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Quits, Worker Recruitment, and Firm Growth: Theory and Evidence*

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Abstract

We use establishment data from the Job Openings and Labor Turnover Survey (JOLTS) to study the micro-level behavior of worker quits and their relation to recruitment and establishment growth. We find that quits decline with establishment growth, playing the most important role at slowly contracting firms. We also find a robust, positive relationship between an establishment's reported hires and vacancies and the incidence of a quit. This relationship occurs despite the finding that quits decline, and hires and vacancies increase, with establishment growth. We characterize these dynamics within a labor-market search model with on-the-job search, a convex cost of creating new positions, and multi-worker establishments. The model distinguishes between recruiting to replace a quitting worker and recruiting for a new position, and relates this distinction to firm performance. Beyond giving rise to a varying quit propensity, the model generates endogenously-determined thresholds for firm contraction (through both layoffs and attrition), worker replacement, and firm expansion. The continuum of decision rules derived from these thresholds produces rich firm-level dynamics and quit behavior that are broadly consistent with the empirical evidence of the JOLTS data.

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1 Introduction

Quits are a critical part of worker turnover, comprising over half of all separations in the U.S. economy. Much of the existing literature focuses on quits from the perspective of the worker (e.g., Topel and Ward (1992)) and, more specifically, on the wage benefits from switching jobs. Quits, however, also affect the firm that a worker leaves behind. In particular, quits may signify hard times at the employing firm or may simply leave the employing firm with a vacant and profitable position. Our aim in this paper, therefore, is to study quits and their relationship to employment dynamics and recruitment from the perspective of the employer using the recently released Job Openings and Labor Turnover Survey. The JOLTS is the first data set in the United States providing establishment-level information on both worker turnover and vacancies. We start by documenting key characteristics of quits, establishment dynamics and recruitment in the JOLTS data based on our own analysis and on that of Davis, Faberman, and Haltiwanger (2006b) and Davis, Faberman, and Haltiwanger (2006a). We then study how a model anchored in the search and matching tradition fares in accounting for the observed features of the JOLTS data. We find that, once appropriately extended, the model produces rich dynamics that are broadly consistent with the empirical evidence in the JOLTS.

The JOLTS evidence shows that both quits and layoffs increase with the size of an employer contraction but do so at different rates. Layoffs are a much higher share of large contractions, while quits account for the majority of smaller contractions. In addition, even though quits fall and the vacancy and hiring rates rise with employer growth, there is a robust, *positive* relationship between hires, vacancies, and the quit rate. Furthermore, the "vacancy yield" (the number of observed hires per reported vacancy) increases with employer growth. Finally, while recruitment and hiring are concentrated at expanding establishments, contracting establishments also exhibit a non-trivial amount of recruiting.

To capture these regularities, we develop a model where firms and workers meet randomly through a matching process. In the model, firms have heterogeneous productivities that change over time and face a convex cost of position creation (as in Fujita and Ramey (2007)). This results in multi-worker firms experiencing expansions and contractions in response to productivity shocks. To make the distinction between quits and layoffs, we allow for on-the-job search and identify quits through job-to-job transitions. Furthermore, to capture the positive relationship between vacancies and quits, we introduce the notion of replacement hiring. In particular, we assume that the upfront cost of position creation need not be paid if a firm is recruiting to replace a quitting worker. This means that the position itself is not destroyed when a worker quits; rather, the worker leaves the physical and organizational capital behind, making the replacement of a quitting worker less costly than the creation of an entirely new position. This notion of quit replacement is novel to our model. Even though previous matching models address the notion of on-the-job search (Pissarides (1994); Mortensen (1994); and Nagypál (2007a)), the literature is silent on the issue of what happens to a position once a worker leaves. Our model highlights the fact that a firm has a choice when facing this situation: it can post a vacancy to refill the position, or it can let the job disappear.

The probabilities that a worker quits or that a vacant position is filled at a firm are both endogenously determined within the model. High-productivity firms offer higher wages. This decreases the chance that an employee quits the firm for a better offer and increases the chance that a searching worker accepts its own offer. Moreover, we show that firm dynamics in the model can be characterized by three productivity thresholds. When productivity is below the first threshold, firms lay off their entire workforce and shut down. When productivity is between the first and second thresholds, firms choose not to replace workers who leave and, therefore, contract through attrition. When productivity is between the second and third thresholds, firms hire to replace workers who leave but no more. Finally, when productivity is above the third threshold, firms choose to both replace workers and expand.

The model implies that new position creation increases with firm productivity, quits and layoffs decline with firm productivity, and the probability that a worker accepts a new job offer increases with the productivity of that job. Given the positive association between firm productivity and firm growth, the model predicts that the job-filling rate (and the related vacancy yield) increases with firm growth, while the quit rate decreases with firm growth. As in the data, these relationships are nonlinear. Moreover, quits are the dominant form of separations for relatively small contractions (which occur in the "attrition" region of the decision continuum), while layoffs dominate for large contractions. The model also produces a positive relationship between hires and quits, as well as vacancies and quits. We also uncover a tension between fitting different aspects of the data, since our model counterfactually predicts a higher than observed vacancy rate for slowly contracting establishments. Overall, though, the model is able to replicate many of the key cross-sectional patterns in the microdata.

Our process of on-the-job search and worker replacement is closely related to the concept of "vacancy chains" described by Akerlof, Rose, and Yellen (1988). It also builds on models, such as those of Bertola and Caballero (1994) and Bertola and Garibaldi (2001), that incorporate firm size into the search framework. Finally, our model is similar to that of Cooper, Haltiwanger, and Willis (2006), except that we include on-the-job-search and are more concerned with the cross-sectional responses of employers than with aggregate dynamics.

2 Evidence on quits and worker recruitment

2.1 Data and measurement

For our empirical analysis, we use microdata from the Job Openings and Labor Turnover Survey (JOLTS), produced by the BLS. The JOLTS data are a monthly sample of roughly 16,000 establishments. Each month, establishments report their employment level, hires, separations, and vacancies (job openings). The data are ideal for our purposes because separations are reported separately as quits, layoffs and discharges, and other separations (e.g., retirements). In addition, the data are reported directly by establishments and are representative of the U.S. economy. The data include a flow measure of hires and separations; i.e., they include all hires and separations an establishment has over the month. The data include a stock measure of vacancies measured as the number of open vacancies on the last day of the month. The survey includes both a certainty sample (which is surveyed every month) and a random sample (which is surveyed for 18 consecutive months). We use data pooled over the December 2000 to January 2005 period and restrict our sample to establishments with observations in at least two consecutive months to avoid issues with establishment entry and exit. Our final sample contains about 377,000 establishment-month observations.

The way in which the JOLTS data are reported creates some challenges in producing a consistent measure of establishment-level employment growth. To address this, we employ the symmetric measure of growth defined by Davis, Haltiwanger, and Schuh (1996) and apply it to the JOLTS data using the same approach as Davis, Faberman, and Haltiwanger (2006b). The resulting growth rate measure for establishment i in month t is

$$g_{it} = \frac{H_{it} - S_{it}}{\frac{1}{2}(N_{it} + \widetilde{N}_{i,t-1})}, \tag{1}$$

where H_{it} is the number of hires, S_{it} is the number of separations, N_{it} is employment and $\widetilde{N}_{i,t-1} = N_{it} - H_{it} + S_{it}$ is the revised measure of employment in $t-1$ used to ensure consistency between hires, separations, and the growth rate. Using this measure, an establishment's growth rate is its net employment change divided by the average of the current and previous months' employment. We measure hiring, quit, layoff, and vacancy rates using the same denominator.

Even with all of its advantages, the JOLTS data leave us with several empirical challenges. Some are common to all studies of establishment data. First, there exists an endogeneity issue because, theoretically, worker turnover, recruiting effort, and growth are jointly determined by the prospects an establishment faces. Second, heterogeneity in size and in other fixed characteristics of establishments can affect our results. By their nature, smaller establishments have "lumpy" employment changes – their growth rates are often zero, and conditional on being nonzero they tend to be relatively large in absolute value by construction. Some establishments also tend to be high-turnover establishments (because of their industry, demand structure, labor force composition, etc.). We can address these issues with establishment fixed effects and by reporting results separately by industry and establishment size.

Other challenges are specific to the JOLTS data. One of these is observability. Ideally, we would like information relating hires, vacancies, and quits to specific positions but our data are at the establishment level; i.e., the data report only the total number of hires, vacancies, and separations at each establishment. Our other challenge is timing. Since the JOLTS measures vacancies as a stock at the end of the month, vacancies opened and filled during the month do not appear in the data, which creates a time aggregation problem.[1] Timing also influences the sequencing of observed quits, vacancies, and hires. Namely, if a worker quits in month t and the firm decides to replace him, this can show up either as a hire in month t or as a vacancy at the end of month t and a hire in month $t+1$ or later. This, of course, assumes that the firm was not anticipating the quit. We address the timing issue by focusing our analysis on the relationship of quits in a month with contemporaneous hires and vacancies and leading hires.

2.2 Quits, recruitment, and establishment growth

Quits comprise a large fraction of worker turnover, accounting for 54 percent of all separations in the JOLTS data between December 2000 and January 2005.[2] An important part of understanding quits and recruitment behavior is characterizing how each relates to establishment growth. In relating theory to the evidence, one can think of employment growth

[1] Burdett and Cunningham (1998) report that, in the 1982 Employment Opportunity Pilot Project, 44 percent of vacancies ended within seven days and 72 percent of vacancies ended within two weeks.

[2] For comparison, Nagypál (2008) finds that job-related quits account for 55 percent of separations in the Survey of Income and Program Participation.

(particularly the high-frequency changes we observe in our data) as being determined by idiosyncratic shocks to firm profitability (i.e., productivity or demand). Matching models with on-the-job search, such as the one we present below, imply that the likelihood that a worker quits decreases with a firm's profitability, since the probability of receiving a more attractive outside offer decreases as the fortune of the current employer improves. At the same time, adverse shocks to profitability increase the likelihood of a layoff and decrease the payoff from opening new vacancies. Thus, theory suggests different relationships between worker flows, vacancies, and establishment growth.

Davis, Faberman, and Haltiwanger (2006b) and Davis, Faberman, and Haltiwanger (2006a) illustrate some of these relationships. We replicate their findings in Figures 1 through 3 using our pooled establishment-month observations.[3] Figure 1 shows the quit and layoff rates as functions of the establishment growth rate. It illustrates that these rates increase with the size of an employment contraction. They are low and essentially constant in expanding establishments and are lowest for establishments with a small or no employment change. Among stable and expanding establishments, quits slightly outpace layoffs, while among contracting establishments, layoffs increase sharply and almost linearly with the size of a contraction. Quits, however, increase rapidly with smaller contractions but then level off at around 10 percent of employment for larger contractions. Thus, quits are more important than layoffs for small contractions, but layoffs account for an increasing share of separations as a contraction gets larger. When interpreting these figures, note that over 90 percent of employment is at establishments with absolute growth rates of less than 10 percent, implying

[3] Davis et al. (2006a, 2006b) estimate these relationships by calculating the weighted mean values of the noted variables for fine growth rate intervals using the same JOLTS sample as our own. In a semi-parametric estimation, they show that these results are robust to the inclusion of establishment fixed effects.

that quits are a substantial component of separations for most employment changes.

Figure 2 depicts the hiring and vacancy rates as functions of the contemporaneous establishment growth rate. The figure illustrates a small but positive amount of hiring among shrinking establishments. Stable establishments have the lowest hiring rate, while the hiring rate increases almost linearly with the establishment growth rate for expanding establishments (the latter occurs almost by construction). Vacancy rates exhibit a similar nonlinear increasing relation to the establishment growth rate but rise less rapidly than hires. Contracting establishments also have a positive, relatively constant vacancy rate.

Finally, Figure 3 shows the vacancy yield (the number of hires in month t per vacancies open at the end of month $t-1$ for establishments reporting at least one vacancy) as a function of the establishment employment growth rate in month t. Due to the timing difference and the requirement of positive reported vacancies, this is not simply a ratio of the two series in Figure 2. The vacancy yield increases rapidly with the establishment growth rate, particularly among expanding establishments, implying that establishments have more hires per vacancy the larger their expansion. The yield is essentially constant for contractions and is lowest for establishments with no employment change.

