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MACROECONOMIC VOLATILITY AND THE  
EQUITY PREMIUM**

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December 28, 2005

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# Macroeconomic Volatility and the Equity Premium

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\*Federal Reserve Bank of Philadelphia, Ten Independence Mall, Philadelphia, PA 19106. The views expressed in this article are those of the author and not necessarily those of the Federal Reserve Bank of Philadelphia or of the Federal Reserve System. This paper is available free of charge at [www.philadelphiafed.org/econ/wps/index.html](http://www.philadelphiafed.org/econ/wps/index.html)

### **Abstract**

Recent empirical work documents a decline in the U.S. equity premium and a decline in the standard deviation of real output growth. We investigate the link between aggregate risk and the asset returns in a dynamic production based asset-pricing model. When calibrated to match asset return moments, the model implies that the post-1984 reduction in TFP shock volatility of 60 percent gives rise to a 40 percent decline in the equity premium. Lower macroeconomic risk post-1984 can account for a substantial fraction of the decline in the equity premium.

# 1 Introduction

Has increased U.S. macroeconomic stability contributed to a decline in the equity premium? Since the early 1980s, the standard deviation of U.S. real GDP has been about half of what it was in the previous 40 years, suggesting a decline in macroeconomic risk. A number of recent empirical studies have also documented a significant decline in the the U.S. equity premium over the last three decades. While this decline might in part be due to a general reduction in financial market imperfections, it is plausible that some portion of the decline can be attributed to a reduction in macroeconomic volatility. This paper uses a calibrated DSGE model to quantify the relationship between macroeconomic risk and asset returns. Simulations from the model suggest that a decline in aggregate risk on the order of what has been observed in the postwar U.S. economy would lead, *ceteris paribus*, to about a 40 percent reduction in the annualized equity premium.

That the volatility of the aggregate economy has fallen since the early 1980s is well-documented (McConnell and Perez-Quiros (2000), Kim and Nelson (1999), Kim et al. (2001), Stock and Watson (2003)). As reported in Stock and Watson (2003), the volatility decline is manifested in a broad array of macroeconomic variables, including output, consumption, and investment. The volatility decline seems to be best characterized as a break point, rather than a long-term trend decline. While the causes of the increased stability remain under investigation, several studies suggest that "good luck," in the form of smaller shocks to total factor productivity, account for a substantial fraction of the drop in the standard deviation of real GDP (Leduc and Sill (2004a), Arias et al. (2004)). We investigate the implications of this decline in aggregate risk for asset returns.

The decline in the equity premium is somewhat more difficult to verify. As noted in Jagannathan et al. (2000), calculating the equity premium using historical average differences between stocks and bonds may not give good estimates in times when the premium is declining because that calculation does not capture the price change that would accompany an unexpected decline in the equity premium. Jagannathan et al. (2000) use an equity valuation model based on Gordon (1962) to calculate changes in the equity premium over time. Their analysis suggests the equity premium for the U.S. economy averaged 6.8 percentage points over the period 1926-1970 and about 1.5 percentage points, on average, over the 1980s and 1990s. Several other studies, including Blanchard (1993), Wadhvani (1999), and Siegel (1998), also find evidence for a drop in the

equity premium.<sup>1</sup> The studies do not, however, analyze reasons for the decline in the equity premium.

Though the equity premium appears to have begun declining in the 1970s, some 10 years or so prior to the sharp drop in the economy's aggregate volatility, that does not preclude a causal effect from volatility to the equity premium. Indeed, there is evidence for a significant decline in the equity premium post-1980. Jagannathan et al. (2000) calculate an equity premium of 3.3 percent for the 1970s, 0.94 percent for the 1980s, and 2.5 percent for the 1990s. While declining macroeconomic risk may not be the only factor behind a drop in the equity premium, it may well be a contributing factor.

The approach of this paper is to investigate the implications of a standard RBC-asset-pricing model for the effect on asset returns of reductions in aggregate risk. A model is calibrated to approximately match the pre-1980 equity premium and the pre-1984 volatility of total factor productivity shocks. The model has a general process for exogenous TFP that allows for stochastic variance. Consequently, it allows a numerical investigation of links between the persistence of exogenous changes in TFP volatility and asset returns. In addition, we examine stochastic steady-state behavior. That is, we quantify the relationship between levels of TFP shock volatility and asset mean returns and variances in an environment in which the variance of TFP shocks is not expected to change over time. The model suggests that a decline in TFP shock volatility on the order of what has been seen in the postwar data would be accompanied by a drop in the annualized equity premium from a little over 5 percent to about 2 percent.

The model is able to approximately match the mean return on equities, the risk-free rate, and the equity premium. A major failing of the model is that it introduces substantial volatility in asset returns, particularly the risk-free rate – much more so than what is observed in the data. The model implies a near linear relationship between the volatility of the TFP shock and the volatility of asset returns (though the slope is steeper for equities than for the risk-free rate); it remains the case that even for the near halving of TFP shock volatility since 1984, predicted volatility of asset returns remains too high.

A paper closely related to this one is Lettau et al. (2004) who investigate how a decline in aggregate consumption risk affects the equity premium. Their model incorporate a two-state Markov switching process for consumption in

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<sup>1</sup>See Jagannathan et al. (2000) for a review of this literature.

