WORKING PAPERS
RESEARCH DEPARTMENT

WORKING PAPER NO. 03-8
A SHORT-TERM MODEL OF THE FED'S PORTFOLIO CHOICE

Dean Croushore
Federal Reserve Bank of Philadelphia

October 2000,
Revised April 2003

FEDERAL RESERVE BANK OF PHILADELPHIA
Ten Independence Mall, Philadelphia, PA 19106-1574• (215) 574-6428• www.phil.frb.org

WORKING PAPER NO. 03-8

A SHORT-TERM MODEL OF THE FED'S PORTFOLIO CHOICE

Dean Croushore

Federal Reserve Bank of Philadelphia

October 2000
revised, April 2003

The views expressed in this paper do not necessarily represent the views of the Federal Reserve Bank of Philadelphia or the Federal Reserve System. This paper was prepared under the direction of Rick Lang for the Federal Reserve System study that was published as: *Alternative Instruments for Open Market and Discount Window Operations*, Board of Governors, December 2002. This paper is referenced in footnote 3, on page 3-47 in Appendix 3.C, "The Potential Effects on Financial Markets and Institutions of Replacing the SOMA's Treasury Securities with Advances of Federal Reserve Credit."

A SHORT-TERM MODEL OF THE FED'S PORTFOLIO CHOICE

ABSTRACT

What would happen if the Federal Reserve were to change the assets in its portfolio? Suppose that instead of using open-market operations in Treasury securities to increase the monetary base, the Fed were to engage in open-market operations in private securities or to use discount loans via a mechanism that allowed banks to borrow as much as they would like at a fixed discount rate. The analysis in this paper shows the impact on the economy in a static general-equilibrium model.

A SHORT-TERM MODEL OF THE FED'S PORTFOLIO CHOICE

What would happen if the Federal Reserve were to change the assets in its portfolio? Suppose that instead of using open-market operations in Treasury securities to increase the monetary base, the Fed were to engage in open-market operations in private securities or to use discount loans via a mechanism that allowed banks to borrow as much as they would like at a fixed discount rate. The analysis in this paper shows the impact on the economy in a static general-equilibrium model.

This model follows Santomero (1983), adapted to evaluate a change in the Fed's portfolio and how that affects the economy's general equilibrium at a point in time. The nature of the exercise done here is completely static in nature and does not evaluate the economy's response to a disappearance of government debt, analysis of which would require a more complete model that's dynamic in nature and incorporates real effects. The present model focuses on the more narrow issue of the direction of portfolio changes with no real-side economic effects. But the model is general equilibrium in nature and thus performs a reasonable comparative-static exercise.

In what follows, we first describe the model in Section I. Next, we model a situation in which the Fed changes its portfolio in such a way as to keep the interest rate on deposits from changing (Section II). Section III generates results under a special set of assumptions that lock most interest rates together. Section IV attempts to generalize the results to a situation in which the monetary base is unchanged. Section V summarizes the results.

I. The Model Framework

There are 5 agents in the model, banks (*b*), the Fed (*f*), the Treasury (*t*), households (*h*), and firms (*m*). There are 5 financial instruments in the model: (1) deposits at banks (*D*); (2) high-powered money (*H*), in the form of currency (*C*) and reserves (*R*); (3) government bonds (*G*); (4) private bonds (*V*) [you can think of private bonds held by households as corporate debt and private bonds held by banks as corporate loans]; and (5) discount window loans (*W*).

Figure 1 shows the balance sheets of the agents. Note that all financial instruments are liabilities of at least one agent and assets of at least one agent, and the total amount of assets held by all agents must equal the total amount of liabilities for each instrument.

Notation in this model follows the convention of Santomero, using lower-case abbreviations for agents and upper-case for instruments. For instrument *I*, the demand by agent *j* will be denoted I_j^d; for example, the demand for government bonds by banks is G_b^d. A superscript *s* denotes supply. We'll also assume there's only one of each agent to keep things simple. The net worth of each agent *i* is NW_i.

Following Santomero, the budget constraints (from the balance sheets in Figure 1) are:

Bank: $R_b^d + G_b^d + V_b^d = D_b^s + W_b^d + NW_b$

Fed: $G_f^d + V_f^d + W_f^s = H_f^s + NW_f$

Household: $D_h^d + C_h^d + G_h^d + V_h^d = NW_h$

Firm: $K = V_m^s + NW_m$ (*K* is firm's capital.)

Treasury: $D_t^d = G_t^s + NW_t$

Now, if we look at the demand and supply of each asset and equate them to find equilibrium, we get:

Deposits: $D_h^d + D_t^d = D_b^s$ interest rate is r_D

High-powered money: $R_b^d + C_h^d = H_f^s$ interest rate is 0

Government bonds: $G_b^d + G_f^d + G_h^d = G_t^s$ interest rate is r_G

Private bonds: $V_b^d + V_f^d + V_h^d = V_m^s$ interest rate is r_V

Discount window loans: $W_b^d = W_f^s$ interest rate is r_W

Equilibrium comes about by changes in interest rates and quantities of assets. It's a short-run model, so we aren't worried about bigger issues. Imagine an equilibrium and we'll look at some comparative-static exercises. Since we won't be allowing income or wealth to change in these experiments, we won't include those in the model.

