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Money and Finance with Costly Commitment

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ABSTRACT

We develop a variant of Townsend's turnpike model where the trading friction is related to a commitment problem rather than spatial separation alone. Specifically, expenditure on financial services is necessary to ensure commitment. When commitment is costless, the equilibrium allocation is equivalent to that from an Arrow sequential markets equilibrium. When commitment is prohibitively expensive, the allocation is similar to the Townsend equilibrium. We use numerical examples to study the consequences of costly commitment for co-existence of money and credit, asset pricing, welfare implications of currency and variations in its growth rate, and the relationships between income and financial development.

Keywords: Commitment Problems; Asset Pricing; Money and Interest; Financial Development

JEL Classification: E40, G12, D58

1. Introduction

Townsend (1980) provided an economic environment in which the spatial itinerary of agents and limited communication across locations preclude intertemporal contracts. He used the model to explore how currency can acquire value as a substitute for nonexistent private claims. Since his agents cannot commit themselves to intertemporal trade *at any cost*, Townsend formalized the importance of limited commitment for monetary theory. In contrast, obstacles to commitment are absent in Arrow's (1963) "sequential-markets" model where agents can *costlessly* commit themselves to one-period contracts. Since these contracts were shown to provide full intertemporal exchange possibilities, currency has no value. In this paper we attempt to synthesize aspects of these two approaches. We introduce a technology that allows households to commit to one-period contracts at a cost. If the technology is not accessed, the spatial arrangement implies default goes unpunished. The primary objective is to highlight some consequences of such a synthesis.

Specifically, in our environment households remain in the same location long enough to make one-period intertemporal trade physically possible but then move to randomly determined locations, never to meet a previous neighbor, or the subsequent neighbors of previous neighbors, or the subsequent neighbors of subsequent neighbors of previous neighbors, etc. Furthermore, goods and information can be transferred from one location to another only by households traveling between those locations. These features imply that a household's decision to honor or default on its current obligations is private and unverifiable information from the perspective of its future trading partners. Since relationships are transient and the household's trading history is private information with respect to any of its future trading partners, it is rational for households to default on their current obligations. Knowing this, no household lends. We bring trade in private claims back into the model by assuming that there is a commitment technology that households can access at each location that has as its output binding intertemporal contracts and as its input financial services. We assume that these financial (commitment) services take time to produce.

There are two immediate consequences of this structure. First, it is the case that intrinsically worthless but durable objects can be of value to households. The belief that an object will have value in the future and the fact that there is no commitment associated with its acquisition or sale (i.e., the object is not a liability of any

entity) are sufficient to give it value in all periods. Thus, unbacked fiat currency can coexist with interest-bearing private claims in our environment.¹ Second, because the financial services that deliver commitment absorb resources, the environment is consistent with factors of production earning positive returns in the financial services sector. For instance, it is consistent with findings such as those in Diaz-Gimenez *et al.* (1992, Table 3b) that the final product of financial services accounted for over 4 percent of U.S. GNP in 1992. In other words, the environment opens up the possibility of an integrated analysis of monetary and financial issues.

We investigate the implications of costly commitment for prices and resource allocations under three broad categories. Under the first category, we explore how costly commitment affects asset prices. In particular, we show how the commitment costs of trade in claims and the fact that participation in claims-trade is now a choice variable modify the standard representative-agent, consumption-based asset-pricing formula. In the numerical examples studied, these modifications lead to equilibrium fluctuations in asset prices.

Under the second category, we explore some classic issues in monetary theory. First, we explore the consequences of the introduction of currency and find that it may not lead to a Pareto improvement. In particular, the introduction of currency is likely to change interest rates on private claims and leads to a redistribution of wealth between borrowers and lenders. Second, we explore the implications of inflation and currency growth on real interest rates. Because currency and claims are substitutes in the portfolio of savers, it is reasonable to expect a positive association between the real return on currency and the real return on private claims. Yet, our environment implies a Sidrauski (1969)-type superneutrality with respect to long-term (but not necessarily short-term) interest rates. In addition, we provide an example of an economy with low commitment costs in which short-term real interest rates are also virtually unaffected by the inflation rate. Thus, our analysis indicates that commitment costs (or transactions costs more generally) can overturn some standard predictions of “store-of-value” monetary models. Third, these currency-growth experiments also synthesize two distinct benefits of lower inflation. Lower inflation implies that fewer households participate in the claims market and hence less time is

¹For an early argument that a crucial friction for valued fiat currency involves moral hazard and the costs of commitment see Bryant (1980). There are other studies in which currency and contracts coexist in environments similar to Townsend’s, but these do not incorporate costly commitment. See, among others, Blume and Corbae (1994), Manuelli and Sargent (1992), and Mitsui and Watanabe (1989).

spent producing contracts. This is our analog of the "shoe-leather" costs of inflation described by Friedman (1969). Additionally, lower inflation brings the marginal rate of substitution between consumption in adjacent periods closer together for buyers and sellers of currency, which is the benefit noted in Townsend (1980).

Finally, under the third category, we explore the relationship between financial development and income levels. Because trading in claims is costly, the extent to which households engage in it depends on their level of income. Thus, our framework delivers relationships between the extent of financial activity and income levels that reflect a reasonably sophisticated model of the demand for financial services. We show that the costly commitment framework is capable of generating patterns that resemble the cross-country regularities in income and financial development documented by Goldsmith (1969), Kuznets (1971), McKinnon (1973) and others.

2. Model Environment

2.1 Spatial Itinerary of Households and the Timing of Events

There are an infinite number of locations. Each location is inhabited by $N+1$ ($N \geq 2$ and even) types of infinitely lived households indexed by $\alpha \in \{0, 1/N, 2/N, \dots, (N-1)/N, 1\}$. There are a pair of members in each household (we call them a producer-accountant pair) and a large number, J , of households of each type at each location.

During a period, the timing of events in each location is as follows. In the beginning of the period, the producer is exogenously moved (according to a relocation scheme described below) to a new location, while the accountant remains at the old location. At the new location, the producer undertakes production and receives a currency transfer (possibly zero) or pays a currency tax. The producer also has the opportunity to enter into new one-period intertemporal contracts and/or trade currency with representatives of other households. These include the newly arrived producers and the accountants of households that arrived in the previous period. In the middle of the period, the producer can costlessly ship goods back to the household accountant in the old location (and only to that location). At the end of the period, the accountant moves with newly acquired goods to join the producer at the new location. Consumption takes place at the end of the period.