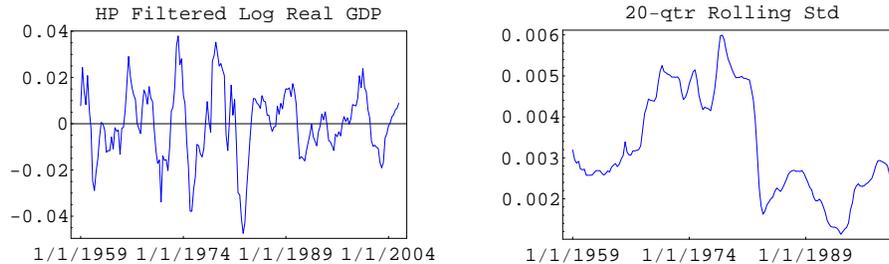
a consumption-based asset pricing model with Epstein-Zin preferences. That analysis suggests a robust correlation between low aggregate volatility and high asset prices, and the estimated probability of being in a low volatility state accounts for 30 to 60 percent of the log dividend-price ratio.

The plan of the paper is as follows. The next section lays out some facts about the increased stability of the U.S. economy since 1984. We then discuss the model and its calibration. Simulation results follow and a final section concludes.

## 2 Postwar Volatility Patterns

A substantial body of empirical evidence indicates that the U.S. economy has become more stable since 1984.<sup>2</sup> Figure 1 plots hp-filtered log real GDP at a quarterly frequency over the period 1947Q1 to 2005Q3. Prior to 1984, real GDP moved in a band of about  $\pm 4$  percent. Since 1984, the range of movement is more on the order of  $\pm 2$  percent. This apparent halving of volatility is visually reinforced by a plot of the rolling 20-quarter standard deviation of hp-filtered real GDP.

Figure 1: Postwar Real GDP



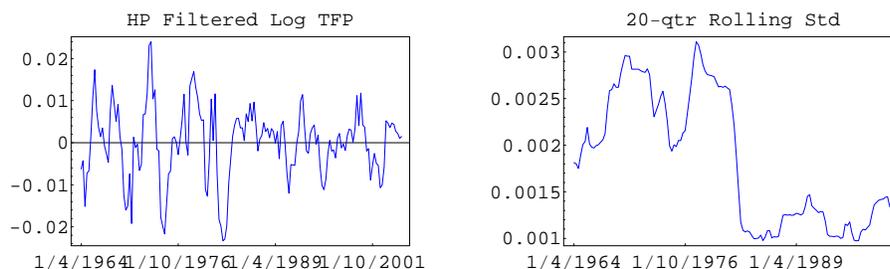
The sharpness of the decline in volatility is striking. Most research on the decline concludes that the drop is best characterized as a one-time break, though Blanchard and Simon (2001) argue that the decline can be thought of a gradual process – one that was slow and steady, interrupted by a temporary volatility

<sup>2</sup>See Kim and Nelson (1999), McConnell and Perez-Quiros (2000), Stock and Watson (2003), and Blanchard and Simon (2001).

increase during the 1970s. What is clear from the studies though is that aggregate volatility is generally lower over the post-1980 sample than in the pre-1980 postwar sample.

Potential causes of the drop in aggregate volatility remain an active area of research.<sup>3</sup> However, standard business-cycle models suggest that lower TFP volatility accounts for a substantial portion of the real output volatility drop, more so than say changes in monetary or fiscal policy (see Leduc and Sill (2004b) and Arias et al. (2004)). Figure 2 plots HP-filtered TFP and its rolling 20-quarter-ahead standard deviation.<sup>4</sup>

Figure 2: Postwar TFP



The plots indicate that real GDP volatility and TFP volatility are closely related, which suggests that there has been relatively less change in the process describing the dynamics of aggregate hours during the postwar period.

The data on postwar real asset returns shows a less clear-cut pattern. Figure 3 plots the realized annual equity premium calculated using the Ibbotson (2003) data on large company stock returns and Treasury bills. The large company return series is based on the S&P500 composite index. The Treasury bill series is derived from a one-bill portfolio containing the shortest term bill with not less than one month to maturity. Inflation is measured using the consumer price index, not seasonally adjusted. The figure suggests a potential decline in the mean and standard deviation of the equity premium since the 1970s.

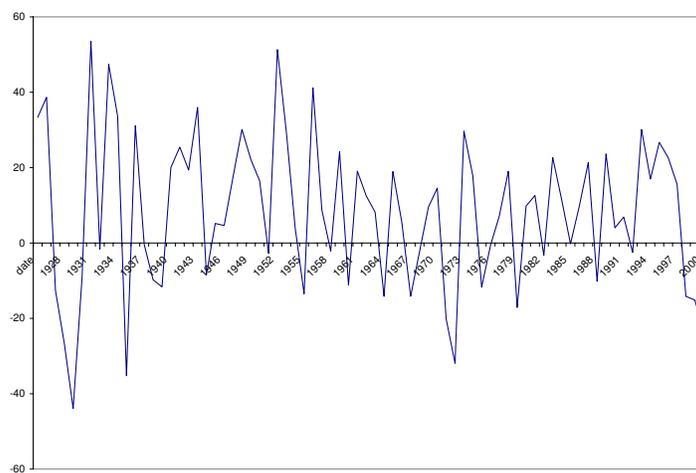
Figure 4 plots the real returns on equities and the Treasury bill series from the Ibbotson dataset. Equity returns appear less volatile post-1970. Over the

<sup>3</sup>See Stock and Watson (2003) for a discussion of the literature

<sup>4</sup>The TFP series calculation follows Cooley and Prescott (1995). TFP growth is calculated as as the quarterly change in log real GDP less 0.64 times the quarterly change in log aggregate hours. The level was calculated as the sum of quarterly changes.

postwar period, the real bill series is marked by a dramatic runup at the end of the 1970s. On average, the post-1980 level of the real riskless rate appears slightly higher than what it was in the 1950s and 1960s. From this graphical evidence though, it is difficult to conclude that there has been a dramatic decline in the average equity premium since the 1970s.