The Fed can increase the amount of high-powered money either by increasing the amount of non-borrowed reserves via open-market operations (buying either government bonds or private bonds in the open market) and via discount window loans (lending directly to banks). Note that we're taking an aggregate view, thus not modeling the interbank (fed funds) market; so any shortage of reserves that banks want must be met by changes in interest rates to reduce the demand for reserves.

Now for each of the demand and supply terms in the model, we need to find how they are related to the various interest rates (and other factors that won't change in this short-run model).

Bank:

Demand for reserves: $R_b^d = \rho D_b^s$ — so reserve requirements are binding and no excess reserves are held.

Demand for government bonds: $G_b^d = G_b^d(\underset{+}{r_G}, \underset{-}{r_V}, \underset{-}{r_D}, \underset{-}{r_W})$ — where the signs indicate the partial response of the variable to an increase in the argument above. Here the demand for government bonds by banks rises when the interest rate on government bonds rises (higher own return), but falls when the interest rate on private bonds rises (banks buy more private bonds and fewer government bonds) or when the bank must pay higher interest rates on deposits or discount loans (since then the cost of funds is higher, so the bank wants a smaller portfolio).

Demand for private bonds: $V_b^d = V_b^d(\underset{-}{r_G}, \underset{+}{r_V}, \underset{-}{r_D}, \underset{-}{r_W})$ -- similar to the case of government bonds.

Supply of deposits: $D_b^s = D_b^s(\underset{+}{r_G}, \underset{+}{r_V}, \underset{-}{r_D}, \underset{+}{r_W})$ -- when returns to bank assets rise, the bank will want more deposits; when the interest rate on deposits rises it will want fewer deposits; when the interest rate on discount loans rises, it will want more deposits to replace discount loans, if it has any outstanding.

Demand for discount loans: $W_b^d = W_b^d(\underset{+}{r_G}, \underset{+}{r_V}, \underset{+}{r_D}, \underset{-}{r_W})$ -- same factors as supply of deposits, but with opposite effects due to changes in rate on deposits or discount rate.

Fed:

We take Fed actions to be exogenous; given some equilibrium, we're going to see what happens in a comparative-static sense when the Fed changes its assets and liabilities, assuming monetary policy targets an interest rate, as discussed below.

Household:

Households substitute between deposits, currency, government bonds, and private bonds, depending on the relative returns to each. A higher own return increases demand, while a higher return on an alternative assets decreases demand. So the demands for each are:

Demand for deposits: $D_h^d = D_h^d(\underset{-}{r_G}, \underset{-}{r_V}, \underset{+}{r_D})$

Demand for currency: $C_h^d = C_h^d(\underset{-}{r_G}, \underset{-}{r_V}, \underset{-}{r_D})$

Demand for government bonds: $G_h^d = G_h^d(\underset{+}{r_G}, \underset{-}{r_V}, \underset{-}{r_D})$

Demand for private bonds: $V_h^d = V_h^d(\underset{-}{r_G}, \underset{+}{r_V}, \underset{-}{r_D})$

Firm:

For now, we take capital and the amount of private bonds issued as exogenous. If we consider more complicated issues, such as the issuance by government-sponsored enterprises of additional bonds when their relative returns decline, this assumption must be changed. But for now, V_m^s is taken to be exogenous.

Treasury:

We assume that Treasury issuance of government bonds, G_t^s, is exogenous. An earlier version of this model assumed that any change in bond issues must be offset by an equal change in deposits, D_t^d. But that's not really appropriate. The decline in government debt occurs because the government has essentially received a surprise increase in net wealth (i.e., tax receipts and unexpectedly higher and expenditures unexpectedly lower than they were before). However, this fact must be reflected in real economic activity (i.e., households or firms must have higher capital or net wealth than before) as well, analysis of which is going to require a more complete model of real activity. Thus, it makes sense at this point to simply do a differential analysis on the Fed's portfolio.

In this version, in order to examine just the differential effect from the Fed's portfolio decision, we'll take the supply of government bonds as exogenous.

Equilibrium:

We assume that an equilibrium exists in which all the equations describing demand and supply are determined, all the budget constraints are satisfied, and there's a unique equilibrium in the return variables, r_D, r_G, r_V, and r_W.

Monetary Policy:

Consider the following comparative statics experiment: The Fed reduces its holding of government bonds by the amount $\alpha = dG_f^d < 0$. What happens in equilibrium depends on what else the Fed does to its portfolio.

We assume for now that, in the short run, monetary policy targets the interest rate on deposits, so $dr_D = 0$. Thus, any changes that occur to r_D are offset by the Fed to return the interest rate to its initial level. Why is it this interest rate that's targeted? Since we aren't modeling banks individually or complicating the model by introducing uncertainty about within-maintenance-period shocks, we don't model a federal funds market explicitly. Instead, thinking of the usual supply and demand diagram for reserves, which is consistent with our model, since reserve demand depends only on deposit demand, we assume the Fed targets the interest rate on deposits in the short run. How can the Fed hit its interest-rate target? Consider two possible methods by which this happens:

(1) The Fed does open-market operations in private bonds to hit its interest-rate target.