2.3 Quits and recruitment, discrete relations

In this section, we look at how quits relate to vacancies posted and hires made subsequently. We report basic summary statistics in Table 1. The upper panel of the table shows quit, layoff, hiring, and vacancy rates for the full sample and the sample broken down by whether a quit occurred during the month. Hiring and vacancy rates are substantially higher when preceded by a quit. The vacancy rate is more than twice as high when a quit occurs in the preceding month. The layoff rate, on the other hand, shows little difference based on quit incidence. The bottom half of Table 1 shows that only 14 percent of all establishments report any quits in the preceding month, but these establishments make up 58 percent of employment, implying that most quits occur at larger establishments. The last two columns show that establishments with at least one quit account for a disproportionate number of vacancies and hires, accounting for 65 percent of all hires and 74 percent of all vacancies. These numbers are particularly striking given the patterns in Figures 1 and 2, which showed the highest hiring and vacancy rates among expanding establishments and the highest quit rates among contracting establishments.

To study the quit-recruitment relationship further, we estimate the probability of having at least one vacancy or at least one hire by the incidence of a prior quit. We report the results in Table 2. For all establishments, the incidence of a quit substantially increases the probability of a subsequent vacancy or hire. The likelihood of each increases by about 50 percentage points. Of course, this could be due to different quit, vacancy, and hiring incidence by industry or, more to the point, by size class. Yet, we find that the same pattern holds across all industries and size classes, though there is considerable variation in the difference by quit incidence. Across establishment size classes, the probabilities of a vacancy or hire increase with size as expected. In all cases, these probabilities are higher when there is a preceding quit, though the difference decreases with establishment size.

As a robustness check, we replicate the results of Tables 1 and 2 by layoff rather than quit incidence. Specifically, we split the data based on whether or not there is a preceding layoff with no preceding quit, so that the exercise is independent of any quit-related outcomes.

Our goal is to see whether the quit-hires and quit-vacancy relationships are unique to quits or are simply a general pattern due to higher turnover. The results in Table 3 suggest that the relations are unique to quits. While the hiring rate is higher when preceded by a layoff, the vacancy rate is not. Overall, layoffs that occur in the absence of a quit account for 6.4 percent of employment. They account for a higher fraction of hires (7.9 percent) but a smaller fraction of vacancies (5.9 percent). One reason for the mixed results may be that the JOLTS definition of layoffs includes not only layoffs but also firings, which could lead to the same worker replacement motive that we attribute to quits. Nevertheless, the discrete probabilities of a vacancy or a hire based on the incidence of a preceding layoff (again, absent a preceding quit) are much different than those observed in Table 2. While the presence of a preceding quit is associated with a 50-percentage-point higher probability of either a vacancy or a hire, the presence of a preceding layoff is associated with a 9-percentage-point *decrease* in the probability of a vacancy and a 3-percentage-point decrease in the probability of a hire. Thus, there is a stark difference in recruitment behavior depending on whether it follows a quit or a layoff.

2.4 Quits and recruitment, continuous relations

Thus far, our evidence has consistently suggested a positive relation between the incidence of a quit and subsequent vacancies and hiring. This relation occurs despite the fact that the quit rate declines with establishment growth and the vacancy and hiring rates increase with establishment growth. For our final analysis, we study the relationship between quits and vacancies and between quits and hires within a regression framework, where we can control for the effects of establishment characteristics and growth.

We regress hires both in the subsequent month $(t+1)$ and current month (t) on the quit rate, since a quit could be replaced within the same month. We also regress vacancies at the end of the month on the quit rate. In each case we run (i) the unconditional OLS regression of the variable on the quit rate, (ii) the same regression controlling for establishment fixed effects, (iii) the regression controlling for establishment fixed effects and the employment growth rate, and (iv) the regression controlling for establishment fixed effects and the growth rate differentiated into positive and negative changes. For the regressions of contemporaneous hires on the quit rate, we face an endogeneity issue when we include the growth rate in the latter two specifications. To account for this, we instrument the contemporaneous growth rate using the prior month's growth rate as the instrument.

Table 4 lists the regression results. It shows that the positive relations of leading hires, contemporaneous hires, and vacancies to quits are robust to controlling for establishment fixed effects and establishment growth. Contemporaneous hires actually have a stronger relation to the quit rate than hires in the subsequent month. These hires can, of course, occur before a quit, confounding the relation we wish to identify. Nevertheless, even when controlling for and instrumenting for the growth rate, the relationship between these hires and the quit rate still holds.

We also replicate the regressions in columns (1), (2), and (3) of Table 4, substituting the continuous quit rate variable with dummy variables for discrete quit rate intervals.[4] We then plot the coefficients on the quit rate graphically (with the means added back where

[4] These dummy variables represent intervals that increase in size with the quit rate. We use a varying interval length to maintain precision because the number of observations declines sharply as the quit rate increases. We also appropriate establishments with zero quits to a separate interval.

appropriate). We again use an IV estimate when including the growth rate in the regression of contemporaneous hires. Our results are in Figure 4. The panels show the results for leading hires, contemporaneous hires, and vacancies, respectively. The unconditional regression specifications (analogous to column (1) in Table 4) illustrate the strongly increasing relationships of hiring and vacancies to quits. The main results of Figure 4 show that when we control for establishment fixed effects and establishment growth, the slope of the relationships becomes flatter (particularly for leading hires), but the positive relations remain in all cases.

To summarize our empirical results, we find that establishments with a quit account for a disproportionately large fraction of subsequent vacancies and hiring. Across a broad set of metrics, we find a positive relationship between the incidence of a quit and subsequent vacancies and hiring. This relation holds up even after controlling for endogeneity issues related to establishment growth, timing, and observability issues that come from studying establishment rather than job-specific data, and fixed establishment characteristics, including industry and size. These findings suggest that quits, and the recruiting behavior they appear to initiate, are important facets of establishment-level employment adjustment, and they account for a significant portion of high-frequency labor-market dynamics.

3 Model

We now formalize a model that fully characterizes the spectrum of employment adjustment by firms and allows for quits and their potential replacement as endogenous decisions by workers and firms, respectively. We consider a matching model with endogenous separations, in the spirit of Mortensen and Pissarides (1994). We allow for on-the-job search, as in Nagypál (2007a), and introduce recruiting costs that differentiate between a sunk job-creation cost and the flow cost of advertising a job (as in Fujita and Ramey (2007)).[5] The former captures the organizational and physical capital required to create a new position, while the latter captures the cost of searching for a suitable worker to fill that position. Convexity of the creation cost gives rise to multi-worker establishments, which in turn allows us to compare data generated by the model to establishment growth data. The creation cost also allows us to differentiate between recruiting to replace a quit and recruiting for a newly created position.

3.1 Model setup

Consider an economy populated by workers and firms who are both risk-neutral and discount future incomes at rate r. There is a unit measure of workers who enjoy flow utility b while unemployed and enjoy the utility of their wage while employed. There is a measure α of potential firms, each of which employs one or more workers if its productivity is high enough. While the set of potential firms is given, variation in productivity generates entry and exit in the economy. We denote by ε a firm's idiosyncratic productivity, which determines the output of a position at the firm. This productivity evolves stochastically over time: at rate γ, a firm draws a new productivity realization from the distribution $F(\varepsilon) : [\underline{\varepsilon}, \overline{\varepsilon}] \rightarrow [0, 1]$. These changes in productivity are what induce changes in a firm's employment and recruitment policy.

In order to hire workers, firms either create new positions or have empty positions come

[5] Note that we use the terms "job" and "position" interchangeably.

available following worker separations. To fill these available positions, firms post vacancies, which are filled independently of one another. The upfront cost of creating n new positions is equal to $C(n)$, where $C(\cdot)$ is a strictly increasing and strictly convex function with $C(0) = 0$ and $C'(0) \geq 0$. The flow cost of advertising a vacant position (regardless of whether it is a new or recently vacated position) is c. The fact that a vacated position does not require the payment of the upfront cost makes it less costly to recruit for it, which is a crucial feature of the model. If a vacant position is closed without being filled or if a firm does not seek to replace a worker at the time of her departure, the position ceases to exist. Once a match is created, there is no interaction between workers within a firm. This is due to the linear production technology. Hence, the payoff of a match to the worker and to the firm is independent of the number of workers a firm employs. In other words, the concept of a firm embedded in this model is that of a multi-worker entity that experiences a common productivity process.

The firm can terminate workers at any time. In addition, while initially productive, positions are destroyed at an exogenous rate δ. Moreover, workers leave the firm for exogenous reasons at rate ρ and quit at an endogenously determined rate $\mu(\varepsilon)$. Following an exogenous separation or a quit, the position remains productive and therefore available to fill.

Workers can search while unemployed and employed. Given that positions at different firms vary in their productivity and therefore in the wage they offer, workers have an incentive to search on-the-job. To keep the model simple, the search intensity of employed workers is fixed at $s \in (0, 1]$, while that of unemployed workers is normalized at 1.

Wages in the model are determined by workers and firms continuously bargaining over and sharing the output from the match without the possibility to commit to future wages. We assume that the disagreement payoff of the worker and the firm is delay in production and that the search opportunities during delay are the same as during production. The assumed lack of commitment has several implications. First, the non-convexity of the Pareto frontier discussed in Shimer (2006) does not arise in this setting, since the worker and the firm bargain only over the current wage that does not affect future turnover decisions. Second, when an employed worker receives an outside offer, she does not have the opportunity to extract the full rent from the less appealing of the two relationships, since she knows that the instant she chooses to forgo one of the employment possibilities, the firm can renegotiate. The outcome of this bargaining game with continuous renegotiation is simply

$$w(\varepsilon) = b + \beta(\varepsilon - b), \tag{2}$$

where β denotes the bargaining share of the worker. This wage function ensures that turnover in the model is monotone in the sense that, when workers have multiple employment opportunities, they always choose to take the job at the most productive firm.[6]

Contacts between vacancies and searching workers are generated by a constant returns to scale matching function that is increasing in its two arguments: the economy-wide measure of vacancies, v, and the measure of workers searching for a job, $u + s(1-u)$, where u denotes the measure of unemployed. We define labor market tightness as $\theta = \frac{v}{u+s(1-u)}$ and denote

[6]For other wage-setting mechanisms, the opportunity to repost vacant positions creates the possibility of the wage declining with productivity over some range. This is due to the presence of search externalities. Our specification of the bargaining game rules out this possibility.

the rate at which workers contact firms as

$$\lambda(\theta) = \frac{m(v, u + s(1-u))}{u + s(1-u)} = m(\theta, 1) \tag{3}$$

and the rate at which firms contact workers as $\eta(\theta)$. Given the assumption of a constant returns to scale matching function, the two are related by $\lambda(\theta) = \theta\eta(\theta)$. Notice that, due to the presence of on-the-job search, not all matches generated by the matching function result in the formation of a new employment relationship, since employed workers will reject offers from firms with a lower productivity than that of their current employer.

3.2 Characterization of the stationary equilibrium

Let us next introduce some notation. Let the probability that a firm of type ε succeeds in filling a vacant position upon contacting a worker be $\xi(\varepsilon)$. Let the unnormalized distribution of firm productivity across vacancies be $H(\varepsilon)$, so that $H(\overline{\varepsilon}) = v$. Then, denote the normalized distribution of productivity across vacancies by $\hat{H}(\varepsilon) = \frac{H(\varepsilon)}{v}$. Finally, let the unnormalized distribution of productivity across filled jobs be $K(\varepsilon)$, so that $K(\overline{\varepsilon}) = 1 - u$.

Due to the linearity of the production function and the fact that positions at a firm are filled independently of one another, the only state variable that affects the value of a position to the worker or the firm is firm productivity, ε.

Given our wage-setting assumption, the worker and the firm need not agree at all times whether to continue an employment relationship. This contrasts with the continuation decision under surplus sharing. Let the set of productivities for which the firm is willing to continue to operate a job be Ω_f (with complement $\overline{\Omega}_f$) and let the set of productivities for which the worker is willing to work in a job be Ω_w (with complement $\overline{\Omega}_w$). Given the requirement of mutual consent, an employment relationship will form and continue as long as $\varepsilon \in \Omega_f \cup \Omega_w$. The value to a worker of a job with productivity $\varepsilon \in \Omega_f \cup \Omega_w$ is

$$\begin{aligned} rW(\varepsilon) &= w(\varepsilon) + s\lambda(\theta) \int_{\Omega_f} \left(\max\left[W(\varepsilon'), W(\varepsilon)\right] - W(\varepsilon)\right) d\hat{H}(\varepsilon') - (\delta + \rho)\left(W(\varepsilon) - U\right) \\ &+ \gamma \left[UF\left(\overline{\Omega}_f\right) + \int_{\Omega_f} \max\left[W(\varepsilon'), U\right] dF\left(\varepsilon'\right) - W(\varepsilon) \right], \end{aligned} \tag{4}$$

where U is the value of unemployment for the worker. The first term is the flow wage the worker receives. The second term reflects the gains of the worker from a quit, which occurs if the worker encounters a job offer with a higher value than her current job. The third term reflects the loss associated with job destruction or exogenous separation. The last term reflects the change in value associated with a new draw of firm productivity and takes into account that continuation requires mutual agreement; i.e., it only takes place if $\varepsilon' \in \Omega_f \cup \Omega_w$. The value of unemployment can be expressed as

$$rU = b + \lambda(\theta) \int_{\Omega_f} \left(\max\left[W(\varepsilon'), U\right] - U\right) d\hat{H}(\varepsilon'), \tag{5}$$

where the first term is the flow utility received by an unemployed worker and the second term reflects the value a worker gains when she encounters a vacant job.

