u'(q^*) = 1$.

⁸We consider the return on investment to be θ^2 instead of θ to guarantee that investment is always positive. In our formulation, the externality is essentially the same as in Angeletos and Pavan (2004) since the individual return on investment also increases in the aggregate level of investment. To see this, notice that we can write the profit from investment as $\theta^2 k_i - (k_i - \gamma K)^2 / 2 = (\theta^2 + \gamma K) k_i - k_i^2 / 2 - \gamma^2 K^2 / 2$.

⁹Assuming that investors' signals are perfectly correlated is for simplicity only. Stages 0 and 1 are interchangeable in what follows. What is important is that the CB's announcement/interest rate choice takes place prior to the investment decision by the private sector. For simplicity, we will assume that only investors (not consumers) receive signals. Morris and Shin (2002) use a similar signalling structure to study the trade-off created by increasing the quality of the information held by the Central Bank.

Good q is sold in a competitive market at price p . Traders are anonymous in this market, so money is used in the exchange of q . In stage 1 of each period, agents have access to a lending facility operated by the CB, where they can borrow money at an interest rate $r \geq 0$. CB loans are uncollateralized. The CB keeps track of all such borrowing and loans are settled in stage 2.¹⁰ The interest rate that will prevail in stage 1 is announced by the central bank at the start of stage 0, before investors make their investment decisions.

Finally, stage-0 investment delivers a return of $\theta^2 k_i$ units of the non-storable stage-2 good, z . This good is traded in a competitive market in stage 2. The utility (disutility) of consumption (production) from z is linear and is denoted by $(-)z$. Stage 2 can be thought of as a settlement stage. Those consumers that borrowed money in order to consume in stage 1 must produce in order to pay off these loans. The investors, who produce in stage 1, will end up consuming in stage 2.¹¹

The sequence of events in each stage is summarized in figure 1 below.

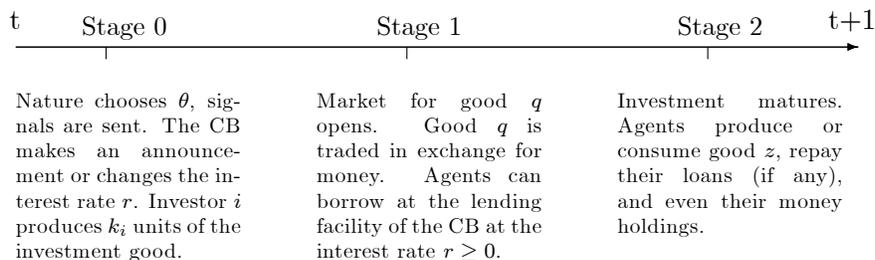


Figure 1: The timing of events within a period

2.1 The Full-Information Benchmark

Throughout the paper we explore efficient ways for the CB to communicate its information to private investors. Such information revelation is potentially beneficial for two reasons.

¹⁰For simplicity, we assume that the CB makes a lump-sum transfer in the settlement market in order to redistribute any profits made by its lending facility.

¹¹Due to linearity, agents will exit stage 2 with equal money holdings. This dramatically improves tractability in what follows.

First, the CB's signal is informative and this information may lead to investments that are closer to best responses under the true fundamentals.¹² Second, the CB takes into account the social cost of investment, which may lead to a better allocation due to the presence of the externality. However, as we shall see, a simple announcement might not be enough, as there are cases where the CB would prefer to communicate false information. Later we will demonstrate that credible information revelation is possible through monetary policy.

Before we study these issues, we introduce the full-information economy as a benchmark. Aggregate period- t welfare, \mathcal{W} , is given by

$$\mathcal{W}(q, k_i, \theta) = \frac{1}{2} \left[u(q) - q + \theta^2 K - \int \frac{1}{2} (k_i - \gamma K)^2 di \right], \quad (1)$$

where $K = \int k_i di$ is the aggregate production of the investment good in stage 0. Imposing that $k_i = K$, the first-order conditions for the full-information efficient allocation (q^*, K^*) in any given period yield

$$u'(q^*) = 1, \text{ and} \quad (2)$$

$$K^* = \frac{\theta^2}{(1 - \gamma)^2}. \quad (3)$$

The amount of good z produced in stage 2 is indeterminate.

As investors are not readily identifiable during stage 1, some type of record-keeping is needed for transactions to take place. Next, we discuss how full-information efficient allocations can be decentralized through monetary trade. Throughout we assume that money is provided exclusively by the CB. We let M denote the per capita supply of money and we let ϕ denote the real price of money in terms of good z . The growth rate of money is given by ρ . Monetary injections are implemented through a transfer, T , in the settlement stage. The net stock of money grows according to $M_+ = M + T$, where subscript $+$ is used to denote next period values and where T is such that $M_+ = \gamma M$. We consider a stationary equilibrium where $\phi M = \phi_+ M_+$, so that $\rho = \phi / \phi_+$.

¹²See Morris and Shin (2002) for a model in which this might not be true for all possible parameter values due to strategic complementarity in agents' actions.

We use $W(k, m, l; \theta)$ to denote the discounted lifetime utility of an agent when he enters stage 2 holding k units of the investment good, m units of money, and l units of loans from the CB, given that the realized productivity shock is θ . The function $V(m)$ denotes the expected discounted lifetime utility from entering stage 0 with money holdings m . Then, $W(k, m, l; \theta)$ is defined by

$$\begin{aligned} W(k, m, l; \theta) &= \max_{z, m_+} \{-z + \beta EV(m_+)\} \\ \text{s.t. } &\phi m_+ = z + \theta^2 k + \phi m - \phi(1+r)l + \phi T, \end{aligned} \quad (4)$$

where z denotes the net production of the stage-2 good and E is the expectation operator over the possible realization of θ_+ . The first-order and envelope conditions give

$$\beta EV_m = \phi, \quad W_k = \theta^2, \quad W_m = \phi, \quad W_l = -\phi(1+r). \quad (5)$$

The discounted lifetime utility of agents entering stage 1 with m units of money is¹³

$$\begin{aligned} V(m) &= \frac{1}{2} \max_{k_i} \left\{ -\frac{(k_i - \gamma K)^2}{2} + E \left[\max_q -q + W(k_i, m + pq, 0) \right] \right\} \\ &\quad + \frac{1}{2} E \left[\max_{q, l \text{ s.t. } pq \leq m+l} u(q) + W(0, m - pq + l, l) \right]. \end{aligned} \quad (6)$$

Using (5), the first-order conditions for investors are

$$p\phi = 1, \text{ and} \quad (7)$$

$$k_i = \theta^2 + \gamma K. \quad (8)$$

Since $k_i = k = K$ for all agents, the equilibrium aggregate production of the investment good under full information is given by $K = \frac{\theta^2}{1-\gamma}$. Using (3), we have $K < K^*$ since $\gamma < 1$. The equilibrium level of production is lower than the level that maximizes aggregate welfare, K^* .

