

# WORKING PAPER NO. 05-9 CONSUMER SEARCH, PRICE DISPERSION, AND INTERNATIONAL RELATIVE PRICE VOLATILITY

George Alessandria Federal Reserve Bank of Philadelphia

May 2005

RESEARCH DEPARTMENT, FEDERAL RESERVE BANK OF PHILADELPHIA

Ten Independence Mall, Philadelphia, PA 19106-1574 • www.philadelphiafed.org/econ/index.html

# Consumer Search, Price Dispersion, and International Relative Price Volatility\*

George Alessandria<sup>†</sup>

Federal Reserve Bank of Philadelphia

May 2005

\*The author thanks Andy Atkeson, Dave Backus, Patrick Kehoe, and Ellen McGrattan as well as numerous participants at seminars and conferences for helpful comments. All remaining errors are my own. This is a substantially revised version of a paper previously circulated in 2000 titled "Price Dispersion and International Relative Price Volatility."

<sup>†</sup>The views expressed here are those of the authors and not necessarily those of the Federal Reserve Bank of Philadelphia or the Federal Reserve System. This paper is available free of charge at www.philadelphiafed.org/econ/wps/index.html.

## Abstract

This paper develops a model of consumer search consistent with the evidence of substantial price dispersion within countries. This model is used to study international relative price fluctuations. Consumer search frictions permit firms to price discriminate across markets based on the local wage of consumers. With price dispersion, the market price of a good does not measure its resource cost. This breaks the tight link between relative quantities and relative prices implied by most models. We show that volatile and persistent fluctuations in relative wages lead to volatile and persistent fluctuations in relative prices at the disaggregate level. These deviations from the law of one price substantially increase international relative price volatility. With productivity and taste shocks, the model generates international business cycles that closely match the data.

**JEL classifications:** E31, F12. **Keywords:** Real Exchange Rate; Pricing-to-Market, Law of One Price; Price Dispersion, Consumer Search.

## 1. Introduction

A central puzzle in international economics is that relative prices across countries are volatile and persistent. This is true at the aggregate level, so that there are large and persistent deviations from PPP. This is also true at the disaggregate level, so that there are large and persistent deviations from the law of one price (LOP). Standard flexible and sticky price models do not generate these relative price fluctuations.<sup>1</sup> One reason these models fail to account for the behavior of international relative prices is that in them firms have no incentive to systematically charge different prices across countries so that deviations from the LOP must be short-lived.

Numerous studies find that persistent deviations from the LOP are due to firms segmenting markets and price discriminating across countries.<sup>2</sup> Krugman (1987) calls this *pricing-to-market* and attributes it to firms facing different local market conditions in each market they serve. Knetter (1989 and 1993) presents evidence that firms vary their markups to foreign markets in response to a change in production costs. In this paper, we develop a model of *pricing-to-market* due to consumer search frictions. Over time, changes in the opportunity cost of search in each country lead firms to vary their markups across countries over time. We show that such a model can better account for the observed fluctuations in international relative prices at the aggregate and disaggregate levels.

Our focus on consumer search as a source of pricing-to-market is motivated by four features of micro price data and consumer purchasing behavior. First, beginning with the work of Stigler (1961) and Stigler and Kindahl (1970), numerous papers document substantial and persistent dispersion in the price of identical products within countries.<sup>3</sup> Such dispersion cannot persist without some

<sup>&</sup>lt;sup>1</sup>See Backus et al. (1995) and Chari et al. (2002).

<sup>&</sup>lt;sup>2</sup>See the work of Mertens and Ginsburgh (1985), Dornbusch (1987), Giovannini (1988), Knetter (1989, 1993), Marston (1990), Feenstra, Gagnon and Knetter (1996), Engel and Rogers (1996). Goldberg and Knetter (1997) provide a detailed summary of the literature on international market segmentation.

<sup>&</sup>lt;sup>3</sup>See for instance Pratt, Wise and Zeckhauser (1979), Dahlby and West (1986), Abbott (1989), and Roberts and Supina (2000).

limits to arbitrage, such as consumer search. Second, time-use studies show that consumers spend a considerable amount of time in search-related activities such as shopping. Third, search intensity is related to the opportunity cost of time, as high-wage earners tend to search less per purchase than low-wage earners. Fourth, an increase in search intensity lowers the expected purchase price of a good. Hence, search is an important consideration for both consumption and price-setting decisions. Even though search frictions are an important source of deviations from the LOP within countries, attempts to explain deviations from the LOP across countries have largely ignored this channel.<sup>4</sup>

There are many theoretical models of imperfect information and search.<sup>5</sup> For our purposes, we extend the static partial equilibrium model of Burdett and Judd (1983) to a two-country, dynamic general equilibrium environment. The two features of the model that lead to price dispersion are 1) the need for consumers to actively search to purchase goods and 2) the nature of search is noisy. In regard to the first feature, search takes time, so its opportunity cost depends on forgone labor income.<sup>6</sup> This links the highest, or reservation, price a consumer is willing to pay for a good with the local wage rate. Search is also noisy, in that some consumers get one price quote while others get multiple quotes. The idiosyncratic elements of search make consumers appear heterogenous to firms. For firms this leads to a trade-off between their price and the number of consumers they attract. In equilibrium, this leads to a distribution of prices with a finite support in each country. The distribution of prices will differ across countries whenever the opportunity cost of search differs. Changes in the international relative wage shift the entire distribution of prices in the home country relative to that in the foreign country. Parsley and Wei (2001) and Crucini, Telmer and Zachariadas (2004) find evidence of precisely these types of movements in international relative prices.

Our model of search and price dispersion can more closely match the properties of international relative price fluctuations for two reasons. First, a change in the relative wage changes the relative

 $<sup>{}^{4}</sup>A$  notable exception is Alessandria (2004) which explores the role of search frictions and market structure for international deviations from the law of one price.

<sup>&</sup>lt;sup>5</sup>For a comprehensive review of the literature on price dispersion see Stiglitz (1989).

<sup>&</sup>lt;sup>6</sup>Search is a very specific form of household production. In this respect, the paper follows Benhabib, Rogerson, and Wright (1991) and Greenwood and Hercowitz (1991) in studying the role of household production for properties of the business cycle.

cost of search so that consumers in the relatively high-wage country will search relatively less intensively. This effectively makes demand less elastic in the high-wage country. Since search gives firms some monopoly power, firms charge relatively high prices in the relatively high-wage country. These relatively high prices are as persistent as relative wages. We show that this effect is present regardless of the source of relative wage fluctuations. Hence, we find that prices are dispersed across countries for the same reason they are dispersed within countries.

Second, with price dispersion the market price of a good no longer measures its resource cost to consumers, as they must incur some costly search activities to purchase a good. Because consumers search for all goods, search and price dispersion break the tight link between relative quantities and relative prices common to most models. Instead, relative quantities depend on the ratio of resource costs. Fluctuations in relative prices have smaller effects on the ratio of resource costs so that it takes relatively large swings in relative prices to change relative quantities. Search makes goods within a country less substitutable. Additionally, with search, the consumption basket in a country is bundled with some non-traded, non-market search activities. This makes consumption across countries less substitutable, which reduces the gains to risk-sharing. In response to productivity and taste shocks, this substantially increases both the absolute volatility of relative prices and the synchronization of business cycles across countries. Backus et al. (1995) have argued that explaining relative price volatility and international comovements are the two major puzzles in international macroeconomics.

The time series properties of international relative prices have been studied in three types of theoretical models. First, there are the flexible price models. Backus et al. (1995), Stockman and Tesar (1995), and Heathcote and Perri (2002) demonstrate that models with perfect competition are unable to generate volatile relative prices with productivity or taste shocks. These models also do not generate international deviations from the LOP. Extending these models to allow for monopolistic competition and constant markup pricing does not alter this finding (Lapham and Vigneault (2001)). Second, there are the sticky price models. Betts and Devereux (2000), Chari et

al. (2002), and Bergin and Feenstra (2001) demonstrate that models incorporating nominal rigidities can increase the volatility of international relative prices following monetary shocks. These models do not generate persistent international relative prices. Moreover, empirically, relative price fluctuations are similar for a broad range of goods, even those for which nominal rigidities are unlikely to be relevant (Engel 1999). Finally, Dumas (1993) and Obstfeld and Rogoff (2000) demonstrate that transaction costs that segment goods markets can increase the volatility of international relative prices. The emphasis in these papers is on iceberg shipping costs, rather than information frictions, that segment goods markets. In these models, the relative prices of traded good only fluctuates with trade costs.

The next section of the paper documents some properties about international relative prices and describes evidence of price dispersion. In section 3, we develop a model of search and pricingto-market. Sections 4 and 5 discuss the calibration of the model and its implications. Section 6 concludes.

## 2. Evidence

In this section we summarize some properties of prices within and across countries. We start by documenting common features of fluctuations in the real exchange rate, terms of trade, and relative unit labor costs of G7 countries. Then, we show that comovements between these different relative prices imply that deviations from the LOP are large, persistent, and central to understanding international relative price fluctuations. Next, we review some evidence of deviations from the LOP within countries. We also summarize some evidence that this dispersion in prices affects consumer search decisions and average transaction prices.

## A. International Relative Price Fluctuations

For expositional purposes, consider a two-country  $\{D, F\}$ , two-good world with each country specialized in the production of a single good. At the aggregate level, we focus on the real exchange rate, which is defined as the ratio of the price of a basket of goods in each country measured in a common currency, or  $RER_t = P_t/P_t^*$ , where  $P_t$  is a CPI-based price index in country D and  $P_t^*$  is a trade-weighted CPI for rest of the world. Next, we measure the terms of trade, defined as  $TOT_t = P_{D,t}^*/P_{F,t}$ , where  $P_{D,t}^*$  measures the price of manufactured goods exported from country D to the rest of the world in period t and  $P_{F,t}$  measures the price of manufactured goods imported into country D in period t. Finally, we also include a measure of the relative cost of producing in country D, denoted by  $RULC_t = ULC_t/ULC_t^*$ , where  $ULC_t$  measures the unit labor costs of producing in country D and  $ULC_t^*$  measures the unit labor cost of country D's trading partners. We use quarterly data from the OECD's International Competitiveness and Main Economic Indicators series to measure these relative prices and GDP for G7 countries from 1975:1 to 2003:1.

Table 1 shows that a number of features of international relative price fluctuations are common across countries. First, relative prices are much more volatile than output. For instance, the real exchange rate is, on average, three times as volatile as GDP. Second, both the real exchange rate and relative unit labor costs are more volatile than the terms of trade, with the real exchange rate about 25 percent more volatile than the terms of trade and relative unit labor costs about 15 percent more volatile than the real exchange rate.<sup>7</sup> Third, these relative price movements are highly correlated with one another with cross correlations in the range of 80 to 90 percent. Finally, relative price fluctuations are persistent, with autocorrelations close to 0.80, which is similar to that of GDP.

To investigate the sources of real exchange rate fluctuations we decompose it into its components. Suppose as an approximation the price index in country D equals  $P_t = (P_{F,t})^{\alpha} (P_{D,t})^{1-\alpha}$ and the price index in country F equals  $P_t^* = (P_{D,t}^*)^{\widehat{\alpha}} (P_{F,t}^*)^{1-\widehat{\alpha}}$  where  $P_{j,t}$  measures the price of a good produced in country j. Changes in the real exchange rate equal

$$\Delta rer_t = \alpha \left( \Delta p_{F,t} - \Delta p_{D,t}^* \right) + (1 - \alpha) \left( \Delta p_{D,t} - \Delta p_{F,t}^* \right) + (\alpha - \widehat{\alpha}) \left( \Delta p_{D,t} - \Delta p_{F,t}^* \right).$$

<sup>&</sup>lt;sup>7</sup>These data do not allow us to distinguish whether relative unit labor costs vary because of changes in relative labor productivity or relative wages. However, given the relatively small fluctuations in labor productivity and relatively large fluctuations in relative unit labor costs at business cycle frequencies (Cooley and Prescott 1995), most relative unit labor cost fluctuations can be attributed to relative wage fluctuations.

