

WORKING PAPER NO. 96-11/R

**MEASURING EFFICIENCY AT U.S. BANKS:
ACCOUNTING FOR HETEROGENEITY IS IMPORTANT**

by

**Loretta J. Mester
Federal Reserve Bank of Philadelphia
and
Finance Department, The Wharton School,
University of Pennsylvania**

**Revised July 1996
Original Draft September 1995**

Forthcoming in the *European Journal of Operational Research*. I thank the editor and referee for helpful comments. Please address correspondence to Loretta J. Mester, Research Department, Federal Reserve Bank of Philadelphia, Ten Independence Mall, Philadelphia, PA 19106-1574; phone: (215) 574-3807; fax: (215) 574-4364; email: lmester@frbphil.org.

The views expressed here are those of the author and do not necessarily represent the views of the Federal Reserve Bank of Philadelphia or the Federal Reserve System.

**MEASURING EFFICIENCY AT U.S. BANKS:
ACCOUNTING FOR HETEROGENEITY IS IMPORTANT**

by

Loretta J. Mester

ABSTRACT

Estimates of bank cost efficiency can be biased if bank heterogeneity is ignored. I compare X-inefficiency derived from a model constraining the cost frontier to be the same for all banks in the U.S. and a model allowing for different frontiers and error terms across Federal Reserve Districts. I find that the data reject the single cost function model; X-inefficiency measures based on the single cost function model are, on average, higher than those based on the separate cost functions model; the distributions of the one-sided error terms are wider for the single cost function model than for the separate cost functions model; and the ranking of Districts by the level of X-inefficiency differs in the two models. The results suggest it is important when studying X-inefficiency to account for differences across the markets in which banks are operating and that since X-inefficiency is, by construction, a residual, it will be particularly sensitive to omissions in the basic model.

JEL Classification Numbers: G2, D2

MEASURING EFFICIENCY AT U.S. BANKS: ACCOUNTING FOR HETEROGENEITY IS IMPORTANT

1. Introduction

U.S. commercial banks have been operating in an increasingly competitive environment. The long-term viability of commercial banks in this environment depends in part on how efficiently they are run. Thus, there has been an increased interest in studies of U.S. bank efficiency and, in particular, in studies that focus on X-inefficiency. X-inefficiency comes in two varieties. A bank is *technically inefficient* if it is using too many inputs to produce its output. In this case, the bank would not be operating on its production frontier but would be at some point in the interior. A bank is said to be *allocatively inefficient* if it is using the wrong mix of inputs to produce its output. In this case, the bank may be operating on its production frontier, but it is not minimizing its production costs.

Previous studies found X-inefficiency on the order of 20 to 30 percent at U.S. banks. These numbers did not seem implausibly high, since banking is a regulated industry in which competitive forces had been restrained until recently. Yet Mester (forthcoming) found X-inefficiency in the 6 to 9 percent range at banks operating in the Third Federal Reserve District.¹ Taking these estimates at face value would suggest that Third District banks are operating more efficiently than banks elsewhere in the nation. One possible contributing factor to the difference, however, is that Mester (forthcoming) estimated a cost model that accounted for the quality and riskiness of a bank's output. Unless such differences are controlled for, some banks might be mislabelled as inefficient because they are operating in a more risk-averse manner than others, while others might be mislabelled as efficient because they are producing lower quality output than others.² Thus, the X-inefficiency measures can be expected to differ between models that control for output risk and quality and those that do not. Another possible explanation for the difference is that fitting a single cost model to the U.S. is not sufficiently flexible to account for differences across markets in which the banks operate, and this could upwardly bias the inefficiency measures in previous national studies. In other words, estimates of bank cost efficiency can be biased if bank heterogeneity is not accounted for in the cost function. Other papers that have studied cost function heterogeneity include Kolari and Zardkoohi (1995), who estimate separate cost functions for banks grouped by product mix, Akhavein, Swamy, and Taubman (forthcoming), who estimate the banks' profit

function using a variable coefficient method, which essentially allows the parameters to vary by bank, and Mester (1993), who allows mutual and stock S&L cost functions and error structures to differ.

In this paper, I compare differences in X-inefficiency measures derived from a model that constrains the cost function to be the same for all banks in the nation and a model that allows the cost functions and error terms to differ across Federal Reserve Districts. As in Mester (forthcoming), both models account for the quality and riskiness of bank output, so that differences in the X-inefficiency measures cannot be attributed to this factor.³ In addition to presenting the usual measure of X-inefficiency—the mean of the conditional distribution of the one-sided component of the cost function's error term—I characterize the spread of the distributions with 90 and 95 percent confidence intervals. These confidence intervals indicate the degree to which the mean of the conditional distribution is a good summary statistic for the level of inefficiency. The wider the distribution, the less able is the mean to characterize the distribution and so to characterize inefficiency. I also calculate approximate standard errors for the estimates of the X-inefficiency measures. These standard errors reflect the accuracy with which the parameters of the error structure are measured and allow me to test for equality of the measures across Federal Reserve Districts.

My results show that both explanations for differences in the findings of Mester (forthcoming) and for previous national studies contain some truth: Third District banks do seem to be operating more efficiently than other banks, but I also find that estimating a single cost function across the nation is insufficiently flexible. In both the single cost function model and the separate cost functions model, average inefficiency is lower in the Third District than in the other Federal Reserve Districts. But my results also show that (1) the data reject the single cost function model; (2) the X-inefficiency measures based on the single cost function model are, on average, higher and more precisely estimated than those based on the separate cost functions model; (3) the distributions of the one-sided error terms on which X-inefficiency measures are based are wider for the single cost function model than for the separate cost functions model (so while the typical measure of bank-specific inefficiency is more precisely estimated in the single cost function model, it is not as good a characterization of bank-specific inefficiency for the

single cost function model as it is in the separate cost functions model); (4) the ranking of the Districts by the average level of X-inefficiency differs slightly, but statistically significantly, in the two models; and (5) the differences in X-inefficiency across Districts reflect more than just differences in bank size, District geographic size, and District population.

