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Philip Jung
University of Mannheim

Keith Kuester
Federal Reserve Bank of Philadelphia

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The (Un)importance of Unemployment Fluctuations for the Welfare Cost of Business Cycles*

Philip Jung
University of Mannheim

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Abstract

This paper studies the cost of business cycles within a real business cycle model with search and matching frictions in the labor market. We endogenously link both the cyclical fluctuations and the mean level of unemployment to the aggregate business cycle risk. The key result of the paper is that business cycles are costly: Fluctuations over the cycle induce a higher average unemployment rate since employment is non-linear in the job-finding rate and the past unemployment rate. We show this analytically for a special case of the model. We then calibrate the model to U.S. data. For the calibrated model, too, business cycles cause higher average unemployment; the welfare cost of business cycles can easily be an order of magnitude larger than Lucas' (1987) estimate. The cost of business cycles is the higher the lower the value of non-employment, or, respectively, the lower the disutility of work. The ensuing cost of business cycles rises further when workers' skills depreciate during unemployment.

JEL Classification System: E32,E24,J64

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* Correspondence: Philip Jung, Department of Economics, University of Mannheim, 68131 Mannheim, Germany, e-mail: p.jung@vwl.uni-mannheim.de, Tel: +49 621 1811854. Keith Kuester, Research Department, Federal Reserve Bank of Philadelphia, Ten Independence Mall, Philadelphia, PA, 19106-1574, USA, e-mail: keith.kuester@phil.frb.org, Tel: +1 215 574-3415. Without implicating, we would like to thank Gadi Barlevy, Wouter den Haan, Shigeru Fujita, Marcus Hagedorn, Michalis Haliassos, Tom Krebs, Dirk Krueger, Lars Ljungqvist, and Makoto Nakajima for comments, and Gisle Natvik, Franck Portier, and Pedro Silos for their discussions of the paper. Earlier versions of this paper circulated under the titles "The (Un)importance of Unemployment Fluctuations for Welfare" and "The Cost of Unemployment Fluctuations Revisited." This paper is a revised version of our Federal Reserve Bank of Philadelphia Working Paper 08-31. Comments from participants at the following conferences and seminars are gratefully acknowledged: Search and Matching Workshop Philadelphia, 2009, New York/Philadelphia Workshop on Quantitative Macroeconomics, 2008, Oslo Workshop on Monetary Policy, 2008, Bank of Canada, Tilburg University, Norges Bank, "Using Dynamic Economic Models to Make Policy Recommendations" in San Sebastian, 2007, and the North American Summer Meeting of the Econometric Society, 2007.

1 Introduction

There is a large literature that computes the cost of business cycles. Most of the studies focus on fluctuations around a given mean level of economic activity, and typically find small estimates of the aggregate cost of cyclical fluctuations; see Lucas (2003) for an overview. The current paper instead puts the effect of the business cycle on average employment front and center. In particular, the paper points out that models with labor market matching frictions along the lines of influential papers by Hall (2005) and Shimer (2005) imply an endogenous link between the business cycle and *both* fluctuations of unemployment risk and the mean of unemployment. When we calibrate the matching model in line with U.S. data, we find a higher average unemployment rate in the stochastic steady state than in the non-stochastic steady state. This renders economic volatility costly and might rationalize why economic volatility ranks so high on the public's agenda.[1]

The previous literature has largely assumed that the business cycle does not affect average unemployment. As a result, if equilibrium prices would not react to the business cycle there would be no welfare cost of such business cycles, or only a very small cost; *e.g.,* Atkeson and Phelan (1994), Krusell and Smith (1999). The effect that we wish to stress, instead, matters precisely because it affects the average unemployment rate. Through the employment-flow equation, we first show that a negative covariance between job-finding and unemployment, as is found in the data, can imply higher mean unemployment. The reason for this is that the number of new matches between firms and workers is the *product* of the job-finding rate and unemployment. A negative covariance between the two implies that unemployed workers are more likely to find a job in a boom, when there are few unemployed workers to start with, than in a recession, when many workers are unemployed. To the extent that the business cycle leaves average job-finding rates untouched, the stronger the business cycle fluctuations are the higher is the average unemployment rate. Thus, a reduction in business cycle volatility leads to higher employment and more consumption; and higher welfare.

We describe a real business cycle model with matching frictions in the labor market. For a special case of our model with risk-neutral consumers and no capital accumulation, we can prove

[1] For example, 70% of the respondents in Shiller's (1997) survey, economists and laymen alike, say that preventing recessions is important. More than 80% of these agree that smoothing out both recessions and booms is preferable to having a business cycle. Wolfers (2003) uses surveys on subjective well-being. He finds that eliminating unemployment volatility would raise well-being by an amount roughly equal to that from lowering the average level of unemployment by a quarter of a percentage point.

analytically that the business cycle indeed does not affect the mean job-finding rate. Using a closed-form second-order approximation for the mean increase in unemployment, we show that the more volatile productivity is, the more persistent it is, and the more the job-finding rate reacts to the cycle, the more does mean unemployment exceed its steady-state level.

When we calibrate the model to U.S. data, in both the special case and in the general case involving risk-aversion and capital, we find a cost of business cycles that is an order of magnitude larger than Lucas' (1987) estimate. For example, for log-utility in our calibrated model the average unemployment rate is 0.21 percentage point higher than in the non-stochastic steady state. For this case, depending on the disutility of work, we obtain a cost of business cycles of up to 0.19% of steady-state consumption. It is interesting to note that the higher mean unemployment in our model causes lower returns to capital. In turn, for lower degrees of risk-aversion the capital stock in the stochastic economy is therefore than in the non-stochastic steady state – by 0.14% for log-utility. This effect contrasts with the previous literature that typically finds that consumers can dissave along the transition path to the non-stochastic steady state; *e.g.,* Krusell and Smith (1999).

Beyond the effects described, the rise in mean unemployment has further repercussions, which we explore. In particular, in an extension we allow for an interaction of the mean effect on unemployment and average skills in the economy. This is motivated by the well-documented long-term earnings losses of displaced workers (see, *e.g.,* Jacobson, LaLonde, and Sullivan, 1993) that have been identified as an important candidate for causing costs of business cycles. Krebs (2007), for example, assumes that business cycles induce a mean-preserving spread in individual income risk; *i.e.*, he emphasizes changes in the *second* moment of individual risk. In his case, this leads to a significant cost of business cycles. In our formulation, instead, the effects are driven by a higher *mean* level of unemployment that translates into a lower mean level of skills due to an increase in skill losses off the job. We show that this composition effect is non-negligible. It results in a cost of business cycles that is about one third higher than the cost that we find in the absence of skill transitions.

The previous literature has typically abstracted from mean effects on employment; *e.g.,* Lucas (1987). In a seminal paper, in an economy with heterogeneous agents, Krusell and Smith (1999) highlight that the cost of business cycles varies with employment and wealth status. They find that on average, however, the cost is small. The reason is that capital/savings typically allow consumers to self-insure against transitory income fluctuations. Recently, Krusell, Mukoyama, Sahin, and Smith (2008) have revisited that calculation and allowed for a greater reduction of idiosyncratic risk when the aggregate shock is eliminated, but retained the assumption that average unemployment is not affected by the size of business cycle fluctuations. They find a welfare cost of business cycles that is on average an order of magnitude larger than Lucas' (1987) estimate. This estimate is similar in size to ours. Interestingly, however, the sources of the cost of business cycles in our papers differ notably. In their model, consumers are imperfectly insured against idiosyncratic income risk, and the reduction in aggregate risk helps to alleviate the obstacles to consumption-smoothing. In our model, instead, as in Merz (1995) and Andolfatto (1996), we assume that workers perfectly insure each others' consumption against idiosyncratic income and unemployment risk. The cost of business cycles that we find arises from an increase in economy-wide average unemployment that is caused by fluctuations in the job-finding rate.[2] Storesletten, Telmer, and Yaron (2001) and Krebs (2003) assume that the cross-sectional variation of idiosyncratic human capital risk increases in recessions and shrinks in booms for all workers. In this paper, we do not account for this pattern and the corresponding component of the cost of business cycles. Beaudry and Pages (2001) analyze the welfare cost of business cycles when the contractual structure in the labor market insures existing workers against wage cuts, while workers who are laid off in a recession enter the labor market at a lower wage level. In our paper, too, wages of re-entrants into the labor market are persistently lower in recessions than in booms when we interact the business cycle with skill transitions, due to a loss of skills off the job.

Fewer papers emphasize that mean effects can generate costly business cycles. In a cross-country study, Ramey and Ramey (1995) document a negative relationship between economic volatility and growth. Barlevy (2004) points out that in an economy with endogenous growth and decreasing returns to investment, eliminating cycles increases average growth rates. Mean effects of business cycles are wider-spread in New Keynesian business cycle models in which real and nominal frictions imply that fluctuations induce an inefficient utilization of resources; *e.g.*, Levin, Onatski, Williams, and Williams (2005). Finally, in complementary independent work Hairault, Langot, and Osotimehin (2008) also exploit the non-linearity of the employment-flow equation in matching models. In a model without capital, they focus on risk-neutral workers

[2] Krusell, Mukoyama, Sahin, and Smith (2008) also examine the cost of business cycles when accounting for two states of unemployment: short-term and long-term. When they allow that the elimination of business cycles significantly reduces the risk of long-term unemployment, a case we do not examine here, they find welfare costs that are two orders of magnitude larger than Lucas' number.

throughout and largely on constant wages, following Hall (2005). The details of the bargaining – and, in parts, the calibration[3] – differ from our paper, yet we capture several of their results in our special case that forms the basis for Propositions 2 and 3. In addition, we make the link between the cost of cycles, the level of mean unemployment and the mean level of skills in the economy explicit. Barlevy (2005) provides a broader overview of the literature on the welfare cost of business cycles, concluding that business cycles are likely costly – as we do. Lucas' (2003) survey touches only marginally on effects of the business cycle on the means of economic variables and arrives at the opposite conclusion.