¹³The following expression uses the fact that producers never borrow from the CB.

The first-order conditions for consumers are

$$u'(q) = \phi p(1 + \lambda), \text{ and} \quad (9)$$

$$W_m + W_l + \phi\lambda = 0, \quad (10)$$

where $\phi\lambda$ is the real Lagrange multiplier on their budget constraint. Using (5) and (7) in (10) we obtain that $\lambda = r$. Then, (9) becomes

$$u'(q) = 1 + r. \quad (11)$$

Consumers equalize the benefit of consumption with the marginal cost of borrowing at the CB. Hence, monetary policy affects the equilibrium allocation through this borrowing channel. Given r , a stationary equilibrium outcome is described by a vector $(q, \{k_i\}_{i \in [0,1]})$ satisfying (8) and (11). Finally, the envelope condition gives the rate of inflation ρ , consistent with the interest rate policy rule:

$$\frac{\rho}{\beta} = 1 + \frac{E[r]}{2}. \quad (12)$$

Using (2) and (11) it follows that, when the true value of θ is publicly observable, the CB follows the Friedman rule. It sets $r = 0$ for all states and $u'(q^*) = 1$. Still the CB does not achieve the efficient allocation as (3) is not satisfied.

3 Information Transmission by the Central Bank

We now turn to the case where the aggregate state of the economy, θ , is unknown. Both the private sector and the CB receive informative signals regarding the true value of θ . The private sector receives signal x with precision δ . The CB receives signal y with precision α .

Suppose the CB announces that its signal is y_a and that the precision of the signal is α_a .

The decision by the private sector is then given by

$$\begin{aligned}
V(m) &= \frac{1}{2} \max_{k_i} E \left[-\frac{(k_i - \gamma K)^2}{2} + E \left[\max_q -q + W(k_i, m + pq, 0) \mid x, y_a, \alpha_a \right] \right] \\
&+ \frac{1}{2} E \left[\max_{q,l \text{ s.t. } pq \leq m+l} u(q) + W(0, m - pq + l, l) \right]. \tag{13}
\end{aligned}$$

Using (5), the first-order conditions for investors are

$$p\phi = 1, \text{ and} \tag{14}$$

$$k(x, y_a, \alpha_a) = E(\theta^2 + \gamma K \mid x, y_a, \alpha_a). \tag{15}$$

Since all investors have the same information, we concentrate on symmetric outcomes where they all produce the same amount, $k(x, y_a, \alpha_a) = K(x, y_a, \alpha_a)$. Hence,

$$k(x, y_a, \alpha_a) = \frac{E(\theta^2 \mid x, y_a, \alpha_a)}{1 - \gamma} = \frac{1}{1 - \gamma} \left[\left(\frac{\alpha_a y_a + \delta x}{\alpha_a + \delta} \right)^2 + \frac{1}{\alpha_a + \delta} \right]. \tag{16}$$

Taking as given the behavior by the private sector, the CB maximizes expected period- t welfare given its signal, y , and precision, α . This is given by

$$\mathcal{W}(q, k_i) = \frac{1}{2} E \left[u(q) - q + \theta^2 K - \int \frac{(k_i - \gamma K)^2}{2} di \mid \alpha, y \right]. \tag{17}$$

Since the CB's signal contains information about the true value of θ , it might be beneficial if this information reaches the private sector. How should the CB transmit its information to the investors? We will consider two possibilities. First, the CB could make a direct "announcement." For example, one could think of a press statement or an interview delivered, say, by the chairman of the CB. Alternatively, the CB could indirectly transmit its information through monetary policy, i.e., through the interest rate, r . The main finding of our paper is that these two ways of transmitting information can have very different consequences. Recall that the objective of the benevolent CB and that of an individual investor in the economy are not completely aligned. Hence, the CB might have an incentive to

misrepresent its information if it believes that this will lead to private investment decisions that improve expected social welfare. Realizing this, the investors might choose to ignore such announcements. In contrast, transmission through changes in the interest rate is not “cheap-talk,” as it reduces welfare through creating a distortion associated with a violation of the Friedman rule. We will demonstrate that this adds a sufficient amount of “credibility” that can induce the investors to rationally take into account the information provided by the CB.

3.1 Public Announcements

For simplicity, we assume that the precision of the CB’s signal, α , is *iid* across periods and that it can take on two values: $\{\alpha_L, \alpha_H\}$, where $\alpha_L = \alpha_H - \varepsilon$ for some $\varepsilon > 0$.¹⁴ The probability that $\alpha = \alpha_L$ is denoted by π , and the probability that $\alpha = \alpha_H$ by $1 - \pi$. The realization of the CB’s signal precision is observed only by the CB. To further simplify the analysis, we will assume that y is publicly observable. In this case, the public announcement (possibly untruthful) concerns only the value of CB’s confidence in its information. We denote such announcement by α_a .¹⁵

We derive the expression for the expected social welfare given the CB’s information, $\mathcal{W}(k_i, K)$, in the Appendix. The following asserts that there are cases such that if the investors believe the CB’s announcement, expected social welfare is increased if the CB misrepresents its confidence.

Proposition 1 *There exists an $\varepsilon > 0$ such that announcing $\alpha_L = \alpha_H - \varepsilon$ is preferred by the CB to announcing α_H . There is no equilibrium where the CB announces its precision truthfully and where the investors use the CB’s announcement.*

The Proof is given in the Appendix. Intuitively, announcing lower precision boosts expected aggregate production of the investment good, K , and hence brings the production

¹⁴The arguments in this paper generalize to the case of any finite number of precision values.

¹⁵We interpret the public announcement as the result of a public speech, say, by the chairman of the CB. The contents of the announcement then become common knowledge among investors.

level closer to the social optimum (we showed in Section 2 that, due to the externality, equilibrium production of the investment good is lower than the level that maximizes aggregate welfare under full information). That is, we have

$$E[K | \alpha, y] = \frac{1}{1 - \gamma} \left[y^2 + \frac{\delta^2 + 2\alpha\delta + \alpha\alpha_a}{\alpha(\alpha_a + \delta)^2} \right]$$

and

$$\frac{\partial}{\partial \alpha_a} E[K | \alpha, y] = -\frac{1}{\alpha(1 - \gamma)(\delta + \alpha_a)^3} (3\alpha\delta + \alpha\alpha_a + 2\delta^2) < 0.$$

In addition, the bigger the wedge between social and private welfare (the higher the level of the externality γ), the higher is the CB's incentive to lie ($\frac{\partial W}{\partial \alpha_a \partial \gamma} < 0$). On the other hand, the CB's incentive to lie decreases as the precision of the private signal increases ($\frac{\partial W(q, k_i)}{\partial \alpha_a \partial \delta} |_{\alpha = \alpha_a} > 0$ for $\delta > \alpha$).