(2) The Fed substitutes a new discount loan mechanism by which discount loans substitute for government bonds on the Fed's balance sheet and are supplied to banks elastically at the interest rate r_W.

The Discount Window: To keep things simple, we assume that case (1) involves mechanisms in place today, and that the discount window is prohibitively expensive for banks. Thus there is no discount window borrowing. In case (2), we assume the Fed has in place a simple discount window mechanism allowing banks to borrow all they want at the discount rate (the NACF, non-administered credit facility).

Spell it out in the model:

Now, let's think about what goes on in each market:

Deposits: $D_h^d(\underset{-}{r_G},\underset{-}{r_V},\underset{+}{r_D}) + D_t^d = D_b^s(\underset{+}{r_G},\underset{+}{r_V},\underset{-}{r_D},\underset{+}{r_W})$

An increase in the return to government or private bonds decreases household demand for deposits and increases bank supply of deposits, so excess demand for deposits decreases. A higher interest rate on deposits increases household demand for deposits and decreases the bank's supply of deposits, so the excess demand for deposits rises. An increase in Treasury deposits increases excess demand. Thus, we can summarize the deposit market with the excess demand function (ignoring the effect of changes in the discount rate):

$$\underset{-}{D}(\underset{-}{r_G}, \underset{+}{r_V}, r_D, \underset{+}{D_t^d}, \underset{-}{r_W}) = 0 .$$

$$D(r_G, r_V, r_D, D_t^d, r_W) = 0 .$$
$$\quad - \quad - \quad + \quad\; + \quad -$$

High-powered money: $R_b^d + C_h^d = H_f^s$

$$\rho D_b^s(r_G, r_V, r_D, r_W) + C_h^d(r_G, r_V, r_D) = H_f^s$$
$$+ \quad + \quad - \quad + \qquad\qquad - \quad - \quad -$$

The demand for high-powered money is split into two parts: currency demand by households and reserve demand by banks. The Fed operates by affecting the supply of high-powered money, thus influencing interest rates. In the market for high-powered money, the excess demand is given by:

$$H(r_G, r_V, r_D, r_W, H_f^s) = 0 .$$
$$? \quad ? \quad - \quad + \quad\; -$$

Note that higher returns to government or private bonds lead banks to earn a higher return on their assets, so they want to attract more deposits, so they demand more reserves, hence more high-powered money, but households demand less currency and deposits, hence less high-powered money; so the signs of the first two arguments are ambiguous. Higher deposit interest rates lead banks to offer fewer deposits, so they need fewer reserves, and households want to hold less currency, so the demand for high-powered money is lower. And, of course, when the Fed supplies more reserves, excess demand for reserves declines.

Government bonds: $G_b^d(r_G, r_V, r_D, r_W) + G_f^d + G_h^d(r_G, r_V, r_D) = G_t^s$
$$\quad\quad\quad + \; - \; - \; - \quad\quad\quad\quad\quad + \; - \; -$$

Higher returns to government bonds will increase the demand by both banks and households for the bonds. Higher returns to private bonds will have the opposite effect. When the return to deposits rises, households and banks want to hold fewer government bonds. Thus the excess demand for government bonds is:

$$\underset{}{G}(\underset{+}{r_G}, \underset{-}{r_V}, \underset{-}{r_D}, \underset{+}{G_f^d}, \underset{-}{G_t^s}, \underset{-}{r_W}) = 0$$

Private bonds: $V_b^d(r_G, r_V, r_D, r_W) + V_f^d + V_h^d(r_G, r_V, r_D) = V_m^s$
$$\quad\quad\quad - \; + \; - \; - \quad\quad\quad\quad\quad - \; + \; -$$

Higher returns to private bonds will increase the demand by both banks and households for the bonds. Higher returns to government bonds will have the opposite effect. When the return to deposits rises, households and banks want to hold fewer private bonds. Thus the excess demand for private bonds is:

$$V(\underset{-}{r_G}, \underset{+}{r_V}, \underset{-}{r_D}, \underset{+}{V_f^d}, \underset{-}{V_m^s}, \underset{-}{r_W}) = 0 .$$

What we have is really a system of 4 excess demand equations [D=0, H=0, G=0, and V=0] in 3 endogenous variables [r_D, r_G , and r_V]. But if three of the excess demands are zero, the fourth must also be zero because budget constraints hold (Walras's Law). That means one excess demand equation is redundant.

Solving:

In all three cases, the Fed's reduction in demand for government debt is modeled as:

$$\alpha = dG_f^d < 0 .$$

This change in the Fed's balance sheet is offset by a Fed action to keep the interest rate on deposits unchanged:

(1) Open-market operations in private securities:

$$\gamma = dV_f^d \ni \alpha + \gamma = dH_f^s$$

or (2) Discount window loans:

$$\delta = dW_f^s \ni \alpha + \delta = dH_f^s$$

Method: In both cases, we'll totally differentiate the excess demand equations and see if we can solve for a deterministic equilibrium.

