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VACANCY PERSISTENCE

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Abstract

This paper reevaluates the quantitative performance of the standard labor-market matching model developed by Mortensen and Pissarides [28] with special attention to the behavior of vacancies, one of the key variables in the model. I first estimate trivariate vector autoregressions with gross worker flows and vacancies and identify an aggregate shock by imposing only minimal sign restrictions on the responses of worker flows and employment growth and no restrictions on the response of vacancies. The data strongly suggest a hump-shaped and persistent response of vacancies. The calibrated model, on the other hand, predicts that vacancies respond to aggregate shocks with no delay and are not persistent even though an aggregate productivity shock is assumed to be highly persistent. These problems in vacancy behavior also cause gross flow series to exhibit counterfactual cyclical properties.

JEL codes: E24, J63, J64

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1 Introduction

The labor-market matching model developed by Pissarides [30], [32], and Mortensen and Pissarides [28] provides a coherent framework to analyze dynamic behavior of gross worker (or job) flows, employment, and job vacancies, and thus has become popular in macro-labor literature. Reflecting this popularity, there have been a number of attempts in the literature to quantitatively evaluate the performance of this class of models. For example, Cole and Rogerson [14] examine the model's ability to deliver plausible cyclical properties of job flows and employment in a reduced-form framework. Collard et al. [15] estimate the structural parameters and undertake formal statistical tests of the model. Overall, the conclusion from these studies is that the framework does a good job in explaining important empirical regularities regarding labor-market flows and employment in the U.S. There are also attempts that embed the labor-matching friction into dynamic stochastic general equilibrium (DSGE) models with capital and a risk-averse household. Andolfatto [2], Merz [26], and den Haan et al. [19] integrate the matching framework into otherwise-standard real business cycle models and show that their extensions significantly improve the models' performance in propagating the underlying technology shocks. Cooley and Quadrini [16] develop a monetary DSGE model with the matching friction and show that it helps produce realistic Phillips curve dynamics as well as labor-market dynamics.

The main purpose of this paper is to reevaluate the model's quantitative performance. I show that the model encounters serious problems in its vacancy dynamics and that the problems in vacancy behavior cause counterfactual dynamic behavior of gross flows as well. These problems result from one of the key equilibrium conditions in this class of models: the free-entry condition into the matching market. Although this condition is widely used in the literature, none of the papers has paid much attention to the implications of the condition. The free-entry condition states that firms can immediately enter the matching market by simply posting vacancies when doing so is expected to yield positive returns. In equilibrium, the expected returns to posting a vacancy are equalized to a vacancy posting cost. Suppose that a negative aggregate productivity shock hits the economy. The negative shock decreases the expected returns from posting a vacancy, and therefore, the number of vacancies initially drops. However, the incentive to post vacancies quickly rises as the adverse shock increases unemployment, since this raises the chance that the firm will successfully find a worker from the pool of unemployed. The "echo effect" caused by the increases in unemployment has several important implications for the model's cyclical properties. A direct implication is that vacancies in the model are not persistent even if one assumes a highly persistent aggregate productivity shock, such as the one used in the real business cycle literature; the adverse effect of the low aggregate productivity on firms' hiring effort is mitigated by the higher probability of finding a worker. Second, the recovery of vacancies puts upward pressures on the number of matches. Higher unemployment during a recession directly contributes to increasing the number of matches, and this increase is enhanced by the recovery in vacancies. Finally, the surge in firms' hiring motivation, in turn, pushes up the job finding rate for unemployed workers. This promotes separations of matched pairs as it is relatively easy for separated workers to find subsequent employment opportunities.[1]

To test these predictions, I provide stylized facts about cyclical properties of job vacancies and gross worker flows by estimating trivariate vector autoregressions (VARs).[2] An "aggregate shock" is identified by using Uhlig's [38] agnostic Bayesian method that imposes only the least controversial inequality constraints on the patterns of impulse responses. Specifically, I impose only minimal and sensible restrictions on the responses of gross worker flows and employment growth and *no* restrictions on the behavior of vacancies.[3] One of the advantages of this approach over the conventional exact-identification scheme is that the method allows one to find all possible responses that satisfy the sign restrictions and thus gives us a better sense about robustness of the empirical findings.

The main findings are as follows. While the VAR exercises show that the empirical responses of vacancies clearly exhibit a hump-shaped pattern, the model fails to produce this pattern. Further, even though the model matches *qualitative* patterns of responses of gross flows, the model's responses greatly exaggerate their empirical analogues. When a recessionary shock arrives, it is the case both in the model and the data that, after an initial decline, the creation rate surges to a level higher than the pre-shock level owing to higher unemployment. However, the extent of the increases in the creation rate is much greater in the model. This is because vacancies bounce back quickly in the model, while they are persistently low in the data. Further, although the model and the data both predict a persistently high separation rate after the recessionary shock, the model's response is "too persistent" compared to the empirical responses. Again, the lack of persistence in vacancies is the source of this behavior as I described above.

[1] Cooley and Quadrini's [16] results show that the productivity shock and monetary shock produce virtually the same responses of job flows and vacancies. Thus, the description in this paragraph appears to apply to the responses to the monetary shock as well.

[2] Note that the behavior of stock of employment is implied by gross flows.

[3] Notice the contrast with the identifying assumption used in an influential paper by Blanchard and Diamond [4]. To identify the aggregate shock, they assume that vacancies decline for nine months following a negative aggregate shock. This is, however, not necessarily consistent with the model's prediction.

Another important symptom of the model's problems with vacancy dynamics is that correlation patterns between unemployment and vacancies are at odds with empirical evidence. The U.S. data show that cyclical components of these series are strongly negatively correlated *not only contemporaneously but also at leads and lags.* This relationship may be referred to as the dynamic Beveridge curve. It is not surprising, however, that the model fails to generate an empirically plausible dynamic Beveridge curve because of the counterfactual behavior of vacancies. In particular, the model predicts that vacancies are positively correlated with lagged unemployment.

A recent paper by Shimer [36] considers a version of the matching model that does not allow for endogenous separations, in contrast to the model considered here. He shows that his model produces a strong contemporaneous correlation between unemployment and vacancies but fails to generate sufficient volatility of unemployment and vacancies for reasonable productivity shocks. The trivariate VAR estimates reported below, however, reveal that the separation rate responds sharply and persistently to aggregate shocks. This raises the question of whether Shimer's findings depend on his counterfactual assumption of a constant separation rate. As shown by den Haan et al. [19], endogenizing the separation rate improves the model's ability to magnify and propagate underlying shocks. Owing to shifts in the separation margin, realistic unemployment responses can be generated using the standard process of productivity shocks. Thus, insufficient magnification of shocks is not an issue under the more realistic specification of the matching model. With endogenous separations, however, the stronger echo effect on the vacancy posting decision greatly reduces vacancy persistence, leading to a much lower negative correlation between unemployment and vacancies.

To enhance my claim, I also replicate Shimer's results by fixing the separation rate, and I show that the model implies insufficient vacancy persistence even under this specification. In particular, even though one can indeed obtain a strong negative *contemporaneous* correlation between unemployment and vacancies, the model lacks the ability to generate a hump-shaped and persistent response of vacancies and, consequently, is unable to produce plausible unemployment-vacancy dynamics.

This paper is organized as follows. Section 2 first overviews the general ideas about the agnostic identification scheme and then applies the method to worker flows and job vacancies, thus providing grounds for evaluating the quantitative performance of the model.[4] Section 3 lays out the discrete time version of the standard Mortensen and Pissarides [28] labor-matching model and calibrates it. This section also presents the calibration for the model

[4]Note that the responses of worker flows imply the response of employment.

that assumes the fixed separation rate. Section 4 shows that the model's cyclical properties are not supported by the empirical evidence given in Section 2 in either version of the model. Section 5 summarizes the results and offers some possible remedies for the model.

2 Empirical Evidence

This section presents the stylized facts about dynamic behavior of gross worker flows and job vacancies in the U.S. Before presenting the results, I first give an overview of the general ideas about the agnostic identification scheme proposed by Uhlig [38].[5] The key elements of this approach are to impose only the least controversial *qualitative* (or sign) restrictions on patterns of impulse responses and to uncover *all possible* responses that are consistent with those restrictions. Following Uhlig, uncertainty about the estimated parameters is taken into account in a Bayesian manner.[6]

2.1 Identification Scheme and Its Mechanics

Let Y_t be a vector of n endogenous variables containing time-t values whose dynamic relationships are described by the following vector autoregression of order p (VAR(p)):

$$\Phi(L)Y_t = \nu_t, \tag{1}$$

where ν_t is an $n \times 1$ vector containing time-t values of reduced-form disturbances whose variance-covariance matrix is written as $E\nu_t\nu_t' = \Sigma$, and $\Phi(L) = I - \Phi_1 L - \Phi_2 L^2 - \cdots - \Phi_p L^p$. Assuming that $\Phi(L)$ is invertible, the VAR(p) has a Wold moving-average representation,

$$Y_t = \Psi(L)\nu_t, \tag{2}$$

where $\Psi(L) = \Phi(L)^{-1} = \Sigma_{j=0}^{\infty}\Psi_j L^j$. Let ω_t be an $n \times 1$ vector containing time-t values of structural disturbances. The reduced-form residuals and structural disturbances are linked through

$$\nu_t = A\omega_t, \tag{3}$$

where it is assumed that the structural disturbances are mutually independent as is standard in the literature. Also, I adopt the normalization that $E\omega_t\omega_t' = I$. Using Equation (3) in Equation (2) implies that

$$Y_t = \Psi(L)A\omega_t.$$

[5]Faust [20] also adopts a similar identification scheme.

[6]Much of the presentation below follows Burnside [9].

Thus, Ψ_j can be constructed from Φ_j, which can be estimated by ordinary least squares, and knowledge about A allows one to fully characterize the process of Y_t in terms of the structural disturbances ω_t. The variance-covariance structure of the reduced-form residuals puts constraints on the matrix A:

$$AA' = \Sigma, \tag{4}$$

where least squares give us a mean estimate of Σ. The identification problem is therefore to uncover the $\frac{n(n-1)}{2}$ free elements in A by imposing identifying restrictions.

An important result in Uhlig's [38] paper is that the matrix A can always be written as

$$A = X\Lambda^{1/2}Q, \tag{5}$$

where X is an orthogonal matrix whose columns are the orthonormal eigenvectors of Σ, Λ denotes a diagonal matrix with the eigenvalues of Σ on its principal diagonal, and Q denotes some orthogonal matrix (i.e., $QQ' = I$). Equation (5) shows that determining the free elements in A can be conveniently transformed into the problem of choosing elements in an orthonormal set. Furthermore, if one is interested only in responses to one particular shock, say, an aggregate shock, then the problem amounts to determining an orthonormal vector q in the following expression:

$$a = X\Lambda^{1/2}q, \tag{6}$$

where a is a column of A (which Uhlig calls an *impulse vector*) containing the contemporaneous responses of n endogenous variables to the structural shock of our interest, and q is a column of Q in the corresponding location. The main idea of the identification scheme is to impose a set of inequality constraints on $\Psi_j a$. This, of course, does not uniquely identify a but gives us ranges of possible responses consistent with the inequality constraints.