These results suggest that it is important when doing studies of X-inefficiency to account for differences across the markets in which the banks are operating. They also illustrate a more general lesson: Since X-inefficiency is, by construction, a residual, it will be particularly sensitive to omissions in the basic model. Perhaps even more care than is usual should be taken in specifying the cost model on which the X-inefficiency measures will be based.

The rest of the paper is organized as follows. Section 2 presents the stochastic econometric frontier models to be estimated. The inefficiency measures, including the confidence intervals for the bank-specific measures, are derived. Section 3 discusses the functional form of the cost function, data, and variables. Section 4 presents the empirical results, and Section 5 concludes.

2. The models

I use the stochastic econometric cost frontier methodology to derive measures of X-inefficiency.⁴ A bank is labeled as inefficient if its costs are higher than the costs predicted for an efficient bank producing the same output/input-price combination and the difference cannot be explained by statistical noise. The cost frontier is obtained by estimating a cost function with a composite error term, ε_i , which is the sum of a two-sided error, v_i (which represents random fluctuations in cost) and a one-sided positive error, u_i (which represents X-inefficiency). Measures of X-inefficiency are derived from this one-sided component of the error term.

For N firms in the sample,

$$\ln C_i = \ln C(\mathbf{y}_i, \mathbf{w}_i, q_i, k_i; \mathbf{B}) + u_i + v_i, \quad i=1, \dots, N, \quad (1)$$

where C_i is observed cost of bank i , \mathbf{y}_i is the vector of output levels for bank i , \mathbf{w}_i is the vector of input prices for bank i , q_i is a variable characterizing the quality of bank i 's output, k_i is the level of financial capital at bank i , \mathbf{B} is a vector of parameters, $\ln C(\mathbf{y}_i, \mathbf{w}_i, q_i, k_i; \mathbf{B})$ is the predicted log cost function of a

cost-minimizing bank operating at $(\mathbf{y}_i, \mathbf{w}_i, q_i, k_i)$, v_i is a two-sided error term representing the statistical noise, and u_i is a one-sided error term representing inefficiency. The v_i are assumed to be independently and identically distributed, and the u_i are assumed to be distributed independently of the v_i .

Note that as in Mester (forthcoming) and Hughes and Mester (1993), output quality, q_i , and the level of financial capital, k_i , are included in the cost model, since both can affect measures of X-inefficiency. For example, if a bank has a large proportion of nonperforming loans, it might mean the bank scrimped on the initial credit evaluation and loan monitoring. This could show up as a short-run cost savings, and the bank might be labeled as more efficient than a bank that spent resources to ensure its loans were of higher quality.⁵ Financial capital is included to account for the probability of default, which again can affect measures of inefficiency, and also because financial capital is an input into the production process; it can be used to fund loans as a substitute for deposits or other borrowed money. The level of financial capital, rather than its price, is included because banks may not be using the cost-minimizing level of financial capital (banks might be risk-averse and there are regulatory minimum capital requirements—see Mester (forthcoming) and Hughes and Mester (1993) for further discussion).

Here, as is typical of most inefficiency studies, it is assumed that the v_i are normally distributed with mean 0 and variance σ_v^2 and the u_i are half-normally distributed, i.e., the u_i are the absolute values of a variable that is normally distributed with mean 0 and variance σ_u^2 . With these distributional assumptions, the log-likelihood function of the model is

$$\ln L = \frac{N}{2} \ln \frac{2}{\pi} - N \ln \sigma - \frac{1}{2\sigma^2} \sum_{i=1}^N \varepsilon_i^2 + \sum_{i=1}^N \ln \left[\Phi \left(\frac{\varepsilon_i \lambda}{\sigma} \right) \right] \quad (2)$$

where N is the number of firms, $\varepsilon_i = u_i + v_i$, $\sigma^2 = \sigma_u^2 + \sigma_v^2$, $\lambda = \frac{\sigma_u}{\sigma_v}$, and $\Phi(\cdot)$ is the standard normal cumulative distribution function. The model can be estimated using maximum likelihood techniques.

Inefficiency measures are calculated using the residuals after the model is estimated.⁶ For the half-normal case, an estimate of the mean inefficiency is given by $\hat{H}(u_i) \equiv \left(\frac{2}{\pi}\right)^{1/2} \hat{\sigma}_u$, where $\hat{\sigma}_u$ is the estimate of σ_u . Since the distribution of the maximum likelihood estimates is known, one can calculate an approximate standard error of $\left(\frac{2}{\pi}\right)^{1/2} \hat{\sigma}_u$.

