



WORKING PAPERS

RESEARCH DEPARTMENT

WORKING PAPER NO. 04-8
Multiple Stages of Processing and the Quantity Anomaly
in International Business Cycle Models*

Kevin X.D. Huang
Federal Reserve Bank of Philadelphia
and
Zheng Liu
Emory University

August 2004

FEDERAL RESERVE BANK OF PHILADELPHIA

Ten Independence Mall, Philadelphia, PA 19106-1574 • (215) 574-6428 • www.phil.frb.org

WORKING PAPER NO. 04-8
**Multiple Stages of Processing and the Quantity Anomaly
in International Business Cycle Models***

Kevin X.D. Huang
Federal Reserve Bank of Philadelphia
and
Zheng Liu
Emory University

August 2004

*Huang: Research Department, Federal Reserve Bank of Philadelphia, Ten Independence Mall, Philadelphia, PA 19106, USA; and CIRPÉE, Canada. Liu: Department of Economics, Emory University, Atlanta, GA 30322, USA, and CIRPÉE, Canada. The authors are grateful to Olivier Blanchard, Henning Bohn, V.V. Chari, Todd Clark, Russell Cooper, Peter Ireland, Robert King, Nobuhiro Kiyotaki, Narayana Kocherlakota, Sylvain Leduc, Lee Ohanian, Alain Paquet, Louis Phaneuf, Cedric Tille, Kei-Mu Yi, Christian Zimmermann, and seminar participants at Boston University, Clark University, CREFE, Emory University, George Washington University, Tufts University, the University of California at Santa Barbara, Utah State University, the European Central Bank, the Federal Reserve Bank of Atlanta, the Federal Reserve Bank of Kansas City, the Federal Reserve Bank of Minneapolis, the 2000 Midwest Macroeconomics Conference, the 2000 World Congress of the Econometric Society, the 2000 Theory and Method in Macroeconomics Conference in Paris, the 2002 Annual Meeting of the Society for Economic Dynamics, the 2002 European Economic Association Annual Congress, the 2002 Federal Reserve System Committee Meeting on International Economics, and the 2003 Missouri Economic Conference for helpful comments. They also thank Sally Burke for editorial assistance. The views expressed herein are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of Philadelphia or the Federal Reserve System. The majority of the research in this paper was conducted when Kevin Huang was at the Federal Reserve Bank of Kansas City.

Abstract

We construct a two-country DSGE model with multiple stages of processing and local-currency staggered price-setting to study cross-country quantity correlations driven by monetary shocks. The model embodies a mechanism that propagates a monetary surprise in the home country to lower the foreign price level while restraining the home price level from rising too quickly. It does so through reducing material costs in terms of the foreign currency unit while dampening the upward movements in the costs in terms of the home currency unit, both in absolute terms and relative to the costs of primary factors. We show that, through this mechanism and a resulting factor substitution effect, the model is able to generate significant cross-country quantity correlations, with correlations in consumption considerably lower than correlations in output, as in the data.

JEL classification: E32, F31, F41

Key Words: Production chain; Vertical international trade; International comovement

1 Introduction

A central challenge to models of international business cycles is to explain the observed patterns in cross-country quantity correlations. While the data typically reveal significant cross-country correlations in consumption and in output, with the former considerably lower than the latter, standard models typically fail to predict these patterns. A one-sector international real business cycle model, for instance, usually generates low or negative international correlations in output and near-perfect correlations in consumption [e.g., Baxter (1995) and Backus, Kehoe and Kydland (1992, 1995)]. Incorporating multiple sectors into this class of models helps raise output correlations to be positive, but consumption correlations remain too high [e.g., Ambler, Cardia, and Zimmermann (2002), and Kouparitsas (forthcoming)]. The standard international monetary business cycle models do not fair better: models with sellers' local currency pricing tend to generate large and positive correlations in consumption and small or even negative correlations in output [see the survey in Lane (2001)], while models with buyers' local currency pricing mostly predict near-zero correlations in both output and consumption, unless shocks are assumed to be highly correlated across countries [e.g., Chari, Kehoe, and McGrattan (2002)]. This mismatch between the models' predictions and the data is often referred to as the "quantity anomaly" in the international business cycle literature.

In this paper, we propose a mechanism that may help resolve the quantity anomaly. Our model incorporates the observation that production of consumption goods in the modern world economy typically involves multiple stages of processing and multiple border-crossing of intermediate goods.¹ In the model, we consider monetary shocks as a driving force of the international quantity correlations, and we stress the role of multiple stages of processing and trade in propagating the shocks via local currency pricing and staggered price contracts.² We

¹For empirical evidence on the vertical patterns of production and trade, see, for example, Hummels, et al. (1998), Hummels, et al. (2001), and Yi (2003). Yi (2003) shows that the multiple "border-crossing" of a same set of goods may be a key for unlocking the mysteries of the large rise in world trade over the past decades and the non-linear pattern in this rise.

²Empirical evidence on local currency pricing behaviors has been documented at least since Page (1981), and a review of this literature can be found in Goldberg and Knetter (1997), among others. For empirical evidence on staggered price contracts, see the survey by Taylor (1999). There is a large body of work that aims at rationalizing the behaviors of local currency pricing and staggered price setting, which are now two standard assumptions adopted by the literature in explaining the observed large and persistent movements in the relative prices of traded goods. This strand of literature shows that the failure of the law of one price among traded goods accounts for a major proportion of the observed real exchange rate fluctuations. See, Engel (1993, 1999), Knetter (1993), Gagnon and Knetter (1995), and Engel and Rogers (1996), among many others.

choose to focus on the role of monetary shocks and nominal rigidities because much empirical evidence suggests a close connection between international monetary regimes and the behaviors of real exchange rates and other macroeconomic variables [e.g., Basu and Taylor (1999), and Kiley (1999)].³ We do not assume a large cross-country correlation in the shocks, because such an assumption does not seem to be supported by empirical evidence, and, after all, it does not help to get the cross-country correlations in consumption and in output into the right order.

The model embodies a mechanism that propagates a monetary surprise in the home country to lower the foreign price level while restraining the home price level from rising too quickly. It does so through reducing material costs and thus marginal costs in terms of the foreign currency unit, while dampening the upward movements in these costs in terms of the home currency unit. In consequence, it tends to amplify and align the movements in the two countries' real aggregate demands and real purchasing powers and to attenuate the terms-of-trade effect, which would otherwise benefit home households and firms at the cost of their foreign counterparts. These all help increase the cross-country quantity correlations. Further, the reduction in the costs of materials relative to the costs of primary factors creates an incentive for firms to substitute intermediate inputs for primary factors. The possibility of factor substitution effectively constrains the tendency of international comovement in labor hours, and with nonseparable preferences in consumption and leisure, the cross-country correlation in consumption as well. Through this mechanism, the model with multiple stages of processing and trade not only helps increase the international quantity correlations, it helps increase the output correlation more than it helps increase the consumption correlation, putting the two quantity correlations into the right order.

The mechanism that we propose sheds light on some observed features of international quantity correlations. For instance, the output correlations between the United States and

Rogoff (1996), Devereux (1997), and Betts and Devereux (2000) provide useful surveys of this literature, while Betts and Devereux (1996, 2000), Chang and Devereux (1998), Devereux and Engel (1998), Bacchetta and van Wincoop (2000), Bergin and Feenstra (2001), and Chari, Kehoe, and McGrattan (2002) develop general equilibrium models of this type.

³Recent years have witnessed a burgeoning interest in developing a new workhorse for open-economy macroeconomics in which monetary shocks are the source of international business cycle fluctuations. Devereux (1997) and Lane (2001) provide useful surveys of this "new open-economy macroeconomics" literature. For a more recent paper that features monetary shocks as a driving force of international business cycle dynamics, see Alvarez, Atkeson, and Kehoe (forthcoming). The recent work by Kehoe and Perri (2002) emphasizes the role of incomplete asset markets in helping establish a correct order between cross-country consumption correlation and output correlation.

other OECD countries are considerably larger than the consumption correlations, and this pattern also holds between the OECD countries in general (see Table 1). The predicted quantity correlations in our model with four stages of processing are broadly consistent with this observation (see Table 10). As we argue in the Calibration section, a number of four stages seems to be an empirically plausible estimate for the production structure in a typical industrialized country such as the United States, since these countries tend to produce and trade with each other a broad range of commodities, from agriculture and mining to manufacturing and services. By contrast, less developed countries, such as the emerging market economies, tend to produce and trade with each other mostly raw materials and simple primary goods.⁴ We argue that a smaller number of processing stages does not seem to be an empirically objectionable estimate for the production of a typical good in these economies. Incidentally, the quantity correlations across the Latin American countries, though also typically higher in output than in consumption, are significantly weaker than those across the OECD countries (see Table 2). Evidently, the predictions of our model with two processing stages are suggestive of the quantity correlations observed between the Latin American economies (see Table 10). In this sense, our model sheds some light on why the quantity correlations between the OECD countries are systematically greater than those within the Latin America region, while within each region the consumption correlations are considerably smaller than the output correlations.

Our work is closely related to several recent contributions. Betts and Devereux (2000) construct a model with buyers' local currency pricing and pre-set prices to address the issue of exchange rate persistence and cross-country quantity co-movements. They emphasize the role of government spending shocks in accounting for the observed order between cross-country output correlation and cross-country consumption correlation. Chari, Kehoe, and McGrattan (2002) present a model with local currency pricing and staggered price-setting, which is similar to the degenerate case of ours with a single stage of processing. They assume a large cross-country correlation in monetary shocks and they aim at generating exchange rate volatility and persistence. Bergin and Feenstra (2001) consider a roundabout input-output structure in conjunction with translog preferences to investigate exchange rate behavior.

⁴According to the OECD (1994), 55 percent of world trade in 1991 occurred between the OECD countries, 82 percent of which was trade in manufactured goods. In contrast, according to the World Development Indicators, about 80 percent of the trade between countries in the Latin America region is accounted for by trade in primary goods.

In what follows, Section 2 sets up the model with multiple stages of processing and trade, Section 3 provides some intuitions to help gain insights into the shock propagation mechanism embodied in the model, Section 4 presents numerical simulations and discusses the model's quantitative implications, and Section 5 concludes. The Appendix presents some analytical results to formally demonstrate the mechanism.

2 A Two-Country Model with Multiple Stages of Processing

The world economy consists of a home country and a foreign country. Each country is populated by a large number of identical, infinitely lived households, each consuming an aggregate consumption good and supplying labor and capital to domestic firms. The production of final consumption or investment goods in each country requires $N \geq 1$ stages of processing, from raw materials to intermediate goods, and then to more advanced intermediate goods, and so on. At each stage, there is a continuum of domestic firms producing differentiated products indexed in the interval $[0, 1]$, with an elasticity of substitution $\theta > 1$. The production of each type of intermediate goods at stage $n \in \{2, \dots, N\}$ uses all types of intermediate goods produced at stage $n - 1$, either domestically produced or imported, along with labor and capital supplied by domestic households. The production of goods at the first stage uses domestic primary factors only.

At each date t , the world economy experiences a realization of shocks s_t . The history of events up to t is denoted by $s^t \equiv (s_0, \dots, s_t)$, with probability $\pi(s^t)$. The initial realization s_0 is given.

