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CAPITAL REQUIREMENTS AND RATIONAL DISCOUNT WINDOW BORROWING

Sherrill Shaffer

Federal Reserve Bank of Philadelphia

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ABSTRACT

When banks face capital regulations and stochastic deposit supply, their decisions to borrow at the discount window will be affected by a broader range of variables than previous theoretical and empirical studies have recognized. Moreover, those decisions can respond discontinuously to changes in market parameters and to the form of rationing rule by which the discount window is administered. Risk aversion can complicate these linkages considerably, even causing some banks to prefer a positive discount rate that may exceed the actual level.

CAPITAL REQUIREMENTS AND RATIONAL DISCOUNT WINDOW BORROWING

1. Introduction

Recent years have witnessed a disruption of historical linkages between the level of discount window adjustment borrowing and spreads between the discount rate and the cost of alternative sources of funds (see Mitchell and Pearce, 1992; Clouse, 1994; Cosimano and Sheehan, 1994). These developments suggest that a deeper understanding is needed of the factors influencing rational borrowing decisions by banks. Few prior studies have addressed this issue from first principles, though several have quantified the historical empirical patterns.¹

Here we analyze rational adjustment (overnight) borrowing decisions by banks in a two-stage stochastic framework that incorporates regulatory capital requirements. Shaffer (1995) demonstrated in a similar framework that capital requirements can have a fundamental influence on banks' profit-maximizing funding strategies, though the focus of that study was on the effects, rather than causes, of borrowing behavior. Four extensions allow us to characterize various determinants of rational borrowing: increasing the number of stochastic factors to allow for unanticipated changes in all interest rates, potentially incorporating the realization of exogenous default risk; imposing a discount window rationing rule similar to the current administration of adjustment credit; allowing arbitrary probability distributions of an additive deposit supply shock; and considering arbitrary degrees of risk aversion among banks.

The results indicate that banks' optimal borrowing decisions depend on a broader set of variables than recognized in prior theoretical and empirical studies. In addition, those decisions can respond discontinuously to changes in market parameters and to the form of nonprice rationing rule by which the discount window is administered. For example, a rationing rule based on the frequency of recent borrowing can, under certain conditions, drive banks

completely away from the discount window unless the discount rate is set at a subsidy level. Finally, risk aversion introduces a linkage between deposit interest rates and banks' choice of funding patterns that does not exist under risk neutrality, and implies conditions under which banks prefer a strictly positive discount rate that may exceed the actual rate.

2. The Model with Expanded Uncertainty

As in Shaffer (1995), we depict banks' decisions as a two-stage process. In the first stage, a bank chooses its level of capital which, in conjunction with regulatory capital ratio requirements, determines an overall asset capacity; at the same time, the bank commits to a particular level of loans.² In the second stage, the bank chooses a deposit interest rate to attract a quantity of deposits sufficient to fund optimal asset levels. The deposit supply function includes a stochastic component that is realized exogenously after interest rates and loan levels have been chosen; the bank may respond *ex post* to various deposit outcomes by making instantaneous adjustments in its securities portfolio or by borrowing at the discount window, as necessary to satisfy its balance-sheet constraint. The bank's choices made at the first stage, in conjunction with the deposit state subsequently realized, determine the feasible set of these balance-sheet adjustments. The model is solved by the standard technique of backward induction, and also permits repeated play.

The basic model incorporates two deposit states, high and low, differing by an additive constant that may take bank-specific but exogenous values. (Section 4 below relaxes the assumption of two states.) It is convenient to categorize the bank's responses to these states in terms of three possible *cases*, or state-contingent patterns of funding, each associated with some

optimal level of capital and loans as derived below. In case 1, the bank plans to ration credit ex post if the high state occurs. In case 2, the bank plans to hold securities ex post if the high state occurs. In case 3, the bank plans to borrow at the discount window ex post if the low state occurs. The bank must select its preferred case ex ante on the basis of expected profitability (or, in section 5 below, on the basis of a utility function incorporating both expected profits and the variability of profits). Thus, the bank's choices of capital levels, loan levels, deposit rates, and case are state-independent. Shaffer (1995) showed that, under the assumptions listed below, the three cases are mutually exclusive--that is, an optimizing bank will always choose a corner solution and will not, for example, both hold securities and ration credit as defined.

Banks are assumed to be price takers in loans, securities, financial capital, and discount loans.³ Except in the analysis of risk-averse banks in section 5, each of these prices will be considered stochastic--a generalization of the model of Shaffer (1995)--and uncorrelated with the deposit state. A stochastic loan rate could reflect changes in the yields of floating-rate loans tied to exogenous market rates; it could also be interpreted as incorporating the realization of exogenous credit risk, though the model does not translate this into an explicit possibility of failure.⁴ Stochastic yield on securities reflects the fact that the yield may change between the time the bank makes its funding decisions and the realization of the demand shock. A stochastic price of financial capital reflects the difficulty that a bank typically has in measuring this price with precision.⁵ A stochastic discount rate is more general than required for realism, and is incorporated primarily to emphasize the model's flexibility. Under the assumed sequence of actions, the results are not affected by the stochastic nature of these variables, but only by the stochastic deposit supply. Analysis is carried out at the level of the banking firm, permitting

a high degree of generality of market structure, strategic interactions among banks, and the degree of product or service differentiation on the deposit side. Resource costs are subsumed into the deposit interest rate.⁶

In each case and in each state, bank i is subject to a balance-sheet constraint, $L_i + S_i \leq K_i + D_i(d_i)(1 - \delta) + F_i$, where L_i is the dollar volume of loans outstanding, $S_i \geq 0$ is the bank's dollar amount of securities owned, K_i is the bank's dollar amount of equity capital, d_i is the interest rate the bank pays for deposits, $\delta \in [0, 1)$ is the fractional reserve requirement, and $F_i \geq 0$ is the dollar amount of the bank's discount window borrowing.⁷ $D_i(d_i)$ is the state-dependent supply of total deposits, assumed to be a continuous, monotone increasing function--which in this context says that a bank is able to attract additional deposits by some combination of actions that may include both price and non-price dimensions (recalling that the interest rate is specified to include resource costs). Although it is convenient to assume that $D_i(d_i)$ is differentiable, such an assumption is nowhere used below and is not necessary so long as $D_i(d_i)$ satisfies some set of conditions that suffice to ensure the existence of equilibrium.

