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Frank Caliendo
Colorado State University
and
Kevin X. D. Huang
Vanderbilt University

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Overconfidence in Financial Markets and Consumption over the Life Cycle*

Frank Caliendo

Department of Economics, Colorado State University, Fort Collins, CO 80523, USA

Kevin X.D. Huang†

Department of Economics, Vanderbilt University, Nashville, TN 37235, USA

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†Corresponding author. Tel.: +1 615 936 7271; fax: +1 615 343 8495.

E-mail address: kevin.huang@vanderbilt.edu (K.X.D. Huang).

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Abstract

Overconfidence is a widely documented phenomenon. Empirical evidence reveals two types of overconfidence in financial markets: investors both overestimate the average rate of return to their assets and underestimate uncertainty associated with the return. This paper explores implications of overconfidence in financial markets for consumption over the life cycle. We obtain a closed-form solution to the time-inconsistent problem facing an overconfident investor/consumer who has a CRRA utility function. We use this solution to show that overestimation of the mean return gives rise to a hump in consumption during the work life if and only if the elasticity of intertemporal substitution in consumption is less than unit. We find that underestimation of uncertainty has little effect on the long-run average behavior of consumption over the work life. Our calibrated model produces a hump-shaped work-life consumption profile with both the age and the amplitude of peak consumption consistent with empirical observations.

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The chance of gain is by every man more or less overvalued, and the chance of loss is by most men undervalued.

The over-weening conceit which the greater part of men have of their own abilities, is an ancient evil remarked by philosophers and moralists of all ages.

—Adam Smith

1. Introduction

Humans can be overconfident in self-perceptions and overrate themselves on many positive personal traits. If John perceives he is better-looking than his classmates, he is not alone: Motley Fool guest columnist Whitney Tilson reports that 86% of his Harvard classmates feel they are better-looking than their peers (who is left to be worse looking?). If John thinks he can get a better grade than his peers, he is not alone either: University of Chicago professor Richard Thaler writes that on the first day of his class every student expects to get an above-the-median grade (half of them are inevitably disappointed).

Overconfidence does not just belong to those elite school students. If you think you are safer and more skillful than your fellow drivers, you are not alone: in Svenson's (1981) study of Texas car drivers, 90% of those assessed believe they have above-average skills and 82% rank themselves among the top 30% of safe drivers. Anything you think you are better at or have better luck with than others, your peers are likely to think the same way: 70% of lawyers in civil cases believe their sides will prevail; doctors consistently overestimate their abilities in detecting certain diseases; parents feel their children are smarter than others'; lottery pickers bet that tickets they choose have greater odds to win than randomly selected ones; professional athletes and military personnel may even be trained to be overconfident and overoptimistic.

The presence of overconfidence in the business world is also well known. A large body of literature documents that managers are prone to the wishful thinking that projects they have command of are bound to succeed.[1] In Copper, Woo, and Dunkelberg's (1988) survey of nearly 3000 new business owners, 81% percent of those sampled believe their

[1]See Kidd and Morgan (1969), Langer (1975), Larwood and Whittaker (1977), Weinstein (1980), Bettman and Weitz (1983), March and Shapira (1987), Russo and Schoemaker (1992), and Malmendier and Tate (2003, forthcoming), among others.

businesses have more than a 70% chance to succeed while 33% believe they will thrive for sure. In actuality, 75% of new ventures do not even survive the first five years.

The phenomena of overconfidence and overoptimism are widespread and have long been documented in the cognitive psychology and behavior science literature based on data from interviews, surveys, experiments, and clinical studies. Perhaps what is more overwhelming than the mere existence of overconfidence itself is the fact that the degree of overconfidence is rather persistent and generally does not wane over time. In the car-driver example, Camerer (1997) notes that even after suffering serious car accidents, drivers still rate themselves as above-average, and Bob Deierlein reports in a 2001 issue of WasteAge that more experienced drivers can develop a higher degree of overconfidence in their ability to avoid accidents but can in fact have accidents more frequently.

When it comes to the financial markets the phenomena of persistent overconfidence and overoptimism are even more overwhelming. Empirical studies suggest that investors persistently overestimate the average rate of return to their assets and underestimate uncertainty associated with the return. A thesis of the present paper is that investor overconfidence can be important for understanding consumption over the life cycle. Our main finding is that overestimation of the mean return can give rise to a hump-shaped work-life consumption profile, although underestimation of uncertainty can have very little effect on the long-run average behavior of consumption over the work life.[2]

It has been a challenge to explain the life-cycle consumption behavior. The empirical fact is that the work-life consumption profile of a typical consumer is hump-shaped, with peak consumption occurring between 45 and 55 years of age and with the ratio of peak consumption to consumption when first entering the workforce generally above 1.1 (e.g., Gourinchas and Parker, 2002, Attanasio, Banks, Meghir, and Weber, 1999, Browning and Crossley, 2001, and Feigenbaum, 2005). Such a hump-shaped work-life consumption profile is regarded as puzzling in the lens of the standard theory, which predicts that

[2]A number of studies show that underestimation of uncertainty associated with asset returns can have a significant effect on short-run volatilities in financial markets. See De Long, Shleifer, Summers, and Waldman (1991), Benos (1998), Odean (1998a, 1999), Statman and Thorley (19989), Gervais and Odean (2001), Barber and Odean (2000, 2001, 2002), Hirshleifer and Luo (2001), Deaves, Luders, and Luo (2004), Shiller (2005), Scheinkman and Xiong (2005), Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subrahmanyam (1998, 2001), Hong and Stein (1999), and Englmaier (forthcoming).

consumption should either increase (if the agent is patient) or decay (if the agent is impatient) monotonically over the life cycle.

Several mechanisms have been proposed to meet this challenge. One assumes that a "hand-to-mouth" agent simply consumes a constant fraction of his wage income that is hump-shaped. Optimization-based mechanisms rely on various features to generate a hump in the life-cycle consumption profile. These features include family size dynamics, uncertain lifetime, consumption-leisure substitutability, labor-income uncertainty, and consumer durables.[3]

Our optimization-based model featuring investor overconfidence generates a hump in the work-life consumption profile even for a single individual, with certain lifetime, without any tension between contemporaneous consumption-leisure substitutability or uncertainty in labor income, and with only nondurable goods in the consumption basket. Our calibrated model produces a hump-shaped work-life consumption profile with both the age and the amplitude of peak consumption roughly in line with the data.