The representative household in the home country has preferences represented by the expected utility function

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) U(C(s^t), \frac{M(s^t)}{\bar{P}_N(s^t)}, L(s^t)),$$

where $\beta \in (0, 1)$ is a subjective discount factor, $C(s^t)$ and $M(s^t)$ denote consumption and money balances, respectively, $L(s^t)$ is labor hours, and $\bar{P}_N(s^t)$ denotes the home price level. The household faces a sequence of budget constraints

$$\begin{aligned} \bar{P}_N(s^t) Y_N(s^t) + \sum_{s^{t+1}} D(s^{t+1}|s^t) B(s^{t+1}) + M(s^t) \\ \leq W(s^t) L(s^t) + R(s^t) K(s^{t-1}) + \Pi(s^t) + B(s^t) + M(s^{t-1}) + T(s^t) \end{aligned} \quad (1)$$

for all t and all s^t , where $B(s^{t+1})$ is a one-period nominal bond that costs $D(s^{t+1}|s^t)$ units of home currency at s^t and pays off one unit of home currency at $t + 1$ if s^{t+1} is realized, $W(s^t)$

is the nominal wage rate, $R(s^t)$ is the capital rental rate, $K(s^{t-1})$ is the beginning-of-period capital stock, $\Pi(s^t)$ is the household's claim to firms' profits, and $T(s^t)$ is a nominal lump-sum transfer from the home government. The term $Y_N(s^t)$ in the budget constraint is the purchase of final goods to be used for consumption or investment. In particular, it is given by

$$Y_N(s^t) = C(s^t) + K(s^t) - (1 - \delta)K(s^{t-1}) + \psi \frac{(K(s^t) - K(s^{t-1}))^2}{K(s^{t-1})}, \quad (2)$$

where $K(s^t)$ denotes the end-of-period capital stock, $\delta \in (0, 1)$ is the depreciation rate of capital, and $\psi > 0$ is a capital adjustment cost parameter.

The final goods are an aggregate composite of domestic and imported finished goods (i.e., produced at stage N), with the aggregation technology given by

$$Y_N(s^t) = \bar{Y}_{NH}(s^t)^\gamma \bar{Y}_{NF}(s^t)^{1-\gamma}, \quad (3)$$

where $\bar{Y}_{NH} = \left(\int_0^1 Y_{NH}(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}$ is a composite of goods produced by home firms and $\bar{Y}_{NF} = \left(\int_0^1 Y_{NF}(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}$ is a composite of goods imported from the foreign country, both produced at stage N . The parameter θ determines the steady state markup of price over marginal cost, and the parameter γ measures the share of expenditures on domestically produced goods in total expenditures on all goods.

The household maximizes utility subject to (1)-(3) and a borrowing constraint $B(s^t) \geq -\bar{B}$, for some large positive number \bar{B} , for each s^t and each $t \geq 0$, with initial conditions $K(s^{-1})$, $M(s^{-1})$, and $B(s^0)$ given. The resulting demand functions for a type i finished good produced in the home country and imported from the foreign country are respectively given by

$$Y_{NH}^d(i, s^t) = \frac{\gamma \bar{P}_N(s^t)}{\bar{P}_{NH}(s^t)} \left[\frac{P_{NH}(i, s^t)}{\bar{P}_{NH}(s^t)} \right]^{-\theta} Y_N(s^t), \quad (4)$$

$$Y_{NF}^d(i, s^t) = \frac{(1 - \gamma) \bar{P}_N(s^t)}{\bar{P}_{NF}(s^t)} \left[\frac{P_{NF}(i, s^t)}{\bar{P}_{NF}(s^t)} \right]^{-\theta} Y_N(s^t), \quad (5)$$

where $\bar{P}_{NH}(s^t) = \left(\int_0^1 P_{NH}(i, s^t)^{1-\theta} di \right)^{\frac{1}{1-\theta}}$ is the price index of finished goods produced and sold in the home country, and $\bar{P}_{NF}(s^t) = \left(\int_0^1 P_{NF}(i, s^t)^{1-\theta} di \right)^{\frac{1}{1-\theta}}$ is the price index of finished goods imported from the foreign country. The overall price level in the home country is an average of the two price indices, that is,

$$\bar{P}_N(s^t) = \bar{\gamma} \bar{P}_{NH}(s^t)^\gamma \bar{P}_{NF}(s^t)^{1-\gamma}, \quad (6)$$

where $\bar{\gamma} = \gamma^{-\gamma} (1 - \gamma)^{\gamma-1}$.

A defining feature of the model is that the production of finished goods in each country involves multiple stages of processing and multiple boarder-crossing of intermediate goods. The production of a stage-1 good of type $i \in [0, 1]$ in the home country requires home primary factors as inputs, with a standard Cobb-Douglas production function given by

$$Y_{1H}(i, s^t) + Y_{1H}^*(i, s^t) = K_1(i, s^t)^\alpha L_1(i, s^t)^{1-\alpha}, \quad (7)$$

where K_1 and L_1 are home capital and labor inputs, and the parameter $\alpha \in (0, 1)$ measures the cost share of capital. The output is either sold in the home country ($Y_{1H}(i)$) or exported ($Y_{1H}^*(i)$) to the foreign country.

To produce a stage- n good of type $i \in [0, 1]$, for $n \in \{2, \dots, N\}$, requires not only home primary factors but a composite of stage- $(n-1)$ goods, both domestically produced and imported. The production function is a constant-return-to-scale technology given by

$$Y_{nH}(i, s^t) + Y_{nH}^*(i, s^t) = \bar{Y}_n(i, s^t)^\phi [K_n(i, s^t)^\alpha L_n(i, s^t)^{1-\alpha}]^{1-\phi}, \quad (8)$$

where K_n and L_n are home capital and labor inputs, and $\bar{Y}_n = \bar{Y}_{n-1,H}(s^t)^\gamma \bar{Y}_{n-1,F}(s^t)^{1-\gamma}$ is an aggregate of two composites of intermediate goods, $\bar{Y}_{n-1,H} = \left(\int_0^1 Y_{n-1,H}(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}$, purchased from domestic firms, and, $\bar{Y}_{n-1,F} = \left(\int_0^1 Y_{n-1,F}(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}$, imported from foreign suppliers.

Firms at each processing stage are price-takers in their input markets and monopolistic competitors in their output markets, where they set prices in the buyers' local currency (e.g., Betts and Devereux (1996, 2000) and Chari, et al. (2002)), with pricing decisions staggered between firms within each processing stage (e.g., Taylor (1980)). More specifically, at each date, and on each stage of processing and trade, half of the home producers cannot adjust their prices, while the other half can each choose a pair of new prices: $P_{nH}(i, s^t)$ in the home currency unit for its product to be sold in the home market and $P_{nH}^*(i, s^t)$ in the foreign currency unit for its product to be exported to the foreign market. Once a new price is set, it will remain in effect for two periods.⁵

⁵We set the duration of price contracts to two periods so as to minimize the amount of exogenous staggering in price-setting. In this sense, the model's equilibrium dynamics are mostly driven by the endogenous propagation mechanism embodied in the production and trading chain rather than by the exogenous nominal staggering *per se*.

At each date t , upon the realization of s^t , a home firm $i \in [0, 1]$ at stage $n \in \{1, \dots, N\}$ that can set new prices chooses $P_{nH}(i, s^t)$ and $P_{nH}^*(i, s^t)$ to maximize

$$\sum_{\tau=t}^{t+1} \sum_{s^\tau} D(s^\tau | s^t) \{ [P_{nH}(i, s^t) - V_n(i, s^\tau)] Y_{nH}^d(i, s^\tau) + [e(s^\tau) P_{nH}^*(i, s^t) - V_n(i, s^\tau)] Y_{nH}^{*d}(i, s^\tau) \}, \quad (9)$$

taking as given the unit cost function $V_n(i, s^\tau)$, the output demand schedules $Y_{nH}^d(i, s^\tau)$ and $Y_{nH}^{*d}(i, s^\tau)$, and the nominal exchange rate $e(s^t)$, measured by units of domestic currency per unit of foreign currency.

The unit cost function V_1 for a firm at stage 1 can be derived from minimizing the cost $WL_1 + RK_1$ subject to the production function (7). In particular, V_1 is given by

$$V_1(s^t) \equiv V_1(i, s^t) = \bar{\alpha} R (s^t)^\alpha W (s^t)^{1-\alpha}, \quad (10)$$

where $\bar{\alpha} = \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)}$. The unit cost function V_n , for $n \geq 2$, can similarly be derived from minimizing $\int_0^1 P_{n-1,H}(j) Y_{n-1,H}(i, j) dj + \int_0^1 P_{n-1,F}(j) Y_{n-1,F}(i, j) dj + WL_n + RK_n$ subject to (8). The resulting unit cost function is given by

$$V_n(s^t) \equiv V_n(i, s^t) = \bar{\phi} \bar{P}_{n-1}(s^t)^\phi V_1(s^t)^{1-\phi}, \quad (11)$$

where $\bar{\phi} = \phi^{-\phi} (1 - \phi)^{-(1-\phi)}$, and $\bar{P}_{n-1}(s^t)$ is the price index for all goods produced by home and foreign firms at stage $n - 1$ and used by i at stage n as inputs. In particular, the price index of all stage- n goods is given by

$$\bar{P}_n(s^t) = \bar{\gamma} \bar{P}_{nH}(s^t)^\gamma \bar{P}_{nF}(s^t)^{1-\gamma}, \quad (12)$$

where $\bar{P}_{nH} = \left(\int_0^1 P_{nH}(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}$ and $\bar{P}_{nF} = \left(\int_0^1 P_{nF}(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}$ are the price indices of stage- n home goods and of stage- n imported goods, respectively.

The output demand schedules resulting from the cost-minimization problems are

$$Y_{nH}^d(i, s^t) = \left[\frac{\phi}{1-\phi} \right]^{1-\phi} \frac{\gamma \bar{P}_n(s^t)}{\bar{P}_{nH}(s^t)} \left[\frac{P_{nH}(i, s^t)}{\bar{P}_{nH}(s^t)} \right]^{-\theta} \left[\frac{\bar{P}_n(s^t)}{V_1(s^t)} \right]^{-(1-\phi)} \tilde{Y}_{n+1}(s^t), \quad (13)$$

$$Y_{nF}^d(i, s^t) = \left[\frac{\phi}{1-\phi} \right]^{1-\phi} \frac{(1-\gamma) \bar{P}_n(s^t)}{\bar{P}_{nF}(s^t)} \left[\frac{P_{nF}(i, s^t)}{\bar{P}_{nF}(s^t)} \right]^{-\theta} \left[\frac{\bar{P}_n(s^t)}{V_1(s^t)} \right]^{-(1-\phi)} \tilde{Y}_{n+1}(s^t), \quad (14)$$

for $n \in \{1, \dots, N - 1\}$, where $\tilde{Y}_{n+1} \equiv \int_0^1 [Y_{n+1,H}(j) + Y_{n+1,H}^*(j)] dj$ is a linear aggregate of all goods produced at stage $n + 1$ in the home country. We can similarly derive the demand schedules $Y_{nH}^{*d}(i, s^t)$ and $Y_{nF}^{*d}(i, s^t)$ for the foreign country. Equation (13) says that the demand for a type i good produced at stage n in either country is higher if its price relative to the

price index of all such goods is lower, if the price index of these goods relative to the overall price index of stage- n goods is lower, or if the cost of materials relative to the cost of primary factors is lower.