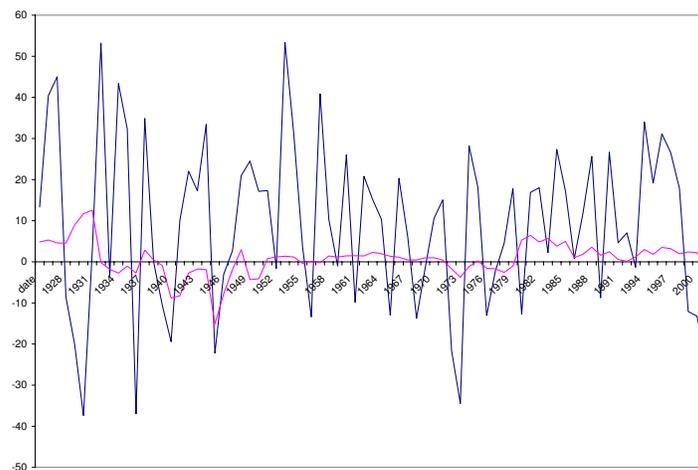
Figure 3: Historical Equity Premium



Instead, our primary evidence on the decline in the equity premium comes from Jagannathan et al. (2000). As mentioned above, their study uses a valuation model based on Gordon (1962) that calculates the equity premium as a function of the bond yield, the stock dividend yield, and the expected growth rate in dividends. The model is applied to several alternative measures of the aggregate U.S. stock portfolio and several alternative assumptions about stock dividends and bond yields. Table 1 reproduces a subset of the numbers from Table 4 of their paper.

All of their alternative formulations suggest a significant decline in the equity premium over the last three decades. Over a longer time span, the authors calculate an equity premium of slightly less than 6 percent over 1926-1999, and about 4.5 percent over 1946-1999. Note that they measure the equity premium

Figure 4: Risky and Riskless Returns



as the difference between the stock yield and long-term bond yield, a measure that is different from the expected return on equities in excess of the short-term risk-free rate, as in Mehra and Prescott (1985).

Table 1: Jagannathan et al. (2000) U.S. Equity Premium

Period	S&P <sup>1</sup>	CRSP <sup>2</sup>	BOG <sup>3</sup>
1950-1959	8.93	8.73	8.69
1960-1969	5.23	5.14	5.05
1970-1979	3.30	3.16	3.30
1980-1989	0.94	0.71	1.67
1990-1999	2.51	1.31	2.50

<sup>1</sup> Standard & Poors Composite Index.

<sup>2</sup> Value-weighted market index from Center for Research in Security Prices.

<sup>3</sup> Federal Reserve Board of Governors stocks held by U.S. residents.

### 3 Model

We modify a standard real business cycle model along the lines suggested in Boldrin et al. (2001) and Jermann (1998) to give it a chance at matching the equity premium without resorting to extreme risk aversion in preferences. These modifications amount to adding habit persistence in consumption and frictions in the adjustment of capital and labor to shocks. We examine two models, one with consumption and leisure in preferences and one with consumption only. The introduction of variable labor supply is problematic for the model's ability to generate an equity premium. This happens because variable labor affords households another channel by which to smooth consumption. As a consequence, there is less variability in the capital stock and not much variation in equity prices. Uhlig (2004) highlights how labor market frictions of some sort need to be introduced if the model is to have chance in matching the equity premium. We follow Boldrin et al. (2001) and require that households choose hours worked prior to the realization of the current-period productivity shock.

In addition to habits in preferences, and labor market frictions, the model includes a capital adjustment cost, which induces variability in the capital gains component of the return to equity. This basic model has been demonstrated somewhat successful in matching the equity premium and business cycle moments for the U.S. economy over the postwar period.<sup>5</sup> In order to more fully analyze the relationship between aggregate stability and asset returns, we modify the model slightly by introducing an exogenous, time-varying TFP volatility component.

Since the basic models are analyzed in Jermann (1998) and Boldrin et al. (2000), we describe a basic version only briefly here. A representative household has preferences that are separable over consumption ( $c_t$ ) and hours worked ( $h_t$ ):

$$E_t \sum_{t=0}^{\infty} \beta^t [\ln(c_t - bc_{t-1}) - \phi \ln(1 - h_t)]$$

In order to match the equity premium, it is important to introduce habit persistent in preferences, though this by itself is not sufficient to generate an equity premium in the standard RBC framework. Essentially, this is because the flexibility of hours worked and the linear capital accumulation process allow households to smooth consumption very effectively and dramatically reduce the variability of the return on equity (see Boldrin et al. (2000) for a full discus-

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<sup>5</sup>Note that this version of the model is discussed in Boldrin et al. (2000).

sion). In order to mitigate the hours worked channel on consumption smoothing, we assume that current-period hours are chosen prior to the realization of the current-period technology shock.