Households are relocated randomly. All J producers of a given type from each location are mixed

together with the groups of J producers of the same type from all other locations, formed into new groups of J producers, and then re-distributed back across locations.² This random relocation scheme maintains the same distribution of household types in each location and allows enough mixing of households so that the probability of a household meeting a previous neighbor, or the subsequent neighbors of previous neighbors, or the subsequent neighbors of subsequent neighbors of previous neighbors, and so on, is arbitrarily small.³ Thus households can rationally act as if they will never meet a previous neighbor or subsequent neighbors of previous neighbors, etc., in future locations. Also, it is assumed that the only way information can be transferred from one location to another is if it is carried by households traveling between those locations.

Although producers leave for a different location in the beginning of the period, the accountants can honor obligations entered into (in the previous period) by producers. They can do so because producers have the ability to ship goods from their new location to the old location before the end of the period. On the other hand, producers need not ship anything, and accountants could move to their producers' location without honoring their (joint) household obligations.

This spatial itinerary of households allows for the possibility of claims-trade, although such trade (if it occurs at all) must be short-lived since any group of households cannot remain in touch for more than two successive periods. If there is no possibility of obtaining commitment, the spatial itinerary precludes one-period claims trade as well; households with obligations will default, since they know with certainty that they will not encounter a household in any future location that might know of its default. Hence, the key impediment to one-period intertemporal trade is the inability to commit. Below, we introduce a commitment technology that permits trade in one-period claims.

²While the random-matching mechanism bears some resemblance to that in Kiyotaki and Wright (1989), since we have a large number of households at each location we study multilateral matching (where all households in a location can trade with one another) rather than a bilateral pairing.

³After t periods, the maximum number of households that could have received a signal about a household's default is given by $(2J-1)(2J)^t + (2J)^{N(t+1)}$. Since the number of locations is infinite, after any finite amount of time the probability of encountering a household that received the signal (the above expression divided by the population of agents) is zero. If the set of locations is dense in R , the probability is zero at $t = \infty$ as well.

2.2 Preferences and Production Technologies

All households have identical preferences over consumption of the nonstorable good:

$$U(c_0, c_1, c_2, \dots) = \begin{cases} \sum_{t=0}^{\infty} \beta^t \ln(c_t) & \text{if there is joint consumption,} \\ -\infty & \text{otherwise.} \end{cases} \quad (1)$$

All households have one unit of time available each period, which they can use for two productive purposes. First, they can use it to produce consumption goods. Letting n_t^C denote the time input by household-type α into the production of consumption goods, we assume

$$C = y_t(\alpha) n_t^C \quad (2)$$

where $y_t(\alpha)$ denotes the productivity of household-type α 's time in period t . Second, households can also use their time to produce a financial service that is an intermediate input into the technology that allows a group of households to commit to intertemporal trade at a given location. Letting n_t^F denote the time input into the production of financial services, we assume

$$F = \frac{y_t(\alpha)}{\tau} n_t^F \quad (3)$$

We assume that household-types differ in terms of their productivity. Specifically,

$$y_t(\alpha) = \begin{cases} \alpha y & t = 0, 2, 4, \dots \\ (1-\alpha)y & t = 1, 3, 5, \dots \end{cases} \quad (4)$$

Thus, for all households except $\alpha = 1/2$, productivity fluctuates over time and household-types with α greater (less) than $1/2$ are more productive in both activities in even (odd) periods.

Substituting (2) and (3) into the time constraint $n_t^C + n_t^F = 1$ yields the production possibility frontier between consumption goods and financial services for each household: $C + \tau F = y_t(\alpha)$. Because variations in productivity affect both activities uniformly, the slope of the frontier is independent of α and constant over time. Therefore, no household has a comparative advantage in the production of consumption goods or financial

services relative to any other household. However, except for household-type $\alpha = \frac{1}{2}$, the position of the frontier fluctuates periodically over time, with the fluctuations being more extreme for household-types with α farther away from $\frac{1}{2}$. Note that, given (4), the *aggregate* production possibility frontier, which is simply the sum of the individual frontiers, shows no fluctuations at all.

2.3 Markets and the Commitment Technology

We assume that each location has three markets: a one-period claims market for future consumption, a market for financial services, and a currency market. The price, in terms of the current consumption good, of a claim to one unit of future consumption is denoted by $1/(1+r_t)$. The price of a unit of financial service, again in terms of the consumption good, is τ (we elaborate on this below). Finally, the price of a unit of fiat currency, in terms of current consumption, is denoted by $1/P_t$. We discuss, in order, the salient features of each of these markets.

We assume that households wishing to trade one-period claims can form financial coalitions that ensure commitment. The enforcement mechanism underlying this commitment technology is that if a household deviates in any way from its obligations, the coalition has the power to prevent the accountant of the deviating household from joining the producer. Since this gives both members of the deviating household unboundedly small ($-\infty$) utility, households that participate in a financial coalition will always honor their obligations. It is also assumed that if the household does *not* deviate from its obligations, the coalition cannot prevent the accountant from joining the producer.

In these financial coalitions, borrowers receive current consumption goods from their coalition in return for a commitment to repay the loan at the market interest rate, and lenders give up current consumption goods in return for a commitment, by the coalition, for repayment at the market interest rate. If S is the amount of financial services needed to commit a coalition of $K \geq 2$ households to a set of binding intertemporal trades, then the commitment technology specifies:

$$1 = \frac{1}{K} S \tag{5}$$

where the left hand side is the final product of the commitment technology: one set of binding intertemporal trades among K households. Thus, the commitment technology specifies the financial services needed to form a coalition of K households with the power to enforce each participating household's rights and obligations. Equations (3) and (5) together imply that low τ environments require less time to obtain commitment than high τ environments.

Turning to the market for financial services, note that with the opportunity cost of the time spent in supplying financial services equal to τ for all α , all households are equally efficient suppliers of this service up to the limits imposed by the productivity of their time. Therefore, it is natural to assume that in each location a competitive market exists in which this service can be bought at a price (in terms of the current consumption good) of τ . Our framework is therefore consistent with a situation where a subset of households specializes in the production of all the financial services needed in a location. In this case, coalitions resemble financial institutions where, through the financial service provided by intermediaries (i.e., the households that have specialized in the production of financial commitment services), borrowers and lenders can come together. For instance, suppose there is a single coalition in each location made up of K households. If all household-types that supply financial services specialize in the production of financial services and A is the set of such household-types, then:

$$\sum_{\alpha \in A} J\left(\frac{y_i(\alpha)}{\tau}\right) = S = K \quad (6)$$

Here, the coalition of K households is the "financial institution" and the set A is the "intermediaries" who run it. In some key respects this resembles Townsend's (1978) treatment of financial intermediation in terms of costly coalition formation with full information.