Now, if the LOP holds, so that the same good sells for the same price everywhere,<sup>8</sup> then

(1) 
$$\Delta rer_t = (1 - \alpha - \widehat{\alpha}) \Delta tot_t,$$

so that the real exchange rate should be both perfectly correlated and less volatile than the terms of trade. This second feature is clearly not true.

We use equation (1) to construct a measure of the contribution of terms of trade fluctuations to real exchange rate movements. To do so requires a measure of import and export shares. We measure  $\alpha$  as the average ratio of the value of imports to nominal GDP. To derive  $\hat{\alpha}$  we measure the value of each country's exports as a fraction of each trading partner's GDP and then take a tradeweighted average of these measures. The Data Appendix fully describes the process. In general, our results are not very sensitive to different measures of  $\hat{\alpha}$ .

Table 2 reports these trade shares as well as the volatility of this terms of trade based statistic relative to the volatility of the actual real exchange rate. By this measure, only about 42 percent of real exchange rate fluctuations can be attributed to fluctuations in the terms of trade. Thus, over half of the movements in the real exchange rate must be due to changes in deviations from the LOP. This is consistent with what Engel (1999) finds using disaggregated data and Chari et al. (2002) find using producer price indices.

When the LOP does not hold, the real exchange rate decomposition includes two more terms,

$$\Delta rer_t = (1 - \alpha - \widehat{\alpha}) \Delta tot_t + (1 - \alpha) \Delta lop_{D,t} + (1 - \widehat{\alpha}) \Delta lop_{F,t},$$

where  $\Delta lop_{j,t} = \Delta p_{j,t} - \Delta p_{j,t}^*$  measures the size of deviations from the LOP in goods from country *j*. Engel (1993 and 1999) finds the behavior of disaggregated international relative prices to be similar across many goods. Based on this evidence, we set  $\Delta lop_{D,t} = \Delta lop_{F,t}$ , to derive a measure of average

<sup>&</sup>lt;sup>8</sup>More generally, if prices are proportional across countries, for instance because of an iceberg shipping cost, then this condition also holds.

changes in deviations from the LOP as

$$\Delta lop_t = \frac{\Delta rer_t - (1 - \alpha - \widehat{\alpha}) \, \Delta \tau_t}{(2 - \alpha - \widehat{\alpha})}.$$

Table 2 reports the properties of deviations from LOP. Fluctuations in these disaggregated relative prices are highly correlated with and smaller than fluctuations in other international relative prices. For instance, deviations from the LOP are about 34 percent the size of real exchange rate fluctuations. They are quite persistent with an autocorrelation of 0.87.

Our evidence of fluctuations in deviations from the LOP is consistent with studies that find at the disaggregate level relative prices are also volatile and persistent across countries. In particular, Engel and Rogers (1996) and Parsley and Wei (2001) find that the relative price of almost identical goods consumed in different countries is nearly as volatile and persistent as aggregate relative prices. Using highly disaggregated data on German exports, Knetter (1989 and 1993) finds these same fluctuations in the relative price of exports. These fluctuations in export relative prices occur at the border, before any local, non-traded inputs are added. We interpret these data as evidence that firms set prices differently across countries and adjust their prices in a different manner over time in each market.

The used-car market provides clear evidence of these changes in deviations from the law of one price across countries. Figure 1 displays the ratio of the price of used cars in the U.S. to the price of used cars in Canada as well as the ratio of U.S. to Canadian average weekly wages from 1998 to 2005. A linear trend has been removed from both series.<sup>9</sup> These series exhibit substantial comovement with a correlation of 0.92. By the nature of the purchase, which tends to occur either as negotiated sales or through auctions, nominal rigidities are unlikely to be important. Thus, this common explanation for international relative price fluctuations based on nominal rigidities and

<sup>&</sup>lt;sup>9</sup>The U.S. used-car price index is from the BLS. For Canada we use the ADESA Canada used-vehicle price index converted to the same currency using the average monthly U.S.-Canadian exchange rate. The U.S. wage series is the BLS's average weekly earnings for private industry, and for Canada, it is Statistics Canada's average weekly earnings for the total economy. All series but exchange rates are seasonally adjusted.

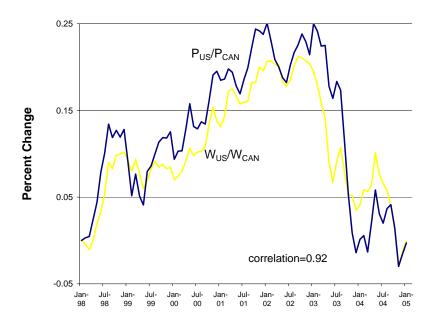


Figure 1: Ratio of Used Car Prices and Wages Between U.S. and Canada

monetary shocks does not seem to apply here. Moreover, as many sales in this market result from bilateral matches, search frictions are likely to matter for price determination. In what follows, we develop a model in which relative prices fluctuate for goods such as used cars because the relative opportunity cost of search fluctuates across countries.

#### **B.** Domestic and International Price Dispersion

Here we briefly summarize some evidence of dispersion in the price of nearly identical goods within tightly defined geographic boundaries. We then discuss evidence that this dispersion affects the search and purchase decisions of consumers.

In a study of 37 basic household goods and services in the Boston area, Pratt et al. (1979) find that for 17 of these products the highest price quote is at least twice the lowest price quote.<sup>10</sup> In a study of the airline industry, Borenstein and Rose (1994) find between two passengers on a route an expected absolute difference in fares of 36 percent of the airline's average ticket price. Using plant

<sup>&</sup>lt;sup>10</sup>Some of the goods included a Raleigh Grand Prix 10 speed bicycle, a 20 gallon aquarium, one hour of horoscope reading including charting, a Texas Instruments SR-50 calculator, and a Dennman styling brush. On average, the authors collected 12 price quotes per good.

	Volatili	ty relat	te to	) RER	Aut	ocorre	lation	
	tot	rulc	y		tot	rulc	rer	y
U.S.	0.92	1.08	0.37		0.80	0.84	0.82	0.86
Japan	0.65	1.03	0.14		0.74	0.85	0.85	0.80
Germany	0.98	1.11	0.45		0.82	0.82	0.82	0.74
France	0.79	1.15	0.36		0.70	0.80	0.78	0.86
Italy	0.96	1.23	0.30		0.76	0.77	0.81	0.80
U.K.	0.69	1.21	0.24		0.69	0.86	0.81	0.84
Canada	0.66	1.31	0.44		0.78	0.90	0.87	0.89
Average	0.81	1.16	0.33		0.76	0.83	0.82	0.83
				Comov	rements			
	tot, rulc)	(tot, r	rer)	(tot, y)	(rulc, rer	$\cdot$ ) ( $ri$	$\iota lc, y)$	(rer,
U.S.	0.81	0.8	8	-0.25	0.93		0.09	-0.
Japan	0.92	0.9	3	-0.31	0.97		0.47	-0.
Germany	0.88	0.9	2	-0.05	0.90		0.15	-0.
France	0.80	0.8	34	-0.14	0.90	_	0.28	-0.
Italy	0.62	0.7	6	-0.06	0.72	_	0.46	0.
U.K.	0.86	0.8	37	-0.07	0.97	_	0.17	-0.
Canada	0.68	0.7	'1	0.20	0.93	_	0.33	-0.
Average	0.80	0.8	34	-0.10	0.90	_	0.28	-0.
ee notes at the end of the tables.								

Table 1: Properties of International Relative Price Fluctuations for G-7 Countries

Table 2:	Accounting	for R	Real Ex	change	Rate	Fluctuations

		Comovements							
	Imports	Exports	$\frac{\sigma_{\widehat{rer}}}{\sigma_{rer}}$	$rac{\sigma_{lop}}{\sigma_{rer}}$	$\rho_{lop}$	(lop, rer)	(lop, rulc)		
U.S.	0.122	0.092	0.367	0.283	0.821	0.722	0.684		
Japan	0.086	0.018	0.254	0.269	0.901	0.904	0.853		
Germany	0.266	0.057	0.464	0.28	0.835	0.832	0.672		
France	0.226	0.028	0.442	0.342	0.873	0.847	0.715		
Italy	0.219	0.017	0.733	0.368	0.860	0.685	0.405		
U.K.	0.277	0.030	0.286	0.371	0.883	0.926	0.880		
Canada	0.332	0.017	0.436	0.46	0.919	0.914	0.836		
Average			0.419	0.339	0.870	0.834	0.721		

\*See notes at the end of the tables.

level data from the Census of Manufactures for 13 industries, Roberts and Supina (2000) find the coefficient of variation exceeds 25 percent for nine of the industries. For goods sold through the Internet, Brynjolfsson and Smith (1999) find that prices differ by an average of 33 percent for books and 25 percent for CD's. In a recent study of retail prescription drug prices in upstate New York, Sorenson (2000) finds a mean coefficient of variation of 22 percent. Given the inherent difficulties in obtaining micro price data,<sup>11</sup> the broad range of goods considered in these studies suggests that price dispersion is a widespread phenomenon.

With price dispersion, consumers can affect the average price of their purchases by being selective about which prices they accept. Two recent studies of the purchasing patterns of individuals find evidence that shopping activities and purchase prices substantially differ across individuals based on their income. In particular, low-wage consumers spend more time shopping per purchase than high-wage consumers, and this extra shopping time results in lower average purchase prices. First, using a unique dataset on consumer purchasing behavior in Argentina, McKenzie and Schargrodsky (2004) find that consumers in the 10th percentile of income spend about 30 percent more time shopping per purchase than consumers in the 90th percentile of income. They also find that a 4 percent increase in shopping time results in about a 4 percent drop in the average price paid for goods. They also show that search effort increased in response to Argentina's real exchange rate depreciation. Second, Aguiar and Hurst (2004) use detailed micro data from U.S. households to show that while consumption expenditures drop with retirement, actual consumption does not change, as retirees increase time shopping thereby lowering their average purchase price. Upon retirement, expenditures drop by 17 percent, while time spent in home production, including shopping, rises by 53 percent.

The evidence of large and persistent price dispersion within countries offers a new interpretation of international deviations from the LOP. Across countries and over time, the ratio of the mean price

<sup>&</sup>lt;sup>11</sup>This is not a problem specific to measuring price dispersion, as the lack of comprehensive disaggregated price data is central to the different views of business cycles.

of identical goods is quite volatile and persistent. This implies that the distribution of prices for a good in one country shifts relative to the distribution of prices in another country. The evidence that wages influence the prices consumer pay for goods implies that fluctuation in relative wages across countries will influence the international relative price of goods. In the remainder of the paper, we develop a model with these features and examine its implications for the volatility of international relative prices.

## 3. Model

Now we develop a two-country model with infinitely lived consumers to study the role of search frictions and pricing-to-market for international relative price fluctuations. In our model, consumers do not know the location of the lowest price for any good and must actively search to purchase goods. Searching is costly in that it takes time away from leisure and work. To minimize the cost of acquiring a good, consumers follow a reservation price strategy and only purchase when they find a good below this reservation price. Because labor is internationally immobile, the opportunity cost of search may differ across countries and lead to different reservation price strategies. Since consumers search only in their own market, firms may face consumers with different reservation prices across countries and will take this into account by pricing-to-market.