Similarly, the value of a filled job to a firm with productivity $\varepsilon \in \Omega_f \cup \Omega_w$ is

$$\begin{aligned} rJ\left(\varepsilon\right) &= \varepsilon - w\left(\varepsilon\right) + \left(\mu(\varepsilon) + \rho\right)\left(R\left(\varepsilon\right) - J\left(\varepsilon\right)\right) - \delta J(\varepsilon) \\ &+ \gamma \left[\int_{\overline{\Omega}_w} R(\varepsilon') dF\left(\varepsilon'\right) + \int_{\Omega_w} \max\left[J(\varepsilon'), R(\varepsilon')\right] dF\left(\varepsilon'\right) - J\left(\varepsilon\right) \right], \end{aligned} \tag{6}$$

where $R(\varepsilon)$ is the firm's expected value of a vacancy. Here, $\varepsilon - w(\varepsilon)$ represents the net flow payoff to the firm from operating a productive job. The second term reflects the loss associated with a worker leaving, either through a quit or an exogenous separation. The third term reflects the loss from a job-destruction shock. Note that a quit or exogenous separation leaves a position with a positive reposting value, while the job destruction shock does not. The last term reflects the change in value associated with a new draw of firm productivity, which again takes into account that continuation requires mutual agreement. The value of a vacant job is determined by

$$\begin{aligned} rR(\varepsilon) &= \max\left[0, -c + \eta(\theta)\xi(\varepsilon)\left(\max\left[J(\varepsilon), R(\varepsilon)\right] - R\left(\varepsilon\right)\right) - \delta R\left(\varepsilon\right)\right. \\ &+ \left.\gamma \int \left[R(\varepsilon') - R(\varepsilon)\right] dF\left(\varepsilon'\right)\right]. \end{aligned} \tag{7}$$

The firm can choose to post no vacancy or close a current opening, in which case its payoff is 0, or it can keep the vacancy open, in which case its payoff is the second expression in the brackets. The first term in this expression is the flow cost of vacancy posting, while the second term reflects the expected gain from meeting a potential new worker. The third term reflects the loss from the open position exogenously becoming unproductive, while the last term reflects the change in value associated with a new draw of firm productivity.

Next, we make the following assumption:

Assumption 1. *The cost of vacancy posting is large enough so that* $\gamma \int R(\varepsilon')dF\left(\varepsilon'\right) < c$.

This assumption states that the option value of keeping a vacancy open, which is present because the productivity of a position can change, is not large enough to justify incurring the cost of keeping the vacancy open. This implies that there exist some vacated jobs that are not profitable to repost and are thus closed by firms. The assumption ensures that any filled vacancy is profitable to operate at its current productivity, that is, for any vacancy $J(\varepsilon) > R(\varepsilon)$.

It is straightforward to show that, given the wage equation in (2), the above functional equations are satisfied by a constant U and three increasing functions J, R, and W. The monotonicity of W has several implications. First, employed workers quit to accept all job offers with a higher productivity than their current job, making the quit rate at a firm with productivity ε

$$\mu(\varepsilon) = s\lambda(\theta)\left(1 - \hat{H}(\varepsilon)\right). \tag{8}$$

As one can see, the quit rate is decreasing in ε: workers are less likely to quit more productive jobs. Second, a firm with productivity ε will have all unemployed workers, as well as all workers employed at firms with productivity lower than ε, accept their job offer, making their job-filling probability:

$$\xi(\varepsilon) = \frac{u + s(1-u)\frac{K(\varepsilon)}{K(\overline{\varepsilon})}}{u + s(1-u)} = \frac{u + sK(\varepsilon)}{u + s(1-u)}, \tag{9}$$

where workers are weighted by their search intensity. Last, the job-continuation decision of the worker has the reservation property – the worker chooses unemployment over working at any firm with productivity below some $\tilde{\varepsilon}_w$, so that $\Omega_w = [\tilde{\varepsilon}_w, \overline{\varepsilon}]$, where $\tilde{\varepsilon}_w$ satisfies $W\left(\tilde{\varepsilon}_w\right) =$

U, or

$$(1-s)\lambda(\theta)\int_{\tilde{\varepsilon}_w}(W(\varepsilon')-U)\,d\hat{H}(\varepsilon')=\beta\,(\tilde{\varepsilon}_w-b)+\gamma\int_{\tilde{\varepsilon}_w}(W(\varepsilon')-U)\,dF\,(\varepsilon')\,. \tag{10}$$

Equation (10) states that the difference between b and $\tilde{\varepsilon}_w$ comes from two sources: the higher option value of search while unemployed (due to the higher search intensity while unemployed), reflected by the term on the left-hand side, and the option value of changing productivity, reflected by the second term on the right-hand side.

Given the monotonicity of J, the job-continuation decision of the firm also has a reservation property. A firm chooses to destroy any job whose productivity falls below some $\tilde{\varepsilon}_f$, so that $\Omega_f=[\tilde{\varepsilon}_f,\bar{\varepsilon}]$, where $\tilde{\varepsilon}_f$ satisfies $J\,(\tilde{\varepsilon}_f)=0$. The optimality of $\tilde{\varepsilon}_f$ requires that

$$0=(1-\beta)\,(\tilde{\varepsilon}_f-b)+\gamma\int_{\tilde{\varepsilon}_f}J\,(\varepsilon')\,dF\,(\varepsilon')\,. \tag{11}$$

Equation (11) states that the flow loss from a job with productivity $\tilde{\varepsilon}_f$ is just offset by the option value of continuing to operate the job in expectation of an improvement in its productivity. Given the continuation set of the worker and the firm, the mutually acceptable job-destruction threshold then is $\tilde{\varepsilon}=\max\,(\tilde{\varepsilon}_f,\tilde{\varepsilon}_w)$.

Given the monotonicity of the function R, the vacancy reposting decision of the firm also has the reservation property – firms post (or repost) a vacancy when its productivity is above some threshold $\hat{\varepsilon}$. This threshold satisfies $R\,(\hat{\varepsilon})=0$ or

$$c=\eta(\theta)\xi\,(\hat{\varepsilon})\,J\,(\hat{\varepsilon})+\gamma\int_{\hat{\varepsilon}}R(\varepsilon')dF\,(\varepsilon')\,. \tag{12}$$

Notice that, given Assumption 1, $J(\hat{\varepsilon}) > 0$, and therefore $\tilde{\varepsilon} < \hat{\varepsilon}$. The presence of search frictions drives a wedge between the two thresholds. Firms with productivity $\varepsilon \in (\tilde{\varepsilon}, \hat{\varepsilon})$ find it profitable to continue existing employment relationships but do not find it profitable to go through costly recruiting if a worker leaves. When workers quit, these firms contract by attrition.

The fixed cost of creating a new job, $C(n)$, implies that $R(\varepsilon)$ is positive for firms with $\varepsilon > \hat{\varepsilon}$, even with a free entry of new jobs. Given the upfront cost of creating new jobs, a firm with productivity ε opens $n(\varepsilon)$ new jobs to the point where

$$C'(n(\varepsilon)) \geq R(\varepsilon) \tag{13}$$

with complementary slackness, $n(\varepsilon) \geq 0$. Given the assumption that $C'(0) \geq 0$, new jobs are created by firms that have productivity above some threshold, $\check{\varepsilon}$, where

$$C'(0) = R(\check{\varepsilon}). \tag{14}$$

Monotonicity ensures that $\check{\varepsilon} \geq \hat{\varepsilon}$. Moreover, given the properties of $C(\cdot)$ and $R(\cdot)$, $n(\cdot)$ is increasing in ε. Thus, firms with productivity $\varepsilon \in (\hat{\varepsilon}, \check{\varepsilon})$ find it profitable to replace workers who leave but do not find it profitable to open new jobs because of their upfront cost. Over time, such firms will be slowly shrinking due to the destruction of some of their positions. Firms with $\varepsilon > \check{\varepsilon}$ not only replace lost workers but also expand by creating new jobs.

Finally, one can derive the distributions of productivity across vacancies and across filled jobs in a stationary equilibrium from the appropriate balance equations. In particular, we can equate the flow into $H(\varepsilon)$, which is made up of new vacancies, reposted vacancies, and existing vacancies that had a change in their productivity, to the flow out of $H(\varepsilon)$, which is made up of exogenously destroyed vacancies, filled vacancies, and vacancies that had a change in their productivity, to get

$$\begin{aligned} &\alpha \int_{\hat{\varepsilon}}^{\varepsilon} n(\varepsilon')dF(\varepsilon') + \int_{\hat{\varepsilon}}^{\varepsilon} (\mu(\varepsilon') + \rho)\, dK(\varepsilon') + \gamma H(\overline{\varepsilon})\left[F(\varepsilon) - F(\hat{\varepsilon})\right] \\ = &\int_{\hat{\varepsilon}}^{\varepsilon} \left[\delta + \eta(\theta)\xi(\varepsilon')\right] dH(\varepsilon') + \gamma H(\varepsilon). \end{aligned} \tag{15}$$

Similarly, we can equate the flow into $K(\varepsilon)$, which is made up of newly filled vacancies and jobs that had a change in their productivity, to the flow out of $K(\varepsilon)$, which is made up exogenously destroyed jobs, jobs made vacant by a quit or exogenous separation, and jobs that had a change in their productivity, to get

$$\int_{\tilde{\varepsilon}}^{\varepsilon} \eta(\theta)\xi(\varepsilon')dH(\varepsilon') + \gamma K(\overline{\varepsilon})\left[F(\varepsilon) - F(\tilde{\varepsilon})\right] = \int_{\tilde{\varepsilon}}^{\varepsilon} (\delta + \mu(\varepsilon') + \rho)\, dK(\varepsilon') + \gamma K(\varepsilon). \tag{16}$$

Notice that the total number of jobs filled and the total number of jobs taken have to be equal to each other so that

$$\lambda(\theta)u + \int_{\tilde{\varepsilon}}^{\overline{\varepsilon}} \mu(\varepsilon')\, dK(\varepsilon') = \int_{\hat{\varepsilon}}^{\overline{\varepsilon}} \eta(\theta)\xi(\varepsilon')dH(\varepsilon'), \tag{17}$$

which can be verified by integration by parts. Equation (16) evaluated at $\varepsilon = \overline{\varepsilon}$ together with Equation (17) implies the steady-state relationship that equates the flow out of and into unemployment:

$$\lambda(\theta)\, u = (\delta + \rho + \gamma F(\tilde{\varepsilon}))\,(1-u)\,. \tag{18}$$

The equilibrium of the model is characterized by eight functions $W(\varepsilon)$, $J(\varepsilon)$, $R(\varepsilon)$, $H(\varepsilon)$, $K(\varepsilon)$, $\xi(\varepsilon)$, $\mu(\varepsilon)$, and $n(\varepsilon)$ and eight constants $U, \widehat{\varepsilon}, \widetilde{\varepsilon}_f, \widetilde{\varepsilon}_w, \check{\varepsilon}, u, v$, and θ. The eight functions are defined by Equations (4), (6), (7), (8), (9), (13), (**??**), and (16), while the eight constants are defined by Equations (5), (10), (11), (12), (14) and the relationships $K(\overline{\varepsilon}) = 1-u$, $H(\overline{\varepsilon}) = v$, and $\theta = v/(u + s(1-u))$. To solve for this set of equations, it is useful to derive differential equations characterizing the functions $W(\varepsilon)$, $J(\varepsilon)$, $R(\varepsilon)$, $H(\varepsilon)$, and $K(\varepsilon)$, which is done in the Appendix.