The deposit supply function in the high state exceeds that in the low state by a positive, bank-specific amount u_i . The high state occurs with exogenous probability α , the low state with probability $(1 - \alpha)$. We assume that all banks face the same state (high or low) on a given date, to avoid the need to model an interbank funds market; the model therefore reflects banks' funding shocks remaining after any interbank funds market has cleared (i.e., net of fed funds transactions).⁸ The assumption of a vertical supply shift (additive uncertainty) means that the bank is unable to influence the size of its random deposit outflows by its choice of deposit interest rate, even though its overall level of deposits (taking high and low states together) may

respond to that choice; this assumption seems appropriate as a first approximation. Allowing bank-specific deposit shocks means that the model is able to accommodate the realistic situation in which larger banks may be subject to larger absolute deposit shocks than smaller banks.⁹

The regulatory leverage constraint is $K_i \geq k(L_i + S_i)$ where $k \in (0, 1)$. The appendix considers an alternative risk-based capital constraint, $K_i \geq kL_i$. Although both forms of constraint may be enforced simultaneously, as under current policy, only one of these can be binding at any one time. The state-dependent profit function is $\pi_i(d_i) = rL_i + sS_i - e_iK_i - d_iD_i(d_i) - fF_i$, where r is the interest rate on loans, s is the yield on securities, e_i is the price of equity capital, and f is the discount rate (all exogenous and stochastic). We use Greek letters to denote the expected values of these interest rates: ρ , σ , ϵ_i , and ϕ , respectively. We abstract from fixed costs, which do not alter the profit-maximizing conditions.

The exogeneity of r , s , and e_i imposes a linear structure on associated components of the bank's objective function, implying that the optimizing bank will operate at a corner solution with respect to S_i and F_i ; we depict this situation by considering three separate cases, as described below. We assume that discount officers administer access to the discount window in a way that precludes arbitrage, so that banks never borrow in the high state.¹⁰ This policy requires a nonprice rationing rule whenever the discount rate is set at a subsidy level on average (in this model, whenever $\phi < \sigma$), although the form of that rule would be more sophisticated than a simple restriction on the frequency of borrowing as embodied in several recent studies. (Section 3 below explores the implications of the latter form of rationing rule.) Although e_i is exogenous, it may vary across banks, reflecting investors' diverse informational costs and banks'

unequal earnings volatility and (unmodelled) insolvency risk; it may also take different values at different phases of the business cycle.

We solve the model separately for each case and characterize the pattern of rational borrowing by comparing cases. Bank subscripts are suppressed hereafter for brevity.

Case 1: Credit Rationing in the High-Deposit State

In this case, the bank lends only the amount that is funded by the supply of deposits in the low state. The bank holds uninvested deposits of amount u in the high state; these may be thought of as vault cash or non-earning excess reserves. Combining the balance-sheet constraint and leverage constraint, which are both binding in the low state, we find:

$$(1) \quad L = D_{\text{low}}(d)(1 - \delta)/(1 - k)$$

$$(2) \quad K = kD_{\text{low}}(d)(1 - \delta)/(1 - k).$$

The resulting levels of profit in the high and low states, respectively, are:

$$(3) \quad \pi(d) = D_{\text{low}}(d)[(r - ek)(1 - \delta)/(1 - k) - d] - du$$

$$(4) \quad \pi(d) = D_{\text{low}}(d)[(r - ek)(1 - \delta)/(1 - k) - d]$$

where profit in the high-deposit state is less than in the low-deposit state by the amount paid as interest on the extra deposits, du . Expected profit is:

$$(5) \quad E\pi(d) = D_{\text{low}}(d)[(\rho - \epsilon k)(1 - \delta)/(1 - k) - d] - \alpha du.$$

Case 2: Buying Securities in the High-Deposit State

In this case, the bank chooses a level of loans according to the level of deposits it receives in the low state, as in case 1. However, to avoid the opportunity cost of holding uninvested funds in the high state, the bank chooses a level of equity that permits it to invest its excess deposits u in securities ($S = u(1 - \delta)$) during the high state. The leverage constraint will therefore be binding only in the high state. Combining the balance-sheet and leverage constraints, we find:

$$(6) \quad K = k[D_{\text{low}}(d) + u](1 - \delta)/(1 - k)$$

$$(7) \quad L = [D_{\text{low}}(d) + ku](1 - \delta)/(1 - k).$$

The associated profits in the high and low states, respectively, are:

$$(8) \quad \pi(d) = D_{\text{low}}(d)[(r - \epsilon k)(1 - \delta)/(1 - k) - d] + u[s(1 - \delta) - d] + uk(r - e)(1 - \delta)/(1 - k)$$

$$(9) \quad \pi(d) = D_{\text{low}}(d)[(r - \epsilon k)(1 - \delta)/(1 - k) - d] + uk(r - e)(1 - \delta)/(1 - k)$$

with expected profit:

$$(10) \quad E\pi(d) = D_{\text{low}}(d)[(\rho - \epsilon k)(1 - \delta)/(1 - k) - d] + uk(\rho - \epsilon)(1 - \delta)/(1 - k) + \alpha u[\sigma(1 - \delta) - d].$$

