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ON THE OPTIMALITY OF ELIMINATING SEASONALITY IN NOMINAL INTEREST RATES

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Abstract

Optimal monetary policy for an economy with seasonal fluctuations and a cash-in-advance requirement on the purchase of consumption goods is studied. The short delay in the availability of newly acquired funds for consumption purchases (the hallmark of cash-in-advance models) typically makes the seasonal steady state inefficient. Monetary policy can overcome this inefficiency induced by the payment-system friction by keeping nominal interest rates constant over the seasons. An analytical model is also presented to explore the effects of seasonal smoothing of nominal interest rates on the seasonal amplitude of other, closely related, variables.

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On the Optimality of Eliminating Seasonality
in Nominal Interest Rates

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1 The Seasonal Monetary Policy Puzzle

Seasonal fluctuations in economic activity result in seasonal variations in the demand for money. Generally speaking, most central banks follow a policy of accommodating seasonal movements in the demand for money. The growth rate of money supply is raised when the demand for money is seasonally high and lowered when that demand subsides. Since seasonally high money demand, if not accommodated, leads to an increase in nominal interest rates, such a policy is rightly seen as one that reduces seasonal fluctuations in nominal interest rates.[1]

Despite the prevalence of seasonal monetary policy, the literature on the appropriate stance of monetary policy toward seasonal fluctuations in money demand is surprisingly sparse. The only authors to discuss this issue in any depth are Mankiw and Miron (1991).[2] For the most part, Mankiw and Miron focus on the effects of seasonal monetary policy in a sticky-price IS-LM model, but they do touch upon the role of seasonal monetary policy when prices are assumed to be flexible.

Their brief discussion of effects of seasonal monetary policy for the flexible-price case is carried out in the context of a model in which money is superneutral. They argue that such a model implies that welfare is improved by a policy that keeps the nominal interest rate (rather than the money supply) constant over the seasons. The key idea underlying their argument is the one familiar from Friedman's discussion of the optimum quantity of money, namely, that lower nominal interest rates increase welfare by reducing the economy's need to economize on cash. By lowering nominal interest rates at a time when the demand for cash is high and raising them at a time when the demand for cash is low, a policy that smooths seasonal fluctuations in nominal interest rates will raise welfare.

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[1]The central bank's seasonal smoothing of nominal interest rates has been documented for the U.S. (Diller (1969), Shiller (1980), Barsky and Miron (1989), among others), for a group of OECD countries, and Japan (Beaulieu and Miron (1990)). Still, such policies are not universal. For instance, short-term interest rates in India display pronounced seasonality.

[2]In a different vein, researchers have explored the role of U.S. seasonal monetary policy in eliminating bank runs and financial panics in turn-of-the-century United States (see, among others, Miron (1986), Calomiris and Gorton (1990), and Champ, Smith, and Williamson (1996)).

However, a justification of seasonal monetary policy based on the logic of the optimum quantity of money is not entirely convincing because it carries with it the awkward implication that the optimal nominal interest rate is zero. From the perspective of this logic, the fact that central banks stand ready to eliminate seasonality in nominal interest rates but do not display any comparable readiness to reduce nominal interest rates on short-term risk-free assets to very low levels is a puzzle. Why is it sensible for central banks to eliminate the additional transaction costs induced by seasonality in nominal interest rates but not the transactions costs induced by positive (and for some countries, high) nominal interest rates?[3]

Mankiw and Miron suggest two possible resolutions to this puzzle. One resolution, which they do not pursue in detail, is along lines familiar from the optimal taxation literature. Since inflation generates seignorage revenue, a government that can levy only distortionary taxes might find it optimal to keep nominal interest rates positive. Thus, a central bank might well eliminate seasonality in interest rates to reduce transactions costs but, for revenue reasons, balk at reducing nominal interest rates to zero.

Since elimination of seasonality in nominal interest rates is unlikely to have important effects on government revenue, this suggestion appears plausible. But, to be persuasive, the suggestion should factor in the possible deadweight loss from distortionary taxation when the inflation tax rate (i.e., the nominal interest rate) is held constant in the face of seasonal fluctuations in fundamentals. The issue is this: Under what conditions do Ramsey-style optimal tax rules prescribe constant tax rates over time when there are fluctuations (seasonal or otherwise) in preferences and technology? Existing derivations of optimal tax rules for a monetary economy do not shed light on this issue because they assume either that all fundamentals are constant over time (see the discussion in Woodford (1990)) or that government expenditure is the only factor that varies over time (Chari, Christiano, and Kehoe (1993)).[4] Furthermore, it is not clear that existing derivations of optimal

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[3]Note, however, that some central banks do pay interest on bank reserves and so eliminate some of the transactions costs induced by positive nominal interest rates. Also, in recent years several central banks (e.g., Canadian and Mexican) have eliminated reserve requirements on commercial bank deposits. This move may have been motivated by a desire to reduce transactions costs.

[4]Mankiw and Miron cite Mankiw's (1987) article on the optimal collection of seignorage in support of their assertion. However, the focus of that paper did not require a derivation

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tax rules are appropriate for the problem at hand because it is unrealistic to assume that income tax rates can be varied over the seasons. Thus, the relevant optimal taxation problem is a "third-best" one where the government can change the inflation tax rate (i.e., the nominal interest rate) over the seasons but not the income tax rate. If it turns out that these optimal tax considerations dictate some seasonality in nominal interest rates, then the deadweight loss induced by a policy of seasonal smoothing of interest rates would have to be set against the benefit from reduced transactions costs. Since these (potential) costs and benefits are likely to be of a similar order of magnitude, the prospect of justifying seasonal monetary policy along optimal taxation lines is uncertain.