Totally differentiate the functions *G, D,* and *V*:

$$D_1 dr_G + D_2 dr_V + D_3 dr_D + D_4 dD_t^d + D_5 dr_W = 0$$
$$G_1 dr_G + G_2 dr_V + G_3 dr_D + G_4 dG_f^d + G_5 dG_t^s + G_6 dr_W = 0$$
$$V_1 dr_G + V_2 dr_V + V_3 dr_D + V_4 dV_f^d + V_5 dr_W = 0$$

Note that the excess demand functions are affected one-for-one by changes in supply or demand, so $D_4 = 1$, $G_4 = 1$, $G_5 = -1$, and $V_4 = 1$.

II. Fed Portfolio Shift with No Change in Interest Rate on Deposits

In this section, we model the results when the Fed's portfolio shifts in such a way that the deposit interest rate is unchanged.

Case 1: Open-Market Operations in Private Securities

$\alpha = dG_f^d < 0$, $\gamma = dV_f^d$, $dH_f^s = \alpha + \gamma$, such that $dr_D = 0$.

System of 3 equations in 3 unknowns:

$$D_1 dr_G + D_2 dr_V = 0$$
$$G_1 dr_G + G_2 dr_V + \alpha = 0$$
$$V_1 dr_G + V_2 dr_V + \gamma = 0$$

Use the first equation to find dr_G in terms of dr_V, then use that in the second equation to solve for dr_V. Use that in the third equation to solve for γ, to see which direction monetary policy goes.

Results of derivations:

$$dr_V = -\alpha D_1 / (D_1 G_2 - D_2 G_1) < 0$$

$$dr_G = \alpha D_2 / (D_1 G_2 - D_2 G_1) > 0$$

$$\gamma = -\alpha (D_2 V_1 - D_1 V_2) / (D_1 G_2 - D_2 G_1) > 0$$

$$\alpha + \gamma = \frac{D_1(G_2 + V_2) - D_2(G_1 + V_1)}{D_1 G_2 - D_2 G_1}$$

The sign of $\alpha + \gamma$ is unclear, since it depends on the relative size of different effects.

Thus we don't know if high-powered money increases or decreases. But we do know that the Fed buys private bonds and sells government bonds; we just don't know which quantity is larger.

Overall effects: The remaining effects can be found from the direction of change of dr_G and dr_V, and the agents' budget constraints. Since r_G and r_V move in opposite directions, some things can't be signed.

Deposits: dD_h^d ?, dD_b^s ?, $dr_D = 0$

High-powered money: dR_b^d ?, dC_h^d ?, dH_f^s ?

Government bonds: $dG_b^d > 0$, $dG_f^d < 0$, $dG_h^d > 0$, $dr_G > 0$

Private bonds: $dV_b^d < 0$, $dV_f^d > 0$, $dV_h^d < 0$, $dr_V < 0$

Case 2: Discount Window Loan—NACF

In this case, the Fed opens the discount window and lets banks borrow all they want at the discount rate, with no administrative costs. Monetary policy is conducted by changing the discount rate as the Fed changes its portfolio. Discount window loans and high-powered money supply clear the market such that the Fed's balance constraint is satisfied and the interest rate on deposits doesn't change.

$\alpha = dG_f^d < 0$, $\beta = dr_W$, such that $dr_D = 0$. Let $\delta = dW_f^s \ni \alpha + \delta = dH_f^s$.

Because there's one more market now (discount window loans), the system grows by an order of magnitude.

The excess demand function for discount loans is:

$$\underset{+}{W(r_G}, \underset{+}{r_V}; \underset{+}{r_D}; \underset{-}{W_f^s}; \underset{-}{r_W}) = 0$$

System of 4 equations in 4 unknowns:

$$D_1 dr_G + D_2 dr_V + D_5 \beta = 0$$
$$G_1 dr_G + G_2 dr_V + \alpha + G_6 \beta = 0$$
$$V_1 dr_G + V_2 dr_V + V_5 \beta = 0$$
$$W_1 dr_G + W_2 dr_V - dW_f^s + W_5 \beta = 0$$

Solve the first 3 equations simultaneously to find β , dr_G, and dr_V, then use those in the fourth equation to solve for dW_f^s .

Results of derivations:

$$X_1 dr_G = -\alpha D_5 (D_2 V_5 - D_5 V_2),$$

$$\text{where } X_1 \equiv D_2 D_5 (G_1 V_5 - G_6 V_1) + D_5^2 (G_2 V_1 - G_1 V_2) + D_1 D_5 (G_6 V_2 - G_2 V_5)$$

If we assume that $G_1 V_2 > G_2 V_1$, then $X_1 < 0$, so $dr_G > 0$.

$$dr_V = -\frac{\alpha D_5 (D_5 V_1 - D_1 V_5)}{X_1}$$

$$\beta = \frac{\alpha D_5 (D_2 V_1 - D_1 V_2)}{X_1} < 0.$$

The term dr_V can't be signed, so we don't know if the return to private bonds will rise or fall.

$$\begin{aligned} dW_f^s &= W_1 dr_G + W_2 dr_V + W_5 \beta \\ &= \frac{\alpha D_5}{X_1} [-W_1 (D_2 V_5 - D_5 V_2) - W_2 (D_5 V_1 - D_1 V_5) + W_5 (D_2 V_1 - D_1 V_2)], \end{aligned}$$

which is positive if the main effect is from the W_5 term.