Section 2 reviews some of the empirical studies that document persistent investor overconfidence in financial markets along with some available explanations, in particular, an optimal-expectations story offered by Brunnermeier and Parker (BP) (2005).

To isolate the effect of overestimation of the mean return to investment from that of underestimation of uncertainty associated with the return on life-cycle consumption, we consider in Section 3 a basic environment that abstracts from uncertainty where an agent overestimates the rate of return to his asset at each age, in the spirit of BP (2005). We obtain a closed-form solution in Section 4 to the time-inconsistent problem facing this overconfident agent who has a CRRA utility function. We use this solution to show that the overestimation of his asset return leads to a hump in consumption during his work life if and only if the elasticity of intertemporal substitution in consumption is less than 1. Section 5 provides some intuition behind this result. Section 6 shows that the calibrated version of the model generates a hump-shaped work-life consumption profile

[3]See, respectively but not exhaustively, Tobin (1967), Browning, Deaton, and Irish (1985), Attanasio and Browning (1995), Attanasio et al. (1999), Browning and Ejrnaes (2000), Bütler (2001); Yaari (1965), Bütler (2001), Hansen and İmrohoroğlu (2005), Feigenbaum (2005); Heckman (1974), Bütler (2001), Bullard and Feigenbaum (2005); Nagatani (1972), Hubbard, Skinner, and Zeldes (1994), Carroll (1994, 1997), Gourinchas and Parker (2002); and Fernández-Villaverde and Krueger (forthcoming). Browning and Crossley (2001) provide a survey.

where both the location and the amplitude of peak consumption are consistent with the empirical observations.

Section 6 also extends the baseline model to an environment with uncertainty where the actual rate of return to assets follows some stationary stochastic process. We examine first the case in which the agent has an unbiased estimation of the mean return but underestimates uncertainty associated with the return, and then the case in which he both overestimates the mean return and underestimates the uncertainty, as in BP (2005). In the first case, the agent's work-life consumption profile is virtually flat, with some bumpy noise in the short run. In the second case, his work-life consumption profile is essentially the same as the one in the baseline model that abstracts from uncertainty. We conclude that it is the overestimation of the mean return that explains the hump-shaped work-life consumption path, while underestimation of uncertainty has very little effect on the long-run average behavior of consumption over the work life.

2. Overconfidence in financial markets

The recent literature has documented a large body of empirical evidence on persistent overconfidence in financial markets. The evidence reveals that people persistently believe they have superior abilities in investing and good fortunes happen more often to them than to others, while they systematically overestimate the average rate of return to their assets and underestimate uncertainty associated with the return.[4] An infamous example is Taylor and Brown's (1988) survey which indicates that only depressed people tend to become less overconfident and more realistic, even in activities like investing. In a recent study based on a sample of nearly $15,000$ individual investors surveyed by the Gallup Organization, Barber and Odean (2001) find that men estimate the rate of return to their investments by nearly 3 percentage points higher than the market average return and women by almost 2 percentage points higher. According to Daniel Houston, a senior vice president for retirement and investor services at the Principal Financial Group Inc., a survey on retirement planning released in April 2005 indicates that Americans are way too confident about the future performance of their assets.

[4]See, among others, Presson and Benassi (1996), Odean (1998a, 1999), Barber and Odean (2002), Chuang and Lee (2003), Statman, Thorley, and Vorkink (2004), Allen and Evans (2005), and Biais, Hilton, Mazurier, and Pouget (2005).

In another study of 78,000 individual investors, Barber and Odean (2000) also find substantial persistence of investor overconfidence, which results in high trading volume and high turnover rates in the face of repeatedly lower-than-expected realizations of asset returns. A Gallup poll conducted in 2001 indicates that, even after an unprecedented stock market bubble peak and subsequent burst, investors still remain overconfident and expect to beat the market return by more than 1.5 percentage points (Fisher and Statman, 2002).[5] Based on a monthly survey of 350 financial market specialists, Deaves, Luders, and Schroder (2005) find that even professional market analysts are persistently overconfident and the degree of their overconfidence even increases with their longevity (see also Atkins and Sundali, 1997, among others).

With growing empirical evidence on persistent overconfidence, much attention has been paid to the question of why people are overconfident and experience does not lead them to become more realistic, especially in activities like investing where results can be calculated ex post. Existing studies demonstrate that self-serving attribution bias (past successes tend to exacerbate overconfidence as people take too much credit for their successes, while past failures tend to be ignored as people blame their failures on forces beyond their control), confirmatory bias and cognitive dissonance (tendency to overweigh data confirming prior beliefs while to dismiss data contradicting prior beliefs), illusion of control, and forces related to evolution and tournaments or contests, can all contribute to generating persistent overconfidence throughout the life cycle.[6] Gervais and Odean (2001) find that, with attribution bias at work, people may even learn to become more overconfident rather than more realistic over the life cycle.

In an important analysis, Brunnermeier and Parker (2005) show that overestimation of the mean of future income and underestimation of uncertainty associated with future income can be the outcomes of optimization by agents who choose subjective beliefs to maximize the average of their expected felicity over time. They provide a stereotypical

[5]As Shiller (2005) notes, speculative bubbles were not new and had persisted over the entire last century, with pronounced peaks in 1901, 1929, 1966, and 2000; yet, even after serious bubbles in stock prices have popped, investors remain persistently overconfident.

[6]See, among others, Odean (1998a, 1998b, and the references therein) and Barber and Odean (1999). A classic example can be found in Shefrin and Statman (1985), where investors judge their performance by returns realized rather than returns accrued, and by holding "losers" and selling "winners" they persistently overestimate the rate of return to their assets. See Englmaier (forthcoming) for a survey.

scenario where optimal beliefs lead to an extremity of overconfidence under which future income is always perceived as certain, even if such belief is repeatedly contradicted by realizations. The agent merely observes one income realization at each age and believes he was unlucky, so he continues to be overconfident looking forward.