We assume that each coalition member shares equally in the costs of commitment. Thus, each household wishing to trade claims would prefer (all else remaining the same) to participate in only one coalition. Since the per capita cost of commitment is independent of the number of households (the commitment technology is constant returns to scale), many different groupings of households can ensure this. For instance, we could imagine that all households wishing to trade claims participate in a single coalition; at the other extreme, we could imagine an

arrangement where participating households sort themselves out into a finite number ($\leq J$) of self-contained coalitions. In each case, a household trading in claims need only participate in one coalition. An implication is that the size of financial coalitions is indeterminate.

Turning finally to the market for fiat currency, note that since currency is not backed by a promise to redeem it in consumption goods, it is unnecessary to access the commitment technology when trading in currency. Provided households believe that currency bought in one location will be accepted in return for goods in any future location, the fact that it is costless to acquire gives it value. Costs of maintaining the currency system (e.g., enforcing counterfeit laws) are ignored.

2.4 The Government

We allow for the possibility of common government activity in each location. Let g_t be the amount of real goods absorbed by the government in each location in period t , and let $\phi_t(\alpha)$ be a lump-sum currency transfer (possibly negative) given to household-type α in each location in period t . The government may issue claims, b_t^g , that are redeemable only in the location of issue. To simplify the analysis we assume that all households that deal with the government form one coalition, which implies that the government pays the commitment cost only once. Finally, the government may sell or buy currency in the amount $(M_t - M_{t-1})$ in each location at the price $1/P_t$. We assume that prior to period $t=0$ there is no currency or outstanding government claims in any location, i.e. $b_{-1}^g = M_{-1} = 0$. Letting $\chi(\cdot)$ be an indicator function that is 1 if its argument is non-zero and 0 otherwise, the government must satisfy the following per-period budget constraint:

$$g_t + \frac{b_t^g}{1+r_t} + \sum_{\alpha=0}^1 \phi_t(\alpha) + \tau \chi(b_t^g) = b_{t-1}^g + \frac{M_t - M_{t-1}}{P_t} \quad (7)$$

given $M_{-1} = b_{-1}^g = 0$, and $g_t \geq 0 \quad \forall t \geq 0$

2.5 Optimization

Household-type α chooses a pair of claims and currency sequences $\{b_t(\alpha)\}_{t=0}^{\infty}$, $\{m_t(\alpha)\}_{t=0}^{\infty}$ to solve:

$$\begin{aligned}
& \max \sum_{t=0}^{\infty} \beta^t \ln(c_t) \quad s.t \\
& c_t + \frac{b_t}{1+r_t} + m_t(1+\pi_t) + \tau\chi(b_t) = y_t(\alpha) + b_{t-1} + m_{t-1} + \phi_t(\alpha), \quad \forall t \geq 0, \\
& c_t \geq 0, \quad m_t \geq 0, \quad b_t + m_t \geq - \sum_{j=1}^{\infty} \left(\frac{y_j(\alpha) - \tau}{\prod_{i=0}^{j-1} (1+r_i)} \right), \quad b_{-1}(\alpha) = 0, \quad m_{-1}(\alpha) = 0.
\end{aligned} \tag{8}$$

The indicator function $\chi(\cdot)$ ensures that the household pays the fixed commitment cost τ only if its end-of-period holdings of claims (b_t) is non-zero; we call the sequence $\{\chi(b_t)\}_{t=0}^{\infty}$ the household's (claims-market) *participation profile*. Note that when $\tau/y = 0$, the sequence of budget constraints faced by the household reduces to the standard intertemporal budget constraint. When τ/y is prohibitively high, and as a consequence b_t is always zero, the sequence of household budget constraints reduces to those in Townsend (1980).

While the fixed cost makes a full characterization of optimal behavior difficult, two features of optimality can be discerned. First, if $\chi_t = 0$, then $c_{t+1}/\beta c_t \geq (1+\pi_t)^{-1}$; otherwise $c_{t+1}/\beta c_t = (1+r_t)$. Second, the optimal participation profile is homogeneous of degree 0 in τ , y and ϕ , i.e., what matters for claims-market participation are the ratios τ/y and ϕ/y .

Since the set of participation profiles is the set of all infinite sequences that can be formed with elements in $\{0,1\}$, it is equivalent to the set $[0,1]$. Specifically, let $i = \sum_{t=0}^{\infty} (\chi_t/2^{t+1})$ where $\chi_t \in \{0,1\}$ map each distinct infinite sequence of zeros and ones into a distinct real number in $[0,1]$. For instance, participation in every period corresponds to $i=1$, no participation at all corresponds to $i=0$, participation every even period corresponds to $i=2/3$, and participation every odd period corresponds to $i = 1/3$. Thus, we can index participation profiles by $i \in I = [0,1]$. Let $V(\alpha, i; \{r_t\}, \{\pi_t\})$ denote the *maximum* lifetime utility from $t=0$ of household-type α conditional on the participation profile i , the interest rate sequence $\{r_t\}$, and the inflation sequence $\{\pi_t\}$. Let $\{b_t(\alpha, i; \{r_t\}, \{\pi_t\})\}_{t=0}^{\infty}$ and $\{m_t(\alpha, i; \{r_t\}, \{\pi_t\})\}_{t=0}^{\infty}$ denote the sequence of bond and money holdings that achieves $V(\alpha, i; \{r_t\}, \{\pi_t\})$. Thus, optimization involves choosing over $i \in I$ to maximize $V(\alpha, i; \{r_t\}, \{\pi_t\})$.

2.6 Equilibrium

Because of the non-convexity in households' budget sets, equilibrium may require that some household-

types be distributed over distinct, but equally desirable, participation profiles. To take this into account, we develop notation for the optimal choice of a participation profile and the distribution of a given household-type across equally desirable participation profiles.

Definition 1. Given interest rate $\{r_t\}$ and inflation rate $\{\pi_t\}$ sequences, let $A^*(\alpha; \{r_t\}, \{\pi_t\}) \subset I$ be the finite set of undominated participation profiles for household α . That is, $V(\alpha, i; \{r_t\}, \{\pi_t\}) \geq V(\alpha, j; \{r_t\}, \{\pi_t\}) \forall j \in I, \forall i \in A^*(\alpha; \{r_t\}, \{\pi_t\})$. Further, let $\lambda(\alpha, i; \{r_t\}, \{\pi_t\})$ be the fraction of household-type α that follow the undominated participation profile $i \in A^*(\alpha; \{r_t\}, \{\pi_t\})$.