A bank-level measure of inefficiency is usually given by the mean of the conditional distribution of u_i given ε_i . For the normal–half-normal stochastic model, the conditional distribution of u_i given ε_i is a normal distribution, $N(\mu_*, \sigma_*^2)$ truncated at zero, where $\mu_* \equiv \frac{\varepsilon_i \sigma_u^2}{\sigma^2}$ and $\sigma_*^2 \equiv \frac{\sigma_u^2 \sigma_v^2}{\sigma^2}$.⁷ The density function is

$$f(u_i | \varepsilon_i) = \frac{\frac{\sigma}{\sigma_u \sigma_v} \varphi \left(\frac{\sigma}{\sigma_u \sigma_v} (u_i | \varepsilon_i) - \frac{\varepsilon_i \lambda}{\sigma} \right)}{1 - \Phi \left(-\frac{\varepsilon_i \lambda}{\sigma} \right)}, \quad (u_i | \varepsilon_i) > 0. \quad (3)$$

The mean of this conditional distribution is

$$E(u_i | \varepsilon_i) = \left(\frac{\sigma_u \sigma_v}{\sigma} \right) \left[\frac{\varphi \left(\frac{\varepsilon_i \lambda}{\sigma} \right)}{\Phi \left(\frac{\varepsilon_i \lambda}{\sigma} \right)} + \frac{\varepsilon_i \lambda}{\sigma} \right]. \quad (4)$$

$E(u_i | \varepsilon_i)$ is an unbiased but inconsistent estimator of u_i , since regardless of the number of observations, N , the variance of the estimator remains nonzero [see Greene (1991), p. 18]. To get estimates, $\hat{H}(u_i | \varepsilon_i)$, of these measures, we evaluate (4) at the estimates of σ_u and σ_v . One can calculate an approximate standard error for these estimates by linearizing (4): the approximate standard error of $\frac{1}{n} \sum_{i=1}^n \hat{E}(u_i | \varepsilon_i)$ is $\frac{1}{n} \left(\sum_{i=1}^n \sum_{j=1}^n \text{Cov}(\hat{E}(u_i | \varepsilon_i), \hat{E}(u_j | \varepsilon_j)) \right)^{1/2}$. These approximate standard errors allow us to test whether the average of the estimates of $\hat{H}(u_i | \varepsilon_i)$ in any two Districts are statistically different.⁸ These standard errors give an indication of how precisely estimated $\hat{H}(u_i | \varepsilon_i)$ is, which, of course, depends on how precisely the parameters of the underlying distributions of u_i and v_i are estimated.

But mean is only one aspect of the conditional distribution of u_i given ε_i . How well the mean of the distribution characterizes the distribution and serves as a summary statistic for bank-specific inefficiency will depend on how widely spread the distribution is. The spread of the distribution can be characterized by confidence intervals for $(u_i | \varepsilon_i)$.⁹ It is important to remember, however, that these bands do not account for the fact that the parameters of the conditional distribution are estimated;¹⁰ they describe the conditional distribution, but do not indicate how precisely $E(u_i | \varepsilon_i)$ is estimated. As we will see below, there will often be a tradeoff: a larger sample size might allow $E(u_i | \varepsilon_i)$ to be more precisely

estimated, but because heterogeneity is likely to increase with sample size, $\hat{H}(u_i|\varepsilon_i)$ will become a poorer summary statistic for bank-specific inefficiency.

As shown in Mester (forthcoming), the 90 percent “confidence interval” for $(u_i|\varepsilon_i)$ is $[(u_i|\varepsilon_i)_{0.05}, (u_i|\varepsilon_i)_{0.95}]$ and the 95 percent “confidence interval” for $(u_i|\varepsilon_i)$ is $[(u_i|\varepsilon_i)_{0.025}, (u_i|\varepsilon_i)_{0.975}]$, where

$$(u_i|\varepsilon_i)_\alpha \equiv \sigma_* \Phi^{-1} \left[\alpha \left(1 - \Phi \left(-\frac{\mu_*}{\sigma_*} \right) \right) + \Phi \left(-\frac{\mu_*}{\sigma_*} \right) \right] + \mu_* . \quad (5)$$

Below, I compare X-inefficiency measures for the *National Model* and the *District Model*. The *National Model* restricts the cost frontier and the distribution of the error terms to be the same for all banks in the nation. That is, equation (1) is estimated including all the banks in the sample. This is comparable to models estimated in previous studies of X-inefficiency at U.S. banks.

However, the National Model might be too restrictive. If economic conditions differ across the nation, a single cost frontier might not be able to capture these differences. X-inefficiency measures based on such a model might be contaminated, since they are derived from the residuals in the estimation. That is, the better the specification of the cost frontier model, the better the measures of X-inefficiency can be expected to be. The *District Model* allows the cost frontier and error distribution to differ across Federal Reserve Districts.¹¹ This is equivalent to estimating equation (1) separately for each District. Of course, one drawback of the District Model compared to the National Model is that there are fewer observations per District than for the nation as a whole. But since the National Model is a restricted version of the District Model, a likelihood ratio test can be used to test the National Model vs. the District Model.

3. Functional form, data, and variables

I specify the translog functional form for the cost frontier $\ln C(\mathbf{y}_i, \mathbf{w}_i, q_i, k_i; \mathbf{B})$:

$$\begin{aligned} \ln C = & a_0 + \sum_i a_i \ln y_i + \sum_j b_j \ln w_j + \frac{1}{2} \sum_i \sum_j s_{ij} \ln y_i \ln y_j + \frac{1}{2} \sum_i \sum_j g_{ij} \ln w_i \ln w_j \\ & + \sum_i \sum_j d_{ij} \ln y_i \ln w_j + f_k \ln k + f_q \ln q + \frac{1}{2} r_{kk} \ln k \ln k + r_{kq} \ln k \ln q \\ & + \frac{1}{2} r_{qq} \ln q \ln q + \sum_j h_{kj} \ln k \ln y_j + \sum_j h_{qj} \ln q \ln y_j \end{aligned}$$

where: $s_{ij} = s_{ji}$ and $g_{ij} = g_{ji}$ by symmetry,

$$\sum_j b_j = 1, \quad \sum_j g_{ij} = 0, \quad \forall i, \quad \sum_j d_{ij} = 0, \quad \forall i,$$

$$\sum_j t_{kj} = 0, \quad \text{and} \quad \sum_j t_{qj} = 0 \quad \text{by linear homogeneity.}$$