3. The basic environment

Time is continuous and begins at 0. An agent enters the workforce at $t = 0$, earns wage income at rate w during his work life, retires at $t = T$, and passes away at $t = \bar{T}$. When entering the workforce the agent is endowed with an initial stock of asset $S^*(0)$ which, without loss of generality, is assumed to be 0. The actual law of motion for the agent's asset position will be governed by the actual rate of return r^* to the asset. When making a consumption/investment plan the forward-looking agent will base his decision on an estimated rate of return r. At each time t the agent derives utility from his actual consumption $C^*(t)$ according to

$$U(C^*(t)) = \frac{C^*(t)^{1-\sigma} - 1}{1 - \sigma} \tag{1}$$

where σ is the inverse elasticity of intertemporal substitution in consumption. The agent has an instantaneous subjective discount rate ρ.

We assume the agent is the sole member in the family, the dates of his retirement and death are both certain, the supply of his labor during the work life is inelastic (as he does not value leisure), the wage rate is constant during the work life, the good that he consumes is perishable, and the actual rate of return to his asset is equal to the discount rate. These assumptions effectively shut off all the channels for generating a hump-shaped life-cycle consumption profile that are already known in the literature.

The defining characteristic of our model is that the agent is overconfident and he overestimates his asset returns. Just as in the scenario described by Brunnermeier and Parker (2005), the agent merely observes one return realization at each age and believes he was unlucky, so he continues to be overconfident or overoptimistic looking forward. With the assumptions made in the last paragraph, investor overconfidence is the only mechanism in the model that is potentially capable of producing a hump in the life-cycle consumption profile.

4. An analytical result

It turns out that whether the agent's overestimation of his asset returns can give rise to a hump in his work-life consumption profile depends on whether the elasticity of intertemporal substitution in consumption (σ^{-1}) is smaller than 1, or equivalently, whether the inverse of the elasticity (σ) is greater than 1.

Claim: *Suppose the agent overestimates the rate of return to his asset so that $r > r^*$. There is a hump in his consumption path during the work life if and only if the inverse elasticity of intertemporal substitution in consumption σ is greater than 1.*

Because of his overconfidence, the dynamic optimization problem faced by the agent is time-inconsistent. In planning his consumption and asset holding for the future, the agent relies on an estimated rate of return to his assets. Since he overestimates the rate of return, his consumption plan so made is not sustainable and must be revised later. To prove the claim is to establish a connection from the planned consumption paths to the actual consumption path. The proof takes three steps.

Step 1: Derive the planned consumption paths

At any time t_0 during his work life, the agent makes a consumption/investment plan for the rest of his lifetime by solving the following control problem

$$\max \quad \int_{t_0}^{\bar{T}} e^{-\rho(t-t_0)}\, U(C(t))\, dt \tag{2}$$

$$\text{s.t.} \quad \dot{S}(t) = rS(t) + w - C(t), \quad \text{for } t \in [t_0, T] \tag{3}$$

$$\dot{S}(t) = rS(t) - C(t), \quad \text{for } t \in [T, \bar{T}] \tag{4}$$

$$S^*(t_0) \text{ given}, \quad S(\bar{T}) = 0 \tag{5}$$

where $C(t)$ and $S(t)$ are the agent's planned consumption and planned asset holding at t, and $\dot{S}(t)$ is the time derivative of $S(t)$. Note that the plan is made based on the agent's estimated rate of return to his asset r and with his actual asset position at t_0, denoted $S^*(t_0)$, taken as an initial condition.

The program defined by (2)-(5) is a two-stage fixed-endpoint control problem with a switch in the state equation from (3) to (4) at the agent's retirement age T. We use the Maximum Principle for two-stage problems to solve this dynamic program. To begin, we define two multiplier functions, $\lambda_1(t)$ for $t \in [t_0, T]$ and $\lambda_2(t)$ for $t \in [T, \bar{T}]$, by two laws of motion and a matching condition

$$\dot{\lambda}_1(t) = -r\lambda_1(t), \quad \text{for } t \in [t_0, T] \tag{6}$$

$$\dot{\lambda}_2(t) = -r\lambda_2(t), \quad \text{for } t \in [T, \bar{T}] \tag{7}$$

$$\lambda_1(T) = \lambda_2(T) \tag{8}$$

where (8) is a continuity or matching condition that links the two multiplier functions at the switch point. Given (6)-(8), the solution to (2)-(5) must satisfy

$$e^{-\rho(t-t_0)}C(t)^{-\sigma} = \lambda_1(t), \quad \text{for } t \in [t_0, T] \tag{9}$$

$$e^{-\rho(t-t_0)}C(t)^{-\sigma} = \lambda_2(t), \quad \text{for } t \in [T, \bar{T}]. \tag{10}$$

Now, solve (6) and (7) to obtain $\lambda_1(t) = x_1 e^{-rt}$ for $t \in [t_0, T]$ and $\lambda_2(t) = x_2 e^{-rt}$ for $t \in [T, \bar{T}]$, respectively, where x_1 and x_2 are constants of integration. Using (8), we show that $x_1 = x_2$. Thus we can write the solution compactly as $\lambda(t) = xe^{-rt}$ for $t \in [t_0, \bar{T}]$. In consequence, we can also write (9) and (10) in a compact way,

$$e^{-\rho(t-t_0)}C(t)^{-\sigma} = xe^{-rt}, \quad t \in [t_0, \bar{T}]. \tag{11}$$

Solving (11) gives rise to

$$C(t) = ye^{gt+\frac{\rho}{\sigma}t_0}, \quad t \in [t_0, \bar{T}] \tag{12}$$

where $y \equiv x^{-1/\sigma}$ and $g \equiv (r - \rho)/\sigma$.

We now proceed to pin down y. First, substituting (12) into (3) yields

$$S(t) = \left(d_1 + w\int^t e^{-rs}ds - y\int^t e^{(g-r)s+\frac{\rho}{\sigma}t_0}ds\right)e^{rt}, \quad \text{for } t \in [t_0, T] \tag{13}$$

where d_1 is a constant of integration. Evaluating (13) at $t = t_0$, using the initial condition $S(t_0) = S^*(t_0)$, and solving for d_1, we get

$$S(t) = S^*(t_0)e^{r(t-t_0)} + w\int_{t_0}^t e^{r(t-s)}ds - y\int_{t_0}^t e^{(g-r)s+\frac{\rho}{\sigma}t_0+rt}ds, \quad \text{for } t \in [t_0, T]. \tag{14}$$