Case 3: Borrowing from the Discount Window in the Low-Deposit State

In this case the bank chooses to hold no securities, lends according to the level of deposits received in the high state, and borrows in the low state an amount $F = u(1 - \delta)$ from the discount window as necessary to satisfy the balance-sheet constraint. It chooses a level of financial capital sufficient to satisfy the leverage constraint given the chosen level of lending. Combining the balance-sheet and leverage constraints, we find:

$$(11) \quad K = k[D_{\text{low}}(d) + u](1 - \delta)/(1 - k)$$

$$(12) \quad L = [D_{\text{low}}(d) + u](1 - \delta)/(1 - k).$$

Profits are:

$$(13) \quad \pi(d) = [D_{\text{low}}(d) + u][(r - ek)(1 - \delta)/(1 - k) - d]$$

$$(14) \quad \pi(d) = D_{\text{low}}(d)[(r - ek)(1 - \delta)/(1 - k) - d] + u[(r - ek)/(1 - k) - f](1 - \delta)$$

in the high and low states, respectively, yielding expected profits of:

$$(15) \quad E\pi(d) = D_{\text{low}}(d)[(\rho - ek)(1 - \delta)/(1 - k) - d] + u[(\rho - ek)(1 - \delta)/(1 - k) - \alpha d - (1 - \alpha)\phi(1 - \delta)].$$

The Bank's Choice of Cases

Once the bank has committed itself to a particular capitalization and balance-sheet structure, ex post realizations of r , s , e , and f cannot alter the bank's choices. Even if the

outcomes cause the bank to prefer a different case, the binding balance-sheet and capital constraints along with the short-run fixity of loans and capital prevent the bank from changing cases. The optimal choice of cases for a risk-neutral bank will depend on the expected values of r , s , e , and f . Depending on the nature of the binding capitalization constraint, the bank must choose among cases 1, 2, and 3 or among cases 1, 3, and 4.

Comparing equations (5), (10), and (15), we see that expected profits differ across cases only in terms that do not involve d . This means that the bank's profit-maximizing choice of deposit rate is invariant across cases, a neutrality result that permits further comparisons to be drawn. From equations (5) and (10), *the bank prefers case 2 to case 1 if and only if:*

$$(16) \quad \alpha\sigma + k(\rho - \epsilon)/(1 - k) \geq 0.$$

For typical values of the parameters, this expression is positive, indicating that the bank will elect to buy securities rather than ration loans. For example, if $\alpha = 0.9$, $\sigma = 0.06$, $k = 0.06$, $\rho = 0.08$, $\epsilon = 0.12$, then expression (16) equals 0.0514.¹¹

From equations (10) and (15), *the bank prefers case 3 to case 2 if and only if:*

$$(17) \quad \phi \leq (\rho - \alpha\sigma)/(1 - \alpha).$$

This condition is satisfied by typical historical values of the parameters. For instance, at the values given above, the right-hand side of (17) equals 0.26, which substantially exceeds

historical or plausible discount rates. Thus, banks can prefer to borrow at the discount window even if the discount rate is set much higher than a subsidy level.

From equations (10) and (5), *the bank prefers case 3 to case 1 if and only if:*

$$(18) \quad \phi \leq (\rho - \epsilon k) / [(1 - k)(1 - \alpha)].$$

Whenever the bank prefers case 2 to case 1, as is true for typical historical values of the parameters, condition (18) will be even less restrictive than condition (16), leaving banks to prefer case 3. For instance, at the values given above, the right-hand side of (18) equals 0.774.

The conditions (16) and (17) indicate that rational borrowing decisions respond not only to the actual and expected spreads between f and the cost of alternative funding, as implied by the model of Goodfriend (1983) and incorporated in many recent empirical studies (such as Mitchell and Pearce, 1992; Dutkowsky, 1993; Peristiani, 1994; Cosimano and Sheehan, 1994), but also to additional variables including the interest rate earned on loans, the cost of financial capital, the regulatory capital requirement, and the probability of an adverse deposit supply shock. Condition (17), comparing cases 1 and 3, is even independent of the yield on securities and therefore independent of the spread between f and s or between ϕ and σ . This set of results suggests that recent studies have omitted potentially important explanatory variables, a conclusion that might be a factor in explaining the difficulty such models have had in predicting recent borrowing behavior.

It is also striking that, for certain combinations of parameter values, conditions (16) and (17) indicate that banks will choose to borrow from the discount window even if the discount

rate equals or somewhat exceeds the yield on securities, and in that sense is not set at a subsidy rate. However, as shown in the next section, this result is sensitive to the form of the nonprice rationing rule by which the discount window is administered.

3. Discount Window Rationing

The previous section characterized the administration of the discount window as precluding arbitrage, without placing restrictions on the allowable frequency of access to the discount window per se. Footnote 9 documents that this interpretation is consistent with the written policy and regulations. However, other studies have described the actual practice of discount window administration as more akin to imposing a ceiling on the frequency of access (Goodfriend, 1983; Meulendyke, 1992). In this section, therefore, we explore how this alternative rationing rule would affect the bank's borrowing decision.

If the maximum allowable frequency of borrowing is comparable to the average frequency of low-deposit states, the rule would have a similar effect on banks as the no-arbitrage policy, apart from the possibility that an abnormally high number of consecutive low states could occur from time to time. It is even plausible that a judiciously chosen frequency rule might be used in place of, rather than in addition to, a no-arbitrage rule to achieve the same outcome without requiring discount officers to monitor the deposit state, at least in a two-state world with known α ; section 4 below briefly explores how a continuously distributed deposit shock might alter this conclusion.¹²

Here, we focus on the contrasting case in which banks are permitted to borrow with maximum frequency $\beta < (1 - \alpha)$. Then, in repeated play within the framework of the previous

section, a bank would expect to experience a low-deposit state more often than it could fund through the discount window. Its choice of capitalization and lending levels must therefore allow for alternative funding patterns (cases 1 or 2), with the result that the bank would then be in a position to operate independently of the discount window. At that point, whether it ever borrows at all depends on which funding pattern the bank chooses and on whether the discount rate is set at a subsidy rate ($f < s$).