I used 1991-92 data from the Consolidated Reports of Condition and Income that commercial banks must file each quarter.¹² There were 6630 banks in the sample. I excluded nonbank banks, banks in unit banking or restricted branch banking states, the special purpose banks in Delaware (legislated under the Financial Center Development Act and the Consumer Credit Bank Act), de novo banks (i.e., banks less than five years old as of December 1992, which have start-up costs that more mature banks do not have), banks that were involved in a merger in 1991-92, banks that bought branches from failed banks in 1991-92, banks that changed charter type in 1991-92, banks that changed holding company status in 1991-92, banks that changed Federal Reserve System membership in 1991-92, and banks with assets over \$4 billion in 1992, since larger banks very likely use a different production technology than smaller banks. The \$4 billion cutoff was chosen so that the Third District banks included in this sample were the same banks included in the sample used in Mester (forthcoming). The median asset size of banks included was \$59 million, and the average asset size was \$144 million.

The outputs and inputs used here are the same as those in Mester (forthcoming) and are based on the intermediation (also known as the asset) approach to the banking firm, which views the bank as using labor, physical capital, and funding to produce earning assets [see Sealey and Lindley (1977)]. The three outputs included were y_1 = real estate loans, y_2 = commercial and industrial loans, lease financing receivables, agricultural loans, loans to depository institutions, acceptances of other banks, loans to foreign governments, obligations of states and political subdivisions, and other loans, and y_3 = loans to individuals. Each was measured by the average of the dollar volumes reported in December 1992 and

December 1991. These three outputs account for nearly all of a bank's non-securities earning assets. The average volume of each of these three outputs at banks in the sample was about \$43 million, \$23 million, and \$14 million, respectively. Thus, about 54 percent of the average bank's loan portfolio is in real estate, about 29 percent is business loans, and the rest is loans to individuals.

The inputs whose prices were used to estimate the cost frontier included labor, physical capital, and borrowed money (including deposits, federal funds, and other borrowed money) used to fund the outputs. The wage rate w_1 was proxied by [salaries and benefits expenses in 1992/number of full-time equivalent employees at year-end 1992]. The unit price of physical capital, w_2 , was constructed as the [premises and fixed assets (net of rental income) expenses in 1992/average value of premises and fixed assets in 1992]. The borrowed money price, w_3 , was constructed as [(interest expense on deposits, net of service charges + interest expense of fed funds purchased and securities sold under agreements to repurchase + interest on demand notes issued to the U.S. Treasury and on other borrowed money + interest on subordinated notes and debentures in 1992)/average dollar volume of these types of funds in 1992]. Total costs, C , were labor, premises, and fixed assets expense, and (interest expense on deposits, net of service charges + interest expense of fed funds purchased and securities sold under agreements to repurchase + interest on demand notes issued to the U.S. Treasury and on other borrowed money + interest on subordinated notes and debentures in 1992) \times (real estate loans, commercial and industrial loans, lease financing receivables, agricultural loans, loans to depository institutions, acceptances of other banks, loans to foreign governments, obligations of states and political subdivisions, other loans, and loans to individuals/total earning assets).¹³

Loan output quality, q , was proxied by the average volume of nonperforming loans in 1992. Nonperforming loans are loans that are 30 or more days past due but still accruing interest plus loans that are not accruing interest. The volume of a bank's nonperforming loans relative to the level of bank output is inversely related to the bank's loan quality. Note that while the national macroeconomy can affect nonperforming loans, its effect is felt equally across banks. The differences in nonperforming loans across banks capture not only differences in quality across banks, but also differences in local

economic conditions to the extent that banks operate in local loan markets. Finally, financial capital, k , was measured as the average volume of equity capital in 1992. A summary of the data and the maximum likelihood parameter estimates of the models are available from the author.

4. Empirical results

Table 1 reports estimates of the inefficiency measures discussed in Section 2 for the National Model, along with approximate standard errors, and the minimum and maximum values of $\hat{H}(u_i|\varepsilon_i)$ for the entire sample and by District. The National Model shows average X-inefficiency at U.S. banks to be around 16 percent. That is, given its particular output level and mix, on average, if a bank were to use its inputs as efficiently as possible, it could reduce its production cost by roughly 16 percent.¹⁴ The table also shows that the average level of inefficiency, as measured by average $\hat{H}(u_i|\varepsilon_i)$, varies over the Districts, from 13.5 percent in the Third District to 18.8 percent in the Eleventh District. These measures are statistically significantly different from zero, and all are statistically significantly different from one another at the 10 percent or better level, except that the First and Fifth, Third and Eighth, and Sixth and Twelfth Districts' estimates do not statistically differ from one another, respectively. As shown in the table, the results are similar for $\hat{H}(u_i|\varepsilon_i)$ evaluated at the average levels of cost, outputs, and input prices in each District. This can be thought of as the inefficiency of the “average bank” in the District. Thus, there is significant variation in inefficiency across most Districts. That the average level of inefficiency is somewhat lower than that found in previous national studies might be due to the fact that here I control for output risk and quality or because I am using more recent data.