The ranges can be easily computed numerically. For each fixed set of the reduced-form VAR coefficients $\Phi = [\Phi_1', \Phi_2', \cdots, \Phi_p']$ and the error variance-covariance matrix Σ, I draw candidate vectors q from a unit sphere and keep only the draws that satisfy the sign restrictions. Following Uhlig, I deal with the sampling uncertainty about the VAR parameters in a Bayesian manner. The parameters Φ and Σ are jointly drawn from a Normal-Wishart posterior distribution.[7] In my application, I examine 400 equally spaced q's on a unit sphere for each set of the VAR parameters (Φ, Σ) and keep the draws that satisfy the sign restrictions. I draw the posterior 100 times, and therefore, a total of $40,000$ q's are examined.

[7] See Appendix B of Uhlig for the collection of formulas used for my estimation and inferences. Also, I use uninformative priors following Uhlig.

2.2 Sign Restrictions

Now, specifically, let $Y_t = [cre_t, des_t, v_t]'$ be a vector of the creation rate, the separation rate, and vacancies. Because the creation rate and the separation rate are defined as the number of matches created and destroyed normalized by the number of employed, the difference between the two gives the growth rate of employment, namely:

$$cre_t - des_t = \Delta e_t, \tag{7}$$

where Δe_t denotes the growth rate of employment in period t. Since the main focus of this paper is to examine the behavior of vacancies in response to an aggregate shock, I impose restrictions only on the responses of gross flows and employment growth and *no restrictions on the response of vacancies.*

The benchmark identification imposes the following three sign restrictions:[8]

1. Employment growth is not positive for at least K periods following a negative aggregate shock.

2. The negative shock leads to a non-negative response in the separation rate in the impact period.

3. The negative shock leads to a non-positive response in the creation rate in the impact period.

There seems to be no room to debate over the validity of the first restriction except for the choice of K. The benchmark case sets K at four quarters. To ensure robustness of the results, I also try the choice of $K = 2$. The second and third restrictions are taken from Davis and Haltiwanger [17]. According to them, these two restrictions are consistent with a wide range of theoretical models and alternative views about business cycles and, therefore, would be widely accepted. As we will see later, the calibrated labor-matching model satisfies these restrictions. Note that these two restrictions imply that employment growth is not positive in the impact period.

However, a potential problem with the third restriction is that the creation rate could possibly *increase* in the impact period of a negative shock. Suppose that a negative shock induces a spike in the separation rate, inducing large flows into the unemployment pool.

[8] I write the restrictions in terms of the responses to a negative aggregate shock for convenience of discussion. The restrictions on the responses to a positive shock can be written symmetrically.

Then the increase in job seekers would have positive impacts on the number of matches.[9] Although a recessionary shock is likely to depress firms' hiring effort, causing a negative effect on job creation, we do not know ex ante which effect dominates. Therefore, imposing the third restriction puts us at risk of eliminating dynamics possibly driven by aggregate shocks. This concern is larger especially if workers' job finding rate is so high that a spike in the separation rate may be immediately followed by increases in the creation rate. It is then possible that the separation rate and the creation rate are both observed to increase in the initial period of the negative shock. To deal with this concern, I also consider the case in which the third restriction is dropped. Note that although dropping the third restriction allows the creation rate to increase in the impact period, the first and second restrictions imply that the increase in the separation rate in the impact period must be larger than that in the creation rate, so that employment growth is negative in the impact period.

Notice that the sign restrictions above are much less restrictive than the conventional exact-identification scheme, which typically takes the form of either short-run zero restrictions or long-run zero restrictions (e.g., Blanchard and Quah [6] and Shapiro and Watson [35]). In particular, the exact-identification scheme generally requires restrictions on the effects of the shocks that we are not interested in. In our case, restrictions about the effects of other shocks such as an "allocative" shock would be additionally required to uncover the effects of the aggregate shock. This can be seen from Equation (4) where elements in one column are related (non-linearly) with elements in other columns. Although Davis and Haltiwanger [17] provide several "reasonable" long-run restrictions on the effects of an aggregate shock and allocative shock, none of them are comparable to the qualitative restrictions used here in terms of simplicity and plausibility.

2.3 Data

The VAR, Equation (1), is estimated by using the Conference Board's help-wanted index and CPS worker flow data. The former series is an index of counts of help-wanted advertisements in 51 major newspapers in the U.S. There are several pieces of evidence that this series closely tracks actual job vacancies in the U.S.[10] It is well known that CPS worker flows are subject to several serious statistical biases. There have been several attempts to

[9]Recall that the creation rate is defined as the number of matches formed normalized by the level of employment. The decreases in employment thus also contribute to raising the creation rate.

[10]See Shimer [36] for this point. The BLS recently started a comprehensive survey on job vacancies (Job Openings and Labor Turnover Survey; JOLTS). Shimer compares the help-wanted index with this series over the recent three-year period after 2000:Q4 and finds that they move very closely with each other.

correct these biases (e.g., Abowd and Zellner [1] and Poterba and Summers [33]). Bleakley et al. [7] have recently updated the Abowd-Zellner adjusted series and made the quarterly series publicly available.[11] The data set includes flows among the three states: employment (E), unemployment (U), and not in the the labor force (N). In the following estimation results, I use total flows into employment (from the unemployment and out-of-the-labor-force states) normalized by employment for the creation rate, and total flows out of employment (to the unemployment and out-of-the-labor-force states) normalized by employment for the separation rate. Alternative measures can be constructed by focusing on the flows between unemployment (U) and employment (E). The results below are, however, robust with respect to using the alternative measures for creation and separation.

The sample period is restricted to 1967:Q3−1999:Q1 owing to the availability of the worker flow data. The order of the VAR is set to 3 suggested by the BIC. The results, however, are not sensitive to the choice of lag length. No deterministic components except constant terms are included in the estimation. The ADF tests reject the null of a unit root at a 15% significance level for all the series.

2.4 Results

Figure 1 presents the impulse responses to a one-standard-deviation negative aggregate shock when all three restrictions are imposed and K is set to 4. The responses of the creation rate and separation rate (and thus implied employment growth and the level of employment) exhibit patterns that are *qualitatively* consistent with the finding in the existing literature that stresses the role of job destruction in propagating shocks (e.g., Ramey and Watson [34], den Haan et al. [19], Gomes et al. [23]); the creation rate recovers quickly after the initial decline contributing to *increasing* employment, whereas the separation rate remains higher than its steady-state level for about $2-3$ years after the shock. Note also that the empirical finding that the aggregate shock has a long-lasting effect on the separation rate suggests the rejection of the model that assumes the fixed separation rate. Figures 2 through 4 present the results for the cases in which K is set to 2 and/or the restriction on the initial response of the creation rate is lifted. The results are virtually unchanged. As I will show later, although the overall patterns of the empirical responses of gross flows are qualitatively consistent with the model responses, the model responses greatly exaggerate these patterns.

Consider now the responses of vacancies. The lower left panels of these figures show that

[11] The data set is available through http://weber.ucsd.edu/~bleakley/eunflows.txt. Although original CPS data are collected at monthly frequency, only the quarterly data are available on the web page.

vacancies clearly exhibit hump-shaped patterns. Although this may not be a surprising result intuitively, the model's prediction is not consistent with the results as we will see shortly. Note, however, that given that the bounds are computed by ordering the responses in each period, it is possible that these hump-shaped responses are generated by the combinations of responses that are not hump shaped individually. To meet this concern, I will later examine the responses individually.

2.5 How Much Variation Do Aggregate Shocks Explain?

Given that the main interest of this paper is to evaluate the model's ability to replicate the empirical impulse responses to an aggregate shock, it is important to make sure that the aggregate shocks account for significant variations in the variables of interest. There has been a long discussion in the literature addressing the question of how much of employment fluctuations can be attributed to aggregate shocks. A number of papers have tried to answer this question by using a wide variety of identifying assumptions in the VAR framework. The results are quite mixed and strongly depend on their identifying assumptions (e.g., Davis and Haltiwanger [17] and Campbell and Kuttner [11]). The sign-restrictions approach seems to have advantages over the conventional exact-identification scheme for addressing this issue because the restrictions imposed in this paper are simple and there appears to be little room to debate over the plausibility of the restrictions. The upper panel of Figure 5 displays the result obtained by imposing the three sign restrictions, and K is set to 4. According to the median estimates, the data indicate that the shock accounts for about 60% of employment growth for all forecast horizons. Compared to the results in the previous studies, this estimate appears to be on the high side. Furthermore, the 80% error bands are quite narrow, given the nature of this type of exercise.

The lower panel of the figure present the results for vacancies. In this case, the median estimate is around 40%, and the 80% band covers a wide range that includes the values close to zero. However, this result may be due to the lack of restrictions on the behavior of vacancies, and thus the valid draws actually include uninteresting cases that a negative shock yields increases in vacancies. Figures 1 through 4 indicate that the 80% bands of the responses cover such cases. To eliminate such dynamics in vacancies, I impose an additional sign restriction that forces vacancies not to increase in the impact period. Figure 6 presents the result. Although the lower band is still less than 10%, the median estimate has risen substantially to 60%.

3 The Mortensen and Pissarides Model

This section lays out a discrete time version of the Mortensen and Pissarides [28] model.[12] There is a continuum of identical workers with total mass equal to one in this economy, along with a continuum of potential firms, potentially having an infinite mass. Further, each firm consists of only one job to which only one worker is attached. Workers are assumed to be risk neutral, with discount factor β lying between zero and one. Time spent working is restricted to be either zero or one, meaning that workers provide one unit of labor when employed and zero when unemployed. Labor is the only input for production.[13] To hire a worker, a firm first must open a vacancy that imposes a cost c per period. Other important assumptions are that workers search for their jobs only when unemployed and that workers' decision about labor-force participation is ignored.

3.1 Employment Relationship

Each worker-firm pair that engages in production produces output according to the production technology:

$$z_{it} y_t,$$

where z_{it} is a random productivity shock that is specific to ith pair in period t and y_t indicates a random aggregate productivity shock in period t, which follows a first order Markov process. The idiosyncratic productivity shock is assumed to be i.i.d. across jobs and time.[14] The distribution of z_{it} is described by a cumulative distribution function $H(z_{it})$ whose support is assumed to be $[0, \infty)$. Worker-firm pairs can be destroyed for either exogenous or endogenous reasons, as in den Haan et al. [19]; matched pairs are exogenously destroyed with constant probability ρ^x per period, and those that do not experience exogenous separation may choose to separate endogenously.

[12] Although there are some variations in my particular version of the model from the original Mortensen and Pissarides model (for example, they formulate the model in continuous time), these differences should not alter the conclusion of this paper.

[13] Andolfatto [2], Merz [26] and den Haan et al. [19] embed the job-matching friction into dynamic stochastic general equilibrium models with capital and a risk-averse household. Since these models with capital encounter the same problems addressed in this paper, I focus on the problems by ignoring capital. In principle, adding capital should not alter the conclusions.

[14] den Haan et al. [19] (p. 495) examine the robustness of their propagation results with respect to the presence of persistence in idiosyncratic productivity and conclude that their results do not depend on the assumption that idiosyncratic productivity follows an i.i.d. process.