Clearly, if the bank chooses case 1, there is no role for the discount window regardless of the spread between f and s . In that circumstance the rationing rule would have the discontinuous effect of driving the bank entirely away from the window. If instead the bank chooses case 2, then it would be permitted to borrow at the window in a percentage of the low states; whether it does so would depend on whether its profit in the low state exceeds that given by equation (9) above. As the bank's balance sheet would be configured for case 2 (i.e., equations (6) and (7) above), its profit in the event of borrowing would be given by:

$$(19) \quad \pi(d) = D_L(d)[(r - ek)(1 - \delta)/(1 - k) - d] + u(1 - \delta)[k(r - e)/(1 - k) + s - f].$$

This expression exceeds the value given by equation (9) if and only if $u(s - f)(1 - \delta) > 0$, or $s > f$ (that is, whenever the discount rate is set at a subsidy rate). In this case, the bank will borrow up to the allowable fraction β of days unless case 1 yields higher expected profit. Note that, because securities holdings and discount loans can both be adjusted instantaneously, the bank is able in this case to respond ex post to realized values of the various parameters in choosing between selling off its securities holdings in the low state versus borrowing at the

discount window to sustain its asset portfolio. Therefore, there is some possibility that the bank may end up borrowing less often than β within any given number of days, depending on the pattern of outcomes of r , e , s , and f . Also note that borrowing in this case would have the effect of sustaining the bank's securities holdings rather than loans; this outcome is consistent with current Federal Reserve policy and practice, as the borrowing bank is not actually expanding its holdings of securities or other assets at the time it borrows.

To compare the expected profitability of this case with case 1, calculate the expected profit of "case 2 with borrowing" as:

$$(20) \quad E\pi(d) = D_L(d)[(\rho - \epsilon k)(1 - \delta)/(1 - k) - d] + \alpha u[\sigma(1 - \delta) - d] + k u(1 - \delta)(\rho - \epsilon)/(1 - k) \\ + \beta u(\sigma - \phi)(1 - \delta)$$

which exceeds equation (5) if and only if:

$$(21) \quad \phi < \sigma(1 + \alpha/\beta) + k(\rho - \epsilon)/[\beta(1 - k)].$$

Thus, if the discount window is administered according to a binding, frequency-based rationing rule ($\beta < 1 - \alpha$), *banks will never choose to borrow unless $\phi < \sigma$ and condition (21) is satisfied*. Whether (21) provides a more or less restrictive upper bound on ϕ than the simple subsidy rate condition depends on other parameter values; we should normally expect it to be less restrictive because $(1 + \alpha/\beta) > 2$ for all $\beta < (1 - \alpha) \leq 1/2$, whereas the rightmost term in (21) will have a smaller order of magnitude for realistic parameter values.

Considering both forms of capital requirements, we see that the principal finding of this section is that *if the discount window is administered with a binding, frequency-based rationing rule, banks will choose not to borrow at the discount window at all unless the discount rate is set at a subsidy level.* Here, the rationing rule is considered "binding" when it is more restrictive than a no-arbitrage condition, and a "subsidy level" is defined relative to yields on investment securities rather than relative to the federal funds rate. In practice, the intent of either definition of "subsidy"--to identify arbitrage opportunities in which the bank could borrow at a low rate from the discount window and simultaneously lend at a higher rate in a liquid market--would be the same.

4. General Additive Deposit Shocks

Several of the results derived above appear to be driven by the model's propensity to generate corner solutions. That propensity, in turn, arises from a combination of assumptions including price-taking behavior by banks on the asset side and the existence of just two deposit states. Although footnote 3 pointed to empirical evidence supporting the price-taking assumption, the two-state feature is clearly a stylized abstraction. In this section we relax the two-state assumption, positing a more general additive deposit shock of the form $D(d) = D_L(d) + \theta$ where θ is a random variable with p.d.f. $g(\theta)$, $\theta \in [0, u]$. Then each bank can choose a threshold θ separating different funding strategies, most of which correspond to the funding cases analyzed in section 2. If the bank chooses the loan rationing equilibrium (as in case 1 above), its chosen financial capital will only support a maximum amount of assets corresponding to $\theta = 0$, making any other funding choice irrelevant; thus, case 1 goes through unaltered in the

presence of continuously distributed deposit shocks. In case 2 (buying securities without borrowing at the discount window), the bank can instantaneously adjust its securities holdings as needed, so this case goes through as in section 2 except that expected profit includes a term reflecting the expected or average deposit shock.