The remainder of the paper is organized as follows. In our model, the key to the welfare cost of business cycles is that business cycles cause a higher unemployment rate on average. Section 2 shows how the non-linearity of the employment-flow equation can generate these mean effects. Section 3 describes a real business cycle (RBC) model with search and matching frictions. Section 4 shows that in a special case with risk-neutral workers, and in the absence of capital, the mean job-finding rate is indeed not affected by the degree of cyclical fluctuations. It also shows analytically that the more volatile the economy is, the more does mean unemployment exceed the steady-state level of unemployment. Section 5.1 discusses the calibration of the model to U.S. data. Section 5.2 discusses how we measure the welfare cost of business cycles. Section 5.3 presents estimates of the welfare cost of business cycles, and Section 5.4 conducts sensitivity analyses. Section 6 discusses how the results are affected when allowing for employment-dependent skill transitions, highlighting the effects of the business cycle on the average skill level. A final section concludes.

2 The employment-flow equation and mean employment

The business cycle is costly in our model because cyclical fluctuations cause a higher average unemployment rate, and thus lower consumption, than would be observed in the absence of business cycles. Before going to the general equilibrium model, we here seek to highlight the importance of the non-linearity of the employment-flow equation and the behavior of the job-finding rate. In the following, we assume that the mass of workers is normalized to one and that workers can either be employed or unemployed. There is a mass u_t of unemployed workers who

[3] For example, Hairault, Langot, and Osotimehin (2008) calibrate their model to a steady-state unemployment rate that is twice as big as in our calibration. As a result, they obtain a higher welfare cost of business cycles. Our Proposition 3 explains the reason for this.

are looking for a job. An unemployed worker will be matched with a firm with probability f_t. New matches are effective from $t+1$ onward. The remaining mass of workers, $e_t = 1-u_t$, is employed. Each period, as in Shimer (2005) and Hall (2005), a constant share ϑ of employed workers becomes unemployed. As a result, employment evolves according to the following employment-flow equation

$$e_t = (1-\vartheta)(e_{t-1}) + f_{t-1}u_{t-1}. \tag{1}$$

It is important to note that unemployment and the job-finding rate enter non-linearly in the final term of equation (1).[4] We are interested in comparing the unconditional average of unemployment, $E\{u_t\}$, in an economy with aggregate fluctuations ("mean unemployment" henceforth) to the non-stochastic steady state of that same economy when aggregate fluctuations are eliminated (the "steady state" henceforth). The steady-state value of any variable is denoted by dropping the time index; e.g., u for steady-state unemployment. We have the following sufficient (but not necessary) condition for the mean effects that we discuss in this paper:

Proposition 1. *Suppose that the employment-flow equation* (1) *holds. Suppose further that all variables in the employment-flow equation are covariance stationary. Then the following are sufficient conditions for mean unemployment to exceed steady-state unemployment,* i.e., *for* $E\{u_t\} > u$*: i) the job-finding rate and the unemployment rate are non-positively correlated,* $Cov(u_t, f_t) \leq 0$*, ii) the average job-finding rate does not exceed the steady-state job-finding rate,* $E\{f_t\} \leq f$*, and at least one of the inequalities in* i) *and* ii) *holds strictly.*

Proof. Using the assumed stationarity, (1) implies $\vartheta E\{1-u_t\} = COV(f_t, u_t) + E\{f_t\}E\{u_t\}$. Subtracting the steady-state version of (1) from both sides of the above equation, we have that $-\vartheta[E\{u_t\}-u] = COV(f_t,u_t) + E\{f_t\}E\{u_t\} - fu$, or equivalently $-\vartheta[E\{u_t\}-u] = COV(f_t,u_t) + [E\{f_t\}-f]E\{u_t\} + f[E\{u_t\}-u]$, so

$$E\{u_t\} - u = -\frac{1}{\vartheta + f}\left(COV(f_t,u_t) + [E\{f_t\}-f]E\{u_t\}\right).$$

The proposition follows by recognizing that $E\{u_t\} > 0$. □

Proposition 1 highlights that mean unemployment can be affected by business cycle fluctuations. Condition i) in the proposition is not particularly demanding. Empirically, it holds with a strict inequality. Indeed, the sign of the covariance in i) is at the core of the search and matching model with exogenous separation that employment-flow equation (1) forms part of; *e.g.*, Hagedorn and

[4] An alternative representation of (1), that also makes the non-linearity in unemployment apparent, is:

$$u_t = \vartheta + u_{t-1}(1-\vartheta-f_{t-1}).$$

Manovskii (2008). Namely, unemployment fluctuations arise in the model since unemployed workers are more likely to find a job in a boom – when there are fewer unemployed workers to start with – than in a recession, when many workers are unemployed. Condition ii) is less general. While we will present special cases below in which the job-finding rate is exactly linear in the technology shock, so $E\{f_t\} = f$, and highlight cases in which the inequality in ii) holds strictly, one cannot make a model-free claim for this condition. This is the reason why we build a general equilibrium model in Section 3, to quantitatively evaluate the effect of the business cycle on mean unemployment, and to subsequently assess the welfare cost of business cycles. In this sense, Proposition 1 serves an illustrative purpose: it presents sufficient conditions for the business cycle to cause higher mean unemployment. These conditions are not necessary conditions, though. In particular, mean unemployment can also exceed the steady-state level of unemployment if $E\{f_t\} > f$, as is the case in some of the calibrations in Section 5.3.

3 The model

The precise interplay of the job-finding rate, unemployment, and aggregate cyclical risk is important for the cost of business cycles in this paper. This Section lays out a real business cycle model with matching frictions along the lines of Pissarides (1985) and Shimer (2005) that generates this interplay endogenously. The wage bargaining follows Hall and Milgrom (2008). Section 4 proves that for a special case of this model, with linear utility and in the absence of capital, the job-finding rate is indeed linear in productivity, so Proposition 1 holds. We derive a closed-form expression for the effect of the business cycle on the mean of unemployment for this case.

To be able to focus most clearly on the mean effects that we wish to stress, we introduce a family structure as in Merz (1995) and Andolfatto (1996). Family members pool their assets, and incomes from market work. They thereby perfectly insure individual consumption streams against individual unemployment risk.

Consumption and investment

There is a measure of size one of identical families in the economy. Each family consists of a unit measure of members. In period t, a measure e_t of these are employed and a measure $u_t = 1 - e_t$

of family members are unemployed. Individual family members, $i \in [0,1]$, have preferences

$$E_t \left\{ \sum_{j=0}^{\infty} \beta^j \left[\mathtt{u}(c_{i,t+j}) - \zeta I(i \text{ is employed in } t+j) \right] \right\},$$

where

$$\mathtt{u}(c_t) = \begin{cases} \log(c_t) & \text{if } \sigma = 1. \\ \frac{c_t^{1-\sigma}}{1-\sigma} & \text{if } \sigma \geq 0,\ \sigma \neq 1. \end{cases}$$

Above, $\zeta \geq 0$ indexes the disutility of work, and $I(\cdot)$ denotes the indicator function. We later seek to illustrate that, through the effects on mean (un)employment, the welfare cost of business cycles depends crucially on the society's value of non-employment relative to the value of market work. We use ζ to trace out the gap between the utility from market work and non-employment. Alternatively, we could have modeled the gap between employment and non-employment through introducing home production.[5] The family collects and distributes all income, maximizing the sum of expected utilities of its individual members. As a result, the family equates the marginal utility of consumption of each member, and, as a consequence of additive separability of the utility function, all members consume the same amount. The welfare of the family is given by

$$W_t = \mathtt{u}(c_t) - e_t\zeta + \beta E_t\{W_{t+1}\}. \tag{2}$$

We use this welfare function to calculate the cost of business cycles. The family's budget constraint is

$$c_t + i_t = e_t w_t + r_t k_t + \Psi_t. \tag{3}$$

Here i_t marks real investment. The terms $e_t w_t$ are the real wages earned by employed household members. k_t is the amount of physical capital owned by the family at the beginning of the period. The real rental rate of capital is r_t. Ψ_t denotes income arising from the firms' profits, described below in equation (7). Capital evolves according to

$$k_{t+1} = k_t(1-\delta) + i_t, \tag{4}$$

where $\delta \geq 0$ is the rate of depreciation. The family maximizes its objective (2) by choosing investment, i_t, and consumption, c_t, subject to (3) and (4). The first-order condition for investment is:

$$1 = E_t \left\{ \beta_{t,t+1} \left[(1-\delta) + r_{t+1} \right] \right\}.$$

[5] For the exercises that we will show below, varying the disutility of work, ζ, neither changes the steady state of the economy nor the dynamics. We believe that this is the cleanest way to illustrate the dependence of the welfare cost of business cycles on the gap between the value of market work and non-employment. The results would be similar, however, if we used home production to motivate that gap.