The worker who is separated from a job, whether exogenously or endogenously, obtains $b + U_t$, where b is the current-period unemployment benefit, and U_t denotes the continuation value of the unemployed worker net of the current-period unemployment benefit.[15]

Given the outside options for the worker and firm, the separation decision of the matched pair can be described as follows. Let G_t denote the joint continuation value of the worker-firm pair in period t. The surplus of the matched pair over the outside options in period t is then written as:

$$S_{it} = z_{it} y_t + G_t - (U_t + b). \tag{8}$$

The worker and firm bargain over this joint surplus. The negotiation is resolved according to the Nash bargaining solution, where the firm and the worker take a fixed proportion of S_{it}, π and $1 - \pi$, respectively. Since the current-period return becomes lower as z_{it} declines, there exists a level $\hat{z}_{it}$ such that $S_{it} < 0$ for $z_{it} < \hat{z}_{it}$, where both parties agree to abandon their relationship, while $S_{it} \geq 0$ for $z_{it} \geq \hat{z}_{it}$, where both parties agree to maintain their relationship and engage in production in this period. The level of $\hat{z}_{it}$ is referred to as the separation margin. Associated with $\hat{z}_{it}$ is the endogenous separation rate:

$$\rho_{it} = \int_0^{\hat{z}_{it}} dH(z_{it}).$$

The overall separation rate is given by $\rho^x + (1 - \rho^x)\rho_{it}$.

3.2 Matching Market

Unemployed workers and firms with vacant jobs engage in search activity in a matching market, which is characterized by a constant-returns-to-scale aggregate matching function:[16]

$$m_t = m(u_t, v_t), \tag{9}$$

where m_t denotes the number of matches formed in the period-t matching market, u_t denotes unemployment, and v_t denotes vacancies. The matching function $m(.)$ is increasing in both arguments. On average, an unemployed worker finds a firm each period with probability

$$\frac{m(u_t, v_t)}{u_t} \equiv \lambda_t^w. \tag{10}$$

[15] The symbol b is referred to as "unemployment benefit" simply for convenience, even though it is not a transfer from the government. More precisely, b should be considered as home production or utility from leisure that unemployed workers enjoy. The firm's outside option alternative to production is zero in the benchmark model, as we will see shortly.

[16] This assumption is supported by numerous empirical studies. Previous work for the U.S. includes Blanchard and Diamond [4] and Bleakley and Fuhrer [8]. See Petrongolo and Pissarides [29] for an extensive survey on this issue.

Similarly, a vacant job is filled with probability

$$\frac{m(u_t, v_t)}{v_t} \equiv \lambda_t^f. \tag{11}$$

Specifically, the matching function takes the following form proposed by den Haan et al. [19]:

$$m_t = \frac{u_t v_t}{\left(u_t^l + v_t^l\right)^{1/l}}. \tag{12}$$

A major advantage of this functional form over the widely used Cobb-Douglas specification is that the matching probabilities defined in Equations (10) and (11) take on values between zero and one for all $u_t \in [0, 1]$ and $v_t \in [0, \infty)$.

3.3 Equilibrium

Consider now the situation facing a firm and a worker in the matching market. When the firm and the worker meet in period t, which occurs with probabilities λ_t^f and λ_t^w, respectively, the pair draws the idiosyncratic productivity shock from the distribution H at the beginning of period $t + 1$ and decides whether to start producing or not. The pair faces exactly the same decision problem as that of the ongoing firm-worker pairs, whose decision problem is described in Subsection 3.1. The newly formed pair is also subject to the exogenous and endogenous separation, and if it survives the separation process, production takes place. When the pair decides to produce, the firm and worker share the surplus over their outside options by a fixed proportion as before.

The firm's outside option, which is the value of having a vacant job, is zero in every period as a consequence of free entry into the matching market. The free-entry condition is written as:[17]

$$0 = -c + \beta \lambda_t^f (1 - \rho^x) \pi E_t \int_{\hat{z}_{t+1}}^{\infty} S_{t+1} dH(z_{t+1}). \tag{13}$$

The condition states that, in equilibrium, the vacancy posting cost equals the expected returns from posting a vacancy.

On the other hand, the worker finds a firm with probability λ_t^w, and if he accepts the relationship, he obtains the share $1 - \pi$ of surplus S_{t+1} in addition to his outside option $U_{t+1} + b$. When he does not meet with a firm, or the relationship is rejected after the meeting, he obtains $U_{t+1} + b$. The continuation value for the unemployed worker is thus written as:

$$U_t = \beta E_t \left[\lambda_t^w (1 - \rho^x)(1 - \pi) \int_{\hat{z}_{t+1}}^{\infty} S_{t+1} dH(z_{t+1}) + U_{t+1} + b \right]. \tag{14}$$

[17]In what follows, the i subscripts are suppressed because the idiosyncratic productivity shocks are assumed to be independent across time and worker-firm pairs.

Next, consider the joint returns of a worker-firm pair that produces in period t. Given that the firm's outside option is zero, the joint outside option for the relationship equals $U_{t+1} + b$. If the relationship survives the separation process at the beginning of period $t+1$, it receives S_{t+1} in addition to $U_{t+1}+b$. Thus the following continuation-value equation holds:

$$G_t = \beta E_t \left[(1-\rho^x) \int_{\hat{z}_{t+1}}^{\infty} S_{t+1} dH(z_{t+1}) + U_{t+1} + b \right]. \tag{15}$$

Finally, unemployment evolves according to:

$$u_t = u_{t-1} + \left[\rho^x + (1-\rho^x)\rho_t\right](1-u_{t-1}) - (1-\rho^x)(1-\rho_t)m(u_{t-1}, v_{t-1}), \tag{16}$$

where the second and third terms on the right-hand side are the flows into and from the unemployment pool, respectively.

The period-t aggregate state variables of the economy consist of aggregate productivity and the number of unemployed at the beginning of period t, $u_{t-1} - m_{t-1}$. Letting $\mathbf{s}_t = \{y_t, u_{t-1} - m_{t-1}\}$ be a set of the period-t aggregate state variables, the recursive equilibrium is defined by a list of functions, $G(\mathbf{s}_t), U(\mathbf{s}_t), v(\mathbf{s}_t)$ and $\hat{z}(\mathbf{s}_t)$, such that (i) equations for continuation values for a vacant job (13), an unemployed worker (14), and a operating job (15) hold; (ii) the separation margin $\hat{z}(\mathbf{s}_t)$ is determined by $\hat{z}_t y_t + G(\mathbf{s}_t) - U(\mathbf{s}_t) - b = 0$; and (iii) these conditions are satisfied under the evolution of aggregate productivity y_t (specified below), and unemployment (Equation (16)).[18]

3.4 Calibrating the Model

This section describes the model calibrations. I first calibrate the model just laid out, which features the endogenous separation decision. I also present the calibration for the specification that assumes the fixed separation rate. This latter specification can be calibrated as a special case of the former. Although the latter specification is not supported by the data as shown in Subsection 2.4, examining the performance of the latter model serves to clarify the fundamental nature of the problems of the model.

I assume the following aggregate productivity process for both specifications:

$$\ln y_{t+1} = \xi \ln y_t + \varepsilon_{t+1}, \tag{17}$$

where ε_t is taken to be independently and identically distributed (i.i.d.) normal with zero mean and standard deviation σ_ε. This process for aggregate productivity is commonly used in

[18] See the appendix for the solution algorithm.

the RBC literature. Following the literature, I set ξ and σ_ε at 0.95 and 0.007, respectively. I also assume for both specifications that the discount factor β is 0.99, which implies an annual interest rate of 4 percent, and that the bargaining parameter for firms π is 0.5, as is standard in the literature.

3.4.1 Benchmark Specification

I start with the steady-state version of the law of motion for unemployment (Equation (16)):

$$[\rho^x + (1-\rho^x)\rho](1-u) = (1-\rho^x)(1-\rho)\lambda^w u. \tag{18}$$

To calibrate the above equation, I make use of the empirical evidence on the worker matching probability and the overall separation rate $\rho + (1-\rho^x)\rho$. The implied unemployment rate in the model economy is then computed. I refer to CPS worker flow data to pin down the overall separation rate. Using Bleakley et al.'s [7] data set, I find the quarterly separation rate from employment at around 0.10 over the available sample period. The worker matching probability is determined from the unemployment duration estimated by Clark and Summers [13]. They show that the measured unemployment duration is substantially downward biased because of reporting errors induced by the presence of the out-of-the-labor-force state. As also discussed by Cole and Rogerson [14], the main issue is that measured unemployment durations do not capture the expected time between employment spells but rather the expected time before leaving the unemployment state to either the employment state or the out-of-the-labor-force state, which is problematic in the face of the fact that there is a large flow out of the labor force into employment every period.[19] Given that the model abstracts from the out-of-the-labor-force state, it appears appropriate to treat the expected time between employment spells as the "unemployment" duration in the model economy.[20] Clark and Summers estimate average unemployment duration at 19.9 weeks in 1974, which is translated into the matching probability of 0.65 per quarter. Taking $\rho + (1-\rho^x) = 0.1$ and $\lambda^w = 0.65$ as given, we can compute the implied unemployment rate at 0.146. This is obviously much higher than the measured unemployment rate in the U.S. whose historical average is around 6 percent. The higher implied unemployment rate here simply reflects that I measure unemployment duration as the expected time between employment spells, including those who are looking for a job being out of the labor force. In fact, referring to

[19] As Bleakley et al.'s [7] data set indicates, the flows between N and E are roughly of the same magnitude as those between U and E. See also Blanchard and Diamond [5].

[20] Note that the correction is essentially equivalent to treating as "unemployed" those who are identified as being out of the labor force but who "want a job," as well as those who are officially unemployed.

Blanchard and Diamond [5], den Haan et al. [19] identify 11.2 million job seekers and 93.2 million employed workers, on average, for the period of 1968-1986. These estimates yield

$$u = \frac{11.2}{93.2 + 11.2} \simeq 0.11.$$

Using this unemployment rate together with either $\rho + (1 - \rho^x) = 0.1$ or $\lambda^w = 0.65$ gives two other sets of estimates that are consistent with Equation (18). My results are based on the first set of estimates. But the results presented below are insensitive with respect to the choice of the other two sets of estimates.

To break down the overall separation rate into exogenous and endogenous parts, I adopt the interpretation that the model's endogenous separation rate corresponds to the permanent layoff rate.[21] Valletta [39] calculates the share of permanent layoffs out of the total incidence of separation over the period 1976−1998 and shows that it fluctuates around 25 percent. This evidence allows me to set ρ at 0.025.[22] Using $\rho = 0.025$ together with the total separation of 0.1, the exogenous separation rate is set equal to 0.083.

Next, the steady-state matching probability for firms λ^f is set equal to the same value as the worker matching probability 0.65. This choice is made simply because the equal matching probabilities imply the equal steady-state elasticities of the number of matches with respect to unemployment and vacancies, i.e., 0.5, under den Haan et al.'s [19] matching function used in this paper (Equation (12)).[23] I could alternatively refer to den Haan et al.'s [19] estimate of the firm matching probability equal to 0.71. The numerical results below are again robust to this alternative choice.

The parameters l, b, and c are uniquely determined by requiring the steady-state version of the model to match the empirical measures of the unemployment rate, the separation rate, and the matching probabilities obtained above. Finally, the idiosyncratic productivity shock is assumed to be i.i.d. lognormal with mean zero and standard deviation σ_z, following den Haan et al. [19]. Finally, σ_z is selected so that the model matches the observed variability of the separation rate.

[21] This interpretation for the endogenous separation rate can be found in den Haan et al. [19] and Collard et al. [15].

[22] Topel [37] calculates the quarterly permanent layoff rate at 0.018 based on the PSID in 1985. This is broadly in line with the the value used here.

[23] den Haan et al.'s matching function implies variable elasticities in contrast to the widely used Cobb-Douglas specification. This is a natural consequence of the matching probabilities being bounded between zero and one for all $u_t \in [0, 1]$ and $v_t \in [0, \infty)$ as I mentioned before.

