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Private-Sector Bailouts

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Financial Networks: Contagion, Commitment, and Private-Sector Bailouts

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Abstract

I develop a model of financial networks where linkages not only spread contagion, but also induce private-sector bailouts in which liquid banks bail out illiquid banks because of the threat of contagion. Introducing this bailout possibility, I show that linkages may be optimal ex-ante because they allow banks to obtain some mutual insurance even though formal commitments are impossible. However, in some cases (for example, when liquidity is concentrated among a small group of banks), the whole network may collapse. I also characterize the optimal network size and apply the results to joint liability arrangements and payment systems.

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Linkages among agents in financial markets are a concern because of the risk of financial contagion, that is, the risk that a small shock to one agent will spread to other agents in a domino effect. Whether we should worry about these linkages is an open question. The main goal of this paper is to show that a network in which agents are closely interlinked may be optimal because of and despite the potential for contagion. Another goal is to characterize optimal networks, i.e., whether and how agents should be linked to one another.

The basic intuition is as follows: Linkages present the threat of contagion. To prevent collapse of the whole network, agents who are lucky ex-post may be willing to “bail out” those who are not. In this way agents obtain the benefits of mutual insurance even though they cannot precommit to making payments. Linkages, however, may have a cost. In some cases (e.g., when the aggregate endowment is not high enough), the whole network may collapse in a contagious fashion.

The paper shares some common features with other papers that model financial contagion. Examples are Allen and Gale (2000), Kiyotaki and Moore (1997), Lagunoff and Schreft (2001), and Rochet and Tirole (1996). As in my paper, agents in these papers are linked to one another through some financial network, and because of these linkages, shocks can spread from one agent to another.¹ In my model the threat of contagion may be part of an optimal network design, while other papers focus on the negative aspects of contagion. In addition, previous papers do not fully address the issue of an optimal network design while this paper does.²

The main contribution is to show that linkages that create the threat of contagion may be optimal. This idea can be applied, for example, to the design of payment systems. Suppose we can choose between two stylized systems: gross, in which settlements occur as soon as transactions are agreed upon, and net, in which settlements occur at the end of

the day, so banks grant intraday credit to one another. Existing literature, such as Freixas and Parigi (1998) and Kahn and Roberds (1998), shows that net payments systems may be optimal because they reduce the costs of holding non-interest-bearing reserves. My theory, on the other hand, implies that net payment systems can be optimal even if holding reserves is not costly. Net systems induce linkages that create the threat of contagion. This, in turn, can motivate banks to help one another, even where they could not precommit to do so.

Another contribution is the analysis of the tradeoff between risk sharing and the potential for collapse in the design of optimal financial networks. Such an analysis can be used, for example, to calculate the optimal number of groups and the optimal number of agents within a group in joint liability arrangements such as the ones used by the Grameen Bank.³ In particular, I show that the optimal group size may be finite, say five, even in infinitely large economies with *iid* endowments. I do not need to assume that members of small groups know more about one another, nor that large groups have free-rider and coordination problems.⁴

To model linkages, I assume that the project of agent i can succeed only if he and all the agents to whom he is linked make a minimum level of investment in their projects. This setting is very simple, but it can be applied to more complex situations. For example, in Allen and Gale's setting, investing could mean meeting demand for liquidity by early consumers, and success could mean not going bankrupt. Since endowments are random variables, an agent may not have enough cash to make the necessary investment. In addition, his inability to precommit to pay prevents him from borrowing against future cash flows or from entering an insurance (forward) contract before endowments are realized. When agents are not linked to one another, agents who realize high endowments have no incentive ex-post to help out those who realize low endowments. Thus, some positive net present

value investments do not take place (or in Allen and Gale's setting, some banks that are solvent but illiquid go bankrupt). On the other hand, when agents are linked to one another, agents with high endowments are willing to bail out those with low endowments. The reason is that if they do not, all projects fail by contagion.

I illustrate two cases in which a network in which all agents are linked to one another can break down. The first case is obvious: the aggregate endowment is not high enough to take all projects. The second case is less obvious: the aggregate endowment is high enough but concentrated among a small number of agents. The intuition is that the threat of losing future income may not be sufficient to induce an agent with a lot of cash to voluntarily hand over his entire endowment. In other words, the amount that an agent may be willing to contribute to a bailout cannot be more than what he loses by not participating. The paper, therefore, implies that fluctuations in the distribution of endowments can cause collapse even without fluctuations in the aggregate endowment. This adds to Rampini's (forthcoming) result that default correlation may be caused by fluctuations in the aggregate endowment.

To find optimal allocations capturing the idea that agents cannot precommit to pay out of their endowments, I solve a planning problem, imposing a restriction that each agent prefers the proposed allocation to autarky. One way to implement an optimal allocation is as follows: Once endowments are realized, a central planner (or one of the agents) proposes a bailout, and each agent can either accept or reject. A bailout takes place only if all agents accept. Thus, a necessary condition for linkages to be optimal is the presence of some coordinating mechanism. For example, if we allowed for coordination in the framework of Allen and Gale, the threat of contagion could induce some banks to voluntarily liquidate some of their long-term assets and transfer cash to banks with unusually high demand for liquidity.

The only role of the central planner here is to coordinate voluntary transfers. This is different from other models of bailouts, such as Freixas' (1999), in which the Fed injects cash into a failing bank because of its obligations as a lender of last resort. One example of a private-sector bailout, as in my model, is the case of Long Term Capital Management (LTCM), where the New York Fed acted as a coordinator. The theory here implies that the bailout of LTCM may have been optimal from an ex-ante point of view. It also gives a different perspective on the Fed's inability to commit not to bail out. Existing literature, e.g., Freixas (1999), Mailath and Mester (1994), and Rochet and Tirole (1996) has focused on inefficiencies that arise when the Fed cannot commit to certain policies regarding bank closure. In my paper, bailouts are optimal even from an ex-ante point of view, and inefficiencies arise because the Fed cannot commit to help.⁵

The paper is organized as follows: In Section I, I present an example that relates my paper to Allen and Gale's, and in Section II, I present my model. In Section III, I state the planning problem, and in Section IV, I show one way to implement an optimal solution. In Section V, I compare two networks: one in which each agent is linked to every other agent, and one in which each agent stands on his own. In Section VI, I characterize optimal networks. One of the main results of this section is that a network in which each agent is linked to every other agent may be optimal. In Section VII, I show how the theory can apply to the design of payments systems and joint liability arrangements. I conclude in Section VIII. There are also two appendices: The first shows that the nature of the results does not change if endowments are private information, and the second contains proofs.

I. Example

In the next example, I use Allen and Gale's (2000) model of financial contagion to illustrate how financial linkages can *stop* contagion as they create the incentives for a healthy bank to help a troubled bank once the threat of contagion arises. In particular, I show how a key result of Allen and Gale is altered if financially linked banks can coordinate ex-post when a threat of contagion arises.

There are n banks that are identical ex-ante. There is also a continuum of consumers who are identical ex-ante and who have Diamond-Dybvig (1983) preferences

$$u(c_1, c_2) = \begin{cases} u(c_1) & \text{with probability } \omega \\ u(c_2) & \text{with probability } 1 - \omega \end{cases} \quad (1)$$

where c_i denotes consumption at date i and $u(\cdot)$ satisfies $u' > 0$ and $u'' < 0$. The banks' role is to make investments on behalf of consumers and to insure them against the risk of not knowing when they will need to consume.

At date 0, each consumer deposits his initial endowment of one dollar in one of the banks (deposits are evenly spread across banks). In exchange, he obtains a deposit contract that allows him to withdraw either c_1 dollars at date 1 or c_2 dollars at date 2. Banks use the money obtained from deposits to invest x dollars per capita in a long-term asset and y dollars per capita in a short-term asset. The short-term asset is equivalent to storage: each unit invested at date 0 yields one dollar at date 1. The long-term asset yields a higher return: each unit invested at date 0 yields $R > 1$ dollars at date 2; it can also be liquidated at date 1, but then it yields only $r < 1$ dollars. One can think of the short-term asset as non-interest-bearing reserves and of the long-term asset as bank loans.

At date 0, banks can create linkages among themselves through cross holdings of deposits. In Allen and Gale, they do so to insure themselves against idiosyncratic liquidity

shocks. Banks with high demand for liquidity by early consumers liquidate their deposits with other banks at date 1, while banks with low demand wait until date 2. This arrangement works if there is no aggregate uncertainty in the fraction of early consumers. Otherwise, a small increase in this fraction can generate financial contagion; in particular, Allen and Gale show that for some parameter values, any equilibrium involves bankruptcy of all banks because of spillover effects. This is the case I am interested in, but in my framework, the ability to coordinate a bailout will sometimes prevent the spread of a crisis.

To focus on the role of linkages in enhancing commitment, I construct an example in which linkages are unnecessary in Allen and Gale's setting, and whose only role is to induce banks to help one another. I assume that there are two possible scenarios: a good scenario in which no bank realizes a liquidity shock, and a bad scenario in which some banks realize a liquidity shock and some don't. More specifically, I assume that the fraction of early consumers in bank i is $\gamma + \tilde{\varepsilon}_i$, where $\tilde{\varepsilon}_i$ is a random variable that can be either 0 or ε . In the good scenario, $\tilde{\varepsilon}_i = 0$ for $i = 1, \dots, n$, and in the bad scenario, one bank chosen randomly realizes a liquidity shock ($\tilde{\varepsilon}_i = \varepsilon$) while the other banks do not. By hoarding reserves, the banking system can avoid costly liquidations in the bad scenario. However, if the probability of the bad scenario is low enough and the potential for loss is bounded (e.g., $u(0)$ has a lower bound), as is assumed here, it is optimal for the system as a whole to hoard less reserves and invest more in high yield but less liquid instruments. More specifically, the first best is to choose $y = \gamma c_1$. In the good scenario, banks can satisfy demand for liquidity by early consumers using only the short-term assets, whereas in the bad scenario, banks must liquidate some of their long-term assets to meet the extra demand.

Whether banks will achieve the first-best depends on the linkages they formed at date 0. Suppose first banks are not linked to one another. A bank that realizes a liquidity

shock can meet the excess demand for liquidity by liquidating some of its long-term assets. The problem is that the bank will not have enough cash to meet the demands of its late consumers at date 2. In fact, if the liquidation value r is small enough, the bank may not be able to pay the late consumers even c_1 . Then the late consumers will prefer to withdraw at date 1 and store the cash until date 2, thus causing a bank run. Another potential solution is to borrow from other banks. However, to come up with the funds, the other banks will need to liquidate some of *their* assets. They will be willing to do so only if the interest rate charged on the loans is high enough to compensate them for the cost of liquidation. But then the cost of borrowing for the illiquid bank will be just as high as the cost of liquidating its own assets and will not provide any additional help. More explicitly, I assume that

$$b(\varepsilon) < \varepsilon c_1, \tag{2}$$

where $b(\xi)$ is the maximum amount that a bank with $\tilde{\varepsilon}_i = \xi$ can raise at date 1 by liquidating its long-term assets without inducing a run. The inequality says that this amount is not high enough to meet the extra demand for liquidity. As in Allen and Gale, $b(\xi)$ is given by $b(\xi) \equiv r[x - \frac{c_1(1-\gamma-\xi)}{R}]$ and is referred to as the buffer of bank i .