where $\beta_{t,t+j} = \beta\frac{\lambda_{t+j}}{\lambda_t}$ is the stochastic discount factor, and $\lambda_t := c_t^{-\sigma}$ is the family's marginal utility of consumption. The optimal consumption plan satisfies the transversality condition

$$\lim_{j\to\infty} E_t \left\{\beta_{t,t+j} k_{t+j}\right\} = 0, \ \forall t.$$

Goods markets

There are two competitive sectors of production. One sector produces a homogeneous "labor good." The other, final sector uses the labor good and physical capital to produce "output," y_t. Output is produced according to

$$y_t = A_t k_t^{\alpha} l_t^{1-\alpha}, \ \alpha \in (0,1).$$

Technology, A_t, is given by

$$A_t - A = \rho(A_{t-1} - A) + \epsilon_t^A, \ \rho \in [0,1),$$

where $\epsilon_t^A \overset{iid}{\sim} N(0, \sigma_A^2)$. Final good firms can rent capital and the labor good in competitive markets at rates r_t and x_t, respectively. The demand functions for capital and the labor good are, respectively, $k_t = \frac{\alpha}{r_t} y_t$, and $l_t = \frac{1-\alpha}{x_t} y_t$. Each firm-worker match constitutes a one-worker labor firm that produces one unit of the labor good. As a result, total supply of the labor good is given by the number of employed workers

$$l_t = e_t.$$

Labor market

The timing of the labor market is as follows. Workers who are already matched with firms bargain about wages. Production takes place. New matches are determined. Separations occur. As a result, employment evolves according to employment-flow equation (1). Period profits from production of a labor firm, Υ_t, are given by

$$\Upsilon_t = x_t - w_t,$$

where x_t denotes the price of the labor good and w_t is the wage paid to the firm's worker. At the end of the period, after production has taken place, the match is severed with probability ϑ. As a result, the value of a labor-firm in period t, J_t, is given by

$$J_t = \Upsilon_t + (1-\vartheta) E_t\{\beta_{t,t+1} J_{t+1}\}.$$

Firms and workers bargain about their share of the overall match surplus. In this paper, we adopt a bargaining mechanism analyzed by Hall and Milgrom (2008), who assume that the outside option in the bargaining process is to delay the bargaining by one period,[6] a "strike." A worker, whose bargaining is delayed, faces the same disutility, ζ, from striking than if he were working productively. In addition, the worker receives a stream of income in the periods in which the bargaining is delayed, labeled $\pi \geq 0$. In equilibrium, under complete information rational firms and workers would never delay the bargaining but instead they would agree on a wage immediately. A strike thus would never actually occur. We follow den Haan, Ramey, and Watson (2000) in assuming that the family bargains on behalf of its workers. The surplus for the family of having a marginal member employed rather than on strike, Δ_t, is

$$\Delta_t = w_t - \pi.$$

When working, the worker earns the wage but loses the strike payment. The firm's surplus from settling the bargaining in period t rather than deferring it to $t+1$ is given by period profits Υ_t. Each period, wages are determined by means of bargaining over the match surplus, where $\eta \in (0,1)$ denotes the family's bargaining power

$$\max_{w_t} (\Delta_t)^{\eta} (\Upsilon_t)^{1-\eta}.$$

The first-order condition for wages yields that earnings are a convex combination of the firm's revenue and the terms determining the bargaining position (remuneration when delaying the bargaining):

$$w_t = \eta x_t + (1-\eta)\pi. \tag{5}$$

This wage equation resembles the standard Nash bargaining solution when the outside option is unemployment rather than delaying the bargaining, except for two differences. First, labor-market tightness would enter the wage equation. Second, a term in disutility of work, $\frac{\zeta}{\lambda_t}$, would enter instead of the strike value, π. Parameter π captures a shift in the bargaining position of the worker not related to utility flows in equilibrium. There are two reasons for this way of modeling: first, as we will discuss further below, the fluctuation of the job-finding rate is key for the mean effect on unemployment. In order to achieve sufficient fluctuations of unemployment, we will rely on the mechanism stressed in Hagedorn and Manovskii (2008), and set a high strike

[6] Hall and Milgrom (2008) also allow for a small exogenous probability that firms and workers fall back to unemployment when no agreement is reached. We abstract from this here.

value. Yet, in contrast to their paper, this does not mean that workers are close to indifferent about either working and not working. In particular, the disconnect between the outside option in the bargaining and the value of unemployment allows us, second, to trace out the welfare cost for different levels of disutility of work, ζ, without affecting the cyclical or steady-state properties of the model. We consider this interesting because, as stressed repeatedly, in our model business cycles cause higher mean unemployment. The gap between the value of market work and the value of non-employment (determined here by the disutility of work) therefore is key for the size of the welfare cost of business cycles. New matches arise according to

$$m_t = \chi u_t^{\xi} v_t^{1-\xi}, \ \chi > 0, \ \xi \in (0, 1). \tag{6}$$

Here m_t is the number of new matches and v_t is the number of vacancies posted. With probability $q_t = \frac{m_t}{v_t}$ a firm with a vacant position finds a worker in period t. Unemployed workers always search for a job. With probability $f_t = \frac{m_t}{u_t}$ an unemployed worker will find a job. In order to find a worker, firms need to post a vacancy. As a result of free entry into the vacancy posting market, in equilibrium the cost of posting a vacancy, $\kappa > 0$, equals the discounted expected value of a labor firm

$$\kappa = q_t E_t \left\{ \beta_{t,t+1} J_{t+1} \right\}.$$

Market clearing and equilibrium

In equilibrium, the final goods market and the labor and capital markets clear. The final good is used for consumption and investment. Also vacancy posting activity requires resources, so output is used according to

$$y_t = c_t + i_t + \kappa v_t.$$

Finally, total period profits that accrue to the family are given by

$$\Psi_t = \Upsilon_t e_t - \kappa v_t. \tag{7}$$

4 The mean effect on unemployment: a special case of the model

Section 2 argued that the business cycle can induce a higher mean unemployment rate. In particular, Proposition 1 showed that in the matching model the mean unemployment rate exceeds its steady-state level if cyclical fluctuations lead to a negative correlation of job-finding and unemployment, and if the mean job-finding rate does not increase due to the fluctuations.

This section looks at a special case of the model that we just laid out. Proposition 2 below shows that the job-finding rate is indeed exactly linear in the technology shock, A_t, if there is no capital, workers are risk-neutral and the matching function gives equal weight to vacancies and unemployment ($\xi = 0.5$). For this case, Proposition 3 further below in this section gives a closed-form approximation for the mean of unemployment, showing the factors that determine the size of the effect that the business cycle has on mean unemployment.

Proposition 2. *Suppose that the economy is described by the model shown in Section 3. Suppose further that labor is the only factor of production, that labor enters the production function linearly ($\alpha = 0$), that consumers are risk-neutral, $\sigma = 0$, and that the matching function gives equal weight to vacancies and unemployment (the elasticity of the matching function is $\xi = 0.5$). Then*

$$f_t = f + \phi_f(A_t - A),$$

where $f = (1-\eta)\frac{\chi^2}{\kappa}\frac{\beta}{1-(1-\vartheta)\beta}(A - \pi)$ and $\phi_f = (1-\eta)\frac{\chi^2}{\kappa}\frac{\beta\rho}{1-(1-\vartheta)\beta\rho}$. Hence, the job-finding rate is linear in the technology shock; so $E(f_t) = f$.

Proof. Without capital, $x_t = A_t$. By (5), $w_t = \eta A_t + (1-\eta)\pi$. Since $\sigma = 0$, $\beta_{t,t+1} = \beta \ \forall t$. This means that the value of the firm is linear in A_t as well. Guessing and verifying yields

$$J_t = \frac{1-\eta}{1-(1-\vartheta)\beta}(A - \pi) + \frac{1-\eta}{1-(1-\vartheta)\beta\rho}(A_t - A). \tag{8}$$

The vacancy posting condition reads $\kappa/q_t = \beta E_t \{J_{t+1}\}$. Using matching function (6), and the worker and job-finding rates, $q_t = \frac{m_t}{v_t} = \chi\left(\frac{v_t}{u_t}\right)^{-\xi}$ and $f_t = \frac{m_t}{u_t} = \chi\left(\frac{v_t}{u_t}\right)^{1-\xi}$, yields that

$$f_t = \chi\left(\frac{\chi}{\kappa}\right)^{\frac{1-\xi}{\xi}} \left[\beta E_t \{J_{t+1}\}\right]^{\frac{1-\xi}{\xi}}. \tag{9}$$

If $\xi = 0.5$, we have that $f_t = \chi\left(\frac{\chi}{\kappa}\right)\beta E_t\{J_{t+1}\}$. Using equation (8) to substitute for $E_t\{J_{t+1}\}$, and rearranging, proves the proposition. □

Proposition 2 illustrates that for a special case of the model, the job-finding rate will be exactly linear in the technology shock. As a result, by Proposition 1, if the calibration of the model accounts for the negative correlation of unemployment and the job-finding rate, mean unemployment is higher in the economy with business cycles than in the steady state, making business cycles costly. The proof of the proposition suggests that several observations are in order: first, the elasticity of the matching function with respect to unemployment, ξ, is a crucial parameter for determining whether condition ii) in Proposition 1 holds. In particular, equations (8) and (9) suggest that the job-finding rate is a concave function of productivity if $\xi > 0.5$ (so $E\{f_t\} < f$) and convex if $\xi < 0.5$.[7] While the estimates of this elasticity show some variation in the literature, including estimates of ξ below 0.5, Petrongolo and Pissarides (2001) argue that a reasonable range for the elasticity seems to be larger, $\xi \in [0.5, 0.7]$. In addition to the elasticity of the matching function, the proof of Proposition 2 relied on two further, crucial elements that show why more generally the effect of the business cycle on the mean of the job-finding rate, and on mean unemployment, is difficult to prove. First, even with the bargaining that we entertain, once capital is present, wages will typically not be linear in the technology shock, A_t, since the price of the labor good, x_t, ceases to be linear in the shock. Second, even if labor is the only factor of production with $\alpha = 0$, once consumers are risk-averse the stochastic discount factor varies over time, so in general $\beta_{t,t+1} \neq \beta$. This means that the value of the firm, J_t, would cease to be linear in productivity, too.