With the unit cost functions and output demand schedules derived from the embedded cost-minimization problem, maximizing (9) gives rise to the optimal price-setting rules

$$P_{nH}(i, s^t) = \frac{\theta}{\theta - 1} \frac{\sum_{\tau=t}^{t+1} \sum_{s^\tau} D(s^\tau | s^t) V_n(s^\tau) Y_{nH}^d(i, s^\tau)}{\sum_{\tau=t}^{t+1} \sum_{s^\tau} D(s^\tau | s^t) Y_{nH}^d(i, s^\tau)}, \quad (15)$$

$$P_{nH}^*(i, s^t) = \frac{\theta}{\theta - 1} \frac{\sum_{\tau=t}^{t+1} \sum_{s^\tau} D(s^\tau | s^t) V_n(s^\tau) Y_{nH}^{*d}(i, s^\tau)}{\sum_{\tau=t}^{t+1} \sum_{s^\tau} D(s^\tau | s^t) e(s^\tau) Y_{nH}^{*d}(i, s^\tau)}, \quad (16)$$

where $n \in \{1, \dots, N\}$. The price-setting rule in (15) says that the optimal price set for the home market in home currency unit is a constant markup over a weighted average of the firm's marginal costs within the duration of its price contract, where the weights are the normalized quantity of demand for its output in the corresponding periods. The price-setting rule in (16) can be interpreted similarly, where the currency units are appropriately converted using the nominal exchange rate.

The problems facing the households and firms in the foreign country are analogous.

We now specify monetary policy processes. There is a monetary authority in each country. Newly created money by the monetary authority in one country is injected into the domestic economy via a lump-sum transfer to domestic households so that $T(s^t) = M(s^t) - M(s^{t-1})$ and $T^*(s^t) = M^*(s^t) - M^*(s^{t-1})$. The money stocks in the two countries grow according to $M(s^t) = \mu(s^t)M(s^{t-1})$ and $M^*(s^t) = \mu^*(s^t)M^*(s^{t-1})$, where the money growth rates $\mu(s^t)$ and $\mu^*(s^t)$ follow stationary stochastic processes given by

$$\ln \mu(s^t) = \rho_\mu \ln \mu(s^{t-1}) + \varepsilon_t, \quad \ln \mu^*(s^t) = \rho_\mu \ln \mu^*(s^{t-1}) + \varepsilon_t^*, \quad (17)$$

where $\rho_\mu \in (0, 1)$, and ε_t and ε_t^* are uncorrelated Gaussian processes with zero mean and finite variance σ_μ^2 .

Finally, the market clearing conditions for the primary factors in the home country requires that $\sum_{n=1}^N \int_0^1 K_n^d(i, s^t) di = K(s^{t-1})$ and $\sum_{n=1}^N \int_0^1 L_n^d(i, s^t) di = L(s^t)$. The market clearing conditions for the primary factors in the foreign country are similar. The world bond market clearing condition requires that $B(s^t) + B^*(s^t) = 0$.

An equilibrium for this economy is a collection of allocations $\{C(s^t), K(s^t), L(s^t), M(s^t), B(s^{t+1})\}$ for households in the home country; allocations $\{C^*(s^t), K^*(s^t), L^*(s^t), M^*(s^t), B^*(s^{t+1})\}$ for households in the foreign country; allocations $\{Y_{nH}(i, s^t), Y_{nH}^*(i, s^t), K_n(i, s^t), L_n(i, s^t)\}$

and prices $\{P_{nH}(i, s^t), P_{nH}^*(i, s^t)\}$ for firms in the home country, where $i \in [0, 1]$ and $n \in \{1, \dots, N\}$; allocations $\{Y_{nF}(i, s^t), Y_{nF}^*(i, s^t), K_n^*(i, s^t), L_n^*(i, s^t)\}$ and prices $\{P_{nF}(i, s^t), P_{nF}^*(i, s^t)\}$ for firms in the foreign country, where $i \in [0, 1]$ and $n \in \{1, \dots, N\}$; price indices $\{\bar{P}_n(s^t), \bar{P}_n^*(s^t)\}$, for $n \in \{1, \dots, N\}$; wages $\{W(s^t), W^*(s^t)\}$; capital rental rates $\{R(s^t), R^*(s^t)\}$; and bond prices $D(s^{t+1}|s^t)$; that satisfy the following four conditions: (i) taking wages, capital rental rates, and prices as given, households' allocations solve their utility maximization problems; (ii) taking wages, capital rental rates, and all prices but its own as given, each firm's allocations and price solve its profit-maximization problem; (iii) domestic capital, labor, and money markets and world asset markets clear; (iv) monetary policies are as specified.

In what follows, we focus on a symmetric equilibrium in which all firms in the same price-setting cohort at the same stage of production and trade in the same country make identical pricing decisions. In such an equilibrium, firms are identified by the country in which they operate, the stage at which they produce and trade, and the time at which they can change prices. Thus, from now on, we can drop the individual firms' indices i and j , and denote, for example, by $P_{nH}(t)$ the price set for the home market by a firm that operates in the home country, produces at stage n , and gets the chance to change its price at date t .

3 International Monetary Transmission: Some Intuitions

This section illustrates the basic intuitions behind the mechanism through which multiple stages of processing help propagate monetary shocks across countries to generate the observed patterns of international quantity correlations. The main idea is that a monetary expansion in, say, the home country, through home currency depreciation, tends to generate a fall in the foreign price level and a rise in the home price level when there are multiple stages of processing; the fall is larger and the rise is smaller, the greater is the number of the processing stages. The key to understanding how this mechanism works is understanding how, at a more advanced processing stage, material costs and thus marginal costs facing firms fall more in terms of the foreign currency unit while rising less in terms of the home currency unit, and complete adjustment of these variables takes a longer period of time. These patterns of marginal cost adjustments imply that the fall in the foreign price level is magnified and the rise in the home price level is attenuated on a period-by-period basis and complete adjustment of the two price levels requires a longer period of time.

The attenuation, through multiple processing stages, in the upward movements of the marginal costs in terms of the home currency unit, and thus in the home price level, following a home monetary expansion may sound intuitive and less surprising to the reader who is familiar with the closed-economy version of the model [e.g., Huang and Liu (2001)]. What is new in this open-economy setup here is the fall in the marginal costs in terms of the foreign currency unit, and thus in the foreign price level, and the magnification of its magnitude through the multiple processing stages following the home monetary expansion. Indeed, this is a unique feature and the novelty of the present open-economy model. It is therefore worth spending some effort to understand the intuitions behind this new feature.

We note first, as we show formally in the Appendix, that, under fairly general specifications of households' preferences, the assumption of competitive domestic factor and international asset markets imply that a monetary expansion in the home country, while resulting in a full rise in home factor prices immediately, leaves foreign factor prices unchanged and leads to a complete home currency depreciation. Since stage-1 production requires only primary factors, firms at this stage in the foreign country face unchanged marginal costs and so do firms at this stage in the home country once their marginal costs are converted into the foreign currency unit using the spot nominal exchange rates. As a consequence, neither of these firms has an incentive to change its price set for the foreign market. If all production and trade occurred at this single stage, the foreign price level would remain unchanged, and foreign aggregate demand would also remain unchanged. This is why the degenerate version of the model with a single processing stage fails as an international monetary transmission mechanism.

Consider next the case with two stages of processing. Stage-2 production requires not only primary factors but also material inputs from stage 1. Firms at this second stage in the foreign country still face untacked marginal costs, since their material suppliers do not change prices. These firms thus do not have incentives to change their prices set for the foreign market either. Meanwhile, firms at this second stage in the home country face only a partial rise in their marginal costs on impact, since half of their material suppliers cannot yet change prices. Because of the complete home currency depreciation, these marginal costs, once converted into the foreign currency unit, fall partially in effect. Hence, these firms, if they can set new prices, would partially lower their prices set for the foreign market. The foreign price level therefore declines partially both on impact and in the subsequent date owing to the two-period staggered price contracts.

Assume now there are three stages of processing. Firms at the third stage in the foreign country face a partial fall in their marginal costs both on impact and in the subsequent date owing to the fall in their material costs in these two periods. Thus, these firms, when they can set new prices, would partially lower their prices set for the foreign market. Further, the marginal costs facing firms at the third stage in the home country rise even less in terms of the home currency unit and thus fall even more in terms of the foreign currency unit than those at the second stage. In consequence, these firms, when they can set new prices, would lower their prices set for the foreign market by even more than would the home firms at the second stage. The foreign price level thus declines in not only the first two but also the third periods following the home monetary expansion, and by more than in the case with two stages of processing.

The patterns in foreign and home price dynamics at different processing stages following a home monetary expansion under empirically reasonable parameter values are illustrated in Tables 3 and 4. These patterns of price dynamics are the key to understanding how multiple stages of processing help propagate monetary shocks to generate the observed patterns of international quantity correlations. As is evident from the tables, with more than one stage of processing, the home monetary expansion tends to generate smaller rises in the home prices and larger falls in the foreign prices, and thus more aligned movements in both the home country's and the foreign country's real aggregate demands (Tables 5 and 6), real money balances and real purchasing powers (Table 7), and smaller terms-of-trade effect that would otherwise benefit home households and firms at the cost of their foreign counterparts (Table 8).⁶ These all help

⁶The foreign terms of trade of stage- n goods are defined as the price of its exported goods (adjusted for currency units) relative to the price of its imported goods at that stage. In a standard two-country sticky price model with buyers' local currency pricing [e.g., Betts and Devereux (2000) and Chari, et al. (2002)], as in the case of our model with a single processing stage, a home monetary expansion worsens the foreign terms of trade on impact and has no further effect on terms of trade in the subsequent periods (see the bottom row in Table 8). Thus, even though the demand for foreign's exported goods is boosted by the rise in real aggregate demand in the home country (see the bottom row in Table 5), and thus the foreign households have to work harder and invest more to meet the increased demand for foreign goods, the resulting increase in foreign's factor incomes is offset by its worsened terms of trade, leaving unchanged its real aggregate demand or real purchasing power (see the bottom rows in Tables 6 and 7, respectively). This is why the degenerate version of our model with a single stage of processing fails to generate cross-country quantity correlations. With multiple stages of processing, not only are the foreign's terms of trade less worsened at more advanced stages on impact, but they are actually reversed in sign in subsequent periods, with the improvements being more significant on a period-by-period basis and over longer periods of time (see the second to the fourth rows in Table 8). Consistently, real aggregate demand and real purchasing power in the foreign country increase and become more aligned with those in the

increase the cross-country quantity correlations. Furthermore, the dampened rises in material costs facing home firms in the face of the full rises in home factor prices, and the magnified falls in material costs facing foreign firms in the face of the untacked foreign factor prices, create incentives for firms to substitute intermediate inputs for primary factors — such incentives become stronger at a more advanced processing stage. This tends to restrain the cross-country correlation in hours worked from rising too much. Thus, and as we will show below through numerical simulations, if consumption and leisure are nonseparable in households' preferences, then our model with multiple stages of processing not only helps increase the cross-country quantity correlations, but it also helps increase the output correlation more than it helps increase the consumption correlation.