The second resolution suggested by Mankiw and Miron, and the one they stress in their paper, is based on the possibility that a central bank is more efficient in bringing about seasonal changes in the quantity of real money supply than the market. The idea is this: If a central bank did not increase the nominal money supply in the high demand season or lower it in the low demand season, the economy would attempt to "produce" the desired change in real balances through changes in the price level in the opposite direction. The price level would tend to fall in the high money demand season and tend to rise in the low money demand season. But if strategic considerations (or physical costs of changing prices) make firms reluctant to vary prices, the required seasonal adjustment in the real value of the money supply may not happen. While the idea is plausible, Mankiw and Miron do not provide a model that demonstrates this possibility.

In this paper, a different resolution is offered for the seasonal monetary policy puzzle. The basic point is this: If it were the case that some benefit of a seasonal monetary policy rose from a factor unconnected with lowering of transactions costs, then central banks' readiness to eliminate seasonal fluctuations wouldn't seem at odds with their reluctance to reduce transactions costs (by lowering nominal interest rates to very low levels). The first (of two) objective of this paper is to point out that one doesn't have to look far for a monetary model with such a property: a cash-in-advance monetary model of the type analyzed in Stockman (1981), and in more depth by Abel (1985), does the trick. In this type of a model, the monetary equilibrium with sea-

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of the welfare cost of inflation from primitive considerations. Thus, arguments presented there are suggestive but not definitive.

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sonal fluctuations in fundamentals (preferences and technology) is typically inefficient, but this inefficiency is not a consequence of positive nominal interest rates. It derives, instead, from an intertemporal distortion created by the interaction of seasonal fluctuations in fundamentals and the delay in the availability, for consumption purposes, of newly acquired funds. Monetary policy can overcome this inefficiency induced by the payment-system friction by keeping the nominal interest rate constant over the seasons. Once the nominal interest rate is constant, there is no further welfare gain to reducing it to zero.

The other objective of this paper is to present a parametric version of this model for which the seasonal steady state can be analytically worked out. The aim is to trace the full general equilibrium consequences of moving from a policy that keeps the money supply constant over the seasons to an optimal policy that keeps the nominal interest rates constant. In particular, the focus is on how the removal of seasonality in nominal interest rates affects the seasonality in other variables, such as the real interest rate and the price level.

2 Seasonality in a Cash-in-Advance Monetary Model

This section describes a cash-in-advance model of the type presented in Stockman (1981), augmented with seasonal fluctuations in fundamentals.

Preferences

The lifetime utility function of the representative agent is:

$$U(c_1, c_2, ...) = \sum_{t=0}^{\infty} \beta^t u(c_t, s_t) \tag{1}$$

where c_t is the single consumption good available in the economy and s_t is an indicator variable that varies with the seasons.

Technology

The technology for producing the single good is:

$$y_t = f(k_t, s_t) \tag{2}$$

where y_t is the output in period t, and k_t is the beginning of period capital stock. Technological opportunities are also affected by the season. Holding fixed s_t, f is assumed to be differentiable and strictly concave in k_t.

The evolution of s_t is periodic:

$$s_t = s_{t+J} \tag{3}$$

where $J \geq 1$ is the length of the seasonal cycle. J is 12 if a period is a month, J is 4 if a period is a quarter, etc.

The technology for accumulating capital over time is:

$$k_{t+1} = i_t + (1 - \delta)k_t \tag{4}$$

where i_t is gross investment in period t and δ is the rate of depreciation of capital.

Markets

As is typical in cash-in-advance models, there is a specific sequence of allowable trades within a period. The agent enters a period with a portfolio of currency, privately issued debt, and capital stock. In the first half of the period he participates in a centralized asset market where he can adjust his portfolio of financial assets at competitively determined prices. In the second half of the period he uses his stock of capital to produce output and participates in the goods market simultaneously as a buyer and a seller. His purchase of consumption goods is constrained by the real value of his currency holdings at the end of the first half of the period. His purchase of investment goods is not subject to this constraint. The currency accumulated from sale of his goods cannot be used for purchases until the next period. Therefore, he exits the period with a portfolio of currency, privately issued debt, and a new stock of capital.

Monetary Authority

It is assumed that the monetary authority maintains a constant stock of money supply. Later on, we allow it to alter the money stock through lump-sum taxes.

Individual Optimization

With only one agent, private indebtedness must be zero in equilibrium. To conserve on notation, the agent's optimization problem is stated without reference to the centralized asset market. Once the equilibrium path of consumption and price level are determined, the asset market can be put back into the analysis and the "no-trade" condition used to infer the equilibrium path of real and nominal interest rates.

Thus, the agent's optimization problem reads:

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t, s_t)$$

subject to:

$$c_t + M_{t+1}/P_t + k_{t+1} \leq f(k_t, s_t) + (1-\delta)k_t + M_t/P_t$$
$$c_t \leq M_t/P_t$$
$$0 \leq k_{t+1}$$
$$0 \leq M_{t+1}$$
$$0 \leq c_t$$

given $\{P_t\}, \{s_t\}, M_0$ and k_0 .

Anticipating the fact that equilibrium consumption, money holdings, and capital stock must be positive every period, the first order necessary conditions reduce to the following:

$$u_1(c_t, s_t) = \lambda_t + \mu_t \tag{5}$$
$$\lambda_t = \beta[P_t/P_{t+1}][\lambda_{t+1} + \mu_{t+1}] \tag{6}$$
$$\lambda_t = \beta[f_1(k_{t+1}, s_{t+1}) + (1-\delta)]\lambda_{t+1} \tag{7}$$

Market Balance

The sequences $\{c_t\}, \{k_{t+1}\}$, and $\{M_{t+1}\}$ must satisfy the following two market clearing conditions

$$M_{t+1} = M \qquad (8)$$
$$c_t + k_{t+1} = f(k_t, s_t) + (1 - \delta)k_t \qquad (9)$$

For future reference, note that the nominal interest rate in the (beginning-of-period) asset market is implicitly defined by the condition that the equilibrium marginal rate of substitution in consumption be equal to the real interest rate:

$$u_1(c_t, s_t) = \beta(1 + R_{t+1})(P_t/P_{t+1})u_1(c_{t+1}, s_{t+1}) \qquad (10)$$

Optimal Allocation

An optimal allocation is a pair of sequences, $\{c_t\}$ and $\{k_{t+1}\}$, that solve the following programming problem:

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t, s_t)$$

subject to:

$$c_t + k_{t+1} \leq f(k_t, s_t) + (1 - \delta)k_t$$
$$0 \leq k_{t+1}$$
$$0 \leq c_t$$

given $\{s_t\}$ and k_0.