Of course, the averages obscure some of the variation in inefficiency across the sample, and there is actually quite a lot of variation: from 2.2 percent to 89.2 percent. In the National Model, the most efficient bank is located in the Third District; its estimated inefficiency measure, $\hat{H}(u_i|\varepsilon_i)$, is 0.0224, with approximate standard error 3.56×10^{-4} . The 90 percent “confidence interval” is [0.00123, 0.0685]; the 95 percent “confidence interval” is [0.000609, 0.0835]. The least efficient bank is located in the Tenth District; its estimated inefficiency measure, $\hat{H}(u_i|\varepsilon_i)$, is 0.892, with approximate standard error 1.42×10^{-2} . The 90 percent “confidence interval” is [0.691, 1.09]; the 95 percent “confidence interval” is [0.653,

1.13].

Table 2 reports similar information for the District Model. Note here, though, that the estimates are based on cost frontiers and error structures that are allowed to differ across Districts. The first thing to note is that for three of the Districts—2, 5, and 12—the degree of skewness of the ordinary least squares residual of the cost function estimation was negative. As shown by Waldman (1982) this means that the maximum likelihood estimates of the model are the ordinary least squares estimates; in other words, the one-sided component of the error term is zero. More generally, this suggests that the frontier model with normal–half-normal error term does not fit the data in these Districts. Also note that convergence was not achieved in District 1; the difficulty might be a lack of observations.

One striking thing to note is that based on the District Model, two of the Districts—3 and 4—have average inefficiency levels, measured by average $\hat{H}(u_i|\varepsilon_i)$ or by $\hat{H}(u_i)$, under 10 percent, while two other Districts—8 and 9—have average inefficiency levels below 14 percent. These estimates are quite a bit lower than ones found in the National Model or in previous studies. The average inefficiency, by either measure, for each District is statistically significantly different from zero, and the measures are statistically significantly different from one another, at the 10 percent or better level, for the following pairs of Districts: 3&6, 3&10, 3&11, 4&6, 4&7, 4&10, 4&11, 6&8, 8&10, and 8&11. Similar results are obtained for $\hat{H}(u_i|\varepsilon_i)$ at the “average bank” in each District, with stronger statistical differences across Districts.

As was true for the National Model, the average levels of inefficiency across Districts in the District Model also obscure substantial variation in inefficiency across banks in the sample. In the District Model, the most efficient bank is located in the Seventh District; its estimated inefficiency measure, $\hat{H}(u_i|\varepsilon_i)$, is 0.0263, with approximate standard error 1.26×10^{-3} . The 90 percent “confidence interval” is [0.00148, 0.0751]; the 95 percent “confidence interval” is [0.000731, 0.0901]. The least efficient bank is located in the Tenth District; its estimated inefficiency measure, $\hat{H}(u_i|\varepsilon_i)$, is 0.926, with approximate standard error 2.89×10^{-2} . The 90 percent “confidence interval” is [0.740, 1.11]; the 95 percent “confidence interval” is [0.704, 1.15].

Table 3 presents some measures that can be used in comparing the National and District Models. As indicated in the top panel, the average level of inefficiency over the eight Districts for which estimates were obtained in the District Model is about 2 percentage points lower when measured by $\hat{H}(u_i)$, and 1 percentage point lower when measured by average $\hat{H}(u_i|\varepsilon_i)$, for the District Model than for the National Model, i.e., 14 or 15 percent compared to 16 percent. A t-test indicates that this difference is statistically different for $\hat{H}(u_i)$; t-tests also indicate that the Fourth District's average $\hat{H}(u_i|\varepsilon_i)$ estimated from the National Model differs significantly from that estimated from the District Model.

Another thing to note is that there is a tradeoff between the precision of the estimates $\hat{H}(u_i|\varepsilon_i)$ and the ability of $\hat{H}(u_i|\varepsilon_i)$ to serve as a summary statistic for bank-specific inefficiency. Since the sample size is larger when estimating the National Model, the parameters of the error structure are more precisely estimated, and this, in turn, means that the bank-specific measures of inefficiency, $\hat{H}(u_i|\varepsilon_i)$, are more precisely measured than they are in the District Model. However, the spreads of the conditional distributions $(u_i|\varepsilon_i)$ (as indicated by the widths of the 90 and 95 percent confidence intervals) are generally wider for the National Model than for the District Model.¹⁵ These spreads are wider because the banks in the National sample are more heterogeneous than the banks in each District. Thus, while the $\hat{H}(u_i|\varepsilon_i)$'s are more precisely estimated for the National Model, they are poorer summary statistics for bank-specific inefficiency in the National Model than in the District Model.

The middle panel lists the Districts ranked by average inefficiency (i.e., by average $\hat{H}(u_i|\varepsilon_i)$ and by $\hat{H}(u_i|\varepsilon_i)$ at the "average bank" in the District), from most efficient to least efficient. The arrows under the Districts indicate where the null hypothesis of the Districts having the same level of inefficiency cannot be rejected: the District at the base of an arrow (\pm) has an insignificantly different level of inefficiency from every District covered by the arrow. For example, in the National Model, the null hypothesis of equal $\hat{H}(u_i|\varepsilon_i)$'s is rejected for all pairs of Districts except for Districts 3 and 8. For the District Model, the null hypothesis of equal average $\hat{H}(u_i|\varepsilon_i)$'s cannot be rejected for Districts 3 and 4, 3 and 8, 3 and 9, and 3 and 7, but can be rejected for 4 and 7.