Therefore, more generally, the effect of the business cycle on mean unemployment, and the welfare cost of business cycles need to be derived numerically. For the case underlying Proposition 2, however, we can derive the following closed-form approximation for the mean of unemployment:

Proposition 3. *Under the conditions of Proposition 2, the unemployment rate, up to a second-order approximation, has a mean of*

$$E\{u_t\} = u + \frac{\phi_f^2}{1-(1-\vartheta-f)\rho}\,\frac{u}{\vartheta+f}\,\frac{\rho}{1-\rho^2}\,\sigma_A^2.$$

Proof. See Appendix A. □

Proposition 3 shows for the assumptions underlying Proposition 2 (namely, risk-neutrality, no capital and $\xi = 0.5$) that whenever there is persistence in productivity shocks, $\rho > 0$, the mean unemployment rate in the cyclical economy exceeds the steady-state level, and increasingly so the more volatile innovations to productivity are (the higher σ_A).[8] In addition, the more the job-finding rate reacts to the technology shock, captured by the term ϕ_f, and the higher the steady-state unemployment rate, u, is, the stronger is the effect of business cycles on mean unemployment. When we fill Proposition 3 with the parameter values implied by our calibration with linear utility and labor as the only factor of production in Section 5.3.1, we find that the business cycle makes mean unemployment rise by 0.115 percentage point above its steady-state level.[9]

[7] Rather than "showing" that $E\{f_t\} < f$ if $\xi > 0.5$ and $E\{f_t\} > f$ if $\xi < 0.5$ under the other conditions of the proposition, the proof only "suggests" this. The reason why we did not include these cases in Proposition 2 is that such a proof would rely on $J_t > 0$ for all t. By a wide margin, this was the case in all the simulations that we conducted with the model, but the model does not currently guarantee this more formally.

[8] Note that if productivity was not persistent, *i.e.*, $\rho = 0$, due to the one-period lag between vacancy posting and production, the job-finding rate and unemployment would not respond to the shock but would stay at their steady state levels indefinitely. $\rho = 0$ is therefore not sensible.

5 The welfare cost of business cycles

We first present the calibration strategy for different variants of the model laid out in Section 3, and then present the welfare measure that we use and the estimates for the welfare cost of business cycles for these variants.

5.1 Calibration of the baseline

One period in the model is one month. We match hp-filtered quarterly data from simulations of the model to hp-filtered quarterly U.S. data from 1951Q1 to 2007Q1. As in Shimer (2005), we use the Hodrick-Prescott filter with a weight of 10^5 to separate fluctuations and trends. All data are in logs and seasonally adjusted. Nominal variables are deflated by the GDP deflator. Consumption is from the national accounts. Our measure of output is GDP net of government spending. We use the civilian unemployment rate among those 16 years old and older. Vacancies are measured by the Conference Board's index of help-wanted advertising. The job-finding rate in our model is the hazard rate of transition from unemployment to employment in any given month. A measure of this time series is taken from Shimer (2007). Wage and labor productivity data are from the BLS private non-farm business series. Table 1 reports the second moments in the hp-filtered data.

To compare the results for different degrees of risk-aversion and to explore the role of capital in the model, we fix a set of targets that we want our model to replicate across different variants of the model. This implies that some parameters will remain constant while others will be adjusted in each comparison. Table 2 summarizes the set of parameters that are the same in each of the model variants, or are altered only when considering the model with or without capital.

The time-discount factor targets an annual rate of return of 4%. The elasticity of the matching function with respect to unemployment is set to $\xi = 0.5$, the lower bound of what Petrongolo and Pissarides (2001) suggest as reasonable. The value of the separation rate in the economy is set to 2.67% per month. This along with the target level for steady-state unemployment of 5.6% ensures that the steady-state job-finding rate per month is 45%; the mean value in the data. When labor is the only factor of production in the model, we assume that output is constant-returns-to-scale in the labor good, $\alpha = 0$. When capital is present, the elasticity of

[9] As it should be, this is the same result that we obtain later in Section 5.3.1. More precisely, the first entry in Table 4 is 0.115 percentage point, rounded up to 0.12 percentage point. The underlying parameters for this example imply $\phi_f = 2.50$, the separation rate is $\vartheta = 0.0267$, the steady-state job-finding rate is $f = 0.45$, the persistence of the shock is $\rho = 0.97$ and the standard deviation of the innovation is $\sigma_A = 0.683/100$.

Table 1: Second moments in the data – 1951Q1-2007Q1

	Std. deviation	Corr. with $\widehat{y/e}_t$	AR(1)
Output and consumption			
$\widehat{y}_t$	2.77	0.61	0.91
$\widehat{c}_t$	2.00	0.66	0.93
Labor market: Wages and labor productivity			
$\widehat{w}_t$	1.62	0.63	.93
$\widehat{y/e}_t$	1.96	1.00	.89
Labor market: Job finding, unemployment, and vacancies			
$\widehat{f}_t$	11.56	0.38	0.91
$\widehat{u}_t$	18.55	-0.40	0.94
$\widehat{v}_t$	19.65	0.38	0.95

Notes: The table reports second moments of the data. All data are quarterly, in logs, HP(10^5) filtered and multiplied by 100 in order to express them in percent deviation from steady state. Labor productivity is real output per person employed, and wages are real compensation per employee in the non-farm business sector. The first column reports the standard deviation. The next column reports the correlation with labor productivity. The final column reports first-order autocorrelation coefficients.

Table 2: Parameters that are the same across variants

Preferences		
β	0.997	Annual real rate of 4 percent.
Labor market - matching and separation		
ξ	0.5	Petrongolo and Pissarides (2001).
ϑ	0.0267	Job-finding rate of 45% for given a target for u.
If no capital		
α	0	Production of output constant-returns-to-scale.
If capital		
α	0.33	Conventional configuration.
δ	0.0052	Steady-state investment/output ratio of 20%.

Notes: This table presents the parameters that are the same across versions of the model, and the targets for these.

output with respect to capital is set to $\alpha = 0.33$. The depreciation rate of $\delta = 0.0052$ targets an investment to output ratio of 20%, recalling that we measure output as GDP net of government consumption.

Finally, some parameters will be adjusted jointly to put the different model variants on a similar footing when computing the welfare cost of business cycles. These are summarized in Table 3. The steady-state level of technology, A, is set so as to normalize output to unity. The vacancy posting costs are set so as to ensure that the steady-state unemployment rate is $u = 5.6\%$.

Table 3: Common targets for parameters

A	Normalizes output to unity.
κ	Targets steady-state unemployment rate of $u = 5.6\%$.
χ	Targets $q = 0.33$, den Haan, Ramey, and Watson (2000).
π	Targets standard deviation of the job-finding rate.
η	Targets an elasticity of wages with respect to labor productivity of 0.446, Hagedorn and Manovskii (2008).
ρ	Targets autocorrelation of labor productivity.
σ_A	Targets standard deviation of labor productivity.

Notes: This table presents the targets for parameters that are not necessarily the same across modeling variants. In each of the model variants analyzed below, the targets are the same.

The efficiency of matching, χ, is set such that firms with a vacancy find a worker with a 33% probability within a month's time; as in den Haan, Ramey, and Watson (2000). The model replicates the cyclical volatility of the job-finding rate, which is the key to the mechanism that we wish to stress, by appropriately setting the strike position, parameter π. Given that the strike value mainly determines the equilibrium profits, we can then use the bargaining power, η, to match the elasticity of the wage with respect to labor productivity to the value of 0.446 reported in Hagedorn and Manovskii (2008). As regards the calibration of the productivity shock, for comparability we follow the literature and compute labor productivity from output per person in the non-farm business sector; see, *e.g.*, Hagedorn and Manovskii (2008). This labor productivity has an hp-filtered autocorrelation of 0.89 in the hp-filtered quarterly data. For each of the models, we choose the autocorrelation of the technology shock, ρ, such that the hp-filtered series of labor productivity, $\log(y_t/e_t)$, from simulations of the model shows the same autocorrelation as in the data. Finally, the standard deviation of the technology shock, σ_A, is chosen so as to replicate the standard deviation of labor productivity. This standard deviation is 1.96 in the data.

Two parameters are left undetermined in the previous tables. First, in the analysis that follows the risk-aversion parameter will be set to several values in the range $\sigma \in [0, 4]$, as in the previous literature. Second, the disutility of work parameter ζ has not been specified. Given the setup of the model, different values of this parameter do not have any bearing on the equilibrium dynamics or the steady state. Rather, we vary ζ from zero to its upper bound to illustrate how the estimate of the welfare cost of business cycles depends on the gap between the value of market work and non-employment.

