WORKING PAPERS
RESEARCH DEPARTMENT

WORKING PAPER NO. 07-10
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Ronel Elul
Federal Reserve Bank of Philadelphia
and
Piero Gottardi
Università Ca' Foscari di Venezia

March 2007

RESEARCH DEPARTMENT, FEDERAL RESERVE BANK OF PHILADELPHIA
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Bankruptcy: Is It Enough to Forgive or Must We Also Forget?*

Ronel Elul†and Piero Gottardi‡

This Version: May 9, 2008

Abstract

In many countries, lenders are not permitted to use information about past defaults after a specified period of time has elapsed. We model this provision and determine conditions under which it is optimal.

We develop a model in which entrepreneurs must repeatedly seek external funds to finance a sequence of risky projects under conditions of both adverse selection and moral hazard. Forgetting a default typically makes incentives worse, ex-ante, because it reduces the punishment for failure. However, following a default it may be good to forget, because by improving an entrepreneur's reputation, forgetting increases the incentive to exert effort to preserve this reputation.

Our key result is that if (i) borrowers' incentives are sufficiently strong, (ii) their average quality is not too low, (iii) the output loss from low effort is not too large, and (iv) agents are sufficiently patient, then the optimal law would prescribe some amount of forgetting — that is, it would not permit lenders to fully utilize past information. We also argue that forgetting must be the outcome of a regulatory intervention by the government — no lender would willingly agree to ignore information available to him. Finally, we show that the predictions of our model are consistent with the cross-country relationship between credit bureau reporting regulations and the provision of credit, as well as Musto (2004)'s evidence on the impact of these regulations on individual borrower and lender behavior.

Keywords: Bankruptcy, Information, Incentives, Fresh Start

JEL Classification Numbers: D86 , G33, K35.

*The authors thank Mitchell Berlin, Philip Bond, Subir Bose, Satyajit Chatterjee, Kevin Huang, Dirk Krueger, Nisan Langberg, Rafael Rob, and Nicholas Souleles, as well as seminar participants at Ben-Gurion University, Finance and Accounting in Tel Aviv, Midwest Theory and Trade, Philadelphia Fed, Stanford (SITE), Wharton School.

†Federal Reserve Bank of Philadelphia, Ten Independence Mall, Philadelphia, PA 19106-1574. E-mail: ronel.elul@phil.frb.org. The views expressed in this paper are those of the authors and do not necessarily represent policies or positions of the Federal Reserve Bank of Philadelphia, nor the Federal Reserve System. This paper is available free of charge at http://www.philadelphiafed.org/econ/wps/index.html.

‡Dipartimento di Scienze Economiche, Università Ca' Foscari di Venezia, Fondamenta San Giobbe, Cannaregio 873, 30121 Venezia, Italy. E-mail: gottardi@unive.it.

I Introduction

In studying the "fresh start" provisions of personal bankruptcy law, economists typically focus on the *forgiveness* of debts. However, another important feature is the *forgetting* of past defaults. In many countries, lenders are not permitted to use information about past defaults after a specified period of time has elapsed.

In the United States, the Fair Credit Reporting Act (FCRA) prescribes that a personal bankruptcy filing may be reported by credit bureaus for up to ten years, after which it must be removed from the records made available to lenders.[1] Similar provisions exist in most other countries. In figure 1 we summarize the distribution of credit bureau regulations governing the time period of information transmission across countries.[2] Of the 113 countries with credit bureaus as of January 2007, over 90 percent of them had provisions for restricting the reporting of adverse information after a certain period of time. Also note that this fraction has remained stable over time, even as countries have set up credit bureaus for the first time (twelve countries introduced bureaus from 2003 to 2007).[3]

Differences in information-sharing regimes across countries — whether a credit-reporting system exists, and whether there are time limits on reporting past defaults — are associated with differences in the provision of credit. In figure 2 we graph the average ratio of Private Credit to GDP according to whether the country restricts the time period of information sharing. Countries with no information sharing at all (i.e. no credit bureau) have low levels of credit; this is well-established and will be discussed below. On the other hand, it is interesting to note that countries in which defaults are always reported tend to have *lower* provision of credit than those countries in which defaults are not reported ("erased") after a certain period of time.[4]

Musto (2004) studies the effect of these provisions on lenders and individual borrowers, using U.S. data. He shows that (i) these restrictions are binding — access to credit increases significantly when the bankruptcy "flag" is dropped from credit files;[5] and (ii) these indi-

[1]Other derogatory information can be reported for a maximum of seven years; see Hunt (2006) for a discussion of the history and regulation of consumer credit bureaus in the United States. This time period is often even shorter in other countries; Jappelli and Pagano (2004) report several specific examples.

[2]Source: Doing Business Database, World Bank, 2004 and 2008. Throughout, we use the term "credit bureau" to refer both to private credit bureaus, as well as public credit registries.

[3]See also Jappelli and Pagano (2006).

[4]Private credit/GDP is constructed from the IMF International Financial Statistics for year-end 2006. As in Djankov, McLiesh and Shleifer (2007), private credit is given by lines 22d and 42d (claims on the private sector by commercial banks and other financial institutions). The credit bureau regulations are current as of January 2007 (source: Doing Business database 2008).

[5]That is, after 10 years. This effect is most significant for former bankrupts who are relatively creditwor-

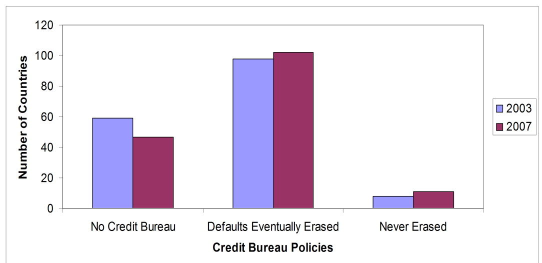

Figure 1: Credit Bureau Policies over Time

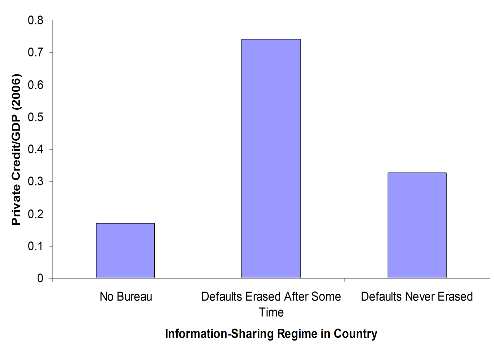

Figure 2: Impact of Information-Sharing Regime on Provision of Credit

viduals who subsequently obtain new credit are subsequently likelier to default than those with similar credit scores (and thus their credit scores tend to decline in the future). He interprets this as evidence that these laws are suboptimal.

In this paper we analyze these restrictions on reporting past defaults in the framework of a model of repeated borrowing and lending, and determine conditions under which they are optimal. Our model will also be consistent with many of these stylized facts. In particular, we consider a model in which entrepreneurs must repeatedly seek external funds to finance a sequence of risky projects under conditions of both adverse selection and moral hazard. We have in mind a world of small entrepreneurs who finance their business ventures with loans for which they are personally liable.[6] In this setup, an entrepreneur's reputation, or *credit history*, as captured by the past history of successes and failures of his projects, can encourage him to exert high effort.

In a typical equilibrium of the model, however, reputation may not be efficacious until agents have accumulated a sufficiently good credit history to make default unattractive. Conversely, those agents who fail will have a poor reputation — and hence weak incentives — and so will not be able to obtain financing.

We then consider the impact of restricting the availability to lenders of information on entrepreneurs' past defaults. Such a restriction leads to a tradeoff in our model. On the one hand, "forgetting" a default makes incentives weaker, *ex-ante*, because it reduces the punishment from failure. On the other hand, forgetting a default improves an entrepreneur's reputation, *ex-post*. This allows him to obtain financing when he otherwise would not be able to. Moreover, this improvement in his reputation also strengthens incentives, and may induce him to exert a higher level of effort than in the absence of forgetting. Our key result is that if (i) borrowers' incentives are sufficiently strong, (ii) their average risk-type is not too low, (iii) the output loss from low effort is not too large, and (iv) agents are sufficiently patient, then welfare is higher in the presence of a limited amount of forgetting, that is, by restricting the information available to lenders on borrowers' credit history. We also argue that forgetting must be the outcome of a regulatory intervention by the government — no lender would willingly agree to ignore the information available to him.

The effects of "forgetting" on lenders' and individual borrowers' behavior in our model are consistent with the empirical evidence presented by Musto (2004). However, we argue

thy; dropping the bankruptcy flag has little effect for those with many other derogatory indicators in their credit file.

[6] And indeed, Avery et al (1998) use the NSSBF and SCF to show that "[l]oans with personal commitments comprise a majority of small business loans."

that these restrictions may be optimal. In addition, our results on the relation between presence of a forgetting clause and the aggregate volume of credit are consistent with the international evidence reported above.

In the Congressional debate surrounding the adoption of the FCRA (U.S. House, 1970 and U.S. Senate, 1969), the following arguments were put forward in favor of forgetting past defaults: (i) if information was not erased the stigmatized individual would not obtain a "fresh start" and so would be unable to continue as a productive member of society, (ii) old information might be less reliable or salient, and (iii) limited computer storage capacity. On the other hand, the arguments raised against forgetting this information were (i) it discourages borrowers from repaying their debts by reducing the penalty for failure, (ii) it increases the chance of costly fraud or other crimes by making it harder to identify (and exclude) seriously bad risks, (iii) it could lead to a tightening of credit policies (which would affect the worst risks disproportionately), and finally, (iv) it forces honest borrowers to subsidize the dishonest ones. We will show that our model captures many of these arguments, and will use it to assess the tradeoffs between the positive and negative effects of forgetting.

The paper is organized as follows. In section II we present the model and the strategy sets of the entrepreneurs and lenders. In the following section we show that a Markov Perfect Equilibrium of the model exists, characterize the properties of the equilibrium strategies, and show that this equilibrium is efficient. In section IV we study the effects of introducing a forgetting clause on equilibrium outcomes and welfare. We derive conditions under which forgetting a default can be socially optimal — and relate them to the empirical evidence and the policy debate surrounding the adoption of the FCRA. Section V provides examples that illustrate these results. We consider an extension of the model in section VI. Section VII concludes, and the proofs are collected in the Appendix.

Previous Literature

Our basic model is one of reputation and incentives, like those of Diamond (1989), Mailath and Samuelson (2001), and Fishman and Rob (2005). In these models, principals and agents interact repeatedly under conditions of both adverse selection and moral hazard. The equilibrium in our model shares many similarities with the ones in these papers, in that agents build reputations over time. There are nevertheless some differences between our model and theirs — in both the setting, and in the structure of markets and information — which are discussed below.

The positive effects that a credit bureau can have through increasing the information publicly available on borrowers' histories have been widely discussed. One noteworthy paper that focuses on lenders' incentives to voluntarily share information is Pagano and Jappelli (1993). In recent empirical work, Djankov, McLiesh and Shleifer (2007) and Brown, Jappelli, and Pagano (2007) have found that credit bureaus are positively associated with increased credit.

Our main focus, however, is on the possible benefits of limiting the information available on borrowers' past histories. This has also been explored in a few other papers. Padilla and Pagano (2000) show, in the framework of a two-period model, that it may be optimal to restrict the type of information shared, because if information about an agent's type is too precise, then a borrower's effort choice will have no effect on his reputation; this eliminates the incentive to exert effort. This effect is also important in our model. Vercammen (1995) also presents an example in a dynamic setting that suggests that the optimal policy might involve restricting the memory of a credit bureau.

Another implication of restricting the availability of information on agents' past behavior is that this may affect the punishments a principal can impose on an agent. This effect has been explored by Crémer (1995), who shows that when the principal cannot commit not to renegotiate, then using an inefficient monitoring technology can sometimes improve incentives, because it limits the potential for renegotiation, and hence allows for stronger punishments. By contrast, in our model forgetting facilitates financing after failures, thereby making punishments *weaker*.

Finally, while we consider the effect of restricting credit histories on entrepreneurs' incentives and access to credit, Chatterjee, Corbae and Rios-Rull (2007) explore the risk-sharing and redistributive impact of these laws on consumers.

II The Model

Consider an economy made up of a continuum (of unit mass) of risk-neutral *entrepreneurs*, who live forever and discount the future at the rate $\beta \leq 1$. In each period $t = 0, 1, \ldots$ an entrepreneur receives a new project, which requires one unit of financing in order to be undertaken. This project yields either R (success) or 0 (failure). Output is non-storable, so entrepreneurs must seek external financing in each period. In addition, there is limited liability, so if a project fails in the current period, then the entrepreneur is not required to make payments out of any future income.

We assume that there are two types of entrepreneurs. There is a set of measure $p_0 \in (0, 1)$ of *riskless* agents, whose projects always succeed (i.e., their return is R with probability one),[7] and a set of *risky* agents, with measure $1 - p_0$, for whom the project may fail with some positive probability. The returns on the risky agents' projects are independently and identically distributed among them. The success probability of a risky agent depends on his effort choice. He may choose to exert high effort (h), at a cost $c > 0$ (in units of the consumption good), in which case the success probability will be $\pi_h \in (0, 1)$. Alternatively, he may choose to exert low effort. Low effort (l) is costless, but the success probability under low effort is only $\pi_l \in (0, \pi_h)$.

We assume:

Assumption 1. $\pi_h R - 1 > c$, $\pi_l R < 1$;

i.e., the project has a positive NPV if high effort is exerted (even when the cost of exerting high effort is taken into account), while it has a negative NPV under low effort.

In addition, we require the cost of effort c to be sufficiently high, so that entrepreneurs face a nontrivial incentive problem. The following condition implies that high effort cannot be implemented in the absence of reputational incentives (e.g. in a static framework) when the entrepreneur is known for certain to be risky.

Assumption 2. $\frac{c}{\pi_h - \pi_l} > R - 1/\pi_h$

Finally, we introduce one further parameter restriction, requiring that π_h and π_l not be too far apart. This condition is used to ensure the existence of an equilibrium.

Assumption 3. $\pi_h^2 \leq \pi_l$

In addition to entrepreneurs, there are lenders, who provide external funding to entrepreneurs in the loan market. More specifically, we assume that in each period there are N profit-maximizing risk-neutral lenders (where N is large) who compete among themselves on the terms of the contracts offered to borrowers. Each lender lives only a single period, and is replaced by a new lender in the following period. Since lenders live only a single period, they cannot write long-term contracts.[8] This is consistent with actual practice in U.S. unsecured credit markets, where borrowers often switch between lenders.

A contract is then simply described by the interest rate r at which an entrepreneur is offered one unit of financing at the beginning of a period (if the entrepreneur is not offered financing in this period then we set $r = \emptyset$). If the project succeeds, the entrepreneur makes the required interest payment r to the lender. On the other hand, if the project fails, the

[7] We discuss the role that this assumption plays in remark 5 below.

[8] This assumption is discussed further in Remark 3 below.

entrepreneur is unable to make any payment (since there is no storage) and we assume that the debt that was incurred is forgiven, i.e., discharged. Since borrowers have no funds to repay lenders other than the proceeds from their project in this period, with no loss of generality r can be taken to lie in $[0, R] \cup \emptyset$.

We assume that an entrepreneur's type, as well as the effort he undertakes, is his private information. Furthermore, since under Assumption 1 it is only profitable to lend to a risky agent if he exerts high effort, there is also incentive problem: a risky entrepreneur may in fact prefer to exert low effort even though the total surplus in that case is lower (indeed negative). The loan market is thus characterized by the presence of both adverse selection and moral hazard.

At the same time, in a dynamic framework such as the one we consider, the history of past financing decisions and past outcomes of the projects of an agent may convey some information regarding the agent's type. In addition, this history will also affect his incentives. Since lenders do not live beyond the current period, we assume that there is a *credit bureau* that records this information in every period and makes it available to future lenders.