To generate enough variability in capital gains, we assume that capital adjustment is subject to a cost. Following Jermann (1998), the capital stock evolves according to:

$$k_{t+1} = (1 - \delta) k_t + G\left(\frac{x_t}{k_t}\right) k_t \quad (1)$$

The function  $G(\cdot)$  is a convex adjustment cost function. We follow convention in parameterizing this function so that  $G'(\delta) = 1$  and  $G(\delta) = \delta$ . Consequently, in steady state, there is no cost of adjusting the capital stock. The adjustment cost function is parameterized as:

$$G\left(\frac{x_t}{k_t}\right) = \frac{\omega}{v} \left(\frac{x_t}{k_t}\right)^v + \kappa \quad (2)$$

Our assumptions on  $G$  then imply  $\omega = \delta^{1-v}$  and  $\kappa = \frac{v-1}{v} \delta$ . The capital adjustment cost is added to the model in order to induce additional variation in the price of capital, and hence equity returns. With this specification, it is easily shown that the return on equity (assuming no leverage) is given by:

$$R_{t+1}^e = G'\left(\frac{x_t}{k_t}\right) \left( \alpha e^{z_{t+1}} k_{t+1}^{\alpha-1} h_{t+1}^{1-\alpha} - \frac{x_{t+1}}{k_{t+1}} + \frac{(1 - \delta) + G\left(\frac{x_{t+1}}{k_{t+1}}\right)}{G'\left(\frac{x_{t+1}}{k_{t+1}}\right)} \right)$$

Note that when  $v = 1$ , this expression reduces to the familiar one-period return to capital:  $mpk_{t+1} + (1 - \delta)$ , where  $mpk_t$  is the marginal product of capital at time  $t$ .

The economy's resource constraint is given by:

$$c_t + x_t \leq e^{z_t} k_t^\alpha h_t^{1-\alpha} \quad (3)$$

where  $z_t$  is an exogenous technology shock. Stochastic volatility is introduced by assuming the technology shock follows the process:

$$z_t = \rho_z z_{t-1} + e^{s_t} \varepsilon_t^z \quad (4)$$

$$s_t = (1 - \rho_s) \bar{s} + \rho_s s_{t-1} + \eta \varepsilon_t^s \quad (5)$$

Thus, we allow the variance of the exogenous TFP shock to vary over time with persistence governed by the parameter  $\rho_s$ . Note that in this specification, a volatility shock does not directly affect the level of  $z_t$  but does so only indirectly by scaling the effect of  $\varepsilon_t^z$ . That is, impulse responses to a shock to  $\varepsilon_t^s$ , holding  $\varepsilon_t^z$  constant, will be flat for all model variables.

## 4 Calibration and Solution

We begin by calibrating a baseline version of the model to see how well it does in matching asset-return moments. As we will see below, this version of the model generates a somewhat low return on equity. However, it is initially investigated because it represents a commonly used calibration of the model’s parameters.

The baseline model sets the average volatility of the TFP shock to 0.0072, a common number in the RBC literature. We discuss how  $\rho_s$  is calibrated below. For the remainder of the model parameters, the calibrated values are standard for this class of models and are reported in Table 2.

Table 2: Baseline Parameterization

$\beta$	$\tau$	$\delta$	$\alpha$	$\theta$	$v$	$\rho_z$	$\sigma_z$
0.9945	0.90	0.025	0.36	2.18	-3.478	0.9219	0.0072

The choice of the habit parameter  $\tau$  is the same as that estimated in Boldrin et al. (2001) and somewhat higher than the value of 0.83 that Jermann (1998) needed to match the equity premium. A higher value of  $\tau$  is needed in models with variable hours because variation in hours worked are used to smooth consumption. The capital adjustment cost parameter  $v$  is the same as that used in Jermann (1998), Boldrin et al. (2001), and Uhlig (2004).

The model is solved using a second-order perturbation method. That is, a second-order Taylor series approximation is taken around the nonstochastic steady state. The solution method is described in Swanson et al. (2005) and is implemented using the perturbationAIM *Mathematica* code available on Eric Swanson’s website: <http://www.ericswanson.us/>. The code allows for higher-order approximations than 2nd, but we have not yet studied the implications of 2nd order versus higher order solutions for the findings of this paper.

## 4.1 TFP Process

We take two approaches in assessing the consequences for changes in TFP shock volatility in the model. The first approach is to split the postwar sample into pre-1983 and post-1983 subsamples and calculate the standard deviation of TFP shocks in each subperiod. The model is then solved and simulated under each standard deviation and asset return moments are calculated. This experiment is one for which households were completely surprised by the increased stability of the economy and do not expect a return to a higher volatility regime. This approach provides no information about the transition from high volatility episodes to low volatility episodes.

The second approach attempts to get at transition dynamics in a modest way. We assume that TFP shock volatility follows an AR(1) process. The empirical model is estimated using postwar data and the economic model is solved and simulated using the estimated parameters.

To begin, a series for log TFP was generated using data on real GDP and aggregate hours worked. This series was then detrended with a linear time trend. An AR(1) process was estimated on the residuals from that regression to get an estimate of the technology persistence parameter  $\rho_z = 0.9219$ . The calculated standard deviation of the residual from this AR(1) regression is 0.0075 over the period 1964:1-1983:4, and 0.0045 over 1984:1-2005:2. These estimates of the standard deviation and of  $\rho_z$  are used in the steady-state analysis.