3.4.2 Fixed-Separation-Rate Specification

The benchmark model is modified by letting the endogenous separation rate be zero. Accordingly, idiosyncratic uncertainty is eliminated by setting $z_t = 1$ and $\sigma_z = 0$. Under this specification, the steady-state version of the evolution of unemployment reduces to:

$$\rho^x(1-u) = (1-\rho^x)\lambda^w u. \tag{19}$$

As in the benchmark specification, the total separation rate and the worker matching probabilities are set equal to 0.1 and 0.65, respectively, implying that the steady-state unemployment rate is 0.146 as in the benchmark case. The worker matching probability and the unemployment rate deviate from Shimer's [36] calibration, which sets the steady-state worker matching probability at 0.34 per month and the steady-state unemployment rate at 0.057. These differences are due to the fact that Shimer takes reported unemployment as the empirical analogue to unemployment in his model, and accordingly, the matching probability is computed from the unemployment duration data. On the other hand, I lump together those who are still looking for jobs being out of the labor force and those officially unemployed and compute the matching probability from the evidence on non-employment duration. However, these differences did not have significant impacts on the model's quantitative properties. In other words, Shimer's results are well approximated under my calibration despite these differences.

The steady-state matching probability for firms λ^f and the parameter l for the matching function are set to the same values as in the benchmark specification.

The unemployment benefit b is chosen to be 0.4, following Shimer. This value corresponds to the upper end of the range of income replacement rates in the U.S. This parameter plays a crucial role in determining the model's ability to magnify aggregate productivity shocks, as shown analytically by Shimer. To see this point intuitively in my framework, first note that surplus S_t gets smaller as b gets larger, and thus an aggregate shock has a larger impact on the surplus in a percentage term. Vacancies then must change by more in a percentage term in order to ensure that the free-entry condition holds (see Equation (13)). The larger volatility in vacancies also makes unemployment more volatile as unemployment evolves according to Equation (16). Note that in the benchmark specification where the separation decision is endogenous, the parameter b must be assigned to achieve the target level of the endogenous separation rate, which yields a much higher level of the outside option $b = 0.87$ than Shimer's choice. This implies that the benchmark model exhibits larger volatilities even apart from endogenous fluctuations in the separation rate. The next section shows that the "leverage effect" together with endogenous fluctuations in the separation rate makes it possible for the benchmark model to perform well along the volatility dimensions whereas the low value of b and the fixed separation rate lead to a serious problem along these dimensions. Rather than exploring the issue of which specification is more reasonable, I will show that the model encounters basically the same problems in its vacancy dynamics no matter which specification is used.

4 Performance of the Model

4.1 Benchmark Specification

Figure 7 presents the model's impulse responses to a one-standard-deviation negative aggregate shock. The upper right panel shows that vacancies respond to shocks immediately; the negative shock lowers firms' expected returns, and thereby vacancies, eventually raising the matching probability up to the point where the free-entry condition is restored. However, the initial effects largely disappear in a couple of quarters. From then on, vacancies slowly converge back to the steady-state level. The behavior after the initial decline is due to the echo effect; as unemployment rises, the matching probability for the firm becomes higher, eliminating firms' incentive to keep cutting vacancies any further. Another important feature of the model's responses, as illustrated in the lower left panel of the figure, is that the initial decline in vacancies produces the lower creation rate, but immediately after the decline, the creation rate surges to a much higher level than the pre-shock level, reflecting the strong influence of unemployment.

Figure 8 compares the model and empirical responses. First, observe in the lower right panel that the model has no trouble in generating a volatility in employment (equivalently unemployment) that is comparable to the data (actually the model generates too much volatility). Next, consider the patterns of responses of gross flows behind the employment stock. As mentioned above, the model's responses greatly exaggerate the empirical responses; increases in the creation rate are too large, and the separation rate is too persistent. Finally, the lower left panel compares the responses of vacancies. As we saw in Section 2, the observed data strongly favor a hump-shaped and persistent response. Neither of these can be observed in the model. This problematic behavior of vacancy is the source of the overshooting behavior in gross flows. Clearly, the immediate recovery of vacancies, due to the echo effect discussed above, pushes up the creation rate to a higher level than otherwise. Moreover, it also makes the separation rate too persistent because the surge in vacancies causes the worker matching probability to be higher than otherwise. Unemployed workers then find it easier to find next employment opportunities and thus, in turn, have larger incentives to separate. Observe also that the initial declines in vacancies in the model and the data are roughly of the same magnitude. This indicates that lack of volatility of vacancies is not a first-order issue under the benchmark specification. The more fundamental problem lies in the model's inability to produce a hump-shaped and persistent response in vacancies.

Another symptom of the model due to the behavior of vacancies is its inability to generate an empirically plausible Beveridge curve. The middle row (labelled "Benchmark") of Table 3 presents the cross correlations between unemployment and vacancies in the model.[24] The model generates only a small negative correlation between the two, whereas the observed data display a correlation coefficient of -0.95. Observe also that in the model, increases in unemployment predict future recovery of vacancies, and this relationship shows up as positive correlations between period-t unemployment and future vacancies. In the observed data, however, we do not observe this pattern. The data display strong negative correlations at all leads and lags with some indication that vacancies lead unemployment.

4.2 Fixed-Separation-Rate Specification

Figure 9 displays impulse responses of the model economy where the separation rate is fixed. An important observation here is that the echo effect in vacancies is still present in the model but weaker than in the preceding case; vacancies recover only half way initially and then slowly converge to the steady-state level. In the benchmark case, on the other hand, almost all the initial response is corrected within a few quarters. This weaker echo effect is explained by the fact that unemployment volatility is much smaller under this alternative specification because of the lower outside option and the fixed separation rate. Figure 10 puts together the model and empirical responses under the alternative specification and clearly illustrates Shimer's [36] point that the model generates only small variabilities relative to the data. As discussed in Subsection 3.4.2, this volatility issue arises because of the choices that the separation rate is fixed and the outside option for the matched pair is set low.

The less substantial echo effect makes it easier for the model to generate negative correlations between unemployment and vacancies, which we can see in the last row of Table

[24] The statistics of the model economies are based on 100 simulated samples, each of which consists of 327 periods, where only the last 127 observations are used to compute the statistics. The number of observations corresponds to the available sample period of CPS worker flow data. The first 200 observations are ignored to randomize initial conditions. The data are logged and HP filtered with the smoothing parameter 1600.

3. Note that given the timing of the model, unemployment responds to the shock only in the next period under the fixed-separation-rate specification. It is therefore appropriate to compare the correlation between period-t unemployment and one-period-lagged vacancies with the contemporaneous correlation in the observed data. As expected, current-period unemployment is highly negatively correlated with one-period-lagged vacancies in the model. This result corresponds to Shimer's simulation result.[25] However, even though the model is able to generate the strong negative correlation under this alternative specification, the correlation patterns of the model still substantially differ from the observed pattern. Importantly, the echo effect is still present, though weaker, making it impossible for the model to generate a hump-shaped response, which is found to be a robust feature of the data.

4.3 Looking at Individual Responses

I have argued so far that the model has a serious problem in its vacancy dynamics, by visually comparing the model and empirical responses; the lower left panels of Figures 8 and 10 clearly indicated that the median empirical responses of vacancies are of quite different shape from the model responses. Recall, however, that the bounds are computed for each period rather than for the entire functions. Given that the model responses are actually within the bands almost all the time, it is important to examine the individual responses. To confirm that my claim is valid, I calculate the fraction of the responses that roughly match the model's predictions about the vacancy behavior to the total number of responses that satisfy the sign restrictions. Specifically, I select the responses that meet the following two criteria in addition to the sign restrictions:

$$\frac{\partial v_t}{\partial \omega_{agg,t}} > 0,$$

$$\left|\frac{\partial v_{t+i-1}}{\partial \omega_{agg,t}}\right| > \left|\frac{\partial v_{t+i}}{\partial \omega_{agg,t}}\right| \text{ for } i = 1, 2, 3.$$

where $\omega_{agg,t}$ denotes an aggregate shock in period t. The first criterion indicates that vacancies must drop (increase) in the impact period of the negative (positive) aggregate shock. The second criterion says that the response of vacancies then approaches toward the steady-state level in the subsequent two periods.[26] Note that these criteria actually allow for a much wider class of responses than the model responses shown in Figures 7 and 9. In particular, these criteria do not exclude the possibility that vacancies slowly move back toward the

[25]Since Shimer works on a continuous time model, there is no timing issue in his model.

[26]The results are not sensitive with respect to setting the horizon longer.

steady-state level following an initial drop. The idea here is to pick up the responses that are not hump shaped and to consider them as consistent with the model. Table 4 presents the results. Surprisingly, only 4 draws meet the above criteria out of the total of 3,149 draws that satisfy the three sign restrictions. This exercise clearly confirms my claim.

5 Conclusion

This paper has shown that the cyclical behavior of vacancies in the standard labor-matching model is counterfactual, implying that the model's key equilibrium condition is not supported by the observed data. Whereas the model predicts that vacancies respond to shocks with no delay and are not persistent, the data strongly favor a hump-shaped and persistent response of vacancies. I have also found that the problems in vacancy behavior cause gross flows in the model to exhibit counterfactual properties.

I propose two possible modifications to the model. First, recall that in the model, rises in unemployment produce an incentive for firms to open up vacancies even though aggregate conditions are not favorable. Notice that the statement is valid only under the assumption that workers engage in search activities only when they are unemployed. That is, countercyclical unemployment directly implies countercyclical search activities under this assumption, thus causing a strong echo effect. It is then clear that anything that generates procyclical search activities may reduce the echo effect caused by countercyclical unemployment. Allowing for on-the-job search could be one such candidate as the outside option for on-the-job seekers is higher (lower) during booms (recessions).[27] Barlevy [3] develops a matching model with on-the-job search and, in fact, emphasizes the mechanism in his model that fewer vacancies during recessions give workers a difficult time in reallocating themselves into better employment relationships. Although he does not explore the quantitative implications for vacancy dynamics, this story appears to be in line with observed persistence in vacancies. Mortensen [27] examines the quantitative implications of on-the-job search extending the standard Mortensen and Pissarides [28] framework. His simulation result in fact displays a somewhat higher negative correlation between unemployment and vacancies

[27]Endogenizing the labor market participation decision may also introduce procyclical search activities into the model (e.g., in the form of a discouraged worker effect). In this case, however, unemployment must be procyclical in order to eliminate the echo effect. Veracierto [40] shows that introducing the participation decision into the Lucas-Prescott island model counterfactually produces procyclical unemployment. He also argues that the result holds in the Mortensen and Pissarides labor-matching framework.

than his earlier result with Pissarides.[28] A remaining problem with this approach, however, is that there appears to be no reason for vacancies to respond to shocks slowly, displaying a hump-shaped pattern.

Second, it may be possible to interpret the persistent and hump-shaped response of vacancies in the data as the evidence suggesting presence of some frictions in creating new positions. Caballero and Hammour [10] develop a vintage model in which job creation requires relation-specific investment as well as a search cost and show that the specific investment component serves to produce a negative comovement between unemployment and vacancies and to "decouple" the positive comovement between creation and destruction.[29] Fujita and Ramey [21] examine cyclical implications of heterogeneity in a job creation process building upon the standard Mortensen and Pissarides framework. Specifically, they augment the model by introducing costly planning for brand-new jobs and the option to mothball preexisting jobs. These modifications are shown to greatly improve the model's quantitative performance.