Let σ_t^i denote the *credit history* of agent $i \in [0, 1]$ at date t, describing for each previous period $\tau < t$ whether the agent's project was funded and if so, whether it succeeded or failed. Hence, denoting by S a success, F a failure, and $\emptyset$ the event where the project is not funded (either because the agent is not offered financing or because he does not accept any offers), σ_t^i is given by a sequence of elements out of $\{S, F, \emptyset\}$: $\sigma_t^i \in \Sigma_t \equiv \{S, F, \emptyset\}^t$.

We show below that only pooling equilibria can exist in this economy;[9] that is, lenders are unable to separate borrowers by offering a menu of contracts to entrepreneurs with the same credit history. Note, however, that they may optimally choose to differentiate the terms of the contracts offered on the basis of entrepreneurs' credit histories. Hence, without loss of generality we can focus our attention on the case where a lender offers a single contract $r(\sigma_t)$ to borrowers with a given credit history σ_t. We let $\mathcal{C}(\sigma_t)$ denote the set of contracts offered at date t by the N lenders to entrepreneurs with credit history σ_t, and let $\mathbb{C}_t \equiv \cup_{t,\sigma_t \in \Sigma_t} \mathcal{C}(\sigma_t)$ be the set of contracts offered by lenders for any possible history up to date t.

We assume that while lenders present at date t know $\mathbb{C}_t$, i.e., the set of contracts which were *offered* to borrowers in the past, they do not know the particular contracts which were *chosen* by an *individual* borrower. This in line with actual practice; while credit bureaus do not report the actual contracts adopted by individual borrowers, the set of contracts generally offered to borrowers is available from databases such as "Comperemedia".

[9]To be precise, we focus on Markov Perfect Equilibria, and show that these must be pooling.

As discussed earlier, the focus of our paper is the effect of restrictions on the transmission of credit bureau records. We model the *forgetting policy* in this economy as follows: when an entrepreneur's project fails, with probability q the credit bureau ignores the failure and updates the entrepreneur's record as if his project succeeded in that period.[10] That is, S now represents either a success or a failure that is forgotten, and F represents a failure that has not been forgotten. The parameter $q \in [0, 1]$ then describes the forgetting policy in the economy. Note that we take q as being fixed over time, which is in line with existing laws.

We adopt this representation of forgetting to make the analysis simpler, though it is somewhat different from existing institutions. In practice, defaults are erased with the passage of time, rather than probabilistically. We intend to argue however that the effects on borrowers' incentives and access to credit are similar; in particular, that the consequences of higher values of q are analogous to those of allowing for a shorter period until negative information is forgotten.[11]

The timeline of a single period is then as follows. Each entrepreneur must obtain a loan of 1 unit in the market in order to undertake his project. Lenders simultaneously the rate at which they are willing to lend in this period to an agent with a given credit history, and do this for all possible credit histories at that date. If an entrepreneur is offered financing, and if he chooses to be financed, he undertakes the project (funds lent cannot be diverted to consumption), and if he is risky he also chooses his effort level. The outcome of the project is then realized: if the project succeeds the entrepreneur uses the revenue R of the project to make the required payment r to the lender, while if the project fails the entrepreneur defaults and makes no payment (since his default is forgiven). Note that — purely for convenience — we assume that entrepreneurs repay at the end of the same period in which they borrow.

The credit history of the entrepreneur is then updated. If the project was financed, a S is added to the sequence if the project succeeded in the period (or, with probability q, if it failed), and a F otherwise. If the project was not financed then a $\emptyset$ is added. This timeline is illustrated in the figure 3 for the case of high and low effort (when $q = 0$).

Next period, the same sequence is repeated: for each updated credit history, lenders choose the contracts they will offer, each entrepreneur then freely chooses the best contract among the ones he is offered,[12] and so on for every t.

Since the updated credit history may affect the contracts the agent will get in the future,

[10] A similar approach is also taken by Padilla and Pagano (2000).

[11] This is indeed exactly so for the polar cases of $q = 0$, which implies that a failure is remembered forever, and $q = 1$, which is equivalent to forgetting immediately, i.e., not keeping any record of failures.

[12] We assume entrepreneurs are unable to commit to any future choice of contract.

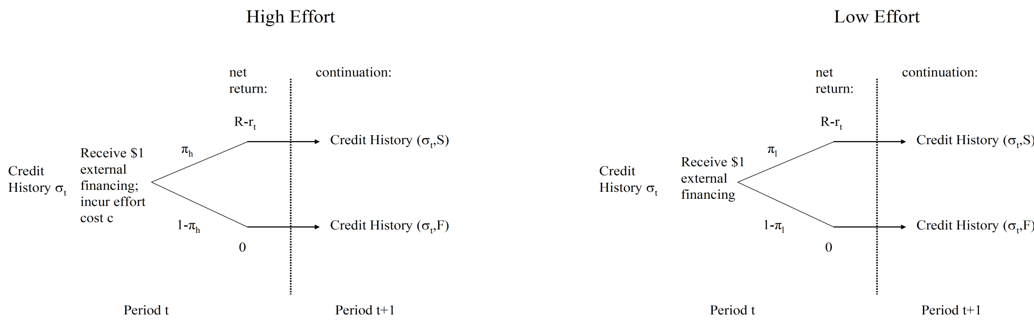

Figure 3: Timeline: $q = 0$

and hence his future expected utility, and since for a risky agent such history is partly affected by his current effort choice, this will affect the agent's incentives to choose high versus low effort. In particular, the agent may care for having a good credit history (i.e., a good reputation), as this might improve his future funding prospects, and this may strengthen the agent's incentives with respect to the case of a static contracting problem. Indeed, we will show that incentives may be sufficiently poor that we *need* reputational effects to elicit high effort (and as a result financing cannot take place at all nodes).

To summarize, a lender's strategy consists in the choice of contract to offer to entrepreneurs at any given date, for any possible credit history. The strategy of an entrepreneur specifies, in every period and for every possible credit history, the choice of the contract among the ones he is offered and, if the entrepreneur is risky, also his choice of effort.

To evaluate the expected profit of a loan offered to an entrepreneur with credit history σ_t, an important role is played a lender's belief $p(\sigma_t)$, that the entrepreneur is a *safe* type.[13] At the initial date such belief is given by the prior probability p_0. The belief is then updated over time on the basis of the knowledge of the credit history σ_t as well as of the contracts $\mathbb{C}_t$ offered up to such date, and of the entrepreneurs' borrowing and effort strategies, as we describe in detail below. We will term $p(\sigma_t)$ the *credit score* of the entrepreneur.

[13]We will sometimes drop the reference to the borrower's credit history and refer simply to p.

III Equilibrium

A Markov Perfect Equilibrium

In what follows we will focus on *Markov Perfect Equilibria* (MPE) in which players' strategies depend on past events only through credit scores. A key appeal of such equilibria is not only that players' strategies are simpler, but also that they resemble actual practice in consumer credit markets, where many lending decisions are conditioned on credit scores, most notably the "FICO score" developed by Fair Isaac and Company. In addition, we will discuss below the differences between MPE and other equilibria and argue that in the latter players' behavior is less plausible.[14]

In particular, we will establish the existence and analyze the properties of *symmetric, sequential MPE*, where all agents of a given type (i.e., all lenders, or all safe entrepreneurs, or all risky entrepreneurs with the same credit score) optimally choose the same strategies. In addition, the restriction to sequential equilibrium means that beliefs are determined by Bayes' Rule whenever possible and, when this is not possible, beliefs must be *consistent*. We can now describe players' strategies more formally for the Markov Perfect Equilibria that we consider.

Let $\mathcal{C}(p)$ denote the collection of contracts r offered by lenders in equilibrium to entrepreneurs with credit score p.

The strategy of an entrepreneur, whatever his type, consists in the choice, for every credit score p he may have, and given that he is offered a set of contracts $\mathcal{C}'$, whether or not to accept any of the loan contracts offered, and if so, which one to accept. For the safe entrepreneurs we denote this choice by $r^s(p, \mathcal{C}') \in \mathcal{C}' \cup \emptyset$, and for the risky by $r^r(p, \mathcal{C}')$. In addition, a risky entrepreneur has to choose the effort level $e^r(p, \mathcal{C}')$ he exerts. We will allow for mixed strategies with regard to effort and hence denote the effort level by $e \in [0, 1]$, where e signifies the probability with which the entrepreneur exerts high effort.[15] Thus $e = 1$ corresponds to a pure strategy of high (h) effort, $e = 0$ to a pure strategy of low (l) effort, and $e = 1/2$ (for example) corresponds to mixing between high and low effort with equal probability.

Since an entrepreneur's choice depends not only on his immediate payoff (which depends on the current contract), but also on how his project outcome will affect the contracts he

[14]Restricting attention to MPE to rule out implausible equilibria is common in the analysis of reputation games; see Mailath and Samuelson (2001), for example.

[15]This is the only form of mixed strategies that we allow; we demonstrate below that mixing only occurs for at most a single period along the equilibrium path.

is offered in the future, we need to specify how lenders update their beliefs concerning the agent's type in light of the outcome of the current project.

Let $p^S(p, \mathcal{C}')$ specify how lenders update their beliefs in case of success (or forgotten failure) of the project of a borrower with credit score p and facing current contracts $\mathcal{C}'$. Analogously, $p^F(p, \mathcal{C}')$ denotes the updated belief in case of a failure (which is not forgotten) and $p^{\emptyset}(p, \mathcal{C}')$ when the entrepreneur is not financed.[16] The updated beliefs will be computed according to Bayes' rule whenever possible; when this is not possible they will be required to be *consistent* in the Sequential Perfect Equilibrium sense.

Observation 1. Since only risky agents can fail, we must have $p^F(p, \mathcal{C}') = 0$ for any p and $\mathcal{C}' \neq \emptyset$.

Furthermore, when entrepreneurs are not offered any loan ($\mathcal{C}' = \emptyset$) and hence are not financed, it is immediate that beliefs remain unchanged: $p^{\emptyset}(p, \emptyset) = p$ for all p.

We are now ready to write the formal choice problem for the entrepreneurs. Each period they have to choose which of the offered loan contracts to accept, if any, and their effort level. Let $v^r(p, \mathcal{C}')$ denote the maximal discounted expected utility that a risky entrepreneur with credit score p, facing a set of contracts $\mathcal{C}'$, can obtain, given the lenders' updating rules $p^S(\cdot), p^F(\cdot), p^{\emptyset}(\cdot)$ and their strategies $\mathcal{C}(\cdot)$, determining future offers of contracts (to simplify the notation we do not make the dependence of v^r on these terms explicit). Observe that $v^r(\cdot)$ is recursively defined as the solution to the following problem:

$$v^r(p, \mathcal{C}') = \max_{e \in [0,1], r \in \mathcal{C}' \cup \emptyset} \begin{cases} (e\pi_h + (1-e)\pi_l)(R - r) - ec \\ +\beta[e(\pi_h + (1-\pi_h)q) + (1-e)(\pi_l + (1-\pi_l)q)]v^r(p^S, \mathcal{C}(p^S)) \\ +\beta[(e(1-\pi_h) + (1-e)(1-\pi_l)][1-q]v^r(0, \mathcal{C}(0)), \quad \text{if} \quad r \neq \emptyset; \\ \beta v^r(p^{\emptyset}, \mathcal{C}(p^{\emptyset})), \text{ if } r = \emptyset. \end{cases} \tag{1}$$

When the agent is financed ($r \neq \emptyset$), the first line in (1) represents the expected payoff from the current project, the second the discounted continuation utility when the project succeeds, and the third line gives the discounted continuation utility following failure. Note that in writing this expression we have used the fact that, by Observation 1, $p^F(\cdot) = 0$. When the agent is not financed ($r = \emptyset$), then his utility is simply the discounted utility of being financed next period, with his credit score appropriately updated. We denote the solution of problem (1) by $e^r(p, \mathcal{C}'), r^r(p, \mathcal{C}')$, which describes the risky entrepreneur's strategy as p and $\mathcal{C}'$ vary.

Analogously, letting $v^s(p, \mathcal{C}')$ be the maximal discounted expected utility for a safe en-

[16] We will sometimes omit the arguments and write simply $p^S, p^F, p^{\emptyset}$.

trepreneur, we have:

$$v^s(p,\mathcal{C}') = \max_{r\in\mathcal{C}'\cup\emptyset} \begin{cases} R - r + \beta v^s(p^S, \mathcal{C}(p^S)) \text{ if } \ r \neq \emptyset; \\ \beta v^s(p^\emptyset, \mathcal{C}(p^\emptyset)), \text{ if } r = \emptyset. \end{cases} \tag{2}$$

The solution to this problem is denoted by $r^s(p, \mathcal{C}')$.

Since lenders cannot observe the specific contract chosen by an individual borrower in any given period, but only whether or not he was financed, we have:

Observation 2. Whenever an entrepreneur accepts financing, he will choose the contract with the lowest interest rate: i.e., for all $p, \mathcal{C}'$ we have $r^j(p, \mathcal{C}') \in \min(\mathcal{C}') \cup \emptyset$, for $j = s, r$.

Next, we determine the expected profits for an arbitrary lender n from a loan with interest rate r offered to a unit mass of entrepreneurs with credit score p, given the entrepreneurs' strategies, $r^s(\cdot), r^r(\cdot)$, and $e^r(\cdot)$, and the contracts $\mathcal{C}^{-n}$ offered by the *other* lenders. The expression for lender n's profits will depend on which types of entrepreneurs accept his offer (if any):

1. No entrepreneur accepts the offer. This will be the case either if he offers no contract, or if his offer is higher than the lowest contract offered by other lenders (observation 2), or if both types' strategies are to reject all offers on the table. In this case his profit will be zero. More formally:

$$\begin{aligned} &\Pi(r, p, \mathcal{C}^{-n}, r^s(\cdot), r^r(\cdot), e^r(\cdot)) = 0, \\ &\text{if either } r > \min(\mathcal{C}^{-n}), \text{ or } r = \emptyset, \text{ or } r^s(p, \mathcal{C}^{-n} \cup r) = \emptyset \text{ and } r^r(p, \mathcal{C}^{-n} \cup r) = \emptyset. \end{aligned} \tag{3}$$

2. Only safe entrepreneurs accept. If the risky types reject all offers on the table, then his profit is determined by payments from the safe entrepreneurs, who have measure p. In order for his offer to be accepted, however, it must be at least as low as all of the lenders' contracts. In addition, he must share the profits with other lenders offering r (if any); we let $\#(r^{n'} \in \mathcal{C}^{-n} \text{ s.t. } r^{n'} = r)$ denote the number of other such lenders. So his profit is given by:

$$\begin{aligned} &\Pi(r, p, \mathcal{C}^{-n}, r^s(\cdot), r^r(\cdot), e^r(\cdot)) = pr/[1 + \#(r^{n'} \in \mathcal{C}^{-n} \text{ s.t. } r^{n'} = r)], \\ &\text{if } r \leq \min(\mathcal{C}^{-n}), r^s(p, \mathcal{C}^{-n} \cup r) \neq \emptyset, \text{ and } r^r(p, \mathcal{C}^{-n} \cup r) = \emptyset \end{aligned} \tag{4}$$

3. Only risky entrepreneurs accept. Similarly, if the safe types reject all offers on the table, then his profit accrues from the risky entrepreneurs. In this case, the profit also depends on the risky entrepreneurs' effort choice $e^r(p, \mathcal{C}^{-n} \cup r))$. Recall that $e^r(\cdot) = 0$ corresponds to their exerting low effort, in which case their success probability is π_l, that $e^r(\cdot) = 1$ corresponds to high effort, with success probability π_h, and that $e^r(\cdot) \in (0, 1)$ corresponds to mixing over high and low effort with probability $e^r(\cdot)$. We have:

$$\begin{aligned}&\Pi(r, p, \mathcal{C}^{-n}, r^s(\cdot), r^r(\cdot), e^r(\cdot)) = \\ &(1-p)\left\{e^r(p, \mathcal{C}^{-n} \cup r)\pi_h + (1 - e^r(p, \mathcal{C}^{-n} \cup r))\pi_l\right\} r/[1 + \#(r^{n'} \in \mathcal{C}^{-n} \text{ s.t. } r^{n'} = r)], \\ &\text{if } r \leq \min(\mathcal{C}^{-n}) \text{ and } r^s(p, \mathcal{C}^{-n} \cup r) = \emptyset, \text{ and } r^r(p, \mathcal{C}^{-n} \cup r) \neq \emptyset\end{aligned} \tag{5}$$

4. All entrepreneurs accept. This will be the case along the equilibrium path. Then his profit will simply be the sum of (4) and (5), and we have:

$$\begin{aligned}&\Pi(r, p, \mathcal{C}^{-n}, r^s(\cdot), r^r(\cdot), e^r(\cdot)) = \\ &\left\{p + (1-p)\left[e^r(p, \mathcal{C}^{-n} \cup r)\pi_h + (1 - e^r(p, \mathcal{C}^{-n} \cup r))\pi_l\right]\right\} r/[1 + \#(r^n \in \mathcal{C}^{-n} \text{ s.t. } r^n = r)], \\ &\text{if } r \leq \min(\mathcal{C}^{-n}), \text{ and } r^S(p, \mathcal{C}^{-n} \cup r) \neq \emptyset, \text{ and } r^r(p, \mathcal{C}^{-n} \cup r) \neq \emptyset\end{aligned} \tag{6}$$

Since a lender lives only a single period, his objective is to choose r so as to maximize his expected profits given by (3)-(6). Given our focus on symmetric MPE, we can denote the solution simply by $r(p)$.