Other effects:

1. If $dr_V = 0$:

   Deposits: dD_h^d<0, dD_b^s<0 , $dr_D = 0$

   High-powered money: dR_b^d<0, dC_h^d<0, dH_f^s<0

   Government bonds: dG_b^d>0, dG_f^d<0, dG_h^d>0, $dr_G > 0$

   Private bonds: dV_b^d>0, dV_h^d<0

   Discount loans: $dW_f^s = dW_b^d > 0$, $dr_W < 0$

2. If $dr_V < 0$:

Deposits: dD_h^d?, dD_b^s?, $dr_D = 0$

High-powered money: dR_b^d?, dC_h^d?, dH_f^s?

Government bonds: dG_b^d>0, dG_f^d<0, dG_h^d>0, $dr_G > 0$

Private bonds: dV_b^d>0, dV_h^d<0

Discount loans: $dW_f^s = dW_b^d > 0$, $dr_W < 0$

3. If $dr_V > 0$:

Deposits: dD_h^d<0, dD_b^s<0, $dr_D = 0$

High-powered money: dR_b^d<0, dC_h^d<0, dH_f^s<0

Government bonds: dG_b^d>0 (probably), dG_f^d<0, dG_h^d>0 (probably), $dr_G > 0$

Private bonds: dV_b^d?, dV_h^d?

Discount loans: $dW_f^s = dW_b^d > 0$ (probably), $dr_W < 0$

III. Special Assumptions Locking Interest Rates Together

Under one scenario under consideration, the returns to most assets are locked together. That scenario involves a situation in which banks view discount loans and deposits as perfect substitutes, so $r_W = r_D$. Banks are willing to loan as much as firms desire to borrow from them at a constant spread, so $r_D + m = r_V$. Given these assumptions, all our solution methods are changed somewhat, because now quantities supplied and demanded, rather than interest rates, will change to achieve equilibrium. Letting $r = r_W = r_D$, the analysis can be restructured so that demands and supplies depend just on r_G and *r*, with some demands or supplies adjusting completely, when there are perfect substitutes.

Deposits: $D_h^d(\underset{-}{r_G},\underset{+}{r}) + D_t^d = D_b^s$, where D_t^d is exogenous and D_b^s is endogenous but banks supply all the deposit accounts that are opened at a given deposit rate.

High-powered money: $R_b^d + C_h^d = H_f^s$

$$\text{so} \qquad \rho[D_h^d(\underset{-}{r_G},\underset{+}{r}) + D_t^d] + C_h^d(\underset{-}{r_G},\underset{-}{r}) = H_f^s .$$

Government bonds: $G_b^d + G_f^d + G_h^d(\underset{+}{r_G},\underset{-}{r}) = G_t^s$.

Private bonds: $V_b^d + V_f^d + V_h^d(\underset{-}{r_G},\underset{+}{r}) = V_m^s$, where V_b^d is determined passively such that the return on deposits plus a constant equals the return on private bonds.

Discount loans: $W_b^d = W_f^s$, where W_b^d adjusts passively such that $r_W = r_D$.

Included in the special assumptions are that $G_b^d = 0$.

Case 1a: OMO in Private Securities

With the special assumptions in place, when the Fed reduces its demand for government bonds and increases its demand for private bonds, doing so in a way that leaves the interest rate on private bonds unchanged (because $dr_V = dr_D = 0$), the following results are obtained:

The changes in the budget constraints of the agents are:

Bank: $dR_b^d + dV_b^d = dD_b^s$

Fed: $dG_f^d + dV_f^d = dH_f^s$

Household: $dD_h^d + dC_h^d + dG_h^d + dV_h^d = 0$

Results:

Government bonds: $dG_h^d(r_G, r) + dG_f^d = 0$

Since $\alpha = dG_f^d < 0, -\alpha = dG_h^d > 0,$ so $dr_G > 0.$

Since $dr_G > 0, dD_h^d < 0, dC_h^d < 0, dV_h^d < 0.$

Deposits: $dD_h^d = dD_b^s < 0,$ so $dR_b^d < 0.$

High-powered money: Since $dR_b^d < 0$ and $dC_h^d < 0,$ then $dH_f^s < 0.$

Private bonds: From the bank's budget constraint, since reserves decline less than deposits, banks must hold fewer private bonds, so $dV_b^d < 0.$ Thus the bank's portfolio shrinks.

Additional Restrictions:

Under another set of assumptions considered, demand by households for currency and deposits is exogenous. Imposing those conditions here would mean that $dH_f^s = 0$ and $dV_f^d = -dG_f^d > 0.$

Case 2a: Discount Window Loans—NACF

The experiment is one in which the Fed reduces its holdings of government bonds, with an NACF in place, and the Fed supplies however many advances banks want at a fixed discount rate, which in this case equals *r*.