Evidently, an optimal allocation takes only physical resource constraints into account. Its relevance to the monetary economy is that the monetary authority can implement this allocation (in the monetary economy) if it has access to non-distorting taxes. This claim is verified below.

The two necessary conditions for an allocation to be optimal are:

$$u_1(c_t, s_t) = \beta[f_1(k_{t+1}, s_{t+1}) + (1 - \delta)]u_1(c_{t+1}, s_{t+1}) \tag{11}$$

$$c_t + k_{t+1} = f(k_t, s_t) + (1 - \delta)k_t \tag{12}$$

In addition, if the following transversality condition is satisfied, the three conditions together are sufficient as well:

$$\lim_{t \to \infty} \beta^t u_1(c_{t+1}, s_{t+1})k_{t+1} = 0 \tag{13}$$

3   Steady States

We focus on equilibria that have a periodicity of J, i.e., equilibria in which the equilibrium path of any endogenous variable z_t satisfies $z_t = z_{t+J}$. To begin with, we study economies for which $J = 1$. In this case there is no seasonal variability in s at all, and the corresponding equilibrium is referred to as the *steady state*. For economies with $J > 1$, the periodic equilibrium is referred to as the *seasonal steady state*.

The Steady State

Since consumption is constant in a steady state, equation (5) implies that $\lambda_t + \mu_t$ is constant. Since the price level is constant, equation (6) implies that λ_t is constant. Hence μ_t is constant. Since consumption is positive, both λ_t and μ_t are positive.

Equation (7), along with the constancy of λ_t, implies that the constant level of capital satisfies the following condition:

$$f(k, s) = 1/\beta - 1 + \delta \tag{14}$$

Then equation (9) implies that the constant level of consumption satisfies

$$c = f(k, s) - \delta k \tag{15}$$

Since the multiplier μ_t is constant and positive each period, the cash-in-advance constraint binds in every period. Hence, the constant price level satisfies

$$P = M_0/c \tag{16}$$

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Efficiency of the Steady State

We now turn to the issue of whether the steady state is efficient. We ask, if starting with an initial capital stock of k (the capital stock in the steady state) there is an optimal allocation that coincides with the consumption and capital stock sequence in the seasonal steady state.

By substituting c and k in equations (11) and (12) we see that the allocation from the steady state satisfies the necessary and sufficient conditions for an optimal allocation. Thus the steady state is efficient. The fact that the steady state is efficient when the cash-in-advance constraint applies only to purchase of consumption goods was noted in Stockman (1981).

The Seasonal Steady State with Small Seasonal Fluctuations

We now consider the case where $J > 1$. The analysis of the seasonal steady state is more difficult because the cash-in-advance constraint need not bind every period. However, since the cash-in-advance constraint binds every period in the steady state, it will continue to do so in the seasonal steady state provided the seasonal fluctuations in preference and technology are small. This section considers only small seasonal fluctuations and assumes that the cash-in-advance constraint binds every period.

Denote the periodic values c, k, and P by c_j, k_j, and P_j, where $j = 1, 2, ...J$. Equations (6) and (7) now imply:

$$\left(\frac{P_j P_{j+2}}{P_{j+1}^2}\right)\left(\frac{u_1(c_{j+1}, s_{j+1})}{u_1(c_{j+2}, s_{j+2})}\right) = \beta\left(f_1(k_{j+1}, s_{j+1}) + (1 - \delta)\right) \text{for } j = 1, 2, ...J$$

(17)

where the indices $J + 1$ and $J + 2$ refer to the indices 1 and 2.

Since the cash-in-advance constraint is assumed to bind every period, equation (8) implies:

$$P_j = M_0/c_j \text{ for } j = 1, 2, ...J$$

(18)

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Equation (17) can then be written as:

$$\left(\frac{c_{j+1}^2}{c_j c_{j+2}}\right)\left(\frac{u_1(c_{j+1}, s_{j+1})}{u_1(c_{j+2}, s_{j+2})}\right) = \beta f_1(k_{j+1}, s_{j+1}) + (1-\delta) \text{ for } j = 1, 2, ..J \quad (19)$$

The goods market clearing condition is:

$$c_j + k_{j+1} = f(k_j, s_j) + (1-\delta)k_j \text{ for } j = 1, 2, ...J \quad (20)$$

In principle, equations (19) and (20) provide $2J$ equations to determine the $2J$ unknowns $\{c_j\}$ and $\{k_j\}$ and equation (18) can be used to determine J remaining unknowns, P_j.

Inefficiency of the Seasonal Steady State

We now turn to the issue of the efficiency of a seasonal steady state. Once again, we ask if starting in any season j, the optimal program with $k_0 = k_j$ coincides with the ensuing (periodic) consumption and capital stock sequence in the seasonal steady state.