6 Appendix: Solution Algorithm

This appendix presents the solution algorithm of the model. The algorithm applied here is a non-linear global projection method. General discussions on the same class of methods can be found in Judd [24], [25].[30]

Before presenting the algorithm, I first rewrite the model by following the convention that an unprimed variable denotes its current-period value, and a primed variable denotes its next-period value. Recall that the current-period state variables consist of $\mathbf{s} = \{y, u - m\}$. The recursive equilibrium is a list of functions $G(\mathbf{s}), U(\mathbf{s}), v(\mathbf{s})$ and $\hat{z}(\mathbf{s})$ such that (i) the continuation-value equations hold:

$$G(\mathbf{s}) = E\left[\beta(1-\rho^x)\int_{\hat{z}(\mathbf{s}')}^{\infty} S(z';\mathbf{s}')dH(z') + U(\mathbf{s}') + b \mid y\right], \tag{20}$$

$$U(\mathbf{s}) = E\left[\beta\lambda^w(1-\rho^x)(1-\pi)\int_{\hat{z}(\mathbf{s}')}^{\infty} S(z';\mathbf{s}')dH(z') + U(\mathbf{s}') + b \mid y\right], \tag{21}$$

where $S(z';\mathbf{s}') = z'y' + G(\mathbf{s}') - U(\mathbf{s}') - b$ and $\lambda^w = m(u', v(\mathbf{s}))/u'$; (ii) the free-entry condition

[28] Pissarides [31] and Chapter 4 of Pissarides [32] also allow for on-the-job search, but they focus on steady-state analysis.

[29] They show that the model with the search cost alone predicts positive comovements between these variables, as is consistent with the prediction of the standard labor-matching model.

[30] See also Gasper and Judd [22] and Christiano and Fisher [12].

holds:

$$\frac{c}{\lambda^f} = E\left[\beta\pi \int_{\hat{z}(\mathbf{s}')}^{\infty} S(z';\mathbf{s}')dH(z') \mid y\right], \tag{22}$$

where $\lambda^f = m(u', v(\mathbf{s}))/v(\mathbf{s})$; (iii) the separation condition $\hat{z}y + G(\mathbf{s}) - U(\mathbf{s}) - b = 0$ defines $\hat{z}(\mathbf{s})$; (iv) these conditions are satisfied under the evolution of aggregate productivity and unemployment:

$$\ln y' = \xi \ln y + \varepsilon', \tag{23}$$

$$u' = u + [\rho^x + (1-\rho^x)\rho(\hat{z}(\mathbf{s}))]\,(1-u) - (1-\rho^x)(1-\rho(\hat{z}(\mathbf{s}))m. \tag{24}$$

The first step in numerically solving the model is to approximate the right-hand side of Equations (20), (21), and (22) by a tensor product of second-order Chebyshev polynomials of each state variable. Note that each function has $3^3 = 27$ unknown coefficients, so there are a total of $27 * 3 = 81$ unknown coefficients. I use fixed-point iteration to solve for these coefficients. The iteration proceeds as follows; using some initial guess for the 81 unknown coefficients, the current period separation margin can be computed by the separation condition, which also gives the separation rate from $\rho(\hat{z}(\mathbf{s})) = \int_{\hat{z}(\mathbf{s})}^{\infty} dH(z)$. The integral is computed by Simpson's rule with 15 nodes. One can then compute u' from the evolution of unemployment (24). Next, making use of the approximating function for $E\left[\beta\pi \int_{\hat{z}(\mathbf{s}')}^{\infty} S(z';\mathbf{s}')dH(z') \mid y\right]$ in the free-entry condition (22) allows one to obtain the equilibrium level of vacancies $v(\mathbf{s})$. The matching technology then reveals the outcome of the matching market $m(u', v(\mathbf{s}))$ given the levels of vacancies and unemployment. Given the next-period values of unemployment u' and the distribution of the aggregate productivity shock ε', one can actually compute the conditional expectations appearing on the right-hand side of Equations (20), (21), and (22). The integral inside the bracket in Equation (20) is again computed by Simpson's rule with 15 nodes. The conditional expectations associated with the aggregate productivity shock are numerically computed by Gauss-Hermite quadrature with 5 nodes. These conditional expectations are evaluated at 27 grid points that are chosen by finding three zeros of Chebyshev polynomials for each state variable and taking all possible combinations of the roots. The new set of coefficients of the approximating functions is obtained by equating the values of the right-hand sides of Equations (20), (21), and (22) to the values of the approximating functions at 27 grid points. Since there are 27 coefficients in each approximating function, this uniquely pins down the new set of coefficients (i.e., orthogonal collocation method). The iteration continues until convergence of the 81 Chebyshev coefficients is achieved.

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Table 1: Parameter Values

Symbol	Concept	Benchmark	Exogenous Separation
ξ	$AR(1)$ coefficient of aggregate productivity	0.950	0.950
σ_ε	Standard deviation of the aggregate productivity shock	0.007	0.007
b	Unemployment benefit	0.872	0.400
σ_z	Standard deviation of the idiosyncratic productivity shock	0.250	0.000
ρ^x	Exogenous separation rate	0.083	0.100
π	Bargaining weight of the firm	0.500	0.500
l	Parameter in the matching function	1.609	1.609
β	Discount factor	0.990	0.990
c	Vacancy posting cost per period	0.124	0.436

Table 2: Steady-State Values

Symbol	Concept	Both Calibrations
λ^w	Worker's matching probability	0.650
λ^f	Firm's matching probability	0.650
u	Unemployment rate	0.156
v	Vacancy rate	0.156

Table 3: Cross Correlations Between Unemployment and Vacancies

$Corr(v_{t+k}, u_t)$	-3	-2	-1	0	1	2	3
US data	-0.60	-0.80	-0.94	-0.95	-0.81	-0.59	-0.35
	(0.068)	(0.054)	(0.014)	(0.014)	(0.046)	(0.058)	(0.109)
Benchmark	-0.43	-0.52	-0.51	-0.17	0.24	0.34	0.33
Exo. Separation	-0.58	-0.77	-0.82	-0.33	-0.10	0.04	0.13

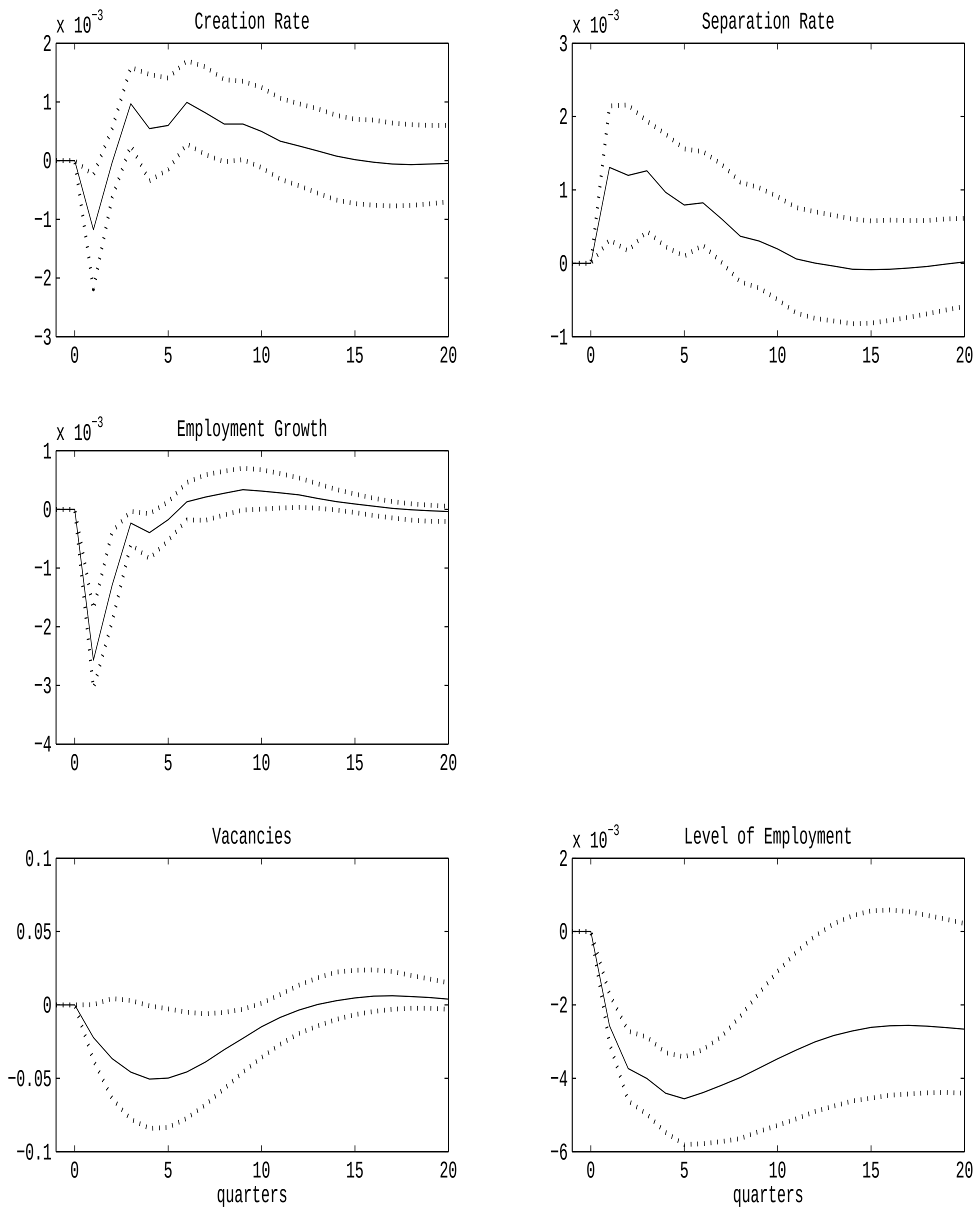
Notes: All series are logged and HP filtered. Standard errors presented in parentheses are computed by the den Haan and Levin's [18] GMM-VARHAC procedure. $u = LHUR \cdot LHPAR/10000$, $v = LHELX \cdot LHUR \cdot LHPAR/10000$, where $LHUR$: unemployment rate, $LHPAR$: labor-force participation rate, $LHELX$: help-wanted ads as percentage of unemployed. These data are taken from DRI-Webstract (former CITIBASE). Sample period is 1972Q1-1993Q1.