To have some benefits from mutual insurance, I also assume that by pooling resources, the liquid banks can come up with the extra funds required for helping the illiquid bank, so that in the first best no bank, whether liquid or illiquid, goes bankrupt. More explicitly, I assume that

$$\varepsilon c_1 \leq (n-1)b(0) + b(\varepsilon), \tag{3}$$

i.e., the aggregate buffer of the banking system as a whole is high enough to meet the liquidity shock. This suggests that banks could benefit if at date 0 they entered insurance contracts, according to which at date 1 the liquid banks transfer cash to the illiquid banks.

However, in many cases such contracting may not be possible because it may be difficult to ascertain with enough precision what is the liquidation value of the banks' assets. Federal Reserve Board Chairman Alan Greenspan captures this view, saying that "bank loans are customized, privately negotiated agreements that, despite increases in availability of price information and in trading activity, still quite often lack transparency and liquidity. This unquestionably makes the risks of many bank loans rather difficult to quantify and to manage" (Greenspan, 1996). Therefore, even though banks may know which banks are liquid and which banks are not, a court may not be able to tell.⁶ In our example, this friction can be modeled by assuming that the liquidation value is a random variable \tilde{r}_i that can take two values 0 and r , that only some of the liquid banks have a realization of $\tilde{r}_i = r$, and that \tilde{r}_i is not verifiable in court.

Banks can still insure themselves in the bad scenario by creating linkages through the (seemingly unnecessary) exchange of deposits. For example, at date 0 bank 1 deposits z dollars in bank 2, bank 2 deposits z dollars in bank 3, \dots , and bank n deposits z dollars in bank 1.⁷ The threat of contagion may then induce liquid banks to bail out illiquid banks. In particular, suppose a central planner proposes that each of the liquid banks transfer $\frac{\varepsilon c_1 - b(\varepsilon)}{n-1}$ dollars to the illiquid bank, so that it will have enough cash to avoid bankruptcy. If the alternative is autarky, there exists an equilibrium in which all banks agree to participate in the bailout. Bailing out is costly to the participating bank because it needs to liquidate some of its long-term assets, but not participating is more costly because all banks will go bankrupt in contagion, and the bank will need to liquidate *all* its assets.

Note that without linkages, the illiquid bank goes bankrupt, but the other banks survive. On the other hand, with linkages but without coordination (as in Allen and Gale), all banks go bankrupt in contagion; so in our example, they will not create the linkages in the first

place.

To summarize, the example shows how linkages can induce banks to help one another in situations where official contracts cannot do so. In practice, banks create linkages not only through cross holdings of deposits but also through more complicated financial claims, such as derivative contracts. Yet, the main point remains: these linkages and the potential for contagion they create allow banks to obtain insurance for events that cannot be contracted upon. The example also shows that private-sector bailouts may be a feature of an optimal risk-sharing. If we changed the example to include potential realizations in which the aggregate buffer is not sufficient to cover the liquidity shock, all banks could sometimes fail in a contagious fashion, making linkages suboptimal ex-post. Nonetheless, if the benefits of creating commitment to help one another are higher than the negative effects of contagion, it may be optimal ex-ante to design a network that is fragile.

II. The model

There are three dates $t = 0, 1, 2$, one divisible good called “dollars” or simply cash, and a set $N = \{1, \dots, n\}$ of risk-neutral agents who are identical ex-ante, obtain an expected utility $E(c_1 + c_2)$ from consuming c_1 and c_2 dollars at dates 1 and 2, respectively, and have limited liability (i.e., $c_t \geq 0$ for $t = 1, 2$).

At date 1, agent i is endowed with \tilde{e}_i dollars and access to a project that requires an investment of one dollar. Project cash flows are realized at date 2. Each project can either succeed and yield R dollars or fail and yield nothing. It is assumed that $R > 1$. Thus, a project that succeeds with probability 1 has a positive NPV.

Linkages are formed at date 0. The project of agent i can succeed only if he and all the agents to whom he is directly linked invest one dollar in their projects. More formally, let

$I_i \in \{0, 1\}$ denote the amount that agent i invests in his project, $I = (I_1, \dots, I_n)$ the vector of investments, and $p_i(I)$ the probability that the project of agent i will succeed. Then

$$p_i(I) = \begin{cases} 1 & \text{if } I_j = 1 \text{ for every } j \in K_i \cup \{i\} \\ 0 & \text{otherwise,} \end{cases} \quad (4)$$

where K_i is the set of agents to whom agent i is directly linked.⁸ The vector (K_1, K_2, \dots, K_n) fully captures the interdependence among agents and is called the *financial network*.

It is assumed that there is a positive probability that \tilde{e}_i is less than 1, so an agent's endowment may be less than what he needs to invest in his project. In addition, there is a positive probability that \tilde{e}_i is more than 1. This suggests potential benefits from trade. However, agents are unable to enter contracts because they cannot precommit to make payments. More specifically, I assume that:

(A1) Agents cannot commit to pay out of their initial endowments.

(A2) Agents cannot commit to pay out of their projects' cash flows.

The second assumption implies that agents who want to invest in their positive NPV projects cannot borrow at date 1 against the date 2 cash flows from their projects. This suggests that agents may want to share the risk associated with their date 1 endowments by entering forward contracts at date 0, according to which agents with high realizations of endowments transfer cash to those with low realizations. Such an arrangement is ruled out, however, by the first assumption.

For simplicity, I consider an extreme situation where agents cannot precommit to pay anything. However, the nature of the results would not change if I assumed the inability to precommit was only partial. For example, I could assume that agents could borrow against future cash flows but not against the full amount.

One way to motivate the second assumption, following Hart and Moore (1994), is to assume that the agent who owns a project (the entrepreneur) needs to provide his human capital for the project to succeed. For example, the human capital of LTCM's owners might have been essential for implementing their sophisticated hedging techniques. Similarly, as in Diamond and Rajan (2000, 2001), the human capital of a bank that monitors a loan may be essential for collecting the loan. When the original owner cannot be replaced costlessly because of special skills, he can always threaten to repudiate a loan contract by withdrawing his human capital. To induce him to provide his human capital, outside investors must promise him a portion of future cash flows meaning that he cannot borrow against the full amount.

Another way to motivate the second assumption is to assume that cash flows are uncertain and it is impossible to make financial contracts explicitly contingent on realized cash flows (see, for example, Bolton and Scharfstein, 1990). One interpretation is that the agent who runs the project observes realized cash flows privately and therefore can divert resources away from other investors to himself. This interpretation is plausible, for example, when we think of international crisis and foreign lending. Another interpretation is that cash flows are observable but not verifiable. In other words, although the parties to a contract can observe realized cash flows, a court cannot. This interpretation is plausible in the case of domestic banking systems because banks by their very nature deal with projects or borrowers that lack transparency and liquidity.

A company may also have problems borrowing against future cash flows when it is hard to determine their present value. For example, for LTCM, borrowing against future positions involved disclosure of what those positions were and the risk inherent in them. And even if both were disclosed, different investors might have had different perceptions as

to what the risk truly was.

The first assumption can be motivated by assuming that endowments are not verifiable. This is plausible if we think of an agent's endowment as the amount of cash he can raise on short notice, possibly by liquidating some of his assets, and if we assume, as in the previous section, that liquidation values are not verifiable in court.

To avoid problems of asymmetric information, I assume that endowments are observable, but the main idea holds even if endowments are private information (see Appendix A). Finally, I assume that the financial network is common knowledge and that:

(A3) Agents cannot commit to invest in their projects.

Thus, an agent invests only if his project can succeed, i.e., only if all the agents to whom he is directly linked invest as well.

The following example illustrates the inefficiency that may arise from the inability to precommit and shows how linkages can mitigate it.

Example 1 There are two agents. One agent (chosen randomly) has two dollars, and the other agent has zero. Since each project requires exactly one dollar, the efficient allocation requires each agent to end up with one dollar. This allocation is also Pareto optimal from an ex-ante point of view.

Consider without loss of generality the realization in which agent 1 has two dollars and agent 2 has nothing. To achieve the efficient allocation, agent 1 needs to transfer one dollar to agent 2 without getting anything in return. The question is whether agent 1 will be willing to do so.

Suppose the two agents are not linked. Since the probability of success of agent 1's project does not depend on whether agent 2 invests, agent 1 can invest one dollar, consume the

second dollar he has, and obtain a utility $1 + R$. If instead agent 1 transfers one dollar to agent 2, agent 1's utility is only R . Therefore, agent 1 is better off not bailing out agent 2. Agent 2 then ends up with nothing to invest and with a utility of zero.

Suppose now the two agents are linked. Agent 1 can gain from investing only if agent 2 invests as well. If agent 1 keeps the two dollars for himself, he is better off consuming his entire endowment, thereby obtaining a utility of 2. If instead he transfers one dollar to agent 2, agent 1 can invest one dollar, thereby obtaining a utility R . The optimal action for agent 1 depends on the value of R . When $R \geq 2$, it is optimal to bail out agent 2; otherwise, it is not.

To summarize, when $R \geq 2$, being linked achieves an efficient allocation, while not being linked doesn't.

III. Planning problem

I characterize optimal risk sharing as a solution to a planning problem. Without loss of generality, we can assume that the central planner can make transfers only at date 1 (Assumption A2). Thus, the sequence of events is as follows:

$t = 0$: A financial network is chosen.

$t = 1$: (a) Endowments are realized.

(b) Transfers are made.

(c) Investments are made.

$t = 2$: Project cash flows are realized.

Denote by $e = (e_1, e_2, \dots, e_n)$ the vector of realized endowments, and by T_i the net transfer to agent i . Choosing a vector of transfers $T = (T_1, T_2, \dots, T_n)$ is equivalent to choosing an allocation $x = (x_1, x_2, \dots, x_n)$ where $x_i = e_i + T_i$.

I rule out outcomes in which the choice of investment levels is not optimal given the chosen allocation (this can be motivated by inability to precommit). Therefore, I solve the problem of choosing an allocation and investment levels in two steps. First, I choose an *investment rule*, that is, a vector of investments as a function of an allocation. Then I choose an *allocation rule*, that is, an allocation as a function of the vector of realized endowments.