For the seasonal steady state to be optimal, c_j and k_j must satisfy these conditions that follow from equations (11) and (12):

$$\left(\frac{u_1(c_j, s_j)}{u_1(c_{j+1}, s_{j+1})}\right) = \beta\left(f_1(k_{j+1}, s_{j+1}) + (1-\delta)\right) \text{for } j = 1, 2, ..J \quad (21)$$

$$c_j + k_{j+1} = f(k_j, s_j) + (1-\delta)k_j \text{ for } j = 1, 2, ...J \quad (22)$$

Obviously, the second set of conditions is satisfied by the seasonal steady state. For the first set of conditions to be satisfied, it must be the case that:

$$\left(\frac{c_{j+1}^2}{c_j c_{j+2}}\right)\left(\frac{u_1(c_{j+1}, s_{j+1})}{u_1(c_{j+2}, s_{j+2})}\right) = \left(\frac{u_1(c_j, s_j)}{u_1(c_{j+1}, s_{j+1})}\right) \quad (23)$$

In other words, it must be the case that:

$$\frac{(u_1(c_{j+1}, s_{j+1})c_{j+1})^2}{(u_1(c_j, s_j)c_j)(u_1(c_{j+2}, s_{j+2})c_{j+2})} = 1 \quad (24)$$

This condition will be satisfied if the s's and the c's are all constant, which agrees with the fact that the steady state is efficient and it will be satisfied if $u_1(c_j, s_j)c_j$ is a constant for all j, which happens if $u(c_j, s_j)$ is given by $A(s_j) + B \ln(c_j)$.[5] Therefore, in this environment, the steady state is always efficient, but the *seasonal* steady state typically is not.

The fact that the seasonal steady state typically does not coincide with the allocation from the programming problem clarifies a closely related result in the cash-in-advance literature. Abel (1985) has noted that in an equilibrium in which the cash-in-advance constraint on consumption binds every period, the transition path to the monetary steady state typically does not coincide with the transition path to steady state in the programming problem. The result derived above shows that the *seasonal steady state* of a monetary equilibrium in which the cash-in-advance constraint on consumption binds every period need not coincide with the *seasonal steady state* of the programming problem.

4 Optimal Seasonal Monetary Policy

This section investigates the monetary policy rule that implements the efficient allocation. The monetary authority alters the money supply through lump-sum taxes or transfers, denoted by H_t, that take place in the first half of a period. The budget and liquidity constraints in the optimization problem of the representative agent are replaced by

$$c_t + M_{t+1}/P_t + k_{t+1} \leq f(k_t, s_t) + (1 - \delta)k_t + M_t/P_t + H_t/P_t$$
$$c_t \leq M_t/P_t + H_t/P_t$$

It is easily verified that the first-order necessary conditions are again those given by equations (5) - (7). The only other change is that the money market clearing condition now becomes

$$M_{t+1} = M_t + H_t \tag{25}$$

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[5]Actually, since the above calculations are valid for small seasonal fluctuations, the requirement is that the utility function be locally logarithmic for values of c around the steady state value.

To fix ideas, suppose that the economy is in an inefficient seasonal steady state with a constant stock of money. Unbeknown to agents, the government appoints a "monetary commission" to recommend the best monetary policy starting period $j = 1$. Thus, members of the commission know that at the start of the new policy, the beginning-of-period capital stock will be k_1. What path of money supply should the commission recommend?

The commission would want to implement the optimal allocation starting from a capital stock of k_1. This allocation can be found by solving the programming problem described earlier for an initial capital stock of k_1. Let c_t^* and k_t^* denote the solution. Since this solution will not start out being periodic, but will become so only in the limit, the time subscript t rather than j is used.[6] Then, equation (11) implies:

$$u_1(c_t^*, s_t) = \beta \left(f_1(k_{t+1}^*, s_{t+1}) + (1 - \delta) \right) u_1(c_{t+1}^*, s_{t+1}) \tag{26}$$

Furthermore, if this allocation is to be a monetary equilibrium, then equations (5) and (7) imply:

$$\lambda_t + \mu_t = u_1(c_t^*, s_t)$$

and

$$\lambda_t = \beta \left(f_1(k_{t+1}^*, s_{t+1}) + (1 - \delta) \right) \lambda_{t+1} \tag{27}$$

Taken together, these three equations imply:

$$\frac{\lambda_t + \mu_t}{\lambda_t} = \frac{\lambda_{t+1} + \mu_{t+1}}{\lambda_{t+1}} \tag{28}$$

Next, note that the interest rate on nominal bonds in the monetary equilibrium supporting the optimal allocation must satisfy:

$$u_1(c_t^*, s_t) = \beta(1 + R_{t+1})P_t/P_{t+1} u_1(c_{t+1}^*, s_{t+1}) \tag{29}$$

Then, equations (5) and (6) imply:

$$u_1(c_t^*, s_t) - \mu_t = \beta(P_t/P_{t+1}) u_1(c_{t+1}^*, s_{t+1}) \tag{30}$$

which, using equation (29), gives:

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[6] For a proof of the statement that (for small seasonal fluctuations) the allocation converges to a seasonal steady state, see Chatterjee and Ravikumar (1992).

$$R_{t+1}/(1 + R_{t+1}) = \mu_t/u_1(c_t^*, s_t) \tag{31}$$

Rearranging the LHS and using equation (5) on the RHS yields:

$$1 - [(1 + R_{t+1})]^{-1} = \mu_t/(\mu_t + \lambda_t) \tag{32}$$

or, equivalently:

$$(1 + R_{t+1})^{-1} = \lambda_t/(\mu_t + \lambda_t) \tag{33}$$

From equation (28) it follows that R_t must be constant over time.

Thus, a necessary condition for implementing the optimal allocation is that the nominal interest be constant. The commission would recommend moving from a policy that keeps the money supply constant over the seasons to one that keeps the nominal interest rate constant.