We are now ready to give a formal definition of a MPE:

Definition 1. A symmetric, sequential Markov Perfect Equilibrium is a collection of strategies $(r(\cdot), r^s(\cdot), r^r(\cdot), e^r(\cdot))$ and beliefs $p(\cdot)$, such that:

- Lenders maximize profits, given $r^s(\cdot), r^r(\cdot), e^r(\cdot)$: for every p, $r = r(p)$ maximizes (3)-(6), when $\mathcal{C}^{-n} = r(p)$;
- Entrepreneurs' strategies are sequentially rational. That is,
  - for all $p, \mathcal{C}'$, $(r^r(p, \mathcal{C}'), e^r(p, \mathcal{C}'))$ solves (1) when $\mathcal{C}(p) = r(p)$.
  - for all $p, \mathcal{C}'$, $r^s(p, \mathcal{C}')$ solves (2) when $\mathcal{C}(p) = r(p)$.
- Beliefs are computed via Bayes' Rule whenever possible and are consistent otherwise.

Observe that along the equilibrium path, strategies and beliefs can be written solely as functions of the credit score p, i.e., $r(p), r^r(p), r^s(p), \mathcal{C}(p)$ and $\{p^S(p), p^F(p), p^{\emptyset}(p)\}$. Similarly, entrepreneurs' discounted expected utility can be written as $v^s(p), v^r(p)$.

The following notation will also be useful. Let $r_{zp}(p, e)$ denote the lowest interest rate consistent with lenders' expected profits being non-negative on a loan to entrepreneurs with credit score p, when risky entrepreneurs exert effort e, and all agents accept financing at this rate. That is,

$$r_{zp}(p, e) \equiv \frac{1}{p + (1-p)(e\pi_h + (1-e)\pi_l)}. \tag{7}$$

Also let $p_{\text{NF}} \equiv \frac{1-\pi_l R}{(1-\pi_l)R}$ denote the lowest value of p for which this break-even rate is admissible when the risky entrepreneurs exert low effort, i.e. $r_{zp}(p_{\text{NF}}, 0) = R$.

B Existence and Characterization of Equilibrium

The following proposition establishes that a Markov Perfect Equilibrium exists, and characterizes its properties. The proof is constructive, and we show in the subsequent proposition that this equilibrium is the most efficient MPE.

Proposition 1. *Under assumptions 1-3, a (symmetric, sequential) Markov Perfect Equilibrium always exists with the following properties:*

i. Entrepreneurs never refuse financing, and always take the contract with the lowest interest rate offered to them: $r^s(p, \mathcal{C}') = r^r(p, \mathcal{C}') = \min(\mathcal{C}')$*, whenever* $\mathcal{C}' \neq \emptyset$*. If a borrower does refuse financing, lenders' beliefs are that he is the risky type:* $p^{\emptyset}(p, \mathcal{C}') = 0$ *whenever* $\mathcal{C}' \neq \emptyset$*.*

ii. Lenders make zero profits in equilibrium: either $r(p) = r_{zp}(p, e^r(p))$*, or* $r(p) = \emptyset$*.*

iii. Lenders never offer financing to entrepreneurs known to be risky with probability 1: $r(0) = \emptyset$*, and so* $v^r(0) = 0$*.*

Furthermore, along the equilibrium path the players' strategies are as follows:

a. if $\frac{(R-1)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)} \leq \frac{c}{\pi_h-\pi_l}$*, then an entrepreneur will be financed if and only if* $p \geq p_{NF}$*, and if risky exerts low effort* $(e^r(p) = 0)$

b. if $\frac{(R-1/\pi_h)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)} < \frac{c}{\pi_h-\pi_l} < \frac{(R-1)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$*, there exists* $0 < p_l \leq p_m \leq p_h < 1$ *such that:*

- *there is financing if and only if* $p \geq p_l$
- *risky entrepreneurs exert high effort if* $p \geq p_h$*, low effort if* $p \in [p_l, p_m)$*, and mix between high and low effort for* $p \in [p_m, p_h)$ *(with* $e^r(p)$ *strictly increasing for* $p \in [p_m, p_h)$*).*

c. if $\frac{c}{\pi_h-\pi_l} \leq \frac{(R-1/\pi_h)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$*, then there is financing for all* $p > 0$*, and risky entrepreneurs exert high effort* $(e^r(p) = 1)$*.*

When c is high (region a.), incentives are weak, and the risky entrepreneurs exert low effort whenever they are financed. Nevertheless, financing can still obtain as long as p is not too low ($p > p_{\text{NF}}$), since there are enough safe entrepreneurs with credit score p from which the lenders can recoup their losses on lending to the risky agents. By contrast, when c is low (region c.) incentives are strong enough that the risky entrepreneurs exert high effort for all $p > 0$. This makes financing profitable for all $p > 0$. Finally, for intermediate values of c (region b.), incentives depend on p. When p is sufficiently high ($p \geq p_m$), interest rates (both current and future) are low, which makes incentives strong enough that high effort can be sustained. By contrast, when $p < p_m$ interest rates are not sufficiently low to sustain high effort. Moreover, when p is particularly low ($p < p_l$) it is not feasible for lenders to break even, just as in region a.; therefore there will be no financing.

Recall that Markov Perfect Equilibrium requires that lenders use Bayes' Rule to update their beliefs whenever possible. That is,

$$p^S(p, \mathcal{C}') = \frac{p}{p + (1-p)[e^r(p,\mathcal{C}')(\pi_h + (1-\pi_h)q) + (1 - e^r(p,\mathcal{C}'))(\pi_l + (1-\pi_l)q)]},$$

for all $p, \mathcal{C}' \neq \emptyset$. From Observation 1, it follows that when agents fail they are known to be risky: $p^F(p, \mathcal{C}') = 0$, for $\mathcal{C}' \neq \emptyset$. And when lenders do not offer financing then beliefs are unchanged: $p^\emptyset(p, \mathcal{C}') = p$ when $\mathcal{C}' = \emptyset$. When borrowers refuse financing, which only happens off the equilibrium path, Bayes' Rule cannot be applied. In this case, Property i. of the Proposition specifies that lenders' beliefs are that the borrower is risky, and the proof of the Proposition verifies that this is a consistent belief, and that under such beliefs refusing financing is not optimal.[17]

In Figure 4 we plot where regions a., b., and c. lie, in the space of possible values of the effort cost c. Figure 5 then illustrates the equilibrium outcomes obtained in region b., for different values of the credit score p. Recall that $0 < p_l \leq p_m \leq p_h < 1$, so the low-effort and mixing regions may be empty, while the high-effort and no-financing regions must always exist for this case.

In proving the Proposition, we first establish property iii. — that entrepreneurs who are known to be risky are never financed — and show that this is actually a general property of Markov equilibria. The basic intuition is that once an entrepreneur is known to be risky, his

[17] Our result is robust to other specifications of the beliefs off the equilibrium path. In particular, even if lenders were to keep their beliefs unchanged when an entrepreneur refuses an equilibrium offer of financing (i.e., $p^\emptyset(p, r(p)) = p$), it would never be optimal for any type of entrepreneur to refuse financing. And even for offers of contracts off-the-equilibrium path (i.e., $\mathcal{C}'$ different from $r(p)$), Assumption 3 would ensure that the safe entrepreneurs do not want to refuse financing when $p^\emptyset(p, \mathcal{C}') = p$.

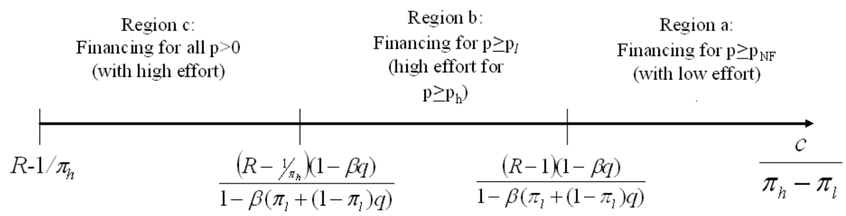

Figure 4: Equilibrium regions

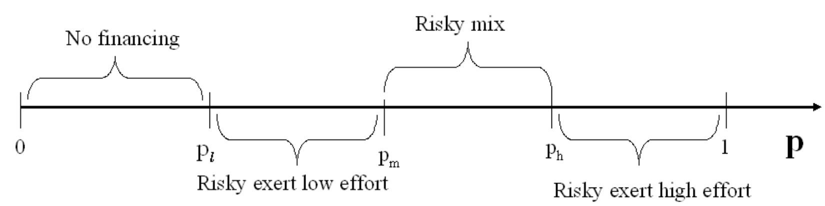

Figure 5: MPE for region b.

continuation utility in a Markov Perfect Equilibrium is not affected by the outcome of his project, which makes it impossible to provide him with incentives to exert high effort.

Lemma 1. *Under assumptions 1 and 2, any Markov Perfect Equilibrium is characterized by no financing when $p = 0$: i.e., $r(0) = \emptyset$ and hence $v^r(0) = 0$.*

This result implies that, in equilibrium, any entrepreneur who fails is excluded forever from financing (unless this failure is "forgotten").

Note that all symmetric MPE are pooling, by definition, since we have restricted lenders to offering a single contract to entrepreneurs with a given credit score p. We now show that this restriction is not binding, and, in particular, that separating Markov Perfect Equilibrium cannot exist.

Lemma 2. *Suppose lenders may offer multiple contracts to entrepreneurs with a given credit score p. Then any (symmetric, sequential) Markov Perfect Equilibrium must be a pooling equilibrium.*

This result is a consequence of the fact that, by Lemma 1, risky entrepreneurs who are separated would not be able to obtain financing.

The rest of the proof of Proposition 1 (in the Appendix) establishes the remaining properties (i. and ii.) of the MPE, and the specific characteristics of the equilibrium we construct for the parameter regions a., b., and c.

Finally, we establish that the equilibrium characterized in Proposition 1 is also the MPE that maximizes welfare. The welfare criterion we consider in this paper is the total surplus generated by entrepreneurs' projects that are financed; given agents' risk-neutrality, this is equivalent to the sum of the discounted expected utilities of all agents in the economy, including lenders.

Proposition 2. *The equilibrium constructed in Proposition 1 is the most efficient MPE.*[18]

To prove the result, we first show that the construction of the equilibrium in Proposition 1 guarantees that the equilibrium implements the highest possible effort at any p. For credit scores $p \geq p_h$ the equilibrium of Proposition 1 then clearly maximizes welfare, since high effort will be exerted in the current period, as well as in any future round of financing. The same is also true for $p < p_m$, as in the equilibrium of Proposition 1 the risky entrepreneurs

[18] When $q \in (0, 1)$ we require an additional condition to prove this result: $\pi_l \geq \pi_h \frac{q}{1+q}$. This condition is implied by Assumption 3 when $\pi_h \geq 1/2$.

exert low effort if financed, and this is the maximal effort level. The result is completed by showing this is true even when $p \in [p_m, p_h)$, i.e. in the mixing region of Proposition 1.

We conclude this section with several remarks concerning the robustness of our results to some of the assumptions.

Remark 1. *(Only Risky Agents Fail)* In our setup, when an entrepreneur fails he is identified as risky and in that case can no longer obtain financing (since he would always exert low effort). This is a consequence of the fact that only risky entrepreneurs can fail; this assumption obviously simplifies the analysis. If "safe" agents could also fail, then the posterior following a failure would be above 0 and so could result in continued financing. This is shown in section VI, where we provide an example in which failure can indeed result in continued financing. We show in that case that the effect of forgetting is nevertheless qualitatively similar to that obtained here; i.e., forgetting may still improve welfare.

Remark 2. *(Non-Markov Equilibria)* Observe that the Markov property of players' strategies only binds at nodes where the entrepreneur is not financed, for example when $p = 0$ after a failure. This is because when the agent is financed the updated belief in case of success will always be higher than the prior one, so p never hits the same value twice.

At non-Markov equilibria, by contrast, lenders' strategies may not be the same each time p is equal to zero. For example, the agent may continue to be financed the first time he fails, as well as at any successor node as long as his project succeeds, but permanently denied financing after a second failure. This threat of exclusion after two failures could be enough to induce high effort and hence to make financing profitable for lenders.

Since these strategies imply that the entrepreneur is not treated identically at different nodes with $p = 0$, they require some coordination among lenders. Such an equilibrium thus seems somewhat fragile, being open to the possibility of breakdowns in coordination, or to renegotiation (which is not the case for the MPE we consider). Moreover, while such non-Markov equilibria have some similarities with the MPE with forgetting, in that a risky entrepreneur who fails may obtain additional periods of financing, they only exist for a limited set of parameter values — when c is low and lies in region c. of Proposition 1, so that incentives are sufficiently strong. By contrast, with forgetting financing with high effort obtains also for intermediate values of c (lying in region b.) This is because forgetting a failure in our setup entails pooling the risky types with the safe entrepreneurs anew, so that financing is granted at a lower interest rate than if their type had been revealed, and this improves their incentives (see also Proposition 4 below).

Remark 3. *(Long-term Contracts)* It is also useful to compare the MPE we consider with the equilibria we would obtain with long-term contracts, assuming that lenders live forever, rather than a single period. In this case lenders only need to break-even over their life-time, and not period-by-period. This would lead to rather extreme and unrealistic contracts in equilibrium, since the equilibrium contract would postpone any net revenue from the projects financed as far into the future as possible, so as to minimize the cross-subsidy from safe to risky entrepreneurs (the benefit to safe entrepreneurs would be that fewer risky entrepreneurs survive to share in the future surplus). That is, the interest payments would be equal to R in the initial periods, and subsequently zero. We conjecture that such an equilibrium, while preferred by the safe entrepreneurs, would be less efficient (total surplus will be lower) than that considered in Proposition 1, because of the negative effect that postponing payments has on incentives.

If the Markov property were also relaxed, a separating equilibrium might obtain. The reason is that the risky entrepreneurs might be able to obtain some financing if separated,[19] and the postponement of payments might make the safe entrepreneurs' contract unattractive to the risky entrepreneurs.

IV Optimal Forgetting

In this section we derive conditions under which forgetting entrepreneurs' failures is a socially optimal policy. That is when, in the equilibrium characterized in Proposition 1, $q > 0$ dominates $q = 0$. The welfare criterion we use is again the total surplus.