Building on the individual optimization implicit in the definition of λ , an equilibrium is then defined as follows:

Definition 2. Given initial distributions of claim holdings $\{b_{t-1}(\alpha) = 0, \forall \alpha\}$ and money holdings $\{m_{t-1}(\alpha) = 0, \forall \alpha\}$, an *equilibrium* is a set of sequences $\{r_t\}_{t=0}^{\infty}$, $\{\pi_t\}_{t=0}^{\infty}$, $\{g_t\}_{t=0}^{\infty}$, $\{b_t^e\}_{t=0}^{\infty}$, $\{\phi_t\}_{t=0}^{\infty}$, $\{M_t\}_{t=0}^{\infty}$, an initial price level P_0 , and sets $\{\{\phi_t(\alpha)\}_{t=0}^{\infty}, \forall \alpha\}$ and $\{\lambda(\alpha, i; \{r_t\}, \{\pi_t\}), i \in A^*(\alpha; \{r_t\}, \{\pi_t\}), \forall \alpha\}$ such that $\forall t \geq 0$:

- i) $\sum_{\alpha} \{\sum_{i \in A^*(\alpha; \{r_t\}, \{\pi_t\})} \lambda(\alpha, i; \{r_t\}, \{\pi_t\}) b_t(\alpha, i; \{r_t\}, \{\pi_t\})\} + b_t^e = 0$
- ii) $\sum_{\alpha} \{\sum_{i \in A^*(\alpha; \{r_t\}, \{\pi_t\})} \lambda(\alpha, i; \{r_t\}, \{\pi_t\}) (1 + \pi_t) m_t(\alpha, i; \{r_t\}, \{\pi_t\})\} = M_t / (P_0 \prod_{s=0}^{t-1} (1 + \pi_s))$
- iii) $\sum_{\alpha} \phi_t(\alpha) = \phi_t$
- iv) $g_t + b_t^e / (1 + r_t) + \phi_t + \tau \chi(b_t^e) = b_{t-1}^e + (M_t - M_{t-1}) / (P_0 \prod_{s=0}^{t-1} (1 + \pi_s))$

2.7 Periodic Equilibria

For stationary government policies (for instance, one in which the government increases the money supply only at a constant rate), the periodicity of household endowments suggests that the equilibria of the model will eventually display periodic asset returns and periodic asset holdings for all household-types. The qualification "eventually" is important because it is possible that the periodic phase is preceded by a (potentially lengthy) transition phase in which neither asset holdings nor asset returns are periodic. For reasons of tractability, we

focus on equilibria in which asset-returns are periodic from the start and in which fiscal policy offsets any initial non-periodic behavior on the part of households (so that equilibrium asset prices can, in fact, be periodic from the start).

To define such an equilibrium formally, let $\omega(t)$ be a set valued function that assigns e to all even t and o to all odd t . Also, let x_ω denote x as a function of ω . Then:

Definition 3. An equilibrium is *periodic* if

$$i) \exists r_\omega, \pi_\omega, \text{ and } k \text{ such that } r_t = r_{\omega(t)}, \pi_t = \pi_{\omega(t)}, \text{ and } \phi_t(\alpha) = ky_t(\alpha) \forall t \geq 0$$

$$ii) \exists 0 \leq T < \infty, b_\omega(\alpha, i; r_\omega, \pi_\omega), m_\omega(\alpha, i; r_\omega, \pi_\omega), \text{ and } \mu \text{ such that } \forall t \geq T \text{ and } \forall(\alpha, i) \text{ such that } \lambda(\alpha, i, r_t, \pi_t) > 0,$$

$$(a) b_t(\alpha, i; \{r_t\}, \{\pi_t\}) = b_\omega(\alpha, i; r_\omega, \pi_\omega)$$

$$(b) m_t(\alpha, i; \{r_t\}, \{\pi_t\}) = m_\omega(\alpha, i; r_\omega, \pi_\omega)$$

$$(c) b_t^g = 0 = g_{t+1}$$

$$(d) (M_t - M_{t-1}) / M_{t-1} = \mu$$

Notice that the definition requires periodicity in asset holdings only beyond some time period T , although asset returns are required to be periodic from period $t=0$. The definition also requires that the government become inactive from T on except for expanding or contracting the money supply (through $\phi_t(\alpha)$) at a constant rate. Finally, the definition also incorporates a simple currency transfer scheme: the lump-sum currency transfer is proportional to the household's current period productivity, i.e., $\phi_t(\alpha) = ky_t(\alpha)$. Hence, the total currency transfer ϕ_t is a constant and equal to $(k \times y \times (N+1))/2$.

The strategy of using government policy to fix rates of return for analytical and computational tractability is not uncommon (see Diaz-Gimenez *et al.* (1992) and Ireland (1994)). However, an important aspect of our work is that the *level* at which interest and inflation rates are fixed in the non-periodic phase is not known in advance. In particular, these rates of return are determined by equilibrium requirements in the periodic phase. Thus, we let fundamentals like endowments, technology, preferences, and the currency growth rate determine asset prices in the long run (unlike Diaz-Gimenez *et al.* and Ireland, who set asset prices exogenously) but follow

Diaz-Gimenez *et al.* and Ireland in using government policy to ensure market balance along the transition path to the long-run equilibrium.