The changes in the budget constraints of the agents are:

Bank: $dR_b^d + dV_b^d = dD_b^s + dW_b^d$

Fed: $dG_f^d + dW_f^s = dH_f^s$

Household: $dD_h^d + dC_h^d + dG_h^d + dV_h^d = 0$

Results:

Government bonds: $dG_h^d(r_G, r) + dG_f^d = 0$

Since $\alpha = dG_f^d < 0, -\alpha = dG_h^d > 0,$ so $dr_G > 0.$

Since $dr_G > 0, dD_h^d < 0, dC_h^d < 0, dV_h^d < 0.$

Deposits: $dD_h^d = dD_b^s < 0,$ so $dR_b^d < 0.$

High-powered money: Since $dR_b^d < 0$ and $dC_h^d < 0,$ then $dH_f^s < 0.$

Private bonds: $dV_b^d + dV_h^d = 0,$ so $dV_b^d > 0.$

In this situation, the banking system expands.

Discount loans: Assuming the direct effect from the Fed's reduction in demand for government bonds exceeds the impact on the monetary base (a very likely event), then $dW_f^s = dW_b^d > 0.$

Additional Restrictions:

Under another set of assumptions, demand by households currency and deposits is exogenous. Imposing those conditions here would mean that

$dH_f^s = 0$ and $dW_f^s = -dG_f^d > 0$.

IV. Generalizing the Problem with an Unchanged Monetary Base

Suppose we now wish to get rid of the restriction that $dr_D = 0$. This makes the problem much more difficult to solve algebraically, because now there's one more term to solve for. However, we then use a new assumption about Fed policy, which puts another restriction on the system—either we assume the Fed does not allow the monetary base to change, or we assume that the discount rate doesn't change. We consider 3 new cases, related to the 2 previous cases:

Case 1b: Private bonds case:

$$\alpha = dG_f^d < 0,\ \gamma = dV_f^d,\ dH_f^s = \alpha + \gamma = 0.$$

In this case, the Fed demands fewer government bonds and replaces them with private bonds, with no change in the monetary base and no discount window in operation.

Case 2b: Discount window—NACF case:

$$\alpha = dG_f^d < 0,\ \gamma = dW_f^s,\ dr_W = 0,\ dH_f^s = \alpha + \gamma .$$

Case 2c: Discount window—ACF case (ACF = auction credit facility):

$$\alpha = dG_f^d < 0,\ \gamma = dW_f^s,\ dH_f^s = \alpha + \gamma = 0 .$$

Case 1b: Private bonds case:

$$\alpha = dG_f^d < 0,\ \gamma = dV_f^d,\ dr_W = 0,\ dH_f^s = \alpha + \gamma\ = 0.$$

System of 3 equations in 3 unknowns:

$$D_1 dr_G + D_2 dr_V + D_3 dr_D = 0$$
$$G_1 dr_G + G_2 dr_V + G_3 dr_D + \alpha = 0$$
$$V_1 dr_G + V_2 dr_V + V_3 dr_D - \alpha = 0$$

Results of derivations:

$$dr_G = \frac{\alpha[D_2(G_3 + V_3) - D_3(G_2 + V_2)]}{D_1(G_2V_3 - G_3V_2) + D_2(G_3V_1 - G_1V_3) + D_3(G_1V_2 - G_2V_1)}$$

The sign of this is hard to figure out, because the terms point in opposite directions. The numerator is positive if $V_2 > |G_2|$, which seems likely since V_2 is an own effect ($\frac{\partial V}{\partial r_V}$), while G_2 is a cross effect ($\frac{\partial G}{\partial r_V}$). The denominator is much more complicated, consisting of six third-order terms. Of these 6, 5 are negative and 1 is positive. However, the one term that's positive is the only own effect ($D_3 G_1 V_2 = \frac{\partial D}{\partial r_D}\frac{\partial G}{\partial r_G}\frac{\partial V}{\partial r_V}$), so this may be larger in value then all the other cross effects, in which case the denominator is positive. This type of assumption is one that we often make in comparative statics exercises using graphical analysis. But we should be careful because those cross partials here must also reflect budget constraints, so judging their magnitude can be difficult. Nonetheless, if these assumptions hold, then $dr_G > 0$.

Other results:

$$dr_V = \frac{\alpha[D_3(G_1 + V_1) - D_1(G_3 + V_3)]}{D_1(G_2V_3 - G_3V_2) + D_2(G_3V_1 - G_1V_3) + D_3(G_1V_2 - G_2V_1)}$$

If we make the same assumptions that we made above, and if we further assume that $G_1 > |V_1|$ and that $D_3G_1 > |D_3V_1 - D_1(G_3 + V_3)|$, which again may be reasonable if the own effects are much larger in magnitude than the cross effects, then $dr_V < 0$. This is intuitive since the demand for private bonds is increasing.

$$dr_D = \frac{\alpha[D_1(G_2 + V_2) - D_2(G_1 + V_1)]}{D_1(G_2V_3 - G_3V_2) + D_2(G_3V_1 - G_1V_3) + D_3(G_1V_2 - G_2V_1)}$$

Unfortunately, there's no intuition on the sign of the change in the interest rate on deposits. Different effects point in different directions, and the sign depends on relative magnitudes. Banks substitute some government bonds for some private bonds, given the direction of change of returns, but it's impossible to tell whether banks' assets overall increase or decline, since we can't tell if deposits increase or decrease.