Note that the rankings for the National and District Models are quite similar at the top and

bottom—for example, both models indicate that average inefficiency is lowest in the Third District and highest in the Eleventh District. It is in the middle range where there is a discrepancy. In particular, in the National Model, District 7 ranks significantly more efficient than District 4, but in the District Model, District 4 ranks significantly more efficient than District 7.

The bottom panel gives the correlation coefficient between the National and District Models for the point estimates, $\hat{E}(u_i|\varepsilon_i)$, and for the width of the “confidence intervals” for all banks in the eight Districts. In all cases, the correlation is quite high, but is below 90 percent, suggesting that modeling assumptions do matter in efficiency studies.

The final, and most important test, is a likelihood ratio test of the National Model against the District Model. The value of the likelihood ratio test statistic (when both Models were estimated excluding District 1 banks, since convergence was not achieved in District 1 in the District Model) is 1836.15; the number of restrictions is 380. Thus, the National Model is strongly statistically rejected by the data.¹⁶

Perhaps a better way to see the differences is by comparing Figures 1 and 2, which give the frequency distributions of the bank-specific inefficiency measures, $\hat{E}(u_i|\varepsilon_i)$, for the National and District Models, respectively. As can be seen, the left-hand tail (low inefficiency measures) is quite a bit thicker for the District Model.

It remains to determine what accounts for the differences in inefficiency across Districts. While a thorough investigation is beyond the scope of this paper, we can determine whether the differences are accounted for merely by differences in the sizes of banks across Districts, the geographic size of the District, and/or the population of the District. To investigate this question, I employed a two-step procedure for both the National and District Models. In step 1, I regressed the bank-specific inefficiency measures, $\hat{E}(u_i|\varepsilon_i)$, from the model on a constant term, bank size (measured as the average asset size of the bank in December 1991 and December 1992), the land area of the District in which the bank is located, and the population of the District. $\hat{E}(u_i|\varepsilon_i)$ is between 0 and 1 for each bank, so I specified a logistic relationship between the explanatory variables and $\hat{E}(u_i|\varepsilon_i)$.¹⁷ Thus, I estimated

$$\hat{E}(u_i | \varepsilon_i) = \frac{\exp(\mathbf{X}_i' \boldsymbol{\gamma})}{1 + \exp(\mathbf{X}_i' \boldsymbol{\gamma})} + \hat{\xi}_i. \quad (7)$$

In step 2, I regressed the estimated residual from the step 1 estimation, $\hat{\xi}_i$, on a constant term and a set of dummy variables, D_1, \dots, D_{12} for the National Model and $D_3, D_4, D_6, D_7, D_8, D_9, D_{10}$ for the District Model, where $D_i = 1$ if the bank is in District i , and 0 otherwise. I used a likelihood ratio test to test whether the coefficients on the dummy variables equal the coefficient on the constant.¹⁸ If so, one could conclude that the differences in the bank-specific inefficiency measures were explained by bank size, District size, and District population and that any residual differences were unrelated to the District in which the bank was located. However, for both the National and District Models, this null hypothesis can be rejected at the 0.001 level of significance. That is, there are differences in inefficiency across Districts not accounted for by differences in bank size, District size, and District population. (The value of the likelihood ratio test statistic was 325.02 for the National Model, with 11 degrees of freedom, and 80.85 for the District Model, with 7 degrees of freedom.¹⁹) In both Models, in step 1, bank size was insignificantly positively related to inefficiency, while District size and District population were significantly positively related to inefficiency.

5. Conclusions

One could argue that the differences found between the National and District Models are not that large. However, a one-to-two-percentage-point difference in the average level of inefficiency seems significant enough, and the differences between the $\hat{E}(u_i)$ for the nation based on the National Model (i.e., 16 percent), and the $\hat{E}(u_i)$'s for, say, the Third and Fourth Districts based on the District Model (i.e., 7.9 percent and 9.3 percent, respectively) seem quite large. These differences might be large enough to lead one to different conclusions about the health of the banking industry or the potential for inducing further cost efficiency via mergers, for example.

The differences found here also suggest that Mester (forthcoming) found a significantly lower level of inefficiency in her study of Third District banks compared to previous national studies both because Third District banks seem to be operating more efficiently, *on average*, than banks elsewhere in

the nation, and because she focused on a more homogeneous market, while the national studies estimated a single cost frontier across heterogeneous markets.

The general lesson here is that it would appear to make sense to test whether one's model fits well across heterogeneous markets. This is likely to be especially important in inefficiency studies using the stochastic econometric frontier methodology, because the inefficiency measures are, by design, residuals. It is also important to remember that there is often a tradeoff between the precision of the estimates of bank-specific inefficiency, $\hat{H}(u_i|\varepsilon_i)$, and the value of $E(u_i|\varepsilon_i)$, as an indicator of bank-specific inefficiency. While increasing the sample size will likely increase the precision of $\hat{H}(u_i|\varepsilon_i)$, it may also make $E(u_i|\varepsilon_i)$ a poorer summary statistic for bank-specific inefficiency, since bank heterogeneity is likely to increase with sample size.