Under the stochastic volatility specification, the residual for the TFP process residual is assumed to follow:

$$\begin{aligned}u_t &= z_t - 0.9219z_{t-1} \\u_t &= \bar{\sigma} \exp(0.5 * s_t)\epsilon_t \\s_t &= \rho_s s_{t-1} + \eta \epsilon_t^s \\ \epsilon_t &\sim N(0, 1) \\ \epsilon_t^s &\sim N(0, 1) \\ E\epsilon_t \epsilon_t^s &= 0\end{aligned}$$

There is a large extant literature on estimating the Gaussian ARSV model (see Broto and Ruiz (2004) for a survey). We used the *ssfpack* routines written for the *Ox* programming language to estimate the model using maximum

likelihood.<sup>6</sup>

The stochastic volatility estimation program actually estimates transformations of the model variables in order to guarantee that certain desirable properties hold. Table 3 gives the estimated parameters transformed to the model- equivalent parameters, the underlying parameter estimates themselves, and their standard errors. The estimated value of  $\rho_s$ , which governs the persistence of the  $s_t$  process is fairly high. This is expected since, as already discussed, there is substantial evidence for a drop in aggregate volatility post-1984 and such a shift will show up as a high degree of persistence in volatility process specification that is stationary. Note that all of the model parameters are estimated with a fair degree of precision.

Table 3: Stochastic Volatility Estimates

	$\rho_s$	$\bar{\sigma}$	$\eta$
model variable equiv	0.84	0.0056	0.4904
underlying parameter	1.635	-5.1573	-.7126
standard deviation	0.709	0.2968	0.1319

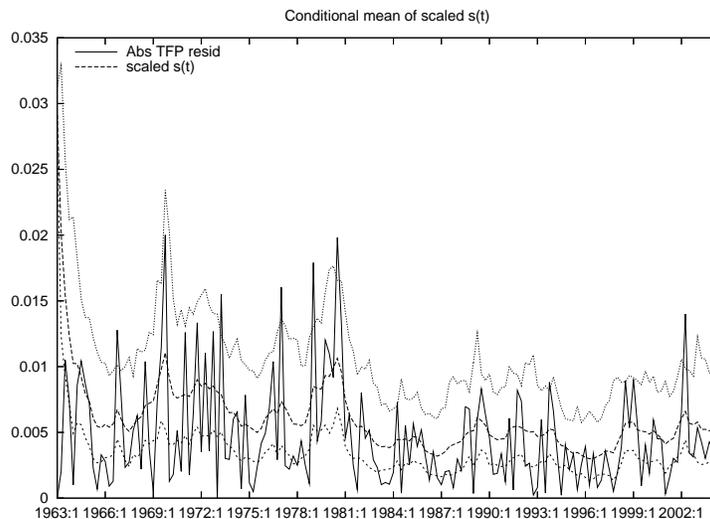
Figure 5 plots the conditional mean of the scaled latent volatility process, two-standard error confidence bands around the mean estimate, and the absolute value of the TFP residual series. The estimated mean is a fairly slow-moving process and is clearly higher, on average, in pre-1984 data than the post-1984 data. The estimated conditional mean drops from about 0.01 in late 1980 to about 0.005 in 1984.

Note that the estimated volatility mean shows a strong decline beginning in 1980Q4 and extending, roughly, through 1984. This timing is somewhat earlier than what is suggested by the break- point analysis conducted in the U.S. volatility literature. We will discuss the implications of volatility shock persistence on the equity premium below. In general though the evidence from this analysis suggests that, for the U.S. economy, high-frequency variation in TFP shock volatility is unlikely to be of first-order importance in determining the relationship between volatility changes and asset returns.

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<sup>6</sup>The GARSV estimation program was written by Siem Jan Koopman, and is available at <http://staff.feweb.vu.nl/koopman/sv>.

Figure 5: Stochastic Volatility

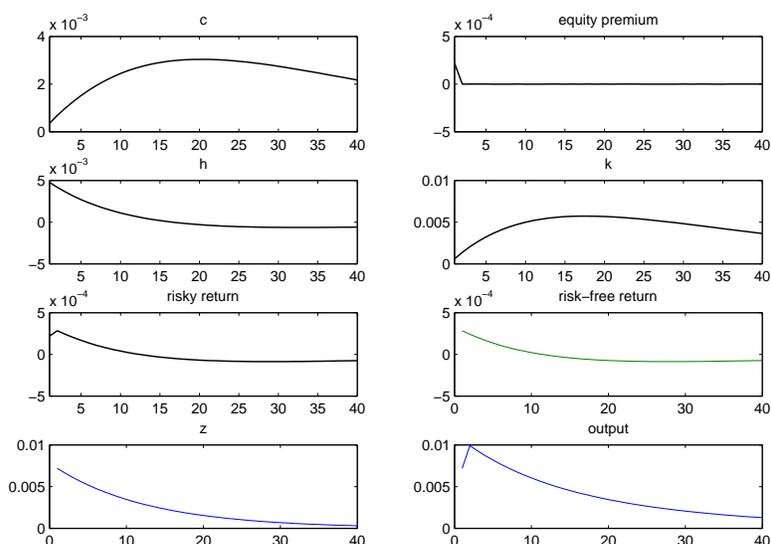


## 5 Impulse Responses to a Technology Shock

To get insight into the workings of the basic model, we start by analyzing a set of impulse responses to a technology shock in the model with variable hours. We first look at how the economy responds to a temporary one-standard-deviation shock to TFP assuming no cost of adjusting the capital stock (but with habit persistence and pre-determined hours worked). The impulse responses for this version of the model, under the baseline calibration in Table 2 are shown in Figure 6.