Specifically, consider a two-country model consisting of a Domestic country and a Foreign country, denoted  $\{D, F\}$ . Each country is populated by a large number of identical, infinitely lived families. Each family consists of many identical agents. In each period of time t, the economy experiences an event  $s_t$ . We denote the history of events up to and including period t as  $s^t = (s_0, ..., s_t)$ . The time-zero probability of any history  $s^t$  is  $\sigma(s^t)$ .

In each period t, the commodities in this economy are labor, a consumption-capital good, a good produced in the Domestic country denoted D, and a good produced in Foreign, denoted F. Each good is produced by a continuum of identical firms. Firms hire labor and capital and sell output through geographically distinct stores. In each country many stores specialize in the sale of either good D or good F. These stores potentially charge different prices. The distribution of prices being charged in stores is common knowledge, but consumers do not know where to find the store with the lowest price. To find the lowest price possible, consumers must search. Because searching takes time, consumers are willing to accept some prices that are higher than the lowest price in the market. In this respect, each firm has some monopoly power over consumers entering its store. This monopoly power allows firms to charge potentially different prices, above marginal cost, and still sell positive quantities of the same good. Firms do not charge the same price because the structure of search enables firms that lower their prices to attract additional customers.

Stores require no local input and can be viewed as part of the producing firm. There are no costs of shipping goods internationally. These stark assumptions isolate the role of consumer search frictions for international deviations from the LOP and international relative price fluctuations.

The search process allows firms to segment markets internationally. With two countries and two differentiated goods, there are four price distributions. The distribution of prices charged by country  $j \in \{D, F\}$  firms in Domestic is denoted by  $G_j(p; s^t)$ . The distribution of prices charged by country j firms in Foreign is denoted by  $G_j^*(p; s^t)$ .

A complete set of one-period state-contingent securities is traded every period. Without loss of generality, these securities are denominated in terms of the Domestic wage (w = 1). For the sake of exposition, we express the Domestic wage as w rather than substituting it out. One security costs  $\chi(s^{t+1}|s^t)$  in state  $s^t$  and pays one unit if and only if the state tomorrow is  $s^{t+1}$ .

## A. Consumer's Problem

Here we describe the problem of a representative Domestic family. The problem of a Foreign family is similar, with variables denoted by an asterisk. The Domestic family is composed of a continuum of identical Domestic agents. This approach eliminates any uncertainty from search and maintains a representative agent framework<sup>12</sup> in each country. Each family divides its members into

<sup>&</sup>lt;sup>12</sup>This is equivalent to allowing agents from the same country to trade a complete set of contingent claims over the uncertainty from searching.

three activities: shopping, working, or leisure.

Shoppers search to reduce the cost of purchasing a good.<sup>13</sup> By searching, consumers shift their purchases toward firms with low prices. The search technology has three characteristics. First, shoppers cannot communicate while searching; second, each shopper can purchase at most one unit;<sup>14</sup> and third, search is noisy in that with probability 1 - q a shopper receives price quotes from two firms and with probability q receives only one price quote. These assumptions imply that shoppers follow a simple reservation price rule: purchase one unit if  $p \leq r$  and none otherwise. With two price quotes, the shopper buys from the firm with the lower price if this price is below the reservation price.

Given a distribution of prices for good j,  $G_j(p; s^t)$ , with finite support  $[\underline{P}_j(s^t), \overline{P}_j(s^t)]$ , the distribution of the lowest price drawn by a shopper equals

(2) 
$$H_j(p;s^t) = qG_j(p;s^t) + (1-q)\left(1 - \left[1 - G_j(p;s^t)\right]^2\right).$$

Clearly,  $H_j(p; s^t)$  is a convex combination of the distribution of prices conditional on a single offer and the distribution of the lowest price conditional on two price quotes.

With a reservation price r, a shopper buys with probability  $H_j(r; s^t)$  at an expected price of

(3) 
$$P_j(r;s^t) = \frac{\int_0^r p \frac{dH_j(r;s^t)}{dp} dp}{H_j(r;s^t)}.$$

A lower reservation price reduces both the expected transaction price and the probability of purchasing. With many identical shoppers, the expected price is also the average purchase price in the market. With noisy search, the average purchase price,  $P_j(r; s^t)$ , is always lower than the average price quoted by firms as more purchases are made at lower prices.

Consumers direct their shoppers toward stores selling either good D or F so that a shopper

<sup>&</sup>lt;sup>13</sup>Allowing consumers to search for a particular variety or quality of product does not influence the results.

<sup>&</sup>lt;sup>14</sup>Relaxing this assumption so that shoppers can purchase multiple goods does not alter our results. What matters is that there is an opportunity cost of searching in terms of work.

sent out for a particular good collects price quotes only from firms selling that good. Shoppers do not return to stores they have visited in previous periods.<sup>15</sup> The family sends out  $n_j(s^t)$  agents to shop for good j and instructs them to use the reservation price rule,  $r_j(s^t)$ . At the end of each period, shoppers return home with their purchases. The total amount of goods purchased equals the number of successful shopping trips so that

(4) 
$$y_D(s^t) = n_D(s^t) H_D(r_D(s^t); s^t),$$

(5) 
$$y_F(s^t) = n_F(s^t) H_F(r_F(s^t); s^t).$$

These goods are combined to produce a composite consumption-capital good using a CES aggregator,

$$A\left(y_D\left(s^t\right), y_F\left(s^t\right)\right) = \left[a_1 y_D\left(s^t\right)^{\frac{\omega-1}{\omega}} + a_2 y_F\left(s^t\right)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}}$$

The parameters  $a_1$  and  $a_2$  determine trade flows and  $\omega$  is the elasticity of substitution between the two goods. The final good is used for consumption and capital accumulation. There is a quadratic adjustment cost on capital,  $\phi(x(s^t)/k(s^{t-1}))$ , leading to the resource constraint

(6) 
$$c(s^{t}) + x(s^{t}) + \phi(x(s^{t})/k(s^{t-1})) = A(y_{D}(s^{t}), y_{F}(s^{t})).$$

Aggregate capital holdings are determined in advance and hence indexed by the period of investment. The law of motion for capital is

(7) 
$$k(s^{t}) = (1-\delta)k(s^{t-1}) + x(s^{t}).$$

Working requires a fixed amount of time denoted by  $h_W$  and search requires a fixed amount of

<sup>&</sup>lt;sup>15</sup>Alessandria (2004) explores the role of long-term relationships and search frictions for firm pricing across countries.

time  $h_S$ . A convenient utility function is then,

$$u(c,\lambda) = \frac{c^{1-\gamma} - 1}{1-\gamma} - \kappa \left[ \frac{\lambda (1-h_W)^{1-\gamma} + \theta (1-\lambda) (1-h_S)^{1-\gamma} - 1}{1-\gamma} \right]$$

where  $\lambda = \{0, 1\}$  is 1 if an agent works and 0 if the agent shops. We let  $\gamma$  measure the inverse of the intertemporal elasticity of substitution,  $\kappa$  measure the value of leisure, and  $\theta$  is the disutility of search relative to work.

Given that work and search are indivisible, it is optimal to have different members of the family specialized in either shopping, working, or relaxing. We follow Rogerson (1988) and assume that agents trade lotteries over these activities and consumption. This generates the following utility function for the family,

$$u(c,l,n_D+n_F;\widehat{\kappa}) = \frac{c^{1-\gamma}-1}{1-\gamma} - \widehat{\kappa}\left(l+\widehat{\theta}\left(n_D+n_F\right)\right),$$

where l is the measure of agents sent out working and  $n_D$  and  $n_F$  are the measures of agents sent out shopping. The weights on leisure and search are rewritten to reflect the different amounts of time used in each activity. For generality, the parameter  $\hat{\kappa}$  is included in the utility function so as to allow for shocks to the marginal utility of leisure as in Hall (1997). Others have shown that this term may fluctuate for reasons other than direct shocks to the marginal utility of leisure. For instance, Chari et al. (2004) demonstrate that this term may vary owing to distortions from changes in marginal labor tax rates. Benhabib et al. (1991) and Chari et al. (2002) show that this term may change as a result of shocks to home production technologies, of which search is only one type. Given that consumption and leisure are separable, all family members share equally in consumption at the end of each period. Income from wages, capital rents, bonds, and profits pay for purchases.

In each period, allocations are chosen after the realization of the aggregate state. Given the distribution of prices, wages, interest rates, and asset prices, each Domestic household directs agents to shop for each good,  $\{n_D(s^t), n_F(s^t)\}$ , or work  $\{l(s^t)\}$ , devises reservation price rules

 $\{r_D(s^t), r_F(s^t)\}$ , chooses consumption and the next period's capital and bond holdings  $\{c(s^t), k(s^t), b(s^{t+1})\}$  to maximize

$$V_{0} = \sum_{t=0}^{\infty} \sum_{s^{t}} \beta^{t} \sigma\left(s^{t} | s_{0}\right) u\left(c\left(s^{t}\right), l\left(s^{t}\right), n_{D}\left(s^{t}\right) + n_{F}\left(s^{t}\right); \widehat{\kappa}\left(s^{t}\right)\right),$$

subject to equations (4) - (7) and the budget constraint:

(8) 
$$\sum_{j \in \{D,F\}} n_j(s^t) H_j(r_j(s^t) | s^t) P_j(r_j(s^t); s^t) + \sum_{s^{t+1} | s^t} \chi(s^{t+1} | s^t) b(s^{t+1}) \le w(s^t) l(s^t) + i(s^t) k(s^t) + \Pi(s^t) + b(s^t),$$

with  $k(s^0)$ ,  $k^*(s^0)$ ,  $b(s^0)$  given. Domestic agents receive profits,  $\Pi(s^t)$ , from owning Domestic firms. We also assume that there is no trade in the shares of firms.

If an interior solution exists, then first order conditions of the problem are

(9) 
$$r_D(s^t) = \frac{\widehat{\theta}w(s^t)}{H_D(r_D(s^t);s^t)} + P_D(r_D;s^t),$$

(10) 
$$r_F(s^t) = \frac{\theta w(s^t)}{H_F(r_F(s^t);s^t)} + P_F(r_F;s^t)$$

(11) 
$$\frac{A_D(s^t)}{A_F(s^t)} = \frac{r_D(s^t)}{r_F(s^t)}$$

(12) 
$$\chi(s^{t+1}|s^t) = \beta\sigma(s^{t+1}|s^t) \frac{u_c(s^{t+1})A_D(s^{t+1})}{r_D(s^{t+1})} \frac{r_D(s^t)}{u_c(s^t)A_D(s^t)},$$

(13) 
$$u_{c}\left(s^{t}\right) = \frac{\beta u_{c}\left(s^{t+1}\right) \left[\frac{i(s^{t+1})G_{D}\left(s^{t+1}\right)}{r_{D}(s^{t+1})} + (1-\delta)\left(1+\phi_{x}\left(s^{t+1}\right)\right) - \phi_{k}\left(s^{t+1}\right)\right]}{(1+\phi_{x}\left(s^{t}\right))}$$

$$(14) \frac{u_c(s^t) A_D(s^t)}{r_D(s^t)} = -\frac{\widehat{\kappa}(s^t)}{w(s^t)}.$$

Here  $u_c$  is the derivative of the utility function with respect to consumption,  $A_j$  is the derivative of the aggregator function with respect to good j,  $\phi_j$  is the derivative of the adjustment cost function with respect to investment or capital, and  $\sigma(s^{t+1}|s^t)$  is the conditional probability of  $s^{t+1}$  given  $s^t$ .