In case 3 (borrowing at the discount window), the bank can apportion its balance-sheet needs among the amount borrowed, the amount invested in securities, and the amount of credit rationing. The possibilities here are more complex, as the bank can borrow from the discount window for all $\theta \in [0, \underline{\theta}]$ and either ration (which we call case 4) or invest in securities (which we call case 5) for $\theta \in [\underline{\theta}, u]$. A final possibility, which we call case 6, is for the bank to invest in securities for all $\theta \in [0, \underline{\theta}]$ and ration loans for all $\theta \in [\underline{\theta}, u]$.¹³

Applying the usual leverage and balance-sheet constraints to case 4, we find:

$$(22) \quad K = k(1 - \delta)[D_L(d) + \underline{\theta}]/(1 - k)$$

$$(23) \quad L = (1 - \delta)[D_L(d) + \underline{\theta}]/(1 - k)$$

$$(24) \quad E\pi = [D_{low}(d) + \underline{\theta}](\rho - \epsilon k)(1 - \delta)/(1 - k) - dD_L(d) - \underline{\theta}\phi(1 - \delta) - \phi(1 - \delta) \int_0^{\underline{\theta}} \theta g(\theta) d\theta \\ - d \int_0^u \theta g(\theta) d\theta$$

where the bank can maximize its expected profit by its choice of $\underline{\theta}$ in the first stage, followed by its choice of d in the second stage. The first-order and second-order conditions at the first stage are found from equation (24) as:

$$(25) \quad \underline{\theta}g(\underline{\theta}) = 1 - (\rho - \epsilon k)/[\phi(1 - k)]$$

$$(26) \quad -\phi(1 - \delta)[g(\underline{\theta}) + \underline{\theta}g'(\underline{\theta})] < 0$$

where the second-order condition reduces to $g(\underline{\theta}) > -\underline{\theta}g'(\underline{\theta})$ or $g'(\underline{\theta})\underline{\theta}/g(\underline{\theta}) > -1$. Since $g'(\underline{\theta})\underline{\theta}/g(\underline{\theta})$ is the elasticity of the p.d.f. at $\underline{\theta}$, the second-order condition amounts to a requirement that the p.d.f. have greater than negative unitary elasticity at $\underline{\theta}$. The first-order condition thus implies an expected profit maximum for all $g'(\underline{\theta}) \geq 0$ as well as for certain ranges of negative $g'(\underline{\theta})$ in which $g'(\underline{\theta}) > -g(\underline{\theta})/\underline{\theta}$. However, the value of $\underline{\theta}$ implied by the first-order condition may lie outside the range $[0, u]$ even in this case, resulting in a corner solution.

For example, let θ be uniformly distributed so $g(\theta) = 1/u$ and $g'(\theta) = 0$. Here the second-order condition is satisfied since $-f(1 - \delta)/u < 0$. Assigning plausible values of the other parameters, $\rho = 0.08$, $\epsilon = 0.12$, $k = 0.06$, and $\phi = 0.05$, we find $\underline{\theta} = -0.549u < 0$. Therefore, a corner solution results at $\underline{\theta} = 0$, in which the bank does not borrow from the discount window but merely rations loans as in case 1 above.

In case 5, the leverage and balance-sheet constraints imply:

$$(27) \quad K = k[D_{\text{low}}(d) + u](1 - \delta)/(1 - k)$$

$$(28) \quad L = [D_{\text{low}}(d) + u](1 - \delta)/(1 - k)$$

$$(29) \quad E\pi = [D_{\text{low}}(d) + \underline{\theta}](\rho - \epsilon k)(1 - \delta)/(1 - k) - dD_L(d) \\ + \sigma(1 - \delta) \int_a^u (u - \theta)g(\theta)d\theta - d \int_0^u \theta g(\theta)d\theta - \phi(1 - \delta) \int_0^{\underline{\theta}} \theta g(\theta)d\theta.$$

The first- and second-order conditions at the first stage are found from equation (29) as:

$$(30) \quad (\rho - \epsilon k)/(1 - k) - \sigma u g(\underline{\theta}) + \underline{\theta}g(\underline{\theta})(\sigma - \phi) = 0$$

$$(31) \quad -\sigma(1 - \delta)g'(\underline{\theta}) + g(\underline{\theta})(1 - \delta)(\sigma - \phi) + \underline{\theta}g'(\underline{\theta})(1 - \delta)(\sigma - \phi) < 0.$$

In the example of a uniformly distributed deposit shock, the second-order condition reduces to $g(\underline{\theta})(1 - \delta)(\sigma - \phi)$, implying a profit *minimum* for all $\phi < \sigma$ (i.e., whenever the discount rate is expected to be a subsidy rate). Then the bank will choose the more profitable of the two corner solutions, $\underline{\theta} = 0$ (case 2) or $\underline{\theta} = u$ (case 3).

Case 6 is identical to case 2 for all $\theta < \underline{\theta}$. Leverage and balance-sheet constraints imply:

$$(32) \quad K = k(1 - \delta)[D_L(d) + \underline{\theta}]/(1 - k)$$

$$(33) \quad L = (1 - \delta)[D_L(d) + k\underline{\theta}]/(1 - k)$$

$$(34) \quad E\pi = D_{low}(d)[(\rho - \epsilon k)(1 - \delta)/(1 - k) - d] + k\underline{\theta}(\rho - \epsilon)(1 - \delta)/(1 - k) - d \int_0^u \theta g(\theta) d\theta \\ + \epsilon(1 - \delta) \int_0^{\underline{\theta}} \theta g(\theta) d\theta$$

with first-stage first- and second-order conditions:

$$(35) \quad \underline{\theta}g(\underline{\theta}) = -k(\rho - \epsilon)/[\sigma(1 - k)]$$

$$(36) \quad \sigma(1 - \delta)[g(\underline{\theta}) + \underline{\theta}g'(\underline{\theta})] < 0$$

where the latter is satisfied for all $g'(\underline{\theta}) < -g(\underline{\theta})/\underline{\theta}$. The uniform distribution violates the second-order condition, so the bank in this case would choose between the two corner solutions $\underline{\theta} = 0$ (which is just case 1) and $\underline{\theta} = u$ (which is essentially case 2).

Overall, a variety of conditions can lead the bank to choose a corner solution even when a continuous distribution of deposit shocks would permit interior solutions in principle. Thus, the simple two-state version of the model in section 2 appears capable of reflecting a more common range of outcomes than might be suspected.