Let $U_i(x, I)$ denote agent i 's utility given the allocation x and the vector of investments I . Then

$$U_i(x, I) = x_i - I_i + p_i(I)R. \quad (5)$$

Optimal investment rule. A vector of investments $I = (I_1, I_2, \dots, I_n)$ is feasible given the allocation x if

$$I_i \leq x_i \text{ for every } i \in N. \quad (6)$$

To capture the idea that agents cannot commit to invest in their projects (Assumption A3), I also require that I form a Nash equilibrium. Formally,

$$\text{For every } i \in N \text{ and for every } \widehat{I}_i \leq x_i, U_i(x, I) \geq U_i(x, I_{-i}, \widehat{I}_i), \quad (7)$$

where (I_{-i}, \widehat{I}_i) denote the vector I in which I_i is replaced with \widehat{I}_i .

Since agents are identical ex-ante, it is natural to assume that the planner's objective is to maximize the unweighted sum of expected utilities. Thus, to find an optimal investment rule, $I(x) = (I_1(x), I_2(x), \dots, I_n(x))$, we need to maximize $\sum_{i=1}^n U_i(x, I)$ subject to equations (6) and (7).

Let $V_i(x) = U_i(x, I(x))$. Lemma 1 below characterizes $I(x)$ and $V_i(x)$. It shows that it is optimal for agent i to invest if and only if all the agents to whom he is linked – either directly or indirectly – have enough cash to invest in their projects. The set of these agents is denoted by L_i . In network terms, $j \in L_i$ if there exists a path that connects agent j to agent i .⁹

Lemma 1 1. $I_i(x) = \begin{cases} 1 & \text{if } x_j \geq 1 \text{ for every } j \in L_i \cup \{i\} \\ 0 & \text{otherwise.} \end{cases}$
 2. $V_i(x) = x_i + (R - 1)I_i(x)$.

Optimal allocation rule. An allocation rule $x(e) = (x_1(e), x_2(e), \dots, x_n(e))$ is feasible if for every realization e , equations (8) and (9) hold.¹⁰

$$\sum_{i=1}^n x_i(e) = \sum_{i=1}^n e_i \quad (8)$$

$$x_i(e) \geq 0 \text{ for every } i \in N. \quad (9)$$

I refer to the benchmark in which agents can precommit to pay out of their endowments as *first best* and to the case in which they cannot precommit to pay (Assumption A1) as *second best*.

The first-best problem is to choose a feasible allocation rule $x(e)$ that maximizes the expected sum of utilities $E \sum_{i=1}^n V_i(x(\tilde{e}))$ where \tilde{e} denotes the vector of random variables $(\tilde{e}_1, \tilde{e}_2, \dots, \tilde{e}_n)$ and $E(\cdot)$ denotes the expectation operator. One can view the first-best allocation rule as a set of forward contracts that are entered at date 0 and specify payments at date 1 as a function of the vector of realized endowments.

Since agents are identical ex-ante, the first-best allocation rule satisfies

$$EV_i(x(\tilde{e})) \geq EV_i(\tilde{e}) \text{ for every } i \in N. \quad (10)$$

In other words, ex-ante all agents prefer the first-best allocation to autarky. However, once endowments are realized, some agents might be better off if no transfers are made.

I say that an allocation rule satisfies the *interim participation constraint* if agents prefer the allocation to autarky even after endowments are realized, that is, if

$$V_i(x(e)) \geq V_i(e) \text{ for every } i \in N \text{ and for every } e. \quad (11)$$

The second-best problem is to choose a feasible allocation rule $x(e)$ that maximizes $E \sum_{i=1}^n V_i(x(\tilde{e}))$ subject to equation (11). This constraint captures the idea that agents cannot precommit to pay, and since it does not necessarily imply equation (10), some first-best allocations may be ruled out.

Optimal networks. Let ψ denote a financial network, let $x(e, \psi)$ denote the second-best allocation rule given ψ , and let $F(\psi) = E[\frac{1}{n} \sum_{i=1}^n V_i(x(\tilde{e}, \psi))]$.

Definition 1 1. A financial network ψ_1 ex-ante Pareto dominates a financial network ψ_2 if $F(\psi_1) \geq F(\psi_2)$ (it strictly dominates if there is a strict inequality).

2. A financial network ψ^* is optimal if $F(\psi^*) \geq F(\psi)$ for every financial network ψ .

IV. Implementation

Suppose x^* is an optimal allocation given some realization e . One way to implement x^* is as follows:

1. A central planner (or one of the agents) proposes x^* and the optimal vector of investments $I(x^*)$ from Lemma 1.
2. Agents $1, \dots, n$ can either accept or reject sequentially after observing the responses of previous agents.

3. If all agents accept, the necessary transfers to implement x^* take place. Otherwise, agents remain in autarky.¹¹

To rule out outcomes in which all agents reject, I use subgame perfection as the equilibrium concept. Then, under the assumption that an agent who is indifferent between accepting and rejecting accepts, we obtain the next proposition.¹²

Proposition 1 *If x^* and $I(x^*)$ are offered, there is a subgame perfect equilibrium whose outcome is that all agents accept. In addition, this equilibrium is unique.*

V. Being linked versus not being linked

This section focuses on two special networks: one in which each agent stands on his own ($K_i = \emptyset$ for $i = 1, \dots, n$), and one in which each agent is directly linked to all other agents ($K_i = N \setminus \{i\}$ for $i = 1, \dots, n$). The first network is referred to as *unlinked*, and the second network is referred to as *fully linked*. Proposition 2 below expands the set of networks that can be thought of as fully linked by showing that the only thing that matters for second-best outcomes is the collection of sets $[L_1, \dots, L_n]$, which specify to whom each agent is linked (either directly or indirectly).¹³ In particular, the outcome of a fully linked network can be obtained even if some agents are not directly linked to all other agents. The next example demonstrates this.

Example 2 Suppose $n = 4$, and consider the financial network $K_1 = \{2\}$, $K_2 = \{3\}$, $K_3 = \{4\}$, $K_4 = \{1\}$. Since every agent is linked to all other agents either directly or indirectly, the second-best outcome given this network is identical to the one obtained given a fully linked network.

Proposition 2 *Financial networks that induce the same sets $[L_1, \dots, L_n]$ have the same second-best outcome (or outcomes).*

Using part 2 in Lemma 1 and equation (8), we obtain

$$\sum_{i=1}^n V_i(x(e)) = \sum_{i=1}^n e_i + (R-1) \sum_{i=1}^n I_i(x(e)). \quad (12)$$

Therefore, maximizing the expected sum of utilities is equivalent to maximizing the expected aggregate level of investment. Also, to find an optimal allocation rule we can solve the planning problem pointwise, that is, solve many problems in which we choose an optimal allocation taking the realization e as given.

First best. An upper bound on $\sum_{i=1}^n I_i(x(e))$ is $\min(n, \lfloor \sum_{i=1}^n e_i \rfloor)$ where $\lfloor \cdot \rfloor$ indicates the integer less than or equal to. If the network is unlinked, this upper bound can be achieved for each realization of e through a set of transfers. If the network is fully linked, the upper bound can be achieved only if there is enough cash to take on all projects, that is, only if $\sum_{i=1}^n e_i \geq n$; otherwise, no agent invests.

Second best. The next proposition shows that if the network is unlinked, it is not possible to do better than autarky. The inability to precommit to help one another eliminates the potential gains from mutual insurance.

Proposition 3 *If the financial network is unlinked, the only feasible allocation rule that satisfies the interim participation constraint is $x_i(e) = e_i$ for every $i \in N$.*

Denote an optimal allocation rule by $x^*(e)$. The following conclusion follows from Proposition 3 and Lemma 1.

Conclusion 1 *If the financial network is unlinked, $I_i(x^*(e)) = \begin{cases} 1 & \text{if } e_i \geq 1 \\ 0 & \text{otherwise.} \end{cases}$*

If the network is fully linked, we may sometimes achieve the upper bound on $\sum_i I_i(x(e))$. In other words, it may be possible to obtain the benefits of mutual insurance.

Proposition 4 *If the financial network is fully linked, the aggregate level of investment (given an optimal allocation rule) is n if $\sum_{i=1}^n \min(e_i, R) \geq n$, and zero otherwise.*

To get the intuition behind the condition in Proposition 4, recall that in Example 1, agent 2 was willing to transfer cash to the other agent only if $R \geq 2$; that is, only if the return on his project was at least as much as he could obtain by consuming his entire endowment. More generally, the amount of cash agent i may be willing to give to the central planner is $\min(e_i, R)$; so the total amount available for investment is $\sum_{i=1}^n \min(e_i, R)$. If this is more than n , all agents can invest. Otherwise, because of linkages, no agent invests. In the latter case, if there exists an agent with $e_i > 1$, a fully linked network is ex-post strictly worse than an unlinked network. A fully linked network leads to no investment, whereas an unlinked network leads to some investment. The next proposition summarizes this.

Proposition 5 *A fully linked network is ex-post strictly worse than an unlinked network if and only if the realization of e is such that $\sum_{i=1}^n \min(e_i, R) < n$ and there exists an agent with $e_i > 1$.*

Example 3 below illustrates two cases in which the condition $\sum_{i=1}^n \min(e_i, R) < n$ in Proposition 5 holds. In the first case the aggregate endowment is not high enough to take all projects. In the second case the aggregate endowment is high enough, but it is concentrated among a small set of agents.

Example 3 Two cases in which linkages are suboptimal ex-post:

1. $\sum_{i=1}^n e_i < n$.

2. $e_1 = n > R$ but $e_j = 0$ for $j = 2, \dots, n$. (note $\sum_{i=1}^n e_i = n$).

It remains to compare the two networks from an ex-ante point of view. It follows from Proposition 4 that if the network is fully linked, the expected aggregate investment is given by

$$E\left(\sum_{i=1}^n I_i(x^*(\tilde{e}))\right) = n \text{Prob}\left(\sum_{i=1}^n \min(\tilde{e}_i, R) \geq n\right) \quad (13)$$

where $\text{Prob}(\cdot)$ denotes the probability of an event. If the network is unlinked, it follows from Conclusion 1 that $E(I_i(x^*(\tilde{e}))) = \text{Prob}(\tilde{e}_i \geq 1)$ and

$$E\left(\sum_{i=1}^n I_i(x^*(\tilde{e}))\right) = \sum_{i=1}^n E(I_i(x^*(\tilde{e}))) = n \text{Prob}(\tilde{e}_i \geq 1). \quad (14)$$

This leads us to Theorem 1.