For an example, let $y = 0.1$, $\delta = 70$, $\alpha_H = 60$, $\alpha_L = 50$ and $\gamma = 0.1$. Note that investors have more precise information than the CB. The welfare under truth-telling when $\alpha = 60$ is $2\mathcal{W}(q, k_i | \alpha = \alpha_a = 60, y = 0.1) = u(q^*) - q^* + 0.7524 \times 10^{-3}$, where q^* denotes the consumption level at the Friedman rule (see equation (2)). On the other hand, if the CB reports $\alpha_a = 50$, welfare increases to $2\mathcal{W}(q, k_i | \alpha = 60, \alpha_a = 50, y = 0.1) = u(q^*) - q^* + 0.7592 \times 10^{-3}$. Thus, the CB with $\alpha = 60$ prefers to announce $\alpha_a = 50$ and follow the Friedman rule; i.e., set $r(\alpha_a) = 0$.

To summarize, the benevolent CB might have an incentive to misrepresent its confidence in its information. This raises the question of whether it is possible for the CB to use a *costly* method in order to credibly communicate its confidence. We investigate this next.

3.2 Credible Monetary Policy

We continue assuming that the value of the CB's signal, y , is known and that its precision, α , is *iid* across periods. For simplicity, we assume that the precision can take on two values: $\{\alpha_L, \alpha_H\}$, $\alpha_L < \alpha_H$, with respective probabilities π and $1 - \pi$. Thus, like before, information transmission concerns the CB's signal precision, α .

The CB chooses an interest rate rule to reveal the precision of its signal, i.e., $r(\alpha)$. In

contrast to a press announcement, the choice of r involves a deviation from the Friedman rule. Thus, it creates a distortion in the economy. At the same time, by “inverting” r , investors can infer the value of α that the CB wishes to communicate.¹⁶ The question is whether, unlike in the case of a pure announcement, information transmission through monetary policy can lead to an equilibrium where the CB reports truthfully and the private sector takes this information into account when making investment decisions.

The problem of the CB is to maximize (17) subject to the equilibrium equations of agents 14 and 15. In addition, in order for monetary policy to *credibly* transmit information about the CB’s confidence, a set of incentive compatibility constraints must hold. More precisely, we require that

$$\mathcal{W}(q, k_i | r(\alpha), \alpha) \geq \mathcal{W}(q, k_i | r(\alpha_a), \alpha), \text{ for } \alpha_a \neq \alpha. \quad (18)$$

In words, the resulting welfare must be higher under truthful revelation by the CB. Consider the case where in any given period, the CB with $\alpha = \alpha_H$ prefers to mimic the CB with $\alpha = \alpha_L$, $\alpha_L < \alpha_H$ (this would be the case under the conditions in Proposition 1). Clearly, the incentive compatibility constraint in this case does not bind if $\alpha = \alpha_L$. However, when $\alpha = \alpha_H$, the CB has an incentive to misrepresent its confidence in its information. Thus, the incentive compatibility constraint must bind

$$\mathcal{W}(q, k_i | r(\alpha_H), \alpha_H) = \mathcal{W}(q, k_i | r(\alpha_L), \alpha_H). \quad (19)$$

Setting $r(\alpha_H) = 0$, we can obtain the corresponding interest rate, $r(\alpha_L)$, implicitly as the solution to

$$\mathcal{W}(q, k_i | 0, \alpha_H) = \mathcal{W}(q, k_i | r(\alpha_L), \alpha_H). \quad (20)$$

We must also verify that

$$\mathcal{W}(q, k_i | r(\alpha_L), \alpha_L) \geq \mathcal{W}(q, k_i | 0, \alpha_L), \quad (21)$$

i.e., inflating in order to truthfully reveal that $\alpha = \alpha_L$ is preferred by the CB to the alternative

¹⁶In the Appendix we show that this idea generalizes to the set-up where the CB announces its signal and the precision by choosing r .

of not inflating and announcing $\alpha = \alpha_H$. In addition, when $\alpha = \alpha_L$, the CB must prefer the corresponding outcome to the case where there is no information revelation (and hence no inflationary distortion) and where investors simply use their priors

$$\bar{\alpha} = \pi\alpha_L + (1 - \pi)\alpha_H.$$

We thus need to ensure that the welfare when the CB chooses an interest rate that reveals the true state exceeds the welfare that results if the CB simply follows the Friedman rule and investors use their priors, i.e.,

$$\mathcal{W}(q, k_i | r(\alpha_L), \alpha_L) \geq \mathcal{W}(q, k_i | r(\bar{\alpha}), \alpha_L), \quad (22)$$

where $r(\bar{\alpha}) = 0$. We have the following.

Proposition 2 *Let π be small. Then, for ε small and $\alpha_L = \alpha_H - \varepsilon$, there is an equilibrium where $r(\alpha_L) > 0$, $r(\alpha_H) = 0$, the CB communicates its precision truthfully and the investors use the CB's announcement.*

The proof is given in the Appendix. For an illustration, fix parameter values to be those in the previous example. Moreover, let $\pi(\alpha_L = 50) = 0.1$. We know that a CB with $\alpha = 60$ has an incentive to report $\alpha_a = 50$. The question is whether a CB with $\alpha = 50$ prefers to costly but credibly communicate the true value of its precision.¹⁷ To prevent a CB with $\alpha = 60$ from communicating the value $\alpha = 50$, the CB must invoke a cost, $\hat{r} > 0$, where \hat{r} solves (20). For the parameter values above, \hat{r} is the solution to

$$u(q^*) - q^* - u(q(\hat{r})) + q(\hat{r}) = 0.6855 \times 10^{-5},$$

where q^* denotes consumption level at the Friedman rule. Since this difference is positive, such an \hat{r} exists. To see whether a CB with $\alpha = 50$ chooses credible but costly communication

¹⁷Note that a CB with $\alpha = 50$ suffers a welfare loss if it cannot reveal its type truthfully and, instead, it is believed to be a CB with $\alpha = 60$: $2\mathcal{W}(k_i, K | \alpha = 50, \alpha_a = 60, y = 0.1) = u(q^*) - q^* + 0.9780 \times 10^{-3} < \mathcal{W}(k_i, K | \alpha = \alpha_a = 50, y = 0.1) = u(q^*) - q^* + 0.1002 \times 10^{-2}$.