Formal analytical results are provided and further intuitions are discussed at length in the Appendix to illuminate in more detail the shock propagation mechanism embodied in multiple stages of processing. We now turn to showing, through simulations, that our model may indeed help resolve the international quantity anomaly present in most international business cycle models.

4 Resolving the International Quantity Anomaly: Simulations

A central challenge to models of international business cycles is that most theories predict cross-country correlations in consumption that are larger than those in output, while the opposite pattern holds in the data. Further, standard monetary business cycle models with local currency pricing and sticky prices typically predict cross-country correlations in consumption and in output close to zero and usually in the wrong order as well. In the previous section, we provided some intuitions as to why our model with multiple stages of processing may potentially raise cross-country quantity correlations and meanwhile create a wedge between the consumption correlation and the output correlation. In this section, we demonstrate this potential of our model through numerical simulations.

home country (see the second to the fourth rows in Tables 5 to 7, respectively). This is why our model with multiple stages of processing may potentially help generate significant cross-country quantity correlations.

4.1 Calibration

We start with calibrating the model's parameter values. We assume that households' period utility function takes the following form:

$$U(C, M/\bar{P}_N, L) = \left\{ \left[bC^\nu + (1-b) \left(\frac{M}{\bar{P}_N} \right)^\nu \right]^{1/\nu} (1-L)^\xi \right\}^{1-\sigma} / (1-\sigma). \quad (18)$$

The parameters to be calibrated include the subjective discount factor β , the preference parameters b , ν , ξ , and σ , the technology parameters α , γ , and θ , the capital depreciation rate δ , the adjustment cost parameter ψ , the number of processing stages N , the share of material input at each stage ϕ , and the monetary policy parameters ρ_μ and σ_μ . The calibrated parameter values are summarized in Table 9.

Since we set the length of each price contract equal to two model periods, and a typical contract in actual economies lasts for one year (as suggested by Taylor's (1999) survey), a period in the model corresponds to one-half of a year in the data. With this in mind, we set $\beta = 0.96^{1/2}$, so that the steady-state annualized real interest rate is equal to 4 percent, as suggested by the standard business cycle literature. The parameter ξ is chosen so that, in the steady state, a household devotes 1/4 of its time endowment to market activity. The parameter σ corresponds to the inverse of intertemporal elasticity of substitution (IES), and we set $\sigma = 3$ so that the IES is about 1/3, which lies in the range of IES estimates obtained by Vissing-Jorgensen (2002) for stock holders. To assign values for b and ν , we use the money demand equation (derived from households' first order conditions)

$$\ln \left(\frac{M(s^t)}{\bar{P}_N(s^t)} \right) = -\frac{1}{1-\nu} \ln \left(\frac{b}{1-b} \right) + \ln(C(s^t)) - \frac{1}{1-\nu} \log \left(\frac{r(s^t) - 1}{r(s^t)} \right),$$

where $r(s^t) = (\sum_{s^{t+1}} D(s^{t+1}|s^t))^{-1}$ is the gross nominal interest rate. The regression of this equation using the U.S. data, as performed in Chari, et al. (2000), suggests that $\nu = -1.56$ and $b = 0.94$.

We next set $\alpha = 1/3$ and $\delta = 0.04$ so that the baseline model predicts an annualized capital-output ratio of 2.6 and an investment-output ratio of 0.21. We vary the capital adjustment cost parameter ψ when computing the equilibrium dynamics for different values of N , so that the standard deviation of investment is three times as large as that of real GDP. In a balanced-trade steady state, $\gamma = Y_H/Y$ corresponds to the share of domestically produced goods in real GDP. We set $\gamma = 0.9$, so that the import share in GDP is 10 percent. The parameter θ determines the steady-state markup by firms at each processing stage. Based on the work

of Basu (1996), Basu and Fernald (1997, 2000), and Chari, et al. (2000), we set $\theta = 13$, corresponding to a markup of $\mu = 1.08$.

A simple autoregression using quarterly M1 data in the postwar U.S. economy results in an AR(1) coefficient of 0.68 in the money growth process. Since a period in the model corresponds to two quarters, we set $\rho_\mu = 0.68^2$. From the same regression, we obtain the standard deviation of ε_t equal to $\sigma_\mu = 0.0092$. We impose no cross-correlation between the two countries' money growth shocks, for two reasons. First, the data in the U.S. and Europe do not support systematic correlations in the money growth rates; second, we would like to see how much of the observed cross-country quantity correlations can be accounted for *endogenously* by the structure of multiple processing stages (see also Footnote 8).

The remaining parameters are ϕ and N , which jointly determine the contribution of intermediate goods in production. According to the BEA's 1997 Benchmark Input-Output Tables, the share of intermediate goods in total manufacturing output is about 0.7. Let η denote the steady-state share of total intermediate inputs (across all stages of processing) in gross sales. Then, from the steady-state relations, we obtain

$$\frac{1}{1 - \eta} = \sum_{n=1}^N \frac{\bar{P}_n \bar{Y}_n}{\bar{P}_N Y_N} = \frac{1 - (\phi/\mu)^N}{1 - \phi/\mu}. \quad (19)$$

Clearly, in addition to the condition that $\eta = 0.7$, we need a second condition to jointly identify ϕ and N . For this purpose, we rely on the empirical evidence produced by Barsky, et al. (2001), which suggests that a lower bound for the gross markup across different stages of production and distribution in the U.S. is at least 1.4. In the model, the gross markup across all processing stages is given by μ^N . Given our calibrated value of the markup $\mu = 1.08$ at each stage and that $\mu^N = 1.4$, the implied value of N is about 4. We thus view $N = 4$ as a reasonable estimate for the OECD countries. The relation in (19) then implies a value for ϕ of about 0.9. Since the OECD countries in general tend to produce a broad range of commodities, from the most simple to the most sophisticated goods, while the emerging market economies tend to produce mostly simple goods, we view $N = 2$ as a reasonable estimate for the Latin American economies (see also Footnote 4).

4.2 Simulations

We examine now the model's quantitative implications for the cross-country correlations in real GDP and in consumption. Real GDP in a country corresponds to the real value added across all stages of processing in that country, which is summarized by the country's wage

income, capital rental income, and profit income. Inspecting the budget constraints facing the home country's households reveals that the country's nominal income from these three sources can be deflated consistently by its consumer price index level $\bar{P}_N(s^t)$. Thus, real GDP in the home country is given by

$$X_N(s^t) = [W(s^t)L(s^t) + R(s^t)K(s^{t-1}) + \Pi(s^t)]/\bar{P}_N(s^t). \quad (20)$$

The foreign country's real GDP can be obtained similarly.

To conduct numerical simulations, we first log-linearize the model's equilibrium conditions and solve this linearized system using standard numerical techniques. We then compute the cross-country correlations in real GDP and in consumption from the simulated data. The detail of the computation procedure is omitted here but available upon request from the authors.

Table 10 presents the simulation results under the calibrated parameter values. To put the results into perspective, we also display the average correlation statistics for the OECD countries as well as for the Latin American countries, which are computed from Tables 1 and 2.

The table shows that, with a single stage of processing, as in the standard monetary business cycle models, monetary shocks cannot explain the observed cross-country quantity correlations. In particular, both the output correlation and the consumption correlation are close to zero, with the latter being slightly larger. Compared to the correlation patterns in the data, this degenerate case of our model with $N = 1$ predicts quantity correlations that are not only too small but also in the wrong order for consumption and for output.⁷

The baseline model with multiple stages of processing is much more promising in generating the observed patterns in cross-country quantity correlations. For N larger than 1, not only do the correlation statistics become larger, but the order between the consumption correlation and the output correlation also comes more into line with what is observed in the actual data. When N equals 2, the output correlation rises to 21 percent, and the consumption correlation also rises, but to a lesser extent, to 16 percent. As N rises further to 3, and then to 4, the output correlation rises to 36 percent, and then to 46 percent, while the consumption correlation rises at a slower pace, first to 23 percent, and then to 30 percent.

⁷We also find that, if we assume that the cross-country correlations in the monetary shocks themselves are large enough, as in Chari, et al. (2002), then the cross-country consumption correlation and output correlation can become proportionally large (not reported). But we choose not to adopt this assumption because it does not seem to be supported by empirical evidence, nor does it help to get the two quantity correlations into the right order.

These results confirm the intuitions provided in Section 3. To reiterate our main findings: the baseline model with multiple stages of processing not only helps increase cross-country quantity correlations, it helps more in increasing output correlation than consumption correlation, thus putting the two quantity correlations into the right order. Comparing the correlation statistics generated from the model to those in the actual data reveals that the model's predicted correlations with $N = 2$ are broadly consistent with the correlations observed between the Latin American economies, and with $N = 4$ are close to those observed between the OECD countries. In both cases, the model is able to generate the correct order between the output correlation and the consumption correlation. As we have argued in the Calibration section, $N = 4$ seems to be an empirically plausible estimation for the OECD countries, and $N = 2$ for the Latin American economies. In this sense, one may interpret our results as providing a possible explanation for why the quantity correlations between the OECD countries are typically higher than those between the emerging market economies, and why, in both regions, the output correlations are systematically larger than the consumption correlations.

5 Concluding Remarks

A central challenge to international business cycle theory is to explain the observed patterns in international quantity correlations. In this paper, we have proposed a mechanism that may help meet this challenge. The novelty of our model with multiple stages of processing is that it propagates a monetary expansion in the home country to lower the foreign price level while containing a smaller rise in the home price level. It does so through reducing material costs and thus marginal costs in terms of the foreign currency unit while dampening the upward movements in the costs in terms of the home currency unit. In consequence, it tends to amplify and align the movements in the countries' real aggregate demands and real purchasing powers and to dampen the effects of the adjustment in the terms of trade, which would otherwise benefit home households and firms at the cost of their foreign counterparts. These all help increase the international quantity correlations. Further, through lowering the relative costs of materials to primary factors, it creates an incentive for firms to substitute intermediate inputs for primary factors. This incentive of factor substitution, which is stronger at a more advanced processing stage, tends to put a constraint on the rise of the cross-country correlation in hours worked and, with nonseparable preferences in consumption and leisure, on the rise in consumption correlation as well. In consequence, the mechanism embodied in the model with

multiple stages of processing and trade helps increase the international quantity correlations, and it helps increase the output correlation more than it helps increase the consumption correlation, putting the two correlations into the right order.

Throughout our analysis, we have assumed that monetary shocks are the only driving force of international business cycle dynamics, and we abstract from other potentially important sources of shocks. To generate a correct order between the cross-country correlations in output and in consumption, our model relies on the factor substitution effect that tends to keep the cross-country correlation in hours worked from rising. Since the international correlation in employment is typically positive and significant [e.g., Backus, et al. (1992, 1995) and Baxter (1995)], there is a tension between matching the employment correlation and matching the consumption correlation. This tension can potentially be relieved by introducing other aggregate demand shocks, such as government spending shocks: an expansion in home government spending tends to reduce home consumption and increase home employment through the standard wealth effect, while boosting foreign's consumption and employment through the stimulating effect on real aggregate demand and real purchasing power identified in the current paper.