Next, substituting (12) into (4) yields

$$S(t) = \left(d_2 - y\int^t e^{(g-r)s+\frac{\rho}{\sigma}t_0}ds\right)e^{rt}, \quad \text{for } t \in [T, \bar{T}] \tag{15}$$

where d_2 is a constant of integration. Evaluating (15) at $t = \bar{T}$, using the terminal condition $S(\bar{T}) = 0$, and solving for d_2, we obtain

$$S(t) = y \int_t^{\bar{T}} e^{(g-r)s+\frac{\rho}{\sigma}t_0+rt} ds, \quad \text{for } t \in [T, \bar{T}]. \tag{16}$$

Finally, evaluating (14) and (16) at $t = T$ and equating one another give rise to

$$y = \frac{(g-r)\left\{S^*(t_0)e^{r(T-t_0)} - \frac{w}{r}\left[1 - e^{r(T-t_0)}\right]\right\}}{e^{r(T-\bar{T})+g\bar{T}+\frac{\rho}{\sigma}t_0} - e^{\left(g-r+\frac{\rho}{\sigma}\right)t_0+rT}}. \tag{17}$$

Substituting (17) into (12) and rearranging yield

$$C(t) = \frac{(g-r)\left[S^*(t_0)e^{-rt_0} - \frac{w}{r}\left(e^{-rT} - e^{-rt_0}\right)\right]}{e^{(g-r)\bar{T}} - e^{(g-r)t_0}} e^{gt}, \quad t \in [t_0, \bar{T}]. \tag{18}$$

This gives the agent's planned consumption path from the planning point t_0 onward.

Step 2: Obtain the actual consumption path

We start to characterize the agent's actual consumption by noting that the agent will actually follow his plan (18) at the initial instant t_0 when the plan is made. That is to say that his actual consumption at t_0 must satisfy (18) in which t is set to t_0. We then note that t_0 is just an arbitrary point in time during the agent's work life. This suggests that the agent's actual consumption path throughout his work life must satisfy a relation characterized by replacing t_0 with t in (18)

$$C^*(t) = \frac{(g-r)\left[S^*(t)e^{-rt} - \frac{w}{r}\left(e^{-rT} - e^{-rt}\right)\right]}{e^{(g-r)\bar{T}} - e^{(g-r)t}} e^{gt}, \quad t \in [0, T] \tag{19}$$

where the law of motion for the agent's actual asset position $S^*(t)$ is governed by the actual rate of return to the asset r^*, rather than his estimated rate of return r,

$$\dot{S}^*(t) = r^* S^*(t) + w - C^*(t), \quad \text{for } t \in [0, T]. \tag{20}$$

Equations (19)-(20), along with the initial condition, $S^*(0) = 0$, completely characterize the agent's actual consumption and actual asset position over his work life. It follows that the time derivative of $C^*(t)$ is

$$\dot{C}^*(t) = \frac{(g-r)\left[e^{(g-r)(\bar{T}-t)}C^*(t) + \dot{S}^*(t) - we^{-r(T-t)}\right]}{e^{(g-r)(\bar{T}-t)} - 1}. \tag{21}$$

Substituting (20) into (21) yields

$$\dot{C}^*(t) = (g-r)\left\{C^*(t) + \frac{r^*S^*(t) + w\left[1 - e^{-r(T-t)}\right]}{e^{(g-r)(\bar{T}-t)} - 1}\right\}. \tag{22}$$

Solving (19) for $S^*(t)$ and substituting the result into (22), we obtain a first order differential equation in $C^*(t)$

$$\dot{C}^*(t) = (g - r + r^*)C^*(t) + \frac{(g-r)(r-r^*)w\left[1 - e^{-r(T-t)}\right]}{r\left[e^{(g-r)(\bar{T}-t)} - 1\right]}. \tag{23}$$

The general solution to the differential equation (23) is

$$C^*(t) = \left[d + \frac{(g-r)(r-r^*)w}{r}\int^t \frac{1 - e^{-r(T-s)}}{e^{(g-r)(\bar{T}-s)} - 1} e^{-(g-r+r^*)s} ds\right] e^{(g-r+r^*)t}, \tag{24}$$

for some constant d. A particular solution is

$$C^*(t) = C^*(0)e^{(g-r+r^*)t} + \frac{(g-r)(r-r^*)w}{r}\int_0^t \frac{1 - e^{-r(T-s)}}{e^{(g-r)(\bar{T}-s)} - 1} e^{(g-r+r^*)(t-s)} ds. \tag{25}$$

Given $S^*(0) = 0$, (19) implies that

$$C^*(0) = \frac{(g-r)w(1 - e^{-rT})}{r\left[e^{(g-r)\bar{T}} - 1\right]}. \tag{26}$$

Substituting (26) back into (25) we obtain

$$C^*(t) = \frac{(g-r)w}{re^{-(g-r+r^*)t}}\left[\frac{1 - e^{-rT}}{e^{(g-r)\bar{T}} - 1} + (r - r^*)\int_0^t \frac{1 - e^{-r(T-s)}}{e^{(g-r)(\bar{T}-s)} - 1} e^{-(g-r+r^*)s} ds\right]. \tag{27}$$

This gives a closed-form solution to the agent's actual consumption during the work life.

Step 3: Establish the claim

We can now use (27) to establish our claim. We proceed by rewriting (27) as

$$C^*(t) = C_1^*(t)\, C_2^*(t), \tag{28}$$

where

$$C_1^*(t) \equiv we^{(g-r+r^*)t}, \tag{29}$$

$$C_2^*(t) \equiv \frac{(g-r)\left(1 - e^{-rT}\right)}{r\left[e^{(g-r)\bar{T}} - 1\right]} + \frac{(g-r)(r-r^*)}{r}\int_0^t \frac{1 - e^{-r(T-s)}}{e^{(g-r)(\bar{T}-s)} - 1} e^{-(g-r+r^*)s} ds. \tag{30}$$

We note that $C_1^*(t)$ is monotone increasing in t if $g - r + r^* > 0$ but monotone decreasing in t if $g - r + r^* < 0$. By contrast, $C_2^*(t)$ is always monotone increasing in t since $r > r^*$. Hence, if $g - r + r^* \geq 0$, consumption would be monotone increasing across all $t \in [0, T]$ and no hump can exist. That is to say that $g - r + r^* < 0$ is a necessary condition for a hump in (27). This necessary condition can be rewritten as $(r - \rho)/\sigma - r + r^* < 0$.