we first check that the welfare under costly communication is higher than the welfare under following the Friedman rule and being taken for an $\alpha = 60$ type:

$$\begin{aligned} & 2\mathcal{W}(q, k_i | \hat{r}(\alpha_L), \alpha = \alpha_a = 50) - 2\mathcal{W}(q, k_i | 0, \alpha = 50, \alpha_a = 60) \\ &= u(q(\hat{r})) - q(\hat{r}) - u(q^*) + q^* + 0.1002 \times 10^{-2} - 0.9780 \times 10^{-3} \\ &= 0.1666 \times 10^{-4}. \end{aligned}$$

Second, we check that welfare under costly communication exceeds welfare under no communication, whereby investors use their priors on α and the CB sets $r = 0$:

$$\begin{aligned} & 2\mathcal{W}(q, k_i | \hat{r}(\alpha_L), \alpha = \alpha_a = 50) - 2\mathcal{W}(q, k_i | 0, \alpha = 50, \alpha_a = \bar{\alpha}) \\ &= u(q(\hat{r})) - q(\hat{r}) - u(q^*) + q^* + 0.1002 \times 10^{-2} - 0.9808 \times 10^{-3} \\ &= 0.1393 \times 10^{-4}. \end{aligned}$$

Taken together, the two Propositions imply that a public announcement alone may not be effective. In order to transmit information to investors, the CB must violate the Friedman rule.¹⁸ In fact, incentive constraints put a lower bound on the inflation level that must be tolerated in order for the information transmission to be credible. Otherwise, like a simple announcement, investors will rationally ignore the supplied information, treating it as “cheap-talk.”¹⁹

How high is the interest rate increase needed to ensure credible information transmission? This question is related to the ongoing debate about the effectiveness of monetary policy in containing developments in the financial sector, which are considered excessive, and can potentially lead to inefficient investment. Should the CB react if it believes that “financial imbalances” are building up? One common argument against a monetary policy reaction is

¹⁸The conditions in the two Propositions above jointly hold for an open set of parameter values. Notice that inflation rates below the Friedman rule are not consistent with the existence of a monetary equilibrium. Thus, in order to communicate its information credibly, the CB *must* choose a positive r , thus creating a costly distortion in the economy.

¹⁹If an announcement by the CB is sufficient to credibly convey its information, inflationary equilibria cannot exist. The reason is that at any positive inflation level the CB can increase social welfare by further lowering inflation.

that the size of the interest rate adjustment that would be necessary to bring about a change in investors' behavior would have to be so large that it would generate too much harm to the real economy. As our approach considers explicitly the costs and benefits of credible information transmission, it might ultimately prove useful in evaluating the magnitude of the interest rate change that is needed for the CB to affect investment decisions and to improve the resulting equilibrium allocation.

For a numerical example, assume a constant relative risk aversion utility function, i.e., $u(q) = \frac{q^{1-\omega}}{1-\omega}$, where ω is the coefficient of the relative risk aversion. For $\omega = 2$, credible information transmission in our example involves setting the interest rate at $\hat{r} = 0.53\%$ (i.e. 53 basis points). It is easy to show²⁰ that as the coefficient of the relative risk aversion increases, credible communication requires larger deviations from the Friedman rule. For higher (and more empirically plausible) values of the coefficient of the relative risk aversion, like $\omega = 4$, credible signaling involves a deviation from the Friedman rule of $\hat{r} = 0.74\%$.

To summarize, we have shown that, at least in some cases, monetary policy achieves a credible transmission of information by the CB, while this credibility is absent when the CB communicates its information through a simple announcement. A natural question to ask is whether the CB can always communicate its signal *as well as* its confidence in the signal through monetary policy. In other words, one can ask whether the one-dimensional set of feasible monetary policies is “rich” enough to communicate all possible values of the two-dimensional set consisting of the CB’s signal and precision. By using a well-known mathematical result, we demonstrate in the Appendix that the answer to this question is affirmative. In other words, in a well-defined mathematical sense, monetary policy can in principle be thought of as nothing but a “translation” of a message revealing the CB’s information and confidence.

²⁰By applying the Implicit Function Theorem to equation (20).

4 Conclusion

We introduced a model in which the CB has some information about the true state of the economic fundamentals (not necessarily better than private agents). Since the objectives of the benevolent CB and that of individual investors are not directly aligned, the CB might have an incentive to misrepresent its information to improve social welfare. We investigated under what conditions the CB can credibly communicate its information to the private sector. Our main finding identifies monetary policy as a tool that can lead to credible information transmission. While other ways to costly communicate information (e.g., taxes) are possible, CBs around the world are restricted in terms of the policy instruments they can use. A costly monetary distortion can accomplish this information transmission. In other words, inflation adds a sufficient amount of “credibility,” thus inducing investors to rationally take into account the information provided by the CB.

One might be tempted to think of ways for the CB to communicate information while avoiding the welfare losses due to inflation.²¹ However, *any* such alternative will (by construction) lack credibility in our model. It is precisely the real cost of the monetary distortion that lends credibility and, as a result, makes this channel of information transmission work in our set-up.

Our approach can be used to derive other properties of optimal monetary policy. For example, the model can be used to quantify the size of the monetary policy response needed to induce investors to take the CB’s information into account when making their investment decisions. Credible information transmission can potentially be achieved with only modest changes in the nominal interest rate.

²¹Two ideas that have been suggested to us are (1) repeated interest rate announcements by the CB (that might cancel each other’s effects), and (2) variations in the timing of the decisions by the CB and by the investors.

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Appendix - Derivation of welfare and proofs

Derivation of welfare: The objective of the CB is given by:

$$\mathcal{W}(q, k_i) = \frac{1}{2} E \left[u(q) - q + \theta^2 K - \int \frac{(k_i - \gamma K)^2}{2} di \mid \alpha, y \right]. \quad (23)$$

The CB takes it as given that investors produce the same amount, $k_i = K$, given by (16).

Thus, we have

$$\begin{aligned} & E \left[\theta^2 K - \int \frac{(k_i - \gamma K)^2}{2} di \mid \alpha, y \right] \\ &= E \left[\theta^2 K - \frac{(1 - \gamma)^2}{2} K^2 \mid \alpha, y \right] \\ &= E [\theta^2 K \mid \alpha, y] - \frac{(1 - \gamma)^2}{2} E [K^2 \mid \alpha, y]. \end{aligned}$$

Using (16) we obtain:

$$\begin{aligned} & E [\theta^2 K \mid \alpha, y] \\ &= \frac{1}{(1 - \gamma)} E \left[\theta^2 \left(\frac{\alpha_a y_a + \delta (\theta + \varepsilon)}{\alpha_a + \delta} \right)^2 + \frac{\theta^2}{\alpha_a + \delta} \mid \alpha, y \right] \\ &= E \left[\theta^2 \frac{\alpha_a + \delta + (\alpha_a y_a)^2 + \delta^2 (\theta + \varepsilon)^2 + 2\delta \alpha_a y_a (\theta + \varepsilon)}{(1 - \gamma) (\alpha_a + \delta)^2} \mid \alpha, y \right] \\ &= \frac{\alpha_a + \delta + (\alpha_a y_a)^2}{(1 - \gamma) (\alpha_a + \delta)^2} E [\theta^2 \mid \alpha, y] + E \left[\theta^2 \frac{\delta^2 (\theta^2 + \varepsilon^2 + 2\theta\varepsilon) + 2\delta \alpha_a y_a (\theta + \varepsilon)}{(1 - \gamma) (\alpha_a + \delta)^2} \mid \alpha, y \right] \end{aligned}$$