Table 4: Probability that the Prediction of the Matching Model is Supported by the Data

number of valid draws	probability (%)	sign restrictions	K
3,149	0.13	1, 2, 3	4
7,662	0.39	1, 2, 3	2
3,986	0.10	1, 2	4
12,431	0.51	1, 2	2

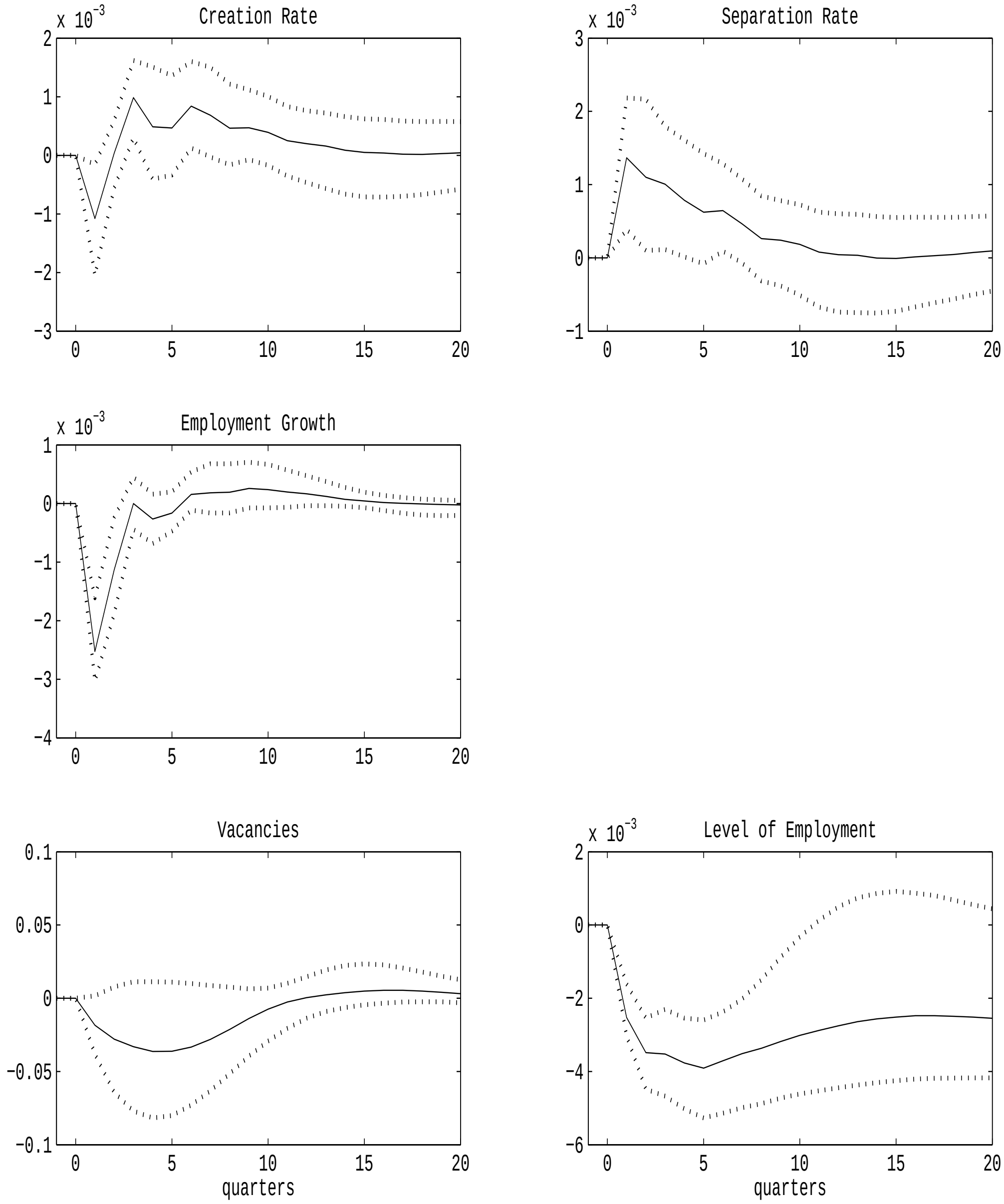
Notes: The first column shows the number of valid draws (from a total of 40,000 draws) that satisfy the sign restrictions listed in the third column. The restrictions are discussed in Subsection 2.2. The third and fourth columns give the probability (%) that the predictions of the Mortensen and Pissarides model regarding the behavior of vacancies are met. See page 19 for the criteria.

Figure 1: Empirical Impulse Responses: Sign Restrictions 1, 2 and 3, $K = 4$



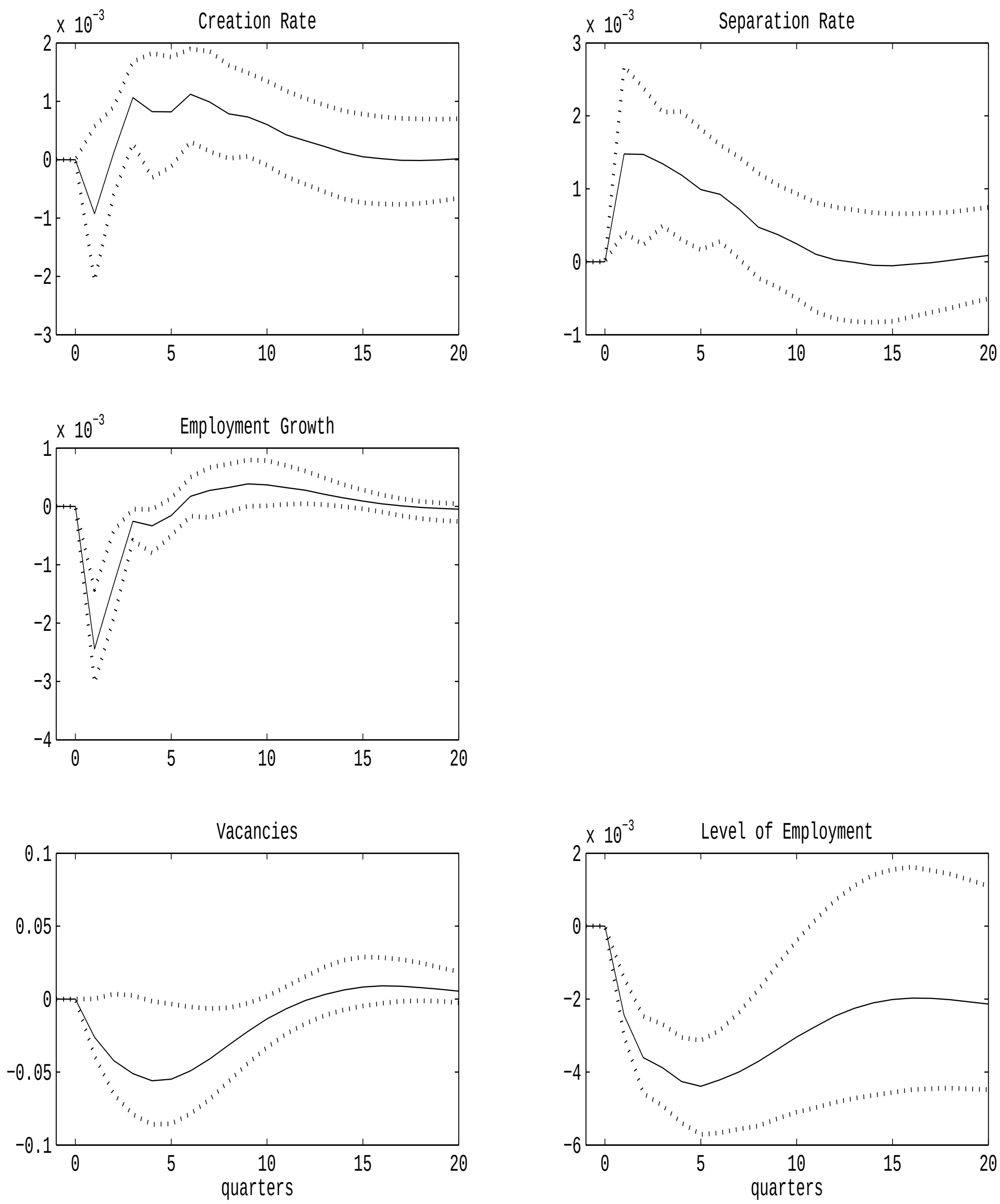
Notes: The three sign restrictions are discussed in Subsection 2.2. The three lines are the 10th percentile, the median, and the 90th percentile of the posterior distribution.

Figure 2: Empirical Impulse Responses: Sign Restrictions 1, 2 and 3, $K = 2$



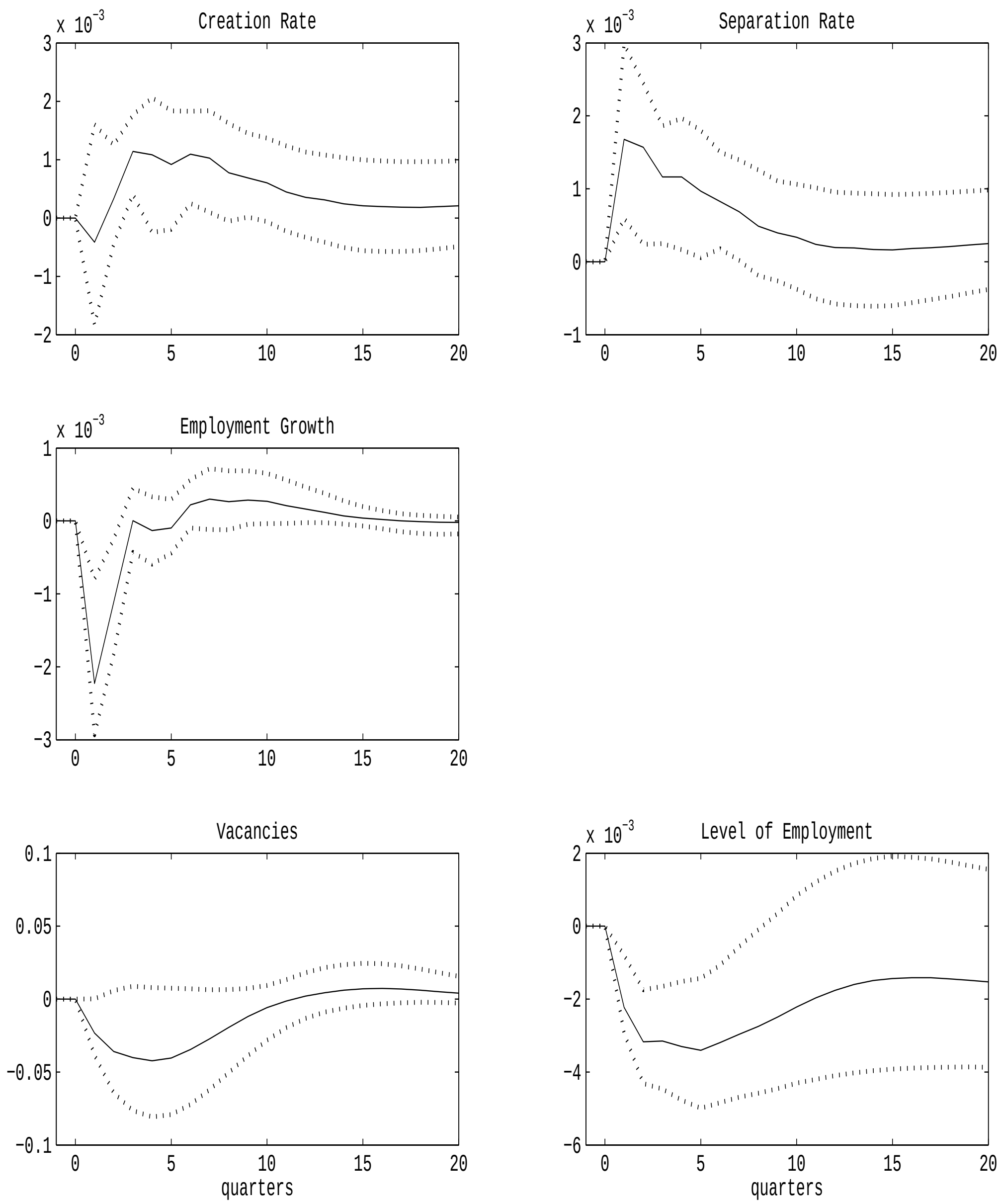
Notes: The three sign restrictions are discussed in Subsection 2.2. The three lines are the 10th percentile, the median, and the 90th percentile of the posterior distribution.