Since $r^* = \rho$, it simplifies to $(r - r^*)(1 - \sigma)/\sigma < 0$, which is equivalent to $\sigma > 1$ given that σ is positive and $r > r^*$. This establishes the "only if" part of the claim.

To establish the "if" part of the claim, we can first evaluate (23) at $t = 0$ to get

$$\dot{C}^*(0) = (g - r + r^*)C^*(0) + \frac{(g-r)(r-r^*)w}{r} \frac{1 - e^{-rT}}{e^{(g-r)\bar{T}} - 1}. \tag{31}$$

Substituting (26) into (31) and combining terms we obtain

$$\dot{C}^*(0) = \frac{g - r}{e^{(g-r)\bar{T}} - 1} \frac{w}{r} \left(1 - e^{-rT}\right) g > 0, \tag{32}$$

where the strict inequality holds since $g > 0$, as is implied by $r > r^* = \rho$, and $g < r$, which follows if $\sigma > 1$. We then evaluate (23) at $t = T$ to get

$$\dot{C}^*(T) = (g - r + r^*)C^*(T) < 0, \tag{33}$$

where the strict inequality holds since $C^*(T) > 0$ and $g - r + r^* = (r - \rho)/\sigma - r + r^* < 0$, which is implied by $r > r^* = \rho$ and $\sigma > 1$. Thus, with the agent's overestimation of his asset returns and a larger-than-unit inverse elasticity of intertemporal substitution in consumption, at the beginning of the work life the rate of growth in his consumption is strictly positive, while the growth rate of his consumption at the date of retirement is strictly negative. These together imply a hump in his consumption during the working period. This establishes the "if" part of the claim.

Our conclusion is fairly general. As long as the inverse elasticity of intertemporal substitution in consumption is greater than 1, a value near the lower end of empirical estimates (e.g., Kocherlakota, 1996) and well below the values commonly used in the life-cycle consumption literature, any degree of overestimation of asset returns will lead to a hump in consumption during the work life.

5. Some intuition

Why does overestimation of the rate of return to assets give rise to a hump-shaped work-life consumption profile if the inverse elasticity of intertemporal substitution in consumption is greater than unit? To help understand the intuition we need to first understand better how the actual consumption path is related to the many planned consumption paths obtained in solving the time-inconsistent dynamic problem.

The problem facing the agent is dynamically inconsistent due to the overestimation of his asset returns and thus his lifetime income, which renders his consumption plan made at any age for the future unsustainable. When the agent first enters the workforce, he plans to increase consumption gradually throughout his lifetime to capitalize on the difference between the estimated rate of return to his assets and the discount rate, as evident from (18). The agent will follow the plan until he observes the actual return was lower than his estimated return for the period. While he believes he was just unlucky and hence continues to be overconfident looking forward, the agent does recognize that the rest of his original plan has become unsustainable. He thus revises down the rest of his old plan and makes a new plan for the remaining of his life span. Throughout his work life, the agent continuously adjusts down his plans made in the past—each being a monotonically increasing consumption path as evident from (18)—and replans for the rest of his life span. This explains why his actual consumption (27) lies below his planned consumption (18), except at the time when the plan is made, at which the two coincide. As a solution to this time-inconsistent dynamic problem, the agent's actual consumption constitutes the envelope of the many initial planned-values. Although the agent's actual consumption also gradually increases through the early stage of his work life, it does so at a lower and lower pace than each of the planned paths, as the agent adjusts his estimated lifetime income lower and lower.

The fact that the agent's actual consumption eventually peaks and turns around to make a hump (rather than ever increasing) has to do with the condition that the inverse elasticity of intertemporal substitution in consumption σ is greater than 1. To understand this, recall that σ governs the curvature of the agent's utility function and therefore affects his willingness to shift consumption across time. If σ is small, marginal utility decreases slowly with the level of consumption and the agent is willing to capitalize on even small differences between his estimated rate of return to investment and the discount rate. If σ is big, marginal utility decreases rapidly with the level of consumption and a given addition to total utility requires a large income from investment. For $\sigma > 1$, the income effect dominates the substitution effect. At some point in time during his work life, the cumulative downward adjustments in the agent's estimated lifetime income become large enough and drive the cumulative downward adjustments in his planned consumption paths so much that his actual consumption reaches a peak. From this point onward, his actual consumption declines monotonically as the agent ages and draws nearer to the end of his work life. This gives rise to a hump in his consumption during the working period.

6. Quantitative results

Figure 1 displays the agent's actual consumption across his work life and some of his planned consumption paths. As is clear from the figure, the actual consumption path is the envelope of the numerous initial planned-levels of consumption. As explained above, this envelop relationship is key to understanding why overestimation of asset returns can lead to a hump-shaped work-life consumption profile. As the figure illustrates, the actual consumption path is indeed hump-shaped, with peak consumption occurring between 45 and 55 years of age and with the ratio of peak consumption to consumption when first entering the workforce greater than 1.1, roughly in line with the data.

In generating Figure 1, we have assumed that the agent enters the workforce at age 25 (corresponding to the beginning point of time in the model), retires at age 65 (corresponding to $T = 40$), and passes away at age 80 (corresponding to $\bar{T} = 55$). We set the actual rate of return r^* to 7% in light of the long-term historical average real return in the U.S. equity market (e.g., Siegel, 1998, 1999)[7]. To calibrate the degree of investor overconfidence, we draw on Barber and Odean (2001) who find that estimation of asset returns by men is about 25% higher than the market average return.[8] This implies a value of r equal to 8.75%. Our result is virtually invariant to the wage rate, a change in which shifts the work-life consumption path up or down in a parallel fashion but affects neither the age of peak consumption nor the ratio of peak consumption to consumption when first entering the workforce. In actuality, we set w to $\$40,000$.

[7]Historical average real equity returns in most other industrialized countries have been about the same as in the U.S., as standard international data from Morgan Stanley Capital International suggest. It is the geometric average return that is referred to, and the arithmetic average return is considerably higher (e.g., Ibbotson, 2001). We could have also used the long-term historical average real return on U.S. Treasury bonds, which is about 3% to 3.5%. None of these choices would actually matter much for our results as it is a just a normalization; what really matters is the degree of investor overconfidence, or, by what percent the estimated return is higher than the actual return. We choose to tell our story using the equity market as a background since this is where investor overconfidence is most reported.