4 Model implications

4.1 Qualitative implications

Our model has a rich set of implications for firm dynamics. The model predicts a direct, positive relationship between firm productivity and firm growth. This relationship is the outcome of the model's implied patterns of worker turnover and recruitment, which are themselves functions of firm productivity. Given the endogenously determined decision thresholds, $\{\tilde{\varepsilon}, \hat{\varepsilon}, \check{\varepsilon}\}$, we can distinguish four regions of productivity, each with a different growth prospect.

We depict these regions in Figure 5. In Region 1, productivity is below the separation threshold, $\tilde{\varepsilon}$, so the firm shuts down, endogenously destroying all of its jobs. In Region

2, firms are between the separation threshold and the replacement threshold, $\hat{\varepsilon}$. When workers separate, either through quits or exogenous separations, firms in this region do not post vacancies to replace them and thereby contract their employment through attrition. In Region 3, firms are between the replacement threshold and the job creation threshold, $\check{\varepsilon}$. These firms post vacancies to replace workers who leave but do not create any new jobs. Finally, firms in Region 4 are above the job creation threshold and therefore have the highest productivity levels. These firms recruit both to replace workers who leave and to fill new jobs.

These regions create a natural relationship between job flows (job creation and job destruction) and worker flows (hires and separations). Firms undergo job destruction in Regions 1 and 2 entirely through the separation rate, though separations are all layoffs in Region 1 and a mix of quits and layoffs in Region 2. In Region 3, firms are essentially stable, since they recruit to replace those who have separated. Firms in this region can end up without any job flows if their hiring rates exactly offset their separation rates. Finally, firms undergo job creation in Region 4. These firms might still have workers separate, but these losses are more than offset by hiring, leading to net job gains. Over time, shocks to firm productivity move firms across these regions, giving rise to rich employment dynamics.

4.2 Model simulation

To examine our model's ability to match the evidence we highlighted in Section 2, we simulate employment turnover data using a mixture of calibrated and plausibly set parameter values. For this exercise, we enrich our model along two dimensions to allow it to replicate important features of the data. First, we assume that the position destruction rate δ decreases with ε. This ensures that not all firms experience the same rate of layoffs, which would be at odds with the data. Varying layoff rates could also be introduced by allowing for decreasing returns to scale in production or for a downward-sloping demand for the firm's output, but these alternatives would substantially complicate the model. Second, we assume that the probability that a position that becomes vacant is available to be reposted is given by the increasing function $\chi(\varepsilon)$ (recall that this probability is unity in the benchmark model). We introduce this feature to remove the discontinuity in the behavior of vacancies that is present in the benchmark model around $\widehat{\varepsilon}$.

In terms of the parameter choices, notice first that the monthly discount rate and the workers' bargaining power do not have an impact on the model-predicted turnover. Therefore, we leave these unspecified. We set the arrival rate of productivity shocks to be 2 percent per month, which implies that establishments are hit with a productivity shock every four years on average. We assume that ε is distributed uniformly on $[0, 1]$ and choose the $\delta(\varepsilon)$ function to be a decreasing and convex cubic spline over $[\widetilde{\varepsilon}, \varepsilon_\delta]$ with level parameter δ_m. The parameter ε_δ defines where $\delta(\varepsilon) = 0$ and is chosen to be 0.5. We choose the $n(\varepsilon)$ function to be an increasing cubic spline over $[\check{\varepsilon}, \overline{\varepsilon}]$ with level parameter n_m (this is equivalent to choosing the cost function $C(\cdot)$ appropriately). The replacement probability function, $\chi(\varepsilon)$, is also set to be a cubic spline such that $\chi(\widehat{\varepsilon}) = 0$ and $\chi(\check{\varepsilon}) = 1$.

We calibrate the parameters s, δ_m, n_m to match three calibration targets: the employment-weighted monthly quit rate in the JOLTS of 1.7 percent , the employment-weighted monthly layoff rate in the JOLTS of 1.1 percent, and the JOLTS average vacancy rate of 2.2 percent. We calibrate to an unemployment rate of 6.0 percent, which, together with the layoff rate, implies a job-finding rate of 0.172.[7] We let the rate of separations for exogenous reasons, ρ, equal a third of the layoff rate in JOLTS. We calibrate α to match the average establishment size of 20 workers in the JOLTS. Finally, we set the productivity thresholds as follows: $\widetilde{\varepsilon} = 0.1, \widehat{\varepsilon} = 0.4$, and $\breve{\varepsilon} = 0.6$. (We can recover the deep parameters b, c, and $C'(0)$ such that these thresholds constitute an equilibrium.) Using these parameter and threshold values, we can calculate the model-implied distribution of vacancies and employment, $H(\varepsilon)$ and $K(\varepsilon)$, respectively, which determine the functions $\mu(\varepsilon)$ and $\xi(\varepsilon)$ that allow us to generate the turnover data.

Figure 6 shows the calibrated $\delta(\varepsilon)$ and $n(\varepsilon)$ functions. Notice that, given the value of ρ, we need a relatively large value of $\delta(\varepsilon)$ for the low productivity firms in order to generate the targeted layoff rate. This is because our simulation has only a small fraction of employment below ε_{δ}. At the same time, firms open a relatively small number of vacancies (exceeding two only for the most productive establishments) because vacancies in the model are easy to fill for very productive establishments (they last for 1.3 weeks on average for the most productive establishment) and the vacancy rate in the data is relatively small. It is worth noting that the model cannot generate the full establishment size distribution. In particular, it does not account for the existence of very large establishments, which in turn explains why there are no establishments with a large number of vacancies. Figure 7 shows the endogenous quit rate, $\mu(\varepsilon)$, and acceptance probability, $\xi(\varepsilon)$, functions. The quit rate is constant at 4.3 percent between $\widetilde{\varepsilon}$ and $\widehat{\varepsilon}$. It then declines steadily until it reaches zero for the

[7]This is somewhat lower than the monthly job-finding rate measure reported by Nagypál (2008) using worker turnover data from the Current Population Survey and the Survey of Income and Program Participation. This is due to the fact that the separation rate in the JOLTS is lower than that measured in the CPS. Hence, to match the unemployment rate, we need a lower job-finding rate. See the discussion in Nagypál (2007b).

highest-productivity establishments. The acceptance probability begins at about 22 percent at the replacement threshold, $\widehat{\varepsilon}$. It then increases with ε, with the highest-productivity establishments always able to fill their jobs.

We next simulate weekly employment dynamics for a single establishment, discarding the first 1000 periods of data to eliminate the effect of initial conditions, to generate a distribution of firm-level turnover data.[8] Given the stationary nature of our model, turnover data over many periods at a single establishment have the same distribution as turnover data across many establishments for a shorter period of time.

4.3 Simulation results

Figures 8 through 12 plot our simulation results. Our empirical evidence presents the cross-sectional patterns of employment dynamics as a function of establishment growth, not productivity. Consequently, we depict the mapping of firm productivity ε into employment growth rates in Figure 8.[9] The mapping is generally consistent with the description of firm dynamics in Figure 5. Growth is negative, but rising, between $\widetilde{\varepsilon}$ and $\widehat{\varepsilon}$. Given the presence of exogenous job destruction and probabilistic availability of vacant positions, this pattern continues between $\widehat{\varepsilon}$ and $\check{\varepsilon}$. For values above $\check{\varepsilon}$, new position creation occurs in addition to replacement hiring, so growth becomes positive and increases somewhat with ε.

Figure 9 plots the simulated quit rate and layoff rate as a function of establishment growth. This figure matches the evidence displayed in Figure 1 along its most important

[8] In our simulation, we assume that events that take place at some monthly rate x in our model take place with weekly probability $1 - e^{-x/4}$ each week. This precludes the possibility of more than one event of a given type taking place within a week, which, given the calibrated arrival rates and the short length of time is not very restrictive.

[9] For Figures 8 through 14, we depict the results using a centered MA(5) process that uses the four closest observations in the moving average. We do this to smooth the noisiness of the stochastic simulation.

dimension. Namely, layoffs dominate quits during large contractions and quits are more prevalent than layoffs during smaller contractions. The model is able to generate a relatively greater importance of quits during small contractions through the decision of firms with productivities between $\widetilde{\varepsilon}$ and $\widehat{\varepsilon}$ to contract through attrition when a worker quits. In turn, the role of quits declines as the size of the contraction gets larger due to the presence of search frictions. While workers desire to quit quickly contracting firms, it takes time for them to locate alternative employment opportunities, which limits the rate at which quitting workers can leave such firms. This finding indicates that our choice of identifying quits with job-to-job transitions is appropriate, since quitting to unemployment would not be limited by search frictions. Given our calibration, the layoff rate decreases monotonically with the establishment growth rate for negative growth rates and is essentially zero for positive growth rates. The quit rate exhibits a similar pattern, though the rate levels off at about 5 percent for contractions of more than 10 percent of employment. This is lower than the 10 percent quit rate observed for rapidly contracting establishments in the data; keep in mind that we match the model only to the aggregate quit rate. Adding an endogenous search intensity choice to the model could overcome this discrepancy, since workers at rapidly contracting establishments would search harder and get more offers.

Figure 10 plots the hiring rate and the vacancy rate as a function of the establishment growth rate. The model matches the behavior of the hiring rate well. It predicts that the hiring rate is small and positive for shrinking establishments, though the magnitude is smaller than in the data. The hiring rate also exhibits a non-monotone pattern around zero growth, implying that establishments with small contractions hire more than stable establishments, and increases with the size of an establishment expansion. Quit replacement plays an important role in creating the non-monotonicity. Establishments with small contractions tend to lose most workers through quits, and although their productivity is high enough to induce them to try and refill these positions, it is not high enough to produce a high job-filling rate. Figure 10 also shows that the model can replicate the non-monotone pattern in the vacancy rate around zero growth, as well as a vacancy rate that increases with the size of an establishment expansion. Again, the incentive to post a vacancy coupled with a relatively low acceptance probability helps generate the non-monotone pattern. Just as with the hiring rate, the predicted vacancy rate for establishments with larger contractions is too low. In turn, the predicted vacancy rate for expanding establishments is too high. The flip side of this finding is that the simulated vacancy yield (the number of hires per vacancy posted at the start of the month) that is plotted in Figure 11 does not rise as much with establishment growth as in the data. This is because while productivity increases with establishment growth among expanding establishments, it does so at a slow pace. This then means that the increase in the job-filling probability is not very pronounced among expanding establishments. The vacancy yield also exhibits a jump around zero growth, which is due to the fact that some of the most productive establishments have grown large and become stable. In sum, the results from the model are broadly consistent with the evidence of Davis, Faberman, and Haltiwanger (2006b) and Davis, Faberman, and Haltiwanger (2006a) that we depicted in Figures 1-3.

Even though our model has a quit rate that declines with growth and hiring and vacancy rates that increase with growth, it is also able to replicate our own findings that the hiring and vacancy rates increase with the quit rate (Figure 4). Figure 12 shows that the model generates both a hiring rate and a vacancy rate that increases with the quit rate. The simulated positive relationships occur because of the incentive for firms with productivity above $\widehat{\varepsilon}$ to replace those who quit.

5 Conclusions

In this paper, we show that the quit-recruitment relationship is a quantitatively important part of establishment-level employment dynamics. While these dynamics may appear too complex for a standard model of labor market search to address, we show that a search model with three extensions – on-the-job search, replacement recruiting, and a fixed creation cost for new jobs – can account for much of the observed micro-level patterns in the data. With respect to these patterns, we focus on findings based on JOLTS microdata from Davis, Faberman, and Haltiwanger (2006b) and Davis, Faberman, and Haltiwanger (2006a). We also present new evidence from the JOLTS data. Our findings show a robust, positive relationship between hires, vacancies, and the incidence of a previous quit. We find that a disproportionate share of hires and vacancies occur at establishments with at least one prior quit. We also find that, at the establishment level, the incidence of a quit substantially increases the probability of a subsequent hire or vacancy in all industries and for all size classes. Across establishments, we find that both the hiring rate and the vacancy rate are increasing functions of the quit rate.

We then develop a matching model that accounts for the empirical patterns we observe. The model builds upon a search and matching framework with endogenous job destruction and on-the-job search by differentiating between the sunk cost of creating a new job and the flow cost of advertising for a vacant position. In the model, multi-worker firms face the standard decision of whether to continue or sever a match after an adverse shock to their productivity. Given the sunk cost of job creation and on-the-job search, firms face additional decisions whether to replace workers who leave the firm and whether to expand employment. This creates three thresholds of firm productivity that define the firm's separation decision, worker-replacement decision, and job-creation decision. The resulting decision rules create rich employment dynamics where quits and layoffs decrease with firm growth, vacancies and hiring increase with firm growth, and complex interactions between these processes emerge when firms must decide whether to replace a worker who leaves or let a job vanish. We find that the dynamics of the model are broadly consistent with the existing evidence from the JOLTS microdata. Namely, the model generates a decision region where relatively small employment contractions occur primarily through quits, and the model produces a job-filling rate that increases with firm growth. The model also produces firm-level hiring and vacancy rates that are positively related to the quit rate. The latter comes about through the potential incentive the model creates for firms to replace workers who leave.