The constancy of the nominal interest rate is the only requirement of optimality. It does not matter at what level the nominal interest rate is set. To see this, suppose that the commission recommends keeping the nominal interest rate constant at $R^* > 0$. Then equation (29) implies that the (gross) inflation rate along the optimal path must be:

$$\frac{P_{t+1}}{P_t} = (1 + \rho_{t+1}^*) = \frac{\beta(1 + R^*)u_1(c_{t+1}^*, s_{t+1})}{u_1(c_t^*, s_t)}$$

Next, note that equation (31) implies that the cash-in-advance constraint binds along the optimal path. Therefore, the path of money supply that supports the optimal allocation solves the difference equation:

$$M_{t+1}^* = (1 + \rho_{t+1}^*)(M_t^*/c_t^*)$$

where the money stock at the start of the period in which the optimal allocation is first implemented is given by history. Given this path of money supply, the equilibrium path of the price level can be inferred from the binding cash-in-advance constraint:

$$P_t^* = M_t^*/c_t^*$$

Clearly, the collection $\{c_t^*, k_t^*, M_t^*, P_t^*, R^*\}$ satisfies all conditions of a monetary equilibrium. Since the choice of nominal interest rate was arbitrary, it

follows that optimality does not require that the nominal interest rate be set at some particular level.

Why does the optimal monetary policy have this property? To see the reason, note that an individual who accumulates an additional unit of capital in period t without altering his consumption in period t reduces his holding of money balances at the beginning of next period by P_t. Against this loss in money balances, the individual has additional income of $P_{t+1}(f_1(k_{t+1}, s_{t+1}) + 1 - \delta)$. Because of the one-period delay in converting income into cash balances, this additional nominal income is worth $[P_{t+1}(f_1(k_{t+1}, s_{t+1}) + 1 - \delta)]/(1 + R_{t+2})$ units of money. At an optimum, these quantities must be exactly equal; otherwise, the individual would be carrying too little or too much capital. Thus, individual optimization requires:

$$(1 + R_{t+2}) = (f_1(k_{t+1}, s_{t+1}) + 1 - \delta)P_{t+1}/P_t \qquad (34)$$

Using equation (10), the above condition may be re-written as:

$$\frac{u_1(c_{t+1}, s_{t+1})}{\beta u_1(c_{t+2}, s_{t+2})} = (f_1(k_{t+1}, s_{t+1}) + 1 - \delta)(P_{t+1}/P_t)(P_{t+1}/P_{t+2}) \qquad (35)$$

In contrast, in the optimal allocation, the marginal rate of substitution between c_{t+1} and c_{t+2} is equated to the marginal product of physical capital in period $t + 2$:

$$u_1(c_{t+1}, s_{t+1}) = \beta u_1(c_{t+2}, s_{t+2})(f_1(k_{t+2}, s_{t+2}) + 1 - \delta) \qquad (36)$$

In general, the path of capital implied by equation (35) will not coincide with the path implied by equation (36). However, if the path implied by equation (35) also happened to satisfy the condition:

$$(f_1(k_{t+1}, s_{t+1}) + 1 - \delta)(P_{t+1}/P_t)(P_{t+1}/P_{t+2}) = (f_1(k_{t+2}, s_{t+2}) + 1 - \delta)$$

then the two paths would coincide. The above condition can be rearranged to yield:

$$(1 + f_1(k_{t+1}, s_{t+1}) - \delta)(P_{t+1}/P_t) = (1 + f_1(k_{t+2}, s_{t+2}) - \delta)(P_{t+2}/P_{t+1})$$

which states that the nominal interest rate in period $t + 1$ be equal the nominal interest rate in period $t + 2$.

Note that without the delay in converting income into cash balances, there is no reason for equation (34) to hold and the rest of the derivation does not follow. Thus, it is the delay in converting cash income into consumption that is fundamental to the result that nominal interest rates must be smoothed to guarantee efficiency.

The result that constant nominal interest rates guarantee efficiency for this class of cash-in-advance models was first noted by Fuerst (1994), although the idea was implicit in Abel's paper. In particular, Abel noted the difference between equations (35) and (36) but did not point out that the difference vanished if nominal interest rates were constant in the monetary economy. However, the significance of this result for the seasonal monetary policy puzzle has not been noted before.

For the above result to be a persuasive justification of seasonal smoothing of interest rates, one issue needs to be addressed. Observe that this result applies to *all* types of fluctuations in preferences and technology, not just seasonal fluctuations.[7] Thus, the implication of this class of models is that nominal interest rates should be kept constant from one period to the next. But what we see central banks actually doing is accommodating *seasonal* pressures on the nominal interest rates but not necessarily ones perceived to be non-seasonal. For instance, in the United States the Federal Reserve typically lowers the short-term interest rate during recessions and raises it in booms. While suggesting a reason for seasonal smoothing of nominal interest rates, the model generates a different puzzle: Why don't central banks keep interest rates constant at all times?

One answer to this question is to say that the current central bank practice of moving interest rates in response to the cyclical phase of business activity is misguided and that central banks ought to keep nominal interest rates constant. There is, however, another answer. This answer notes that the implication of this class of models for monetary policy is that central banks should accommodate pressures on nominal interest rates that originate in shocks to *fundamentals*, i.e., shocks to technology and preferences. Obviously, the model is silent on the correct response of monetary policy to non-fundamental shocks such as shocks to beliefs about future business conditions. Therefore, the appropriate conclusion to draw from this class

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[7]For instance, Carlstrom and Fuerst (1995) exploit this result in a environment where there are stochastic shocks to technology.

of models is that a central bank should accommodate nominal interest rate pressures that stem from shocks that the bank is confident are fundamental. *In practice, this means accommodating seasonal shocks and, perhaps, shocks that are clearly technological or preference-driven.* Thus, the resolution of the puzzle may simply be that central banks hesitate to view non-seasonal shocks as purely fundamental and therefore do not feel impelled to smooth nominal interest rates in response to them. In contrast, they are confident that seasonal pressures on interest rates have a fundamental source and therefore smooth in response to them. The result is a short-term interest rate path that displays (cyclical) variability but little or no seasonality.