Figure 3: Empirical Impulse Responses: Sign Restrictions 1 and 2, $K = 4$



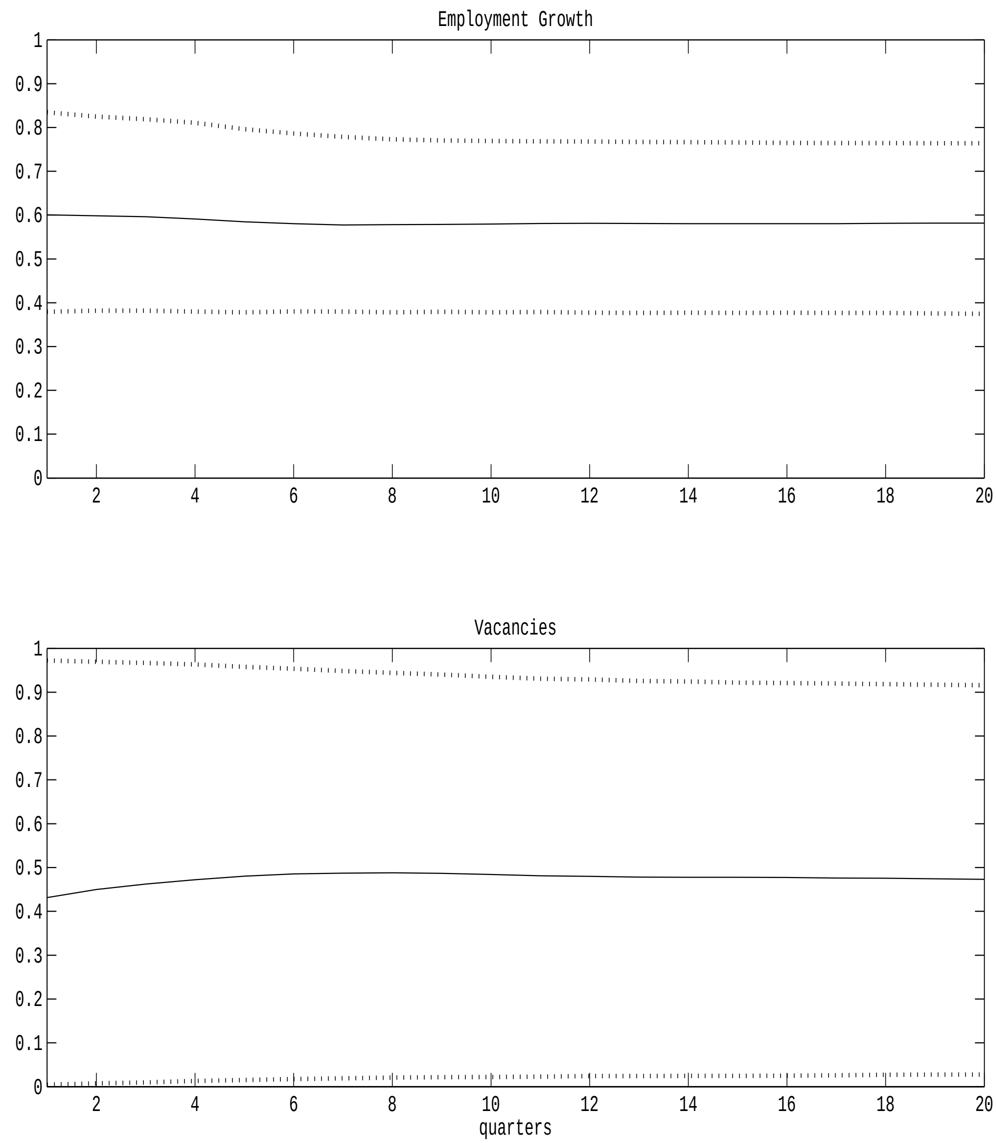
Notes: The three sign restrictions are discussed in Subsection 2.2. The three lines are the 10th percentile, the median, and the 90th percentile of the posterior distribution.

Figure 4: Empirical Impulse Responses: Sign Restrictions 1 and 2, $K = 2$



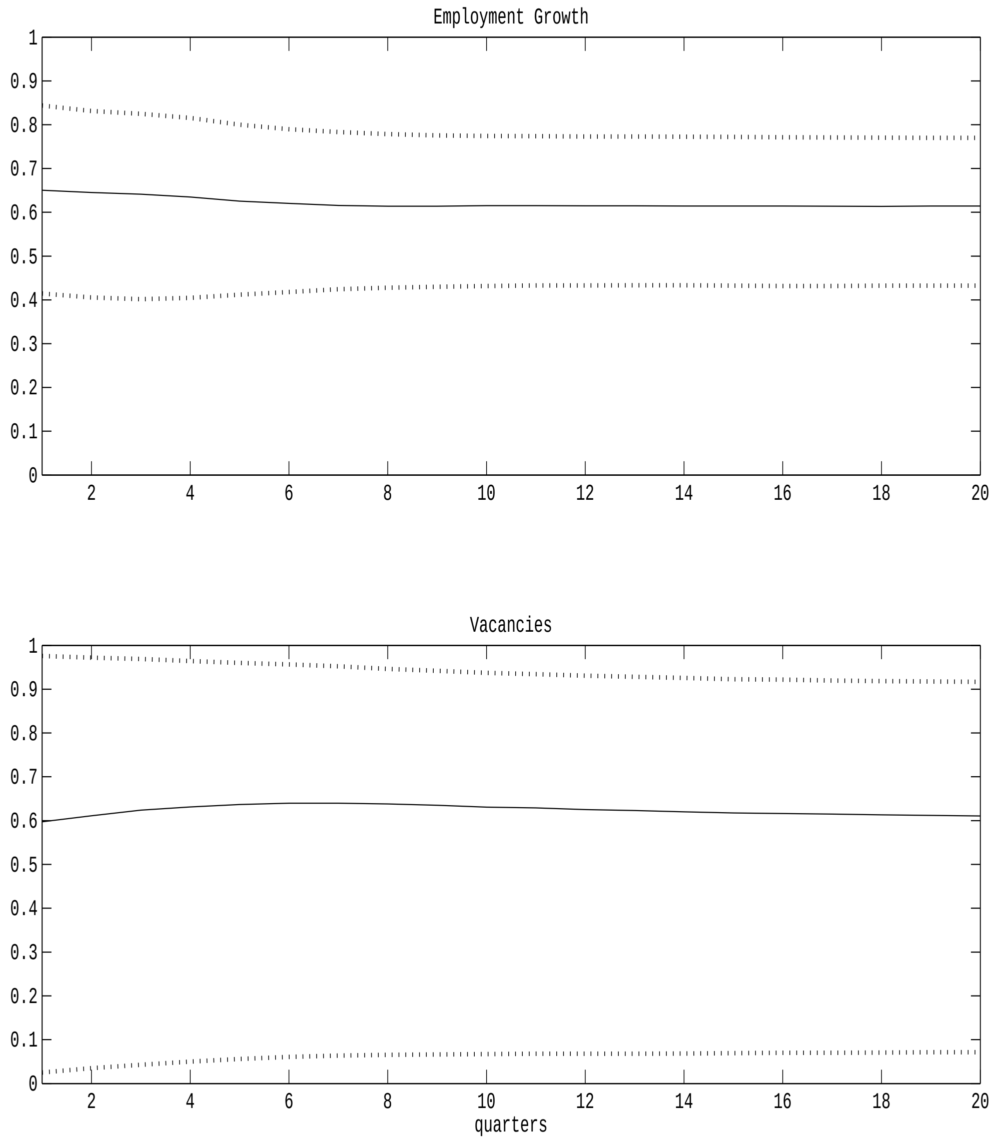
Notes: The three sign restrictions are discussed in Subsection 2.2. The three lines are the 10th percentile, the median, and the 90th percentile of the posterior distribution.

Figure 5: Variance Decomposition for Employment Growth and Vacancies: Sign restrictions 1, 2 and 3 are imposed. $K = 4$.



Notes: The sign restrictions are discussed in Subsection 2.2. The three lines are the 10th percentile, the median, and the 90th percentile of the posterior distribution.

Figure 6: Variance Decomposition for Employment Growth and Vacancies: Sign restrictions 1, 2 and 3 are imposed. $K = 4$. Responses of vacancies in the impact period are restricted to be negative.



Notes: The sign restrictions are discussed in Subsection 2.2. The three lines are the 10th percentile, the median, and the 90th percentile of the posterior distribution.