5.2 Measuring the welfare cost of business cycles

In this paper, removing business cycles means setting the innovation to the technology shock, ϵ_t^A, to zero. The welfare cost of business cycles is defined as the share, γ, of steady-state consumption that would leave the family indifferent between the stochastic economy and the non-stochastic economy. The family's welfare in the stochastic economy is given by equation (2), repeated here for convenience,

$$W_t = \mathrm{u}(c_t) - e_t\zeta + \beta E_t\{W_{t+1}\}.$$

Let $\tilde{W}^s(\gamma)$ be the welfare of the family when, instead, $\epsilon_t^A = 0$ in the current and all future periods, so there are no business cycles and when a share, γ, is deducted from actual consumption in that economy in all periods. Superscript s indexes the current state of the economy. The welfare cost of business cycles is computed *ex ante*: we average over the states of the stochastic economy. The measure that we report includes the transition dynamics to the new steady state. We simulate one million periods of the stochastic economy. These draws represent the stochastic steady state of the economy. Of these, we then draw randomly S =100,000 states. These states are used as initial conditions to compute the welfare in the non-stochastic economy, withdrawing a share γ from consumption in all periods (including those on the transition path to the non-stochastic steady state). We then compute the value of γ that solves

$$E\{W_t\} \equiv \frac{1}{S}\sum_{s=1}^{S}\tilde{W}^s(\gamma).$$

To solve the model, to compute the mean effects and the welfare cost of business cycles, we rely on second-order approximations as in Schmitt-Grohé and Uribe (2004). When simulating series, we apply pruning; see Kim, Kim, Schaumburg, and Sims (2007).[10]

5.3 Results: the welfare cost of business cycles

We next discuss the welfare cost of business cycles implied by the respective variants of the model. We report results separately for the cases with and without capital. Appendix B reports the parameterizations and the implied second moments.

[10] We also calculated the cost of business cycles neglecting the transition path. This measure would have computed γ by equating

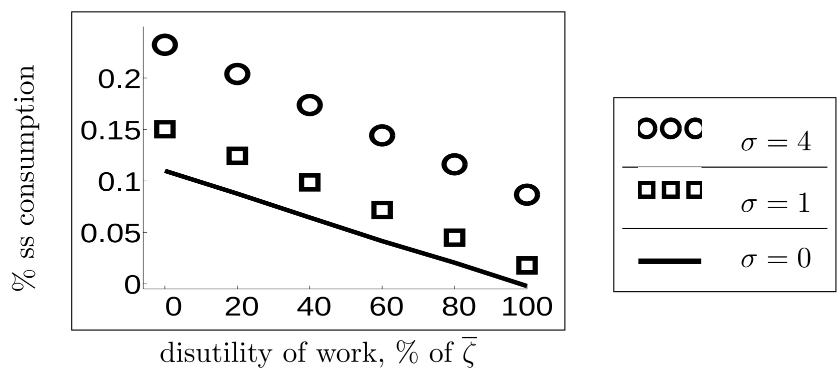
$$E\{W_t\} \equiv \overline{W}(\gamma),$$

where $\overline{W}(\gamma)$ is $\tilde{W}^s(\gamma)$ evaluated at the steady-state values. In general, these measures of the welfare cost were slightly higher than the measure that we report. Qualitatively, however, the results in the paper are not affected by this choice.

5.3.1 Model with labor as the only factor of production

For the model without capital, Figure 1 plots the welfare cost of business cycles for three degrees of risk aversion (risk-neutrality, $\sigma = 0$, log-utility, $\sigma = 1$, and $\sigma = 4$) and for alternative values of the disutility of work. In order to give the level of the disutility of work an interpretable scale, we have chosen the x-axis as follows. Imagine that the outside option of the worker in the bargaining was enjoying a period of leisure before negotiations resume, rather than striking. Define the value $\overline{\zeta}$ by $\pi = \overline{\zeta}/\lambda$. In the above case a value of the disutility of work of $\zeta = \overline{\zeta}$ would generate the same bargaining position of the worker in steady state as the strike value, π, does in our specification of the bargaining process. In this sense, this value for the disutility of work, $\zeta = \overline{\zeta}$, presents an upper bound for the disutility of work that is consistent with the job-finding rate fluctuations in the data. At the same time, and importantly for our paper, this value for ζ results in a lower bound for the welfare cost of business cycles. The x-axis in Figure 1 varies the value of the disutility of work, ζ, between 0% and 100% of the upper bound $\overline{\zeta}$. Two observations

Figure 1: The cost of business cycles – no capital



Notes: Welfare cost of business cycles in percent of steady-state consumption (y-axis) for alternative values of disutility of work, ζ (x-axis). The x-axis varies ζ between 0 and 100% of the value $\overline{\zeta}$ explained in the text. A thick solid line shows the welfare cost for risk-neutral consumers, squares mark the case of log-utility and circles show the case of higher risk-aversion ($\sigma = 4$).

are apparent. First, even for risk-neutral workers business cycles can cost up to 0.11% of steady-state consumption. The reason is that the mean unemployment rate rises considerably if there are business cycles; by 0.12 percentage point, see Table 4. This in turn means that mean consumption falls, and that welfare is negatively affected by business cycle fluctuations. As the proof of Proposition 2 highlighted, risk-aversion changes the evolution of the job-finding rate, both with regard to the mean job-finding rate and with regard to its comovement with

Table 4: Mean effects and welfare cost of business cycles in the baseline without capital

	$\sigma = 0$	$\sigma = 1$	$\sigma = 4$
$100 \cdot (E\{u_t\} - u)$	0.12	0.14	0.15
$100 \cdot (E\{c_t\} - c)/c$	-0.11	-0.12	-0.11
$100 \cdot (E\{f_t\} - f)$	0.00	-0.17	-0.29
$100 \cdot (E\{w_t\} - w)/w$	0.00	0.00	0.00
Welfare cost	0.11	0.15	0.23

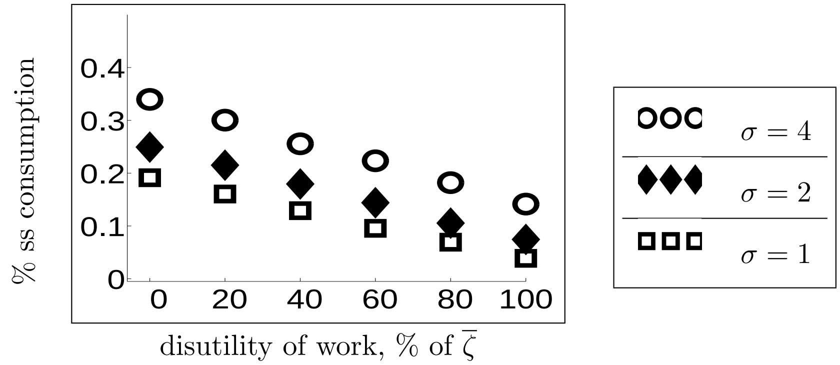
Notes: The table compares the mean values of endogenous variables in the economy with business cycles to the steady-state values. From left to right: three different degrees of risk-aversion.

unemployment. Table 4 documents that the average job-finding rate in the model without labor decreases with the degree of risk-aversion, and that the average unemployment rate increases. Besides the dislike of risk-averse workers for fluctuations *per se*, these mean effects add to their cost of business cycles: to eliminate the business cycle, they would be willing to pay up to 0.15% of steady-state consumption with log-utility (an order of magnitude larger than Lucas', 1987, estimate), and up to 0.23% of steady-state consumption with a degree of risk aversion of $\sigma = 4$. The higher the disutility of work is, the more attractive is unemployment relative to market work, and the lower is the welfare cost of business cycles. In the extreme, with linear utility, workers do not suffer virtually any cost of business cycles at the empirical upper bound of $\zeta = \overline{\zeta}$; see the solid line at 100% in Figure 1. Also for the calibrations with the higher degrees of risk-aversion, the cost of business cycles is notably smaller if ζ is larger. While this paper does not take a stand on the size of the disutility of work, the value of this disutility is crucial for the welfare cost of business cycles. If the surplus from employment in the market relative to the value of unemployment is small, the mean effect on unemployment associated with business cycles does not affect the utility of workers by as much. For log-utility, for example, the maximum welfare cost of business cycles is 0.15% (attained at $\zeta = 0$), while the minimum cost is only 0.018% of steady-state consumption (attained at $\zeta = \overline{\zeta}$).