Finally, while this paper focused entirely on the cross-sectional aspects of the model, the presence of on-the-job search and the distinction between recruiting for a new position and recruiting to replace a worker may be well-suited to replicate the cyclical movements in unemployment and vacancies observed in the data. For example, the quit replacement distinction, coupled with the fixed cost of job creation and flow cost of vacancy posting, can produce a countercyclical recruitment cost, since it would be less costly to replace a quit during booms.

6 Appendix

This appendix details the derivation of the necessary differential equations needed to characterize the stationary equilibrium. Differentiating Equation (4) with respect to ε for $\varepsilon \geq \tilde{\varepsilon}$ and rearranging gives

$$W'(\varepsilon) = \frac{\beta}{r + \delta + \rho + \mu(\varepsilon) + \gamma}. \tag{19}$$

Differentiating Equation (6) with respect to ε for $\varepsilon \geq \tilde{\varepsilon}$, rearranging, and taking into account that $R(\varepsilon) = 0$ for $\varepsilon \in [\tilde{\varepsilon}, \hat{\varepsilon}]$ gives

$$J'(\varepsilon) = \frac{(1-\beta) - \mu'(\varepsilon) J(\varepsilon) + I(\varepsilon \geq \hat{\varepsilon}) \left[(\mu(\varepsilon) + \rho) R'(\varepsilon) + \mu'(\varepsilon) R(\varepsilon) \right]}{r + \mu(\varepsilon) + \rho + \gamma + \delta}. \tag{20}$$

Differentiating Equation (7) with respect to ε for $\varepsilon \geq \hat{\varepsilon}$, then substituting in from Equation (20) and rearranging gives

$$(r + \eta(\theta)\xi(\varepsilon) + \delta + \gamma) R'(\varepsilon) = \eta(\theta)\xi'(\varepsilon) (J(\varepsilon) - R(\varepsilon)) + \eta(\theta)\xi(\varepsilon) J'(\varepsilon). \tag{21}$$

Differentiating Equation (**??**) with respect to ε for $\varepsilon \in [\hat{\varepsilon}, \overline{\varepsilon}]$ and differentiating Equation (16) with respect to ε for $\varepsilon \in [\tilde{\varepsilon}, \overline{\varepsilon}]$ and combining these results give for $\varepsilon \in [\tilde{\varepsilon}, \hat{\varepsilon})$

$$k(\varepsilon) = \frac{\gamma (1-u) f(\varepsilon)}{\delta + \mu(\varepsilon) + \rho + \gamma} \tag{22}$$

and for $\varepsilon \in [\hat{\varepsilon}, \overline{\varepsilon}]$, $h(\varepsilon)$ and $k(\varepsilon)$ are solutions to the system of linear equations

$$\alpha n(\varepsilon) f(\varepsilon) + \gamma v f(\varepsilon) = [\delta + \eta(\theta)\xi(\varepsilon) + \gamma] h(\varepsilon) - (\mu(\varepsilon) + \rho) k(\varepsilon) \quad (23)$$

$$\gamma (1 - u) f(\varepsilon) = (\delta + \mu(\varepsilon) + \rho + \gamma) k(\varepsilon) - \eta(\theta)\xi(\varepsilon) h(\varepsilon). \quad (24)$$

Then one can solve for $W(\varepsilon)$ from Equation (19) together with boundary condition $W(\tilde{\varepsilon}_w) = U$, for $J(\varepsilon)$ and $R(\varepsilon)$ from Equations (20) and (21) together with boundary conditions $J(\tilde{\varepsilon}_f) = 0$ and $R(\hat{\varepsilon}) = 0$, for $H(\varepsilon)$ from Equations (23) together with boundary condition $H(\hat{\varepsilon}) = 0$, and for $K(\varepsilon)$ from Equations (22) and (24) together with boundary condition $K(\tilde{\varepsilon}) = 0$.

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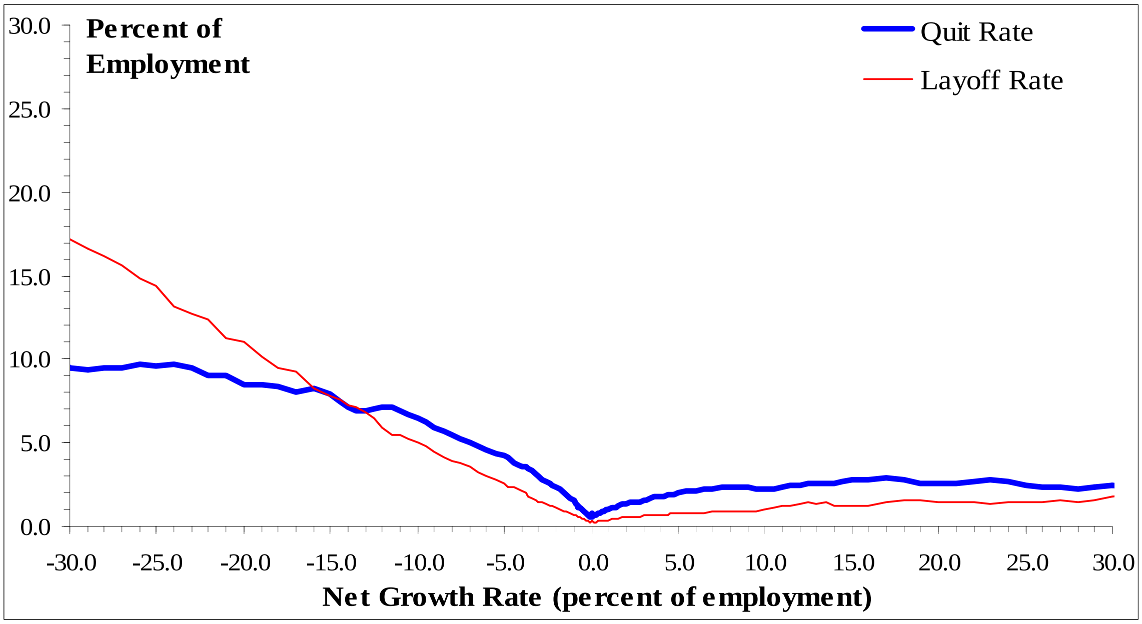
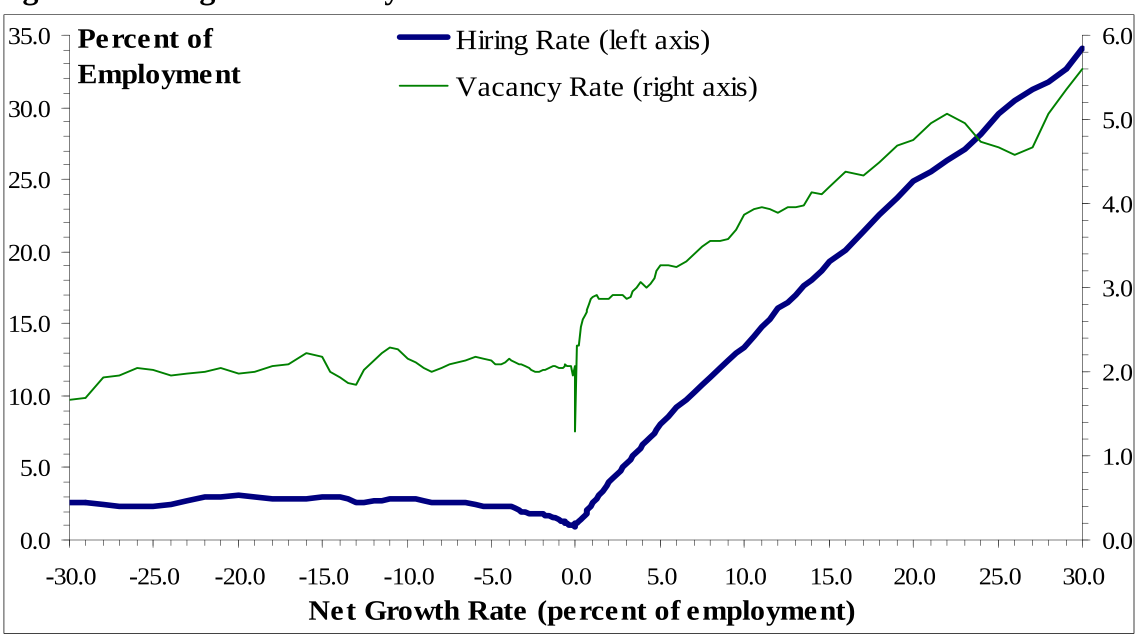
Figure 1. Quit and Layoff Rates as a Function of Establishment Growth



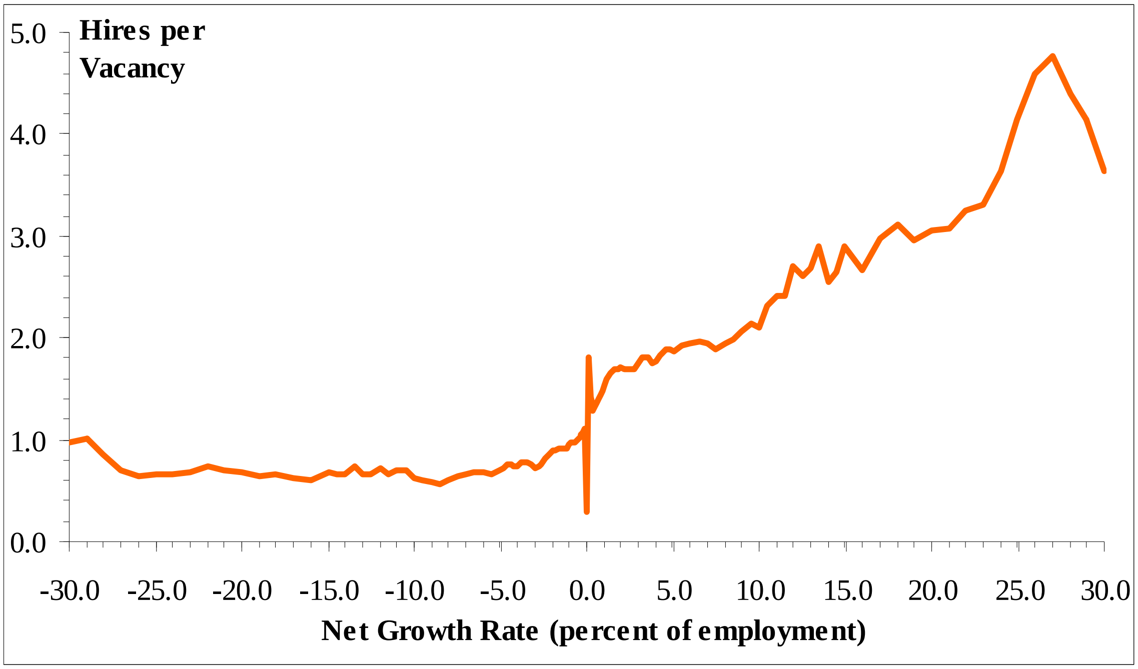
Notes: Figure depicts the quit and layoff rates as functions of the establishment-level employment growth rate (all depicted as fractions of employment), and is taken from Davis, Faberman, and Haltiwanger (2006a, p. 17). Rates are estimated over fine growth rate intervals that increase in size with the magnitude of growth. Estimates use pooled observations of JOLTS microdata, and the figure illustrates rates as a 5-interval centered moving average with a discontinuity allowed at zero growth.

Figure 2. Hiring and Vacancy Rates as a Function of Establishment Growth



Notes: Figure depicts the hiring and vacancy rates as functions of the establishment-level employment growth rate (all depicted as fractions of employment). Rates are estimated over fine growth rate intervals that increase in size with the magnitude of growth. The figure illustrates rates as a 5-interval centered moving average with a discontinuity allowed at zero-growth. Estimates are from Davis, Faberman, and Haltiwanger (2006b), who use pooled observations of JOLTS microdata.