[8]Barber and Odean (2001) find that men estimate their asset returns by nearly 3 percentage points higher than the market average return that is about 12% during the period covered by the data. These numbers are nominal. We assume this overestimation is entirely due to overestimation of the real return rather than inflation. This translates into a 25% lower bound on overestimation of the real return.

Our analytical result in Section 4 shows that the inverse elasticity of intertemporal substitution in consumption is a key parameter, and it must be greater than 1 in order to have a hump-shaped work-life consumption profile. In generating Figure 1, we set σ to 3, a value close to what is commonly used in the literature, which is also the midpoint of two recent calibrations in the life-cycle macroeconomic models (e.g., Bullard and Feigenbaum, 2005, and Feigenbaum, 2005).

Figure 2 displays the agent's work-life consumption profiles normalized to the level of his consumption when first entering the workforce under two alternative values of σ, 2 and 4, against the one under the baseline value 3. As is clear from the figure, all three consumption paths are hump-shaped. In the benchmark case, peak consumption occurs at 52 years of age, with the ratio of peak to initial consumption equal to 1.11. When we lower σ from 3 to 2, the location of peak consumption shifts to near the upper bound of its empirical range, to 56 years of age, while the ratio of peak to initial consumption rises quite a bit, to 1.19. When we raise σ from 3 to 4, the location of peak consumption shifts comfortably to the midpoint of its empirical range, to 50 years of age, while the ratio of peak to initial consumption declines somewhat, to 1.07. In all of these cases, our model explains the data fairly well.

The consumption hump in the above cases arises solely from the overestimation of the mean return to investment, since uncertainty is deliberately abstracted away in the attempt to isolate the effect of this type of investor overconfidence. In order to examine the effect of underestimation of uncertainty on life-cycle consumption, we extend the baseline model to an environment in which the actual return to investment follows some stationary stochastic process. For simplicity, we assume that the actual rate of return to the agent's asset at each point in time is a random draw from a uniform distribution over the support [0%, 14%]. This choice of support imposes the largest volatility in the asset returns while still maintaining the limited-liability property of the assets and also ensuring a mean rate of return of 7% equal to the discount rate. As such, it gives uncertainty the greatest chance to make a difference on the life-cycle consumption profile.

We examine the case in which the agent has an unbiased estimation of the mean return but underestimates uncertainty associated with the return, as well as the case in which he both overestimates the mean return and underestimates the uncertainty. Just as in Brunnermeier and Parker (2005), the agent always perceives the future return to his asset as certain, although this belief may repeatedly be contradicted by realizations. With this extremity of overconfidence the effect of underestimation of uncertainty should be most pronounced. Nevertheless, as Figure 3 reveals, the work-life consumption profile in the former case is virtually flat, with some bumpy noise in the short run. In the latter case, as Figure 4 illustrates, the work-life consumption profile is essentially the same as the one in the baseline model without uncertainty where the agent only overestimates the mean return. We conclude that underestimation of uncertainty plays virtually no role in shaping the long-run average behavior of consumption over the work life, and it is the overestimation of the mean return that accounts for the hump-shaped work-life consumption profile.

7. Concluding Remark

We have shown that overconfidence in financial markets can account for a defining feature of consumption over the life cycle. For the two types of investor overconfidence documented in the empirical literature, we have shown that it is overestimation of the mean return to assets rather than underestimation of uncertainty associated with the return that can explain why the work-life consumption profile is hump-shaped, a stylized fact that is puzzling in the lens of the standard life-cycle theory. The necessary and sufficient condition for overestimation of the mean return to generate a consumption hump established in the paper, that is, the inverse elasticity of intertemporal substitution in consumption being greater than unit, has almost always been validated by empirical estimates. Our calibrated model gives rise to a hump-shaped work-life consumption profile with both the age and the amplitude of peak consumption roughly in line with the data. This leads us to conjecture that overconfidence in financial markets might have been an important contributing factor in shaping the life-cycle consumption profile.

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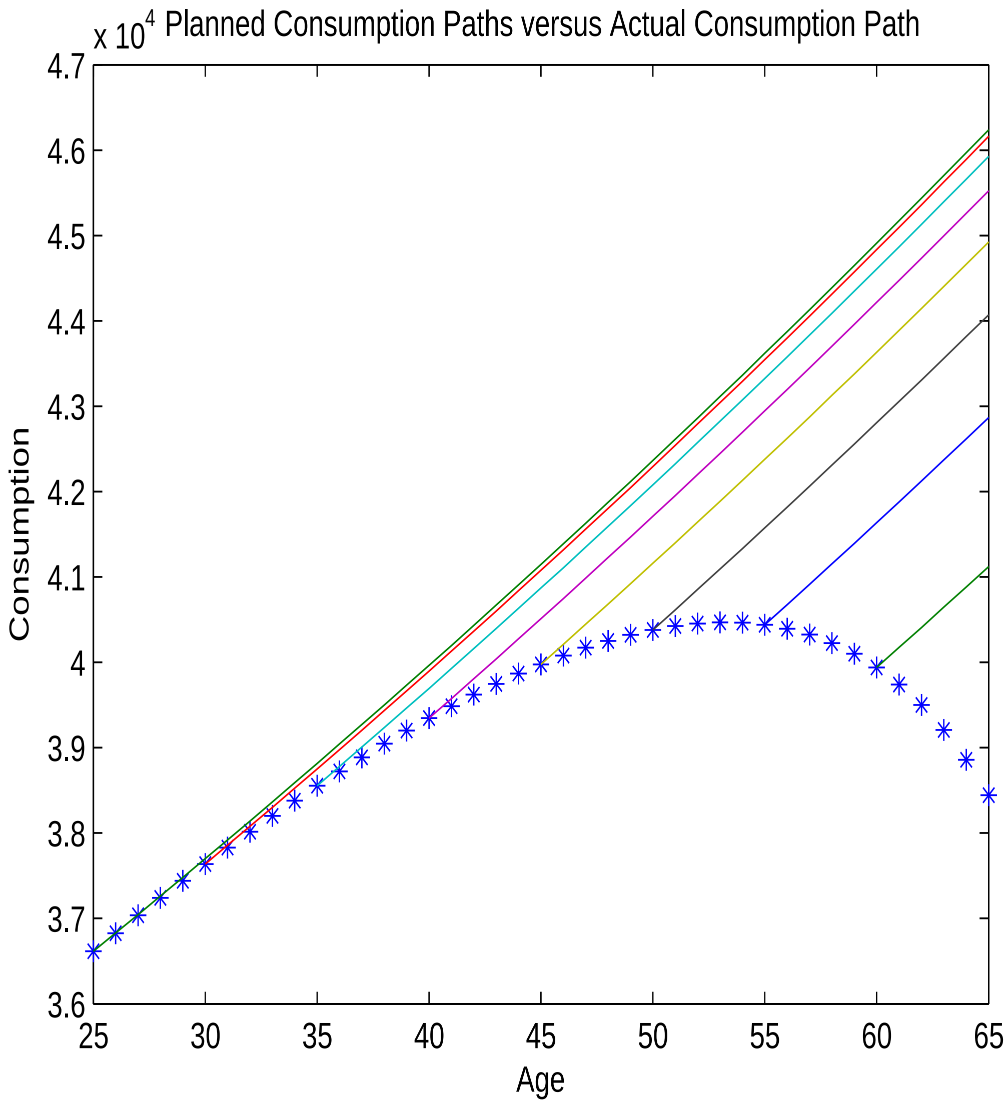