Theorem 1 1. *If $\text{Prob}(\sum_{i=1}^n \min(\tilde{e}_i, R) \geq n) > \text{Prob}(\tilde{e}_i \geq 1)$, a fully linked network ex-ante Pareto dominates an unlinked network.*

2. *If $\text{Prob}(\sum_{i=1}^n \min(\tilde{e}_i, R) \geq n) < \text{Prob}(\tilde{e}_i \geq 1)$, an unlinked network ex-ante Pareto dominates a fully linked network.*

3. *If $\text{Prob}(\sum_{i=1}^n \min(\tilde{e}_i, R) \geq n) = \text{Prob}(\tilde{e}_i \geq 1)$, both a fully linked network and an unlinked network give the same expected utilities (from an ex-ante point of view).*

VI. Optimal networks

A. Characterization

I now characterize optimal networks within a larger set of networks. The only restriction I impose is that agent i is linked to agent j if and only if agent j is also linked to agent i . More formally,

$$\text{For every } i \neq j, \quad i \in L_j \text{ if and only if } j \in L_i. \quad (15)$$

Note that I do not require that $i \in K_j$ implies that $j \in K_i$. Thus, agent i can be directly linked to agent j even though agent j is not directly linked to agent i . Financial networks that satisfy the restriction above specify groups of agents such that agents within a group are linked to one another. More formally, they induce a partition N_1, \dots, N_κ .¹⁴ The same partition may be induced by more than one network, but Proposition 2 says that all these networks are equivalent in terms of optimal allocations and investments. Therefore, the problem of finding an optimal network reduces to finding an optimal partition of N .

I assume that date-1 transfers can occur within groups, but not across groups. Therefore, I find a second-best allocation rule for each group separately. This restriction can arise if we assume that communication at date 1 is easier for those who formed linkages at date 0. But it can also arise *endogenously* by applying Ray and Vohra's (1997) notion of "equilibrium binding agreements" – a refinement of the standard core concept – to our setting.¹⁵

Denote the number of agents in group k by n_k , and the aggregate level of investment in the group by I^k . Proposition 4 implies that

$$I^k = \begin{cases} n_k & \text{if } \sum_{i \in N_k} \min(e_i, R) \geq n_k \\ 0 & \text{otherwise,} \end{cases} \quad (16)$$

and

$$E(I^k) = n_k \text{Prob}\left(\sum_{i \in N_k} \min(\tilde{e}_i, R) \geq n_k\right). \quad (17)$$

Either everyone in the group invests, or no one invests. In the first case I say that the group survives and in the second case I say the group collapses.

Since $E(\sum_{i=1}^n I_i) = E(\sum_{k=1}^\kappa I^k) = \sum_{k=1}^\kappa E(I^k)$, we obtain Theorem 2.¹⁶

Theorem 2 *If the financial network ψ specifies the partition N_1, \dots, N_κ , ψ is optimal if and*

only if the partition N_1, \dots, N_κ maximizes the expression

$$\sum_{k=1}^{\kappa} n_k \text{Prob}(\sum_{i \in N_k} \min(\tilde{e}_i, R) \geq n_k). \quad (18)$$

Since agents are identical ex-ante, we can simplify the maximization problem in Theorem 2. Let $f(v)$ denote the probability that a group of v agents linked to one another will survive.

That is,

$$f(v) = \text{Prob}(\sum_{i=1}^v \min(\tilde{e}_i, R) \geq v). \quad (19)$$

Note that $f(v)$ is also the expected per capita investment in the group. We can find an optimal network by solving the following problem:

$$\max_{\kappa, n_1, \dots, n_\kappa} \sum_{k=1}^{\kappa} n_k f(n_k) \quad (20)$$

subject to

$$\sum_{k=1}^{\kappa} n_k = n. \quad (21)$$

Let $n^* = \arg \max_{v \in N} f(v)$. If $n^* = 1$, an optimal solution is to have n groups with one agent in each group; so an unlinked network is optimal. If $n^* = n$, an optimal solution is to have one group of n agents; so a fully linked network is optimal. In some cases, however, optimal networks are neither fully linked nor unlinked. I use the term *partially linked* to refer to such networks and illustrate in Example 5 below. Before that I show how to calculate $f(v)$ when endowments are *iid* Bernoulli.

Example 4 Suppose endowments are iid, and \tilde{e}_i can take two values: H with probability p and 0 with probability $1 - p$. Then

$$\min(\tilde{e}_i, R) = \begin{cases} \min(H, R) & \text{with probability } p \\ 0 & \text{with probability } 1 - p, \end{cases} \quad (22)$$

and since the sum of iid Bernoulli random variables has a binomial distribution, we obtain

$$f(v) = \text{Prob}(Z_v \geq \frac{v}{\min(H, R)}) \quad (23)$$

where Z_v is a random variable that is distributed $\text{Binomial}(v, p)$.

Example 5 Suppose $N = \{1, 2, 3\}$ and endowments are as in Example 4 with $H = R = 2$. There are five partitions: $p_1 = (\{1, 2, 3\})$, $p_2 = (\{1, 2\}, \{3\})$, $p_3 = (\{1\}, \{2, 3\})$, $p_4 = (\{1, 3\}, \{2\})$ and $p_5 = (\{1\}, \{2\}, \{3\})$. p_1 specifies a fully linked network; p_2, p_3 and p_4 specify a partially linked network, and p_5 specifies an unlinked network. Denote by Q_i the value of (18) given the partition p_i . Using equation (23), we obtain

$$f(1) = \text{Prob}(Z_1 \geq \frac{1}{2}) = \text{Prob}(Z_1 = 1) = p \quad (24)$$

$$f(2) = \text{Prob}(Z_2 \geq \frac{2}{2}) = \text{Prob}(Z_2 = 1) + \text{Prob}(Z_2 = 2) = 2p(1 - p) + p^2 \quad (25)$$

$$f(3) = \text{Prob}(Z_3 \geq \frac{3}{2}) = \text{Prob}(Z_3 = 2) + \text{Prob}(Z_3 = 3) = 3p^2(1 - p) + p^3, \quad (26)$$

and using (20), we obtain

$$Q_1 = 3f(3) = 3[3p^2(1 - p) + p^3] \quad (27)$$

$$Q_2 = Q_3 = Q_4 = 2f(2) + f(1) = 2[2p(1 - p) + p^2] + p \quad (28)$$

$$Q_5 = f(1) + f(1) + f(1) = 3p. \quad (29)$$

The optimal partition depends on the value of p . If the choice is only between p_1 and p_5 , then p_1 is optimal when $p > 0.5$, and p_5 is optimal when $p < 0.5$. If it is also possible to choose p_2, p_3 , and p_4 , these partitions are optimal when $p < 0.83$. This is illustrated in Figure 1.

***** Insert Figure 1 about here. *****

B. *When is it optimal to add an agent to an existing group?*

To get more intuition regarding optimal network design, it is useful to think of the conditions under which adding an agent to an existing group is beneficial for existing group members. The advantage is better risk-sharing due to diversification: a new agent may provide insurance when the existing group is low on cash. The disadvantage is that a new agent may increase the probability of bad outcomes: when the new agent is low on cash, he may become the “weakest link,” thereby bringing the whole group down. The next theorem formalizes this tradeoff. The left-hand side of the inequality captures the advantage and the right-hand side captures the disadvantage.

Theorem 3 *Adding an agent to an existing group of n agents is (strictly) optimal if and only if*

$$\begin{aligned} \int_{y < n} \text{Prob}(Y_{n+1} \geq n + 1 - y \mid \sum_{i=1}^n Y_i = y) dG_n(y) \\ > \int_{n \leq y < n+1} \text{Prob}(Y_{n+1} < n + 1 - y \mid \sum_{i=1}^n Y_i = y) dG_n(y), \end{aligned} \quad (30)$$

where $Y_i = \min(\tilde{e}_i, R)$ and $G_n(y) \equiv \text{Prob}(\sum_{i=1}^n Y_i \leq y)$.

When endowments are *iid* Bernoulli, we obtain:

Corollary 1 *If endowments are iid Bernoulli (\tilde{e}_i equals H with probability p and zero otherwise), it is (strictly) optimal to add an agent to an existing group if and only if*

$$(p)\text{Prob}(n + 1 - h \leq \sum_{i=1}^n Y_i < n) > (1 - p)\text{Prob}(n \leq \sum_{i=1}^n Y_i < n + 1) \quad (31)$$

where $h = \min(H, R)$.

The intuition is as follows: A new agent can benefit the existing group if 1) he does not suffer from a liquidity shock, and 2) the aggregate endowment of the existing group is less

than n , so they do not have enough cash for all their projects, but high enough so that the new agent can help. If the endowment is too low, the group cannot survive even if the new agent helps. The probability these two events will happen is captured by the expression $(p)Prob(n + 1 - h \leq \sum_{i=1}^n Y_i < n)$. In contrast, a new agent can become the weakest link if 3) he realizes a liquidity shock, and 4) the aggregate endowment of the existing group is more than n , so they can survive without the additional agent, but less than $n + 1$, so they cannot support both themselves and the additional agent. The probability of these two events is captured by the expression $(1 - p)Prob(n \leq \sum_{i=1}^n Y_i < n + 1)$.

The two expressions above are not monotonic in p . Thus, it is possible, for example, that when the probability of a liquidity shock increases (and everything else remains the same), the disadvantage of adding an agent actually decreases. The group need not become more worried about a new agent becoming a weakest link because the group is more likely to fail even without him.¹⁷

C. Large economies with iid endowments

In the special case of *iid* endowments (not necessarily Bernoulli), we can use the weak law of large numbers to prove the following lemma.

Lemma 2 *If $\tilde{e}_1, \dots, \tilde{e}_n$ are iid, then,*

$$\lim_{n \rightarrow \infty} f(n) = \begin{cases} 0 & \text{if } E[\min(\tilde{e}_i, R)] < 1 \\ 1 & \text{if } E[\min(\tilde{e}_i, R)] > 1. \end{cases} \quad (32)$$

Combining the results in Lemma 2 and Theorem 1, we obtain:

Theorem 4 *If endowments are iid, there exists an integer \bar{n} such that if the economy consists of more than \bar{n} agents, the following two statements hold:*

1. *If $E[\min(\tilde{e}_i, R)] > 1$, a fully linked network Pareto dominates an unlinked network.*

2. If $E[\min(\tilde{e}_i, R)] < 1$, an unlinked network Pareto dominates a fully linked network.

In addition, if \tilde{e}_i is bounded, and the riskiness of \tilde{e}_i increases in the sense of mean preserving spread,¹⁸ the threshold $E[\min(\tilde{e}_i, R)]$ is weakly decreasing.

While the theorem above says that when n is large enough and $E[\min(\tilde{e}_i, R)] > 1$, a fully linked network Pareto dominates an unlinked network, I am not sure whether a fully linked network must be optimal. This is because the convergence of $f(n)$ to 1 is not necessarily monotone. However, it is not difficult to prove that a fully linked network is “almost optimal.”