What are the effects of the forgetting policy on the equilibrium properties? When we are in regions a. and c. of Proposition 1, q has no effect on the surplus generated in equilibrium by financing to safe entrepreneurs. This follows because, within each region, the set of nodes for which the safe agents are financed does not depend on q: in region c. there is financing for all $p > 0$, and in region a. there is financing for $p > p_{\mathrm{NF}}$, where recall that p_{NF} does not depend on q. So in these cases the only effect of q is on the surplus generated by financing to risky entrepreneurs.

In this regard, a first implication of raising q is that the probability that a risky entrepreneur will be excluded from financing decreases: failure of his project leads to exclusion only with probability $1 - q$. The impact of this on welfare depends on the effort choice of the risky entrepreneur after his failure is forgotten. If he exerts high effort (as will be the case in region c.), then this extra period of financing makes a strictly positive contribution

[19]As discussed above, this could only occur in region c.

to the social surplus, given by $G \equiv \pi_h R - 1 - c > 0$. Under low effort, however (as in region a.), the contribution is strictly negative: $B \equiv \pi_l R - 1 < 0$.

But increasing q has another effect that needs to be taken into account: since exclusion after a project's failure is less likely, the incentives to exert high effort will be weaker. In region a. (in which low effort is always exerted when financing takes place), the weakening of incentives manifests itself in the fact that the lower bound of this region, $\frac{(R-1)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$, is decreasing in q, so that this region expands when q is increased. Analogously, the upper bound of parameter region c. (where high effort is always exerted), $\frac{(R-1/\pi_h)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$, also decreases in q, so that this region becomes smaller when q is higher.

Let $q(p_0)$ denote the welfare maximizing level of q (which clearly depends on the proportion p_0 of risky types in the population, as the equilibrium depends on it). From the above discussion the properties of the optimal forgetting policy when the parameters of the economy are in region a. or c. of Proposition 1 (with $q = 0$) immediately follow:

Proposition 3. *The welfare maximizing forgetting policy respectively for high and low values of c is as follows:*

1. *If $\frac{c}{\pi_h - \pi_l} \geq \frac{R-1}{1-\beta\pi_l}$, no forgetting is optimal for all p_0: $q(p_0) = 0$.*
2. *If $\frac{c}{\pi_h - \pi_l} < \frac{R-1/\pi_h}{1-\beta\pi_l}$, for any $p_0 > 0$ some degree of forgetting is optimal: $q(p_0) > 0$.*

Thus in region c., when incentives are strong and high effort is implemented everywhere, some positive level of forgetting is optimal.

We now turn our attention to region b., the intermediate values of c, where the level of effort varies along the equilibrium path (switching at some point from low to high). The weakening of incentives due to forgetting now manifests itself not only in the change of the boundaries of this region, which again shift to the left as q increases, but also in the change of the points in the equilibrium paths where the switch from low to high effort takes place. Such switching points are identified by the levels of $p_h(q), p_m(q)$, and $p_l(q)$ introduced in Proposition 1.[20] In what follows we will restrict attention to prior probabilities $p_0 > p_{\text{NF}}$, in which case there is financing in the initial period regardless of the level of q; this will allow us to ignore any possible effect of q on $p_l(q)$. These switching points are key to the analysis of the welfare impact of raising q, since an extra period of financing with high effort makes a positive contribution to the social surplus, while one with low effort makes a negative contribution.

[20]The dependence of these switching points on q is now highlighted.

Notice first that when $p_0 > p_h(0)$ high effort is always exerted by a risky entrepreneur when financed. Hence an analogous argument to that used to prove case 2. of Proposition 3 establishes that the socially optimal level of q is above 0 in this case.

On the other hand, when $p_0 \leq p_h(0)$ raising q above 0, while leading to a lower probability of exclusion, does not necessarily increase welfare. There is a tradeoff between the positive effect when high effort is exerted (i.e., when $p > p_h(0)$), and its negative effect when low effort may be exerted (when $p < p_h(0)$). There are in fact two facets to the negative effect when $p < p_h(0)$. First, as discussed above, an agent whose failure is forgotten has an opportunity to exert low effort once again. In addition, raising q will "slow down the updating". That is, $p^S(p)$ will be closer to p, and thus a longer string of successes will be required until the risky entrepreneurs exert high effort.

We will show in what follows that the positive effect of raising q prevails over the negative ones when (i) agents are sufficiently patient (β close to 1), (ii) p_0 is sufficiently close to $p_h(0)$, and (iii) $|B|$ is sufficiently small relative to G. The first two conditions, in particular, are needed because the positive effect follows the negative ones along the equilibrium path. The third condition more generally limits the degree to which low effort reduces welfare.

In addition, we must also take into account that raising q may increase $p_h(q)$ as well, since the fact that failures are less costly can weaken incentives.[21] When β is close to 1, however, we are able to show that $p_h(q)$ does not grow too much, because the positive effect of raising q on the continuation utility in case of success is larger, thereby mitigating the negative effect on incentives from the weaker punishment after failure we have with $q > 0$.

Proposition 4. *For intermediate values of c, $\frac{R-1/\pi_h}{1-\beta\pi_l} \leq \frac{c}{\pi_h-\pi_l} < \frac{R-1}{1-\beta\pi_l}$, the optimal policy might also exhibit forgetting. More precisely:*

1. *If $p_0 > p_h(0)$, welfare is always maximized at $q(p_0) > 0$.*

2. *If $p_0 \in [p_{NF}, p_h(0)]$ and $-\frac{B}{G} < \frac{p_0(1-p_h(0))(1-\pi_l)}{p_h(0)((1-\pi_h)(1+(1-p_0)\pi_h))+\pi_h^2-p_0(1-\pi_h+\pi_h^2)}$, then for β sufficiently close to 1 we also have $q(p_0) > 0$.*

While the condition in case 2. is stated in terms of $p_h(0)$, which is an endogenous variable, it is possible to show that it is not vacuous[22] (this is also evident from the examples in the next section). Figure 6 illustrates the welfare-maximizing forgetting policy, as derived in Propositions 3 and 4, as a function of the cost of effort c.

[21]This, however, may not always be the case, since a higher value of q also increases the continuation utility upon success.

[22]In particular, let $\pi_l \to 1/R$, so that $B \to 0$. If we hold c and R fixed, then it is not hard to show that $p_h(0)$ will be bounded away from 0, so that the condition will be satisfied.

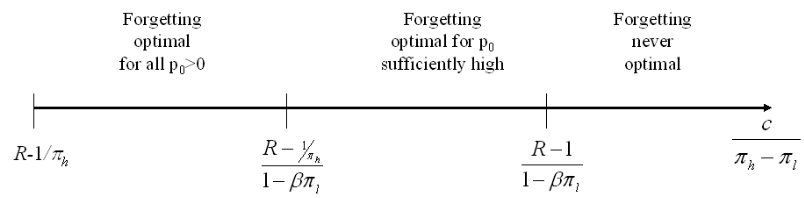

Figure 6: Welfare-maximizing forgetting policy, as a function of c

While the previous results give conditions under which some $q > 0$ maximizes total welfare, we can also determine when $q(p_0) = 1$, i.e., when it is optimal to keep no record of any failure. Evidently, this is the case when $p_h(1) \leq 1$ and $p_0 \geq p_h(1)$. In the next Proposition we show that these conditions are also necessary.

Proposition 5. *$q = 1$ maximizes total welfare if and only if $\frac{c}{\pi_h - \pi_l} \leq (R - 1)$ and $p_0 \geq p_h(1) = \frac{1-\pi_h(R-\frac{c}{\pi_h-\pi_l})}{(1-\pi_h)(R-\frac{c}{\pi_h-\pi_l})}$.*

Note that the restriction on c in this Proposition is always satisfied for c sufficiently close to its minimal value, as defined by Assumption 2.

Remark 4. *(Who Benefits from Forgetting?)* While the above results demonstrate that it is possible to achieve an improvement in total welfare by forgetting past failures, it is useful to distinguish the impact of forgetting across the two types of entrepreneurs. It is easy to see that the risky entrepreneurs must gain whenever forgetting leads to an improvement in total welfare, since the improvement arises precisely because, rather than being excluded from financing after failing, with some probability they are permitted to re-enter the pool of agents who receive financing. By contrast, forgetting generally hurts the safe types, since it slows down the updating, and the lower is p, the higher the interest rate paid. The only way in which forgetting might possibly benefit the safe types is if it were to decrease the high-effort cutoff $p_h(q)$, since the interest rate will be lower when the risky entrepreneurs exert high effort. We will see that this is *not* the case for the examples presented in section V below; so in those cases forgetting, while maximizing total surplus, hurts the safe types.

Remark 5. *(Risky Entrepreneurs Can Fail Even Under High Effort)* As we discussed above, the social benefit of forgetting failures arises from the additional periods of financing under high effort which it permits. In light of this, we can also understand the importance of our assumption that the risky entrepreneur can fail even when he exerts high effort, i.e., that $\pi_h < 1$. When this is not the case and we have $\pi_h = 1$ (as, for example, in Diamond, 1989) then high effort ensures success, and there is no benefit from forgetting a failure, since such failures only result from low effort.

Discussion — Empirical Evidence and Policy Implications

Our model captures many of the key arguments made in the Congressional debate surrounding the adoption of the FCRA, which we summarized in the Introduction. As such, it allows us to determine conditions under which the positive arguments prevail over the negative ones.

Notice first that the main argument put forward in favor of forgetting — that it allows individuals to obtain a true fresh start and hence to continue being productive members of society — is echoed in our model, where the positive effect on welfare of forgetting is that it gives risky entrepreneurs who fail access to new financing. They sometimes exert high effort, and hence this may increase aggregate surplus.[23] Furthermore, all of the arguments made against forgetting operate in our model: (i) forgetting weakens incentives by reducing the penalty for failure — i.e., region c. shrinks, and region a. increases in size, as we raise q; (ii) by erasing the records of those who were bad risks in the past, there is an increased risk that they will commit fraud in the future — the analog in our model is that forgetting "slows down" the weeding out of risky entrepreneurs; (iii) forgetting can lead to tighter lending standards — in our model this may be seen most sharply in the fact that forgetting makes region c. (where there is financing for all $p_0 > 0$ and interest rates are lower) smaller.[24] In addition, while the policy debate suggested that (iv) another negative effect of forgetting is that it forces safe agents to subsidize the risky, this is in fact socially optimal in our model, because it thereby improves the risky entrepreneurs' incentives.[25]

Our results are also consistent with the empirical evidence in Musto (2004). Forgetting clearly leads to increased credit scores for those who fail, and thus to more credit — in our

[23]Two other arguments were also made in favor of forgetting — that old information may be less relevant, and limited storage space — these do not have a role in our model. Furthermore, we conjecture that even if old information were less relevant (as will be the case if the type of an entrepreneur could change), lenders would take this into account and give it less weight in equilibrium.

[24]Just as suggested in the policy debate, the cohorts who are excluded from financing as a result of the introduction of such a policy are those with a low p_0 — i.e. the bad risks (see also example 2. in the next section).

[25]Since only they face a moral hazard problem.

model they would have $p = 0$, and no credit, without forgetting. Moreover, Musto's second point — that those who have their failure forgotten are likelier to fail in the future than those who are observationally equivalent (i.e. with the same score) is also an implication of the model, since only the risky agents ever have their failure forgotten. However, in contrast to Musto's suggestion that these laws are inefficient, Propositions 3 and 4 show that forgetting may be optimal.

Our model can also help us understand the international evidence, and in particular the relationship between forgetting clauses and the provision of credit. An implication of our model is that, if the forgetting clause is optimally determined, then there will be a positive relationship between credit volume and the degree of forgetting (as measured by q). The first reason is that forgetting is optimal when incentives are strong, i.e. for low values of c. Also, in this case, the introduction of a forgetting policy further increases the volume of credit, since it gives entrepreneurs who fail another chance at financing. This relationship is consistent with the empirical evidence reported in Figure 2 for those countries that have a credit bureau in place. Those countries in which information is only reported for a limited period of time have higher provision of credit than those in which the policy is to never forget defaults.

But what about those countries with no credit bureau, i.e., in which there is no information sharing? In our model this would only be optimal for very low values of c, in which case credit would be plentiful. However, these countries actually have the lowest provision of credit in the data. One way to understand this is that the financial systems in these countries are not fully developed, and that, as shown by several authors (see, for example, Djankov, McLiesh and Shleifer 2007, and Brown, Jappelli, and Pagano 2007), the introduction of a credit bureau would be beneficial in such cases. And indeed, from the historical record shown in Figure 1, we can see that the fraction of countries with no credit bureau has been shrinking over time, whereas the relative shares of the other two groups have remained stable.

Finally, while we have shown that forgetting past defaults can be welfare improving, this would never arise in equilibrium as the outcome of the choice of lenders. As shown in Lemma 1, there cannot exist any Markov Perfect Equilibrium in which agents who are known to be risky (as is the case for those who failed) obtain financing. Thus forgetting can only occur through government regulation of the credit bureau's information disclosure policies.

V Examples

In this section we present a few examples to illustrate the results of the previous sections. Let $R = 3, \pi_h = 0.5$, and $\pi_l = 0.32$. With regard to the remaining parameters, c, β and p_0, we consider some alternative specifications, which allow us to obtain the different types of equilibria described in Proposition 1. Note that for assumptions 1 and 2 to be satisfied, the effort cost c must lie in the interval $(0.18, 0.5)$.

1. Let $c = 0.4$ and $\beta = 0.975$. For these values we are in region b. of Proposition 1, for which high effort is implemented when $p \geq p_h(q)$. The threshold $p_h(0)$ above which high effort is exerted when $q = 0$ can be computed from equation (12) in the Appendix, which yields: $p_h(0) = 0.241$.

   When $p_0 > p_h(0) = 0.241$, from Proposition 4 we know that $q(p_0) > 0$ is optimal, because forgetting failures increases the rounds of financing to risky entrepreneurs and in these new rounds they always exert high effort. On the other hand, for $p_0 \in [p_{\text{NF}}, p_h(0)) = [0.0196, 0.241)$ low effort is exerted with at least some positive probability. However, for the parameters of this example B/G satisfies the condition stated in 2. of Proposition 4 whenever $p_0 > 0.205$. Thus some degree of forgetting will be optimal for β sufficiently close to 1; we will verify that this is indeed the case when $\beta = 0.975$. The reason is that for these parameters the increase in surplus $G = \pi_h R - 1 - c = 0.1$ from a project undertaken with high effort is high, relative to the decrease in surplus $B = -0.04$ from a project undertaken with low effort and so, for agents who are sufficiently patient the additional periods of high effort provided by forgetting outweigh the cost of the extra periods of low effort at the start of the game.

   Consider $p_0 = 0.206$. When $q = 0$, we have $p^S(p_0) = 0.448 > p_h$, and so low effort is exerted for the first round of financing along the equilibrium path, and high effort forever after, as long as the projects succeed.[26] However, when $q > 0$, more rounds of financing with low effort may be needed before risky entrepreneurs begin to exert high effort, both because the updating is slower and because $p_h(q)$ is higher. For example, with $q = 0.735$ three periods of financing with low effort followed by success of the project are needed until the posterior exceeds $p_h(0.735) = 0.322$. We now compare welfare levels for different specification of the forgetting policy. In figure 7 we plot the value of the total surplus $\mathcal{W}(q, 0.206)$ as a function of q, when $p_0 = 0.206$. From

[26]The risky entrepreneurs never randomize in their effort choice along the equilibrium path for any of the values of p_0 and q considered in this example.