The model has a hump-shaped response of consumption to the positive technology shock. Hours worked rise in response to greater productive opportunities today as substitution effects dominate income effects. Output jumps, but not as much as in the second period after the shock because households are not allowed to adjust hours worked immediately in response to the shock. The risk-free rate initially rises, and then declines as the growth rate of consumption moderates. Note that equity returns rise, but by only slightly more than the rise in the risk-free rate. Indeed, the ex post equity premium rises only 0.025 percentage points in response to the TFP shock. Clearly, a specification that does not include a mechanism to induce variability in the return to capital will not be able to generate a sizeable equity premium. In this model, the supply curve for capital

Figure 6: Responses to Positive Technology, No Capital Adjustment Cost



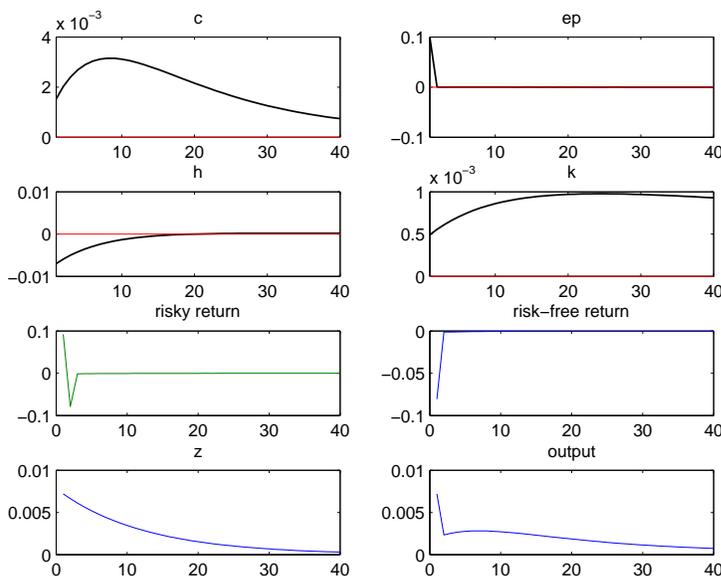
is perfectly elastic. As a result, the price of capital is constant and there is no variation in capital gains. The marginal product of capital is the only source of variation in equity returns, and under standard calibrations, variation in the marginal product of capital are small.

Adding capital adjustment costs makes a striking difference to the magnitude of asset return responses to a technology shock. The impulse responses when capital is costly to adjust are reported in Figure 7.

Note first that with capital adjustment costs, hours worked now respond negatively to the exogenous increase in productivity.<sup>7</sup> This happens because the adjustment cost acts like a tax on labor income. As noted in Boldrin et al. (2001) this tax causes the income effect of the technology shock to dominate the substitution effect – leading to a drop in labor supply. Note as well that now the risk-free rate responds negatively to the positive technology shock, and that equity returns show a strong positive response. Consequently, there is now a strong positive response of the equity premium to a positive technology

<sup>7</sup>Recent research suggests this may be a feature of the data but our model does not explain the positive comovement of hours, consumption, investment, and output. See, e.g., Basu et al. (1999), Shea (1998), Gali (1999) and Francis and Ramey (2001).

Figure 7: Responses to Positive Technology, with Capital Adjustment Cost



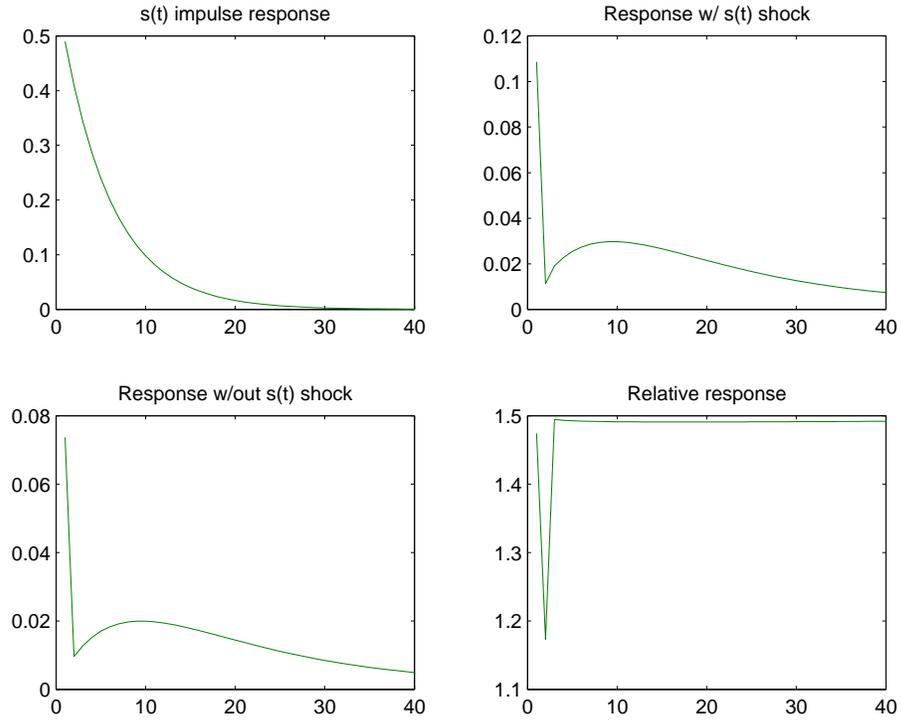
surprise. However, there is relatively little persistence in the response of the equity premium to a technology shock.