Administration of the discount window in this context can still take two forms. The above derivations assume the no-arbitrage restriction of section 2, suitably reinterpreted for the more general deposit shock. Alternatively, a ceiling on the frequency of borrowing can be represented as a ceiling on $\underline{\theta}$ exogenously given to the bank; then the bank would compare the expected profitability of its alternatives at this value versus at its preferred choice of $\underline{\theta}$, if that is lower than the ceiling, or versus $\underline{\theta} = 0$ otherwise. As in the two-state model, the possibility arises that a binding administrative ceiling on the borrowing frequency (i.e., $\underline{\theta}$ set below the bank's preferred value) could have the discontinuous effect that the bank chooses never to borrow at the discount window. Because not all banks will generally face the same distribution of deposit shocks, it is unlikely that a uniform administrative ceiling can be appropriate for all banks. Rather, any given administrative value of $\underline{\theta}$ is likely to be both too high to prevent occasional arbitrage opportunities for some banks and too low to be consistent with appropriate, rational use of the discount window by some other banks. More specific conclusions would require empirical study of the historical distribution of interday deposit shocks.

5. Risk-averse Banks

The analysis thus far has assumed that banks are risk neutral. We can extend this framework to risk-averse banks maximizing an objective function $U(\pi) = E\pi + \gamma\sigma_{\pi}^2$ where $\gamma < 0$ indexes the degree of risk aversion and σ_{π}^2 denotes the variance of profit. In the simplest case we assume that r , s , e , and f are non-stochastic and that there are two deposit states. Then $E\pi = \alpha\pi_H + (1 - \alpha)\pi_L$ while $\sigma_{\pi}^2 = \alpha(1 - \alpha)(\pi_H - \pi_L)^2$ so $U(\pi) = \alpha\pi_H + (1 - \alpha)\pi_L + \gamma\alpha(1 - \alpha)(\pi_H - \pi_L)^2$. For case 1, applying equations (3) and (4), this expression reduces to:

$$(37) \quad U(\pi) = D_{\text{low}}(d)[(r - ek)(1 - \delta)/(1 - k) - d] - \alpha du + \gamma\alpha(1 - \alpha)d^2u^2.$$

For case 2, applying equations (8) and (9), it reduces to:

$$(38) \quad U(\pi) = D_{\text{low}}(d)[(r - ek)(1 - \delta)/(1 - k) - d] + uk(r - e)(1 - \delta)/(1 - k) + \alpha u(s - s\delta - d) \\ + \gamma\alpha(1 - \alpha)u^2(s - s\delta - d)^2.$$

For case 3, applying equations (13) and (14), the bank's objective function reduces to:

$$(39) \quad U(\pi) = D_{\text{low}}(d)[(r - ek)(1 - \delta)/(1 - k) - d] + u[(r - ek)/(1 - k) - f](1 - \delta) \\ - \alpha u(d - f + f\delta) + \gamma\alpha(1 - \alpha)u^2(d - f + f\delta)^2.$$

Interestingly, this expression suggests that a risk-averse bank would not want to see the discount rate set below some level, as the first-order condition $\partial U(\pi)/\partial f = 0$ indicates a maximum at $f = (1 + 2\gamma\alpha ud)/[2\gamma\alpha u(1 - \delta)]$.¹⁴ Larger values of f increase the bank's direct cost of borrowing; smaller values of f increase the variance of profits across states, an effect which--for a risk-averse bank--tends to offset the benefit of reduced borrowing costs.

Because these last three equations (as well as the corresponding equation in the appendix under risk-based capital requirements) differ from each other in terms involving d , the bank's preferred choice of d (and hence the quantity of deposits attracted) will differ in each case. Thus, risk aversion destroys the case invariance of the equilibrium deposit rate that characterizes risk-neutral banks in this framework. Consequently, we cannot identify in general which case

or funding pattern risk-averse banks will choose, or even which cases would yield the higher deposit rates, apart from knowledge of the specific functional form of the deposit supply curve. However, for values of γ sufficiently close to zero (i.e., for sufficiently small degrees of risk aversion), banks' funding preferences would remain unchanged from the risk-neutral choices shown in section 2 above.

For any given value of γ , the difference between the risk-averse and risk-neutral optimal pricing decisions is greatest for $\alpha = 1/2$ and declines as either $\alpha \rightarrow 0$ or $\alpha \rightarrow 1$. That is, the more time the economy spends in a particular deposit supply state, the less difference a bank's risk attitude makes regarding its rational pricing, funding, and borrowing decisions. For sufficiently large or small values of α , risk-averse banks' funding preferences would match the rational choices of risk-neutral banks shown in section 2. Little else can be usefully said about the effects of risk aversion in the absence of more specific information about the distribution of deposit shocks and the functional form of deposit supply.

6. Conclusion

Recent theoretical and empirical studies of discount window borrowing behavior have largely focused on actual and expected spreads between the discount rate and the federal funds rate, and on the frequency of recent borrowing, as explanatory variables. The analysis of rational borrowing decisions in the presence of regulatory capital requirements indicates that a broader range of variables will influence those decisions. Among the additional variables are the interest rate earned on loans, the cost of financial capital, the regulatory capital requirement,

and the probability of an adverse deposit supply shock. Thus, existing studies (both theoretical and empirical) omit potentially important variables.

Moreover, it was shown that borrowing decisions may respond discontinuously to a variety of conditions, even in some cases where all stochastic factors follow a continuous distribution. This finding suggests a further possible reason why it has proven difficult to forecast a smooth relationship between discount borrowing and various financial factors.

It was also shown that a nonprice rationing rule that is more restrictive than a no-arbitrage condition can cause banks to exhibit extreme reluctance to borrow at the discount window if the discount rate is not set at a subsidy level. This finding appears to have no relevance to current practice, but would have implications for any policy decision to combine a market-based discount rate with a ceiling on the frequency of access to the discount window.