Definition 2 A financial network ψ^* is “ ε -optimal” if $F(\psi^*) \geq F(\psi) - \varepsilon$ for every financial network ψ .

Theorem 5 If endowments are iid, then for every $\varepsilon > 0$, there exists an integer \bar{n} such that if the economy consists of more than \bar{n} agents, the following two statements hold:

1. If $E[\min(\tilde{e}_i, R)] > 1$, a fully linked network is ε -optimal.
2. If $E[\min(\tilde{e}_i, R)] < 1$, a fully linked network is not optimal.

When a fully linked network is not optimal, the optimal network can be either unlinked or partially linked. In particular, even if n is very large, it may be optimal to have subgroups of agents who are linked to one another. The next example illustrates this.

Example 6 Go back to Example 4, and consider the following three cases:

1. $p = 0.75$ and $\min(H, R) = 1.6$
2. $p = 0.6$ and $\min(H, R) = 1.5$
3. $p = 0.75$ and $\min(H, R) = 1.2$

Figures 2-4 graph $f(n)$ as a function of n for each case. In case 1, $E[\min(\tilde{e}_i, R)] =$

$1.2 > 1$, $f(n)$ converges to 1, and a fully linked network is ε -optimal. In cases 2 and 3, $E[\min(\tilde{e}_i, R)] = 0.9 < 1$ and $f(n)$ converges to 0. In case 2, $n^* = 3$ and it is optimal to have groups of three agents (because of integer problems, we may have one group of one or two agents). In case 3, $n^* = 1$, so an unlinked network is optimal.

***** Insert Figures 2-4 about here. *****

VII. Applications

A. *Payment systems*

To apply the results to payment systems, I extend the setting in Section I to allow for consumption in different locations, as in Freixas and Parigi (1998). Each bank is in a different location. Late consumers consume in their original location, but early consumers may need to switch location. An early consumer learns between date 0 and date 1 in what location he would like to consume. If he learns that he wants to consume in a different location, he has two alternatives: The first is to withdraw his deposit in the bank in the old location and take the cash with him to the new location. This corresponds to a gross system. The second is to get a guarantee from the bank in the old location that says that if he withdraws cash from the bank in the new location, the bank in the old location will transfer the funds. This corresponds to a net system.

These bank guarantees, which represent intraday credits, create linkages among banks in the same way that interbank deposits do in the setting of Allen and Gale. Thus, the same arguments used in Section I imply that a net payment system may be optimal. This

is despite the fact that the money needed to satisfy the liquidity needs of early consumers is already in storage, so there is no cost of holding reserves.

B. *Joint liability arrangements*

With the interpretation that I_i in the basic model denotes whether agent i repays his loan ($I_i = 1$) or defaults ($I_i = 0$), and R denotes the return on a non-transferable asset (e.g., benefits from future loans) that an agent loses if he defaults, the theory can apply to joint liability arrangements, such as the ones used by the Grameen Bank. The theory suggests that group punishments may be effective because they induce voluntary transfers from successful agents to agents who are about to default.¹⁹ It can also be used to calculate the optimal number of groups and the optimal number of agents within a group (different groups may have different sizes).

VIII. Discussion

The main point of this paper is that linkages that help spread contagion may be optimal because the threat of contagion can motivate agents to help one another, even when they cannot precommit to do so. This result seems robust. It can be applied, for example, to the design of payment systems showing that net payment systems can be optimal even if holding reserves is not costly.

I also characterize optimal networks, focusing on the tradeoff between risk sharing and the potential for extreme outcomes that result in a collapse of the whole network. Doing so, I ignore other issues that may be important in optimal network design, such as moral hazard, coordination problems, and free-riding problems. In some sense, this makes some of the results more interesting, in particular, the result on the optimal group size.

A crucial assumption is the availability of some coordinating device when the threat of contagion arises. Without coordination the results do not hold. This suggests that whether agents should be linked or whether contagion is bad may partially depend on whether it is possible to coordinate.

The paper offers a new perspective on private-sector bailouts: these voluntary transfers may be a form of achieving mutual insurance and may be optimal from an ex-ante point of view. I also demonstrate the role of government: it can increase welfare by coordinating voluntary transfers. In future research, it remains to be explored whether coordination may occur also in decentralized environments.

An open question is whether agents will form optimal networks. The existing literature shows that this may not always be the case. For example, in a related paper, Acharya (2001) shows that banks may choose projects with correlated payoffs in order to fail together. This may not be optimal for the economy, however. In a more general framework, Jackson and Wolinsky (1996) study stable networks, in which a link between a pair of agents exists only if both agents agree to it. They show that a stable network that is also efficient does not always exist.²⁰ The conclusion is that in some cases there may be room for endogenous institutions or rules by regulators that are created in order to help implement efficient networks.

Appendix A Unobservable endowments

When endowments are private information, allocations cannot be contingent on endowments directly, but they can be contingent on announcements agents make. By the revelation principle, we can assume without loss of generality that the announcement space is the space of realized endowments, and we can focus on allocation rules that implement truth telling. Let $F_i(\widehat{e}_i | e_i, x(e))$ denote the expected utility for agent i if given the allocation rule $x(e)$, he announces \widehat{e}_i instead of e_i . Then

$$F_i(\widehat{e}_i | e_i, x(e)) = e_i - \widehat{e}_i + \sum_{e_{-i}} V_i((x(\widehat{e}_i, e_{-i})) \text{ Prob}(\widetilde{e}_{-i} = e_{-i})). \quad (\text{A1})$$

$x(e)$ induces truth telling if for every $i \in N$, for every e , and for every \widehat{e}_i ,

$$F_i(e_i | e_i, x(e)) \geq F_i(\widehat{e}_i | e_i, x(e)). \quad (\text{A2})$$

The planning problem is as in Section III, but we need to add the truth telling constraint.

The next example illustrates that a fully linked network can still be optimal, but for a smaller set of parameters.

Example 7 Suppose $n = 2$, endowments are *iid*, and each \widetilde{e}_i can take two values: 2 with probability p and 0 with probability $1 - p$. Consider the following allocation rule: If $e_1 + e_2 \geq 2$, then $x_1(e) = x_2(e) = \frac{e_1 + e_2}{2}$; otherwise, $x_i(e) = e_i$ for $i = 1, 2$. If agents make truthful announcements, $x(e)$ achieves the second best. In addition, it follows from Example 1 that the participation constraint holds when $R \geq 2$. I will show that the truth telling constraint holds when $R \geq \frac{2-p}{1-p}$ which implies $R \geq 2$.

Consider without loss of generality agent 1. If $e_1 = 0$, he must announce $\widehat{e}_1 = 0$. If $e_1 = 2$, he can choose to announce either 0 or 2. Suppose the other agent announces truthfully. If agent 1 announces truthfully, his utility is R if $e_2 = 0$, and $1 + R$ if $e_2 = 2$. If he announces

$\hat{e}_1 = 0$, his utility is 2 if $e_2 = 0$, and $2 + R$ if $e_2 = 2$. Thus, agent 1 will announce truthfully if

$$(1 - p)R + p(1 + R) \geq (1 - p)2 + p(2 + R). \quad (\text{A3})$$

This is equivalent to $R \geq \frac{2-p}{1-p}$.

Appendix B

Proof of Lemma 1. To prove the first part, we need to show that $I_i(x) = 1$ if and only if $x_j \geq 1$ for every $j \in L_i \cup \{i\}$.

First direction: Suppose $x_j \geq 1$ for every $j \in L_i \cup \{i\}$. Assume by contradiction that $I_i(x) = 0$, and consider the investment rule $\bar{I}(x)$ given by

$$\bar{I}_j(x) = \begin{cases} 1 & \text{if } j \in L_i \cup \{i\} \\ I_j(x) & \text{if } j \notin L_i \cup \{i\}. \end{cases} \quad (\text{A4})$$

$\bar{I}(x)$ satisfies equation (6). Since $K_j \subset L_i$ for every $j \in L_i \cup \{i\}$, it follows that $p_j(\bar{I}(x)) = 1$ for every $j \in L_i \cup \{i\}$; and since $R > 1$, it follows that $\bar{I}(x)$ satisfies equation (7). In addition $\sum_{i=1}^n U_i(x, \bar{I}(x)) > \sum_{i=1}^n U_i(x, I(x))$. This contradicts the optimality of $I(x)$.

Second direction: Suppose that $I_i(x) = 1$, and assume by contradiction that there exists $k \in L_i \cup \{i\}$ such that $x_k < 1$. Equation (6) implies that $I_k(x) = 0$, and equation (4) implies that $p_z(I(x)) = 0$ for every $z \in K_k$. Equation (7) then implies that $I_z(x) = 0$ for every $z \in K_k$. We can continue to show that $I_i(x) = 0$, thereby obtain a contradiction, by induction on the distance (i.e., the length of the shortest path) between k and i .

To prove the second part, note that equation (7) implies that $I_i(x) = 1$ only if $p_i(I(x)) = 1$, and equation (4) implies that $p_i(I(x)) = 1$ only if $I_i(x) = 1$. Thus, $p_i(I(x)) = I_i(x)$, and it follows that $V_i(x) = x_i + (R - 1)I_i(x)$. ■

Proof of Proposition 1. Let σ_i denote whether agent i accepts ($\sigma_i = 1$) or rejects ($\sigma_i = 0$). A (pure) strategy for agent i specifies whether to accept or reject as a function of $(\sigma_1, \dots, \sigma_{i-1})$. I will show by a backward induction that if x^* is offered, any equilibrium strategies are such that agent 1 accepts, and agent $i \in \{2, \dots, n\}$ accepts if agents $1, \dots, i-1$ have accepted. Thus, the unique outcome is that all agents accept.

Start with agent n . Suppose that agents $1, \dots, n-1$ have accepted. If he accepts, he obtains $V_n(x^*)$. If he rejects, he obtains $V_n(e)$. Since x^* satisfies $V_n(x^*) \geq V_n(e)$, and an agent who is indifferent accepts, agent n accepts.