## 5.1 Volatility Shocks

What is the dynamic response of the economy to a volatility shock? The volatility process specification implies that a volatility shock in and of itself has no consequence for the dynamic path of the economy. It is only the interaction of volatility and shocks to TFP that influences model dynamics. Consequently, we consider how the *ex post* equity premium responds to a TFP shock both with and without a concurrent volatility shock. Impulse responses to a positive one standard deviation technology shock and a one standard deviation volatility shock are plotted in figure 8.

The upper right left plot is the response of volatility ( $s_t$ ) to a one-standard deviation shock. The upper right panel plots the equity premium response to simultaneous, positive, one-standard-deviation shocks to TFP ( $z_t$ ) and volatility. The lower left panel plots the response of the equity premium to a one-standard-deviation shock to TFP, with no concurrent volatility shock. The lower right

Figure 8: Ex Post Equity Premium Impulse Response



panel is the relative response of the equity premium to the two cases. Ie, it is the upper right impulse divided by the lower left impulse.

The figure indicates that persistent volatility shocks do not add much to the shape of the dynamic path for the model's equity premium. That is, comparing the cases with and without a volatility shock, the largest difference is in the first period after the shocks hit the economy, when the absence of a volatility shock leads to lower equity premium. For the most part though, the volatility shock increases the height of the impulse response by a factor of about 1.5, which is fairly sizeable. Note that the proportionality factor is not declining over the life of the impulse response.

It would appear then that the stochastic volatility component of the model

has its effect mainly on the level of the equity premium rather than its dynamic response to a technology shock. How does the persistence of shocks to TFP volatility affect the equity premium in the baseline model? Figure 9 plots the mean equity premium as a function of the persistence of the volatility process. The figure was generated by assuming the baseline calibration, varying  $\rho_s$  from 0.5 to 0.99, and simulating the model for 10,000 periods for each value of  $\rho_s$ . The first 1000 simulation periods are dropped and a mean annualized equity premium was calculated from the remaining 9,000 observations.

Figure 9: The Effect of Volatility Persistence: Baseline Calibration

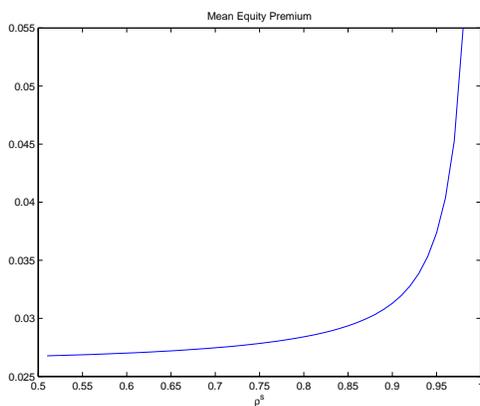


Figure 9 shows that the calculated equity premium rises with the persistence of volatility shocks. However, over a wide range of persistence parameters, there is relatively little change in the equity premium. It is only when the volatility process has persistence above 0.9 that we see a dramatic effect on the mean premium. Under the baseline calibration, the mean equity premium is on the order of 3 percent. This value is lower than common target values for the equity premium. For example, using data from Ibbotson Associates, the equity premium for large stocks over Treasury bills averaged about 8 percent over the period 1926-1929, and about 5 percent over the period 1964-2002, which sample period matches up with our calculated TFP series. For the period 1964-2002, real stock returns averaged 6.7 percent and the real return on Treasury bills averaged about 1.5 percent calculated using the Ibbotson dataset.

Some modification must be made to the baseline calibration if the model is to match the pre-1984 equity premium. In the next section we shut down the

stochastic variance feature of the model and look at standard specifications for the technology shock that have constant variance. To generate a higher mean return on equity, we increase the adjustment cost parameter  $v$  slightly to -4.75, from the baseline value of -3.478. This reparameterized version of the model gives a mean equity return of about 7 percent and a mean riskless return of about 1.5 percent when the standard deviation of TFP shocks is set to 0.0072, a common value for standard deviation used in the RBC literature.

## 6 Aggregate Volatility and Asset Returns

Increasing the mean variance of the TFP shock process and raising the adjustment cost on capital allows the model to approximately match the historical equity premium. We now turn to what the model implies about the relationship between aggregate risk and asset returns. To do so, we simulate the model under a wide range of parameterizations for the volatility of TFP shocks. For a given standard deviation of the TFP shock, the model is solved and simulated for 10,000 periods. The first 1000 observations are dropped, and mean returns are calculated for equities and the riskless asset.