The general framework outlined in this paper can be used to study other important issues. For example, with typical goods going through multiple stages of processing and crossing borders multiple times, a small transportation cost at each stage will generate large impediments to moving the goods across countries. Therefore, while a single-stage model with transportation cost may not be very successful in explaining the puzzle of home-bias in consumption and production [see, for example, the exchange between Obstfeld and Rogoff (2000) and Engles (2000)], a model with multiple stages of processing seems to be more promising. This idea is in the same spirit of Yi (2003), who shows how small tariff cuts in recent decades can serve as an important source of the large and non-linear rise in world trade (in particular, vertical trade). The model presented here can also be used to address the exchange rate persistence and international welfare issues following a country's unilateral monetary expansion [e.g., Huang and Liu (2003)]. In our view, future research along these dimensions can be fruitful. The current paper represents only a small step in this direction.

6 Appendix

In this appendix, we formally demonstrate the mechanism through which multiple stages of processing help propagate monetary shocks to generate international quantity correlations. To help obtain analytical results, we focus here on the case with no capital accumulation. For the same purpose, we assume that the period utility function of the representative household in each country is separable in consumption, real money balances, and labor hours. In particular, the period-utility function of a home household takes the following form:

$$U(C, M/\bar{P}_N, L) = \log(C) + \Phi \log(M/\bar{P}_N) - \Psi L, \quad (21)$$

for $\Phi > 0$ and $\Psi > 0$, and that of the foreign household takes a similar form. As shown by Hansen (1985) and Rogerson (1988), the linearity of the period-utility function in labor hours is a consequence of aggregation when labor is assumed to be indivisible and such a utility function is consistent with any labor supply elasticity at the *individual* level.

6.1 Linearized Equilibrium Conditions

To solve for equilibrium dynamics, we first reduce the equilibrium conditions to $10N + 4$ equations. These include $2N$ pairs of pricing decision equations, one for each firm in a given country on a given stage of processing (i.e., there are $4N$ pricing decision equations). In each pair, one component corresponds to the price set by the firm for the domestic market while the other corresponds to that for the foreign market. There are correspondingly $2N$ pairs of price indices. In addition, there are $2N$ price indices in the two countries for the N processing stages, each being an average of the price indices of domestic goods and imported goods at a given stage. Finally, there is a labor supply equation and a money demand equation of each country's representative household. We log-linearize these equations around a deterministic steady state and use lowercase letters to denote the log-deviations of the corresponding level variables (in uppercase letters) from their steady-state values.

The $2N$ pairs of linearized pricing decision rules are given by

$$p_{nH}(t) = \frac{1}{1+\beta} v_n(t) + \frac{\beta}{1+\beta} E_t[v_n(t+1)], \quad (22)$$

$$p_{nH}^*(t) = \frac{1}{1+\beta} [v_n(t) - e(t)] + \frac{\beta}{1+\beta} E_t[v_n(t+1) - e(t+1)], \quad (23)$$

$$p_{nF}^*(t) = \frac{1}{1+\beta} v_n^*(t) + \frac{\beta}{1+\beta} E_t[v_n^*(t+1)], \quad (24)$$

$$p_{nF}(t) = \frac{1}{1+\beta} [v_n^*(t) + e(t)] + \frac{\beta}{1+\beta} E_t[v_n^*(t+1) + e(t+1)], \quad (25)$$

where E_t is a conditional expectation operator, $n \in \{1, \dots, N\}$, $v_n = \phi \bar{p}_{n-1} + (1 - \phi)w$ and $v_n^* = \phi \bar{p}_{n-1}^* + (1 - \phi)w^*$ are the linearized stage- n marginal costs in domestic currency units facing home firms and foreign firms, respectively, with $\bar{p}_0(t) \equiv w(t)$ and $\bar{p}_0^*(t) \equiv w^*(t)$. Equation (22) says that the optimal price a stage- n firm in the home country would set for the home market is an average of its marginal costs in the current and the next period. If $n \geq 2$, the marginal cost is a weighted average of the price index of stage $n - 1$ goods and the nominal wage rate, since both of these goods and labor are used as inputs for producing the firm's output; if $n = 1$, the marginal cost is simply the nominal wage rate, since labor is the only input used by the firm. Similarly, from equation (23), we see that the optimal price set by a home firm for the foreign market is a weighted average of the exchange-rate-adjusted marginal costs facing the firm within its price contract duration. The optimal pricing rules (24)-(25) of the foreign firms are similarly interpreted.

The ($2N$ pairs of) price indices of goods produced in the two countries and sold in the two markets at each processing stage $n \in \{1, \dots, N\}$ are related to the pricing decisions via the following equations

$$\bar{p}_{nH}(t) = \frac{1}{2}[p_{nH}(t) + p_{nH}(t-1)], \quad \bar{p}_{nF}(t) = \frac{1}{2}[p_{nF}(t) + p_{nF}(t-1)], \quad (26)$$

$$\bar{p}_{nF}^*(t) = \frac{1}{2}[p_{nF}^*(t) + p_{nF}^*(t-1)], \quad \bar{p}_{nH}^*(t) = \frac{1}{2}[p_{nH}^*(t) + p_{nH}^*(t-1)]. \quad (27)$$

Under staggered price contracts, each price index records both the prices set in the current period and those set in the previous period. The price index of stage n in each country is an average of the price index of domestically produced goods and the price index of imported goods. There are $2N$ of these price indices and they are given by

$$\bar{p}_n(t) = \gamma \bar{p}_{nH}(t) + (1 - \gamma) \bar{p}_{nF}(t), \quad \bar{p}_n^*(t) = \gamma \bar{p}_{nF}^*(t) + (1 - \gamma) \bar{p}_{nH}^*(t). \quad (28)$$

Note that the parameter γ corresponds to the steady state share of domestically produced goods in total GDP, and it measures the steady-state home-bias.⁸

The households' labor supply decisions are described by

$$w(t) = \bar{p}_N(t) + y_N(t), \quad w^*(t) = \bar{p}_N^*(t) + y_N^*(t). \quad (29)$$

⁸From equations (4) and (5), the steady-state ratio of home-produced goods to imported goods is given by $\gamma/(1 - \gamma)$. Further, in light of (13) and (14), this is true for all processing stages.

Thus, real wage in each country is proportional to real consumption in that country (since the final good market clearing condition implies that $y_N(t) = c(t)$ and $y_N^*(t) = c^*(t)$).⁹

The money demand equations are given by

$$\bar{p}_N(t) + y_N(t) = (1 - \beta)m(t) + \beta E_t[\bar{p}_N(t+1) + y_N(t+1)], \quad (30)$$

$$\bar{p}_N^*(t) + y_N^*(t) = (1 - \beta)m^*(t) + \beta E_t[\bar{p}_N^*(t+1) + y_N^*(t+1)], \quad (31)$$

where an intertemporal term enters each country's money demand equation because money demand is interest-rate sensitive.

The equilibrium dynamics for this simplified model are described by (22)-(31).

6.2 Analytical Results and Further Intuitions

To gain insights into the monetary transmission mechanism embedded in multiple stages of processing, it is useful to examine the effects on each country's variables of a unilateral monetary expansion in, say, the home country. For this purpose, we assume that money supply in each country follows a random walk process, i.e., $m(t) = m(t-1) + \epsilon(t)$, and $m^*(t) = m^*(t-1) + \epsilon^*(t)$, where the money growth rates $\epsilon(t)$ and $\epsilon^*(t)$ are uncorrelated white noise process, and that there is a one-time shock to the money growth rate in the home country in period 0 and no shocks to the foreign money growth rate. In other words, we consider the case in which $\epsilon(0) = 1$, $\epsilon(t) = 0$ for all $t \geq 1$, and $\epsilon^*(t) = 0$ for all $t \geq 0$. The implied money supply processes are then $m(t) = 1$ and $m^*(t) = 0$ for all $t \geq 0$. We focus on a perfect foresight equilibrium and compute the impulse responses of each country's variables.

Before stating our first proposition, we need to introduce a notation for terms of trade. The foreign terms of trade of stage- n goods are defined as the price of its exported goods (adjusted for currency units) relative to the price of its imported goods at that stage, which is given by

$$\tau_n^*(t) = \bar{p}_{nF}(t) - e(t) - \bar{p}_{nH}^*(t), \quad 1 \leq n \leq N. \quad (32)$$

The following proposition partially characterizes the equilibrium.

Proposition 1. *There is a unique perfect foresight equilibrium in which*

$$w(t) = 1, \quad w^*(t) = 0, \quad \forall t \geq 0, \quad (33)$$

$$e(t) = 1, \quad \forall t \geq 0, \quad (34)$$

⁹Note that in this open-economy setup without capital or government spending, the current account balance accounts for the difference between GDP (aggregate output) and consumption (aggregate demand) in each country.

$$p_{nH}(t) = 1, \quad p_{nF}(t) = 1, \quad \forall t \geq n - 1, \quad 1 \leq n \leq N, \quad (35)$$

$$p_{nF}^*(t) = 0, \quad p_{nH}^*(t) = 0, \quad \forall t \geq n - 1, \quad 1 \leq n \leq N, \quad (36)$$

$$\bar{p}_n(t) = 1, \quad \bar{p}_n^*(t) = 0, \quad \forall t \geq n, \quad 1 \leq n \leq N, \quad (37)$$

$$y_N(t) = 0, \quad y_N^*(t) = 0, \quad \forall t \geq N, \quad (38)$$

$$\tau_n^*(t) = 0, \quad \forall t \geq n, \quad 1 \leq n \leq N. \quad (39)$$

for all $N \geq 1$.

Proof: Using the money demand equations (30) and (31), the pricing decision equations (22)-(25), the equations defining price indices (26)-(28), and proper transversality conditions for home and foreign households' optimization problems, we can show that there is a unique non-explosive perfect foresight equilibrium that satisfies

$$\bar{p}_N(t) + y_N(t) = m(t) = 1, \quad \bar{p}_N^*(t) + y_N^*(t) = m^*(t) = 0, \quad (40)$$

which, along with the labor supply equations (29), leads to (33). The first order conditions for the households' optimization problems also imply that the nominal exchange rate is given by $e(t) = [u_c^*(t) - \bar{p}_N^*(t)] - [u_c(t) - \bar{p}_N(t)] = [\bar{p}_N(t) + y_N(t)] - [\bar{p}_N^*(t) + y_N^*(t)] = 1$, which proves (34). Given (33) and (34), we can prove (35)-(37) by induction and then use (40) to obtain (38). Finally, using (34)-(36), we can establish (39). *Q.E.D.*

Proposition 1 shows that the shock to money supply in the home country drives up the domestic nominal wage rate immediately but has no effect on the foreign nominal wage rate. The shock leads to a complete home nominal exchange rate depreciation. After n periods following the shock, the price index of stage- n goods in the home country rises fully while the price index of stage- n goods in the foreign country returns to the steady state. As a consequence, the stage- n terms of trade return to the steady state n periods following the shock. Finally, after N periods following the shock, the real aggregate demand in each country also returns to the steady state.

The proposition therefore shows, among other things, that the monetary shock affects neither country's terms of trade or real aggregate demand in the long run. It turns out that cross-country quantity correlations in our model are largely determined by the short-run effects on each country's real aggregate demand and terms of trade. How large these effects can be and how long they can last depend on how many processing stages there are. Before proceeding further, it is useful to examine a degenerate version of our model with a single processing stage and to illustrate why it fails to serve as an international monetary transmission mechanism.