Case 2b: Discount window—NACF case:

$$\alpha = dG_f^d < 0\,,\ \gamma = dW_f^s,\ dr_W = 0,\ dH_f^s = \alpha + \gamma\ .$$

System of 4 equations in 4 unknowns:

$$D_1 dr_G + D_2 dr_V + D_3 dr_D = 0$$
$$G_1 dr_G + G_2 dr_V + G_3 dr_D + \alpha = 0$$
$$V_1 dr_G + V_2 dr_V + V_3 dr_D = 0$$
$$W_1 dr_G + W_2 dr_V + W_3 dr_D - \gamma = 0$$

Results of derivations:

$$dr_G = \frac{\alpha(D_2V_3 - D_3V_2)}{D_1(G_2V_3 - G_3V_2) + D_2(G_3V_1 - G_1V_3) + D_3(G_1V_2 - G_2V_1)}$$

Compared to case 1b above, finding signs is a bit easier. The numerator is probably positive, which occurs if $D_3V_2 > D_2V_3$. Again, this would occur if the own effects are bigger than cross effects. The denominator is the same as in case 1b, so under the same assumptions, then $dr_G > 0$.

$$dr_V = \frac{\alpha(D_3V_1 - D_1V_3)}{D_1(G_2V_3 - G_3V_2) + D_2(G_3V_1 - G_1V_3) + D_3(G_1V_2 - G_2V_1)}$$

$$dr_D = \frac{\alpha(D_1V_2 - D_2V_1)}{D_1(G_2V_3 - G_3V_2) + D_2(G_3V_1 - G_1V_3) + D_3(G_1V_2 - G_2V_1)}$$

Both dr_V and dr_D are positive, under the assumptions made above. People's demand for deposit accounts could rise or fall, depending on how they respond in comparing the higher yield on government bonds and private bonds to the higher yield on deposits.

$$\gamma = \frac{\alpha[D_1(V_2W_3 - V_3W_2) + D_2(V_3W_1 - V_1W_3) + D_3(V_1W_2 - V_2W_1)]}{D_1(G_2V_3 - G_3V_2) + D_2(G_3V_1 - G_1V_3) + D_3(G_1V_2 - G_2V_1)}$$

This term is most likely positive, though not definitively (5 of the 6 terms in the numerator have the same sign). But to know whether the monetary base rises or falls, we need to know whether $\alpha + \gamma$ is positive or negative, which is impossible to tell.

With returns on government bonds, private bonds, and deposits all rising, people will hold less currency. But we don't know if the monetary base rises or falls. If it rises, then given that there's less currency held, there must be more reserves held against more deposits. In that case, the banking system must get bigger. But if the monetary base declines, which happens if deposits decline sufficiently, then we wouldn't be able to tell if the banking system's assets would increase or decrease.

Case 2c: Discount window—ACF case:

$$\alpha = dG_f^d < 0,\ \gamma = dW_f^s,\ dH_f^s = \alpha + \gamma = 0.$$

System of 4 equations in 4 unknowns:

$$\begin{aligned}
&D_1 dr_G + D_2 dr_V + D_3 dr_D + D_5 dr_W = 0\\
&G_1 dr_G + G_2 dr_V + G_3 dr_D + \alpha + G_6 dr_W = 0\\
&V_1 dr_G + V_2 dr_V + V_3 dr_D + V_5 dr_W = 0\\
&W_1 dr_G + W_2 dr_V + W_3 dr_D + \alpha + W_5 dr_W = 0
\end{aligned}$$

Results of derivations:

$$dr_G = \frac{\alpha(D_2X_1 + D_3X_2 + D_5X_3)}{D_1X_4 + D_2X_5 + D_3X_6 + D_5X_7},$$

$$\begin{aligned}
X_1 &= V_3(W_5 - G_6) + V_5(G_3 - W_3)\\
X_2 &= V_2(G_6 - W_5) + V_5(W_2 - G_2)\\
X_3 &= V_2(W_3 - G_3) + V_3(G_2 - W_2)\\
X_4 &= G_2(V_3W_5 - V_5W_3) + G_3(V_5W_2 - V_2W_5) + G_6(V_2W_3 - V_3W_2)\\
X_5 &= -G_1(V_3W_5 - V_5W_3) - G_3(V_5W_1 - V_1W_5) - G_6(V_1W_3 - V_3W_1)\\
X_6 &= G_1(V_2W_5 - V_5W_2) + G_2(V_5W_1 - V_1W_5) + G_6(V_1W_2 - V_2W_1)\\
X_7 &= G_1(V_3W_2 - V_2W_3) + G_2(V_1W_3 - V_3W_1) + G_3(V_2W_1 - V_1W_2)
\end{aligned}$$

This is quite a complicated system. If we assume that the own effects dominate, as we did earlier, then it suggests that both numerator and denominator are negative, so $dr_G > 0$.