5 The Effects of Optimal Seasonal Monetary Policy: An Analytical Example

The previous section established that the cash-in-advance monetary model provides a resolution to the seasonal monetary puzzle. This section explores the impact of seasonal monetary policy on the seasonal behavior of other variables. In particular, we are interested in seeing how the seasonal amplitude on closely related variables, such as the real interest rate and the price level, is affected by seasonal smoothing of nominal interest rates.

The model described in the previous section is too general to deliver sharp predictions about the impact of a seasonal smoothing of nominal interest rates on other variables. To make headway, the model needs to be specialized. One possibility is to specialize it by choosing numerical analogs for model primitives. An alternative is to alter it with a view to obtaining analytical results. Both strategies are useful, but in this paper, the latter alternative is explored.

To make the analysis interesting, the model is specialized to explore the impact of seasonal monetary policy on a developing economy. The predominance of agriculture in these economies makes seasonality an important source of fluctuations and the cash-in-advance constraint on the purchase of goods is a reasonable approximation of payment arrangements in many developing countries. In keeping with the "developing economy" perspective, no restriction on the size of seasonal fluctuations is imposed, since seasonal fluctuations in agriculture can be very large. In this sense the model presented

below is more general.

The lifetime utility function is assumed to be:

$$U(c_0, c_1, \ldots) = \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma} - 1}{1 - \sigma} \right) \quad \text{where } \sigma > 0 \text{ and } \beta < 1 \qquad (37)$$

For $\sigma = 1$, the momentary utility function is interpreted to be logarithmic.

The production function is assumed to be:

$$f(k_t, s_t) = y \cdot s_t \qquad (38)$$

where $s_t = 1$ for t even, and $s_t = 0$ for t odd. Thus the output stream is exogenous; it is positive and equal to y in even numbered periods and zero in odd numbered periods. In what follows, odd numbered periods will be referred to as winter and even numbered periods as summer.

Since the output stream is exogenously given, the capital stock is to be interpreted as inventories of goods. The inventory accumulation equation is:

$$k_{t+1} = \Delta k_t + i_t \text{ where } \Delta < \beta < 1$$

Here, Δ is the fraction of goods left over after spoilage (in terms of the notation of the previous section, $\Delta = (1 - \delta)$).

The first order conditions now read:

$$c_t^{-\sigma} = \lambda_t + \mu_t \qquad (39)$$

$$\lambda_t = \beta \left(\frac{P_t}{P_{t+1}} \right) (\lambda_{t+1} + \mu_{t+1}) \qquad (40)$$

$$\lambda_t \begin{cases} = \beta \Delta \lambda_{t+1} & \text{for } t = 0, 2, 4, \ldots \\ \geq \beta \Delta \lambda_{t+1} & \text{for } t = 1, 3, 5, \ldots \end{cases} \qquad (41)$$

Equation (41) reflects the fact that inventory holdings must be positive at the end of the summer (otherwise consumption in winter will be zero) but could be zero at the end of winter.

In addition to the individual optimization conditions, the sequences $\{c_t\}$, $\{k_{t+1}\}$, and $\{M_t\}$ must satisfy the following market balance conditions:

$$M_t = M \qquad (42)$$

$$c_t = y_t + \Delta k_t - k_{t+1} \qquad (43)$$

As mentioned above, it is no longer assumed that seasonal fluctuations are small (i.e., y could be considerably larger then 0). This means that the liquidity constraint cannot be assumed to always bind. However, because the periodicity of seasonal fluctuations is just 2, the seasonal steady state must be one of three types: (i) the liquidity constraint binds only in the summer (ii) it binds in both summer and winter, or (iii) it binds only in the winter. It turns out that all three cases are possible, so the seasonal steady state is solved for each case.

Seasonal Steady State When the Liquidity Constraint Binds Only in the Summer

Denote the summer (winter) values of all endogenous variables by the subscript s (w). Since the agent is unconstrained in winter, it follows that $\mu_w = 0$. Using equation (40), this implies that $\lambda_s = \beta\pi_s\lambda_w$, where $\pi_s = P_s/P_w$ is the anticipated real return on money between summer and winter. However, equation (41) implies that $\lambda_s = \beta\Delta\lambda_w$. Thus, $\pi_s = \Delta$. Therefore, in this case, the real return on money between summer and winter must equal the return on inventories.

Using equation (39) and (40) for winter, and the fact that $\mu_w = 0$, implies that $c_w^{-\sigma} = \beta(P_w/P_s)c_s^{-\sigma} = (\beta/\pi_s)c_s^{-\sigma}$. Therefore:

$$c_s/c_w = (\beta/\Delta)^{1/\sigma} \tag{44}$$

Since $\pi_w = (1/\pi_s) = (1/\Delta) > 1$, it is clear that money dominates inventories as a store of value between winter and summer and the agent will not carry any inventories into summer. Therefore, the market clearing condition (43) implies:

$$c_w = \Delta(y - c_s) \tag{45}$$

Equations (44) and (45) can be used to solve for unique values of c_s and c_w.

Finally, these values will constitute an equilibrium if the money market clearing condition is satisfied. Since the agent is liquidity constrained in the summer, it follows that $P_s c_s = M$. Hence $P_s = M/c_s$. Since $\pi_s = \Delta$, it follows that $P_w = (1/\Delta)M/c_s$. Now, money market equilibrium in winter requires that $P_w c_w \leq M$. Thus, it requires that $(1/\Delta)c_w/c_s \leq 1$. Using equation (44), this requirement can be stated as $(1/\Delta) \leq (\beta/\Delta)^{1/\sigma}$. Therefore, this particular seasonal steady state will arise only if the curvature of the utility function is sufficiently low, i.e., $\sigma \leq \ln(\Delta/\beta)/\ln\Delta$.