Some general properties of a periodic equilibrium should be noted. First, $(1+r_t)$ must exceed $1/(1+\pi_t)$ for all $t \geq 0$, i.e., the nominal interest rate must be positive. A negative nominal interest rate in any period will induce households to borrow an unlimited amount of goods through coalitions and convert them into currency. Since the size of the trade can be made arbitrarily large, fixed commitment costs do not invalidate standard arbitrage arguments. Second, the periodicity in individual real currency holdings and constant currency growth implies that π_{t_0} must satisfy $(1+\pi_{t_0})(1+\pi_{t_0})=(1+\mu)^2$ for $t \geq T$. Third, periodicity of bond holdings for $t \geq T$ implies that only four types of participation profiles are possible in this phase: a household-type could be participating in the claims market every period, only in even periods, only in odd periods, or not participating at all. Finally, periodicity in claim holdings implies that $(1+r_{e,t})(1+r_{o,t})$ must be in a neighborhood (whose size depends positively on τ/y) of $1/\beta^2$. To see this, note that if some households participated continuously in the claims market then $(1+r_{e,t})(1+r_{o,t}) \neq 1/\beta^2$ would imply continual accumulation or decumulation of claims for them and therefore a violation of periodicity. If none of the households participate continuously, $(1+r_{e,t})(1+r_{o,t})$ could be different from $1/\beta^2$. But if this difference is too large (relative to τ/y), it will invite continuous participation and once again a violation of periodicity. Since $(1+r) = \sqrt{(1+r_{e,t})(1+r_{o,t})}$ is the long-term interest rate in our environment, this final point can be summarized as the property that the long-term interest rate is always in a neighborhood of $1/\beta$, with the size of the neighborhood shrinking to zero as τ/y shrinks to zero.⁴

3. Asset Pricing

We explore the influence of commitment costs on the pricing of claims and currency by computing the periodic equilibrium of economies with successively higher values of τ/y . Each economy has $N = 100$ and $\beta =$

⁴ The long-term interest rate faced by a household participating continuously in the claims market for M periods is defined to be the interest rate r such that $(1+r)^M = [(1+r_{e,t})(1+r_{o,t})]^{M/2}$ if M is even or $[(1+r_{e,t})(1+r_{o,t})]^{(M-1)/2}(1+r_{o,t})$ if M is odd. In either case $(1+r)$ tends to $[(1+r_{e,t})(1+r_{o,t})]^{1/2}$ as M tends to infinity.

1/1.1. The key features of the periodic phase of each equilibrium are reported in Table 1.⁵

The first row reports features of the *pure-credit* equilibrium in which $\tau/y = 0$. With costless access to one-period loans and logarithmic preferences, individual excess demands can be aggregated and equilibrium interest rates inferred from the first-order conditions of intertemporal optimality of the representative household. Since the representative household must consume the constant per-capita endowment, the equilibrium interest rate in this costless commitment economy is simply the subjective rate of time preference ($= 0.10$). Furthermore, all households' consumption profiles are perfectly smooth, and, except for household-type $\alpha = 0.5$ (which is the representative household when $\tau/y=0$), all households trade claims in even periods only. It is easy to see that this allocation is identical to that from an Arrow sequential markets world in which all households resided in the same location. There is, of course, no scope for currency in this equilibrium.

In stark contrast to the pure credit equilibrium, the last row of Table 1 reports features of the equilibrium when $\tau/y = 1$. In this case, available output net of commitment costs in any financial coalition with different household-types is negative and such coalitions are not feasible. Hence, while the inability to commit rules out trade in the claims market, households can still save in the form of currency. We assume that the government sells a pre-determined amount of currency in each location at $t=0$, spends the proceeds on the local good, and then becomes forever inactive. The equilibrium inflation rate in this *pure-currency* equilibrium is zero, and the behavior of households resembles, in part, that in the Townsend (1980) turnpike model. In particular, household-types with $\alpha \geq 0.52$ buy currency in even periods and households with $\alpha \leq 0.48$ buy currency in odd periods. In contrast to Townsend, our pure currency equilibrium also has household types, $0.49 < \alpha < 0.51$, which find it *optimal* to simply consume their endowments. We term these autarkic households.

Between these two extremes lie cases in which the commitment cost is positive but not prohibitive so that currency and credit can coexist. The middle two rows in Table 1 report the details of the equilibrium with $\tau/y = 0.002$ and 0.02 . The existence of the fixed cost means that only households with sufficiently variable productivity participate in the claims market. Thus, the set of claims-market participants is larger than that in the pure-

⁵For this and all other tables, the details of the non-periodic phase (i.e., if $T > 0$) are reported in the appendix to Chatterjee and Corbae (1994).

currency equilibrium but smaller than that in the pure-credit equilibrium. For instance, in the $\tau/y = 0.02$ equilibrium, households with $\alpha \leq 0.28$ participate as borrowers and households with $\alpha \geq 0.72$, as well as 91.5 percent of household-type $\alpha = 0.71$ (that is, $\lambda(0.71, 2/3;) = 0.915$), participate as lenders. Among the other household-types, those with moderate fluctuations in their endowments choose to smooth consumption with currency. In particular household-types with $0.52 \leq \alpha \leq 0.70$ as well as 8.5 percent of $\alpha = 0.71$ hold currency in even periods, while household-types $0.29 \leq \alpha \leq 0.47$ hold currency in the odd periods. All other households consume their endowments.

To understand the determination of interest rates in these equilibria observe that each even-period claims-market participant has the same marginal rate of substitution between consumption at different dates as other even-period (claims-market) participants. Further, the total goods consumption of even-period participants is equal to their total group income net of expenditure on financial services. Given logarithmic preferences, aggregation theorems can be applied to this group. Thus, the even-period interest rate can be inferred from the first-order condition of the average even-period participant. More formally, define

$$\bar{\alpha}(2/3; r_\omega, \pi_\omega) \equiv \frac{\sum_\alpha \lambda(\alpha, 2/3; r_\omega, \pi_\omega) \alpha}{\sum_\alpha \lambda(\alpha, 2/3; r_\omega, \pi_\omega)} \quad (9)$$

so that $\bar{\alpha}(2/3; r_\omega)y(1+k)$ and $(1-\bar{\alpha}(2/3; r_\omega))y(1+k)$ is the average even- and odd-period income, inclusive of currency transfers, of even-period participants. Then we can represent the even period interest rate as:

$$(1+r_e) = \frac{1 - \bar{\alpha}(2/3; r_\omega, \pi_\omega)}{\beta \left(\bar{\alpha}(2/3; r_\omega, \pi_\omega) - \frac{\tau}{y(1+k)} \right)} \quad (10)$$

The asset-pricing formula (10) indicates that the usual representative-household, consumption-based asset-pricing formula needs to be modified in two ways when there are commitment costs. First, a distinction needs to be made between consumption expenditures that provide utility directly (e.g., food) and expenditures that provide utility indirectly by altering the scope of household budget sets (e.g., financial services). Only the former belong in consumption-based asset-pricing equations. Thus, commitment costs have to be subtracted from the

consumption expenditure of participating households (the term $-\tau/y(1+k)$). Second, it is the average consumption of *participants* (the term $\bar{\alpha}(2/3;r_\omega)y$) rather than that of the *population* that is important for asset pricing. Since commitment costs can skew the distribution of participants, the average consumption of participants may not be the same as per capita consumption (i.e., $\bar{\alpha}(2/3;r_\omega)y(1+k)$ may be different from $1/2y(1+k)$).