In the case with $N = 1$, all production and trade occur at a single stage, and firms' marginal cost is simply given by the domestic wage rate. Note that the home nominal wage rate rises fully in all periods following the shock (i.e., (33)). Thus, firms in the home country that can set new prices will respond by fully increasing their prices for the home market (i.e., (35)). On the other hand, although the foreign nominal wage rate in terms of the foreign currency unit is unaffected by the shock (i.e., (33)), it rises fully in terms of the home currency unit due to home currency depreciation (i.e., (34)). Thus, firms in the foreign country that can set new prices will fully raise their prices for the home market (i.e., (35)) as well. In consequence, the price level in the home country, which is a weighted average of the price index of domestic goods and the price index of imported goods, will rise fully as soon as all firms have had the chance to adjust their prices (i.e., (37)). It then follows from (40) that home's real aggregate demand rises only in the impact period when the shock occurs, and it goes back to the steady state upon the expiration of the initial price contracts (i.e., (38)).

In the foreign country, since nominal wage rate is unaffected by the shock, firms will choose not to adjust their prices set for the domestic market even if they can do so. On the other hand, since the rise in the home nominal wage rate is exactly offset by the home currency devaluation, the home nominal wage rate in terms of the foreign currency unit remains unchanged and thus firms in the home country will also choose not to adjust their prices set for the foreign market even if they can choose new prices. Thus, in all periods following the shock, the foreign price level remains unchanged and, in light of (40), so does its real aggregate demand. That is, we have both $\bar{p}_N^*(t) = 0$ and $y_N^*(t) = 0$ for all $t \geq 0$ if $N = 1$. This in particular implies that the home monetary expansion does not affect real aggregate demand in the foreign country in the case with a single processing stage. This is so since, even though the demand for foreign's exported goods increases and thus the foreign household has to work harder to meet the demand, the increase in the foreign household's income is offset by its worsened terms of trade (see (39) and Proposition 2 below), leaving unchanged the real aggregate demand in the foreign country. This is why the degenerate version of our model with $N = 1$ fails to generate cross-country quantity correlations.

We now show that our model with multiple stages of processing implies a dampened terms-of-trade effect and an enhanced effect on real aggregate demand of the shock, both of which are important for generating cross-country quantity correlations. We establish first a key result regarding the patterns of short-run price adjustments across different processing stages in the two countries.

Theorem 1. *Suppose that $N \geq 2$. In the perfect foresight equilibrium, the following inequalities hold for all $n \in \{1, \dots, N-1\}$:*

$$0 < p_{n+1,H}(t) < p_{nH}(t), \quad 0 < p_{n+1,F}(t) \leq p_{nF}(t), \quad 0 \leq t \leq n-1, \quad (41)$$

$$p_{n+1,F}^*(t) \leq p_{nF}^*(t) \leq 0, \quad p_{n+1,H}^*(t) < p_{nH}^*(t) \leq 0, \quad 0 \leq t \leq n-1, \quad (42)$$

$$0 < \bar{p}_{n+1}(t) < \bar{p}_n(t), \quad \bar{p}_{n+1}^*(t) < \bar{p}_n^*(t) \leq 0, \quad 0 \leq t \leq n. \quad (43)$$

Proof: Given the solution for the nominal exchange rate $e(t) = 1$ as in (34), the pricing decision equations (22)-(25) imply that

$$p_{nH}^*(t) = p_{nH}(t) - 1, \quad p_{nF}^*(t) = p_{nF}(t) - 1. \quad (44)$$

Thus, (42) will be an immediate corollary if we can establish (41).

To prove (41), we first use (22) and (25), along with the definitions of the marginal cost terms $v_n = \phi \bar{p}_{n-1} + 1 - \phi$ and $v_n^* = \phi \bar{p}_{n-1}^*$ (where we have used the conditions $w(t) = 1$ and $w^*(t) = 0$ based on (33)) and the relations between home prices and foreign prices described in (44) to obtain the following recursive relations:

$$\begin{aligned} p_{n+2,H}(t) = & 1 - \phi + \frac{\phi}{2} \{ \gamma [p_{n+1,H}(t) + ap_{n+1,H}(t-1) + (1-a)p_{n+1,H}(t+1)] \\ & + (1-\gamma)[p_{n+1,F}(t) + ap_{n+1,F}(t-1) + (1-a)p_{n+1,F}(t+1)] \}, \end{aligned} \quad (45)$$

$$\begin{aligned} p_{n+2,F}(t) = & 1 - \phi\gamma + \frac{\phi}{2} \{ \gamma [p_{n+1,F}(t) + ap_{n+1,F}(t-1) + (1-a)p_{n+1,F}(t+1)] \\ & + (1-\gamma)[p_{n+1,H}(t) + ap_{n+1,H}(t-1) + (1-a)p_{n+1,H}(t+1)] \}, \end{aligned} \quad (46)$$

where $a \equiv 1/(1+\beta)$. We then prove (41) by induction. It's straightforward to verify that the inequalities in (41) hold for $n = 1$. This establishes the result for $N = 2$. Suppose $N > 2$ and assume (41) holds for $n \in \{1, \dots, N-2\}$. Fix an arbitrary t with $0 \leq t \leq n$. By the induction hypothesis and (35), we have

$$p_{n+1,H}(t-1) \leq p_{nH}(t-1), \quad p_{n+1,H}(t) \leq p_{nH}(t), \quad p_{n+1,H}(t+1) \leq p_{nH}(t+1),$$

with at least one strict inequality, and

$$p_{n+1,F}(t-1) \leq p_{nF}(t-1), \quad p_{n+1,F}(t) \leq p_{nF}(t), \quad p_{n+1,F}(t+1) \leq p_{nF}(t+1),$$

with at least one strict inequality if and only if $n > 1$. It follows from (45) and (46) that $p_{n+2,H}(t) < p_{n+1,H}(t)$ and $p_{n+2,F}(t) \leq p_{n+1,F}(t)$. This completes the proof of (41). Equation (44) then implies that the inequalities in (42) also hold. Finally, the inequalities in (43) follow from the definitions of the price indices specified in (12) and (41)-(42). *Q.E.D.*

According to Theorem 1, individual prices (for both domestic goods and imported goods) and price indices rise in the home country and fall in the foreign country in the short run at all processing stages. The rises are smaller and the falls are greater, while both are spread over longer periods of time, at more advanced processing stages.

Theorem 1 lays out a foundation for establishing our next two results concerning the short-run effects on real aggregate demand and terms of trade of the shock. It is therefore worth spending some effort to understand the intuition behind this theorem.

The key to understanding the patterns of price adjustments prescribed in Theorem 1 is to understand the patterns of marginal cost dynamics across different processing stages. First, consider stage 1. The marginal cost facing home firms is the home nominal wage rate, which rises fully in terms of the home currency unit, but stays unchanged in terms of the foreign currency unit due to the foreign currency appreciation. Thus, these firms, whenever they can set new prices, would fully raise their prices $p_{1H}(t)$ for the home market, but keep unchanged their prices $p_{1H}^*(t)$ for the foreign market. The marginal cost facing foreign firms is the foreign nominal wage rate, which, although unchanged in terms of the foreign currency unit, rises fully in terms of the home currency unit due to the home currency devaluation. Thus, these firms, whenever they can set new prices, would fully increase their prices $p_{1F}(t)$ for the home market, but keep unchanged their prices $p_{1F}^*(t)$ for the foreign market. Combining these individual pricing decisions implies that, in the home market, both the price index of home goods $\bar{p}_{1H}(t)$ and the price index of foreign goods $\bar{p}_{1F}(t)$ rise fully for $t \geq 1$, but only partially at $t = 0$ due to staggered price contracts; however, in the foreign market, both the price index of home goods $\bar{p}_{1H}^*(t)$ and the price index of foreign goods $\bar{p}_{1F}^*(t)$ stay unchanged for all $t \geq 0$.

Next, consider stage 2. The marginal cost facing home firms records not only the home nominal wage rate, but also the stage-1 home market price index $\bar{p}_1(t)$, which is an average of $\bar{p}_{1H}(t)$ and $\bar{p}_{1F}(t)$, and thus rises partially in terms of the home currency unit and falls partially in terms of the foreign currency unit at $t = 0$ due to the foreign currency appreciation. Thus, these firms, if they can set new prices at $t = 0$, would only partially raise their prices $p_{2H}(0)$ for the home market and partially lower their prices $p_{2H}^*(0)$ for the foreign market. The marginal cost facing foreign firms records both the foreign nominal wage rate and the stage-1 foreign market price index $\bar{p}_1^*(t)$, which is an average of $\bar{p}_{1H}^*(t)$ and $\bar{p}_{1F}^*(t)$, and thus stays unchanged in terms of the foreign currency unit but rises fully in terms of the home currency unit due to the home currency devaluation. Thus, these firms, whenever they can set new prices, would fully raise their prices $p_{2F}(t)$ for the home market but keep unchanged their prices $p_{2F}^*(t)$ for

the foreign market. Combining these individual pricing decisions implies that, in the home market, the price index of home goods $\bar{p}_{2H}(t)$ not only rises less than does $\bar{p}_{1H}(t)$ at $t = 0$, but also rises only partially at $t = 1$ due to staggered price contracts (as opposed to the latter, which rises fully at $t = 1$); however, in the foreign market, the price index of home goods $\bar{p}_{2H}^*(t)$ declines partially at $t = 0$ and 1 due to staggered price contracts. The behaviors of $\bar{p}_{2F}(t)$ and $\bar{p}_{2F}^*(t)$, on the other hand, are similar to those of $\bar{p}_{1F}(t)$ and $\bar{p}_{1F}^*(t)$, respectively.

Now, consider stage 3. The marginal cost facing home firms records both the home nominal wage rate and the stage-2 home market price index $\bar{p}_2(t)$, which is an average of $\bar{p}_{2H}(t)$ and $\bar{p}_{2F}(t)$, and thus not only rises less than does $\bar{p}_1(t)$ in terms of the home currency unit and falls more than does $\bar{p}_1(t)$ in terms of the foreign currency unit at $t = 0$, but also rises only partially in terms of the home currency unit and falls partially in terms of the foreign currency unit at $t = 1$. Thus, these firms, when they can set new prices, would raise their prices $p_{3H}(t)$ for the home market by less than the rise in $p_{2H}(t)$, and lower their prices $p_{3H}^*(t)$ for the foreign market by more than the cut in $p_{2H}^*(t)$ for $t = 0$ and 1. The marginal cost facing foreign firms records both the foreign nominal wage rate and the stage-2 foreign market price index $\bar{p}_2^*(t)$, which is an average of $\bar{p}_{2H}^*(t)$ and $\bar{p}_{2F}^*(t)$, and thus falls partially in terms of the foreign currency unit and rises only partially in terms of the home currency unit at $t = 0$ and 1. Thus, these firms, when they can set new prices, would only raise their prices $p_{3F}(t)$ partially for the home market, and lower their prices $p_{3F}^*(t)$ partially for the foreign market for $t = 0$ and 1. Combining these individual pricing decisions implies that, in the home market, the price index of home goods $\bar{p}_{3H}(t)$ not only rises less than does $\bar{p}_{2H}(t)$ at $t = 0, 1$, but also rises only partially at $t = 2$, and the price index of foreign goods $\bar{p}_{3F}(t)$ rises only partially for $t = 0, 1$, and 2; however, in the foreign market, the price index of home goods $\bar{p}_{3H}^*(t)$ not only declines more than does $\bar{p}_{2H}^*(t)$ at $t = 0, 1$, but also declines partially at $t = 2$, and the price index of foreign goods $\bar{p}_{3F}^*(t)$ declines partially for $t = 0, 1$, and 2. It follows that the rise in the home market price index $\bar{p}_3(t)$ is smaller than the rise in $\bar{p}_2(t)$ for all $t \leq 1$ while $\bar{p}_3(t)$ does not rise fully until $t = 3$ (as opposed to $\bar{p}_2(t)$, which rises fully at $t = 2$), and the fall in the foreign market price index $\bar{p}_3^*(t)$ is greater than the fall in $\bar{p}_2^*(t)$ for all $t \leq 1$, while $\bar{p}_3^*(t)$ does not return to 0 until $t = 3$ (as opposed to $\bar{p}_2^*(t)$, which returns to 0 at $t = 2$).