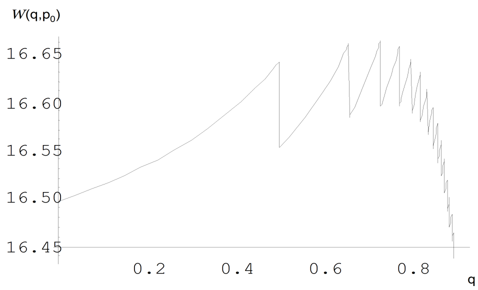

Figure 7: Example 1: total welfare as a function of q (when $p_0 = 0.206$)

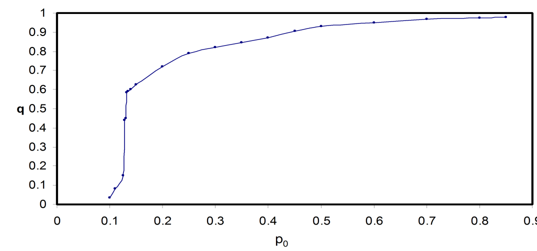

Figure 8: Example 1: welfare-maximizing value of q

this figure one can see that the optimum obtains at $q(0.206) = 0.77$, in which case $\mathcal{W}(0.77, 0.206) = 16.648$.

We also plot in figure 8 the optimal level $q(p_0)$ of the forgetting policy[27] as we vary the prior probability p_0.[28]

2. Consider next $c = 0.26$ and $\beta = 0.975$. We are now in region c. of Proposition 1, for which high effort is exerted for all $p > 0$. As long as $q \leq 0.359$ (i.e., as long as q is sufficiently low that we remain in region c.), forgetting provides additional opportunities for projects to be undertaken with high effort, and so is clearly efficient. Hence, as we can see in figure 9, we have $\mathcal{W}(0.359, p) > \mathcal{W}(0, p)$ for all $p > 0$.

   As we raise q further, incentives become so weak that we move into region b.; it is then no longer the case that the risky entrepreneurs exert high effort for all p.[29] We know from Proposition 4, however, that as long as $p_0 > p_h(q)$, raising q continues to improve welfare, as the risky agents will exert high effort when they are financed again. For example, with $q = 0.975$ this is the case for all $p_0 > p_h(0.975) = 0.1139$.

   By contrast, for $p_0 \in (p_{\text{NF}}, p_h(q)]$ we face the same tradeoff discussed in example 1 above. A higher q leads to more rounds of financing where both low and high effort are exerted. When p_0 is sufficiently close to $p_h(q)$, the time spent in the low effort region will be relatively short, and thus increasing the level q of forgetting above

[27] We discretize the domains of p_0 and q. For each point in the grid we compute $p_h(q)$ and then the welfare $\mathcal{W}(q, p_0)$. We assign $q(p_0)$ to be the value of q that maximizes this surplus, given p_0.

[28] Although the condition in 2. of Proposition 4 is violated for $p_0 \leq 0.205$, we can nevertheless still have $q(p_0) > 0$, since the condition is only sufficient, not necessary.

[29] For the values of c and β under consideration we are never in region a., no matter how high q.

0.359 may still increase surplus. As we see in figure 9, when $p_0 > 0.066$ we have $\mathcal{W}(0.975, p_0) > \mathcal{W}(0.359, p_0)$. On the other hand, when p_0 is closer to $p_{\text{NF}} = 0.0196$ the cost of additional rounds of financing with low effort dominates, in which case welfare is higher for $q = 0.359$.

Finally, for very low values of p_0 (in particular $p_0 < p_{\text{NF}}$) there will be no financing when q is sufficiently high. The reason is that there is no feasible interest rate which would allow lenders to break even for these values of p_0; the lenders make losses on the risky entrepreneurs because they exert low effort, and there are too few safe entrepreneurs from which to recoup these losses. In other words, raising q too much can lead to a tightening in lending standards, as discussed in the previous section. For example, when $p_0 < p_{\text{NF}}$ there is no financing if $q = 0.975$; hence the optimal value of q is clearly lower.

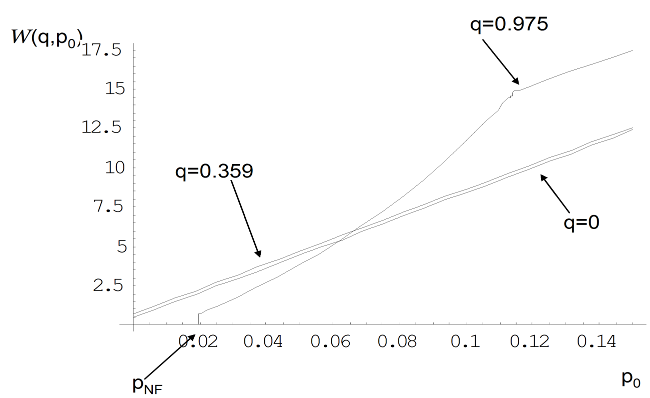

Figure 9: Example 2: total welfare

3. Finally, consider $\beta = 0.8$, $c = 0.48$ and a slightly lower value for π_l: $\pi_l = 0.3$. While these parameters are in region b., as in example 1 above, the contribution G to total surplus of a project undertaken with high effort is now much lower and agents are less patient. As a consequence, the condition stated in 2. of Proposition 4 never holds. In this case we find that welfare is decreasing in q for p_0 sufficiently low, as the cost of less frequent exclusion in the low effort region dominates the benefit in the high effort

region. This is illustrated in figure 10 for the case $p_0 = 0.2$.

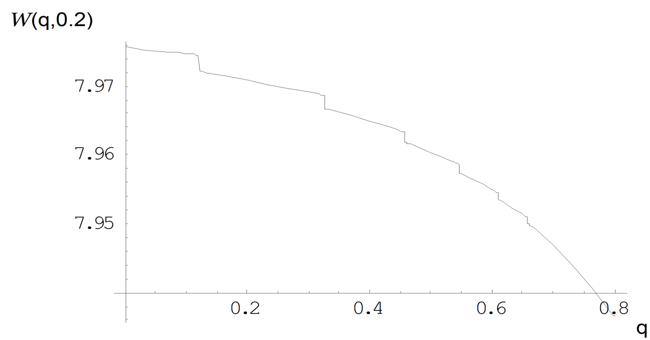

Figure 10: Example 3: total welfare

VI Extension — Both Types can Fail

We extend the model to allow the projects of both the risky and safe types to fail, and we present an example in which our central finding — that forgetting defaults may be welfare-enhancing — continues to hold. We conjecture that the main qualitative features of our previous results remain valid in this case, although a formal analysis of it is beyond the scope of the current paper.

When both types can fail an agent who defaults can no longer be identified for certain as risky. As discussed in Remark 1 above, he may thus be able to obtain additional periods of financing even without forgetting. As a result, one might think that forgetting would be superfluous. In this example, however, forgetting continues to provide a benefit even though agents may obtain some financing after they fail.

Let $\pi \in (\pi_h, 1]$ denote the probability that the project of a safe entrepreneur fails. Consider the following parameter values: $R = 3, \pi_h = 0.5, \pi_l = 0.32, \beta = 0.975, c = 0.35$. When $\pi = 1$ (the projects of safe types always succeed) these parameters fall in region b. of Proposition 1, where (for $q = 0$) high effort is exerted for all $p \geq p_h(0) = 0.113$, and agents are financed for all $p \geq p_{\text{NF}} = 0.0196$. The situation is thus analogous to Example 1 in the previous section. Consider the prior belief $p_0 = 0.1$; by similar computations to that in the example we derive the optimal forgetting policy: $q(0.1) = 0.77$.

Next, suppose projects of safe entrepreneurs only succeed with probability $\pi = 0.99$. We find that the equilibrium strategies exhibit, in most respects, analogous properties to those found in Proposition 1 (i.e. when $\pi = 1$).[30] When $q = 0$ there is an MPE where high effort is exerted as long as $p \geq p_h(0) = 0.1065$, and entrepreneurs are financed for $p \geq p_l = p_{\text{NF}} = 0.0199$.[31] So long as $p < 0.58$, we have $p^F(p) < p_l$ and so a single failure still results in exclusion; however, for higher values of p an agent *will* be able to obtain financing following a failure. Comparing theis value of $\pi_h(0)$ with the one found above for the case $\pi = 1$, we see that the region of p for which high effort is exerted is larger. The failure of a project does not necessarily lead to exclusion, and this has two effects on incentives. First, when p is high a failure is not punished by exclusion, which weakens incentives. In addition, however, the fact that the agent may be financed following a failure in the future raises his continuation utility upon success, which has a positive effect on incentives. Such efect is present no matter what is the current level of p. Thus for a relatively low value of p, this second, positive, effect clearly prevails.

Proceeding along the same lines, we also find a MPE for positive values of q, and compute the surplus function $\mathcal{W}(q, p_0)$. In figure 11 we have plotted the improvement in total surplus (relative to its level when $q = 0$) for various values of q: we see that when $p_0 = 0.1$ total surplus is maximal when $q = 0.80$.

An interesting feature of this extension is that forgetting may now also increase the surplus generated by the projects of safe entrepreneurs who are financed. Recall that, when $\pi = 1$ this surplus was either unaffected, or decreased, by the introduction of forgetting. Now, however, since safe entrepreneurs are also at risk of failing and hence of being excluded, forgetting may benefit them by increasing the likelihood that their projects will be financed in the future.

[30] To construct a Markov Perfect Equilibrium we must however follow a different procedure, because the continuation utility for an agent who fails no longer need be equal to zero. Hence we discretize the domain of p and, for each pair of candidates values for $p_l \geq p_{\text{NF}}$ and $p_h < 1$, we compute the value function for the risky entrepreneurs, using value function iteration. We then determine whether these values are indeed associated with an equilibrium by verifying that no deviation is profitable, neither by borrowers nor lenders. Finally, we select the pair with the lowest value of p_h.

[31] In this example high effort is implemented for all $p \geq p_h$, and so the equilibrium is qualitatively similar to that of the main model of the paper. For higher values of c, however, high effort could no longer be sustained for p very close to 1; see Mailath and Samuelson (2001) for further discussion.

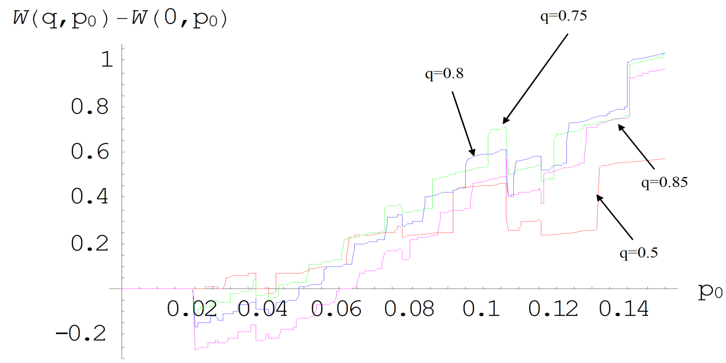

Figure 11: Change in surplus with forgetting when both types can fail

VII Conclusion

In this paper we have investigated the effects of restrictions on the information available to lenders on borrowers' past performance. These restrictions may facilitate a "fresh start" for borrowers in distress, but also clearly have an effect on their incentives. To this end, we have considered an environment where borrowers need to seek funds repeatedly, and the borrower-lender relationship is characterized by the presence of both moral hazard and adverse selection. In such a framework we have determined the effects of such restrictions on borrowers' incentives as well as on lenders' behavior, and hence on access to credit and overall welfare. We found that imposing limits on the information available to lenders is desirable when (i) borrowers' incentives are sufficiently strong, (ii) the average risk type is not too low, (iii) low effort is not too inefficient, and (iv) agents are sufficiently patient. In this case imposing such limits is welfare improving and increases credit volume, otherwise the reverse may obtain. We also show that these findings may help to explain the empirical evidence.

As noted in the Introduction, there are some cross-country differences in the laws governing the memory of the credit reporting system; in general, European countries tend to allow defaults to be forgotten more quickly. In addition, bankruptcy laws, which govern the extent to which defaulting borrowers can shield assets and income, can also differ dramatically across countries. It would be interesting to study how these features of credit markets interact, and how they are related to differences in the economic environments in such countries.

VIII Appendix A — Proofs

Lemma 1 — No financing when known to be risky

If $p = 0$, we must have $p^S(p, \mathcal{C}') = 0 = p^F(p, \mathcal{C}')$ whatever $\mathcal{C}'$, i.e., the agent will be known to be risky in the future as well.

Furthermore, under assumption 1, if the agent is known to be the risky type, he can only be financed in a given period if he exerts high effort with some probability, as otherwise lenders cannot break even. But for high effort (or mixing) to be incentive compatible, the utility from high effort must be no less than that from low effort, i.e., the interest rate r offered must be such that:

$$\pi_h(R-r) - c + (\pi_h + (1-\pi_h)q)\beta v^r(p^S(0)) + (1-\pi_h)(1-q)\beta v^r(p^F(0)) \geq$$
$$\pi_l(R-r) + (\pi_l + (1-\pi_l)q)\beta v^r(p^S(0)) + (1-\pi_l)(1-q)\beta v^r(p^F(0)),$$

which simplifies to the static incentive compatibility condition:

$$\frac{c}{\pi_h - \pi_l} \leq R - r, \tag{8}$$

since when $p = 0$ we have $p^S = p^F = 0$.

By assumption 2, this can only be satisfied if $r < 1/\pi_h$, in which case lenders cannot break even. Thus the agent cannot be financed in equilibrium if he is known to be risky. Finally, since this agent is never financed, it is immediate that $v^r(0) = 0$.■

Lemma 2 — All MPE are pooling

Suppose this is not the case; consider a candidate separating equilibrium. Let r^s denote the contracts chosen by the safe types and r^r those chosen by the risky in such an equilibrium. From Lemma 1 we know that in a separating MPE the risky types cannot be financed, i.e. we must have $r^r = \emptyset$ for all nodes along the equilibrium path, and so their utility is $v^r = 0$. Hence for the risky entrepreneurs not to pretend to be safe, we must have either $r^s = R$ in every period, or $r^s = \emptyset$ in every period (the contract must be the same in every period by the Markov property). But if $r^s = \emptyset$ the equilibrium would in fact be pooling, contrary to the stated claim. We now argue that $r^s = R$ cannot be an equilibrium strategy for lenders, because each lender would have an incentive to undercut and offer $R - \epsilon$.

Consider, in particular, some future period $t > 0$. In such period a lender can deviate and offer $R - \epsilon$ (for ϵ small) to the safe entrepreneurs. Note that this offer can be made to the safe agents alone because the credit history of a safe agent differs from that of a risky one by virtue of the fact that only the safe agents are financed in the initial period in the proposed equilibrium. Such a deviation would clearly be profitable, thus overturning the proposed equilibrium. ■

Proposition 1 — Characterization of the Equilibrium

To complete the proof of Proposition 1, we establish the remaining properties of the MPE, i. and iii., and the specific features of this equilibrium for parameter regions a., b., and c.

We begin by verifying property i. First note that the second part of property i. follows immediately from Observation 2. It is also easy to verify the third part of property i.: that a consistent belief for lenders is that an entrepreneur is risky if he refuses financing. To see this, simply let the risky entrepreneurs refuse financing at some node with probability $\epsilon > 0$, and the safe ones with probability ϵ^2, and let $\epsilon \to 0$. Consistency of the above belief can then readily be verified using Bayes' Rule. This immediately demonstrates the first part of property i. as well, since from Lemma 1 refusing financing would give an entrepreneur a utility of 0.

We now verify the characterization of the equilibrium strategies provided for each region, and show that there are no profitable deviations by lenders.

a. To show that the strategies specified in the Proposition constitute an MPE when $\frac{c}{\pi_h - \pi_l} \geq \frac{(R-1)(1-\beta q)}{1-\beta(\pi_l + (1-\pi_l)q)}$, we need to demonstrate that (a-i) low effort is incentive compatible for $p \geq p_{\text{NF}}$; (a-ii) $r(p) = r_{zp}(p, 0) \leq R$ for $p \geq p_{\text{NF}}$, i.e., it is admissible; and (a-iii) there are no profitable deviations by lenders.

a-i. Given the above strategies and beliefs, from (1) we get:

$$v^r(p) = \pi_l(R - r_{zp}(p, 0)) + (\pi_l + (1 - \pi_l)q)\beta v^r(p^S(p)), \tag{9}$$

since from Lemma $1 v^r(p^F(p)) = v^r(0) = 0$.