The first order conditions in this model differ from a model without search and price dispersion along two dimensions. First, there are two additional equations which determine the reservation price for each good. Second, at the margin it is the reservation price rather than the average price that measures the consumer's resource cost of acquiring an additional unit of a good. Given that the marginal cost to the consumer of an additional unit of a good is summarized by the reservation price, equations (11) - (14) are quite standard.

Equations (9) and (10) implicitly determine the reservation prices for each good. At the margin, a family is indifferent between paying the reservation price for a unit of good j or sending out more shoppers. To purchase an additional unit of good j the family must send out  $1/H_j(r_j)$  more shoppers and expects to buy at a price of  $P_j(r_j)$ . The cost of sending out each additional shopper depends on the disutility of shopping relative to working,  $\hat{\theta}$ , and the market value of working, which depends on the wage. These equations are arbitrage conditions that closely tie the highest price a consumer is willing to pay with the local wage and the average price in the market. The reservation price is increasing in the market wage and the disutility of shopping relative to working. Equation (10) shows that the price of Foreign goods sold in country D will depend in part on the wage of Domestic consumers.

## International Risk-Sharing

In nearly all models with complete asset markets, the international risk-sharing condition equates the marginal rate of substitution (MRS) of agents across countries to the relative price of their consumption bundles. This implies that the ratio of marginal utilities across countries should equal the ratio of prices, or the real exchange rate. With utility that is separable in consumption and labor the following condition should hold,

(15) 
$$\left(c\left(s^{t}\right)/c^{*}\left(s^{t}\right)\right)^{-\gamma} = P\left(s^{t}\right)/P^{*}\left(s^{t}\right),$$

When countries can trade only a nominal bond, this equation holds in expected terms. Backus and Smith (1993) find no evidence that this holds for a broad cross-section of countries. Consequently, Chari et al. (2002) argue that a key challenge in international macroeconomics is breaking the tight link between these relative quantities and relative prices. We now show this link is broken in the search model and thus can potentially resolve the Backus-Smith puzzle.

First, we note that to a consumer the minimum expected resource cost of a unit of consumption<sup>16</sup> in country D equals  $R(s^t) = \left(a_1^{\omega}R_D(s^t)^{1-\omega} + a_2^{\omega}R_F(s^t)^{1-\omega}\right)^{\frac{1}{1-\omega}}$  and in country F equals  $R^*(s^t) = \left(a_2^{\omega}R_D^*(s^t)^{1-\omega} + a_1^{\omega}R_F^*(s^t)^{1-\omega}\right)^{\frac{1}{1-\omega}}$ , where uppercase reservation prices are for the aggregate economy. These resource costs exceed the market price of a unit of consumption by the value of search services used in collecting the consumption bundle. Given the resource costs, the risk-sharing condition can be derived from the Foreign family's first order condition with respect to bond holdings which is

$$\chi\left(s^{t+1}|s^{t}\right) = \beta\sigma\left(s^{t+1}|s^{t}\right)\frac{u_{c}^{*}\left(s^{t+1}\right)}{R^{*}\left(s^{t+1}\right)}\frac{R^{*}\left(s^{t}\right)}{u_{c}^{*}\left(s^{t}\right)}.$$

Substituting out the bond price from equation (12) and iterating yields the risk-sharing condition<sup>17</sup>

(16) 
$$\left(c\left(s^{t}\right)/c^{*}\left(s^{t}\right)\right)^{-\gamma} = R\left(s^{t}\right)/R^{*}\left(s^{t}\right),$$

which equates the ratio of marginal utilities to the ratio of resource costs per unit of consumption across countries. Comparing equations (15) and (16), introducing search frictions can potentially resolve the Backus-Smith puzzle. In particular, whereas risk-sharing in most models require the MRS across agents to equal the relative price of their consumption goods, in the search model the MRS is equated to the relative cost of acquiring the two goods. If resource costs do not move one for one with market prices, the real exchange rate and consumption ratios will not be perfectly negatively correlated. Essentially, search frictions introduce a non-traded input into the final consumption bundle. Unlike other non-traded goods, such as services or retail, and distribution margins though, the price of search services is not captured in the price of the composite consumption bundle because

<sup>&</sup>lt;sup>16</sup>These resource costs solve the consumer's cost minimization problem, taking into account the value of search efforts or  $R = \min y_D R_D + y_F R_F$  subject to  $A(y_D, y_F) \ge 1$ .

<sup>&</sup>lt;sup>17</sup>This assumes that countries are ex-ante identical.

this costs is borne by the consumer.

#### **B.** Firm's Problem

In each country there are many firms producing exclusively for either the Domestic or Foreign market. These firms hire labor and rent capital to produce output. They sell this output to shoppers through stores. Shoppers choose a store based on the price quotes they have received.

We focus on the problem of Domestic firms selling to Domestic consumers. There are a large number of these firms (normalized to a continuum of unit mass) selling the same good. Each firm producing in the same country is identical. We assume that each firm can adjust labor and capital within the period, and thus each firm faces the same per unit cost of production of  $mc_j$ . Given the structure of search and production, the firm's problem is static, and so we suppress the state variable in the description of the problem. Upper case variables denote aggregate decisions.

The firm's problem is similar to a problem examined in Burdett and Judd (1983). A firm quotes a price to  $(q + 2(1 - q)) N_j$  consumers. Of these consumers, a fraction  $\frac{q}{q+2(1-q)}$  receive one quote, while  $\frac{2(1-q)}{q+2(1-q)}$  get two quotes.<sup>18</sup> The firm cannot distinguish between shoppers with one or two price quotes and can quote only one price.

The price charged does not affect the number of shoppers that receive price quotes or the cost of production so that the firm's pricing decision is summarized by the problem of maximizing profits per shopper. All shoppers are identical in their reservation price, so the maximum price a firm can charge is the reservation price, denoted by  $R_j$ . While each shopper has the same reservation price, noisy search implies that shoppers may differ in their outside option. Some shoppers have multiple price quotes, and among these shoppers, their second price quote will differ. For firms, this leads to a trade-off between price and the probability that a shopper with two quotes makes a purchase.

<sup>&</sup>lt;sup>18</sup>The probability that a customer has one price quote is less than the probability that a shopper gets one price quote as each shopper expects 2 - q quotes.

Based on this trade-off, the probability that a shopper purchases from a firm charging p is

$$Q_{j}(p) = \begin{cases} 0 & \text{if } p > R_{j} \\ \frac{q}{q+2(1-q)} + \frac{2(1-q)[1-G_{j}(p)]}{q+2(1-q)} & \text{if } p \le R_{j} \end{cases}$$

If the firm charges a price above the reservation price, it will not make a sale. If the firm charges a price below the reservation price, the firm makes a sale for certain if the shopper has one price quote. If the shopper has two price quotes, then the firm will make a sale only if its price is less than the shopper's other price quote, which is drawn from  $G_j(p)$ .

Combining the demand per shopper and the number of shoppers to which a firm quotes a price generates the following demand curve per shopper of  $\hat{Q}_j(p) = q + 2(1-q)(1-G_j(p))$ . Now, the elasticity of substitution between firms depends on search frictions and the distribution of prices of other firms. In this respect, the model can generate interactions between firms different from standard models of monopolistic competition and constant elasticity of demand.

Given the distribution of prices in the economy,  $G_j(p)$ , consumers' reservation price,  $R_j$ , the demand per shopper,  $\hat{Q}_j(p)$ , the mass of shoppers,  $N_j$ , and the unit cost of production,  $mc_j$ , a firm's problem is to charge a price that maximizes the profits per shopper,

$$\pi_{j} = \max_{p} \left( p - mc_{j} \right) \widehat{Q}_{j} \left( p \right).$$

This is a well-defined problem with at least one solution. Furthermore, Burdett and Judd (1983) demonstrate that if firms have the same cost of production,<sup>19</sup> each firm earns the same profit  $\pi_j$  on the support  $[\underline{P}_j, \overline{P}_j]$  of  $G_j(p)$  and that the highest price charged is equal to the reservation price

<sup>&</sup>lt;sup>19</sup>Allowing firms to have different costs leads to more price dispersion but does not change the model's predictions.

 $(\overline{P}_j = R_j)$ . These are equal to

$$\begin{aligned} \pi_j &= q \left( \overline{P}_j - mc_j \right), \\ G_j \left( p \right) &= \begin{cases} 0 & p < \underline{P}_j, \\ 1 - \frac{\overline{P}_j - p}{p - mc_j} \frac{q}{2(1 - q)} & p \in [\underline{P}_j, \overline{P}_j] \\ 1 & p > \overline{P}_j, \end{cases} \\ \underline{P}_j &= \frac{2 \left( 1 - q \right) mc_j + q \overline{P}_j}{2 - q}. \end{aligned}$$

Finally, in total, individual and aggregate firm profits are  $\Pi_j = \pi_j N_j$ .

Because firms are indifferent between charging any price on the support of the distribution, they will randomize. With a continuum of firms, the law of large numbers holds, and there is a continuous distribution of prices,  $G_j(p)$ .

The assumption that stores can charge only one price implies that the model may overstate dispersion across stores in the data. In particular, if stores quote different prices to different consumers within the same period, for instance by having periodic sales, then measured price dispersion in average prices across stores understates true price dispersion.

Since firms choose prices by randomizing over the support of the distribution, firm pricing will generally not be persistent. For example, firms charging a low price on average over time will not have chosen a low-price strategy, but rather will have repeatedly drawn a low price in the process of randomizing. In a recent study, Lach (2002) finds precisely this type of behavior across a set of stores in Israel. Similarly, some firms may draw the same, or nearly the same, price from one period to the next, even in the face of a large shock to their production costs. In this respect, some prices may appear not to adjust to large shocks to costs.

On the production side, firms fulfill demand by hiring labor and renting capital. Given L workers and K units of capital, a firm can produce,  $y = ZK^{\alpha}L^{1-\alpha}$ , units of output where Z is an aggregate technology shock common to all firms. Firms adjust capital and labor within the period

so that the marginal cost of firms from the same country is determined by the cost minimization problem:  $mc_j = \min_{L,K} w_j L + i_j K$  subject to  $y \ge 1$ .

Given the reservation price,  $R_j$ , and marginal cost,  $mc_j$ , it is possible to solve analytically for some moments of the posted and transacted distribution of prices. In particular, the mean transaction and mean posted price are

(17) 
$$P_j(R_j) = mc_j + q(R_j - mc_j),$$

(18) 
$$P_j^{Posted}(R_j) = mc_j + q \frac{\ln\left(\frac{2}{q} - 1\right)}{2(1-q)} (R_j - mc_j).$$

The mean transaction price is quite intuitive. If firms could differentiate between consumers with one or two offers, they would play Bertrand with those agents with two price quotes and charge the highest acceptable price to those with one price quote. This implies that firms would charge the (1-q) agents with two offers,  $mc_j$ , and the q agents with one quote would pay  $R_j$ . As firms cannot distinguish between the two types of consumers, they choose a convex combination of these prices. A key feature of the model is then that with q = 0, the model collapses to a model of Bertrand competition with no price dispersion.

Comparing the two average prices, the mean posted price is more sensitive to changes in the reservation price, while the mean transacted price is more sensitive to changes in marginal cost. The distribution of transacted prices places a greater weight on firms with prices that compete to attract consumers with multiple price quotes, while the distribution of posted prices places a greater weight on those firms that seek to exploit single-quote consumers.