Finally, the funding decisions of risk-averse banks were explored, but with relatively sparse conclusions. Risk aversion introduces a linkage between the bank's funding decision and its choice of deposit interest rate that does not exist for risk-neutral banks, and which implies that the specific deposit supply function and other parameter values must be known before meaningful comparisons can be drawn among alternative funding patterns. It was shown that risk-averse banks prefer a particular value for the discount rate that, for some parameter values, would be positive; this characterization contrasts with the preferences of risk-neutral banks for unboundedly negative discount rates, and results from the tradeoff between lower borrowing costs and a higher variance of profits.

Appendix: Risk-Based Capital Requirements

If the regulatory capital constraint is risk-based, $K \geq kL$, then cases 1 and 3 remain unchanged but the securities equilibrium (which, under a risk-based capital constraint, we call case 7) differs from case 2. The balance-sheet and risk-based capital constraints together imply:

$$(A1) \quad K = kD_{low}(d)(1 - \delta)/(1 - k)$$

$$(A2) \quad L = D_{low}(d)(1 - \delta)/(1 - k)$$

Profits in the high and low states, respectively, are:

$$(A3) \quad \pi(d) = D_{low}(d)[(r - ek)(1 - \delta)/(1 - k) - d] + u[s(1 - \delta) - d]$$

$$(A4) \quad \pi(d) = D_{low}(d)[(r - ek)(1 - \delta)/(1 - k) - d]$$

yielding expected profits of:

$$(A5) \quad E\pi(d) = D_{low}(d)[(\rho - ek)(1 - \delta)/(1 - k) - d] + \alpha u[\sigma(1 - \delta) - d].$$

Because equations (A5), (5), and (15) differ only in terms that do not involve d , the bank's profit-maximizing choice of deposit rate is invariant across cases, just as under a simple leverage-based capital constraint. The following conclusions may then be drawn.

From equations (15) and (A5), *the bank prefers the securities equilibrium to the borrowing equilibrium if and only if:*

$$(A6) \quad \phi \leq (\rho - \epsilon k)/[(1 - k)(1 - \alpha)] - \alpha\sigma/(1 - \alpha).$$

As in conditions (16) and (17), condition (A6) indicates that a bank's decision to borrow from the discount window is a function of additional variables beyond the expected spread and frequency of recent borrowing. The right-hand side of (A6) is less than that of (18) by the positive amount $\alpha\sigma/(1 - \alpha)$, so that discount-window borrowing is preferred under a narrower range of discount rates when the capital requirement is risk-based than when it is a simple leverage ratio. Even so, condition (A6) is satisfied at historical values of the parameters, so that banks would typically prefer to borrow at the discount window in the low-deposit state when the capital requirement is risk-based. For the sample parameter values listed in section 2, the right-hand side of (A6) equals 0.234.

Equation (A4) is the same as equation (4), indicating that profits in the low state are identical in cases 1 and 7, while equation (A3) shows the high-state profit to exceed that of case 1 (equation (3)) by the positive amount $us(1 - \delta)$. Therefore, *the bank always prefers case 7 to case 1* and will never choose to ration loans if the risk-based capital constraint is binding.

If the rationing rule of section 3 is combined with a risk-based capital requirement, the expected profit in "case 7 with borrowing" (corresponding to equation (20) under the alternative capital requirement) is:

$$(A7) \quad E\pi(d) = D_L(d)[(\rho - \epsilon k)(1 - \delta)/(1 - k) - d] - \alpha du + (\alpha + \beta)u\sigma(1 - \delta) - \beta u\phi(1 - \delta)$$

which exceeds equation (A5) if and only if $\sigma > \phi$, or the discount rate is set at a subsidy rate on average. Equation (A7) exceeds equation (5) (case 1) if and only if:

$$(A8) \quad \phi < (\alpha + \beta)\sigma/\beta.$$

For all positive α and β , the right-hand side of expression (A8) exceeds σ , so the simple subsidy condition $\sigma > \phi$ always provides a tighter upper bound on allowable discount rates under which the bank will ever choose to borrow in the presence of a binding rationing rule and risk-based capital constraints.

From equations (18) and (19), a risk-averse bank's objective function in case 7 is:

$$(A9) \quad U(\pi) = D_{\text{low}}(d)[(r - ek)(1 - \delta)/(1 - k) - d] + \alpha u[s(1 - \delta) - d] + \gamma \alpha(1 - \alpha)u^2[s(1 - \sigma) - d]^2.$$

As under a simple leverage requirement, this equation differs from equations (37) and (39) in terms involving d . Thus, again, the bank's preferred choice of d (and the associated quantity of deposits) will differ across cases.