Since $E(\varepsilon \theta^k \mid \alpha, y) = E(\varepsilon \mid \alpha, y) E(\theta^k \mid \alpha, y) = 0$ for any k , the above expression simplifies

to

$$\begin{aligned}
& E [\theta^2 K \mid \alpha, y] \\
&= \frac{\alpha_a + \delta + (\alpha_a y_a)^2}{(1 - \gamma)(\alpha_a + \delta)^2} E [\theta^2 \mid \alpha, y] + E \left[\theta^2 \frac{\delta^2 (\theta^2 + \varepsilon^2) + 2\delta \alpha_a y_a \theta}{(1 - \gamma)(\alpha_a + \delta)^2} \mid \alpha, y \right] \\
&= \frac{\alpha_a + \delta + (\alpha_a y_a)^2}{(1 - \gamma)(\alpha_a + \delta)^2} E [\theta^2 \mid \alpha, y] + \frac{\delta^2 E [\theta^4 \mid \alpha, y] + \delta E [\theta^2 \mid \alpha, y] + 2\delta \alpha_a y_a E [\theta^3 \mid \alpha, y]}{(1 - \gamma)(\alpha_a + \delta)^2},
\end{aligned}$$

where we have used that $E [\varepsilon^2 \mid \alpha, y] = 1/\delta$. Hence, we have

$$E [\theta^2 K \mid \alpha, y] = \frac{\alpha_a + (\alpha_a y_a)^2 + 2\delta}{(1 - \gamma)(\alpha_a + \delta)^2} E [\theta^2 \mid \alpha, y] + \frac{\delta^2 E [\theta^4 \mid \alpha, y] + 2\delta \alpha_a y_a E [\theta^3 \mid \alpha, y]}{(1 - \gamma)(\alpha_a + \delta)^2}.$$

Next, we use the following expressions in order to compute $E [K^2 \mid \alpha, y]$.

$$\begin{aligned}
E [\theta^2 \mid \alpha, y] &= y^2 + \frac{1}{\alpha}, \\
E [\theta^3 \mid \alpha, y] &= y \left(y^2 + 3\frac{1}{\alpha} \right), \\
E [\theta^4 \mid \alpha, y] &= y^4 + 6y^2 \frac{1}{\alpha} + 3\frac{1}{\alpha^2}.
\end{aligned}$$

Thus,

$$\begin{aligned}
& E [K^2 \mid \alpha, y] \\
&= \frac{1}{(1 - \gamma)^2} E \left[\left[\left(\frac{\alpha_a y_a + \delta x}{\alpha_a + \delta} \right)^2 + \frac{1}{(\alpha_a + \delta)} \right]^2 \mid \alpha, y \right] \\
&= \frac{1}{(1 - \gamma)^2 (\alpha_a + \delta)^2} E \left[\frac{(\alpha_a y_a + \delta x)^4}{(\alpha_a + \delta)^2} + 2 \frac{(\alpha_a y_a + \delta x)^2}{(\alpha_a + \delta)} + 1 \mid \alpha, y \right] \\
&= \frac{1}{(1 - \gamma)^2 (\alpha_a + \delta)^2} E \left[\frac{(\alpha_a y_a)^4 + \delta^4 x^4 + 4\alpha_a y_a \delta^3 x^3 + 4(\alpha_a y_a)^3 \delta x + 6(\alpha_a y_a)^2 \delta^2 x^2}{(\alpha_a + \delta)^2} \right. \\
&\quad \left. + 2 \frac{(\alpha_a y_a)^2 + \delta^2 x^2 + 2\alpha_a y_a \delta x}{(\alpha_a + \delta)} + 1 \mid \alpha, y \right].
\end{aligned}$$

Hence,

$$\begin{aligned}
& (1 - \gamma)^2 (\alpha_a + \delta)^2 E [K^2 | \alpha, y] \\
&= \frac{(\alpha_a y_a)^4 + 2 (\alpha_a + \delta) (\alpha_a y_a)^2}{(\alpha_a + \delta)^2} + 1 + E \left[\frac{\delta^4 x^4 + 4 \alpha_a y_a \delta^3 x^3 + 4 [(\alpha_a y_a)^3 + (\alpha_a + \delta) \alpha_a y_a] \delta x}{(\alpha_a + \delta)^2} \right. \\
& \left. + \frac{[6 (\alpha_a y_a)^2 + 2 (\alpha_a + \delta)] \delta^2 x^2}{(\alpha_a + \delta)^2} | \alpha, y \right]. \tag{24}
\end{aligned}$$

In addition, we have

$$\begin{aligned}
E(x | \alpha, y) &= E(\theta | \alpha, y) + E(\varepsilon | \alpha, y) = y, \\
E(x^2 | \alpha, y) &= E((\theta + \varepsilon)^2 | \alpha, y) \\
&= E(\theta^2 + \varepsilon^2 + 2\theta\varepsilon | \alpha, y) \\
&= E_{|\alpha, y} \theta^2 + \frac{1}{\delta}, \\
E(x^3 | \alpha, y) &= E((\theta + \varepsilon)^3 | \alpha, y) \\
&= E(\theta^3 + \varepsilon^3 + 3\theta^2\varepsilon + 3\varepsilon^2\theta | \alpha, y) \\
&= E(\theta^3 + 3\varepsilon^2\theta | \alpha, y) \\
&= E_{|\alpha, y} \theta^3 + \frac{3}{\delta} y, \\
E(x^4 | \alpha, y) &= E((\theta + \varepsilon)^4 | \alpha, y) \\
&= E(\theta^4 + \varepsilon^4 + 4\theta^3\varepsilon + 4\varepsilon^3\theta + 6\theta^2\varepsilon^2 | \alpha, y) \\
&= E(\theta^4 + \varepsilon^4 + 4\varepsilon^3\theta + 6\theta^2\varepsilon^2 | \alpha, y) \\
&= E_{|\alpha, y} \theta^4 + E_{|\alpha, y} \varepsilon^4 + 4E_{|\alpha, y} \varepsilon^3 E_{|\alpha, y} \theta + 6E_{|\alpha, y} \theta^2 E_{|\alpha, y} \varepsilon^2 \\
&= E_{|\alpha, y} \theta^4 + \frac{3}{\delta^2} + \frac{6}{\delta} E_{|\alpha, y} \theta^2
\end{aligned}$$

where we have used the moments of the Normal distribution, $E_{|\alpha, y} \varepsilon^4 = \frac{3}{\delta^2}$ and $E_{|\alpha, y} \varepsilon^3 = 0$.