To see the effects of these two modifications on equilibrium interest rates observe that if participation in the even-period claims market is symmetric and $\mu=0$, then $\bar{\alpha}(2/3;r_\omega,\pi_\omega)=1/2$ and $r_e = \{1/[\beta(1-2\tau/y(1+k))]\}-1$. This is the case in the $\tau/y = 0.002$ economy for which $r_e = 0.1044$. Thus, the only reason r_e exceeds the interest rate implied by the usual asset-pricing formula is that expenditure on financial services have to be subtracted. For the $\tau/y = 0.02$ economy, symmetric participation would imply $r_e = 0.1458$, but there is an excess supply of loans at this interest rate. Thus, the equilibrium interest rate is lower ($r_e = 0.1361$). The difference between 0.1458 and 0.1361 can be traced to the asymmetric participation of households in the even-period claims-market; almost all (91.5 percent) of household-type $\alpha=0.71$ participate as lenders, while their "counterparts" (household-type $\alpha = 0.29$) hold currency. It is of some interest to note that the discrepancy between the even-period interest rate predicted by the representative-agent, consumption-based asset-pricing formula and that noted in the equilibria in Table 1 increases with the size of τ/y .

A striking feature of the two costly commitment equilibria is that $r_e \neq r_o$; interest rates fluctuate over time. Recall that in a periodic equilibrium without continuously participating households, $(1+r_e)(1+r_o)$ is constrained to be in a neighborhood (whose size depends positively on the magnitude of τ/y) of $1/\beta^2$. Since commitment costs make r_e larger than $(1/\beta)-1$ (for the reasons noted above), they tend also to make r_o smaller than $(1/\beta)-1$.⁶ If some households were to participate in the claims market in all periods, the same logic would apply, with the difference that $(1+r_e)(1+r_o)$ must now exactly equal $1/\beta^2$. Thus, the fact that households do not participate in the claims market in all periods is not the key to fluctuations in interest rates in these equilibria; rather the requirement that the long-term interest rate be close to $1/\beta$ is.

The possibility that the pricing of private claims may follow a modified consumption-based asset-pricing

⁶If there had been participation in the odd period claims market, an asset-pricing formula analogous to (10) would apply to r_o (see Chatterjee and Corbae (1994) for details).

formula in a monetary economy has also been noted by Lucas (1984) and Townsend (1987). However, their modifications arise from the requirement that purchases of certain goods satisfy a cash-in-advance constraint. Since such constraints are not a part of our environment, there are no analogs of these modifications in our pricing formula.

Asymmetric participation in the even-period claims market spills over to the currency market. Because participation in the currency market is then asymmetric, the equilibrium inflation rate fluctuates as well. Notice in the example that the asymmetry implies an excess supply of money in even periods, which raises even-period prices and decreases the inflation rate (defined as $\pi_e = (P_e/P_o) - 1$) to -0.0183. If participation had been symmetric, with zero money growth the inflation rate would have been zero as in the prohibitively costly (Townsend) equilibrium.

The fluctuation in currency prices in the costly commitment equilibrium is related, in its origins, to the asset price fluctuation in the environment studied by Scheinkman and Weiss (1986). They studied (in effect) currency price fluctuations in a Townsend turnpike model in which the common times at which the two households switched between being productive and unproductive (and vice versa) were random. We have shown that currency price fluctuations can also occur if these switch times are known in advance but households can relax the borrowing constraint at a cost.

It is worth noting that, as in Townsend (1980), the logic of consumption-based asset pricing does *not* apply to the pricing of currency. Because the non-negativity constraint on currency holdings always binds for sellers of currency, their marginal rate of substitution between current and the next period's consumption is never equal to that of the buyers of currency. Consequently, the ratio between the aggregate consumption of currency-market participants in adjacent periods does not bear a simple relationship to the return on currency.

4. Currency Experiments

We now turn to the role of fiat currency in influencing real interest rates, allocations, and welfare.

4.1 Introduction of Currency and Welfare

In our costly commitment environment, the introduction of currency affects welfare by providing an alternative means of transferring wealth over time. Clearly, household-types that consume their endowments in a no-currency equilibrium can never lose, and possibly gain, from the introduction of currency.⁷ On the other hand, household-types that participate in the claims market in the no-currency equilibrium may be adversely affected by currency. For instance, if the availability of currency results in fewer lenders in the claims market, borrowers are adversely affected. Whether the introduction of currency leads to a Pareto improvement thus depends entirely on how participation in the claims market is affected.

To explore this issue, the top-half of Table 2 displays no-currency equilibria for economies with τ/y equal to 0.002, 0.02, and 1. Comparing this with Table 1, it is easy to see that the low cost ($\tau/y = 0.002$) and high cost ($\tau/y = 1$) have similar properties with respect to the welfare consequences of currency. In either case, the introduction of currency does not change the participation profile of households. This is trivially true in the high cost case (there is no participation in the claims market before or after the introduction of currency). It is also true in the low cost case; all that happens in the low-cost monetary equilibrium is that some previously autarkic households begin to hold currency. As a result, the introduction of currency is a Pareto improvement in either case.

In contrast, the impact of currency is more intrusive in the $\tau/y = 0.02$ equilibrium. When there is no currency, the even- and odd-period interest rates are 0.1360 and 0.0647, and there is participation in both even- and odd-period claims markets. When currency is introduced, the equilibrium interest rates in both even and odd periods become slightly higher, and there is no participation in the odd-period claims market. With higher interest rates, household-types that are borrowers in both equilibria are worse off with the introduction of currency. This provides another example where the opening of a market (currency) does not lead to a Pareto improvement because of the general equilibrium pricing effects emphasized in Hart (1975).⁸

⁷ No-currency periodic equilibria are defined analogously to periodic (monetary) equilibria.

⁸ Since the periodic phase does not begin immediately but at $T=2$, the new equilibrium reflects the combined effects of the introduction of currency and an active fiscal policy in periods $t=0$ and $t=1$.

4.2 Currency Growth and “Superneutrality”

We study the effects of variation in the growth rate of currency for the low commitment cost economy ($\tau/y = .002$). The bottom half of Table 2 reports our findings for currency growth rates ranging from $\mu = .03$ to $\mu = -.06$. For a currency growth rate of 3 percent (and higher), periodic monetary equilibria do not exist because all households drop out of the currency market. Since the demand for currency is related to the costliness of commitment, there is an upper bound on the size of the currency growth rate. As the currency growth rate drops, periodic monetary equilibria become possible.