5.3.2 Model with labor and capital

Figure 2 reports the welfare cost of business cycles in the model variants with both capital and labor. We look at degrees of risk-aversion between $\sigma = 1$ and $\sigma = 4$. The welfare costs in the calibrated model variants with capital are notably larger than the welfare costs in the versions without capital. In the absence of disutility of work, $\zeta = 0$, the welfare cost is 27% larger for the case of log-utility and 46% larger for $\sigma = 4$, at 0.19% and 0.34% of steady-state

Figure 2: Cost of business cycles – with capital



Notes: Welfare cost of business cycles in percent of steady-state consumption (y-axis) for alternative values of disutility of work, ζ (x-axis). The x-axis varies ζ between 0 and 100% of the value $\overline{\zeta}$ explained in the text. Squares mark the case of log-utility, diamonds mark $\sigma = 2$, and circles show the case of higher risk-aversion ($\sigma = 4$).

consumption, respectively. Table 5 shows the implied changes in the means for these cases. The

Table 5: Mean effects and welfare costs in the baseline with capital

	$\sigma = 1$	$\sigma = 2$	$\sigma = 4$
$100 \cdot (E\{u_t\} - u)$	0.21	0.25	0.29
$100 \cdot (E\{c_t\} - c)/c$	-0.19	-0.23	-0.26
$100 \cdot (E\{f_t\} - f)$	-0.02	0.02	0.24
$100 \cdot (E\{k_t\} - k)/k$	-0.14	-0.08	0.16
$100 \cdot (E\{w_t\} - w)/w$	0.01	0.02	0.05
Welfare cost	0.19	0.25	0.34

Notes: The table compares the mean values of endogenous variables in the economy with business cycles to the steady-state values. From left to right: three different degrees of risk-aversion.

mean unemployment rate is between 0.21 percentage point (for log-utility) and 0.29 percentage point (for $\sigma = 4$) larger than the steady-state unemployment rate. The lower employment that results from business cycle fluctuations reduces the return to capital, and has a negative effect on the capital stock. This effect is not present in the previous literature that abstracts from the effects that the business cycle has on mean unemployment. Krusell and Smith (1999), for example, find that precautionary savings induce the mean level of capital to exceed its steady-state level. In contrast, in our economy, for lower degrees of risk-aversion, the negative effect on the rental rate induced by lower employment can dominate the effect caused by precautionary savings. In the presence of business cycles the average capital stock therefore can be *lower* in our economy than in the steady state while the precautionary savings effect alone would have meant more savings and thus more capital and higher wages.[11] Interestingly, Table 5 also shows two cases that violate condition ii) of Proposition 1. With $\sigma = 2$ and $\sigma = 4$, the average job-finding rate in the stochastic economy is higher than in steady state, $E\{f_t\} > f$. Nevertheless, the mean unemployment rate in that case considerably exceeds the steady-state level. This shows that the conditions in Proposition 1 were sufficient conditions for the detrimental effect of the business cycle on mean unemployment, but that they were not necessary conditions.

5.4 Sensitivity

This section conducts a sensitivity analysis of the above results. We analyze the dependence of our results on the value of the elasticity of the matching function, ξ, and on the bargaining power of workers, η. The sensitivity analysis focuses on the model with capital, and restricts itself to the case of log-utility.

5.4.1 Varying the elasticity of matches with respect to unemployment, ξ

Table 6: Mean effects and welfare costs when varying ξ

	Model with capital, $\sigma = 1$						
	$\xi = 0.2$	$\xi = 0.3$	$\xi = 0.4$	$\xi = 0.5$	$\xi = 0.6$	$\xi = 0.7$	$\xi = 0.8$
$100 \cdot (E\{u_t\} - u)$	0.14	0.15	0.17	0.21	0.26	0.35	0.53
$100 \cdot (E\{c_t\} - c)/c$	-0.12	-0.13	-0.15	-0.19	-0.24	-0.34	-0.53
$100 \cdot (E\{f_t\} - f)$	0.82	0.57	0.31	-0.02	-0.49	-1.26	-2.79
$100 \cdot (E\{k_t\} - k)/k$	-0.07	-0.08	-0.11	-0.14	-0.20	-0.29	-0.48
Welfare cost	0.14	0.15	0.16	0.19	0.23	0.29	0.44

Notes: The table compares the mean values of endogenous variables in the economy with business cycles to the steady-state values. The final row reports the welfare cost of business cycles for $\zeta = 0$. From left to right: different values for the elasticity of the matching function, ξ. The underlying model is the baseline model with capital.

Proposition 2 suggested that the match elasticity would be key for the sign and size of the effect of business cycle fluctuations on the mean job-finding rate and on the mean unemployment rate. In particular, the proof of the proposition suggested that volatility in the technology shock, A_t, would mean a higher mean job-finding rate than in steady state if $\xi < 0.5$ and a lower mean

[11] This has two offsetting effects on our estimate of the welfare cost. On the one hand, the lower capital stock renders the non-stochastic equilibrium more attractive, and thus business cycles more costly. On the other hand, a lower capital stock means that the consumers have to save up on the transition path to the non-stochastic steady state. This makes eliminating cycles less attractive.

job-finding rate if $\xi > 0.5$. This is borne out by Table 6. For low values of the match elasticity, ξ, the mean job-finding rate exceeds the steady-state value, and the opposite holds for high values of ξ. For $\xi = 0.2$, for example, therefore the rise in mean unemployment associated with business cycle fluctuations is not as pronounced as in the baseline ($E\{u_t\} - u = 0.14$ percentage point), while for $\xi = 0.8$, for example, it is more pronounced (0.53 percentage point).[12] In line with this, the larger ξ is, the larger is the welfare cost of business cycles. Yet, regardless of the precise value of the match elasticity, the welfare cost of business cycles is notably positive throughout.

5.4.2 Varying the bargaining power of workers, η

The results above followed Hagedorn and Manovskii (2008) by using a target for the wage-elasticity for determining the bargaining power of workers, η. This section now drops this target, but retains the remaining targets, and checks the robustness of the previous findings with respect to different parameterizations for η. Of course, by itself the bargaining power is important for the response of wages to technology shocks, and would thereby be expected to matter for fluctuations of the job-finding rate and the mean effects. In Table 7, however, we reconfigure other parameters as well, especially the strike value π, to match the remaining calibration targets (especially the standard deviation of the job-finding rate). The table shows that the bargaining power does have a bearing on the mean effects and the welfare cost of business cycles, but that the sensitivity is not particularly strong.

Table 7: Mean effects and welfare costs when varying η

	Model with capital, $\sigma = 1$		
	$\eta = 0.1$	$\eta = .5$	$\eta = 0.9$
$100 \cdot (E\{u_t\} - u)$	0.20	0.21	0.22
$100 \cdot (E\{c_t\} - c)/c$	-0.19	-0.19	-0.19
$100 \cdot (E\{f_t\} - f)$	-0.04	-0.01	-0.00
$100 \cdot (E\{k_t\} - k)/k$	-0.15	-0.14	-0.12
Welfare cost	0.18	0.19	0.20

Notes: The table compares the mean values of endogenous variables in the economy with business cycles to the steady-state values. The final row reports the welfare cost of business cycles. From left to right: different values for the bargaining power of workers, η. The underlying model is the baseline model with capital.

[12] The results do not yield that $E\{f_t\} = f$ for $\xi = 0.5$ since the model features capital and risk-averse consumers, which violates the conditions in Proposition 2.

6 Welfare cost when skills depend on employment

Our results so far have focused purely on how the business cycle affects average (un)employment. This section shows that the mean effects on (un)employment studied above can interact with the skill distribution in a way that exacerbates the welfare cost of business cycles. In particular, we assume that workers lose skills off the job, and gain skills from work-experience. Then, the higher mean unemployment rate induced by business cycle fluctuations means that more workers are likely to lose their good skills in an unemployment spell, and fewer workers are likely to gain good skills through work experience; therefore, the business cycle does not only reduce employment but it also reduces the average level of skills. We now assess how important these effects are quantitatively.

It is well-documented that workers who are displaced can face severe and long-lasting earnings losses; *e.g.*, Jacobson, LaLonde, and Sullivan (1993), and Farber (2005). Krebs (2007) observes that the size of these losses is countercyclical. He assumes that business cycles cause a mean-preserving spread of long-run earnings risk, and finds that this can generate a sizable cost of business cycles. In this paper we abstract entirely from individual risk considerations and focus instead on mean shifts. We show that if the business cycle induces higher mean unemployment, as it does in our model, it can also induce a deterioration of the composition of skills in the economy. This in turn exacerbates the cost of business cycles.

6.1 Model with skill transitions

We generate longer-term earnings losses by allowing for an accumulation of skills when workers are employed and a loss of skills when workers are unemployed. Towards this end, we introduce two levels of individual workers' productivity states: Skills can either be "good" or "bad." Transitions across skills depend on the employment status of the worker. For simplicity, transitions are stochastic, and – conditional on the employment state – independent across time and aggregate state.

A worker, i, who is matched with a firm produces an amount of

$$l_{i,t} = \begin{cases} \epsilon_g, & \text{if worker } i \text{ has good skills,} \\ \epsilon_b, & \text{if worker } i \text{ has bad skills,} \end{cases}$$

of the labor good, where $\epsilon_g = 1 + \omega/2$ and $\epsilon_b = 1 - \omega/2$, $\omega \geq 0$. There is an interaction with the mean effect on unemployment since we assume that the transition probabilities between skill states depend on the state of employment of a worker. The skill transition probabilities are summarized by:

$$P^e = \begin{bmatrix} p^e(g,g) & 1-p^e(g,g) \\ p^e(b,g) & 1-p^e(b,g) \end{bmatrix} \text{ if employed, } P^u = \begin{bmatrix} p^u(g,g) & 1-p^u(g,g) \\ p^u(b,g) & 1-p^u(b,g) \end{bmatrix} \text{ if unemployed.}$$

Rows mark the current skill level, $[g,b]'$, and columns mark the next period's skill level of the worker, $[g,b]$. So $p^e(b,g)$, for example, denotes the probability that a currently employed worker who has bad skills will have good skills in the next period.