Figure 1. The actual work-life consumption path (line with stars) constitutes the envelope of many initial planned-values of consumption

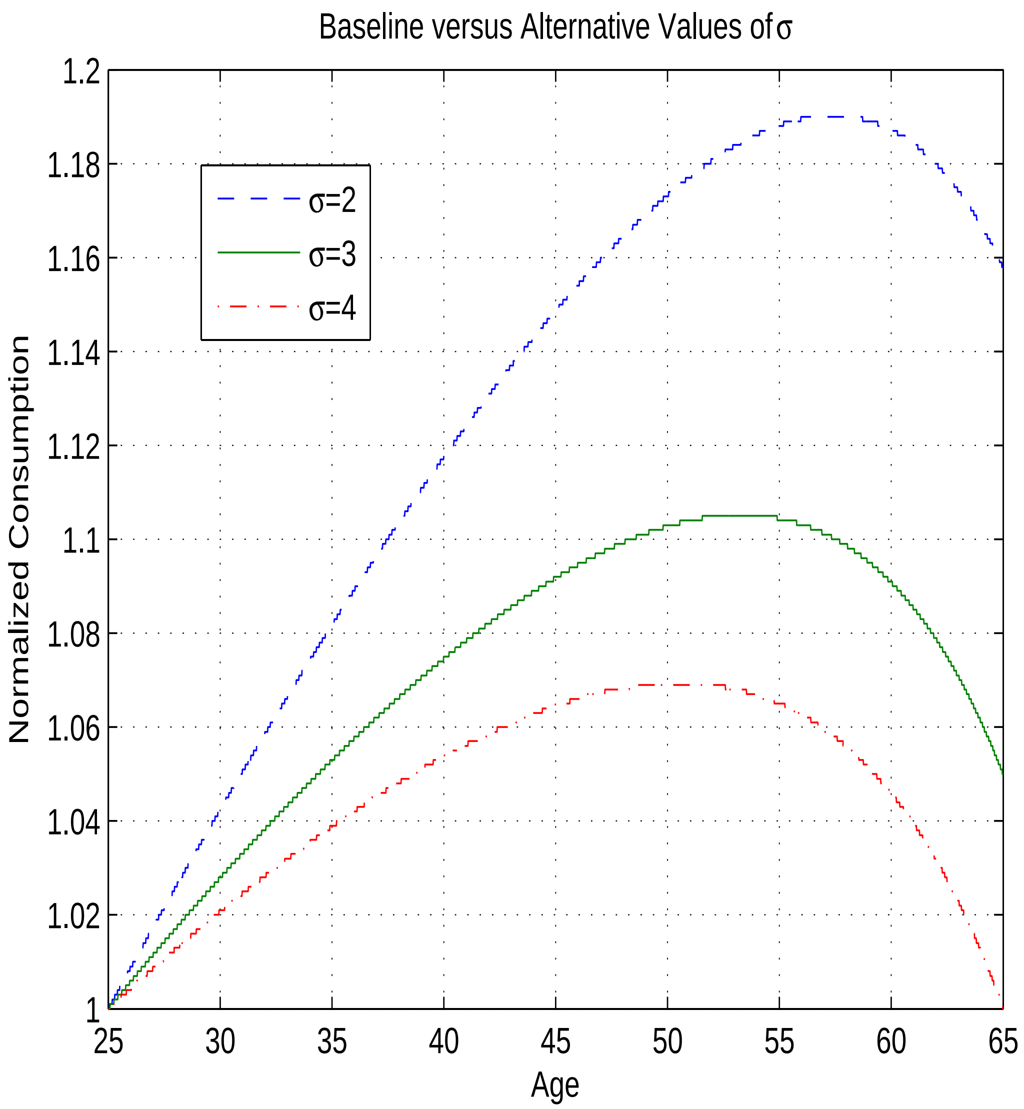

Figure 2. The work-life consumption profile normalized to the level of consumption when first entering the workforce: Under alternatives values of σ