Seasonal Steady State When the Liquidity Constraint Binds in Summer and Winter

Since the agent is constrained in both seasons, $\mu_s > 0$ and $\mu_w > 0$. Using equations (39) and (40) for summer yields $\Delta\beta\lambda_w = \beta\pi_s(\lambda_w + \mu_w)$, i.e., $\lambda_w = (\pi_w/\Delta)(\lambda_w + \mu_w)$. Using equation (40) for winter yields $\lambda_w = (\beta/\pi_s)(\lambda_s + \mu_s)$. Using equation (39) for summer then gives:

$$\frac{c_w}{c_s} = \left(\frac{\pi_s^2}{\beta\Delta}\right)^{1/\sigma} \tag{46}$$

Note that $\mu_w > 0$ in conjunction with $\Delta\beta\lambda_w = \beta\pi_s(\lambda_w + \mu_w)$ implies that $\pi_s < \Delta$. Also, $\mu_s > 0$ implies that $c_s^{-\sigma} > \beta\pi_s c_w^{-\sigma}$. Using equation (46) this implies that $\pi_s > \Delta\beta^2$. Therefore, in this case, the return on money must lie in the open interval $(\Delta\beta^2, \Delta)$.

Once again, $\pi_w \ (= 1/\pi_s)$ exceeds 1. Thus, money dominates inventories as a store of value between winter and summer and the agent will not carry inventories into summer. Therefore, the goods market clearing is the same as in equation (45).

Since the agent is liquidity constrained in both periods, the money market clearing condition requires that $P_s c_s = M = P_w c_w$. This yields $c_w/c_s = P_s/P_w = \pi_s$. Therefore, equation (46) yields:

$$\pi_s = (\Delta\beta)^{1/(2-\sigma)} \tag{47}$$

Note that $(\Delta\beta)^{1/(2-\sigma)}$ is a decreasing function of σ over the range $(0, 2)$. It can be solved for two values σ_1 and σ_2 such that if σ lies in the interval (σ_1, σ_2) then π_s lies in the open interval $(\Delta\beta^2, \Delta)$. Clearly, $\sigma_1 = \ln(\Delta/\beta)/\ln\Delta$ and $\sigma_2 = \ln(\Delta\beta^3)/\ln(\Delta\beta^2)$ and $\sigma_1 < 1 < \sigma_2$.

Once the value of π_s has been obtained for a value of σ between σ_1 and σ_2, equations (45) and (46) can be solved for equilibrium values of c_s and c_w.

Seasonal Steady State When the Liquidity Constraint Binds Only in Winter

Since the agent is unconstrained in the summer, it follows that $\mu_s = 0$. Using equations (39) and (40) for summer gives $c_s^{-\sigma} = \beta\pi_s c_w^{-\sigma} = \lambda_s$. Using

equations (39) and (40) for winter yields $\lambda_w = (\beta/\pi_s)c_s^{-\sigma}$. Using equation (41) for summer yields $\lambda_s = \beta\Delta\lambda_w$. Therefore, in this case $\pi_s = \Delta\beta^2$ and

$$\frac{c_w}{c_s} = \left(\Delta\beta^3\right)^{1/\sigma} \tag{48}$$

Once again, note that $\pi_s < 1$ and hence $(1/\pi_s) > 1$. Therefore the agent will not carry inventories into summer and the market clearing condition in equation (45) applies. Equations (45) and (48) can be used to solve for the unique equilibrium values of c_s and c_w.

Since the agent is constrained in winter only, money market clearing requires that $P_w c_w = M$. Therefore, $P_s = \Delta\beta^2 M/c_w$. For money market balance in the summer, it is sufficient that $c_s P_s \leq M$. This requires that $c_s/c_w \leq (1/(\Delta\beta^2)$. Using equation (48), this requires $\sigma \geq \ln(\Delta\beta^3)/\ln(\Delta\beta^2) = \sigma_2$. Thus, this case will happen if the curvature of the utility function is sufficiently high.

5.1   Key Properties of the Seasonal Steady State

The three cases worked out above correspond to a three-way partition of the σ space, with the low σ case corresponding to $\sigma \leq \sigma_1$, the medium σ case corresponding to $\sigma \in (\sigma_1, \sigma_2)$, and the high σ case corresponding to $\sigma \geq \sigma_2$. The following table displays the key properties of the seasonal steady state with a constant money supply.

Table 1

Variable	Low σ	Medium σ	High σ
inventories	$\frac{y}{\left[1+\Delta(\beta/\Delta)^{1/\sigma}\right]}$	$\frac{y}{\left[1+\Delta(\Delta\beta/\pi_s^2)^{1/\sigma}\right]}$	$\frac{y}{\left[1+\Delta(1/\beta^3\Delta)^{1/\sigma}\right]}$
π_s	Δ	$(\Delta\beta)^{1/(2-\sigma)}$	$\Delta\beta^2$
c_s/c_w	$(\beta/\Delta)^{1/\sigma}$	$(\Delta\beta/\pi_s^2)^{1/\sigma}$	$(1/\Delta\beta^3)^{1/\sigma}$
$(1+R_s)$	$1/\beta^2$	$\pi_s/\Delta\beta^2$	1
$(1+R_w)$	1	Δ/π_s	$1/\beta^2$
$(1+r_s)$	Δ/β^2	$\pi_s^2/\Delta\beta^2$	$\Delta\beta^2$
$(1+r_w)$	$1/\Delta$	Δ/π_s^2	$1/\Delta\beta^4$

There are several common themes across the three type of equilibria. First, note that c_s/c_w is always greater than 1. Because it is costly to store

goods over time and because utility is discounted (i.e., β and Δ are both less than 1), the seasonality in output imparts a similar seasonality to consumption.

Second, the value of π_s is always less than or equal to Δ. Thus, the nominal price of goods is always higher in winter and the inflation rate between summer and winter is at least $1/\Delta$. The lower bound on inflation between summer and winter makes sense because if the inflation rate were to be any lower, money would dominate inventories as store of value and no inventories would be held.