Figure 7: Impulse Responses: Endogenous Separation

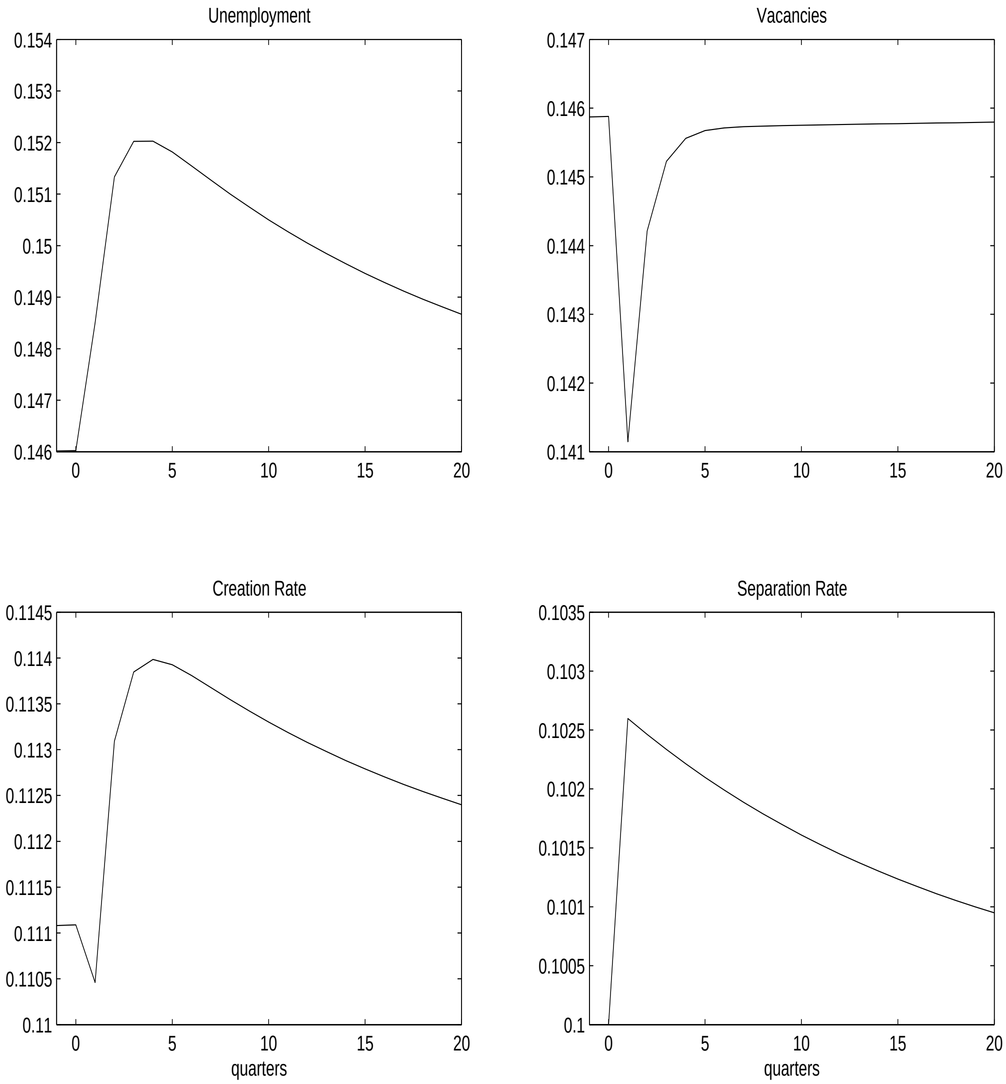
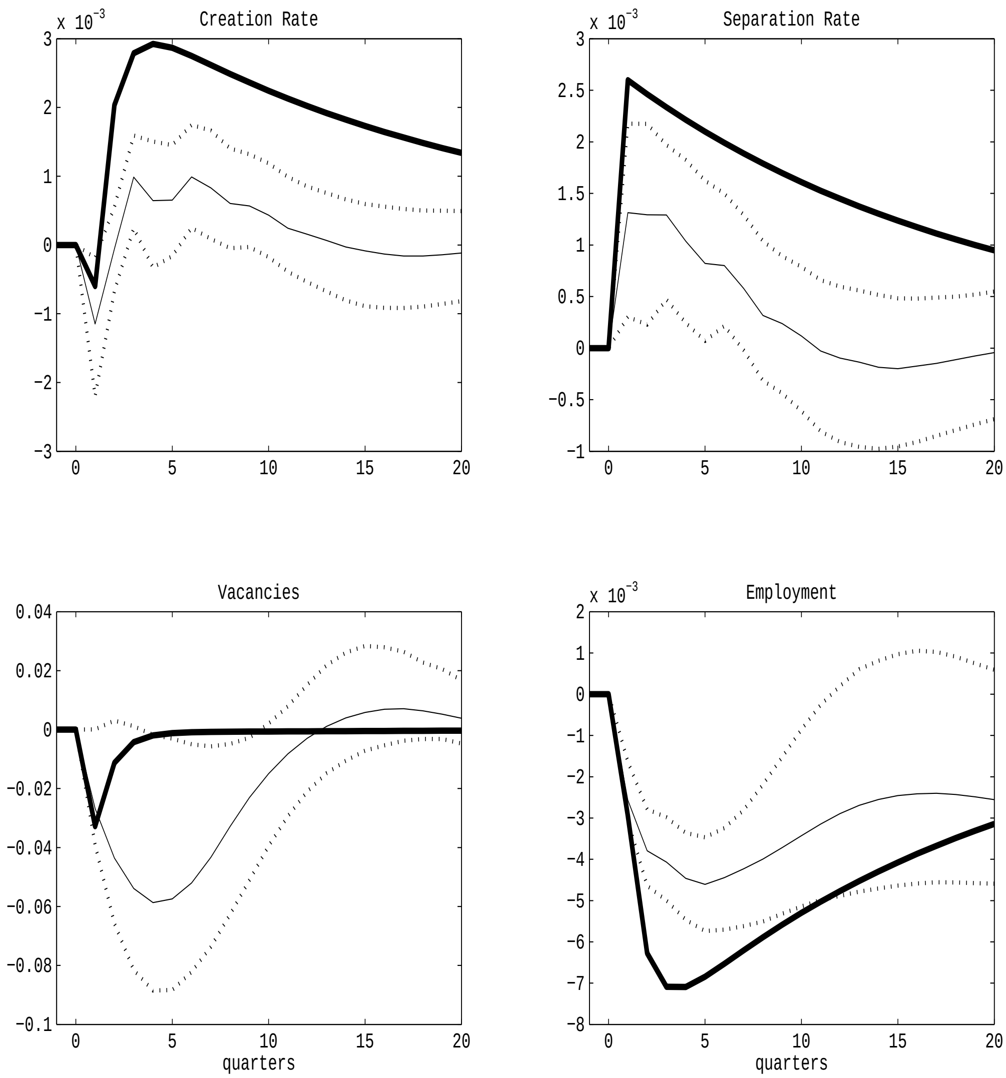


Figure 8: Comparison between Model Impulse Responses and Empirical Impulse Responses: Endogenous Separation



Notes: Responses in the model economy are shown by the thick solid lines. Other lines plot the empirical responses that are taken from Figure 1. All responses are expressed as deviations from the steady-state levels. Responses of vacancies and employment are expressed as log deviations.

Figure 9: Impulse Responses: Fixed Separation Rate

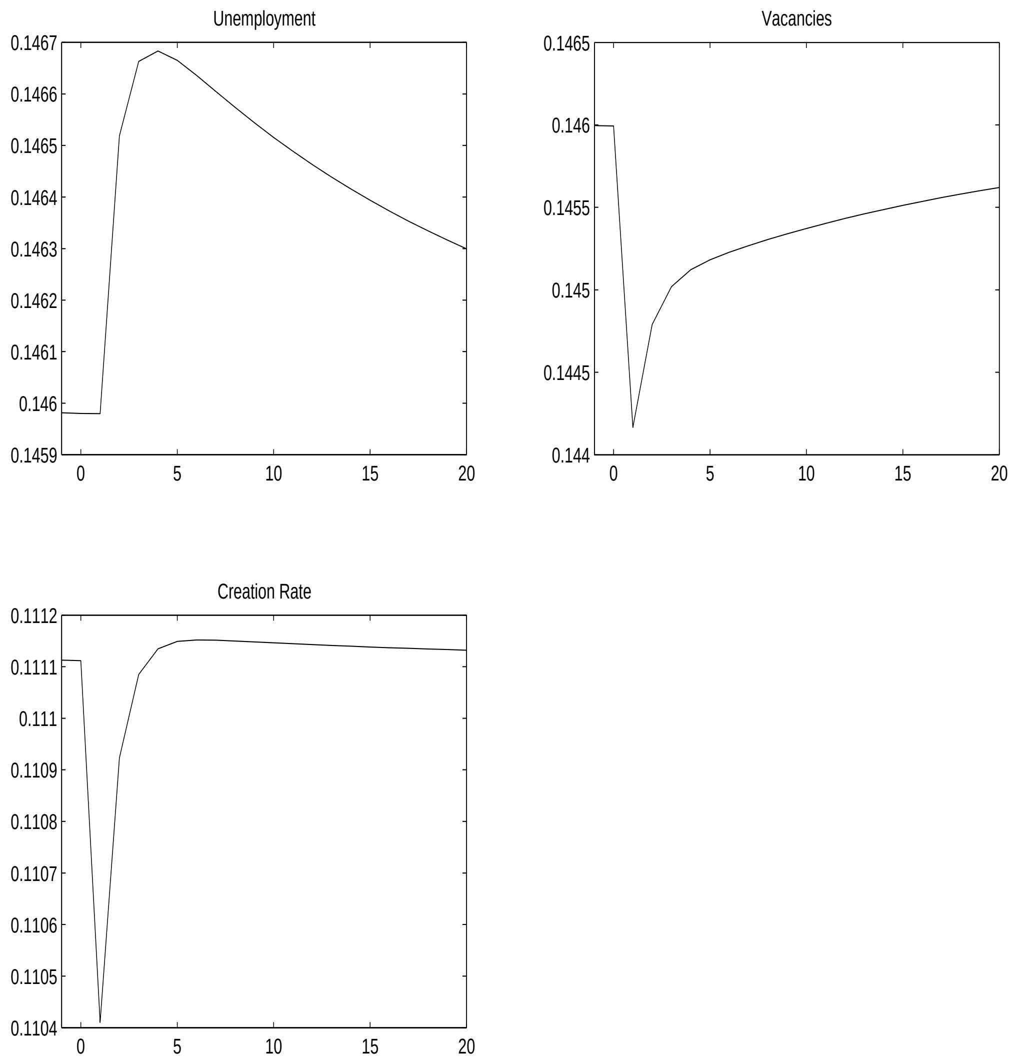
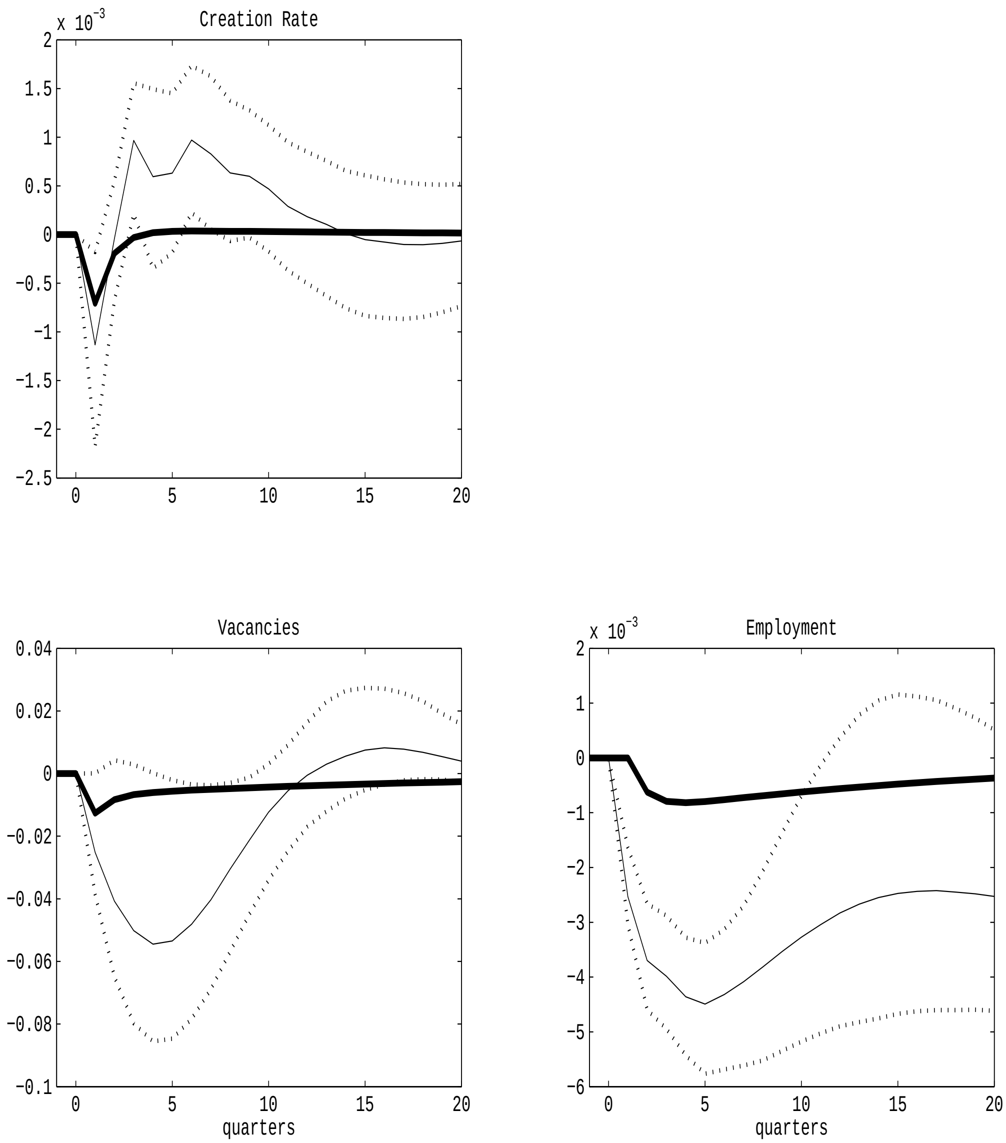


Figure 10: Comparison between Model Impulse Responses and Empirical Impulse Responses: Fixed Separation Rate



Notes: Responses in the model economy are shown by the thick solid lines. Other lines plot the empirical responses that are taken from Figure 1. All responses are expressed as deviations from the steady-state levels. Responses of vacancies and employment are expressed as log deviations.