We assume the following timing of labor market events: Idiosyncratic skill shocks materialize. Knowing this period's skill level, the family and the firms bargain about wages for the two skill types. Production takes place. At the same time, unemployed workers search for a job and firms without a match post vacancies that, for parsimony, cannot target certain skills. Matches are formed in a common matching market for both types. Therefore workers of both skill levels have the same job-finding rate. At the end of the period, new matches are determined and separations occur. Appendix C.1 describes the necessary adjustments to the model described in Section 3.

6.2 Calibration and results for the model with skill transitions

We continue to calibrate the model as laid out in Tables 2 and 3. We set the strike values to target fluctuations of the job-finding rate, with the restriction that $\pi_g/\epsilon_g = \pi_b/\epsilon_b$. This condition ensures that the strike payment relative to productivity is the same among workers with good and bad skills.

For the quantitative results, the difference between the productivity in the two skill groups, ω, is important, as is the share of workers in the respective skill groups. We calibrate the model as follows. We assume that workers never lose good skills while they are employed, $p^e(g,g) = 1$. In addition, unemployed workers with bad skills never move towards good skills, $p^u(b,g) = 0$. This leaves us with three free parameters: The transition rate from good skills to good skills when unemployed, $p^u(g,g)$, the transition rate from bad skills to good skills when employed, $p^e(b,g)$, and the gap between good and bad skills, ω. We set $p^e(b,g) = 1/96$, so on average it takes a worker eight years of employment to move from bad to good skills. We set $p^u(g,g) = 0.7$, and $\omega = 0.4$. These values were chosen to be consistent with three observations: First, using the National Longitudinal Survey of Youth, Keane and Wolpin (1997) estimate that skills of white-collar workers depreciate by 30% for one year of unemployment, and by

10% for blue-collar workers; our parameters imply a loss of 21.7%. Second, using the PSID Kambourov and Manovskii (2007) argue that there are significant returns to tenure, but that what matters is occupational tenure more than tenure with an employer. The simple model in this section cannot distinguish to what extent workers switch occupations when they lose their jobs, so we add up the returns to occupational, industry, and employer tenure in their paper. This yields a return of 5% for two years of tenure; the above parameter values imply a return to tenure of 5.1%. Third, the literature finds that a displaced worker five years after a displacement has earnings that are about 10-15% lower than that of a worker who was employed continuously; see, *e.g.*, the literature overview in Krebs (2007). Our parameters position us at the lower bound, with a long-run earnings loss of 10.3%.[13] This calibration implies that in steady state 44% of the employed workers and 33% of the unemployed workers have good skills. The final row of Table 8 reports the maximum welfare cost of business cycles; *i.e.*, for $\zeta = 0$.

Table 8: Mean effects and welfare costs when there are skill transitions

	Labor only			With capital		
	$\sigma = 0$	$\sigma = 1$	$\sigma = 4$	$\sigma = 1$	$\sigma = 2$	$\sigma = 4$
$100 \cdot (E\{u_t\} - u)$	0.10	0.13	0.17	0.21	0.26	0.30
$100 \cdot (E\{c_t\} - c)/c$	-0.14	-0.18	-0.21	-0.25	-0.29	-0.28
$100 \cdot (E\{f_t\} - f)$	0.03	-0.20	-0.39	0.03	0.11	0.59
$100 \cdot (E\{k_t\} - k)/k$	–	–	–	-0.18	-0.09	0.37
$100 \cdot (E\{e_{g,t}\} - e_g)$	-0.14	-0.21	-0.28	-0.24	-0.26	-0.19
$100 \cdot (E\{e_{b,t}\} - e_b)$	0.03	0.08	0.12	0.02	0.001	-0.11
$100 \cdot (E\{u_{b,t}\} - u_b)$	0.10	0.13	0.16	0.21	0.26	0.31
Increase in welfare cost relative to baseline	18%	33%	46%	30%	32%	38%
Welfare cost	0.13	0.20	0.34	0.25	0.33	0.47

Notes: The table compares the mean values of endogenous variables in the economy with business cycles to the steady-state values. The fifth through seventh rows show the difference between means and steady-state values for the number of employed of workers with good skills, $e_{g,t}$, and bad skills, $e_{b,t}$, and the number of unemployed workers with bad skills, $u_{b,t}$ (the increase in the number of unemployed workers with good skills is the negative of the sum of the aforementioned numbers). The final row reports the welfare cost of business cycles (in percent of steady-state consumption) for $\zeta = 0$, the second to last row reports the percent increase in the welfare cost of business cycles relative to the baseline without skill transitions. From left to right: different values for risk aversion in the model without/with capital. The underlying model is the model with skill transitions.

[13] We view this as positive, as it is well-understood that our calibration strategy mixes up displacement and flows from employment to unemployment. Job displacements are job-losses not related to the worker's performance. Therefore, the *monthly* job separation rate of $\vartheta = 2.7\%$ that we use in this paper dwarfs the *annual* job displacement rate of about 4% that the literature finds. Also the return to tenure is relatively high in our calibration. In this sense, the results described here present an upper bound for the importance of skill losses.

The second to last row reports the percent increase in the welfare cost relative to the baseline without skill transitions. The cost of business cycles is between 18% and 46% larger than in the baseline model; corresponding to increases in the welfare cost estimates to between 0.13 and 0.47 percentage point. The business cycle continues to induce higher mean unemployment. Now, however, this has a further negative effect on welfare: Higher unemployment means fewer workers with good skills. Namely, the higher mean unemployment implies that more workers are likely to lose their good skills in an unemployment spell, and that fewer workers are likely to gain good skills through continuous experience. This is borne out by our results. For log-utility, for example, the share of workers with bad skills rises by 0.21 percentage point in the model with labor only (adding the fourth to last and the third to last rows, 0.08+0.13), and by 0.23 percentage point in the model with capital (0.02+0.21).

7 Conclusions

This paper developed a real business cycle model with search and matching frictions in the labor market. We calibrated the model to U.S. data and used it to compute the welfare cost of business cycles. We computed the cost for different values of the disutility of work, which we understood as a catch-all term for determining the value of market work relative to non-employment. Importantly, we let the model govern how both the fluctuations and the levels of labor market risk change when the business cycle risk is eliminated. In a second step, we extended the model to allow for employment-dependent skill levels of workers that lead to longer-term earnings losses upon separation.

General equilibrium effects apart, in the model unemployment fluctuations by themselves would only have minor implications for the welfare cost of business cycles. Nevertheless, our estimates for the cost of business cycles are easily an order of magnitude larger than the estimates provided by Lucas (1987). This is due to the fact that besides fluctuations in unemployment and consumption, which have been the focus of the previous literature, the model also implies significantly higher mean unemployment rates in the presence of a business cycle. These mean effects arise as a direct consequence of the non-linearity between unemployment and the job-finding probability in the employment-flow equation. Therefore business cycles are costly even for workers who are well-insured against idiosyncratic fluctuations in income and unemployment risk. Reducing business cycle fluctuations reduces average unemployment risk and increases welfare. In the economy without skill transitions, we find that for log-utility, for example, mean unemployment is 0.21 percentage point higher than in steady state, and that the potential gain from eliminating business cycles is up to 0.19% of steady-state consumption.

We then assessed the cost of business cycles when unemployment spells increase the risk of losing skills acquired through previous work experience. In our calibrated model, the interaction of skills and the mean effect on unemployment caused by the business cycle is quantitatively important. Again for log-utility, the maximum welfare cost rises by a third relative to a model without skill transitions, to 0.25% of steady-state consumption.

Our estimates of the cost of business cycles focused on the business cycle's effect on mean unemployment and mean skills while we clearly have omitted further sources for costly business cycles. Most important to us, a number of authors have pointed out that the risk of infrequent disasters linked to cyclical phenomena significantly raises the cost of business cycles. These authors typically appeal to a (once in a lifetime) Great Depression scenario; see Chatterjee and Corbae (2007) and Salyer (2007). In the current paper, not only do we abstract from such aggregate disasters, but in the same vein we limit the damage that unemployment can do to skills. In particular, regardless of the length of the unemployment spell, in the paper skills never fall below a certain level. Business cycles would be more costly if very long-term unemployment – which is much more likely to occur when there are lasting deep recessions – were associated with a very deep (disastrous) loss of skills, or with the absence of unemployment insurance. Needless to say that this would point to an even higher cost of business cycles.

In sum, in a model with labor market search and matching frictions we found that business cycles increase the average unemployment risk, and that cycles reduce the skill level of the workforce. Accordingly, business cycles are considerably more costly than the mere degree of aggregate fluctuations suggests, and this cost affects a wide range of consumers (in the model, all consumers).

For future work, it would be interesting to assess the welfare cost of business cycles when, realistically, workers with lower skills have a lower job-finding rate, and face a higher unemployment rate. Our approximation for the mean effects in Proposition 3 suggests that for the latter group mean employment may be more adversely affected than for the population as a whole.