Replacing these expressions back into (24): $(1 - \gamma)^2 (\alpha_a + \delta)^2 E [K^2 | \alpha, y]$, and after some

algebra, we obtain:

$$\begin{aligned}
& (1 - \gamma)^2 (\alpha_a + \delta)^2 E [K^2 | \alpha, y] \\
&= \frac{(\alpha_a y_a)^4 + 2(\alpha_a + \delta)(\alpha_a y_a)^2}{(\alpha_a + \delta)^2} + 1 + \frac{3\delta^2}{(\alpha_a + \delta)^2} + \frac{[6(\alpha_a y_a)^2 + 2(\alpha_a + \delta)] \delta}{(\alpha_a + \delta)^2} \\
&+ \frac{4\delta [(\alpha_a y_a)^3 + (\alpha_a + \delta)\alpha_a y_a + 3\alpha_a y_a \delta]}{(\alpha_a + \delta)^2} y \\
&+ \frac{[6\delta + 6(\alpha_a y_a)^2 + 2(\alpha_a + \delta)] \delta^2}{(\alpha_a + \delta)^2} [E_{|\alpha, y} \theta^2] \\
&+ \frac{4\alpha_a y_a \delta^3}{(\alpha_a + \delta)^2} [E_{|\alpha, y} \theta^3] + \frac{\delta^4}{(\alpha_a + \delta)^2} [E_{|\alpha, y} \theta^4].
\end{aligned}$$

Thus, expected welfare is given by:

$$\begin{aligned}
2\mathcal{W}(q, k_i) &= E \left[u(q) - q + \theta^2 K - \int \frac{(k_i - \gamma K)^2}{2} di | \alpha, y \right] \\
&= u(q) - q + E \left[\theta^2 K - \frac{(1 - \gamma)^2}{2} K^2 | \alpha, y \right] \\
&= u(q) - q + E [\theta^2 K | \alpha, y] - \frac{(1 - \gamma)^2}{2} E [K^2 | \alpha, y] \\
&= u(q) - q + \frac{\alpha_a + (\alpha_a y_a)^2 + 2\delta}{(1 - \gamma)(\alpha_a + \delta)^2} [E_{|\alpha, y} \theta^2] + \frac{\delta^2 [E_{|\alpha, y} \theta^4] + 2\delta \alpha_a y_a [E_{|\alpha, y} \theta^3]}{(1 - \gamma)(\alpha_a + \delta)^2} \\
&- \frac{1}{2(\alpha_a + \delta)^2} \left[\frac{(\alpha_a y_a)^4 + 2(\alpha_a + \delta)(\alpha_a y_a)^2}{(\alpha_a + \delta)^2} + 1 + \frac{3\delta^2}{(\alpha_a + \delta)^2} + \frac{[6(\alpha_a y_a)^2 + 2(\alpha_a + \delta)] \delta}{(\alpha_a + \delta)^2} \right] \\
&- \frac{4\delta [(\alpha_a y_a)^3 + (\alpha_a + \delta)\alpha_a y_a + 3\alpha_a y_a \delta]}{2(\alpha_a + \delta)^4} y \\
&- \frac{[6\delta + 6(\alpha_a y_a)^2 + 2(\alpha_a + \delta)] \delta^2}{2(\alpha_a + \delta)^4} [E_{|\alpha, y} \theta^2] \\
&- \frac{4\alpha_a y_a \delta^3}{2(\alpha_a + \delta)^4} [E_{|\alpha, y} \theta^3] - \frac{\delta^4}{2(\alpha_a + \delta)^4} [E_{|\alpha, y} \theta^4].
\end{aligned}$$

Combining terms, we get the following expression for the expected welfare:

$$\begin{aligned}
2\mathcal{W}(q, k_i) &= u(q) - q + F(\alpha_a, \alpha, y) = u(q) - q \\
&- \left[\frac{(\alpha_a y_a)^4 + 2(\alpha_a + \delta)(\alpha_a y_a)^2}{2(\alpha_a + \delta)^4} + \frac{1}{2(\alpha_a + \delta)^2} + \frac{3\delta^2}{2(\alpha_a + \delta)^4} + \frac{[3(\alpha_a y_a)^2 + (\alpha_a + \delta)]\delta}{(\alpha_a + \delta)^4} \right] \\
&- \frac{2\delta [(\alpha_a y_a)^3 + (\alpha_a + \delta)\alpha_a y_a + 3\alpha_a y_a \delta]}{(\alpha_a + \delta)^4} y \\
&+ \left[\frac{\alpha_a + (\alpha_a y_a)^2 + 2\delta}{(1 - \gamma)(\alpha_a + \delta)^2} - \frac{[3\delta + 3(\alpha_a y_a)^2 + (\alpha_a + \delta)]\delta^2}{(\alpha_a + \delta)^4} \right] [E_{|\alpha, y} \theta^2] \\
&+ \left[\frac{2\delta \alpha_a y_a}{(1 - \gamma)(\alpha_a + \delta)^2} - \frac{2\alpha_a y_a \delta^3}{(\alpha_a + \delta)^4} \right] [E_{|\alpha, y} \theta^3] \\
&+ \left[\frac{\delta^2}{(1 - \gamma)(\alpha_a + \delta)^2} - \frac{\delta^4}{2(\alpha_a + \delta)^4} \right] [E_{|\alpha, y} \theta^4],
\end{aligned}$$

where

$$\begin{aligned}
E[\theta^2 | \alpha, y] &= y^2 + \frac{1}{\alpha}, \\
E[\theta^3 | \alpha, y] &= y \left(y^2 + 3\frac{1}{\alpha} \right), \\
E[\theta^4 | \alpha, y] &= y^4 + 6y^2 \frac{1}{\alpha} + 3\frac{1}{\alpha^2}.
\end{aligned}$$

Proof of Proposition 1: The derivative of the welfare function with respect to α_a ,