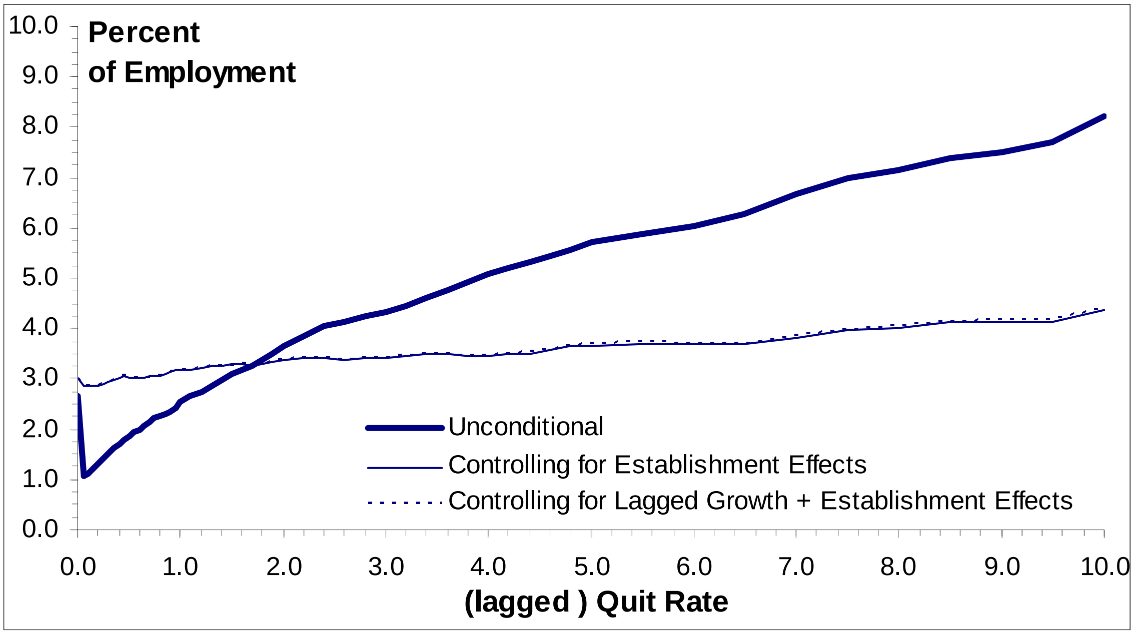
Figure 3. The Vacancy Yield as a Function of Establishment Growth



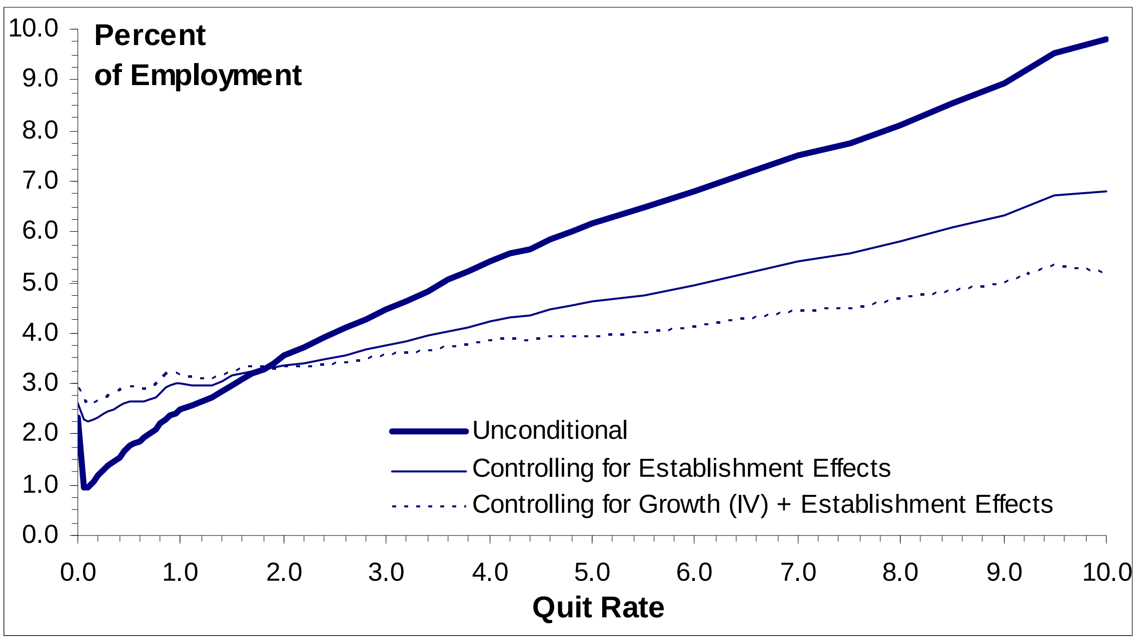
Notes: Figure depicts the vacancy yield (measured as hires per reported vacancy for establishments with at least one vacancy) as a function of the establishment-level employment growth rate. The yield is estimated over fine growth rate intervals that increase in size with the magnitude of growth. The figure illustrates rates as a 5-interval centered moving average with a discontinuity allowed at zero-growth. Estimates are from Davis, Faberman, and Haltiwanger (2006b), who use pooled observations of JOLTS microdata.

Figure 4. Hiring and Vacancy Rates vs. the Quit Rate, Regression Results

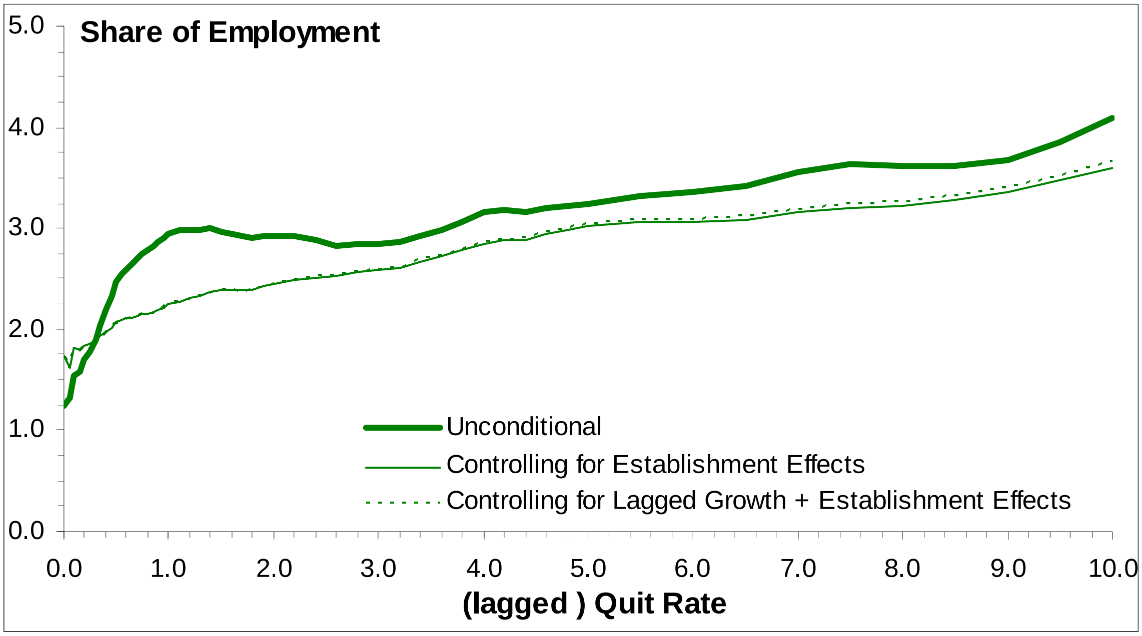
(a) Leading Hires vs. Quits



(b) Contemporaneous Hires vs. Quits

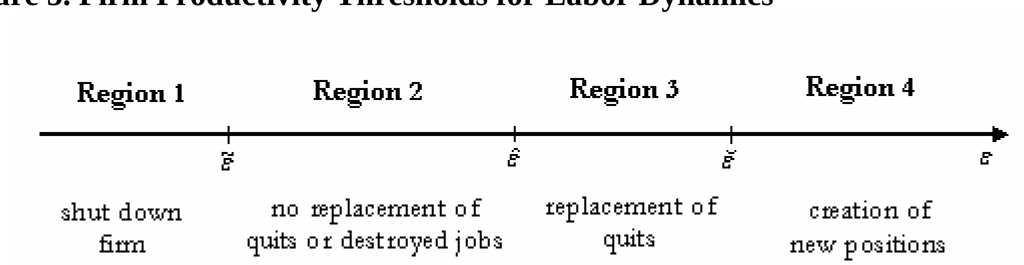


(c) Vacancies vs. Quits



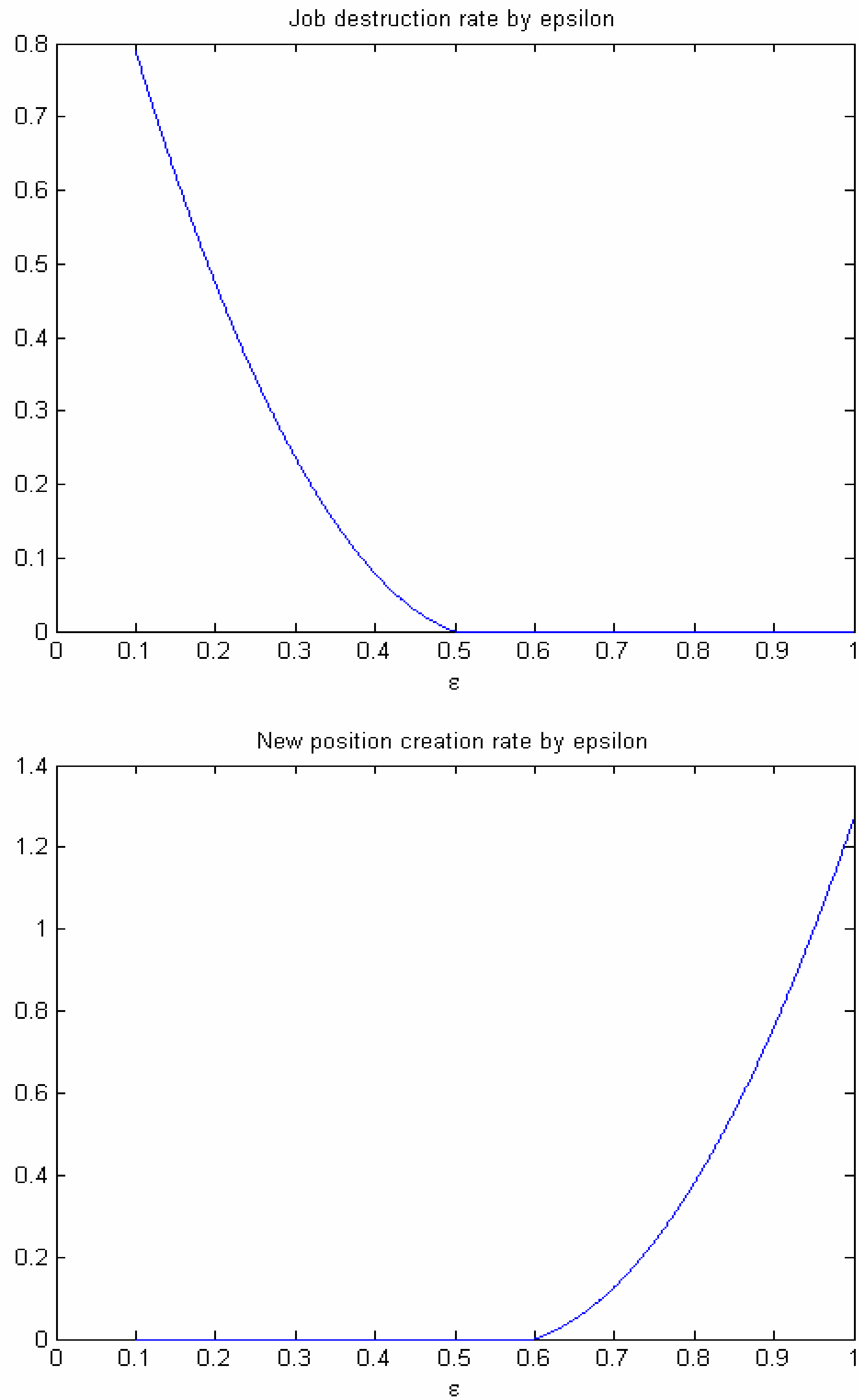
Notes: Figures depict the (leading) hires rate (top panel) and vacancy rate (bottom panel) as a function of the establishment-level quit rate (all depicted as fractions of employment) and broken out by type of establishment-level employment growth (expanding, contracting, no change). Rates are estimated over fine quit rate intervals that increase in size with the rate. Estimates are from authors' tabulations using pooled observations of JOLTS microdata, and the figure illustrates rates as a 5-interval centered moving average.