By the same argument used to derive (8) above, for low effort to be incentive compatible we need:

$$\frac{c}{\pi_h - \pi_l} \geq R - r_{zp}(p,0) + \beta(1-q)v^r(p^S(p)), \tag{10}$$

Since $r_{zp}(p,0) > r_{zp}(1,0) = 1$ for all $p < 1$,

$$v^r(p) < \frac{\pi_l(R-1)}{1-\beta(\pi_l + (1-\pi_l)q)},$$

where the term on the right-hand side is the present discounted utility of a risky entrepreneur who is financed in every period (until he has a failure that is not forgotten), exerting low effort, and at the rate $r = 1$.

So for any $p \in (p_{\text{NF}}, 1)$, we have

$$R - r_{zp}(p,0) + \beta(1-q)v^r(p^S(p)) \quad < R - 1 + \beta(1-q)\tfrac{\pi_l(R-1)}{1-\beta(\pi_l+(1-\pi_l)q)}$$
$$= \tfrac{(R-1)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)} < \tfrac{c}{\pi_h-\pi_l}$$

where the last inequality follows from definition of region a. This verifies (10).

a-ii. Note that $r_{zp}(p,0) \leq R$ if and only if $\frac{1}{p+(1-p)\pi_l} \leq R$, or equivalently $p \geq p_{\text{NF}}$.

a-iii. Consider a deviation by a lender. First note that lenders make zero profits in equilibrium, so refusing to offer a contract would never be profitable. So consider a deviation consisting of the offer of a contract r' to entrepreneurs with credit score p. Without loss of generality we can restrict attention to $r' > 1$, since otherwise the deviation could never be profitable. Let the new set of contracts (which includes the deviation r') be $\mathcal{C}'$. But then by the same argument as in a-i. above we can show that since $r' > 1$, the optimal response by risky entrepreneurs who accept r' is to exert low effort, i.e., $e^r(p, \mathcal{C}') = 0$. This implies that lenders cannot profit from r'. To see this, first note that if $r' \leq r_{zp}(p,0)$ this deviation could not be profitable, since low effort is exerted. Alternatively, suppose that $r' > r_{zp}(p,0)$. If $p \geq p_{\text{NF}}$, then this would imply that $r' > r(p)$ and so no borrower would accept this contract. If $p < p_{\text{NF}}$, however, then we must have $r' > R$ by the definition of p_{NF}, and this deviation would not be admissible.

b. Next, we show that for intermediate values of c, $\frac{(R-1/\pi_h)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)} < \frac{c}{\pi_h-\pi_l} < \frac{(R-1)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$, an MPE exists characterized by $0 < p_l \leq p_m \leq p_h < 1$ such that: for $p \geq p_l$ entrepreneurs are always financed, $e^r(p) = 1$ for $p \geq p_h$, $e^r(p) \in (0,1)$ and is (strictly) increasing in p for $p \in [p_m, p_h)$, $e^r(p) = 0$ for $p \in [p_l, p_m)$ and $r(p) = r_{zp}(p, e^r(p))$.

We begin by characterizing the values of (b-i) p_h, (b-ii) p_m and (b-iii) p_l, showing that the effort choices specified above for the risky entrepreneurs are optimal. In (b-iv) we demonstrate that there are no profitable deviations for lenders.

b-i. Let $\tilde{p}^S(p,e) \equiv \frac{p}{p+(1-p)[e(\pi_h+(1-\pi_h)q)+(1-e)(\pi_l+(1-\pi_l)q)]}$; this is the posterior belief, following a success, that an entrepreneur is risky, when the prior belief is $p \in (0,1)$ and the effort undertaken if risky is e, calculated via Bayes' Rule. Also, let $\tilde{v}^r(p,1)$ denote the discounted expected utility for a risky entrepreneur with credit score p when he is financed in every period until experiencing a failure that is not forgotten, he exerts high effort ($e = 1$), beliefs are updated according to $\tilde{p}^S(p,1)$ and the interest rate is $r_{zp}(p',1)$ for all $p' \geq p$. Then $\tilde{v}^r(p,1)$ satisfies the following equation:[32]

$$\tilde{v}^r(p,1) = \pi_h(R - r_{zp}(p,1)) - c + \beta(\pi_h + (1-\pi_h)q)\tilde{v}^r(\tilde{p}^S(p,1),1). \qquad (11)$$

We then define p_h as the value of p that satisfies the following equality:

$$\frac{c}{\pi_h - \pi_l} = R - r_{zp}(p_h,1) + \beta(1-q)\tilde{v}^r(\tilde{p}^S(p_h,1),1) \qquad (12)$$

Observe that, since $\tilde{p}^S(p,1)$ is strictly increasing in p, and $r_{zp}(p,1)$ is strictly decreasing, $\tilde{v}^r(p,1)$ is strictly increasing in p. Thus the term on the right-hand side of (12) is increasing in p, and so (12) has at most one solution.

By a continuity argument, it can be verified that:

Claim 1. *A solution $p_h \in (0,1)$ to (12) always exists.*[33]

Given the monotonicity of the term on the right-hand side of (12), it is then immediate that the incentive compatibility constraint for high effort (8) is satisfied for all $p \geq p_h$.

b-ii. Next, we find p_m, the lower bound of the region where risky entrepreneurs mix over high and low effort, and establish the properties of the equilibrium in this mixing region.

[32] Note that while $\tilde{v}^r(p,1)$ and $\tilde{p}^S(p,e)$ are well defined for all $p \in (0,1)$, they only coincide with the equilibrium values $v^r(p)$ and $p^S(p)$ when both $p \geq p_h$ and $e = e^r(p) = 1$.

[33] The proofs of claims 1-5 can be found in appendix B (http://www.elul.org/papers/forget/appendix_b.pdf).

For mixing to be an equilibrium strategy at p, risky entrepreneurs must be indifferent between high and low effort, i.e.,

$$R - r_{zp}(p,e) + \beta(1-q)v^r(\tilde{p}^S(p,e)) = \frac{c}{\pi_h - \pi_l} \tag{13}$$

for some $e \in [0,1]$. Now, let $\left(\tilde{p}^S\right)^{-1}(p_h, 1)$ denote the preimage of p_h according to the map $\tilde{p}^S(p,1)$, i.e., $\tilde{p}^S\left(\left(\tilde{p}^S\right)^{-1}(p_h,1),1\right) = p_h$.[34] We define p_m to be the lowest value of $p \geq \left(\tilde{p}^S\right)^{-1}(p_h,1)$ for which a solution of (13) can be found for some e. Observe that by the construction of p_h, $e = 1$ is a solution to (13) when $p = p_h$, and so $p_m \leq p_h$. It can be shown that:

Claim 2. *A lowest value p_m always exists and, moreover, $p_m > \left(\tilde{p}^S\right)^{-1}(p_h,1)$.*

This implies that there is at most a single period of mixing along the equilibrium path. It can also be shown that:

Claim 3. *For all $p \in [p_m, p_h]$, there exists a solution $e^r(p)$ to (13), with $e^r(p)$ strictly increasing in p.*

If there is more than one solution to (13) at p, we choose the highest.

b-iii. We still have to determine p_l, the lower bound on the financing region, and demonstrate that low effort is incentive compatible in $[p_l, p_m)$.

⧫ If $p_m \geq p_{\text{NF}}$, set $p_l = p_{\text{NF}}$. By construction, $r_{zp}(p,0) \leq R$ for all $p \geq p_{\text{NF}}$; hence the contract $r_{zp}(p, e^r(p))$ is admissible for all $p \geq p_{\text{NF}}$.

Alternatively, if $p_m < p_{\text{NF}}$ set p_l to be the lowest value of $p \geq p_m$ such that the contract $r_{zp}(p, e^r(p))$ is admissible (i.e., not greater than R). Note that since $r_{zp}(p,e)$ is decreasing in e, we have $r_{zp}(p, e^r(p)) \leq r_{zp}(p,0)$ for all $p \in [p_m, p_{\text{NF}}]$, so this will imply that $p_l \leq p_{\text{NF}}$. In this case we also redefine p_m, with some abuse of notation, to be equal to p_l; following this redefinition the low effort region $[p_l, p_m)$ is then empty in this case.

Observe that in either case we have $p_l > 0$. Furthermore, $p_l \leq p_{\text{NF}}$, which implies that $r_{zp}(p,0) > R$ for $p < p_l$. Finally, $p_l \leq p_m$, with p_m as defined in the preceding paragraphs.

⧫ To prove that $e^r(p) = 0$ for $p \in [p_l, p_m)$ it suffices to consider the case $p_l = p_{\text{NF}}$

[34] That is, when the prior belief is $\left(\tilde{p}^S\right)^{-1}(p_h,1)$, and the entrepreneur exerts high effort if risky, the posterior belief of lenders after observing a success is equal to p_h.

(since when $p_l < p_{\text{NF}}$, we showed above that $p_l = p_m$, in which case there is no low-effort region).

- First consider $p \in \Big[\max[p_l, \big(\tilde{p}^S\big)^{-1}(p_h, 1)], p_m\Big)$. Intuitively, were low effort not incentive compatible in this region, that would contradict the construction of p_m as minimal. This is verified in the following:

  Claim 4. *The contract $r_{zp}(p, 0)$ satisfies the IC constraint for low effort when $p \in \Big[\max[p_l, \big(\tilde{p}^S\big)^{-1}(p_h, 1)], p_m\Big)$.*

- If $\max[p_l, \big(\tilde{p}^S\big)^{-1}(p_h, 1)] = p_l$ we are done. Otherwise, we need to iterate the argument. Consider first $p \in \Big[\max[p_l, \big(\tilde{p}^S\big)^{-1}(p_h, 0)], \big(\tilde{p}^S\big)^{-1}(p_h, 1)\Big)$.[35] We will show that since $r(p) > r(p_h)$, low effort must be incentive compatible for p in this region.

  We begin by showing that $v^r(p^S(p)) < v^r(p^S(p_h))$. To see this, observe first that for all such values of p, we have $p^S(p) = \tilde{p}^S(p, 0) \geq p_h$. Also, by Assumption 3, we have $p^S(p) < p^S(p_h)$. Thus $v^r(p^S(p)) < v^r(p^S(p_h))$, since $v^r(p')$ was shown to be strictly increasing for $p' \geq p_h$ (as $v^r(p') = \tilde{v}^r(p, 1)$ in this region).

  This then implies that low effort is incentive compatible. To see this, first note that, by the definition of p_h, we have $R - r(p_h) + \beta(1-q)v^r(p^S(p_h)) = \frac{c}{\pi_h - \pi_l}$. But we have shown above that $v^r(p^S(p)) < v^r(p^S(p_h))$. Also, $r(p) \equiv r_{zp}(p, 0) > r_{zp}(p_h, 1) \equiv r(p_h)$. Thus,

$$R - r(p) + \beta(1-q)v^r(p^S(p)) < \frac{c}{\pi_h - \pi_l},$$

  and so low effort is incentive compatible at p.

- If $\max[p_l, \big(\tilde{p}^S\big)^{-1}(p_h, 0)] = p_l$ we are done. Otherwise we proceed as follows. It is convenient here to use the shorthand $\tilde{p}^{S^{-1}}$ to denote the term $\big(\tilde{p}^S\big)^{-1}(p_h, 0)$. Consider $p \in \Big[\max[p_l, \big(\tilde{p}^S\big)^{-1}(\tilde{p}^{S^{-1}}, 0)], \tilde{p}^{S^{-1}}\Big)$. To prove that low effort is incentive compatible at p, the following bounds on the risky agent's utility function — obtained in each case by substituting the relevant incentive compatibility constraint into the recursive definition of the risky entrepreneur's utility, given by (1) — will be useful:[36]

[35] Observe that $\tilde{p}^S(p, e)$ in decreasing in e, so $\big(\tilde{p}^S\big)^{-1}(p_h, 0) \leq \big(\tilde{p}^S\big)^{-1}(p_h, 1)$. This property can be easily verified from the expression of $\tilde{p}^S(p, e)$ and can be understood as follows: for any given p, the lower the probability e that the risky entrepreneurs exert high effort, the stronger is success a signal that the entrepreneur is a safe type.

[36] When $e^r(p) = 1$ (1) reduces to $v^r(p) = \pi_h(R - r(p)) - c + \beta(\pi_h + (1-\pi_h)q)v^r(p^S(p))$, and hence we

$$v^r(p) \geq \frac{c(\pi_l + q/(1-q))}{\pi_h - \pi_l} - (R - r(p))\frac{q}{1-q}, \text{ if } e^r(p) = 1; \tag{14}$$
$$v^r(p) \leq \frac{c(\pi_l + q/(1-q))}{\pi_h - \pi_l} - (R - r(p))\frac{q}{1-q}, \text{ if } e^r(p) = 0; \tag{15}$$
$$v^r(p) = \frac{c(\pi_l + q/(1-q))}{\pi_h - \pi_l} - (R - r(p))\frac{q}{1-q}, \text{ if (13) holds (mixing).} \tag{16}$$

For p lying in the interval under consideration, we have $p^S(p) < p_h$. Also, with low effort $p^S(p) \geq \tilde{p}^{S^{-1}}$. Now recall that we have shown immediately above that $e^r(p') < 1$ for all $p' \in [\tilde{p}^{S^{-1}}, p_h)$; and, in particular, $e^r(p^S(p)) < 1$. So (15) and (16) imply that $v^r(p^S(p)) \leq \frac{c(\pi_l+q/(1-q))}{\pi_h-\pi_l} - (R - r(p^S(p))\frac{q}{1-q}$. On the other hand, since the equilibrium implements high effort at p_h, by (14) we have $v^r(p_h) \geq \frac{c(\pi_l+q/(1-q))}{\pi_h-\pi_l} - (R - r(p_h))\frac{q}{1-q}$. Moreover, since we have shown that $v(p')$ is increasing for $p' \geq p_h$, and $p^S(p_h) \geq p_h$, this also implies $v^r(p^S(p_h)) \geq \frac{c(\pi_l+q/(1-q))}{\pi_h-\pi_l} - (R - r(p_h))\frac{q}{1-q}$.
Thus $v^r(p^S(p)) \leq v^r(p^S(p_h)) + \frac{q}{1-q}\left(r(p^S(p)) - r(p_h)\right)$, which implies that

$$R - r(p) + \beta(1-q)v^r(p^S(p)) \leq R - r(p) + \beta(1-q)v^r(p^S(p_h)) + \beta q\left(r(p^S(p)) - r(p_h)\right). \tag{17}$$

But (recalling that $r(\cdot)$ is decreasing) $r(p) > r(p^S(p)) > r(p_h)$, and so $-r(p) + \beta q\left(r(p^S(p)) - r(p_h)\right) < -r(p_h)$. Hence (17) implies that

$$R - r(p) + \beta(1-q)v^r(p^S(p)) < R - r(p_h) + \beta(1-q)v^r(p^S(p_h)) = \frac{c}{\pi_h - \pi_l},$$

where the final equality follows from the definition of p_h. We conclude that low effort is incentive compatible at p.