Assume now that agents $i+1, \dots, n$ accept if all agents who responded before them accept. Consider agent i , and suppose agents $1, \dots, i-1$ have accepted. If agent i accepts, the induction assumption implies that agents $i+1, \dots, n$ will accept as well; thus, agent i ends up with $V_i(x^*)$. If agent i rejects, he ends up with $V_i(e)$, no matter what the other agents do. Since $V_i(x^*) \geq V_i(e)$, it is optimal for him to accept. ■

Proof of Proposition 2. Consider the second-best problem. Lemma 1 implies that $p_i(I)$ does not appear in the constraints nor in the objective function; only L_1, \dots, L_n appear. Therefore, only L_1, \dots, L_n matter. ■

Proof of Proposition 3. Lemma 1 implies that for an unlinked network, $V_i(x)$ depends only on x_i and is increasing in x_i . Thus, the interim participation constraint is satisfied only if $x_i \geq e_i$ for every $i \in N$. Equation (8) then implies that $x_i = e_i$ for every $i \in N$. ■

Proof of Proposition 4. Assume first that $\sum_{i=1}^n \min(e_i, R) \geq n$. If $e_i \geq 1$ for every $i \in N$, the allocation rule $x_i(e) = e_i$ is feasible and satisfies the interim participation constraint; in addition, $\sum_{i=1}^n I_i(x(e)) = n$; so the allocation is optimal. Assume now that $e_i < 1$ for some i , and consider the allocation rule $x(e)$ where $x_i(e) = e_i - \min(e_i, R) +$

$\frac{\sum_{i=1}^n \min(e_i, R)}{n}$. Since $e_i \geq \min(e_i, R)$ and $\sum_{i=1}^n \min(e_i, R) \geq n$, it follows that $x_i(e) \geq 1$. In addition, $\sum_{i=1}^n x_i(e) = \sum_{i=1}^n e_i$; so $x(e)$ is feasible. Now, Lemma 1 implies that for every $i \in N$, $I_i(x(e)) = 1$, $V_i(x(e)) = x_i(e) - 1 + R$ and $V_i(e) = e_i$. Thus,

$$V_i(x(e)) - V_i(e) = -\min(e_i, R) + \frac{\sum_{i=1}^n \min(e_i, R)}{n} - 1 + R \geq 0$$

and the interim participation constraint is satisfied. Finally, $\sum_{i=1}^n I_i(x(e)) = n$; so $x(e)$ is optimal.

Assume now that $\sum_{i=1}^n \min(e_i, R) < n$. Suppose $x(e)$ is an optimal allocation rule. Lemma 1 implies that $\sum_{i=1}^n I_i(x(e))$ equals either n or zero. Assume, by contradiction, that $\sum_{i=1}^n I_i(x(e)) = n$; that is, $I_i(x(e)) = 1$ for every $i \in N$. Let $t_i = -T_i$ be the net amount taken from agent i , i.e., $t_i = e_i - x_i(e)$. Then, $x_i(e) = e_i - t_i$. Since $R > 1$ and $I_i \in \{0, 1\}$, Lemma 1 implies that $x_i(e) \leq V_i(x(e)) \leq x_i(e) + R - 1$. Thus, $V_i(x(e)) \leq e_i - t_i - 1 + R$, and $V_i(e) \geq e_i$. To satisfy equation (11), we must have $e_i - t_i - 1 + R \geq e_i$, which implies that $t_i \leq R - 1$. To have $I_i(x(e)) = 1$, we must have $x_i(e) \geq 1$, which implies that $t_i \leq e_i - 1$. Therefore, $t_i \leq \min(R - 1, e_i - 1)$. Summing over i , we obtain $\sum_{i=1}^n t_i \leq \sum_{i=1}^n \min(R, e_i) - n$. But then, since $\sum_{i=1}^n \min(e_i, R) < n$, it follows that $\sum_{i=1}^n t_i < 0$. This is equivalent to $\sum_{i=1}^n x_i(e) \geq \sum_{i=1}^n e_i$, which violates equation (8). Hence, there is no feasible allocation in which $\sum_{i=1}^n I_i(x(e)) = n$; so we must have $\sum_{i=1}^n I_i(x(e)) = 0$. ■

Proof of Theorem 1. The results follow from equations (13), (14), and the fact that maximizing the expected sum of utilities is equivalent to maximizing the expected aggregate investment. ■

Proof of Theorem 3. Adding an agent is optimal if and only if $f(n + 1) - f(n) > 0$. This is because $f(n)$ is the expected per capita investment in a group of n agents, and maximizing the expected sum of utilities is equivalent to maximizing the expected aggregate

investment. Denote $\bar{F}_n(n+1-y|y) \equiv \text{Prob}(Y_{n+1} \geq n+1-y | \sum_{i=1}^n Y_i = y)$, and note that $\bar{F}_n(n+1-y|y) = 1$ if $y \geq n+1$. The result then follows from:

$$\begin{aligned}
f(n+1) - f(n) &= \text{Prob}\left(\sum_{i=1}^{n+1} Y_i \geq n+1\right) - \text{Prob}\left(\sum_{i=1}^n Y_i \geq n\right) \tag{A5} \\
&= \int_y \text{Prob}\left(\sum_{i=1}^n Y_i + Y_{n+1} \geq n+1 \mid \sum_{i=1}^n Y_i = y\right) dG_n(y) - \int_{y \geq n} dG_n(y) \\
&= \int_y \bar{F}_n(n+1-y|y) dG_n(y) - \int_{y \geq n} dG_n(y) \\
&= \int_{y < n} \bar{F}_n(n+1-y|y) dG_n(y) + \int_{n \leq y < n+1} \bar{F}_n(n+1-y|y) dG_n(y) \\
&\quad + \int_{y \geq n+1} dG_n(y) - \int_{y \geq n} dG_n(y) \\
&= \int_{y < n} \bar{F}_n(n+1-y|y) dG_n(y) + \int_{n \leq y < n+1} \bar{F}_n(n+1-y|y) dG_n(y) \\
&\quad - \int_{n \leq y < n+1} dG_n(y) \\
&= \int_{y < n} \bar{F}_n(n+1-y|y) dG_n(y) - \int_{n \leq y < n+1} (1 - \bar{F}_n(n+1-y|y)) dG_n(y)
\end{aligned}$$

■

Proof of Lemma 2. Denote $Y_i = \min(\tilde{e}_i, R)$, $M_n = |\frac{1}{n} \sum_{i=1}^n Y_i - E(Y_i)|$, and let $\varepsilon = |1 - E(Y_i)|$. Then

$$f(n) = \text{Prob}\left(\frac{1}{n} \sum_{i=1}^n Y_i \geq 1\right) = \tag{A6}$$

$$\text{Prob}\left(\frac{1}{n} \sum_{i=1}^n Y_i \geq 1 \mid M_n < \varepsilon\right) \text{Prob}(M_n < \varepsilon) + \text{Prob}\left(\frac{1}{n} \sum_{i=1}^n Y_i \geq 1 \mid M_n \geq \varepsilon\right) \text{Prob}(M_n \geq \varepsilon)$$

By the weak law of large numbers,

$$\lim_{n \rightarrow \infty} \text{Prob}(M_n < \varepsilon) = 1. \tag{A7}$$

Thus,

$$\lim_{n \rightarrow \infty} f(n) = \lim_{n \rightarrow \infty} \text{Prob}\left(\frac{1}{n} \sum_{i=1}^n Y_i \geq 1 \mid M_n < \varepsilon\right). \tag{A8}$$

$M_n < \varepsilon$ is equivalent to $E(Y_i) - \varepsilon < \frac{1}{n} \sum_{i=1}^n Y_i < E(Y_i) + \varepsilon$. When $E(Y_i) < 1$, we obtain $\varepsilon = 1 - E(Y_i)$, and $\frac{1}{n} \sum_{i=1}^n Y_i < E(Y_i) + \varepsilon = 1$. Thus, $\lim_{n \rightarrow \infty} f(n) = 0$. When $E(Y_i) > 1$, we obtain $\varepsilon = E(Y_i) - 1$, and $\frac{1}{n} \sum_{i=1}^n Y_i > E(Y_i) - \varepsilon = 1$. Thus, $\lim_{n \rightarrow \infty} f(n) = 1$. ■

Proof of Theorem 4. If $E[\min(\tilde{e}_i, R)] > 1$, then $\lim_{n \rightarrow \infty} f(n) = 1$ (Lemma 2); and since $f(1) < 1$, there exists \bar{n} such that $f(n) > f(1)$ for every $n > \bar{n}$. If $E[\min(\tilde{e}_i, R)] < 1$, then $\lim_{n \rightarrow \infty} f(n) = 0$; and since $f(1) > 0$, there exists \bar{n} such that $f(n) < f(1)$ for every $n > \bar{n}$. The first two statements then follow from Theorem 1. The last result follows since $\min(x, R)$ is concave in x and since the statement “ A is more risky than B in the sense of mean preserving spread” is equivalent to “ $EU(A) \leq EU(B)$ for all concave functions U .” See Rothschild and Stiglitz (1970) for a proof of this fact. ■

Proof of Theorem 5. To prove the first part, denote by $G(\psi)$ the value of the objective function in equation (20) given a network ψ . Let $\varepsilon > 0$, and let ψ_1 be a fully linked network. Lemma 2 implies that there exists \bar{n} such that for every $n > \bar{n}$, $G(\psi_1) \geq 1 - \frac{\varepsilon}{R-1}$. Since an upper bound on $G(\psi)$ is 1, it follows that $G(\psi_1) \geq G(\psi) - \frac{\varepsilon}{R-1}$ for every ψ . Since $F(\psi_1) - F(\psi) = (R-1)[G(\psi_1) - G(\psi)]$, it follows that $F(\psi_1) \geq F(\psi) - \varepsilon$ for every ψ . The second part follows from part 2 of Theorem 4. ■

References

- Acharya, Viral V., 2001, A theory of systemic risk and design of prudential bank regulation, Working paper, London Business School.
- Allen, Franklin, and Douglas Gale, 2000, Financial contagion, *Journal of Political Economy* 108, 1-33.
- Bala, Venkatesh, and Sanjeev Goyal, 2000, A noncooperative model of network formation, *Econometrica* 68, 1181-1229.
- Besley, Timothy, and Stephen Coate, 1995, Group lending, repayment incentives and social collateral, *Journal of Development Economics* 46, 1-18.
- Bolton, Patrick, and David S. Scharfstein, 1990, A theory of predation based on agency problems in financial contracting, *American Economic Review* 80, 93-106.
- Bond, Philip, forthcoming, Joint liability among bank borrowers, *Economic Theory*.
- Calomiris, Charles W., and Charles M. Kahn, 1991, The role of demandable debt in structuring optimal banking arrangements, *American Economic Review* 81, 497-513.
- Corbae, Dean, and John Duffy, 2003, Experiments with network economies, working paper.
- Dewatripont, Mathias, and Eric Maskin, 1995, Credit and efficiency in centralized and decentralized economies, *Review of Economic Studies* 62, 541-555.
- Diamond, Douglas W., and Philip H. Dybvig, 1983, Banks runs, deposit insurance, and liquidity, *Journal of Political Economy* 91, 401-419.
- Diamond, Douglas W., and Raghuram G. Rajan, 2000, A theory of bank capital, *Journal of Finance* 55, 2431-2465.
- Diamond, Douglas W., and Raghuram G. Rajan, 2001, Liquidity risk, liquidity creation, and financial fragility: A theory of banking, *Journal of Political Economy* 109, 287-327.