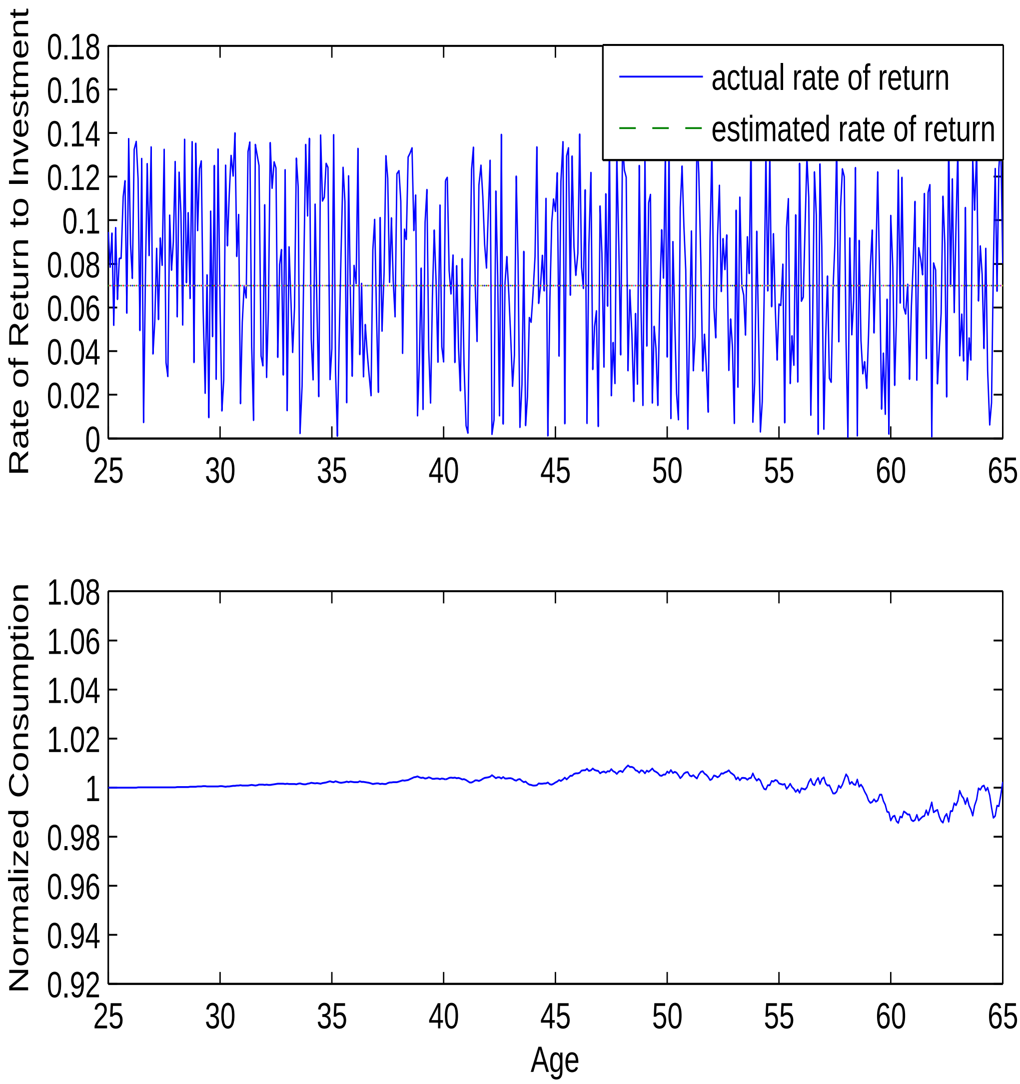

Figure 3. The work-life consumption profile normalized to the level of consumption when first entering the workforce (lower panel) under stochastic return to investment with underestimation in volatility (upper panel)

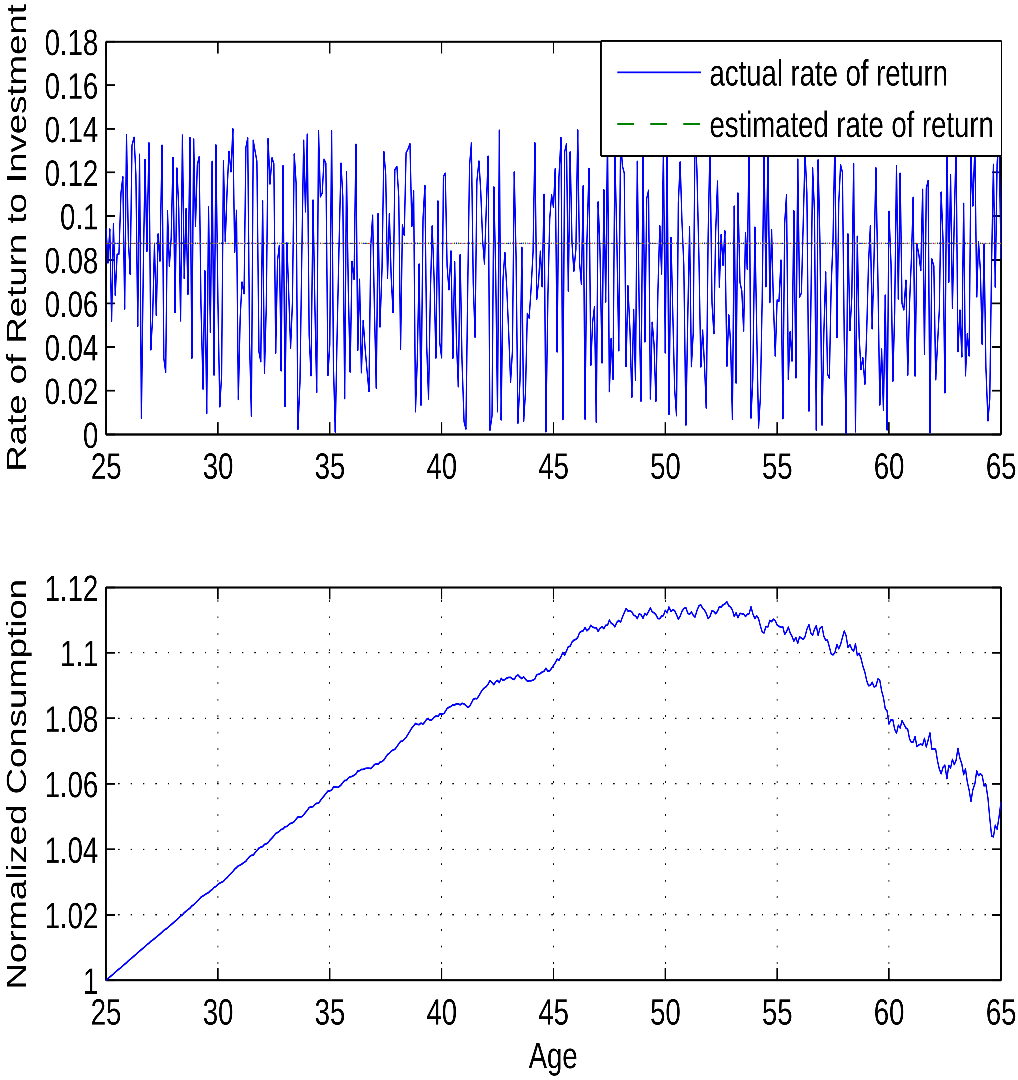

Figure 4. The work-life consumption profile normalized to the level of consumption when first entering the workforce (lower panel) under stochastic return to investment with overestimation in the mean return and underestimation in volatility (upper panel)