- If $\max[p_l, \left(\tilde{p}^S\right)^{-1}(\tilde{p}^{S^{-1}}, 0)] = p_l$ we are done. Otherwise, we proceed by induction, as follows. Redefine $\tilde{p}^{S^{-1}}$ to be $\left(\tilde{p}^S\right)^{-1}(\tilde{p}^{S^{-1}}, 0)]$. Observe that we have established immediately above that $e^r(p') < 1$ for $p' \in \left[\tilde{p}^{S^{-1}}, p_h\right)$. So we can

get $\beta(1-q)v^r(p^S(p)) = \frac{(v^r(p)+c-\pi_h(R-r(p)))(1-q)}{\pi_h+(1-\pi_h)q}$. The high-effort IC constraint (8) can then be rewritten as follows: $\beta(1-q)v^r(p^S(p)) \geq \frac{c}{\pi_h-\pi_l} - (R - r(p))$. Substituting for $v^r(p^S(p))$ from the previous equation, yields $\frac{(v^r(p)+c-\pi_h(R-r(p)))(1-q)}{\pi_h+(1-\pi_h)q} \geq \frac{c}{\pi_h-\pi_l} - (R - r(p))$, or $(v^r(p) + c - \pi_h(R - r(p)))(1-q) \geq \left(\frac{c}{\pi_h-\pi_l} - (R - r(p))\right)(\pi_h + (1-\pi_h)q)$. Simplifying, we get (14). The other expressions are similarly obtained.

iterate the same argument as above, and do so until reaching p_l.

b-iv. As noted in a-iii. above, we can restrict attention to lenders' deviations consisting in the offer of a contract $r' > 1$ to entrepreneurs with credit score p.

Now, for r' to be accepted it must be lower than the equilibrium rate when there is financing in equilibrium. So when $p \geq p_l$, it suffices to consider $r' < r(p) \equiv r_{zp}(p, e^r(p))$. When $p < p_l$ there is no financing in equilibrium, and the deviation can be any contract $r' \in (1, R]$.

In the statement of the Proposition we did not describe the risky entrepreneurs' effort strategy $e^r(p, \mathcal{C}')$ off the equilibrium path. We will do so here, and show that $e^r(p, \mathcal{C}')$ renders any possible deviation r' described in the previous paragraph unprofitable.

♦ We first begin with the simplest case: $p \geq p_h$. Since high effort is implemented for these values of p, it is immediate that no deviation could be profitable, since for r' to be accepted by the entrepreneurs we would need $r' < r(p) = r_{zp}(p, 1)$.

♦ Next consider the case $p \in \left[\left(\tilde{p}^S\right)^{-1}(p_h, 1), p_h\right)$. Now if

$$R - r' + \beta(1-q)v^r(\tilde{p}^S(p, 0)) \leq \frac{c}{\pi_h - \pi_l}, \tag{18}$$

then $e^r(p, r') = 0$ is an optimal effort choice for entrepreneurs when they are offered the rate r' and lenders' belief is that they exert low effort. If in addition $p \geq p_l$ we need $r' < r(p) \leq r_{zp}(p, 0)$ for r' to attract some entrepreneurs, and so the deviation will be unprofitable. On the other hand, if $p < p_l$, from b-iii. above we know that $r_{zp}(p, 0) > R$ (since $p_l \leq p_{\text{NF}}$), while the admissibility of the contract requires $r' \leq R$, implying $r' < r_{zp}(p, 0)$. That is, the deviation is unprofitable in this case as well.

Alternatively, suppose the reverse inequality to (18) holds. This means that low effort is not an optimal response to r'. Nevertheless, the deviation can be shown to be unprofitable. More precisely, we show in what follows that, were a profitable deviation to exist, this would contradict the construction of the equilibrium (in particular, either the definition of p_h, or of p_m, or $e^r(p)$ being maximal in the mixing region).

We begin by determining the effort level and lenders' beliefs associated with r'. First note that, for the values of p under consideration, $\tilde{p}^S(p, e) \geq p_h$ for all e. Then since $\tilde{p}^S(p, e)$ is decreasing with respect to e (footnote 35) and $v^r(p')$ is both increasing and continuous for $p' \geq p_h$, we either have

$$R - r' + \beta(1-q)v^r(\tilde{p}^S(p,e')) \geq \frac{c}{\pi_h - \pi_l}, \text{ for } e' = 1 \tag{19}$$

or

$$R - r' + \beta(1-q)v^r(\tilde{p}^S(p,e')) = \frac{c}{\pi_h - \pi_l} \text{ for some } e' \in (0,1), \tag{20}$$

so that the optimal effort choice of risky entrepreneurs when $\mathcal{C}'$ contains r' and r' is chosen, is $e^r(p,\mathcal{C}') = e'$, and lenders' beliefs $\tilde{p}^S(p,e')$ are consistent with Bayes' Rule.

We will establish that $r' \leq r_{zp}(p,e')$, implying that the deviation to r' is unprofitable. Suppose that this is not the case, i.e. that $r' > r_{zp}(p,e')$; we will prove in what follows that this implies a contradiction.

When $e' = 1$, $r' > r_{zp}(p,e') = r_{zp}(p,1)$ together with (19) imply $R - r_{zp}(p,1) + \beta(1-q)v^r(\tilde{p}^S(p,1)) \geq \frac{c}{\pi_h - \pi_l}$. But since, as we argued, $v^r(p')$ is increasing for $p \geq p_h$ and $r_{zp}(\cdot,1)$ strictly decreasing, this would imply that $R - r_{zp}(p_h,1) + \beta(1-q)v^r(\tilde{p}^S(p_h,1)) > \frac{c}{\pi_h - \pi_l}$, contradicting the construction of p_h in (12).

Consider next $e' < 1$. From $r' > r_{zp}(p,e')$ and equation (20) we get

$$R - r_{zp}(p,e') + \beta(1-q)v^r(\tilde{p}^S(p,e')) > \frac{c}{\pi_h - \pi_l}.$$

Recall that $p \in \left[\left(\tilde{p}^S\right)^{-1}(p_h,1), p_h\right)$, so that $v^r(\tilde{p}^S(p,e)) = \tilde{v}^r(\tilde{p}^S(p,e),1)$ for any e, and, from the definition of p_h,

$$R - r_{zp}(p,1) + \beta(1-q)v^r(\tilde{p}^S(p,1)) < \frac{c}{\pi_h - \pi_l}.$$

By the continuity of $\tilde{v}^r(p,1)$ it follows that there must be a solution $\tilde{e} \in (e',1)$ to (13) for the value of p under consideration. If $p < p_m$ the existence of such a solution contradicts the construction of p_m as the minimal value of p for which a solution e to (13) exists, with $r_{zp}(p,e^r(p)) \leq R$, in the region $p \in [\left(\tilde{p}^S\right)^{-1}(p_h,1), p_h]$, since $r_{zp}(p,\tilde{e}) < r_{zp}(p,e') < r' < r(p)$. Alternatively, consider $p \geq p_m$. If $\tilde{e} > e^r(p)$ this contradicts the construction of $e^r(p)$ as the highest solution of (13) at p (see the proof of Claim 3). On the other hand, if $\tilde{e} \leq e^r(p)$, this implies $e' < e^r(p)$, and thus $r' > r_{zp}(p,e') > r_{zp}(p,e^r(p)) = r(p)$, another contradiction.

♦ Now consider the remaining values: $p \in (0, \left(\tilde{p}^S\right)^{-1}(p_h,1))$. We restrict attention to deviations $r' > r_{zp}(p,1)$; this is without loss of generality, since if this were not the case the deviation could never be profitable, regardless of the risky entrepreneurs' effort choice (since no entrepreneur refuses financing). But recall that, in the proof of b-iii., we showed that for $p < \left(\tilde{p}^S\right)^{-1}(p_h,1)$, low effort will be chosen at r' whenever $r' > r(p_h) = r_{zp}(p_h,1)$. This implies then, just as in the argument immediately following (18) above, that the deviation must be unprofitable.

c. Finally, consider the low values of c: $\frac{c}{\pi_h-\pi_l} \leq \frac{(R-1/\pi_h)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$. Note first that, by Assumption 1, $r_{zp}(p,1) \leq R$ for all $p>0$, so $r(p)=r_{zp}(p,1)$ is always admissible. Also, the argument that there are no profitable deviations for lenders is the same as the one in b-iv., for the case $p \geq p_h$. So it only remains to verify that risky entrepreneurs indeed prefer to exert high rather than low effort for all $p>0$.

For high effort to be incentive compatible for all $p>0$, we need to show that

$$\frac{c}{\pi_h-\pi_l} \leq R - r(p) + \beta(1-q)v^r(p^S(p)). \tag{21}$$

Notice that, for any $p>0$, a lower bound for $v^r(p)$ is given by $\frac{\pi_h(R-1/\pi_h)-c}{1-\beta(\pi_h+q(1-\pi_h))}$, which is the present discounted utility for a risky entrepreneur who is financed in every period (until a failure that is not forgotten) at $r=1/\pi_h$ and exerts high effort.[37]

Thus since $p^S(p)>0$ for all $p>0$, we have

$$R - r(p) + \beta(1-q)v^r(p^S(p)) > R - 1/\pi_h + \beta(1-q)\frac{\pi_h(R-1/\pi_h)-c}{1-\beta(\pi_h+(1-\pi_h)q)}.$$

So to verify (21) it suffices to show that

$$R - 1/\pi_h + \beta(1-q)\frac{\pi_h(R-1/\pi_h)-c}{1-\beta(\pi_h+(1-\pi_h)q)} \geq \frac{c}{\pi_h-\pi_l}.$$

But this follows immediately from the definition of region c.[38]■

[37]This follows immediately from the fact that $v^r(p)$ is the present discounted utility under the same circumstances except that the interest rate is $r(p)=r_{zp}(p,1)<1/\pi_h$ for all $p>0$.

[38]Suppose this were not the case, so that $R-1/\pi_h+\beta(1-q)\frac{\pi_h(R-1/\pi_h)-c}{1-\beta(\pi_h+(1-\pi_h)q)} < \frac{c}{\pi_h-\pi_l}$. If we multiply both sides of this inequality by $(\pi_h-\pi_l)(1-\beta(\pi_h+(1-\pi_h)q))$ and then simplify, this becomes $\frac{c}{\pi_h-\pi_l} > \frac{(R-1/\pi_h)(1-\beta q)}{1-\beta(\pi_l+(1\pi_l)q)}$, contradicting the lower bound on c that defines region c.

Proposition 2 — Efficiency of Equilibrium

We begin by showing that the equilibrium constructed in Proposition 1 maximizes $e^r(p)$, the effort exerted by the risky entrepreneurs, for any p; this will play an important role in the proof of the Proposition. This result is intuitive, as the equilibrium of Proposition 1 was constructed recursively, with effort chosen to be maximal at each stage.

Claim 5. *The equilibrium constructed in Proposition 1 maximizes the risky entrepreneurs' effort $e^r(p)$, across all symmetric sequential MPE, when $q \in \{0,1\}$. When $q \in (0,1)$ this result holds as long as $\pi_l \geq \pi_h \frac{q}{1+q}$.*

The following corollary is immediate, since for lenders to break even when $p < p_l$ a higher level of effort is needed than in the equilibrium of Proposition 1, contradicting Claim 5.

Corollary 1. *No MPE can implement financing when $p < p_l$.*

We now turn to demonstrating that the equilibrium of Proposition 1 is the most efficient MPE. The result follows from the above claim, since surplus in any given period will also be higher, given properties (i)-(iii) of the equilibrium of Proposition 1.

From Corollary 1, we can restrict attention to $p_0 \geq p_l$, without loss of generality. Recall that welfare is given by the total surplus accruing from the agents' projects that are financed. Let $\mathcal{W}(p)$ denote the total surplus at the MPE of Proposition 1 accruing from projects of entrepreneurs with credit score p, and let $\overline{\mathcal{W}}(p)$ denote the total surplus at a different MPE. We will show that we always have $\mathcal{W}(p) \geq \overline{\mathcal{W}}(p)$ for $p \geq p_l$.

Observe that when $p = p_0$ there is a measure p_0 of safe entrepreneurs, and $1 - p_0$ of risky entrepreneurs, while when their credit score is $p > p_0$ there is a measure p_0 of safe entrepreneurs, and a measure $\frac{p_0}{p} - p_0$ of risky entrepreneurs, since the safe types never fail. So total surplus can be defined recursively:

$$\mathcal{W}(p) = p_0(R-1) + \left(\frac{p_0}{p} - p_0\right)\left(\pi_{e(p)}R - 1 - ce^r(p)\right) + \beta\mathcal{W}(p^S(p)),$$

where $\pi_{e(p)} \equiv \pi_h e^r(p) + \pi_l(1 - e^r(p))$ is the risky entrepreneurs' success probability given the equilibrium effort level at p, and similarly for $\overline{\mathcal{W}}(p)$.[39]

Observe that $\mathcal{W}(p)$ is strictly decreasing for $p \geq p_h$.[40] It is then immediate to verify that $\mathcal{W}(p) \geq \overline{\mathcal{W}}(p)$ for all $p \geq p_h$. Now consider $p \in [p_m, p_h)$. If $\bar{r}(p) = \emptyset$ (i.e., there is no financing

[39] Analogously, define $\pi_{\bar{e}(p)} \equiv \pi_h \bar{e}^r(p) + \pi_l(1 - \bar{e}^r(p))$.

[40] Since $e^r(p) = 1$ for all $p \geq p_h$, and $\frac{p_0}{p} - p_0$ (the measure of risky entrepreneurs who have not been excluded thus far) is decreasing in p.

at p in the other MPE under consideration), then $\overline{\mathcal{W}}(p) = 0$, and so clearly $\mathcal{W}(p) \geq \overline{\mathcal{W}}(p)$. Alternatively, suppose that $\bar{r}(p) \neq \emptyset$. Then we know from Claim 5 that $e^r(p) \geq \bar{e}^r(p)$, which also implies that $\bar{p}^S(p) \geq p^S(p) \geq p_h$, and thus that $\mathcal{W}(\bar{p}^S(p)) \geq \overline{\mathcal{W}}(\bar{p}^S(p))$. So

$$\begin{aligned}\overline{\mathcal{W}}(p) &= p_0(R-1) + \left(\frac{p_0}{p} - p_0\right)\left(\pi_{\bar{e}(p)}R - 1 - c\bar{e}^r(p)\right) + \beta\overline{\mathcal{W}}(\bar{p}^S(p)) \\ &\leq p_0(R-1) + \left(\frac{p_0}{p} - p_0\right)\left(\pi_{e(p)}R - 1 - ce^r(p)\right) + \beta\mathcal{W}(\bar{p}^S(p)).\end{aligned}$$

If we replace $\mathcal{W}(\bar{p}^S(p))$ with $\mathcal{W}(p^S(p))$ in the righthand side of the inequality this cannot decrease its value, since we showed that $\mathcal{W}(p')$ is decreasing for $p' \geq p_h$ (and $\bar{p}^S(p) \geq p^S(p)$), thus demonstrating that $\overline{\mathcal{W}}(p) \leq \mathcal{W}(p)$ for $p \in [p_m, p_h)$.

We use induction to establish the result for the remaining values of p: $p \in [p_l, p_m)$. Let $p^* = p_m$ and $p^{**} \equiv \left(\tilde{p}^S\right)^{-1}(p^*, 1)$. Recall that we have shown in Claim 5 that either (i) $\bar{r}(p) = \emptyset$, and hence $\overline{\mathcal{W}}^r(p) = 0$; or else (ii) $\bar{e}^r(p) = e^r(p) = 0$, in which case it is immediate that $\overline{\mathcal{W}}(p) = \mathcal{W}(p) + \beta[\overline{\mathcal{W}}(p^S(p)) - \mathcal{W}(p^S(p))]$. Since we have established above that $\mathcal{W}(p') \geq \overline{\mathcal{W}}(p')$ for $p' \geq p^*$, it thus follows that $\mathcal{W}(p) \geq \overline{\mathcal{W}}(p)$. If $p^{**} > p_l$, redefine $p^* \equiv p^{**}$, and $p^{**} \equiv \left(\tilde{p}^S\right)^{-1}(p^*, 1)$, and repeat the same argument as above. ■

Proposition 3 – Optimal Forgetting (regions a. and c.)

Consider case 1. When $\frac{c}{\pi_h - \pi_l} \geq \frac{R-1}{1-\pi_l\beta}$, since $\frac{(R-1)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$ is decreasing in q, the condition defining region a. in Proposition 1 is satisfied for all q. At the MPE there is financing only when $p_0 \geq p_{\text{NF}}$ and risky entrepreneurs never exert high effort, regardless of the value of q.