$$dr_V = \frac{\alpha(D_1X_8 + D_3X_9 + D_5X_{10})}{D_1X_4 + D_2X_5 + D_3X_6 + D_5X_7},$$

$$\begin{aligned}
X_8 &= -V_3(W_5 - G_6) - V_5(G_3 - W_3)\\
X_9 &= -V_1(G_6 - W_5) - V_5(W_1 - G_1)\\
X_{10} &= -V_1(W_3 - G_3) - V_3(G_1 - W_1)
\end{aligned}$$

This is clearly not signable, and there are no own effects, so it's impossible to sign dr_V.

$$dr_D = \frac{\alpha(D_1X_{11} + D_2X_{12} + D_5X_{13})}{D_1X_4 + D_2X_5 + D_3X_6 + D_5X_7},$$

$$X_{11} = V_2(W_5 - G_6) + V_5(G_2 - W_2)$$
$$X_{12} = V_1(G_6 - W_5) + V_5(W_1 - G_1)$$
$$X_{13} = V_1(W_2 - G_2) + V_2(G_1 - W_1)$$

Again, there are no own effects in the numerator, so it can't be signed.

$$dr_W = \frac{\alpha(D_1X_{14} + D_2X_{15} + D_3X_{16})}{D_1X_4 + D_2X_5 + D_3X_6 + D_5X_7},$$

$$X_{14} = -V_2(W_3 - G_3) - V_3(G_2 - W_2)$$
$$X_{15} = -V_1(G_3 - W_3) - V_3(W_1 - G_1)$$
$$X_{16} = -V_1(W_2 - G_2) - V_2(G_1 - W_1)$$

Here, there's an own effect, and if it dominates the other terms, then $dr_W > 0$.

All the results here are quite tentative, for they depend on somewhat dubious assumptions.

Whether the banking system expands or contracts here is hard to say. We know that the banking system takes out more discount loans, equal to the decrease in the demand for government bonds by the Fed. But do those discount loans displace more or fewer deposits? Since funding is becoming more expensive, banks may wish to reduce the size of their portfolio; on the other hand, returns are higher, so they may wish to expand. Households may substitute government bonds for deposit accounts, as well.

V. SUMMARY

This paper develops a static general-equilibrium model of the Fed's portfolio choice in the short run. Finding the results of a Fed portfolio shift requires a number of assumptions, which may not hold in all cases. The results reported below make use of those assumptions.

When the Fed changes its portfolio by reducing its ownership of government bonds and keeps the interest rate on deposits from changing, the results depend on what the Fed substitutes to replace government bonds. If the Fed buys private securities, the interest rate on such securities decline while the interest rate on government bonds rises. If the Fed uses a non-administered credit facility (NACF), the interest rate on government bonds rises and the discount rate falls.

When the Fed changes its portfolio by reducing its ownership of government bonds and interest rates are locked together, the results also depend on what the Fed substitutes to replace government bonds. If the Fed buys private securities, the interest rate on government bonds rises and banks hold fewer private bonds and banks shrink in size. If the Fed uses an NACF, the interest rate on government bonds rises and the banking system expands.

When the Fed changes its portfolio by reducing its ownership of government bonds but does not fix the deposit rate and interest rates are not locked together, we find the following results. If the Fed buys private bonds and does not allow the monetary base to change, then the interest rate on private bonds declines while the interest rate on government bonds rises. If the Fed uses an NACF and does not allow the discount rate to change, then the interest rates on government bonds, private bonds, and deposits all rise and the demand for currency declines. If the Fed uses an auction credit facility (ACF) and does not allow the monetary base to change, then the interest rates on government bonds and private bonds rise.

REFERENCES

Federal Reserve System Study Group on Alternative Instruments for System Operations. *Alternative Instruments for Open Market and Discount Window Operations*. Washington, D.C.: Federal Reserve Board of Governors, 2002.

Santomero, Anthony M. "Controlling Monetary Aggregates: The Discount Window." *Journal of Finance* 38 (June 1983), pp. 827–843.

Figure 1

Balance Sheets

Bank (b)

Assets		Liabilities + Net Worth	
Reserves	R_b^d	D_b^s	Deposit Accounts
Government Bonds	G_b^d	W_b^d	Discount Loans
Private Bonds	V_b^d	NW_b	Net Worth

Household (h)

Assets		Liabilities + Net Worth	
Deposits	D_h^d		
Currency	C_h^d		
Government Bonds	G_h^d		
Private Bonds	V_h^d	NW_h	Net Worth

Federal Reserve (f)

Assets		Liabilities + Net Worth	
Government Bonds	G_f^d	H_f^s	High-Powered Money
Private Bonds	V_f^d		
Discount Loans	W_f^s	NW_f	Net Worth

Figure 1 (continued)

Balance Sheets

Treasury (t)

Assets		Liabilities + Net Worth	
Deposits	D_t^d	G_t^s	Government Bonds
		NW_t	Net Worth

Firm (m)

Assets		Liabilities + Net Worth	
Capital	K	V_m^s	Private Bonds
		NW_m	Net Worth