Figure 5. Firm Productivity Thresholds for Labor Dynamics



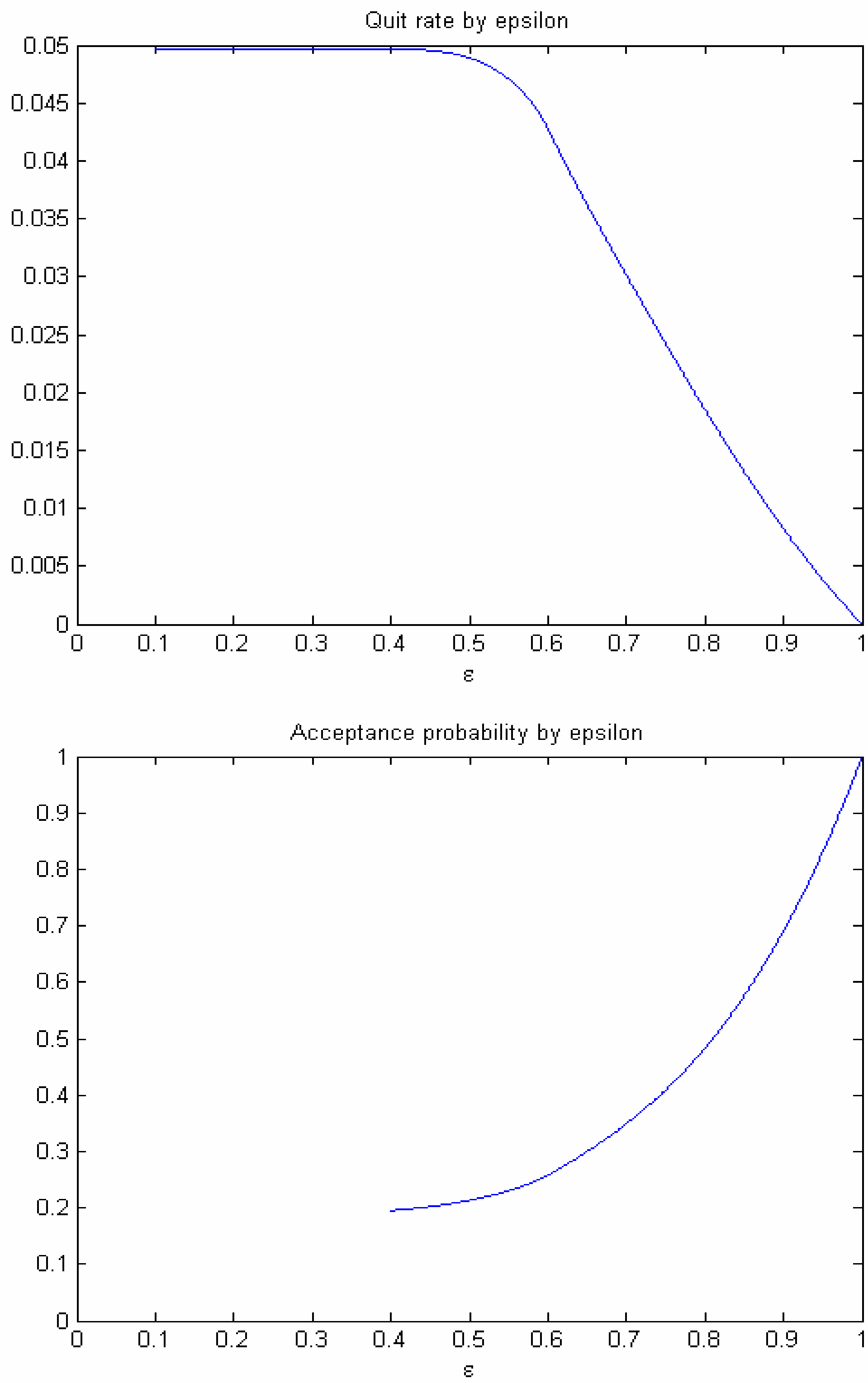
Note: Figure depicts the three endogenously determined thresholds of the model described in the text, with the decision rules between each threshold noted. See text for details.

Figure 6. Calibrated Job Destruction Rate ($\delta(\varepsilon)$) and New Position Creation Rate ($n(\varepsilon)$) Functions



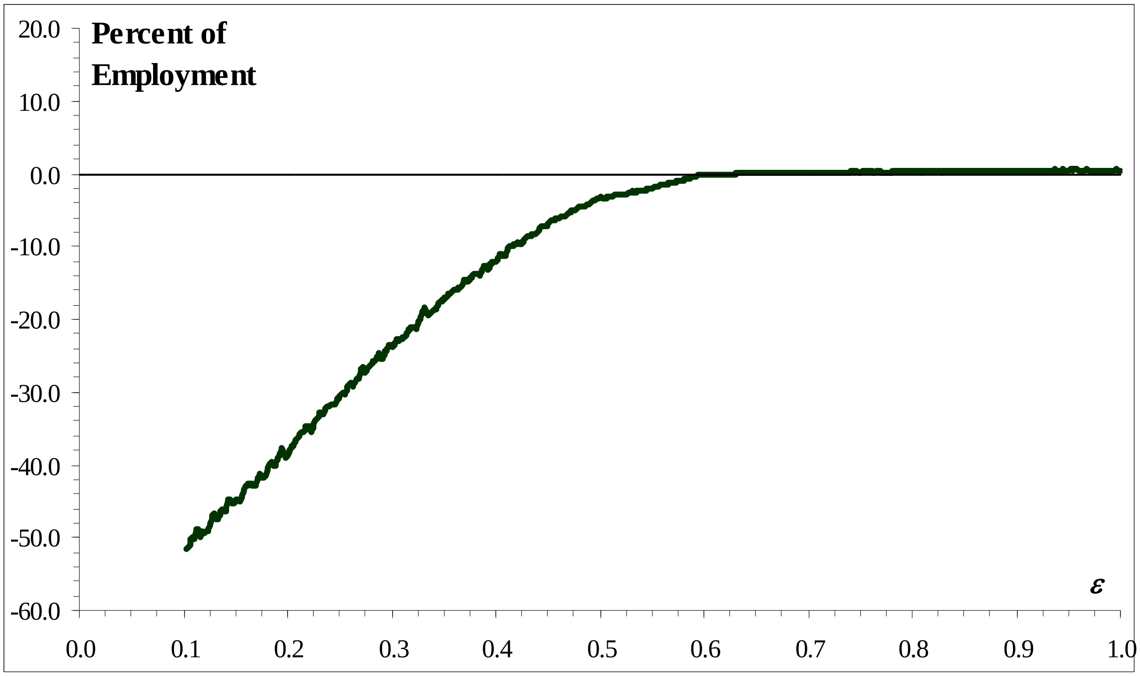
Note: The figure depicts the calibrated exogenous job destruction rate (top panel) and new position creation rate (bottom panel) as a function of firm productivity, ε, used in the model's simulation. See text for simulation details.

Figure 7. Model-Implied Quit Rate ($\mu(\varepsilon)$) and Acceptance Probability ($\xi(\varepsilon)$) Functions



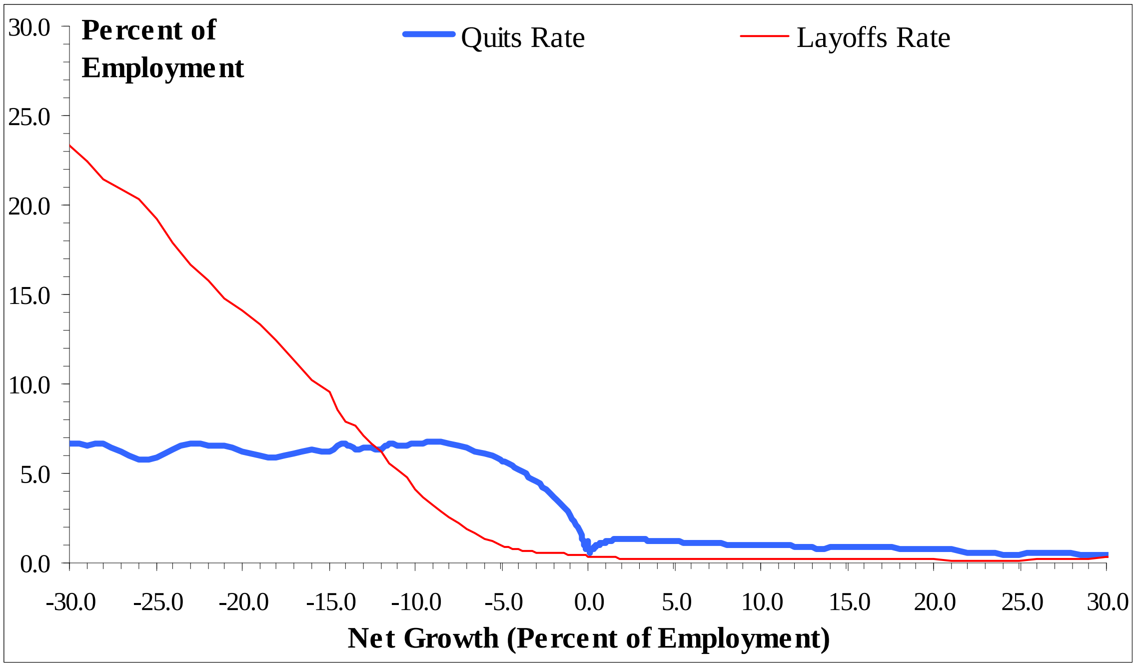
Note: The figure depicts the model implied quit rate (top panel) and acceptance probability (i.e., the job-filling rate, bottom panel) as a function of firm productivity, ε. See text for simulation details.

Figure 8. Simulated Average Net Growth Rate by Firm Productivity (ε)



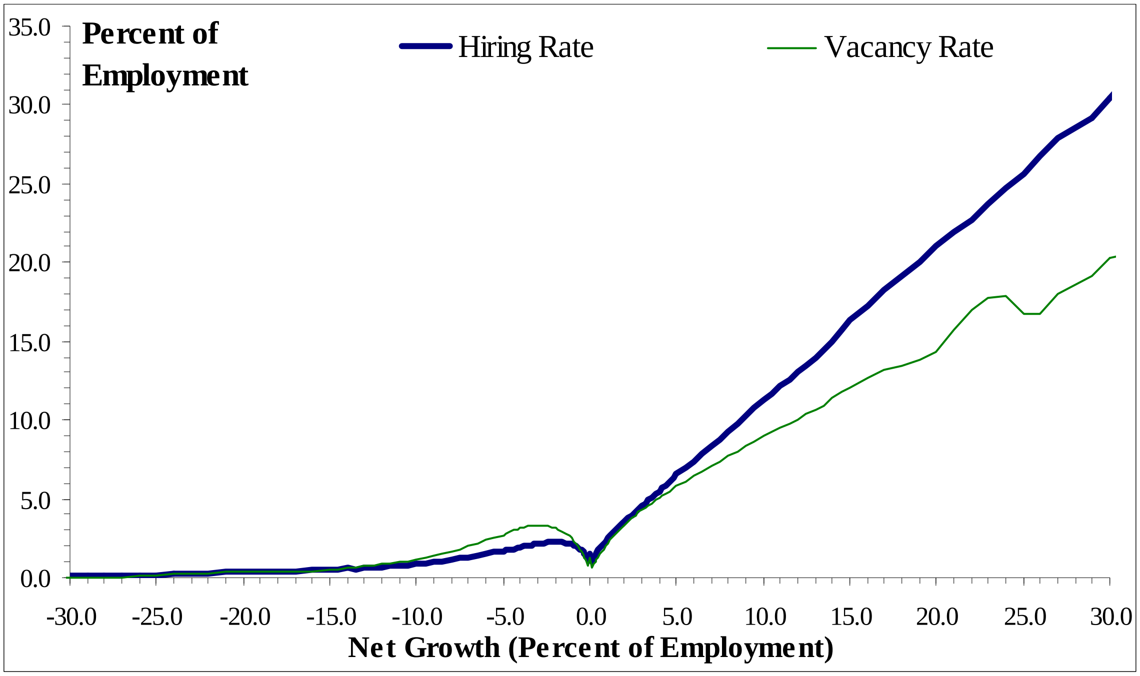
Note: The figure depicts the average simulated firm-level employment growth rate by firm productivity, ε. Estimates are smoothed using a 5-bin centered moving average across growth rate intervals identical to those used for the data analysis. See text for simulation details.