## C. Equilibrium

A competitive equilibrium<sup>20</sup> is characterized by the distribution of prices in each country  $\{G_D(p), G_F(p), G_D^*(p), G_F^*(p)\}$ , wages  $\{w, w^*\}$ , capital rental rates  $\{i, i^*\}$ , prices for securities  $\chi(s^{t+1}|s^t)$ , decision rules  $\{c, k', l, n_D, n_F, r_D, r_F, b'\}$  for agents in country D and decision

<sup>&</sup>lt;sup>20</sup>We have dropped the state  $s^t$  in the definition to save space.

rules  $\{c, k', l, n_D^*, n_F^*, r_D^*, r_F^*, b'^*\}$  for agents in country F and aggregate decision rules  $\{C, C^*, K, K^*, N_D, N_F, R_D, R_F, N_D^*, N_F^*, R_D^*, R_F^*, B^{*'}, B'\}$  such that (i) individual decision rules solve the consumer's problem; (ii) each firm chooses a price to solve its problem; (iii) goods markets and bond markets clear; and (iv) individual and aggregate decisions are consistent.

The key element in solving the model is determining the equilibrium reservation prices in each country and requires the following result.

PROPOSITION 1. The highest price is equal to the shoppers' reservation price  $(R_j = \overline{P}_j)$ .

Proof. Two cases are possible  $R_j > \overline{P}_j$  or  $R_j < \overline{P}_j$ . If  $R_j > \overline{P}_j$ , then consumers are willing to accept a higher price than the highest price on the market. In this case, firms charging  $\overline{P}_j$  can raise their price to  $R_j$  and earn more profits. If  $R_j < \overline{P}_j$ , then we need to check two more cases: (a)  $mc_j \leq R_j < \overline{P}_j$  and (b)  $R_j < \overline{P}_j = mc_j$ . In case a, those firms charging  $\overline{P}_j$  are earning no profits and are at least as well off by charging  $\overline{P}_j = R_j$ . In case b, firms cannot earn higher profits by lowering their price. In this case all firms are charging the same price  $\overline{P}_j = mc_j$ . From the consumer's optimization, we have that  $R_j \geq \hat{\theta}w + mc_j$ , which implies  $R_j > \overline{P}_j$  and a contradiction.

Proposition 1 implies that no consumer returns empty handed so that the amount of a good purchased is equal to the mass of shoppers sent out searching for that good. If some shoppers returned without a good  $(R_j < \overline{P}_j)$  those firms charging  $\overline{P}_j$  can increase profits by charging  $R_j$ . Similarly, if shoppers are willing to pay more than the highest price in the market  $(R_j > \overline{P}_j)$ , then those firms charging  $\overline{P}_j$  can increase profits by raising their price to  $R_j$ .

Based on proposition 1., the reservation price is implicitly defined as the R that satisfies

(19) 
$$R_j = \widehat{\theta}w + P_j \left(R_j; R_j = \overline{P}_j\right),$$

where  $P_j(R_j)$  is as defined in equation (3). In other words, the family is indifferent between purchasing a unit at the reservation price or sending out one more shopper who expects to purchase at the average price and incurs some search costs  $\hat{\theta}w$ . Because of the trade-off between working and search, the opportunity cost of search depends on the forgone wage and the disutility of search relative to work. Combining equations (17) and (19) we can solve for the key prices in the distribution as

$$P_j\left(\overline{P}_j\right) = mc_j + \frac{q\widehat{\theta}w}{1-q}, \qquad \overline{P}_j = mc_j + \frac{\widehat{\theta}w}{1-q}, \qquad \underline{P}_j = mc_j + \frac{\widehat{\theta}wq}{(2-q)(1-q)}$$

These moments of the price distribution hold for  $q \in (0, 1)$ . As  $q \to 0$ , more of the mass of firms charge prices near  $\underline{P}_j(q)$ . When q = 0, all firms play Bertrand and charge their marginal cost so that the price distribution is degenerate. As  $q \to 1$ , more of the firms charge prices closer to reservation price  $\overline{P}_j(q)$  and  $\lim_{q\to 1} \overline{P}_j(q) \to \infty$ . The highest price is unbounded for q = 1 because it is the ability of searchers to purchase from lower priced firms that bound the price distribution. If there were an arbitrarily large but finite exogenous reservation price, when q = 1 firms would charge this reservation price. That the price distribution is discontinuous as  $q \to 0$  and that all firms charge the monopoly price when  $q \to 1$  are both standard features of search models (Diamond 1971).

These prices imply that the local wage matters because it affects 1) firms' cost of production and 2) the opportunity cost of search and hence the consumer's reservation price. Because the distribution of prices is endogenous, the distribution of prices across countries will differ whenever the opportunity cost of search, or wage, differs.

Consider the average transaction price for country D goods in each market,  $\{P_D, P_D^*\}$ . These prices differ whenever the wage differs across countries. As the wage difference increases, so will the price difference as firms with relatively high-wage consumers increase their markup. This is also true for prices from country F firms, since consumers in relatively high-wage countries will be willing to pay relatively high prices, for both goods. This is because the local wage influences the price local consumers are willing to accept. It also implies that shocks that affect the relative wage between countries will shift the distribution of prices being charged for the same goods in different countries and lead to deviations from the LOP.

The equilibrium prices show that search generates pricing-to-market. In particular, firms price discriminate across countries based on the local wage of consumers in the destination market. In related work, Alessandria and Kaboski (2004) use highly disaggregate U.S. export data to show that U.S. exporters systematically charge relatively high prices for goods exported to markets with relatively high wages. Across countries, they find that a doubling of destination wages leads to a 20 percent higher price. The model developed here is consistent with this observation.

## 4. Calibration

In this section, we describe the choice of benchmark parameter values. We report these choices in Table 3. Much of our calibration follows directly from work by Backus et al. (1995) and Chari et al. (2002). The major departure from previous studies is in calibrating the search process.

Consider the costs of search. First, there is a direct resource cost measured as the amount of time spent in search activities relative to work. Second, there is the value of time spent searching relative to work. Last, search gives firms market power so that they can charge a price above marginal cost. We calibrate the ratio of shopping to market production time to be 1/4 to match evidence from the 2003 American Time Use Survey.<sup>21</sup> To measure the value of time spent searching relative to working we match the amount of price dispersion in the data. There are many measures of price dispersion. We focus on the coefficient of variation and set this to equal 25 percent.<sup>22</sup> This is slightly higher than what Sorenson (2000) finds for pharmaceutical drug prices in a small geographic region in upstate New York and slightly lower than what Brynjolfsson and Smith (1999) find for books and music CDs on the Internet. For comparison, in 1987 the coefficient of variation in the price of ready-mix concrete among U.S. manufacturers was 37.2 percent (Roberts and Supina

 $<sup>^{21}</sup>$ The 2003 American Time Use Survey (ATUS) finds that the average American spends 0.81 hours a day shopping and 3.32 hours in market production. Time-use studies in Canada, the U.K, Australia, and New Zealand find that time spent shopping is approximately equal to 1/4 of time spent in market production.

<sup>&</sup>lt;sup>22</sup>Transaction prices are considerably less dispersed than posted prices, since the coefficient of variation of transacted prices is 11 percent in the benchmark economy.

2000). Finally, we set the markup to be 22 percent. This is higher than the 15 percent that Basu and Fernald (1995) find for gross output in the U.S. and lower than the 31 percent that Berry, Levinsohn, and Pakes (1995) estimate for new cars sold in the U.S. Given that cars are an important component of trade flows we set the markup between these two estimates.

Consider the other preferences parameters. The discount factor  $\beta$  is set to give an annual real return of 4 percent. There is a wider range of estimates for the curvature parameter,  $\gamma$ . We set this to 2 as in Backus et al. (1995).

The parameter  $\hat{\kappa}$  determines the fraction of agents that are employed. We set  $\hat{\kappa}_t$  so that 60 percent of the agents are employed in steady state. This is consistent with the U.S. economy. To measure how  $\hat{\kappa}_t$  changes over time, we follow the approach of Chari et al. (2004) and use the laborleisure first order condition (equation 14) to solve for the shocks. A slightly modified version of this condition is

$$\widehat{\kappa}_t = w_t c_t^{-\gamma} / R_t.$$

To derive  $\hat{\kappa}_t$  requires a time series for the consumer's resource cost of consumption. As we lack a measure of  $R_t$  we instead measure

$$\widetilde{\kappa}_t = w_t c_t^{-\gamma} / P_t,$$

using data on real wages from the BLS and per capita consumption from the BEA from 1975:1 to 2003:1. Given our process for  $\tilde{\kappa}_t$ , we then choose a process for  $\hat{\kappa}_t$  so that the measured shocks in the model have the same properties as in the data. In particular, we focus on matching the correlation between our measured taste shock and technology shock and the relative volatility of these two series.

The four shocks in the model take the following process,

$$\log z_t = \rho_z \log z_{t-1} + \varepsilon_z, \qquad \log z_t = \rho_z \log z_{t-1}^* + \varepsilon_z^*,$$
$$\log \hat{\kappa}_t^* = \rho_\kappa \log \hat{\kappa}_{t-1} + \varepsilon_\kappa, \qquad \log \hat{\kappa}_t = \rho_\kappa \log \hat{\kappa}_{t-1}^* + \varepsilon_\kappa^*,$$

where  $(\varepsilon_z, \varepsilon_z^*, \varepsilon_\kappa, \varepsilon_\kappa^*)$  are normally distributed, mean zero shocks. The standard deviation of the technology shock (taste shock) in each country is  $\sigma_z$  ( $\sigma_\kappa$ ). We set  $\rho_z = 0.98$  to match the data for the U.S. in our sample. We follow Chari et al. (2002) and let the technology shocks have a positive correlation of 0.25. We follow Stockman and Tesar (1995) and assume that Domestic technology shocks are not correlated with Foreign taste shocks. In measuring technology and taste shocks in the U.S. we find that they have a negative correlation of -0.45. We assume that Foreign and Domestic taste shocks are uncorrelated.

For the production technology, we set the capital share parameter  $\alpha = 1/3$  and the depreciation rate  $\delta = .025$ . The latter generates an annual depreciation rate of 10 percent. These are standard estimates for the U.S.

We consider a capital adjustment cost function of the form  $\phi (x/k) = v (x/k - \delta)^2 / 2$ . With this specification, in steady state both the total and marginal costs are zero. In all of our experiments the parameter v is chosen so that the standard deviation of investment relative to the standard deviation of output matches the data.

Consider the final goods technology parameters. In our model,  $\omega$  is the elasticity of substitution between Domestic and Foreign goods. There are a wide range of measures of this parameter. We follow Backus et al. (1995) and use an elasticity of 1.5. To set  $a_1$  and  $a_2$ , note that in a symmetric steady state  $y_D/y_F = [a_1/a_2]^{-\omega}$ . We set this ratio to equal 15 percent. This amount overstates the trade share for the U.S. and understates the trade share for other G7 countries. This along with a normalization generates the values of  $a_1$  and  $a_2$  reported in Table 3.