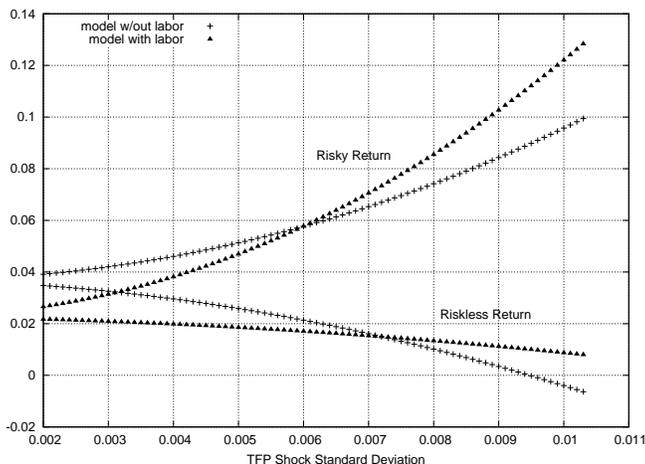
As a robustness check on the findings, we also conduct the same simulation exercise on a model that removes leisure from household preferences (but retains habit persistence and capital adjustment costs. Thus the model is that of Jermann (1998)). In order to facilitate comparison across models, we adjusted the discount factor  $\beta$  for the model without hours so that it matched a mean risk-free rate of about 1.5 percent when the standard deviation of the technology shock is set at 0.0075. The simulation results for the two model specifications are graphed in Figure 10.

There is a modest effect of aggregate risk on the risk-free rate, and much larger effect on the risky rate. The estimated value of TFP shock standard deviation for the pre-1984 period is 0.0075. For the post-1984 period, the estimated standard deviation of the TFP shock is 0.0045.

The steady-state simulations suggest that a drop in aggregate risk on the order of what has been observed for the postwar U.S. economy leads to a sizeable effect on mean equity returns and the equity premium. Varying aggregate risk has a relatively modest effect on the level of the risk-free rate.

The average return on equity falls from about 7 percent when TFP shock volatility is at its post-1984 estimate to a bit under 4 percent when TFP shock

Figure 10: Asset Returns and TFP Shock Volatility

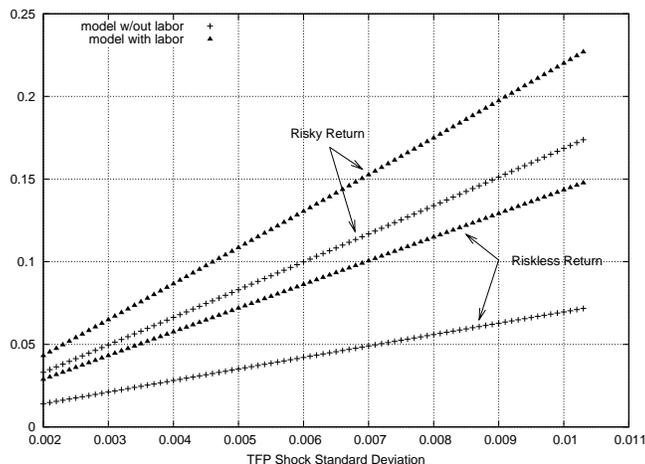


volatility is at its pre-1984 estimate. With only a slight increase in the risk-free rate, the equity premium falls by about 3 percentage points to roughly 2 percent. Thus, we get about a 40 percent reduction in the equity premium entirely from a 60 percent reduction in macroeconomic risk, as measured by the standard deviation of TFP shocks.

We might expect then, that to the extent that the U.S. economy remains in its current low-volatility regime, the equity premium will remain low by historical standards. Our implications of lower macroeconomic risk for the equity premium leave room for the role of reduced market imperfections in accounting for the decline. The model does not predict as large a decline as say Jagannathan et al. (2000) calculate in their model. In addition, the 1984 timing of the aggregate break does not match up with the start of the decline in the equity premium, which may have begun sometime in the 1960s.

While the model gives reasonable predictions on mean returns and the equity premium, it does not do well in matching the volatility of risky and riskless returns. Figure 11 plots return volatility against TFP shock volatility. The methodology for generating this graph is the same as in Figure 10. For the pre-1984 data, the model predicts quarterly equity return volatility on the order of 16 percent, and riskless return volatility on the order of 10 percent. In the data, quarterly real stock return standard deviation is about 6 percent and riskless rate standard deviation is about 0.6 percent. The model does predict

Figure 11: Asset Return Volatility and TFP Shock Volatility



a significant decline in risky and riskless return volatility when aggregate risk declines. For the post-1984 TFP shock volatility estimate, the model predicts an equity return standard deviation of about 9 percent and riskless return standard deviation of about 6 percent. So, the model gets closer on equity return volatility but continues to miss badly on riskless return volatility. However, as noted in Boldrin et al. (2001) this may not necessarily be a fundamental shortcoming of this class of models. Campbell and Chochrane (1999) and Abel (1999) present models with habit persistence that generate more reasonable results for the volatility of the risk-free rate.

## 7 Conclusion

Recent empirical work documents a dramatic decline in the aggregate volatility of the U.S. economy and a sharp decline in the equity premium. We use a dynamic equilibrium model to measure the implied decline in the equity premium from a reduction in macroeconomic risk on the order of what has occurred for the postwar economy. The model implies that lower macroeconomic risk can account for a substantial fraction of the decline in mean stock returns and the equity premium.

While the model does reasonably well in matching first moments of asset returns, it has difficulties matching second moments, especially of the real risk-

free rate. The model overpredicts riskless rate volatility in part because a high value of the parameter that governs habit persistence is needed to generate a sizeable equity premium. In models of this class, if one is willing to increase risk-aversion, less habit persistence is required to match mean asset returns, which simultaneously leads to lower volatility of the risk-free rate. However, Boldrin et al. (2001) point out that higher risk aversion also has adverse implications for employment dynamics in such models.

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