$\frac{\partial 2\mathcal{W}(\alpha_a, \alpha, y)}{\partial \alpha_a}$, is given by:

$$\begin{aligned}
&- \frac{1}{1 - \gamma} \frac{1}{\alpha^2 (\delta + \alpha_a)^5} (6\gamma\delta^4 - 12\alpha\delta^3 + 15\alpha\gamma\delta^3 + \alpha\alpha_a^3 + 12\delta^3\alpha_a + 5\alpha\delta\alpha_a^2 + 4\alpha\delta^2\alpha_a \\
&- 5\alpha^2\delta\alpha_a + 3\alpha\gamma\delta^2\alpha_a + 5\alpha^2\gamma\delta\alpha_a - 10\alpha^2\delta^2 + 10\alpha^2\gamma\delta^2 + 6y^2\alpha\gamma\delta^4 - \alpha^2\alpha_a^2 + 6\delta^2\alpha_a^2 \\
&+ \alpha^2\gamma\alpha_a^2 + 4y^2\alpha\delta\alpha_a^3 + 4y^2\alpha\delta^3\alpha_a + 12y^2\alpha\gamma\delta^3\alpha_a - 4y^2\alpha^2\delta^3 + 7y^2\alpha^2\gamma\delta^3 + 8y^2\alpha\delta^2\alpha_a^2 \\
&+ y^2\alpha^2\gamma\alpha_a^3 - 4y^2\alpha^2\delta\alpha_a^2 - 8y^2\alpha^2\delta^2\alpha_a + 6y^2\alpha\gamma\delta^2\alpha_a^2 + 9y^2\alpha^2\gamma\delta\alpha_a^2 + 15y^2\alpha^2\gamma\delta^2\alpha_a). \quad (25)
\end{aligned}$$

For $\alpha \neq 0$ and $\delta \neq 0$, evaluating this expression at $\alpha = \alpha_a$ yields:

$$-\frac{\gamma}{1 - \gamma} \frac{1}{\alpha^2 (\delta + \alpha)^3} (3\delta\alpha + \alpha^2 + 6\delta^2 + y^2\alpha^3 + 6y^2\alpha\delta^2 + 7y^2\alpha^2\delta) \leq 0. \quad (26)$$

In particular, for $\gamma > 0$, announcing $\alpha = \alpha_a$ is not optimal. Moreover, since

$\frac{\partial}{\partial \alpha_a} \mathcal{W}(q, k_i) |_{\alpha=\alpha_a} < 0$, if the CB believes that investors will use its announcement, it prefers to announce an α_a that is lower than the true α . ■

Proof of Proposition 2: The single crossing property is satisfied if for all y ,²²

$$\frac{\partial \mathcal{W}(\alpha_a, \alpha, y)}{\partial \alpha_a \partial \alpha} > 0,$$

which implies that the marginal benefit of announcing α_a is increasing in the CB's type.

Notice that

$$\mathcal{W}(\alpha_a, \alpha, y) = u(q) - q + F(\alpha_a, \alpha, y).$$

Since $u'(q) = 1 + r(\alpha_a)$, the derivative of $u(q) - q$ is independent of the CB's type α so that

$$\frac{\partial [u(q) - q]}{\partial \alpha_a \partial \alpha} = 0.$$

Therefore, the single crossing property is satisfied whenever

$$\frac{\partial F(\alpha_a, \alpha, y)}{\partial \alpha_a \partial \alpha} > 0.$$

In the case where there are two possible precisions, α_L and α_H , the single crossing property implies:

$$\frac{\partial F(\alpha_a, \alpha_H, y)}{\partial \alpha_a} > \frac{\partial F(\alpha_a, \alpha_L, y)}{\partial \alpha_a}. \quad (27)$$

Evaluated at $\alpha_a = \alpha_L$, this is the condition we are checking.

The first-order derivative of the expected welfare with respect to α_a , $\frac{\partial \mathcal{W}(\alpha_a, \alpha, y)}{\partial \alpha_a}$, is given in

²²See <http://web.mit.edu/athey/www/scpexist.pdf> definition 1 on page 5 and the paragraph that follows.

(25). We have that $\frac{\partial \mathcal{W}(\alpha_a, \alpha_L, y)}{\partial \alpha_a} \Big|_{\alpha_a = \alpha_L}$ is given by

$$-\frac{1}{2} \frac{\gamma}{1 - \gamma} \frac{1}{\alpha_L^2 (\delta + \alpha)^3} (3\delta\alpha_L + \alpha_L^2 + 6\delta^2 + y^2\alpha_L^3 + 6y^2\alpha_L\delta^2 + 7y^2\alpha_L^2\delta) < 0, \quad (28)$$

while $\frac{\partial \mathcal{W}(\alpha_a, \alpha_H, y)}{\partial \alpha_a} \Big|_{\alpha_a = \alpha_L}$ is given by

$$\begin{aligned} & -\frac{1}{2} \frac{1}{1 - \gamma} \frac{1}{(\alpha_L + \varepsilon)^2 (\delta + \alpha_L)^5} (6\gamma\delta^4 - 12(\alpha_L + \varepsilon)\delta^3 + 15(\alpha_L + \varepsilon)\gamma\delta^3 \\ & + (\alpha_L + \varepsilon)\alpha_L^3 + 12\delta^3\alpha_L + 5(\alpha_L + \varepsilon)\delta\alpha_L^2 + 4(\alpha_L + \varepsilon)\delta^2\alpha_L - 5(\alpha_L + \varepsilon)^2\delta\alpha_L \\ & + 3(\alpha_L + \varepsilon)\gamma\delta^2\alpha_L + 5(\alpha_L + \varepsilon)^2\gamma\delta\alpha_L - 10(\alpha_L + \varepsilon)^2\delta^2 + 10(\alpha_L + \varepsilon)^2\gamma\delta^2 \\ & + 6y^2(\alpha_L + \varepsilon)\gamma\delta^4 - (\alpha_L + \varepsilon)^2\alpha_L^2 + 6\delta^2\alpha_L^2 + (\alpha_L + \varepsilon)^2\gamma\alpha_L^2 + 4y^2(\alpha_L + \varepsilon)\delta\alpha_L^3 \\ & + 4y^2(\alpha_L + \varepsilon)\delta^3\alpha_L + 12y^2(\alpha_L + \varepsilon)\gamma\delta^3\alpha_L - 4y^2(\alpha_L + \varepsilon)^2\delta^3 + 7y^2(\alpha_L + \varepsilon)^2\gamma\delta^3 \\ & + 8y^2(\alpha_L + \varepsilon)\delta^2\alpha_L^2 + y^2(\alpha_L + \varepsilon)^2\gamma\alpha_L^3 - 4y^2(\alpha_L + \varepsilon)^2\delta\alpha_L^2 - 8y^2(\alpha_L + \varepsilon)^2\delta^2\alpha_L \\ & + 6y^2(\alpha_L + \varepsilon)\gamma\delta^2\alpha_L^2 + 9y^2(\alpha_L + \varepsilon)^2\gamma\delta\alpha_L^2 + 15y^2(\alpha_L + \varepsilon)^2\gamma\delta^2\alpha_L), \end{aligned} \quad (29)$$

where we wrote α_H as $\alpha_L + \varepsilon$. By continuity, for ε sufficiently small, this expression is also negative. By (27), we are checking that

$$\frac{\partial \mathcal{W}(\alpha_a, \alpha_H, y)}{\partial \alpha_a} \Big|_{\alpha_a = \alpha_L} - \frac{\partial \mathcal{W}(\alpha_a, \alpha_L, y)}{\partial \alpha_a} \Big|_{\alpha_a = \alpha_L} > 0. \quad (30)$$