Table 3: Parameter Values

Preferences		Target
Time Preference	$\beta = 0.99$	4% annual return
Probability of one quote	q = 0.10811	Coefficient of variat $=25\%$
Disutility of Search	$\theta = 3.48634$	Markup = 0.22
Disutility of Work	$\widehat{\kappa}=8.50861$	n/l = 1/4
Market to search productivity	z = 0.2616	l = 0.60
Intertemporal Elasticity	$\gamma = 2$	Backus et al. (1994)
Elasticity of substitution	$\omega = 1.5$	Backus et al. (1994)
Trade weights in final good	$a_1 = 0.897$	imports = 0.15
	$a_2 = 0.282$	
Technology		
Capital's share in production	$\alpha = 0.33$	NIPA
Adjustment cost	v = 5.75	$\frac{\sigma_L}{\sigma_V} = 3.2$
Depreciation	$\delta=0.025$	10% annually
Shocks		
Productivity	$\rho_z = .98, \mathrm{corr}$	$(\varepsilon_z, \varepsilon_z^*) = 0.25, \operatorname{var}(\varepsilon_z) = \operatorname{var}(\varepsilon_z^*) = (0.0062)^2,$
Taste	$\rho_{\kappa} = .98, \mathrm{con}$	$\operatorname{rr}\left(\varepsilon_{\widetilde{\kappa}},\varepsilon_{z}\right) = -0.45, \operatorname{corr}\left(\varepsilon_{\widetilde{\kappa}},\varepsilon_{\widetilde{\kappa}}^{*}\right),\\\operatorname{ar}\left(\varepsilon_{\widetilde{\kappa}}^{*}\right) = 1.4 \cdot \operatorname{var}\left(\varepsilon_{z}\right) = 0$

## 5. Findings

There are multiple notions of average prices in our model. From the standpoint of the consumer the relevant average price is the transaction price; however, statistical agencies such as the BLS primarily sample posted prices. Based on this consideration, we measure the price of a good as its average posted price. We then use these average posted prices to measure ideal price indices that do not include search considerations. The Domestic price level equals  $P(s^t) = (a_1^{\omega} P_D(s^t)^{1-\omega} + a_2^{\omega} P_F(s^t)^{1-\omega})^{\frac{1}{1-\omega}}$  and the Foreign price level equals  $P^*(s^t) = (a_2^{\omega} P_D^*(s^t)^{1-\omega} + a_1^{\omega} P_F^*(s^t)^{1-\omega})^{\frac{1}{1-\omega}}$ where the average prices are determined by the posted price equation (18).

To understand the role of search frictions, the properties of the benchmark model are compared to one in which the time spent in search, price dispersion, and markups are very close to zero. This alternative parameterization, which we denote No Search, is essentially a version of Backus et al. (1995) in which labor adjustment occurs along the extensive margin. We also consider a model in which there is search, but prices are not dispersed, denoted No Dispersion.

Our analysis proceeds in three steps. First, we consider the relationships between relative prices, wages, and costs based only on the pricing equations. Next, we consider the behavior of the model in response to shocks to productivity and the labor wedge similar to those in the data. We finish by examining the sensitivity of the model to the structure of consumer search and international asset trade.

## A. Relative Prices and Relative Costs

Here we examine the relationship between relative cost and relative price fluctuations. We take a sequence of exogenous relative costs and then use the pricing rules to derive international relative prices. We consider two sources of changes to relative costs: either a change in relative productivity or a change in relative wages. For simplicity we assume that labor is the only input to production so that marginal costs move one-for-one with wages and productivity. Table 4 reports the volatility of international relative prices normalized by the volatility of relative costs for the benchmark model. For comparison, we include results when there is no dispersion in prices and when there is no search. We show that it is fluctuations in relative wages that generate fluctuations in disaggregate relative prices as firms systematically vary their markups across countries. This pricing-to-market leads to larger real exchange rate fluctuations and smaller terms of trade fluctuations relative to fluctuations in relative costs.

First, consider the case when there are no search frictions. The market is perfectly competitive so that firms set price equal to marginal cost. The source of fluctuations in relative costs does not matter for relative price fluctuations. Firms pass through changes in marginal cost directly to prices so that there are no deviations from the LOP. We see from the column labelled "No Search" in Table 4 that the terms of trade moves one-for-one with the ratio of marginal costs and the real exchange rate moves 70 percent as much as the terms of trade.

Second, consider the case where there are search frictions but no dispersion in prices within countries. In this case, the source of the shock to marginal cost determines the size of relative price fluctuations. When shocks are due to changes in productivity, the model does not generate any deviations from the LOP. Consequently, all real exchange rate fluctuations are due to changes in the terms of trade. However, compared to the no search case, now the terms of trade moves only 82 percent as much as relative costs. The terms of trade is less volatile because firms endogenously vary their markup in response to changes in costs. Consider a positive shock to foreign productivity. In this case, the cost of producing a foreign good has fallen relative to the cost of searching for it. Since the average price is a convex combination of the search cost and the production cost, the drop in the production cost leads to a less than proportional drop in the market price as firms raise markups.

Next, consider fluctuations in relative costs due to fluctuations in relative wages. In this case, there are fluctuations in disaggregated relative prices, since firms charge relatively high prices in the relatively high-wage country. Consider a decrease in the Foreign relative wage. This lowers both the opportunity cost of time of Foreign shoppers and the cost of producing Foreign goods. In Foreign, the Foreign firms keep their markup constant and pass this reduction in cost fully to Foreign consumers. In Domestic, the Foreign firms lower their prices but raise their markups as their customer's search costs have not changed. Domestic firms also lower their prices to Foreign consumers. This cut in Domestic export prices mitigates the change in the terms of trade and reinforces the change in the real exchange rate. Hence, the terms of trade fluctuates less, only 64 percent of relative costs, and the real exchange rate fluctuates relatively more, close to 75 percent of relative costs.

Finally, consider fluctuations in relative costs when prices are dispersed. Dispersion arises when firms compete to attract additional consumers to their store. Since consumers purchase relatively more at stores with lower prices, the distribution of transaction prices differs from the distribution of posted prices. Since the distribution of posted prices is more responsive to changes in costs than the distribution of transacted prices, the terms of trade (real exchange rate) fluctuations are 74 (51) percent of relative cost fluctuation. When changes in relative costs are due to changes in relative

			No Disper	rsion	Benchmark		
Std dev.	Data	No Search	Productivity	Wages	Productivity	Wages	
lop	0.26	0	0	0.18	0	0.26	
tot	0.86	1	0.82	0.64	0.74	0.47	
rer	0.92	0.7	0.57	0.75	0.51	0.78	
All variables are measured relative to fluctuations in relative unit labor costs.							

Table 4: International Relative Prices and Exogenous Fluctuations in Relative Unit Labor Costs

\*See notes at the end of tables.

wages, disaggregate relative price fluctuations are largest since the mean posted price depends quite a bit on the search cost. Thus, changes in the relative cost of search lead to larger fluctuations in the ratio of posted prices. These larger deviations from the LOP reduce the volatility of the terms of trade to 48 percent and raise the volatility of the real exchange rate to 78 percent.

## **B.** General Equilibrium Shocks

In this section we examine the aggregate implications of the model in response to technology and taste shocks. The HP filtered statistics of the data, the benchmark model, and some variations on that economy are reported in Table 5. The statistics for the U.S. economy are computed for the period 1975:1 to 2003:1 from BEA and BLS data. The statistics on international comovements are an average of comovements between the U.S. and the other G7 countries from 1980:1 to 2002:4. The cross-country data are from the OECD's Main Economic Indicators.

Overall, we find that the benchmark model generates international relative prices that match the data qualitatively in that they are volatile, persistent, and highly correlated. The ordering of relative price fluctuations also matches the data as relative unit labor costs are the most volatile series, followed in descending order by the real exchange rate, terms of trade and disaggregate relative prices. In absolute terms, depending on the series these relative price fluctuations are between approximately 43 and 80 percent as volatile as those in the data. Relative price fluctuations are also as persistent as GDP, with autocorrelations of approximately 0.69. Thus the benchmark model appears successful in accounting for a number of features of international relative price fluctuations.

The benchmark model substantially reduces the gap between theory and data. To begin with, the model generates fluctuations in disaggregate relative prices, which are 0.51 as large as GDP fluctuations. Compared to the model without search, the benchmark model generates terms of trade fluctuations that are almost twice as large (1.36 vs. 0.70) and real exchange rate fluctuations over 3.5 times larger (1.76 vs. 0.49). The benchmark model generates more volatile prices because relative unit labor costs are over 3.5 times more volatile (2.45 vs. 0.70) than in the standard model.

The model with search generates more volatile relative wage fluctuations because search alters the labor supply decision. It is straightforward to manipulate the risk-sharing condition to solve for the relative wage

$$w/w^* = \hat{\kappa}/\hat{\kappa}^*$$
.

This condition makes it clear that the absolute magnitude of relative wage fluctuations is due entirely to the volatility of the taste shocks. The difference in relative volatility across models is then due to the volatility of output, which is considerably lower in the search model (1.02 vs. 2.00). Output is less volatile in the search model because consumption and investment are bundled with search activities. Thus, while productivity shocks are good opportunities to work, they are also relatively good opportunities to search since many low -cost goods are available. Thus, search makes labor supply in the market less responsive to productivity shocks. Taste shocks are good opportunities to both search and work, but because of the diminishing returns to labor, the increase in market production is limited. Both of these effects tend to lower the volatility of output in the search model.

Consider now the rest of the statistics for the benchmark economy in Table 5. The benchmark model generates comovements with output of investment, consumption, employment, and net exports that are consistent with those in the data. The size of consumption fluctuations are similar to the data (0.74 vs. 0.81), although they are considerably more procyclical than in the data (0.99 vs. 0.85). Employment is not as strongly procyclical as in the data (0.55 vs. 0.86). Net exports are also less volatile (0.12 and 0.43) and more countercyclical than in the data (-0.7 vs. -0.47). The volatility of the import ratio, measured as the ratio of real imports to absorption of local goods, is also not volatile enough (1.19 vs. 2.65). Trade flows are also too smooth in the No Search model. Based on the work of Boileau (1999), it is likely that separately modelling trade in investment and consumption goods would substantially increase the volatility of trade flows.

The benchmark model generates comovements in economic activity that are similar to the data. Investment is slightly less synchronized in the model than in the data (0.15 vs. 0.20), while employment is slightly too synchronized (0.28 vs. 0.21). Finally, consumption correlations are quite low in the model (0.21) as in the data (0.23). The largest difference between the model and the data is in the comovement of output, which is essentially uncorrelated in the model but has a correlation of 0.33 in the data.

When there is no search, consumption is highly correlated across countries (0.53), while output, investment, and employment are negatively correlated. This is because there are benefits to smoothing consumption and shifting production across countries. For this reason, Backus et al. (1995) and Baxter (1995) have argued that the large comovements in economic activity in the data are a puzzle. In the benchmark model, international business cycles are closer to the data, since the gains to both risk-sharing and production-shifting are reduced. These channels are less important in the search model because labor is used in market and shopping production. Overall, the gap between the theory and the data is small, particularly compared with the standard model in which there is no search.

The benchmark model generates international relative price volatility similar to the data without generating counterfactual predictions elsewhere. This has proven difficult in other models. In particular, Backus et al. (1995) show that when domestic and foreign goods are highly complementary, the standard RBC model can match the volatile terms of trade in the data but with trade flows that are about 1 percent as volatile as in the data. Chari et al. (2002) show that a sticky price model can generate very volatile and somewhat persistent real exchange rates, but with domestic business cycles that differ substantially from the data.

Finally, the model does not quantitatively resolve the Backus-Smith puzzle. The correlation between the real exchange rate and consumption ratio is now equal to -0.985. The reason that the mechanism in the model does not have much of an impact on the Backus-Smith puzzle is that relative wages and relative prices move together so that the ratio of resource costs is almost perfectly correlated with the ratio of market prices.