Farrell, Joseph, and Garth Saloner, 1985, Standardization, compatibility, and innovation, *Rand Journal of Economics* 16, 70-83

Freixas, Xavier, 1999, Optimal bail-out, conditionality and creative ambiguity, Discussion paper series No. 2238, Centre for Economic Policy Research.

Freixas, Xavier, and Bruno Parigi, 1998, Contagion and efficiency in gross and net interbank payment systems, *Journal of Financial Intermediation* 7, 3-31.

Ghatak, Maitreesh, and Timothy W. Guinnane, 1999, The economics of lending with joint liability: Theory and practice, *Journal of Development Economics* 60, 195-228.

Goldstein, Itay, and Ady Pauzner, forthcoming, Contagion of self-fulfilling financial crises due to diversification of investment portfolios, *Journal of Economic Theory*.

Greenspan, Alan, Remarks delivered at financial markets conference of the Federal Reserve Bank of Atlanta, Coral Gables, Florida, February 23, 1996.

Hart, Oliver, and John Moore, 1994, A theory of debt based on the inalienability of human capital, *Quarterly Journal of Economics* 109, 841-879.

Jackson, Matthew, and Asher Wolinsky, 1996, A strategic model of social and economic networks, *Journal of Economic Theory* 71, 44-74.

Kahn, Charles M., and William Roberds, 1998, Payment system settlement and bank incentives, *Review of Financial Studies* 11, 845-870.

Kiyotaki, Nobuhiro, and John Moore, 1997, Credit chains, mimeo, University of Minnesota and London School of Economics.

Kremer, Michael, 1993, The O-Ring theory of economic development, *Quarterly Journal of Economics* 108, 551-575.

Kyle, Albert S., and Wei Xiong, 2001, Contagion as a wealth effect, *Journal of Finance*, 56, 1401-1440.

Lagunoff, Roger, and Stacey L. Schreft, 2001, A model of financial fragility, *Journal of Economic Theory* 99, 220-264.

Mailath, George J., and Loretta J. Mester, 1994, A positive analysis of bank closure, *Journal of Financial Intermediation* 3, 272-299.

Morgan, Donald P., 2002, Rating banks: Risks and uncertainty in an opaque industry, *American Economic Review* 92, 874-888.

Rai, Ashok S. and Tomas Sjöström, 2004, Is Grameen lending efficient? Repayment incentives and insurance in village economies, *Review of Economic Studies* 71, 217-234.

Ray, Debraj, and Rajiv Vohra, 1997, Equilibrium binding agreements, *Journal of Economic Theory* 73, 30-78.

Rampini, Adriano, forthcoming, Default and aggregate income, *Journal of Economic Theory*.

Rochet, Jean-Charles, and Jean Tirole, 1996, Interbank lending and systemic risk, *Journal of Money, Credit, and Banking* 28, 733-762.

Rothschild, Michael, and Joseph E. Stiglitz, 1970, Increasing risk: I. A definition, *Journal of Economic Theory* 2, 225-243.

Notes

¹In a different framework, Kyle and Xiong (2001) and Goldstein and Pauzner (forthcoming) model contagion across two asset markets that propagates through wealth effects.

²In different contexts, Diamond and Rajan (2001) and Calomiris and Kahn (1991) show that a fragile capital structure for banks is optimal; Rampini (forthcoming) shows that substantial correlation of default may be a result of optimal risk sharing; and Bond (forthcoming) analyzes joint liability among borrowers that results from optimal contractual arrangements.

³Besley and Coate (1995) analyze a similar tradeoff in a different theoretical framework in a model of joint liability between two borrowers. Rai and Sjöström (2004) show that this tradeoff disappears once we allow for cross reporting; in other words, when one agent cannot pay, the other agent is being punished only if he could have helped out, but chose not to.

⁴See Ghatak and Guinnane (1999) for discussion and empirical evidence regarding group size in joint liability arrangements.

⁵In a different context, Dewatripont and Maskin (1995) focus on the inefficiencies that arise from an inability to commit to stop bad projects (i.e., not to bail out). They show that decentralization can sometimes mitigate these inefficiencies because it creates coordination problems.

⁶See Morgan (2002) for empirical evidence consistent with the view that bank loans are opaque.

⁷Since all banks are identical ex-ante, all deposits have the same value at date 0.

⁸Another way to write equation (4) is as follows: $p_i(I) = \prod_{j \in K_i \cup \{i\}} I_j$. This is similar to the O-Ring production function in Kremer (1993).

⁹More formally, $L_i = \{j \in N : \text{there exist a positive integer } m \text{ and agents } i_1, \dots, i_m \text{ such that } i_k \in K_{i_{k-1}} \text{ for } k = 1, \dots, m+1 \text{ where } i_0 = i \text{ and } i_{m+1} = j\}$.

¹⁰With no private information, the results will stay the same if we replace equation (8) with the constraint $\sum_{i=1}^n x_i(e) \leq \sum_{i=1}^n e_i$.

¹¹Footnote 15 proposes an equilibrium concept where a bailout takes place even though some banks, such as Bear Stearns in LTCM's bailout, choose not to participate.

¹²Using a similar logic, Farrell and Saloner (1985) show that if each firm prefers to switch to a new standard given that all other firms switch, the unique subgame perfect equilibrium outcome is that all firms switch.

¹³To apply Proposition 2 to Allen and Gale's setting, you need to make sure that the amount bank i deposits in other banks does not depend on the number of banks to which bank i is directly linked. Allen and Gale obtain a different result because they assume that the *total exposure* of each bank remains constant; so if a bank is directly linked to more banks, the exposure between any two banks becomes smaller, and contagion is less likely to happen.

¹⁴ N_1, \dots, N_κ is a partition of N if $\cup_{k=1}^\kappa N_k = N$ and if for every $k' \neq k''$, $N_{k'} \cap N_{k''} = \emptyset$.

¹⁵I thank the referee for motivating me to think along these lines. Loosely speaking, an allocation x is an equilibrium binding agreement given a coalition structure \wp (a coalition structure is a partition of N , not necessarily the one induced by the linkages) if (1) each coalition in \wp implements the best allocation for them taking as given the allocations agreed upon by the other coalitions; and (2) taking into account the resulting equilibrium actions of the residual members of the coalition, no subgroup of agents can gain by breaking up

a coalition. In our setting, for any \wp , if x is an equilibrium binding agreement given \wp , then $\sum_{i \in N_k} x_i = \sum_{i \in N_k} e_i$, for every $k = 1, \dots, \kappa$. In addition, there exists a second-best allocation \hat{x} and a coalition structure $\hat{\wp}$, such that \hat{x} is a binding agreement given $\hat{\wp}$. Using the core concept can also help to rule out transfers across groups, but it may also raise a free-rider problem: anticipating that the other agents will coordinate a bailout without him, an agent may be better off separating from the group. The notion of equilibrium binding agreements overcomes this problem by allowing us to break each N_j into more than one coalition, so that all members within a coalition are essential to the bailout.

¹⁶Since $R > 1$ implies that $(e_i, R) \geq 1$ iff $e_i \geq 1$, Theorem 1 is a special case of Theorem 2 ($\kappa = n$ and $n_k = 1$).

¹⁷More generally, denote the first expression (advantage) by $\alpha(p)$ and the second one (disadvantage) by $\beta(p)$. As in Example 4, $\alpha(p) = \text{Prob}(\frac{n+1-h}{h} \leq Z_n < \frac{n}{h})$ and $\beta(p) = \text{Prob}(\frac{n}{h} \leq Z_n < \frac{n+1}{h})$, where Z_n is a random variable that is distributed *Binomial*(n, p). Since the length of the interval $[\frac{n+1-h}{h}, \frac{n+1}{h})$ is 1, there is exactly one integer, say k , that falls in that interval. If $k \in [\frac{n+1-h}{h}, \frac{n}{h})$, then $\alpha(p) = \binom{n}{k} p^{k+1} (1-p)^{n-k} > 0$, $\beta(p) = 0$, and there exists \bar{p} such that $\alpha(p)$ is increasing if $p < \bar{p}$ and decreasing if $p > \bar{p}$. If $k \in [\frac{n}{h}, \frac{n+1}{h})$, then $\alpha(p) = 0$, $\beta(p) = \binom{n}{k} p^k (1-p)^{n+1-k} > 0$, and there exists $\bar{\bar{p}}$ such that $\beta(p)$ is increasing if $p < \bar{\bar{p}}$ and decreasing if $p > \bar{\bar{p}}$.

¹⁸Let A be a random variable with a distribution function F_A and let B be a random variable with a distribution function F_B , such that both take values on the interval $[0, \bar{y}]$. A is more risky than B in the sense of mean preserving spread if $E(A) = E(B)$ and $S(y) \equiv \int_0^y (F_A(z) - F_B(z)) dz \geq 0 \forall y \in [0, \bar{y}]$.

¹⁹An alternative explanation is that group members impose additional penalties on a defaulting member, thereby increasing his ability to commit. This is referred to in the

literature as social collateral. See, for example, Besley and Coate (1995).

²⁰See also Bala and Goyal (2000), who study a model in which one agent can impose links on other agents, and Corbae and Duffy (2003), who do some experiments with network formation.

Figures

Figure 1. Optimal networks (An example). The figure plots the expected aggregate investment for three different networks. There are three agents, and endowments are iid: either zero or two. When $p < 0.83$, it is optimal to have two groups: one with one agent and one with two agents. When $p > 0.83$, it is optimal to have the three agents linked to one another.

Figure 2. An example in which a fully linked network is “ ϵ -optimal.” The figure plots the probability that a group of agents linked to one another will survive as a function of the group size. Endowments are iid Bernoulli: 1.6 with probability 0.75 and zero otherwise.

Figure 3. An example in which a partially linked network is optimal. The figure plots the probability that a group of agents linked to one another will survive as a function of the group size. Endowments are iid Bernoulli: 1.5 with probability 0.6 and zero otherwise. The optimal group size is three.

Figure 4. An example in which being unlinked is optimal. The figure plots the probability that a group of agents linked to one another will survive as a function of the group size. Endowments are iid Bernoulli, 1.2 with probability 0.75 and zero otherwise.