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Footnotes

1. Goodfriend (1983) presents the most widely accepted theoretical analysis of a bank's decision to borrow, a dynamic optimization model based on banks' expectations of future federal funds rates and a nonprice discount window rationing rule; Waller (1990) models the interaction between Federal Reserve discount officers and banks. Dutkowsky (1993) and Cosimano and Sheehan (1994) estimate dynamic borrowing models, with mixed results: the former finds support for Goodfriend's model while the latter finds little or no empirical support for it. This unsettled state of affairs suggests a need for a fresh look at the theoretical incentives of individual banks to borrow.
2. Furlong and Keeley (1989) and other studies have similarly construed financial capital as one of the bank's choice variables, noting that the assumption of fixed capital is not appropriate for many larger banking organizations with access to capital markets. Moreover, regulatory constraints on allowable capital-to-asset ratios require that capital and assets be separately adjustable by the bank. The market value of a bank's capital may change *ex post* to reflect profit outcomes, but a bank can at least choose dividend payout ratios to prevent financial capital from rising above its desired levels. In repeated play, adverse shocks to profitability could conceivably reduce *ex post* capital below desired levels in ways not reflected in this type of model.
3. Price-taking behavior has been found empirically on the asset side not only for the U.S. banking industry (Shaffer, 1989, 1996) but also for the much more concentrated Canadian banking industry (Nathan and Neave, 1989; Shaffer, 1993). By contrast, non-price-taking behavior has been found on the deposit side (Hannan and Liang, 1993).
4. Previous models of banks, both with and without the discount window, have similarly neglected the possibility of failure; see for example Sealey and Lindley (1977), Tobin (1982), and Goodfriend (1983). In the U.S. from 1934 through 1994, an average of fewer than 34 banks failed each year out of a total of more than 10,000. These figures imply that the probability of an individual bank's failing within a given day is less than 0.00001. Therefore, ignoring the possibility of failure should not quantitatively bias the model to any great degree. Moreover, the model is consistent with the historical fact that, unless closed by its chartering authority, a bank could continue to operate even if insolvent, as long as it remained liquid (i.e., could meet demands for deposit withdrawal). This possibility was foreclosed by the Prompt Corrective Action provision of the FDIC Improvement Act of 1991.
5. See Friedman and Kuttner (1992) and Hardouvelis and Wizman (1992) for more analysis of the cost of financial capital.
6. See Shaffer (1995) for further discussion and defense of these assumptions. The treatment of resource costs is comparable to that in VanHoose (1985).

7. The reserve requirement is modelled as a true constraint that the bank must satisfy at all times. If the bank instead has the option of violating the requirement in some states and paying an associated penalty, the bank's optimization decision would include an additional step. The model also assumes that reserve requirements apply equally to all banks; however, as the conditions under which banks would self-select into a borrowing equilibrium versus a non-borrowing one are shown to be independent of the reserve requirement, all the results of this paper would hold even if a separate reserve requirement were set for each bank, including a zero requirement for some and positive requirements for others.

8. Thus, the model focuses on flows between the banking sector and other sectors, rather than on interbank flows. This focus is consistent with the role of open market operations in monetary policy, which represents a flow between the central bank and the banking sector; and with the market for banks' equity, which must ultimately be funded from outside the banking sector. The factors affecting banks' optimal choice of cases and their associated use of the discount window are not qualitatively altered by this abstraction.

9. Tobin (1982) also analyzed uncertainty, but for different purposes and assuming multiplicative rather than additive shocks, which requires that a bank be able to influence the size of its shock through its choice of deposit interest rates.

10. Regulation A of the Federal Reserve System restricts adjustment credit to be available from the discount window "only for appropriate purposes and after reasonable alternative sources of funds have been fully used" (12 CFR 201.3 (a)) and requires each Federal Reserve Bank to "keep itself informed...with a view to ascertaining whether undue use is being made of depository-institution credit for the speculative carrying of or trading in securities..." (12 CFR 201.6 (b)(1)), while other Federal Reserve documents define inappropriate use of the discount window as including borrowing "to take advantage of a differential between the discount rate and the rate of alternative sources of funds" and "to support a planned increase in or continued holdings of investments or loans" (Federal Reserve System, 1994, p. 10). Thus, our assumption is consistent with official policy and with actual administration of the discount window.

11. Although Waller (1990) cites 0.5 as a realistic probability that a bank will need to borrow within a given reserve period, current reserve periods are ten business days long. A bank utilizing adjustment credit seldom borrows every day during the period. Thus, accepting Waller's reasoning, 0.5 would constitute an upper bound for the value of $1 - \alpha$ or a lower bound for α , where α is defined relative to one day. The corresponding lower bound for α is found by assuming that borrowing within a reserve period occurs only on a single day; we then find $0.5 = 1 - \alpha^{10}$ or $\alpha = 0.93$.

12. Waller (1990) assumes that discount officers derive disutility from deterring appropriate borrowing requests and from approving inappropriate requests, even if these opposing types of errors are made equally often and thus result in a frequency of actual borrowing that coincides with the frequency of appropriate needs to borrow. Although we do not explicitly model the utility of the discount officer, in this section we implicitly adopt the most optimistic assumption under which to assess rationing rules--namely, that only the frequency of borrowing must be

matched to the frequency of legitimate funding needs, without regard to matching specific events period by period. The justification for this assumption is that, if a bank knows that inappropriate borrowing for arbitrage purposes today is likely to render it ineligible to borrow during a future period of legitimate need, its profit-maximizing decision will be to forego the inappropriate borrowing because of the higher cost of violating the reserve requirement later. If this assumption is not valid, then it becomes harder to justify a frequency-based rationing rule as optimal.

13. Though other permutations of borrowing, investing, and rationing exist on paper, no other such combinations make sense in the bank's operating context. For instance, if the bank chooses to ration credit, it will do so for all $\theta > \underline{\theta}$, not for $0 < \theta < \underline{\theta}$, because of the costly capitalization and funding requirements to support higher levels of lending. If the bank chooses to invest in securities for $\theta < \underline{\theta} < u$, it would ration rather than borrow for $\theta > \underline{\theta}$, because of the funding requirements (and, in the case of a regulatory leverage requirement, the costs of capitalization) to support securities holdings. These two considerations rule out all funding combinations other than those explicitly treated in the text.

14. This value of f is positive for a sufficiently large deposit shock. For example, if $\alpha = 0.1$, $\gamma = -0.5$, and $d = 0.06$, then $f > 0$ for all $u > 167$. The second-order condition shows that the extremum is a maximum, since $\partial^2 U(\pi)/\partial f^2 = 2(1 - \delta)^2 \gamma \alpha (1 - \alpha) u^2 < 0$ for all $\gamma < 0$.