## C. Sensitivity

Consumer search and price dispersion: The properties of the consumer search model without price dispersion<sup>23</sup> are reported in the column titled No Dispersion in Table 5. The model generates the correct ordering of international relative price volatility, but in absolute terms, relative prices are considerably less volatile than in the benchmark economy. To understand the lower volatility, consider the first order condition, which determines the consumer's allocation of the two goods (equation 11). This condition is rewritten with the functional form of the aggregator and substituting out the reservation prices,

$$\left[\frac{y_{D}\left(s^{t}\right)}{y_{F}\left(s^{t}\right)}\right]^{-\frac{1}{\gamma}} = \frac{\widehat{\theta}w\left(s^{t}\right) + P_{D}\left(s^{t}\right)}{\widehat{\theta}w\left(s^{t}\right) + P_{F}\left(s^{t}\right)}.$$

For prices to be dispersed it must be the case that the value of the search effort  $(\hat{\theta}w = R_j - P_j)$  is large relative to the average price. This means that changes in average prices have a relatively small impact on resource costs so that changes in relative prices do not change relative allocations much. This lowers the incentive to increase production following taste or technology shocks. Without price dispersion, relative costs move one-for-one with the ratio of resource costs, and this leads

 $<sup>^{23}</sup>$ The model is actually calibrated so that the reservation price is 0.01 percent larger than the mean transaction price.

			Complete	Incomplete		
Statistics	Data	Benchmark	No	No	Benchmark	No
Statistics	Data	Dencimark	Dispersion	Search	Dencimark	Search
Std dev						
Y	1.43	1.02	1.68	2.00	1.08	1.85
NX/Y	0.43	0.12	0.11	0.12	0.09	0.15
Std dev						
$\mathbf{C}$	0.81	0.73	0.35	0.29	0.67	0.34
Ι	3.20	3.20	3.20	3.20	3.13	3.20
L	0.87	1.02	1.05	1.06	1.02	1.02
tot	2.49	1.36	0.65	0.82	1.55	0.64
rer	2.70	1.76	0.75	0.57	2.22	0.45
rulc	2.92	2.45	0.86	0.82	2.98	0.64
lop	0.84	0.51	0.20	0	0.69	0
IR	2.65	1.19	1.21	1.23	1.45	0.96
Domestic correl w/ Y						
С	0.85	0.99	0.94	0.92	0.97	0.97
Ι	0.93	0.93	0.98	0.99	0.93	0.99
L	0.86	0.58	0.90	0.94	0.64	0.92
NX/Y	-0.47	-0.69	0.40	-0.12	-0.72	-0.50
Int Correl						
Y	0.33	0.01	-0.13	-0.14	-0.11	0
$\mathbf{C}$	0.19	0.24	0.43	0.52	0.30	0.33
Ι	0.20	0.15	-0.12	-0.16	0.07	-0.05
L	0.21	0.28	-0.23	-0.26	0.14	-0.09
$\left(\frac{C^*}{C}, \frac{P}{P^*}\right)$	0.35	-0.985	-1.00	-1.00	-0.991	-0.997
Persistence						
Υ	0.86	0.69	0.68	0.68	0.68	0.68
rer	0.82	0.69	0.72	0.72	0.69	0.72
tot	0.80	0.70	0.70	0.72	0.68	0.72
lop	0.84	0.70	0.72	0.72	0.69	0.72
rulc	0.83	0.69	0.68	-	0.69	-

# Table 5: Business Cycle Statistics

\*See notes at the end of tables.

to much larger changes in allocations. This has two effects on relative price behavior. First, the increased demand for the low-price good leads to increased production, which because of diminishing returns dampens fluctuations in relative unit labor costs. Second, increased production raises GDP volatility, which lowers the relative volatility of the relative wage.

Consider next how price dispersion affects consumption in each country. The modified risksharing condition states that  $(c_t/c_t^*)^{-\gamma} = (R_t/R_t^*)$ . With price dispersion the resource cost of a unit of consumption places a relatively large weight on the value of the non-traded search services, which depends on the local wage. Clearly, then, a change in the relative wage will lead to bigger changes in relative resource costs than in the real exchange rate. This weakens the incentive to smooth consumption across countries and explains the relatively low comovements of consumption across countries in the price dispersion model. The lower comovement in consumption generates larger volatility in the consumption ratio, and this contributes to the increased volatility in the real exchange rate in the price dispersion economy.

Incomplete markets: Now we explore whether the properties of relative prices or the Backus-Smith puzzle depend on the assumption that there is a complete set of contingent claims. We do this by restricting the set of assets available. In theory, market incompleteness should help to resolve the Backus-Smith puzzle since shocks will lead to wealth redistributions. These wealth effects also introduce another channel through which relative wages and hence relative prices will fluctuate. We find that introducing market incompleteness has mixed success. On the one hand, the fit between the model and the data improves since relative prices and output are even more volatile. On the other hand, the model's ability to address the Backus-Smith puzzle actually worsens slightly.

We introduce market incompleteness by assuming that agents are restricted to trading a single nominal bond. This bond is denominated in units of the Domestic wage. The Domestic agent's budget constraint is now

$$\sum_{j \in \{D,F\}} n_j \left(s^t\right) H_j \left(R_j \left(s^t\right) | s^t\right) P_j \left(R_j \left(s^t\right); s^t\right) + \chi \left(s^t\right) b \left(s^t\right) \le w \left(s^t\right) l \left(s^t\right) + i \left(s^t\right) k \left(s^t\right) + \Pi \left(s^t\right) + b \left(s^{t-1}\right),$$

where  $k(s^0)$ ,  $k^*(s^0)$ ,  $b(s^0)$  are given and  $\chi(s^t)$  is the price of the bond. The Foreign consumer's budget constraint is modified similarly. The new risk-sharing condition is then

$$E_t \left[ \frac{u_{ct+1}}{u_{ct}} \frac{R_t}{R_{t+1}} \right] = E_t \left[ \frac{u_{ct+1}^*}{u_{ct}^*} \frac{R_t^*}{R_{t+1}^*} \right].$$

Thus, with incomplete markets, the relationship between relative resource costs and marginal utilities holds in expectation.

Table 3 reports the properties of an incomplete markets economy with the same parameters as the benchmark model.<sup>24</sup> For comparison, we also include the properties of the no search economy. The only sizable difference between the benchmark model with complete and incomplete markets is in the behavior of relative prices. With incomplete markets, relative unit labor costs are 20 percent more volatile (3.05 vs. 2.45). This is a result of larger relative wage fluctuations, which, because of the pricing-to-market, also lead to larger fluctuations in disaggregate relative prices (0.69 vs. 0.51). The increased relative unit labor cost fluctuations result in larger terms of trade<sup>25</sup> (1.56 vs. 1.36) and real exchange rate fluctuations (2.27 vs. 1.75). These relative prices are slightly more persistent than in the complete markets case (0.70 vs. 0.69).

Introducing incomplete markets does not help to resolve the Backus-Smith puzzle. In fact, compared to the complete markets case, the puzzle is even greater in that the correlation has declined from -0.985 to -0.991. This is because, with the larger relative wage fluctuations, there is more pricing-to-market so that the ratio of resource costs moves more closely with the real exchange rate.

## 6. Conclusions

Previous research on fluctuations in international relative prices has focused on models with sticky prices or flexible prices with constant markups. These models do not generate volatile and

<sup>&</sup>lt;sup>24</sup>To ensure stationarity, we include a small adjustment cost on bond holding for Domestic agents.

 $<sup>^{25}</sup>$ The terms of trade is most influenced by the decision to measure average prices as posted prices. If average prices are transaction based, then the terms of trade volatility is 1.70 with complete markets and 2.00 with incomplete markets.

persistent fluctuations in international relative prices at the aggregate and disaggregate levels. Moreover, these models are inconsistent with fluctuations in international relative prices of certain goods such as used cars. These models are also at odds with the evidence of substantial dispersion in the price of identical goods within countries. This paper develops a model of consumer search consistent with price dispersion within countries. These search frictions lead firms to price-to-market based on the opportunity cost of their customers' time, which depends on the local wage. Changes in international relative wages lead firms to endogenously vary their markup across countries so that disaggregate relative prices fluctuate. These fluctuations are as persistent as the fluctuations in relative wages.

Our search-based model of pricing-to-market generates international relative prices that more closely match the data along several dimensions. First, the model generates persistent deviations from the LOP. Second, these deviations from the LOP lead to real exchange rates that are more volatile than the terms of trade as in the data. The volatility of these international relative prices relative to international relative unit labor costs closely matches the data. Finally, in absolute terms, the type of pricing considered here leads to fluctuations in the real exchange rate that are 3.5 times as volatile compared to a model with constant markups and can account for approximately 80 percent of the volatility of the real exchange rate and 80 percent of the deviations from the LOP in the data.

The model studied here makes some progress toward resolving the Backus-Smith puzzle. With search and price dispersion, the resource cost per unit of consumption differs from the market price, since it includes the value of search activities. Thus, we show that in theory, with complete markets the real exchange rate is not perfectly negatively correlated with consumption ratios. However, quantitatively, we find that this modified risk-sharing condition does little to resolve the Backus-Smith puzzle since the value of search moves closely with the local price level so that the real exchange rate and ratio of resource costs move closely together. Perhaps, allowing for heterogeneity in search and work abilities will help to resolve this puzzle by allowing the ratio of resource costs to move differently than the ratio of price levels.

For the sake of exposition, we have made a number of simplifying assumptions. First, we have assumed that all goods are search goods. It is likely that search is relatively more important for some goods than others. For instance consumption goods are likely to be relatively more search intensive than investment goods, although the gains to search may be higher for durable goods. Second, we have assumed all goods are traded. Other work has shown that introducing non-traded goods may be important for understanding international relative price fluctuations. Third, we do not consider monetary shocks. In the data there is a strong relationship between real and nominal exchange rates. The current model is silent on this aspect of the data. A natural extension would include sticky wages with monetary shocks. With pricing-to-market of the form considered here, it is likely that sticky wages may generate persistent relative price fluctuations. Finally, a key issue is how does price dispersion evolve over time? This is central to explaining relative price behavior in all models. Empirical evidence of disaggregate prices and price dispersion would greatly improve our understanding of the pricing decisions of firms as well as the international transmission of business cycles.

## References

- [1] Abbott, T. 1989. "Price Dispersion in U.S. Manufacturing."
- [2] Aguiar, M. and E. Hurst. 2004. "Consumption vs. Expenditure." mimeo Chicago GSB.
- [3] Alessandria, G. 2004. "International Deviations From the Law of One Price: The Role of Search Frictions and Market Share." *International Economic Review*, 45:1263-1291.
- [4] Alessandria, G. and J. Kaboski. 2004. "Violating Purchasing Power Parity." Federal Reserve Bank of Philadelphia Working Paper #04-19.
- [5] Backus, D., P. Kehoe and F. Kydland. 1995. International Business Cycles: Theory and Evidence, in T. Cooley, ed., Frontiers in Business Cycle Research. 331-356.
- [6] Backus, D. and G. Smith. 1993. "Consumption and Real Exchange Rates in Dynamic Economies with Non-Traded Goods." *Journal of International Economics*, 35:297-316.
- [7] Basu, S. and J. Fernald. 1995. "Are Apparent Productivity Spillovers a Figment of Specification Error?" Journal of Monetary Economics, 36:165-188.
- [8] Baxter, M. 1995. "International Trade and Business Cycles." Handbook of International Economics, in G. Grossman and K. Rogoff, vol. 3:1801-64.
- [9] Benhabib, J., R. Rogerson and R. Wright. 1991. "Homework in Macroeconomics: Household Production and Aggregate Fluctuations." *Journal of Political Economy*, 99: 1166-87.
- [10] Bergin, P. and R. Feenstra. 2001. "Pricing-To-Market, Staggered Contracts, and Real Exchange Rate Persistence." *Journal of International Economics*, 54: 333-59.
- [11] Berry, S., J. Levinsohn, and A. Pakes. 1995. "Automobile Prices in Market Equilibrium." Econometrica, 63/4: 841-890.
- [12] Betts, C. and M. Devereux. 2000. "Exchange Rate Dynamics in a Model of Pricing-to-Market." Journal of International Economics, 50: 215-44.
- [13] Boileau, M. 1999. "Trade in Capital Goods and the Volatility of Net Exports and the Terms of Trade." Journal of International Economics, 48: 347-365.
- [14] Borenstein, S. and N. Rose. 1994. "Competition and Price Dispersion in the U.S. Airline Industry." Journal of Political Economy, 103.
- [15] Brynjolfsson, E. and M. Smith. 1999. "Frictionless Commerce? A Comparison of Internet and Convential Retailers." mimeo, M.I.T.
- [16] Burdett, K. and K. Judd. 1983. "Equilibrium Price Dispersion." *Econometrica*, 51: 955-69.
- [17] Chari, V.V., P. Kehoe and E. McGrattan. 2002. "Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?" *Review of Economics Studies*, 69/3 533-63.
- [18] Chari, V.V., P. Kehoe and E. McGrattan. 2004. "Business Cycle Accounting." Federal Reserve Bank of Minneapolis Staff Report 328.
- [19] Cooley, T. and E. Prescott. 1995. Economic Growth and Business Cycles, in T. Cooley, ed., Frontiers in Business Cycle Research. 1-38.