The above expression is given by:

$$\begin{aligned} & \varepsilon \frac{1}{2} \frac{1}{1 - \gamma} \frac{1}{\alpha_L^2 (\delta + \alpha_L)^5 (\alpha_L + \varepsilon)^2} (\alpha_L^5 + 6\gamma\delta^4\varepsilon + 5\delta\alpha_L^4 + \varepsilon\alpha_L^4 + 12\gamma\delta^4\alpha_L \\ & + 5\delta\varepsilon\alpha_L^3 + 15\gamma\delta^3\varepsilon\alpha_L + 16\delta^2\alpha_L^3 + 12\delta^3\alpha_L^2 + 4y^2\delta\alpha_L^5 + 3\gamma\delta^2\alpha_L^3 + 15\gamma\delta^3\alpha_L^2 \\ & + 10\delta^2\varepsilon\alpha_L^2 + 4y^2\delta\varepsilon\alpha_L^4 + 3\gamma\delta^2\varepsilon\alpha_L^2 + 6y^2\gamma\delta^4\varepsilon\alpha_L + 8y^2\delta^2\alpha_L^4 + 4y^2\delta^3\alpha_L^3 \\ & + 6y^2\gamma\delta^2\alpha_L^4 + 12y^2\gamma\delta^3\alpha_L^3 + 6y^2\gamma\delta^4\alpha_L^2 + 8y^2\delta^2\varepsilon\alpha_L^3 + 4y^2\delta^3\varepsilon\alpha_L^2 + 6y^2\gamma\delta^2\varepsilon\alpha_L^3 \\ & + 12y^2\gamma\delta^3\varepsilon\alpha_L^2). \end{aligned} \quad (31)$$

Note that since $\varepsilon > 0$, this expression is positive. Turning to the investors, since the

truthful information revealed by the CB is useful in this case, it is a best response for investors to use this information. ■

An equivalence result: Here we demonstrate how the CB can in principle use monetary policy in order to convey *both* the realized value of its signal and its confidence in the signal through manipulating the interest rate. We consider again the case where the precision of the CB's signal, α , can take on two values: $\{\alpha_L, \alpha_H\}$, where $\alpha_L < \alpha_H$. The probability that $\alpha = \alpha_L$ is denoted by π . As before, the CB receives a signal y about the value of θ . The CB also knows the realization of the precision of its signal, α . It can choose to convey the value of y and α to the private sector through monetary policy. This can be accomplished via a rule that takes the following form:

$$r = h(y, \alpha), \tag{32}$$

with $h(y, \alpha) = 0$, if the CB does not reveal its information in state (y, α) . Can the CB use monetary policy in order to reveal its information about *both* its signal and that signal's precision? Put differently, is the (expanded) information set of the CB equivalent to the set of feasible monetary policies? To give an affirmative answer, we need to demonstrate that there exists a homeomorphism between these two sets.

Since the CB needs to signal both y and α to the private sector, it must be the case that $h(y, \alpha_L) \neq h(y', \alpha_H)$ for any y, y' . Let \hat{r} stand for the minimum level of the interest rate that makes the corresponding information revelation credible. For any $\Delta > \hat{r} > 0$, we have the following.

Proposition 3 *Let $h : \mathbb{R} \rightarrow (0, \Delta)$ be a homeomorphism where $\Delta > 0$. Then, the function $H(y, \alpha) : \mathbb{R} \times \{L, H\} \rightarrow (0, \Delta) \cup (\hat{r}, \hat{r} + \Delta)$ given by*

$$H(y, \alpha) = \begin{cases} h(y) & \text{if } \alpha = \alpha_L \\ h(y) + \hat{r} & \text{if } \alpha = \alpha_H \end{cases}, \tag{33}$$

is a homeomorphism.

Proof. We impose the usual topology on \mathbb{R} , and the usual subspace topologies on $(0, \Delta)$ and on $(0, \Delta) \cup (\hat{r}, \hat{r} + \Delta)$. We impose the discrete topology on $\{\alpha_L, \alpha_H\}$. The resulting topology on $\mathbb{R} \times \{\alpha_L, \alpha_H\}$ is the product topology. It is straightforward to show that $H(y, \alpha)$ satisfies all the conditions for a homeomorphism: $H(y, \alpha)$ is injective, surjective, and continuous, and $H^{-1}(y, \alpha)$ is continuous. ■

The proof readily generalizes to a countable set of possible precision values. This demonstrates the (topological) equivalence between the set describing the information potentially held by the CB and the set of feasible and credible monetary policies at the CB's disposal. At the cost of imposing a distortion associated with some inflation, the CB can use monetary policy in order to reveal both its signal about fundamentals and its confidence in that signal to the private sector.

The figure below plots a candidate monetary policy H as a function of the CB's signal y and its confidence in the signal α . The interest rate path is given by the red line, $h(y)$, if $\alpha = \alpha_L$ and by the blue line, $h(y) + \hat{r}$, if $\alpha = \alpha_H$. If the CB follows such a policy, it can convey both the realized y and α through manipulating the interest rate.

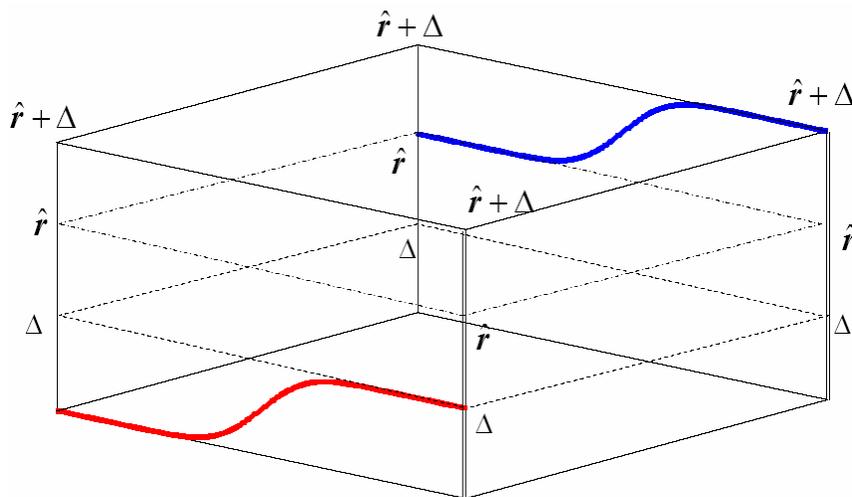


Figure 2: Monetary policy as a homeomorphism