A striking result to emerge from these experiments is that it is possible to generate an example of near-superneutrality of interest rates in a store-of-value monetary model. With currency and claims as alternative stores of value, one might expect the equilibrium real return on currency to be positively associated with the equilibrium real return on claims. Indeed, in an overlapping generations model with costly commitment, where participation decisions are particularly simple given that households live two periods, we found this to be the case (Chatterjee and Corbae (1992)). However, with infinitely lived households, the participation decision is more subtle. As Table 3 shows, a drop in the currency growth rate from $\mu = .02$ to $\mu = -.06$ predictably increases the set of currency holders at the expense of lenders; the set of even-period currency holders expands from $\alpha = 0.53$ to $0.51 \leq \alpha \leq 0.56$. At the same time, the set of even-period borrowers *shrinks* at the expense of *odd-period currency holders*; odd-period currency holders expand from $\alpha = 0.47$ to $0.44 \leq \alpha \leq 0.49$. Because household-types with low α weigh the net benefits of being an even-period borrower with that of being an odd-period currency holder, the increase in the rate of return on currency induces them to withdraw from the claims market as well. Thus, both the demand and the supply of bonds shrink as a result of lower currency growth. In this example, the demand and supply of bonds shrink *symmetrically*, and there is no change in interest rates.

This result can be related back to the asset-pricing formula in (10). Recall that if participation is symmetric, then $\bar{\alpha}(2/3; r_w, \pi_w) = 1/2$ and the interest rate is given by $\{1/[\beta(1-\tau/(y(1+k)))]\}^{-1}$. Although the inflation rate affects the interest rate through k in the financial expenditure term, these terms are so small (see Tables A.1-A.7 in the appendix to Chatterjee and Corbae (1994)) that changes in them leave the interest rate virtually unchanged. Note that this example of approximate superneutrality depends on the commitment technology. For

instance, if the cost of commitment was proportional to the value of the trade, the equilibrium return on currency and claims would have to be identical (see Bryant and Wallace (1979)) and superneutrality would not obtain.

There is an additional (and different) form of real interest rate superneutrality in the model that stems from one of the general properties of periodic equilibria noted earlier. Recall that we argued that in a periodic equilibrium $\sqrt{(1+r_e)(1+r_o)}$ cannot deviate too far from $1/\beta$ because a large deviation would invite continuous participation in the claims market and lead to a violation of periodicity. Thus, restricting attention to the class of periodic equilibria, the long-term interest rate $(1+r) = \sqrt{(1+r_e)(1+r_o)}$ is constrained to be near $1/\beta$ irrespective of the inflation rate. Thus, a form of Sidrauski (1967)-style real interest rate superneutrality obtains in our model without having currency appear in the utility function. This type of real interest superneutrality cannot occur if transaction costs are proportional, or if it is assumed that currency is needed to mediate all transactions as in cash-in-advance models.

The second result to emerge from these experiments concerns the impact on welfare of different currency growth rates. Our model synthesizes two well-known channels through which lower inflation benefits households. First, there is a benefit from a reduction of expenditure on financial services as households switch from claims into currency. In the jargon of monetary theory this is the reduction in the "shoe-leather" costs of inflation noted by Friedman (1969). In addition, as the rate of currency growth falls toward $\beta-1$, welfare improves because the discrepancy in the marginal rate of substitution between even- and odd-period consumption across even- and odd-period *currency* holders is reduced.⁹ This is the gain in allocative efficiency emphasized in Townsend (1980).

5. Income Levels and Financial Development

It has been known, at least since Kuznets (1971) and Goldsmith (1969), that various measures of financial activity are correlated with the level of per capita income in cross-country data. The costly commitment model is well suited for thinking about these regularities for two reasons. First, a relationship between financial activity and aggregate income is delivered without treating financial services as just another commodity in the utility

⁹ The marginal rate of substitution between even- and odd-period consumption for even-period currency holders is $\beta/(1+\pi_e)$, while it is $(1+\pi_o)/\beta$ for odd-period currency holders. When $\pi_e = \pi_o = \pi$, the latter exceeds the former for all $\pi > \beta-1$, with the discrepancy shrinking as π tends toward $\beta-1$.

function. Second, in accordance with the facts of development, poverty and affluence are modeled as differences in the *productivity* of time by varying y in the production functions (2) and (3) (for related results using a representative agent framework see Ireland (1994) and Townsend (1983)).

Table 3 relates measures of financial activity and sophistication to productivity (and hence aggregate income), holding the commodity price of financial services (τ) constant at 0.2. In the first row, productivity (y) is 1 (which implies aggregate income is 101), while in the second and third rows productivity is 10 and 100, respectively, with aggregate income proportionally higher.

The first column records the falling price of financial services in terms of labor time (τ/y). The second column records the associated increase in the fraction of household-types that participate in coalition formation (financial activity). This fraction is zero for the least productive economy (because the commitment cost is prohibitive) and 0.58 and 0.93, respectively, for the more productive economies.

Columns (4) and (5) record the average value of goods exchanged through the currency market and claims market, respectively. Thus, column (4) reveals that the average real value of currency rises a bit from the least to the more productive economy and then falls drastically in the most productive economy. There are two forces at work: as productivity rises, those who continue to use currency hold more of it, but the fall in τ/y makes for fewer currency holders. Thus, there is at first a small increase in currency balances and then, as participation in the currency market dwindles, a decline. In contrast, the effects of increases in income and falling τ/y work in the same direction for the real value of financial claims; as column (5) reveals, it rises steadily with higher productivity.

Column (6) reports the ratio of the value of claims in financial coalitions to aggregate income. Starting with zero for the least productive economy, this ratio increases with productivity. If interest rates and participation were held constant across these economies, the ratio of the value of claims to aggregate income would remain constant. However, interest rates do change, and, more significantly, the fraction of households trading in claims increases with productivity. Therefore, the ratio tends to increase with productivity as well. Thus, the model provides an explanation of Goldsmith's finding of increasing importance of financial institutions' assets as a proportion of national income.

Column (7) displays the ratio of currency to a M2-type monetary aggregate measure. For the latter, we interpreted the value of "deposits" made in financial coalitions as inside money and added it to the real value of currency. The column shows that this ratio is positively related to income. This feature is in accordance with the facts noted by McKinnon and documented in more detail by Ezekiel and Adekunle (1969) and Bordo and Jonung (1987).