Figure 1

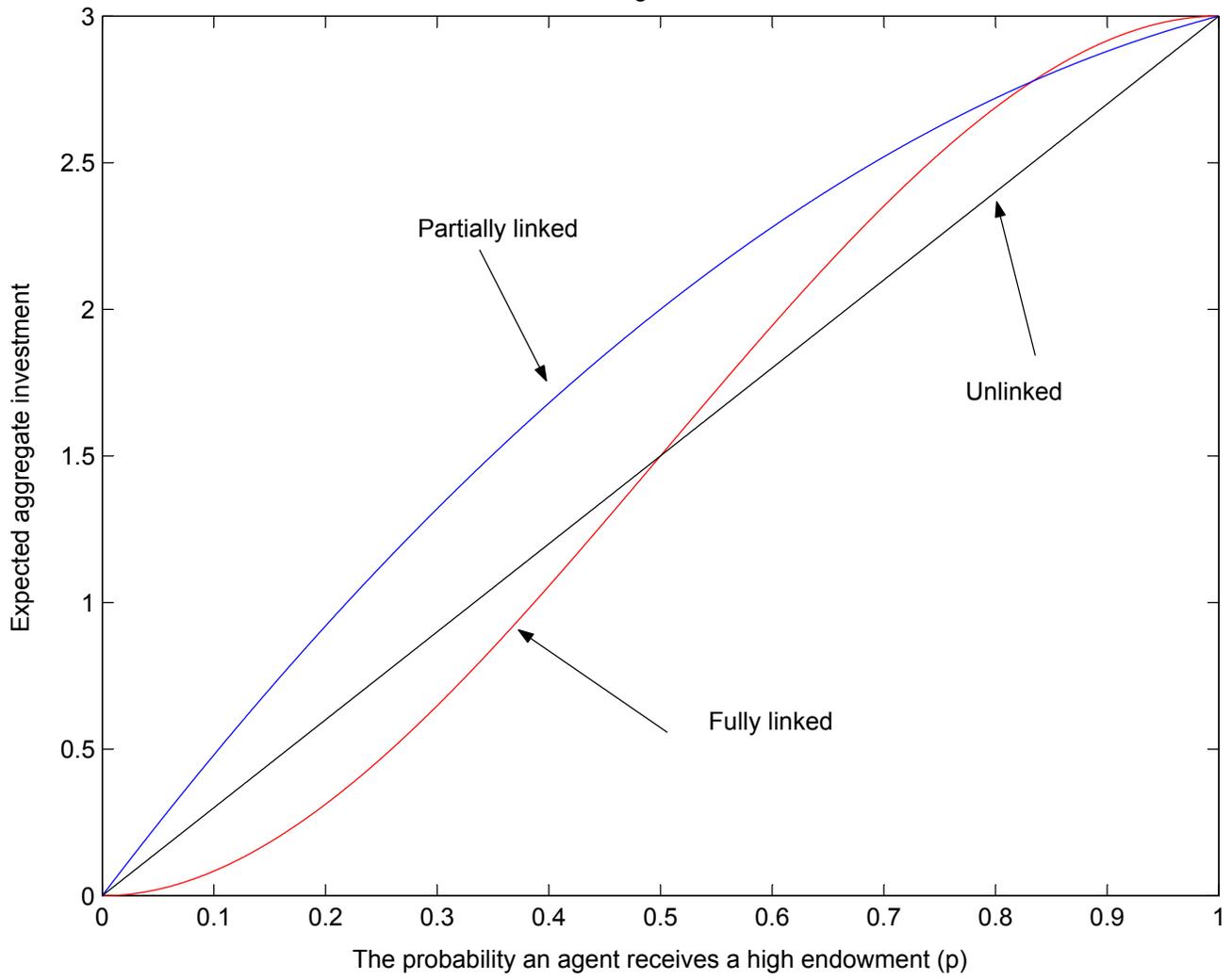


Figure 2

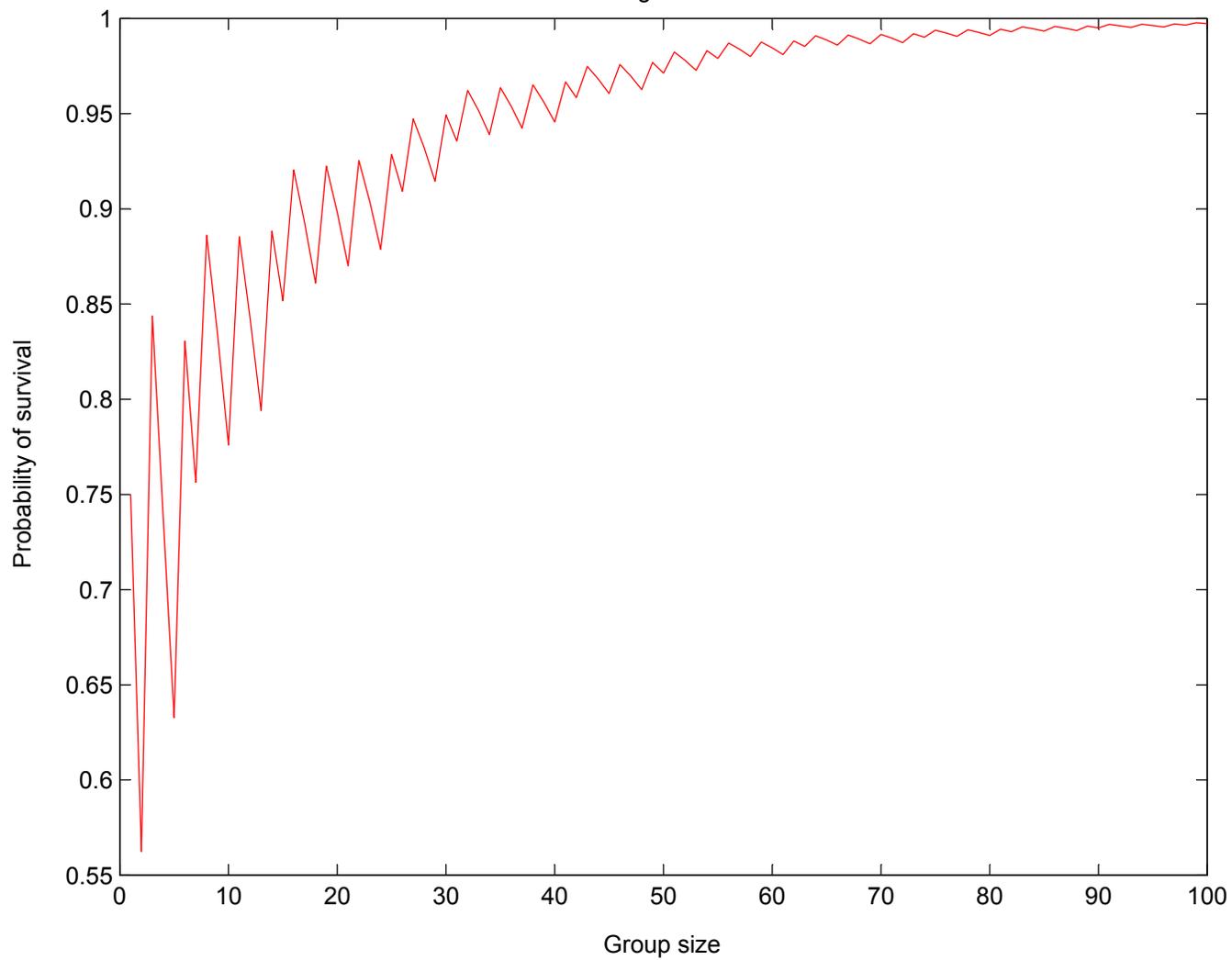


Figure 3

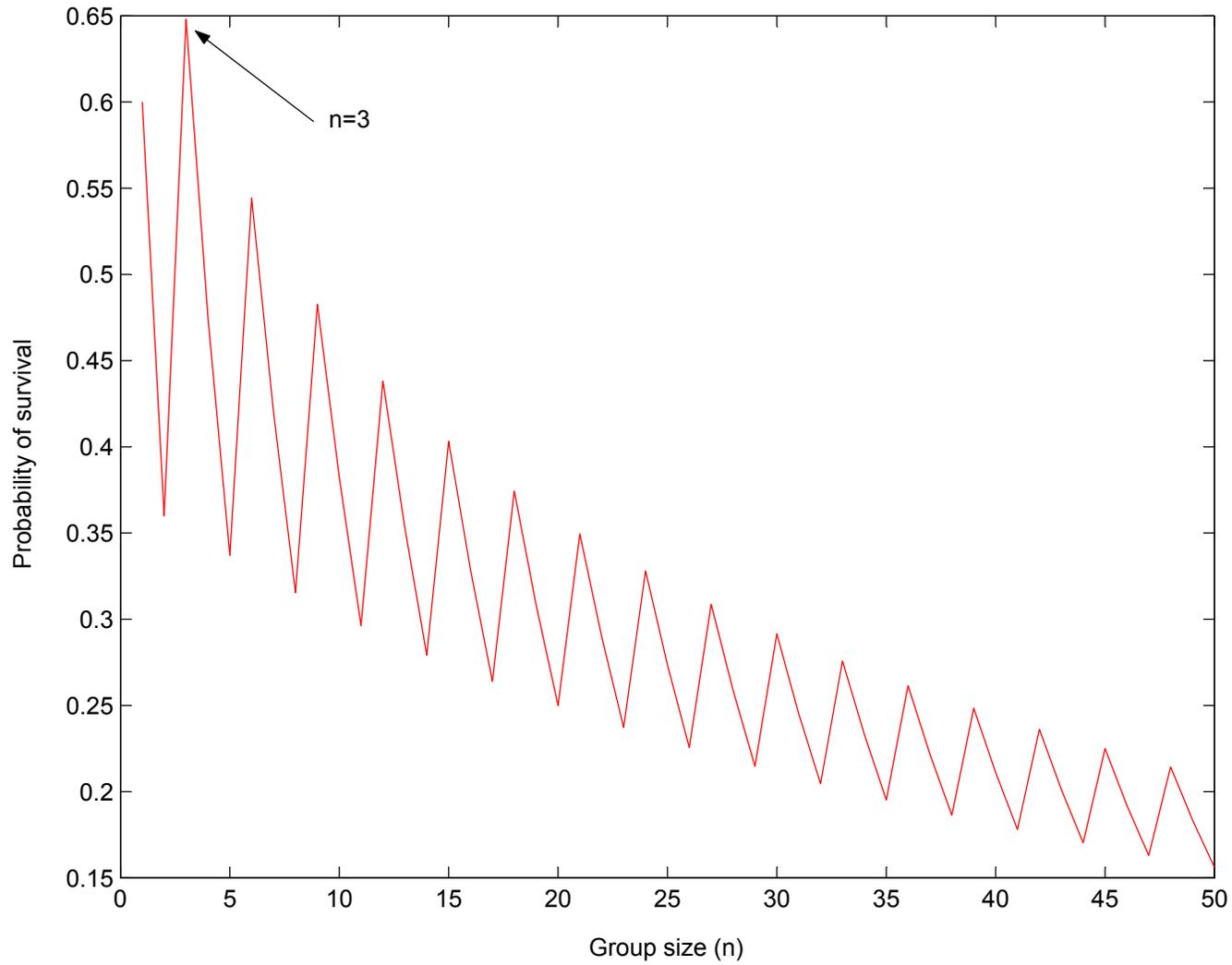


Figure 4