References

- Akhavein, J.D., Swamy, P.A.V.B., and Taubman, S.B. (forthcoming), A general method for deriving the efficiencies of banks from a profit function, *Journal of Productivity Analysis*.
- Berger, A.N., and DeYoung, R. (1995), Problem loans and cost efficiency in commercial banks, Economics and Policy Analysis Working Paper 95-5 (December).
- Berger, A.N., Hunter, W.C., and Timme, S.G. (1993), The efficiency of financial institutions: a review and preview of research past, present, and future, *Journal of Banking and Finance* 17, 221-249.
- Board of Governors of the Federal Reserve System. (1994), *The Federal Reserve System: purposes and functions*, Washington, D.C.
- Greene, W.H. (1991), The econometric approach to efficiency measurement, mimeo, Stern School of Business, New York University.
- Greene, W.H. (1993), *Econometric analysis*, 2nd edition, Macmillan Publishing Co., New York.
- Hughes, J.P., and Mester, L.J. (1993), A quality and risk-adjusted cost function for banks: evidence on the "too-big-to-fail" doctrine, *Journal of Productivity Analysis* 4, 293-315.
- Hughes, J.P., Lang, W., Mester, L.J., and Moon, C-G. (1995), Recovering technologies that account for generalized managerial preferences: an application to non-risk-neutral banks, Federal Reserve Bank of Philadelphia Working Paper No. 95-8/R (September).
- Hughes, J.P., Lang, W., Mester, L.J., and Moon, C-G. (forthcoming), Efficient banking under interstate branching, *Journal of Money, Credit, and Banking*.
- Hunter, W.C., Timme, S.G., and Yang, W.K. (1990), An examination of cost subadditivity and multiproduct production in large U.S. banks, *Journal of Money, Credit, and Banking* 22, 504-525.
- Johnson, N.L., Kotz, S., and Balakrishnan, N. (1994), *Continuous univariate distributions*, 2nd edition, John Wiley and Sons, Inc., New York.
- Jondrow, J., Lovell, C.A.K, Materov, I.S., and Schmidt, P. (1982), On the estimation of technical inefficiency in the stochastic frontier production function model, *Journal of Econometrics* 19,

233-238.

Kolari, J., and Zardkoohi, A. (1995), Economies of scale and scope in commercial banks with different output mixes, Texas A&M Working Paper.

Mester, L.J. (1992), Traditional and nontraditional banking: an information-theoretic approach, *Journal of Banking and Finance* 16, 545-566.

Mester, L.J. (1993), Efficiency in the savings and loan industry, *Journal of Banking and Finance* 17, 267-286.

Mester, L.J. (forthcoming), A study of bank efficiency taking into account risk-preferences, *Journal of Banking and Finance*.

Sealey, C.W., and Lindley, J.T. (1977), Inputs, outputs, and theory of production cost at depository financial institutions, *Journal of Finance* 32, 1251-1266.

Waldman, D.M. (1982), A stationary point for the stochastic frontier likelihood, *Journal of Econometrics* 18, 275-279.

Table 1. National Model: X-Inefficiency Measures

	$\hat{E}(u_i) (2/\pi)^{1/2} \sigma_u$ (Approx. Std. Err.)	Average $\hat{E}(u_i \varepsilon_i)$ (Approx. Std. Err.) ^b	$\hat{E}(u_i \varepsilon_i)$ at “average bank” (Approx. Std. Err.) ^c	Min $\hat{E}(u_i \varepsilon_i)$ (Approx. Std. Err.)	Max $\hat{E}(u_i \varepsilon_i)$ (Approx. Std. Err.)
U.S. (6,630 banks)	0.161 ^a (5.54×10 ⁻³)	0.160 ^a (5.42×10 ⁻³)	0.119 ^a (3.08×10 ⁻³)	0.0224 ^a (3.56×10 ⁻⁴)	0.892 ^a (1.42×10 ⁻²)
District 1: Boston (122 banks)		0.150 ^a (5.06×10 ⁻³)	0.113 ^a (3.28×10 ⁻³)	0.0312 ^a (4.98×10 ⁻⁴)	0.450 ^a (7.17×10 ⁻³)
District 2: New York (173 banks)		0.172 ^a (6.46×10 ⁻³)	0.125 ^a (4.20×10 ⁻³)	0.0383 ^a (6.11×10 ⁻⁴)	0.815 ^a (1.30×10 ⁻²)
District 3: Philadelphia (214 banks)		0.135 ^a (4.02×10 ⁻³)	0.101 ^a (2.55×10 ⁻³)	0.0224 ^a (3.61×10 ⁻⁴)	0.367 ^a (5.85×10 ⁻³)
District 4: Cleveland (417 banks)		0.139 ^a (4.15×10 ⁻³)	0.113 ^a (2.95×10 ⁻³)	0.0384 ^a (6.12×10 ⁻⁴)	0.418 ^a (6.67×10 ⁻³)
District 5: Richmond (462 banks)		0.148 ^a (4.66×10 ⁻³)	0.120 ^a (3.21×10 ⁻³)	0.0321 ^a (5.11×10 ⁻⁴)	0.403 ^a (6.42×10 ⁻³)
District 6: Atlanta (767 banks)		0.181 ^a (6.73×10 ⁻³)	0.133 ^a (3.84×10 ⁻³)	0.0337 ^a (5.37×10 ⁻⁴)	0.717 ^a (1.14×10 ⁻²)
District 7: Chicago (645 banks)		0.137 ^a (3.99×10 ⁻³)	0.106 ^a (2.45×10 ⁻³)	0.0428 ^a (6.78×10 ⁻⁴)	0.534 ^a (8.52×10 ⁻³)
District 8: St. Louis (628 banks)		0.135 ^a (3.83×10 ⁻³)	0.119 ^a (2.90×10 ⁻³)	0.0399 ^a (6.36×10 ⁻⁴)	0.456 ^a (7.27×10 ⁻³)
District 9: Minneapolis (352 banks)		0.143 ^a (4.38×10 ⁻³)	0.099 ^a (1.99×10 ⁻³)	0.0457 ^a (7.29×10 ⁻⁴)	0.473 ^a (7.55×10 ⁻³)
District 10: Kansas City (1353 banks)		0.161 ^a (5.59×10 ⁻³)	0.121 ^a (3.19×10 ⁻³)	0.0384 ^a (6.12×10 ⁻⁴)	0.892 ^a (1.42×10 ⁻²)
District 11: Dallas (989 banks)		0.188 ^a (7.50×10 ⁻³)	0.158 ^a (5.74×10 ⁻³)	0.0416 ^a (6.63×10 ⁻⁴)	0.811 ^a (1.29×10 ⁻²)
District 12: San Francisco (508 banks)		0.180 ^a (7.04×10 ⁻³)	0.124 ^a (3.82×10 ⁻³)	0.0312 ^a (5.00×10 ⁻⁴)	0.620 ^a (9.89×10 ⁻³)