The final column reports the fraction of national income originating in the financial services sector. We interpreted the expenditure on financial services as the value-added by the financial services sector. Naturally, this fraction is zero in the least productive economy and becomes positive as productivity rises. However, in the model and in the real world there is a natural limit to the increase in participation in any one type of financial activity. Thus, while income increases tenfold between the middle economy to the most productive one, participation can increase by a factor of at most 1.72 (from 0.58 to 1). Consequently, there is a tendency for the fraction of national income originating in the financial services sector to fall, and in the most productive economy, this is indeed the case. Thus, the model captures Kuznets' findings to some extent but also hints at forces that may tend to reduce the fraction of aggregate income originating in the financial services sector.

A final comment on Table 3 is that the fall in τ/y noted in the first column can alternatively be interpreted as a reduction in τ instead of an increase in productivity. In this interpretation, the entries in the third through the fifth columns of the second and third rows will get deflated by a factor of 10 and 100, respectively, and all other entries will remain unchanged. The comparison of the rows then would show the impact of "institutional" shocks in which there is a large change in the cost of commitment.

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Table 1

Equilibrium Asset Prices and Participation Profiles

τ/y	T	r_e r_o	π_e π_o	Autarkic Households	Even-period Participants	Even-period Currency holders	Odd-period Participants	Odd-Period Currency holders
0	0	0.1 0.1	N.A. N.A.	$\alpha = 0.5$	$0 \leq \alpha \leq 0.49$ $0.51 \leq \alpha \leq 1$	N.A.	none	N.A.
0.002	0	0.1044 0.0955	0 0	$0.48 \leq \alpha \leq 0.52$	$0 \leq \alpha \leq 0.46$ $0.54 \leq \alpha \leq 1$	0.53	none	0.47
0.02	2	0.1361 0.0702	-0.0183 0.0186	$0.48 \leq \alpha \leq 0.51$	$0 \leq \alpha \leq 0.28$ $0.72 \leq \alpha \leq 1$ and $\lambda(.71, 2/3) = .915$	$0.52 \leq \alpha \leq 0.70$ and $\lambda(.71, 0) = .085$	none	$0.29 \leq \alpha \leq 0.47$
1	0	N.A. N.A.	0 0	$0.48 \leq \alpha \leq 0.52$	none	$0.52 \leq \alpha \leq 1$	none	$0 \leq \alpha \leq 0.48$

Parameters: $\beta = 1/1.1$, $N = 100$, and $\mu = 0$.
Notes: N.A. denotes not applicable.

Table 2
Currency Experiments

No Currency Equilibria (Compare with Table 1)								
τ/y	T	r_e r_o		Autarkic Households	Even-period Participants		Odd-period Participants	
0.002	0	0.1044 0.0955		$0.47 \leq \alpha \leq 0.53$	$0 \leq \alpha \leq 0.46$ $0.54 \leq \alpha \leq 1$		none	
0.02	0	0.1360 0.0647		$0.41 \leq \alpha \leq 0.58$	$0 \leq \alpha \leq 0.37$ $0.62 \leq \alpha \leq 1$ and $\lambda(.61, 2/3) = .60$		$0.38 \leq \alpha \leq 0.39$ $\alpha = 0.60$ and $\lambda(.61, 1/3) = .40$	
1	0	N.A. N.A.		$0 \leq \alpha \leq 1$	N.A.		N.A.	
Equilibrium Asset Prices and Participation Profiles with Currency Growth (with $\tau/y=0.002$)								
μ	T	r_e r_o	π_e π_o	Autarkic Households	Even-period Participants	Even-Period Currency Holders	Odd-period Currency Holders	ϕ
0.03	0	0.1044 0.0955	N.A.	$.47 \leq \alpha \leq .53$	$0 \leq \alpha \leq .46$ $.54 \leq \alpha \leq 1$	N.A.	N.A.	N.A.
0.02	0	0.1044 0.0955	0.02 0.02	$.48 \leq \alpha \leq .52$	$0 \leq \alpha \leq .46$ $.54 \leq \alpha \leq 1$	$\alpha = .53$	$\alpha = .47$	0.000120
0.00	0	0.1044 0.0955	0.00 0.00	$.48 \leq \alpha \leq .52$	$0 \leq \alpha \leq .46$ $.54 \leq \alpha \leq 1$	$\alpha = .53$	$\alpha = .47$	0.000000
-0.02	0	0.1044 0.0955	-0.02 -0.02	$.49 \leq \alpha \leq .51$	$0 \leq \alpha \leq .46$ $.54 \leq \alpha \leq 1$	$.52 \leq \alpha \leq .53$	$.47 \leq \alpha \leq .48$	-0.00126
-0.04	0	0.1044 0.0955	-0.04 -0.04	$.49 \leq \alpha \leq .51$	$0 \leq \alpha \leq .45$ $.55 \leq \alpha \leq 1$	$.52 \leq \alpha \leq .54$	$.46 \leq \alpha \leq .48$	-0.01002
-0.06	2	0.1044 0.0955	-0.06 -0.06	$\alpha = .50$	$0 \leq \alpha \leq .43$ $.57 \leq \alpha \leq 1$	$.51 \leq \alpha \leq .56$	$.44 \leq \alpha \leq .49$	-0.04940
Parameters: $\beta=1/1.1, N=100$.								
Notes: N.A. denotes not applicable.								

Table 3

Relation between Income Levels and Measures of Financial Activity

(1) τ/y	(2) Fraction of population participating in claims market	(3) GNP	(4) Average Real Currency Demand	(5) Average value of goods passing through claims market	(6) Financial depth (5) ÷ (3)	(7) M2/M1 ((4) + (5)) ÷ (4)	(8) Fraction of GNP absorbed by finance (1) x (2)
0.2	0	101	11.57	0	0	1	0
0.02	$\{29+29+.915\} \div 101$ = 0.5838	1,010	17.4	49.484	0.0490	3.84	0.0117
0.002	$\{47+47\} \div 101$ = 0.9307	10,100	0.620	603.07	0.0597	973	0.0019

Parameters: $\beta=1/1.1$, $N=100$, and $\mu=0$
Notes: In column 4, Average Real Currency Demand = $\{(M_e(1+\pi_e)+M_o(1+\pi_o))/2$
In column 5, Average Value of goods passing through asset market = $\{b_e/1+r_e + b_o/1+r_o\}/2$