Figure 9. Simulated Quit and Layoff Rates as Functions of Firm Growth



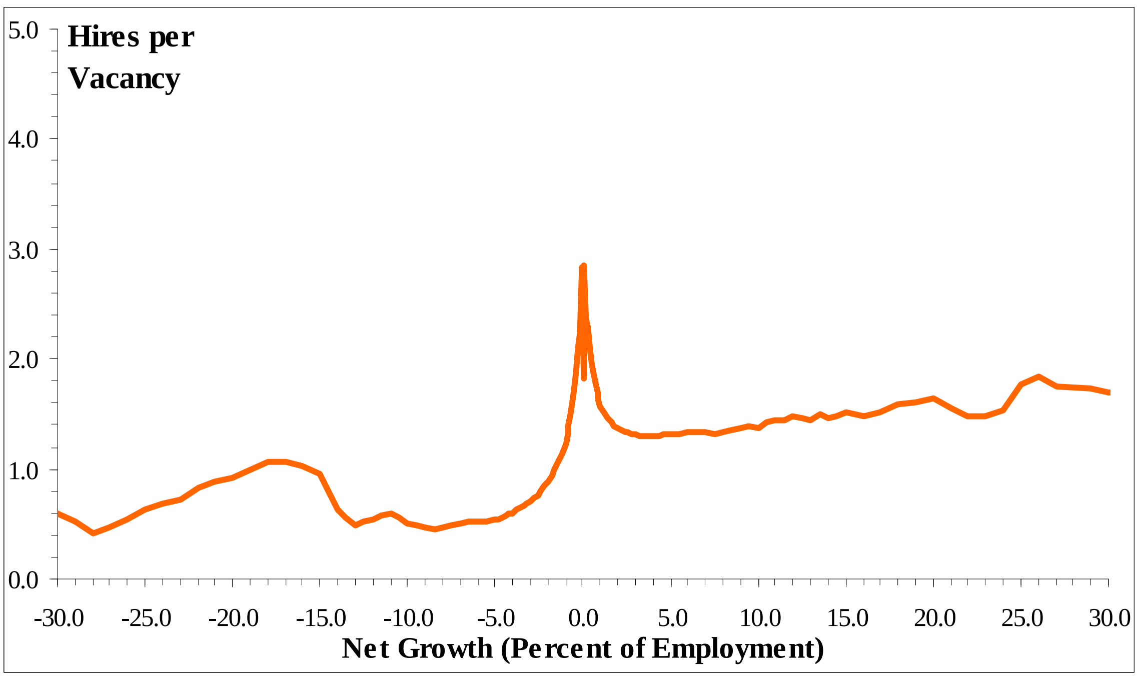
Note: The figure depicts the simulated layoff rate and simulated quit rate as a function of firm-level employment growth. Estimates are smoothed using a 5-bin centered moving average across growth rate intervals identical to those used for the data analysis. See text for simulation details.

Figure 10. Simulated Hiring and Vacancy Rates as Functions of Firm Growth



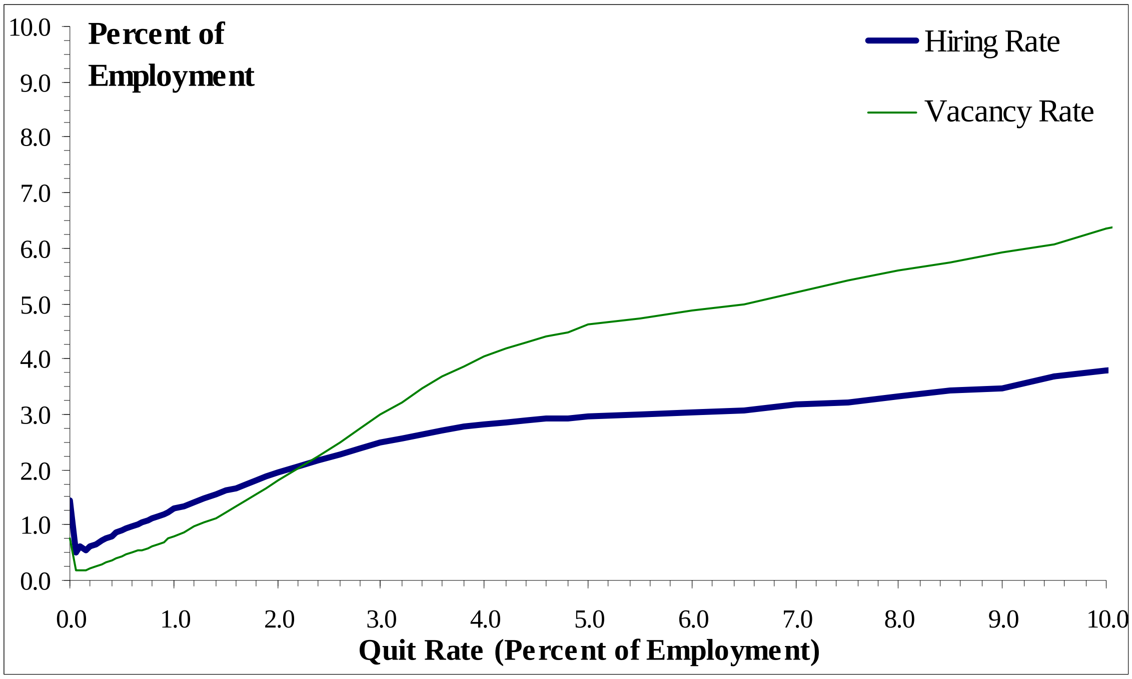
Note: The figure depicts the simulated vacancy rate and hiring rate as a function of firm-level employment growth. Estimates are smoothed using a 5-bin centered moving average across growth rate intervals identical to those used for the data analysis. See text for simulation details.

Figure 11. Simulated Vacancy Yield as a Function of Firm Growth



Note: The figure depicts the simulated vacancy yield (hires per vacancies posted at the beginning of the month) as a function of firm-level employment growth. Estimates are smoothed using a 5-bin centered moving average across growth rate intervals identical to those used for the data analysis. See text for simulation details.

Figure 12. Simulated Hiring and Vacancy Rates as Functions of Firm Quits



Note: The figure depicts the simulated hiring rate and vacancy rate as a function of firm-level quit rate. Estimates are smoothed using a 5-bin centered moving average across growth rate intervals identical to those used for the data analysis. See text for simulation details.

Table 1. Summary Statistics by Incidence of a Quit

	Full Sample	***No Previous Quit*** ($q_t = 0$)	***Previous Quit*** ($q_t > 0$)
Quit Rate (*t*)	0.018	---	0.031
Hiring Rate (*t* + 1)	0.033	0.027	0.037
Vacancy Rate (*t*)	0.022	0.013	0.029
Layoff Rate (*t*)	0.012	0.013	0.011
Share of Establishments	---	0.860	0.140
Share of Employment	---	0.421	0.579
Share of Hires (*t* + 1)	---	0.353	0.647
Share of Vacancies (*t*)	---	0.257	0.743

Notes: Estimates are means (employment-weighted) and fractions across pooled establishment observations from authors' tabulations using JOLTS microdata. Standard errors are all smaller than 0.0001.

Table 2. Frequency of Vacancies and Hiring by Quit Incidence and Establishment Characteristics

(a) Nonfarm Employment

	Pr(v_t > 0)		**Pr(h_{t+1} > 0)**	
	$q_t = 0$	$q_t > 0$	$q_t = 0$	$q_t > 0$
By Incidence of Quit	0.25	0.75	0.36	0.86
For All Establishments	0.54		0.65	

(b) Major Industry

	Pr(v_t > 0)		**Pr(h_{t+1} > 0)**	
	$q_t = 0$	$q_t > 0$	$q_t = 0$	$q_t > 0$
Natural Resources & Mining	0.20	0.53	0.35	0.79
Construction	0.13	0.44	0.37	0.77
Manufacturing	0.30	0.67	0.43	0.79
Transportation & Utilities	0.23	0.72	0.31	0.81
Retail Trade	0.17	0.58	0.34	0.81
Information	0.32	0.81	0.36	0.83
FIRE	0.20	0.82	0.26	0.86
Professional & Business Services	0.28	0.80	0.35	0.87
Health & Education	0.26	0.90	0.33	0.92
Leisure & Hospitality	0.21	0.63	0.43	0.84
Other Services	0.15	0.60	0.22	0.75
Government	0.41	0.88	0.43	0.92

(c) Establishment Size

	Pr(v_t > 0)		**Pr(h_{t+1} > 0)**	
	$q_t = 0$	$q_t > 0$	$q_t = 0$	$q_t > 0$
0-9 Employees	0.06	0.29	0.10	0.36
10-49 Employees	0.18	0.43	0.31	0.59
50-249 Employees	0.38	0.65	0.54	0.82
250-999 Employees	0.61	0.84	0.71	0.91
1000-4999 Employees	0.73	0.93	0.83	0.97
5000+ Employees	0.81	0.92	0.94	0.99

Notes: Estimates are the (employment-weighted) probabilities of a vacancy or a hire based on the incidence of at least one quit. Estimates come from authors' tabulations using pooled observations of JOLTS microdata.

Table 3. Vacancy and Hiring Statistics by Layoff Incidence

(a) Summary Statistics

	No Previous Layoff, or Layoff with a Quit ($l_t = 0 + [l_t > 0, q_t > 0]$)	***Previous Layoff, but No Quit*** ($l_t > 0, q_t = 0$)
Quit Rate (t)	0.019	---
Hiring Rate ($t + 1$)	0.032	0.040
Vacancy Rate (t)	0.022	0.020
Layoff Rate (t)	0.007	0.083
Share of Establishments	0.959	0.041
Share of Employment	0.936	0.064
Share of Hires ($t + 1$)	0.921	0.079
Share of Vacancies (t)	0.941	0.059

(b)Frequency of Vacancies and Hiring

	Pr($v_t > 0$)		**Pr($h_{t+1} > 0$)**	
	$l_t = 0 +$ $l_t > 0, q_t > 0$	$l_t > 0,$ $q_t = 0$	$l_t = 0 +$ $l_t > 0, q_t > 0$	$l_t > 0,$ $q_t = 0$
By Incidence of Layoff (with No Contemporaneous Quit)	0.55	0.46	0.65	0.62
For All Establishments	0.54		0.65	

Notes: The top panel reports the estimated mean rates (employment-weighted) and fractions across pooled establishment observations from JOLTS, conditional on whether the observation had at least one reported layoff, but no quit ($l_t > 0$, $q_t = 0$). The lower panel reports the (employment-weighted) probabilities of a vacancy or a hire based on the same criteria. Standard errors on all estimates are all smaller than 0.0001.

Table 4. Establishment-Level Regressions, Hiring, Vacancies and the Quit Rate

(a) Dependent Variable: $h_{i,t+1}$ (Leading Hires)

	(1)	(2)	(3)	(4)
q_{it}	.222	.050	.069	.056
	[.002]	[.002]	[.002]	[.001]
g_{it}			.012	
			[.001]	
$g_{it} > 0$				.024
				[.001]
$g_{it} < 0$				.003
				[.001]
Establishment Effects?	No	Yes	Yes	Yes
R-squared	.033	.291	.291	.292

(b) Dependent Variable: h_{it} (Contemporaneous Hires)

	(1) OLS	(2) OLS	(3) IV	(4) IV
q_{it}	.592	.347	.153	.665
	[.003]	[.003]	[.090]	[.032]
g_{it}			-.307	
			[.141]	
$g_{it} > 0$				1.170
				[.061]
$g_{it} < 0$				.382
				[.062]
Establishment Effects?	No	Yes	Yes	Yes
R-squared	.120	.319	.319	.320

(c) Dependent Variable: v_{it}

	(1)	(2)	(4)	(5)
q_{it}	.103	.062	.089	.092
	[.001]	[.001]	[.001]	[.001]
g_{it}			.017	
			[.0004]	
$g_{it} > 0$				.015
				[.001]
$g_{it} < 0$				.019
				[.001]
Establishment Effects?	No	Yes	Yes	Yes
R-squared	.022	.410	.413	.413

Notes: Tables report coefficients and standard errors (in brackets) of OLS (or instrumental variables, where noted) for regressions of the noted dependent variable on the noted regressors using pooled establishment-month observations. *N = 377,597*. Regressions include establishment fixed effects where noted. For IV estimates, regressions use the lagged growth rate (column 3) or the lagged growth rates differentiated between rates that are positive or negative (column 4) as the instrument(s).