Hence if $p_0 \geq p_{\text{NF}}$, the total surplus generated in equilibrium by the loans to risky entrepreneurs is $\frac{B}{1-(\pi_l+(1-\pi_l)q)\beta}$, , which is strictly decreasing in q since $B < 0$. Thus $q = 0$ is optimal. If on the other hand $p_0 < p_{\text{NF}}$, such surplus is zero for all q, and so $q = 0$ is also (weakly) optimal.

Consider now case 2. Again notice that $\frac{(R-1/\pi_h)(1-\beta q)}{1-\beta(\pi_l+(1-\pi_l)q)}$ is decreasing in q. Thus when $\frac{c}{\pi_h-\pi_l} < \frac{R-1/\pi_h}{1-\beta\pi_l}$, the condition defining region c. of Proposition 1 is satisfied for all $q \in [0, q^*]$, where $q^* = \frac{(R-1/\pi_h)-\frac{c}{\pi_h-\pi_l}(1-\beta\pi_l)}{\beta\left((R-1/\pi_h)-\frac{c}{\pi_h-\pi_l}(1-\pi_l)\right)} > 0$. Hence at the MPE there is always financing whatever p_0 is, and for all $q \in [0, q^*]$, and risky entrepreneurs always exert high effort. That is, for $q \in [0, q^*]$, the total surplus generated in equilibrium by the loans to risky entrepreneurs is

$$\frac{G}{1-(\pi_h+(1-\pi_h)q)\beta}.$$

Now this is increasing in q since $G > 0$. Thus any $q \in (0, q^*]$ dominates $q = 0$ and the optimal value will be $q(p_0) \geq q^*$.[41]■

Proposition 4 – Optimal Forgetting (region b.)

For case 1 ($p_0 > p_h(0)$) the proof is an immediate corollary of the second case of Proposition 3.

Consider then case 2. Since $p_0 \geq p_{\text{NF}}$, the agents will always be financed at the initial date, irrespective of q. Thus, by the argument given above, it suffices to show that we can increase the surplus generated by the risky entrepreneurs' projects. Letting $\mathcal{W}^r(q, p_0)$ denote the surplus from the risky agents' projects, when the forgetting policy is q and the prior probability of being safe is p_0, we will show that under the conditions stated in the Proposition, we can find some $\bar{q} > 0$ such that $\mathcal{W}^r(\bar{q}, p_0) > \mathcal{W}^r(0, p_0)$.

We proceed as follows. For any $q > 0$ we first find a threshold $\tilde{p}_h(q)$ for $p_h(q)$, relative to $p_h(0)$, such that *if* $p_h(q) < \tilde{p}_h(q)$ then the surplus from risky entrepreneurs' projects is higher at q than at 0. We then show that the parameter restrictions stated in the Proposition ensure the existence of $\bar{q} > 0$ such that $p_h(\bar{q}) \leq \tilde{p}_h(q)$.

Let $n(q, p_0)$ denote the number of successes (or forgotten failures), starting from the prior p_0, until the risky entrepreneurs first exert high effort, when the forgetting policy is q. Then the following upper and lower bounds for the surplus generated by lending to risky entrepreneurs can be shown to hold:[42]

$$\mathcal{W}^r(0, p_0) \leq \frac{B(1 - (\pi_l \beta)^{n(0,p_0)-1})}{1 - \pi_l \beta} + \frac{G(\pi_l \beta)^{n(0,p_0)-1}}{1 - \pi_h \beta} \tag{22}$$

and

$$\mathcal{W}^r(q, p_0) \geq \frac{B(1 - ((\pi_l + (1 - \pi_l)q)\beta)^{n(q,p_0)})}{1 - (\pi_l + (1 - \pi_l)q)\beta} + \frac{G(\pi_l + (1 - \pi_l)q)\beta)^{n(q,p_0)}}{1 - (\pi_h + (1 - \pi_h)q)\beta}. \tag{23}$$

[41] The optimal value of q could be higher than q^*, which would push us out of region c., into region b.

[42] When there is no mixing in equilibrium (i.e. $p_m(q) = p_h(q)$), $\mathcal{W}^r$ is simply equal to the discounted expect surplus generated by consecutive successes of the project (the first $n(q, p_0)$ of which with low effort, the remainder with high effort):

$$\mathcal{W}^r(q, p_0) = \frac{B(1 - ((\pi_l + (1 - \pi_l)q)\beta)^{n(q,p_0)})}{1 - (\pi_l + (1 - \pi_l)q)\beta} + \frac{G(\pi_l + (1 - \pi_l)q)\beta)^{n(q,p_0)}}{1 - (\pi_h + (1 - \pi_h)q)\beta}.$$

With mixing in equilibrium, the exact expression of $\mathcal{W}^r$ depends on the equilibrium level of effort exerted in the mixing region. However, since there can be at most only a single period of mixing in equilibrium, an upper and lower bound for such utility is given by (22) and (23), independent of the mixing probability.

So to show that $\mathcal{W}^r(q,p_0) > \mathcal{W}^r(0,p_0)$, it suffices to show that we can find $q>0$ such that

$$\frac{B(1-(\pi_l\beta)^{n(0,p_0)-1})}{1-\pi_l\beta}+\frac{G(\pi_l\beta)^{n(0,p_0)-1}}{1-\pi_h\beta} < \frac{B(1-((\pi_l+(1-\pi_l)q)\beta)^{n(q,p_0)})}{1-(\pi_l+(1-\pi_l)q)\beta}+\frac{G(\pi_l+(1-\pi_l)q)\beta)^{n(q,p_0)}}{1-(\pi_h+(1-\pi_h)q)\beta}.$$

Letting $\beta\to 1$ and simplifying, the above expression reduces to:

$$\frac{\frac{B}{G}}{1-\pi_l}+\frac{\pi_l^{n(0,p_0)-1}}{(1-\pi_l)(1-\pi_h)}\left[(1-\pi_l)-\frac{B}{G}(1-\pi_h)\right]$$
$$< \frac{\frac{B}{G}}{(1-\pi_l)(1-q)}+\frac{(\pi_l+(1-\pi_l)q)^{n(q,p_0)}}{(1-q)(1-\pi_l)(1-\pi_h)}\left[(1-\pi_l)-\frac{B}{G}(1-\pi_h)\right],$$

since $1-(\pi_l+(1-\pi_l)q)=(1-\pi_l)(1-q)$ and $1-(\pi_h+(1-\pi_h)q)=(1-\pi_h)(1-q)$, or, equivalently, to

$$\pi_l^{n(0,p_0)-1}(1-q)-\frac{\frac{B}{G}(1-\pi_h)q}{(1-\pi_l)-\frac{B}{G}(1-\pi_h)} < (\pi_l+(1-\pi_l)q)^{n(q,p_0)} \tag{24}$$

It will be useful to rewrite (24) in terms of a condition on $p_h(q)$ and $p_h(0)$. To this end, notice that $p_h(q)$ and $n(q,p_0)$ are related by the following expression: $n(q,p_0)$ is the smallest integer for which[43]

$$\frac{p_0}{p_0+(1-p_0)[\pi_l+(1-\pi_l)q]^{n(q,p_0)}} \geq p_h(q), \tag{25}$$

so that $\pi_l^{n(0,p_0)} \leq \frac{p_0}{1-p_0}\left(\frac{1}{p_h(0)}-1\right)$ and $(\pi_l+(1-\pi_l)q)^{n(q,p_0)-1} \geq \frac{p_0}{1-p_0}\left(\frac{1}{p_h(q)}-1\right)$. Thus to satisfy (24) it suffices to show that:

$$\frac{1}{\pi_l}\frac{p_0}{p_h(0)}\left(\frac{1-p_h(0)}{1-p_0}\right)(1-q)-\frac{\frac{B}{G}(1-\pi_h)q}{(1-\pi_l)-\frac{B}{G}(1-\pi_h)} < (\pi_l+(1-\pi_l)q)\frac{p_0}{p_h(q)}\left(\frac{1-p_h(q)}{1-p_0}\right).$$

Simplifying, we obtain the following sufficient condition for q to implement a welfare im-

[43] When there is no mixing in equilibrium, i.e., $p_m(q)=p_h(q)$, the validity of this expression follows immediately from the definition of $p_h(q)$ and $n(q,p_0)$. The fact that it also holds with mixing can be seen by noticing that in such case the probability of success is greater or equal than when low effort is exerted, and so the posterior is $\tilde{p}^S(p,e^r(p)) \leq \tilde{p}^S(p,0)$. Hence $n(q,p_0)$ will be greater or equal than the term satisfying (25). But $n(q,p_0)$ cannot be strictly greater, as this would imply that we mix for more than a single period, which we have shown (in the proof of Proposition 1) cannot happen.

provement as $\beta \to 1$:

$$p_h(q) < \tilde{p}_h(q) \equiv \frac{p_0(\pi_l + (1-\pi_l)q)}{p_0(\pi_l + (1-\pi_l)q) + (1-p_0)\left[\frac{1}{\pi_l}\frac{p_0}{p_h(0)}\left(\frac{1-p_h(0)}{1-p_0}\right)(1-q) - \frac{\frac{B}{G}(1-\pi_h)q}{(1-\pi_l)-\frac{B}{G}(1-\pi_h)}\right]}. \tag{26}$$

We now show that the condition on B/G stated in the Proposition ensures that we can find $\bar{q} > 0$ such that $p_h(\bar{q})$ satisfies (26) and so we can achieve a welfare improvement. We begin by providing a convenient upper bound for the level of $p_h(q)$.

For intermediate values of c, lying in the region where type b. equilibria obtain when $q = 0$, $p_h(0)$ belongs to $(0,1)$ and satisfies equation (12) above. It is then easy to see from the definition of this region in Proposition 1 that, when β is sufficiently close to 1, c will remain in the same region for any $q > 0$.[44] So for β close to 1, $p_h(q)$ also lies in $(0,1)$ and satisfies an expression analogous to (12):

$$\frac{c}{\pi_h - \pi_l} = R - r_{zp}(p_h(q),1) + \beta(1-q)\tilde{v}^r(\tilde{p}^S(p_h(q),1),1;q), \tag{27}$$

where, similarly to (12), $\tilde{v}^r(p,1;q)$ denotes the discounted expected utility of a risky entrepreneur with credit score p, when he exerts high effort for all $p' > p$ and the contracts offered are $r_{zp}(p,1)$, highlighting the dependence of the utility on the forgetting policy q. From (27) and (12) we obtain then:

$$-r_{zp}(p_h(0),1) + \beta\tilde{v}^r(\tilde{p}^S(p_h(0),1),1;0) = -r_{zp}(p_h(q),1) + \beta(1-q)\tilde{v}^r(\tilde{p}^S(p_h(q),1),1;q). \tag{28}$$

By a similar argument to that in the proof of parts a. and c. of Proposition 1, a (strict) upper bound for $\tilde{v}^r(\tilde{p}^S_h(p_h(0),1),1;0)$ is given by the utility of being financed in every period at the constant rate $r = 1$ until a failure occurs, while exerting high effort, i.e., by $\frac{\pi_h(R-1)-c}{1-\beta\pi_h}$. Conversely, when the forgetting policy is q, a (strict) lower bound for $\tilde{v}^r(\tilde{p}^S(p_h(q),1),1;q)$ is given by $\frac{\pi_h(R-r_{zp}(p_h(q),1))-c}{1-\beta(\pi_h+(1-\pi_h)q)}$, that is, the utility of a risky agent when financed at the constant rate $r_{zp}(p_h(q),1)$ until he experiences a failure that is not forgotten, still exerting high effort. Together with (28) this implies that:

$$-r_{zp}(p_h(0),1) + \beta\frac{\pi_h(R-1)-c}{1-\beta\pi_h} > -r_{zp}(p_h(q),1) + \beta(1-q)\frac{\pi_h(R-r_{zp}(p_h(q),1))-c}{1-\beta(\pi_h+(1-\pi_h)q)}.$$

[44]For β close to 1, the boundaries of the region are approximately equal to $\frac{(R-1/\pi_h)}{1-\pi_l}$ and $\frac{(R-1)}{1-\pi_l}$, both independent of q.

When $\beta \to 1$, the above inequality becomes

$$-r_{zp}(p_h(0),1) + \frac{\pi_h(R-1)-c}{1-\pi_h} > -r_{zp}(p_h(q),1) + \frac{\pi_h(R-r_{zp}(p_h(q),1))-c}{1-\pi_h},$$

or, simplifying,

$$r_{zp}(p_h(q),1) > (1-\pi_h)r_{zp}(p_h(0),1) + \pi_h.$$

Using the definition of $r_{zp}(\cdot,\cdot)$ in (7), the previous expression can be rewritten as follows:

$$\frac{1}{p_h(q)+(1-p_h(q))\pi_h} > (1-\pi_h)\frac{1}{p_h(0)+(1-p_h(0))\pi_h} + \pi_h,$$

or

$$\begin{aligned} p_h(0)+(1-p_h(0))\pi_h &> (1-\pi_h)[p_h(q)+(1-p_h(q))\pi_h] + \pi_h[p_h(q)+(1-p_h(q))\pi_h][p_h(0)+(1-p_h(0))\pi_h] \\ &= [p_h(q)+(1-p_h(q))\pi_h]\,[1-\pi_h(1-\pi_h)(1-p_h(0))]\,, \end{aligned} \tag{29}$$

which is in turn equivalent to:

$$p_h(0)(1-\pi_h)+\pi_h > [p_h(q)(1-\pi_h)+\pi_h]\,[1-\pi_h(1-\pi_h)(1-p_h(0))]\,,$$

i.e.,

$$\frac{p_h(0)(1-\pi_h)+\pi_h}{[1-\pi_h(1-\pi_h)(1-p_h(0))]} > [p_h(q)(1-\pi_h)+\pi_h].$$

The above inequality implies that when β is close to 1 the following upper bound on the level of $p_h(q)$ must hold, for all q:

$$p_h(q) < \bar{p}_h \equiv \frac{p_h(0)(1-\pi_h^2)+\pi_h^2}{[1-\pi_h(1-\pi_h)(1-p_h(0))]}. \tag{30}$$

Finally, note that for q close to 1, $\tilde{p}_h(q)$ is approximately equal to $p_0\frac{(1-\pi_l)-\frac{B}{G}(1-\pi_h)}{p_0(1-\pi_l)-\frac{B}{G}(1-\pi_h)}$. Hence, under the condition on B/G stated in the Proposition we have that

$$\frac{p_h(0)(1-\pi_h^2)+\pi_h^2}{[1-\pi_h(1-\pi_h)(1-p_h(0))]} < p_0\frac{(1-\pi_l)-\frac{B}{G}(1-\pi_h)}{p_0(1-\pi_l)-\frac{B}{G}(1-\pi_h)},$$

or equivalently that, for q close to 1 we have $\bar{p}_h < \tilde{p}_h(q)$.

Thus on the basis of the previous discussion we can conclude that there exists $\bar{q}$ yielding a welfare improvement over $q = 0$. ■

Proposition 5 — $q = 1$ optimal

Since $p^S(p) = p$ when $q = 1$, $q = 1$ is optimal if and only if $p_0 > p_h(1)$ and $p_h(1) < 1$. To compute $p_h(1)$, note that for $q = 1$ the continuation utility drops out of (12) and so we have $\frac{c}{\pi_h - \pi_l} = R - r(p_h(1))$. Substituting $r(p_h(1)) = r_{zp}(p_h(1), 1) = \frac{1}{p_h(1)+(1-p_h(1)\pi_h}$, we obtain

$$p_h(1) = \frac{1 - \pi_h(R - \frac{c}{\pi_h - \pi_l})}{(1 - \pi_h)(R - \frac{c}{\pi_h - \pi_l})}.$$

So $p_h(1) < 1$ if and only if $\frac{c}{\pi_h - \pi_l} < (R - 1)$.■

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