Continuing this argument shows that, from early to later processing stages, the marginal costs facing home and foreign firms rise less and less in terms of the home currency unit and fall more and more in terms of the foreign currency unit, while the movements of the marginal costs are spread over a longer and longer period of time. Thus, this cross-country input-output

structure serves both to dampen the upward movements of the marginal costs in the home currency unit and to magnify the downward movements of the marginal costs in the foreign currency unit. As a consequence, the rises in the home market prices are attenuated and the falls in the foreign market prices are magnified on a period-by-period basis and are spread over a longer and longer period of time along the processing stages, as Theorem 1 prescribes.

These patterns of short-run price dynamics propagated by multiple stages of processing have important implications for short-run dynamics in real aggregate demand and terms of trade. First, they imply a positive effect on real aggregate demand in the foreign country of the home monetary expansion. Moreover, the increases in real aggregate demand in the two countries tend to reinforce each other so that they both become greater. The following proposition follows directly from (40) and (43).

Proposition 2. *In the perfect foresight equilibrium, the following inequalities hold for all $N \geq 1$:*

$$y_{N+1}(t) > y_N(t) > 0, \quad y_{N+1}^*(t) > y_N^*(t) \geq 0, \quad 0 \leq t \leq N. \quad (47)$$

One implication of (47) is that real aggregate demand increases not only in the home country but also in the foreign country, even if there are only two stages of processing. In contrast, in the degenerate case with a single processing stage, real aggregate demand in the home country rises but that in the foreign country remains unchanged. A further implication of (47) is that, with more processing stages, real aggregate demands in the two countries tend to reinforce each other so that they both become larger. We show now that such cross-country input-output connections tend to alleviate the negative effect on the terms of trade facing foreign households and firms. In particular, not only are the foreign's terms of trade in the impact period of the shock less worsened at more advanced processing stages, but they are actually reversed in subsequent periods, with the improvement being more significant on a period-by-period basis and over a longer period of time at more advanced stages.

Proposition 3. *Suppose that $N \geq 2$. In the perfect foresight equilibrium, the following inequalities hold:*

$$\tau_n^*(t) > \tau_{n-1}^*(t), \quad \text{for } 0 \leq t \leq n-1, \quad 2 \leq n \leq N, \quad (48)$$

$$\tau_n^*(0) < 0, \quad \text{for } 1 \leq n \leq N, \quad (49)$$

$$\tau_n^*(t) > 0, \quad \text{for } 1 \leq t \leq n-1, \quad 2 \leq n \leq N. \quad (50)$$

Proof: Since the foreign's terms of trade involve only \bar{p}_{nF} and \bar{p}_{nH}^* , to prove (48), we first use the relation (44) and the definitions of price indices (26) - (28) to express the price indices

of stage- n goods in terms of \bar{p}_{nF} and \bar{p}_{nH}^* . In particular, we obtain

$$\bar{p}_n(t) = \begin{cases} \gamma \bar{p}_{nH}^*(0) + (1 - \gamma) \bar{p}_{nF}(0) + \frac{\gamma}{2} & \text{if } t = 0 \\ \gamma \bar{p}_{nH}^*(t) + (1 - \gamma) \bar{p}_{nF}(t) + \gamma & \text{if } t \geq 1. \end{cases} \quad (51)$$

$$\bar{p}_n^*(t) = \begin{cases} \gamma \bar{p}_{nF}(0) + (1 - \gamma) \bar{p}_{nH}^*(0) - \frac{\gamma}{2} & \text{if } t = 0 \\ \gamma \bar{p}_{nF}(t) + (1 - \gamma) \bar{p}_{nH}^*(t) - \gamma & \text{if } t \geq 1. \end{cases} \quad (52)$$

We next use the pricing decision equations (25) and (23), along with the results established in Proposition 1, to obtain

$$p_{nF}(t) = a\phi \bar{p}_{n-1}^*(t) + (1 - a)\phi \bar{p}_{n-1}^*(t + 1) + 1, \quad (53)$$

$$p_{nH}^*(t) = a\phi \bar{p}_{n-1}(t) + (1 - a)\phi \bar{p}_{n-1}(t + 1) - \phi. \quad (54)$$

Then, by combining (51) - (54) and using the definition of the terms of trade (32), we obtain a recursive relation for $\tau_n^*(t)$ across stages of processing:

$$\tau_n^*(t) = \begin{cases} \frac{\phi(2\gamma-1)}{2} [a\tau_{n-1}^*(0) + (1 - a)\tau_{n-1}^*(1)] + \frac{a\phi\gamma-1}{2}, & \text{if } t = 0 \\ \frac{\phi(2\gamma-1)}{2} [\tau_{n-1}^*(1) + a\tau_{n-1}^*(0) + (1 - a)\tau_{n-1}^*(2)] + \frac{a\phi\gamma}{2}, & \text{if } t = 1 \\ \frac{\phi(2\gamma-1)}{2} [\tau_{n-1}^*(t) + a\tau_{n-1}^*(t - 1) + (1 - a)\tau_{n-1}^*(t + 1)], & \text{if } t \geq 2. \end{cases} \quad (55)$$

We can now prove (48) by induction. First, it is easy to verify that $\tau_1^*(0) = -\frac{1}{2}$, $\tau_1^*(t) = 0$ for all $t \geq 1$; $\tau_2^*(0) = -\frac{1}{2} + \frac{a\phi}{4}$, $\tau_2^*(1) = \frac{a\phi}{4}$, and $\tau_2^*(t) = 0$ for all $t \geq 2$. Thus, (48) holds for $N = 2$.

Suppose $N > 2$ and assume (48) holds for an arbitrary $n \in \{2, \dots, N - 1\}$. It suffices to show that

$$\tau_{n+1}^*(t) > \tau_n^*(t), \quad \text{for } 0 \leq t \leq n. \quad (56)$$

To prove (56), we first extend the recursive relation in (55) to stage $n + 1$ and take the difference across stages to get

$$\tau_{n+1}^*(t) - \tau_n^*(t) = \begin{cases} \frac{\phi(2\gamma-1)}{2} [a\Delta_n \tau_n^*(0) + (1 - a)\Delta_n \tau_n^*(1)], & \text{if } t = 0 \\ \frac{\phi(2\gamma-1)}{2} [\Delta_n \tau_n^*(t) + a\Delta_n \tau_n^*(t - 1) + (1 - a)\Delta_n \tau_n^*(t + 1)], & \text{if } t \geq 1, \end{cases} \quad (57)$$

where $\Delta_n \tau_n^*(t) \equiv \tau_n^*(t) - \tau_{n-1}^*(t)$.

If $t = 0$, then the induction hypothesis implies that $\tau_n^*(0) > \tau_{n-1}^*(0)$ and $\tau_n^*(1) > \tau_{n-1}^*(1)$. Thus, (56) holds for $t = 0$.

If $1 \leq t \leq n - 2$, then $t + 1 \leq n - 1$. From the induction hypothesis, we have

$$\tau_n^*(t) > \tau_{n-1}^*(t), \quad \tau_n^*(t + 1) > \tau_{n-1}^*(t + 1), \quad \tau_{n+1}^*(t - 1) > \tau_n^*(t - 1). \quad (58)$$

Thus, (57) implies that (56) holds for $1 \leq t \leq n - 2$.

If $t = n - 1$, then from the induction hypothesis, $\tau_n^*(s) > \tau_{n-1}^*(s)$ for $s = t, t - 1$; and from Theorem 1, $\tau_n^*(t + 1) = \tau_{n-1}^*(t + 1) = 0$. Therefore, (56) holds for $t = n - 1$.

Finally, if $t = n$, then $\tau_n^*(t - 1) > \tau_{n-1}^*(t - 1)$ by the induction hypothesis, and meanwhile, (39) implies that $\tau_n^*(s) = \tau_{n-1}^*(s) = 0$ for $s = t, t + 1$. Thus, (56) also holds for $t = n$.

To prove (49), we first note that $\bar{p}_{nH}^*(0) = \bar{p}_{nH} - \frac{1}{2}$ and thus $\tau_n^*(0) = \bar{p}_{nF}(0) - \bar{p}_{nH}(0) - \frac{1}{2}$. It is easy to verify that $\bar{p}_{1F} = \bar{p}_{1H} = \frac{1}{2}$. These results, coupled with the monotone pattern of price adjustments established in Theorem 1, imply that $0 < \bar{p}_{nF} \leq \bar{p}_{1F} = \frac{1}{2}$ and $0 < \bar{p}_{nH} \leq \bar{p}_{1H} = \frac{1}{2}$ for all $n \in \{1, \dots, N\}$. It follows that $\tau_n^*(0) < 0$ for all $n \in \{1, \dots, N\}$.

Given (48), to prove (50), it suffices to show that $\tau_n^*(n - 1) > 0$ for all $n \in \{2, \dots, N\}$. It is straightforward to verify that $\tau_2^*(1) = \frac{\alpha\phi}{4} > 0$ and thus (50) holds for $n = 2$. Suppose it holds for an arbitrary $n \in \{2, \dots, N - 1\}$. We need to show that $\tau_{n+1}^*(n) > 0$. This last inequality holds, since (48) and (39) imply that $\tau_{n+1}^*(n) > \tau_n^*(n) = 0$. *Q.E.D.*

According to Proposition 3, although the foreign's terms of trade in the impact period of the shock are worsened [e.g., (49)], they are less so at more advanced stages of processing [e.g., (48)]. Further, in light of (50) and (48), the foreign's terms of trade actually improve in the subsequent periods, and the improvements are greater and over longer periods of time at more advanced processing stages.