Third, the real interest rate always reaches its seasonal low in the summer. This is a reflection of the fact that consumption is highest in the summer: the real interest rate must fall in order to sustain the relatively high level of consumption.

The seasonal behavior of nominal interest rates varies across the three cases. For $\sigma < 1$, the nominal interest rate peaks in the summer, but for $\sigma > 1$ it peaks in the winter. For $\sigma = 1$, there is no seasonality in interest rates at all. Thus, one interesting implication of this environment is that the seasonal pattern in real and nominal interest rates need not match.

Why is the curvature of the momentary utility function important for the seasonal pattern of nominal interest rates? Note that in this model, the nominal interest rate in the summer is given by:

$$\beta^{-1} \left(\frac{c_w}{c_s} \right)^{\sigma} \frac{P_w}{P_s} = 1 + R_s$$

Comparing it to the situation where there is no seasonality, so that $c_w = c_s$ and $R_s = R_w$, seasonality makes $c_w < c_s$ and, holding prices fixed, lowers the MRS between consumption in winter and summer. This MRS effect makes $R_s < R_w$. But the movement in consumption makes $P_w > P_s$, which tends to raise R_s above R_w. In the case where σ is low (and consumption is very substitutable over the seasons), the equilibrium change in the MRS is small and consequently the price level effect dominates and the summer nominal interest rate rises above the winter nominal interest rate. On the other hand, when σ is low (and consumption is not very substitutable over the seasons), the change in MRS is large and the MRS effect dominates the price level effect. In this case the winter nominal interest rate rises above the summer nominal interest rate. For a developing economy (with low consumption

levels), one would expect the intertemporal substitution in consumption to be low and therefore σ to be high. For such economies, the model suggests that nominal interest rates should peak in the agriculturally lean season.

5.2 Key Properties of the Seasonal Steady State When Nominal Interest Rates Are Smoothed

The seasonal steady state described in the previous section is generally inefficient for the same reasons the seasonal steady state was inefficient in the general model. However, because agents do not accumulate any capital between winter and summer, the reason is less transparent in this model. As before, the agent's choice of inventory holdings (capital) is governed by the equation $\pi_s \beta u'(c_{w)} = \Delta \beta^2 u'(c_s)/\pi_s$ where we have used the fact that both the price level and consumption have a periodicity of 2. Now note that in the efficient allocation $u'(c_s) = \beta \Delta u'(c_w)$. From these two equations it is easily verified that (i) equilibrium will be efficient only if the nominal interest rate is constant across the seasons and (ii) this will happen without any intervention only if $\sigma = 1$, i.e., the utility function is $\ln(c)$.

For this example, we can also determine the direction of inefficiency in consumption. Clearly, there will be too much consumption in the summer (i.e., too little inventory will be accumulated) if $\pi^2/\Delta\beta$ is less than $\Delta\beta$, i.e., if $(\pi/\Delta\beta)^2$ is less than 1. Similarly, there will be too little consumption in the summer (i.e., too much inventory will be accumulated) if $(\pi/\Delta\beta)^2$ is greater than 1. A glance at Table 1 shows that these two cases occur for σ greater than and less than 1, respectively.

The following table reports the allocation that comes about when the government switches (unexpectedly) into a policy of keeping interest rates constant over the seasons beginning in the summer. It is assumed that the nominal interest rate is chosen in such a way that there is no trend in the path of prices, i.e., the price level continues to remain periodic.

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<div align="center">Table 2</div>

Variables	Values When Seasonal Monetary Policy Is Optimal
Inventories	$\frac{y}{[1+\Delta(1/\beta\Delta)^{1/\sigma}]}$
c_s/c_w	$(1/\beta\Delta)^{1/\sigma}$
$(1+R_s)=(1+R_w)$	$1/\beta$
$(1+r_s)$	Δ
$(1+r_w)$	$1/\Delta\beta^2$
π_s	$\Delta\beta$

The issue of interest is how an optimal seasonal monetary policy might alter the seasonal amplitude in the real interest rate and the price level. In particular, is it possible that the seasonal amplitude in these variables is increased by a policy that smooths nominal interest rates? A comparison of Table 1 with Table 2 shows that it is indeed possible for the seasonal amplitude (measured as the ratio of the seasonal high to the seasonal low) in other variables to rise with the seasonal smoothing of nominal interest rates. The seasonal amplitude increases for the real interest rate when $\sigma < 1$. Recall that in the case where $\sigma < 1$, there is too much inventory accumulated in the seasonal steady state. Therefore, optimal monetary policy increases consumption in the summer and reduces it in winter. However, this has the effect of lowering the real interest rate even more in summer and raising it further in winter. Thus, the seasonal amplitude in the real interest rate is increased. For this case, optimal monetary policy also increases the seasonal amplitude in prices. As noted earlier, the price level is always highest in winter. A glance at Table 1 shows that when σ is in the low range the seasonal amplitude in the price level is $1/\Delta$, whereas under optimal seasonal monetary policy it is $1/\beta\Delta$. It can also be verified that when σ is in the medium range but is less than 1, the seasonal amplitude in the price level is less than $1/\beta\Delta$.

6 Conclusions

This paper studied the nature of optimal monetary policy for an economy with seasonal fluctuations and a cash-in-advance constraint on the purchase of the consumption goods. It showed that the short delay in the availability of newly acquired funds for consumption purchases (the hallmark of cash-

in-advance models) typically makes the seasonal steady state inefficient and that this inefficiency can be removed by a monetary policy that keeps nominal interest rates constant over the seasons. Thus, this class of monetary models can justify the seasonal smoothing of nominal interest rates carried out by many central banks. The paper also presented an analytical model in order to explore the general equilibrium effects of seasonal smoothing of nominal interest rates. An important finding was that the seasonal amplitude of the real interest rate and the price level could increase as a result of seasonal smoothing of nominal interest rates.

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