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A Proofs

Proposition 3. *Under the conditions of Proposition 2, the unemployment rate, up to a second-order approximation, has a mean of*

$$E\left\{u_t\right\} = u + \frac{\phi_f^2}{1-(1-\vartheta-f)\rho}\,\frac{u}{\vartheta+f}\,\frac{\rho}{1-\rho^2}\,\sigma_A^2. \tag{10}$$

Proof. The employment-flow equation (1) can be written as $u_t = \vartheta + (1-\vartheta-f_{t-1})u_{t-1}$. Using $f_t = f + \phi_f(A_t - A)$ from Proposition 2, and rewriting gives

$$\breve{u}_t = (1-\vartheta-f)\breve{u}_{t-1} - \phi_f\breve{A}_{t-1}\breve{u}_{t-1} - \phi_f u\breve{A}_{t-1}. \tag{11}$$

Here a check marks deviations from steady state, *e.g.*, $\breve{u}_t = u_t - u$. Take unconditional expectations of (11), use the covariance stationarity of the model, and that $E\{\breve{A}_t\} = 0$. This gives

$$E\left\{\breve{u}_t\right\} = -\frac{1}{\vartheta+f}\phi_f E\{\breve{A}_{t-1}\breve{u}_{t-1}\}. \tag{12}$$

In order to obtain an expression for $E\{\breve{u}_{t-1}\breve{A}_{t-1}\}$, multiply (11) by $\breve{A}_t$, and expand the right-hand side by using $\breve{A}_t = \rho\breve{A}_{t-1} + \epsilon_t^A$. A second-order approximation of the resulting terms yields $\breve{u}_t\breve{A}_t \approx (1-\vartheta-f)\left[\rho\breve{u}_{t-1}\breve{A}_{t-1} + \breve{u}_{t-1}\epsilon_t^A\right] - \phi_f u\rho\breve{A}_{t-1}^2 - \phi_f u\breve{A}_{t-1}\epsilon_t^A$. Taking unconditional expectations, and using the stationarity, gives that up to second order $E\{\breve{u}_t\breve{A}_t\} \approx -\frac{1}{1-(1-\vartheta-f)\rho}\phi_f u\frac{\rho}{1-\rho^2}\sigma_A^2$. Using this with (12) yields expression (10). This proves the proposition. □

B Parameters and second moments for baseline calibrations

Table 9: Parameters in the respective variants

	Labor only			With capital		
	$\sigma = 0$	$\sigma = 1$	$\sigma = 4$	$\sigma = 1$	$\sigma = 2$	$\sigma = 4$
A	1.059	1.059	1.059	0.310	0.310	0.310
η	0.424	0.422	0.414	0.419	0.419	0.418
κ	0.562	0.615	0.818	0.459	0.471	0.481
χ	0.385	0.385	0.385	0.385	0.385	0.385
π	0.970	0.962	0.932	0.637	0.636	0.634
ρ	0.970	0.970	0.970	0.971	0.972	0.973
$100 \cdot \sigma_A$	0.683	0.683	0.683	0.211	0.211	0.211

Notes: This table presents the parameters for the respective model variants.

Table 10: Standard deviations in the respective variants

	Labor only			With capital		
	$\sigma = 0$	$\sigma = 1$	$\sigma = 4$	$\sigma = 1$	$\sigma = 2$	$\sigma = 4$
Output and consumption						
$\widehat{y}_t$	2.57	2.57	2.56	2.57	2.57	2.56
$\widehat{c}_t$	2.13	2.08	1.89	0.74	0.64	0.56
Wages and labor productivity						
$\widehat{w}_t$	0.87	0.87	0.87	0.87	0.87	0.87
$\widehat{y/e}_t$	1.96	1.96	1.96	1.96	1.96	1.96
Job finding, unemployment, and vacancies						
$\widehat{f}_t$	11.56	11.56	11.56	11.56	11.56	11.56
$\widehat{u}_t$	10.66	10.67	10.72	10.69	10.69	10.69
$\widehat{v}_t$	13.33	13.25	13.06	13.19	13.19	13.19

Notes: The table reports second moments implied by the different calibrations of the baseline model. The second moments are computed from simulated data from the model. All data are quarterly aggregates, in logs, HP(10^5) filtered and multiplied by 100 in order to express them in percent deviation from steady state.

C Skill differences

C.1 Adjustments to the model of Section 3

We discuss the adjustments to the model equations in Section 3 that become necessary when we allow for two skill groups; see also Section 6.1. Let $\iota \in \{g, b\}$ be the individual's current skill and let $\bar{\iota}$ be the opposite skill; *e.g.*, $\iota = g$ and $\bar{\iota} = b$. There continues to be a unit mass of workers. Employment for the two skills evolves as

$$\begin{aligned} e_{\iota,t} &= (1-\vartheta)[e_{\iota,t-1}p^e(\iota,\iota) + e_{\bar{\iota},t-1}p^e(\bar{\iota},\iota)] \\ &+ f_{t-1}[u_{\iota,t-1}p^u(\iota,\iota) + u_{\bar{\iota},t-1}p^u(\bar{\iota},\iota)], \quad \iota \in \{g, b\}. \end{aligned}$$

The laws of motion for unemployment are correspondingly given by

$$\begin{aligned} u_{g,t} &= \vartheta[e_{g,t-1}p^e(g,g) + e_{b,t-1}p^e(b,g)] \\ &\quad + (1-f_{t-1})[u_{g,t-1}p^u(g,g) + u_{b,t-1}p^u(b,g)], \\ u_{b,t} &= 1 - e_{g,t} - e_{b,t} - u_{b,t}. \end{aligned}$$

Let $p_{g,t}$ and $p_{b,t} = 1 - p_{e_t}$ be the period t share of workers with good and bad skills, respectively. In our setup and calibration, the unemployment rate among workers with bad skills, $u_{b,t}/p_{b,t}$ is the same as the unemployment rate of workers with good skills, $u_{g,t}/p_{g,t}$. Period profits of firms depend on the skill level of the worker:

$$\Upsilon_{\iota,t} = x_t \epsilon_\iota - w_{\iota,t}, \quad \iota \in \{g, b\}.$$

The value of the firm with worker of type ι is

$$\begin{aligned} J_{\iota,t} &= \Upsilon_{\iota,t} \\ &\quad + p^e(\iota,\iota)(1-\vartheta)E_t\{\beta_{t,t+1}J_{\iota,t+1}\} \\ &\quad + p^e(\iota,\bar{\iota})(1-\vartheta)E_t\{\beta_{t,t+1}J_{\bar{\iota},t+1}\}. \end{aligned}$$

Let π_ι be the strike value for workers of type ι. The wage equation for these workers that results from bargaining is given by

$$w_{\iota,t} = \eta x_t \epsilon_\iota + (1-\eta)\pi_\iota.$$

The matching function is as in equation (6), but now $u_t := u_{g,t} + u_{b,t}$. Job-finding and matching probabilities are defined the same way as before. The free-entry condition for vacancies becomes

$$\frac{\kappa}{q_t} = \frac{u_{g,t}p^u(g,g) + u_{b,t}p^u(b,g)}{u_t} E_t\left\{\beta_{t,t+1}J_{g,t+1}\right\} + \frac{u_{g,t}p^u(g,b) + u_{b,t}p^u(b,b)}{u_t} E_t\left\{\beta_{t,t+1}J_{b,t+1}\right\},$$

which accounts for the skill transitions. The total amount of the labor good produced is given by

$$l_t = \epsilon_g e_{g,t} + \epsilon_b e_{b,t}.$$

C.2 Parameters and second moments with skill transitions

Table 11: Parameters when there are skill differences

	Labor only			With capital		
	$\sigma = 0$	$\sigma = 1$	$\sigma = 4$	$\sigma = 1$	$\sigma = 2$	$\sigma = 4$
A	1.084	1.084	1.084	0.315	0.315	0.315
η	0.412	0.411	0.405	0.408	0.408	0.408
κ	0.506	0.558	0.745	0.424	0.434	0.442
χ	0.385	0.385	0.385	0.385	0.385	0.385
π	1.004	0.996	0.967	0.659	0.658	0.656
ρ	0.964	0.965	0.965	0.969	0.970	0.971
$100 \cdot \sigma_A$	0.717	0.716	0.714	0.218	0.218	0.218
$p^e(g, g)$	1	1	1	1	1	1
$p^e(b, g)$	1/96	1/96	1/96	1/96	1/96	1/96
$p^u(g, g)$	0.7	0.7	0.7	0.7	0.7	0.7
$p^u(b, g)$	0	0	0	0	0	0
ω	0.4	0.4	0.4	0.4	0.4	0.4

Notes: This table presents the parameters in the model with skill transitions.

Table 12: Standard deviations when there are skill differences

	Labor only			With capital		
	$\sigma = 0$	$\sigma = 1$	$\sigma = 4$	$\sigma = 1$	$\sigma = 2$	$\sigma = 4$
Output and consumption						
$\widehat{y}_t$	2.57	2.57	2.57	2.57	2.57	2.57
$\widehat{c}_t$	2.18	2.13	1.96	0.81	0.75	0.70
Wages and labor productivity						
$\widehat{w}_t$	0.89	0.89	0.89	0.89	0.89	0.89
$\widehat{y/e}_t$	1.96	1.96	1.96	1.96	1.96	1.96
Job finding, unemployment, and vacancies						
$\widehat{f}_t$	11.56	11.56	11.56	11.56	11.56	11.56
$\widehat{u}_t$	10.64	10.66	10.71	10.68	10.68	10.68
$\widehat{v}_t$	13.37	13.29	13.09	13.23	13.23	13.23

Notes: The table reports second moments implied by different degrees of risk-aversion in model with skill transitions. The second moments are computed from simulated data from the model. All data are quarterly aggregates, in logs, HP(10^5) filtered and multiplied by 100 in order to express them in percent deviation from steady state.