References

- Alvarez, F., A. Atkeson, and P. J. Kehoe, Money, interest rates, and exchange rates with endogenously segmented asset markets, *Forthcoming* in Journal of Political Economy.
- Ambler, S., E. Cardia, and C. Zimmermann, 2002, International transmission of the business cycle in a multi-sector model, *European Economic Review* 46, 273-300.
- Bacchetta, P. and E. van Wincoop, 2000, Does exchange-rate stability increase trade and Welfare? *American Economic Review* 90, 1093-1109.
- Backus, D. K., P. J. Kehoe, and F. E. Kydland, 1992, International real business cycles, *Journal of Political Economy* 100, 745-775.
- , 1995, International business cycles: theory and evidence, in: T. F. Cooley, ed., *Frontiers of business cycle research* (Princeton, New Jersey, Princeton University Press), 331-356.
- Barsky, R., M. Bergen, S. Dutta, and D. Levy, 2001, What can the price gap between branded and private label products tell us about markups? National Bureau of Economic Research Working Paper No. 8426.
- Basu, S., 1996, Procyclical productivity: Increasing returns or cyclical utilization? *Quarterly Journal of Economics* 111, 719-751.
- Basu, Susanto, and John G. Fernald, 1997, Aggregate productivity and aggregate technology. Board of Governors of the Federal Reserve System International Finance Discussion Paper No. 593.
- Basu, Susanto, and John G. Fernald, 2000, Why is productivity procyclical? Why do we care? National Bureau of Economic Research Working Paper No. 7940.
- Basu, Susanto, and Alan M. Taylor, 1999, Business cycles in international historical perspective, *Journal of Economic Perspectives* 13, 45-68.
- Baxter, M., 1995, International trade and business cycles, in: G. Grossman and K. Rogoff, eds., *Handbook of international economics*, Vol. 3 (Amsterdam, Elsevier Science), 1801-1864.
- Bergin, P. and R. C. Feenstra, 2001, Pricing-to-market, staggered contracts, and real exchange rate persistence, *Journal of international Economics* 54, 333-359.
- Betts, C., and M. B. Devereux, 1996, The exchange rate in a model of pricing-to-market, *European Economic Review* 40, 1007-1021.
- Betts, C., and M. B. Devereux, 2000, Exchange rate dynamics in a model of pricing-to-market, *Journal of International Economics* 50, 215-244.

- Chang, R. and M. B. Devereux, 1998, Dynamic collusion, pricing-to-market, and real exchange rates, Mimeo, University of British Columbia.
- Chari, V. V., Patrick J. Kehoe, and Ellen R. McGrattan, 2000, Sticky price models of the business cycle: Can the contract multiplier solve the persistence problem? *Econometrica* 68, 1151-1179.
- , 2002, Can sticky price models generate volatile and persistent real exchange rates? *Review of Economic Studies* 69, 533-563.
- Clark, Todd E., 1999, The responses of prices at different stages of production to monetary policy shocks, *Review of Economics and Statistics* 81, 420-433.
- Devereux, Michael B., 1997, Real exchange rates and Macroeconomics: Evidence and theory, *Canadian Journal of Economics* 30, 773-808.
- Devereux, M. B. and C. Engel, 1998, Fixed vs. floating exchange rates: how price setting affects the optimal choice of exchange-rate regime, NBER Working Paper 6867.
- Dornbusch, Rudiger, 1987, Exchange rates and prices, *American Economic Review* 77, 93-106.
- Engel, Charles, 1993, Real exchange rates and relative prices: An empirical investigation, *Journal of Monetary Economics* 32, 35-50.
- , 1999, Accounting for U.S. real exchange rate changes, *Journal of Political Economy* 107, 507-538.
- , 2000, Comments on Obstfeld and Rogoff's "The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?" NBER Working Paper 7818
- Engel, Charles, and John H. Rogers, 1996, How wide is the border? *American Economic Review* 86, 1112-1125.
- Gagnon, J.E., and M.N. Knetter, 1995, Markup adjustment and exchange rate fluctuations: evidence from panel data on automobile exports, *Journal of International Money and Finance* 14, 289-310.
- Goldberg, P. K. and M. M. Knetter, 1997, Goods prices and exchange rates: what have we learned? *Journal of Economic Literature* 35, 1243-1272.
- Hansen, G.D., 1985, Indivisible labor and the business cycle. *Journal of Monetary Economics* 16, 309-328.
- Huang, K. X. D. and Z. Liu, 2000, Vertical international trade as a monetary transmission mechanism in an open economy, CREFE Working Paper 107, UQAM, Canada.
- , 2001, Production chains and general equilibrium aggregate dynamics, *Journal of Monetary Economics* 48, 437-462.

- , 2003, Sellers' local currency pricing or buyers' local currency pricing: Does it matter for international welfare analysis? Manuscript, Federal Reserve Bank of Kansas City and Emory University.
- Hummels, D., D. Rapoport, and K. Yi, 1998, Vertical specialization and the changing nature of world trade, *Federal Reserve Bank of New York Economic Policy Review* 4 (2), 79-99.
- Hummels, D., J. Ishii, and K. Yi, 2001, The nature and growth in vertical specialization in world trade, *Journal of International Economics* (June), 75-96.
- Kehoe, P. J., and F. Perri, 2002, International business cycles with endogenous incomplete markets, *Econometrica* 70(3), 907-928.
- Kiley, M. T., 1999, Endogenous price stickiness and business cycle persistence, *Journal of Money, Credit, and Banking* 32, 28-53.
- Knetter, M. N., 1993, International comparisons of pricing-to-market behavior, *American Economic Review* 83, 473-486.
- Kouparitsas, M. A., North-South Business Cycles. *Forthcoming* in *Journal of Monetary Economics*.
- Lane, P. R., 2001, The new open economy macroeconomics: A survey, *Journal of International Economics* 54, 235-266.
- Obstfeld, Maurice, and Kenneth Rogoff, 2000, New directions for stochastic open economy models. *Journal of International Economics* 50, 117-153.
- OECD, 1994, The OECD jobs study, Part I: Labor market trends and underlying forces of change.
- Page, S. A. B., 1981, The choice of invoicing currency in merchandise trade, *National Institute Economic Review* 98, 60-72.
- Rogerson, R., 1988, Indivisible labor, lotteries and equilibrium. *Journal of Monetary Economics* 21, 3-16.
- Rogoff, K., 1996, The purchasing power parity puzzle, *Journal of Economic Literature* 34, 647-668.
- Taylor, J. B., 1980, Aggregate dynamics and staggered contracts, *Journal of Political Economy* 88, 1-23.
- , 1999, Staggered price and wage setting in macroeconomics, in: John B. Taylor and Michael Woodford, eds., *Handbook of Macroeconomics* (Amsterdam, North Holland: Elsevier Science).

- Vissing-Jorgensen, A., 2002, Limited asset market participation and the elasticity of intertemporal substitution, *Journal of Political Economy* 110, 825-853.
- Yi, K. M., 2003, Can vertical specialization explain the growth of world trade? *Journal of Political Economy* 111, 52-102.

Table 1
International Quantity Correlations in OECD Countries^a

	Correlation with US		Correlation with the rest of OECD ^b	
	$Corr(y, y^*)$	$Corr(c, c^*)$	$Corr(y, y^*)$	$Corr(c, c^*)$
Australia	0.72	0.08	0.50	-0.11
Canada	0.80	0.48	0.69	0.41
France	0.30	0.15	0.40	0.43
Italy	0.33	-0.15	0.41	0.03
Japan	0.21	0.24	0.22	0.36
Switzerland	0.38	0.30	0.41	0.38
UK	0.69	0.55	0.68	0.65
US	1.00	1.00	0.50	0.31
Average ^c	0.60	0.38	0.47	0.31

^aThe data are annual per capita real GDP and consumption from 1973 to 2000, taken from the World Development Indicators.

^bCorrelations in output and consumption between a particular country and an OECD aggregate constructed using all other countries' data.

^cThe first two entrants in this row are the correlations of output and consumption between the US and an EU aggregate, taken from Chari, et al. (2002). The other two entrants are the averages of the correlation statistics in the last two columns.

Table 2
International Quantity Correlations in Latin American Countries^a

	$Corr(y, y^*)$	$Corr(c, c^*)$
Argentina	0.28	0.13
Brazil	0.23	-0.01
Chile	0.32	0.18
Ecuador	0.35	0.35
Mexico	0.09	0.03
Nicaragua	-0.08	-0.15
Peru	0.57	0.19
Venezuela	0.12	-0.01
Average	0.24	0.09

^aThe data are annual per capita real GDP and consumption from 1973 to 2000, taken from the World Development Indicators. The correlations are between a particular country and an aggregate of the rest of the Latin American countries. We have also examined the pairwise correlations and obtained similar results.

Table 3.

Price indices in the home country

$\bar{p}_n(t)$	$t = 0$	$t = 1$	$t = 2$	$t = 3$	$t = 4$
$n = 4$	0.3337	0.7941	0.9558	0.9953	1
$n = 3$	0.3532	0.8304	0.9772	1	1
$n = 2$	0.3987	0.8987	1	1	1
$n = 1$	0.5	1	1	1	1

Table 4.

Price indices in the foreign country

$\bar{p}_n^*(t)$	$t = 0$	$t = 1$	$t = 2$	$t = 3$	$t = 4$
$n = 4$	-0.0276	-0.0379	-0.0118	-0.0015	0
$n = 3$	-0.0204	-0.0250	-0.0046	0	0
$n = 2$	-0.0113	-0.0113	0	0	0
$n = 1$	0	0	0	0	0

Table 5.

Real aggregate demand in the home country

$Y_N(t)$	$t = 0$	$t = 1$	$t = 2$	$t = 3$	$t = 4$
$N = 4$	0.6663	0.2059	0.0442	0.0047	0
$N = 3$	0.6468	0.1696	0.0228	0	0
$N = 2$	0.6013	0.1013	0	0	0
$N = 1$	0.5	0	0	0	0

Table 6.

Real aggregate demand in the foreign country

$Y_N^*(t)$	$t = 0$	$t = 1$	$t = 2$	$t = 3$	$t = 4$
$N = 4$	0.0276	0.0379	0.0118	0.0015	0
$N = 3$	0.0204	0.0250	0.0046	0	0
$N = 2$	0.0113	0.0113	0	0	0
$N = 1$	0	0	0	0	0

Table 7.

Real balances in the foreign country

$m^*(t)$	$t = 0$	$t = 1$	$t = 2$	$t = 3$	$t = 4$
$N = 4$	0.0276	0.0379	0.0118	0.0015	0
$N = 3$	0.0204	0.0250	0.0046	0	0
$N = 2$	0.0113	0.0113	0	0	0
$N = 1$	0	0	0	0	0

Table 8.

Foreign country's terms of trade

$\tau_n^*(t)$	$t = 0$	$t = 1$	$t = 2$	$t = 3$	$t = 4$
$n = 4$	-0.3265	0.21	0.0407	0.0041	0
$n = 3$	-0.3870	0.1128	0.0003	0	0
$n = 2$	-0.3875	0.1125	0	0	0
$n = 1$	-0.5	0	0	0	0

Table 9
Calibrated parameters

Preferences:	$\beta = 0.96^{1/2}, b = 0.94, \nu = -1.56, \xi = 1.6, \sigma = 3$
Technologies:	$\alpha = 1/3, \theta = 13, \gamma = 0.9, \phi = 0.9$
Capital accumulation:	$\delta = 0.04, \psi$ adjusted
Money growth process:	$\rho_\mu = 0.68^2, \sigma_\mu = 0.0092$

Table 10
International Correlations: Model versus Data^a

	$Corr(y, y^*)$	$Corr(c, c^*)$
OECD	0.47	0.31
Latin America	0.24	0.09
$N = 1$	-0.05	-0.03
$N = 2$	0.21	0.16
$N = 3$	0.36	0.23
$N = 4$	0.46	0.30

^aThe model's correlation statistics are averages over 300 simulations of 90 periods each (the first and the last 20 observations in each simulated series are discarded to avoid dependence on initial and terminal conditions).