- [20] Crucini, M., C. Telmer and M. Zachariadas. 2004. "Understanding European Real Exchange Rates." Vanderbilt University Working Paper No. 01-W20R.
- [21] Dahlby, B. and D. West. 1986. "Price Dispersion in an Automobile Insurance Market." Journal of Political Economy, 94: 418-38.
- [22] Diamond, P. 1971. "A Model of Price Adjustment." Journal of Economic Theory, 3: 156-68.
- [23] Dornbsuch, R. 1987. "Exchange Rates and Prices." American Economic Review, 77: 93-106.
- [24] Dumas, B. 1993. "Dynamic Equilibrium and the Real Exchange Rate in a Spatially Separated World." *Review of Financial Studies*, 5: 153-180.
- [25] Engel, C. 1993. "Real Exchange Rates and Relative Prices: An Empirical Investigation." Journal of Monetary Economic, 32: 35-50.
- [26] Engel, C. 1999. "Accounting for U.S. Real Exchange Rate Changes." Journal of Political Economy, 107: 507-538.
- [27] Engel, C. and J. Rogers. 1996. "How Wide Is the Border?" American Economic Review, 86: 1112-25.
- [28] Feenstra, R., J. Gagnon and M. Knetter. 1996. "Market Share and Exchange Rate Pass-Through in World Automobile Trade." *Journal of International Economics*, 40: 187-207.
- [29] Giovannini, A. 1988. "Exchange Rates and Traded Goods Prices." Journal of International Economics, 24: 45-68.
- [30] Goldberg, P., and M. Knetter. 1997. "Goods Prices and Exchange Rates: What Have We Learned?" *Journal of Economic Literature*, 35: 1243-72.
- [31] Goldberg, P. and F. Verboven. 2001. "The Evolution of Price Dispersion in The European Car Market." *Review of Economic Studies*, 68: 811-48.
- [32] Greenwood, J. and Z. Hercowitz. 1991. "The Allocation of Capital and Time Over the Business Cycle." *Journal of Political Economy*, 99: 1188-214.
- [33] Hall, R. 1997. "Macroeconomic Fluctuations and the Allocation of Time." Journal of Labor Economics, 15: 223-250.
- [34] Heathcote, J. and F. Perri. 2002. "Financial Autarky and International Business Cycles." Journal of Monetary Economics, 49/3, 601-627.
- [35] Kravis, I. and R. Lipsey. 1977. "Export Prices and The Transmission of Inflation." American Economic Review, 67: 155-63.
- [36] Krugman, P. 1987. "Pricing to Market When the Exchange Rate Changes." in S.W. Arndt and J.D. Richardson. *Real Financial Linkages Among Open Economies*. pp 49-70.
- [37] Knetter, M. 1989. "Price Discrimination by U.S. and German Exporters." American Economic Review, 79: 198-210.
- [38] Knetter, M. 1993. "International Comparisons of Pricing-to-Market Behavior." American Economic Review, 83: 473–86.

- [39] Lach, S. 2002. "Existence and Persistence of Price Dispersion: An Empirical Analysistextquotedblright NBER Working Paper 8737.
- [40] Lapham, B. and M. Vigneault. 2001. "National Markets and International Relative Prices." manuscript, Queen's University.
- [41] Obstfeld, M. and K. Rogoff. 2000. "The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?" NBER Macroeconomics Annual 2000, 339-390.
- [42] Marston, R. 1990. "Pricing to Market in Japanese Manufacturing." Journal of International Economics, 29: 217-36.
- [43] McKenzie, D. and E. Schargrodsky. 2004. "Buying Less, But Shopping More: Changes in Consumption Patterns During A Crisis." mimeo.
- [44] Mertens, Y. and V. Ginsburgh. 1985. "Product Differentiation and Price Discrimination in the EC: The Case of Automobiles." *Journal of Industrial Economics*, 34: 151-66.
- [45] Mussa, M. 1986. Nominal Exchange Rate Regimes and the Behavior of Real Exchange Rates: Evidence and Implications. In K. Brunner and A. Meltzer, eds., *Carnegie Rochester Series on Public Policy* 25: 117-213.
- [46] Parsley, D. and S.J. Wei. 2001. "Explaining the Border Effect: the Role of Exchange Rate Variability, Shipping Costs, and Geography." *Journal of International Economics*, 55:87-105.
- [47] Pratt, J., D. Wise and R. Zeckhauser. 1979. "Price Differences in Almost Competitive Markets." Quarterly Journal of Economics, 93:189-211.
- [48] Roberts, M. and D. Supina. 2000. "Output Price and Markup Dispersion in Micro Data: The Roles of Producer Heterogeneity and Noise." textitAdvances in Applied Microeconomics, Vol. 9, Industrial Organization, Baye, Michael, ed.
- [49] Rogerson, R. 1988. "Indivisible Labor, Lotteries and Equilibrium." Journal of Monetary Economics, 21: 3-16.
- [50] Rogoff, K. 1996. "The Purchasing Power Parity Puzzle." Journal of Economic Literature, 34: 647-668.
- [51] Sorenson, A. 2000. "Equilibrium Price Dispersion in Retail Markets for Prescription Drugs." Journal of Political Economy, 108: 833-850.
- [52] Stockman, A. and L. Tesar. 1995. "Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements." *American Economic Review*; 85: 168-85.
- [53] Stigler, G. 1961. "The Economics of Information." Journal of Political Economy, 64: 213-225.
- [54] Stigler, G. and J. Kindahl. 1970. The Behavior of Industrial Prices. NBER, New York.
- [55] Stiglitz, J. 1989. Imperfect Information in the Product Market. Handbook of Industrial Organizations, Chapter 13.

#### Data Appendix

Computing International Relative Prices and Comovements. Relative prices are from the OECD's International Trade and Competitiveness Indicators. The three series are Index of Relative Unit Labour Cost Manufacturing Sector, Common Currency (Overall Competitiveness), Index of Relative Export Price Manufactured Goods in a Common Currency (Overall Competitiveness), Index of Relative Consumer Price in a Common Currency, (Overall Competitiveness). International comovements are based on output, investment, employment, and consumption from the OECD's Main Economic Indicators from 1980:1 to 2003:1.

Computing Trade Shares for the G7. Here we describe how we construct import  $(\alpha_i)$  and export shares  $(\hat{\alpha}_i)$  for G7 countries.

- For each OECD country *i*, we calculate the value of imports as a share of nominal GDP denoted by  $\theta_{it}$ . These data are from the Main Economic Indicators at current prices. For the G7 countries, we define  $\alpha_{it} = \theta_{it}$  as the period *t* import share.
- For the G7, we measure exports by each destination which we define as  $\alpha_{it}^{j}$  where *i* is the source country and *j* is the destination country. The data is from the OECD's STAN Bilateral Trade Database.
- For each destination country, we define  $\eta_{it}^j$  as the fraction of country j's imports which are from country i.
- Given these three series  $(\theta_{it}, \hat{\alpha}_{it}, \alpha_{it}^j)$ , we compute a country's exports as a share of its trading partner's GDP  $\hat{\alpha}_{it} = \sum_{j \in OECD} \alpha_{it}^j \cdot \eta_{it}^j \cdot \theta_{jt} / \sum_{j \in OECD} \alpha_{it}^j$ .
- Import and export shares are then computed as an average over the period.
- The data are not comprehensive of all exports, but cover between 50 and 75 percent of exports depending on the country.

Computing Taste and Technology Shocks. We measure technology shocks as is standard in the literature. In particular, given a series  $\{y_t, l_t, k_t\}$  we solve for  $\log z_t = y_t - (1 - \alpha) l_t - \alpha k_t$ . Since comparable capital stocks are not available across countries, we follow Backus et al. (1995) and omit the capital stock portion of the expression. To solve for the shocks to the marginal utility of leisure we use  $\log \tilde{\kappa}_t = w_t - p_t - \gamma c_t$ . Real wages are measured as nominal average hourly earnings of production workers (CES050000006) deflated by the U.S. City Average Consumer Price Index (LNS1000000). Employment is measured as Civilian Employment (LNS12000000) and real Output and Consumption are from the BEA. Consumption is converted into a per-worker series. The time period is 1975:1 to 2004:4. All series are seasonally adjusted. From the residuals, we estimate the following bivariate AR(1) with a linear trend using the Seemingly Unrelated Regression procedure

$$\begin{bmatrix} z_t \\ \widetilde{\kappa}_t \end{bmatrix} = \begin{bmatrix} \alpha_z \\ \alpha_{\widetilde{\kappa}} \end{bmatrix} + \begin{bmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{bmatrix} \begin{bmatrix} z_{t-1} \\ \widetilde{\kappa}_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{z,t} \\ \varepsilon_{\widetilde{\kappa},t} \end{bmatrix}, \begin{bmatrix} \varepsilon_{z,t} \\ \varepsilon_{\widetilde{\kappa},t} \end{bmatrix} = N(0,\Sigma)$$

The elements of the linear trend and A are below. Standard errors are in parentheses.

$$\begin{bmatrix} \alpha_z \\ \alpha_{\widetilde{\kappa}} \end{bmatrix} = \begin{bmatrix} -0.11 \ (0.144) \\ -0.34 \ (0.257) \end{bmatrix}, \qquad A = \begin{bmatrix} 0.977 \ (0.031) & 0.008 \ (0.009) \\ 0.058 \ (0.054) & 0.981 \ (0.016) \end{bmatrix}$$

*Computing Moments of U.S. Economy.* Quantities are from the BEA from 1975:1 to 2003:1. Net exports are measured in nominal terms as a fraction of nominal GDP. Employment is measured as non-farm employees (CES0000000001) from the BLS.

## Footnotes to the Tables

- Table 1. \*The statistics are based on logged and HP-filtered quarterly data with a smoothing parameter of 1600.
- Table 2. \*The statistics are based on logged and HP-filtered quarterly data with a smoothing parameter of 1600.
- Table 4. \*The statistics are based on logged and HP- filtered quarterly data with a smoothing parameter of 1600.
- Table 5. \*The domestic statistics are for the US in the period 1975:1 to 2003:1. The statistics on international comovements are an average of comovement between the U.S. and G7 countries from 1980:1 to 2002:4. The model statistics are computed from a simulation of 1000 iterations with 120 periods. All series but net exports have been logged and HP filtered with a smoothing parameter of 1600. The TOT is measured using relative manufactured export unit values. The correlation between the consumption ratio and real exchange rate is for the U.S. and a European aggregate from Chari et al. (2002).