^aSignificantly different from zero at the 5 percent level, two-tailed test.

^bEach District's average $\hat{E}(u_i | \varepsilon_i)$ is statistically different from another's at the 10 percent or better level, two-tailed test, except for the following pairs: 1&5, 3&8, and 6&12.

^cEach District's $\hat{E}(u_i | \varepsilon_i)$ for the “average bank” is statistically different from another's at the 10 percent or better level, two-tailed test, except for the following pairs: 1&4, 2&12, 3&9, 5&8, 5&10, and 8&10.

Table 2. District Model: X-Inefficiency Measures

	$\hat{E}(u_i) = (2/\pi)^{1/2} \sigma_u$ (Approx. Std. Err.)	Average $\hat{E}(u_i \varepsilon_i)$ (Approx. Std. Err.) ^c	$\hat{E}(u_i \varepsilon_i)$ at “average bank” (Approx. Std. Err.) ^d	Min $\hat{E}(u_i \varepsilon_i)$ (Approx. Std. Err.)	Max $\hat{E}(u_i \varepsilon_i)$ (Approx. Std. Err.)
District 1: Boston (122 banks)	Lack of Convergence				
District 2: New York (173 banks)	Skewness < 0				
District 3: Philadelphia (214 banks)	0.0790 ^b (4.80×10 ⁻²)	0.0789 ^a (4.70×10 ⁻²)	0.0532 ^a (1.67×10 ⁻²)	0.0294 ^a (5.63×10 ⁻³)	0.230 ^a (4.39×10 ⁻²)
District 4: Cleveland (417 banks)	0.0926 ^a (2.79×10 ⁻²)	0.0923 ^a (2.74×10 ⁻²)	0.0666 ^a (1.27×10 ⁻²)	0.0266 ^a (2.70×10 ⁻³)	0.299 ^a (3.05×10 ⁻²)
District 5: Richmond (462 banks)	Skewness < 0				
District 6: Atlanta (767 banks)	0.171 ^a (2.20×10 ⁻²)	0.170 ^a (2.09×10 ⁻²)	0.118 ^a (1.01×10 ⁻²)	0.0362 ^a (2.28×10 ⁻³)	0.773 ^a (4.87×10 ⁻²)
District 7: Chicago (645 banks)	0.151 ^a (1.14×10 ⁻²)	0.150 ^a (1.05×10 ⁻²)	0.0904 ^a (6.62×10 ⁻³)	0.0263 ^a (1.26×10 ⁻³)	0.773 ^a (3.72×10 ⁻²)
District 8: St. Louis (628 banks)	0.124 ^a (1.60×10 ⁻²)	0.124 ^a (1.54×10 ⁻²)	0.109 ^a (1.27×10 ⁻²)	0.0293 ^a (1.90×10 ⁻³)	0.524 ^a (3.40×10 ⁻²)
District 9: Minneapolis (352 banks)	0.137 ^a (1.73×10 ⁻²)	0.135 ^a (1.60×10 ⁻²)	0.0948 ^a (1.04×10 ⁻²)	0.0279 ^a (2.02×10 ⁻³)	0.643 ^a (4.69×10 ⁻²)
District 10: Kansas City (1353 banks)	0.164 ^a (1.05×10 ⁻²)	0.163 ^a (1.05×10 ⁻²)	0.125 ^a (7.00×10 ⁻³)	0.0318 ^a (1.00×10 ⁻³)	0.926 ^a (2.89×10 ⁻²)
District 11: Dallas (989 banks)	0.174 ^a (1.85×10 ⁻²)	0.174 ^a (1.80×10 ⁻²)	0.148 ^a (1.33×10 ⁻²)	0.0417 ^a (2.14×10 ⁻³)	0.668 ^a (3.43×10 ⁻²)
District 12: San Francisco (508 banks)	Skewness < 0				

^aSignificantly different from zero at the 5 percent level, two-tailed test.

^bSignificantly different from zero at the 10 percent level, two-tailed test.

^cThe average $\hat{E}(u_i | \varepsilon_i)$'s in the following pairs of Districts are statistically different from one another at the 10 percent or better level, two-tailed test: 3&6, 3&10, 3&11, 4&6, 4&7, 4&10, 4&11, 6&8, 8&10, and 8&11.

^dEach District's $\hat{E}(u_i | \varepsilon_i)$ for the “average bank” is statistically different from another's at the 10 percent or better level, two-tailed test, except for the following pairs: 3&4, 6&8, 6&9, 6&10, 7&8, 7&9, 8&9, 8&10, and 10&11.

Figure 1

National Model Inefficiency Distribution

(Districts 3, 4, 6, 7, 8, 9, 10, 11)

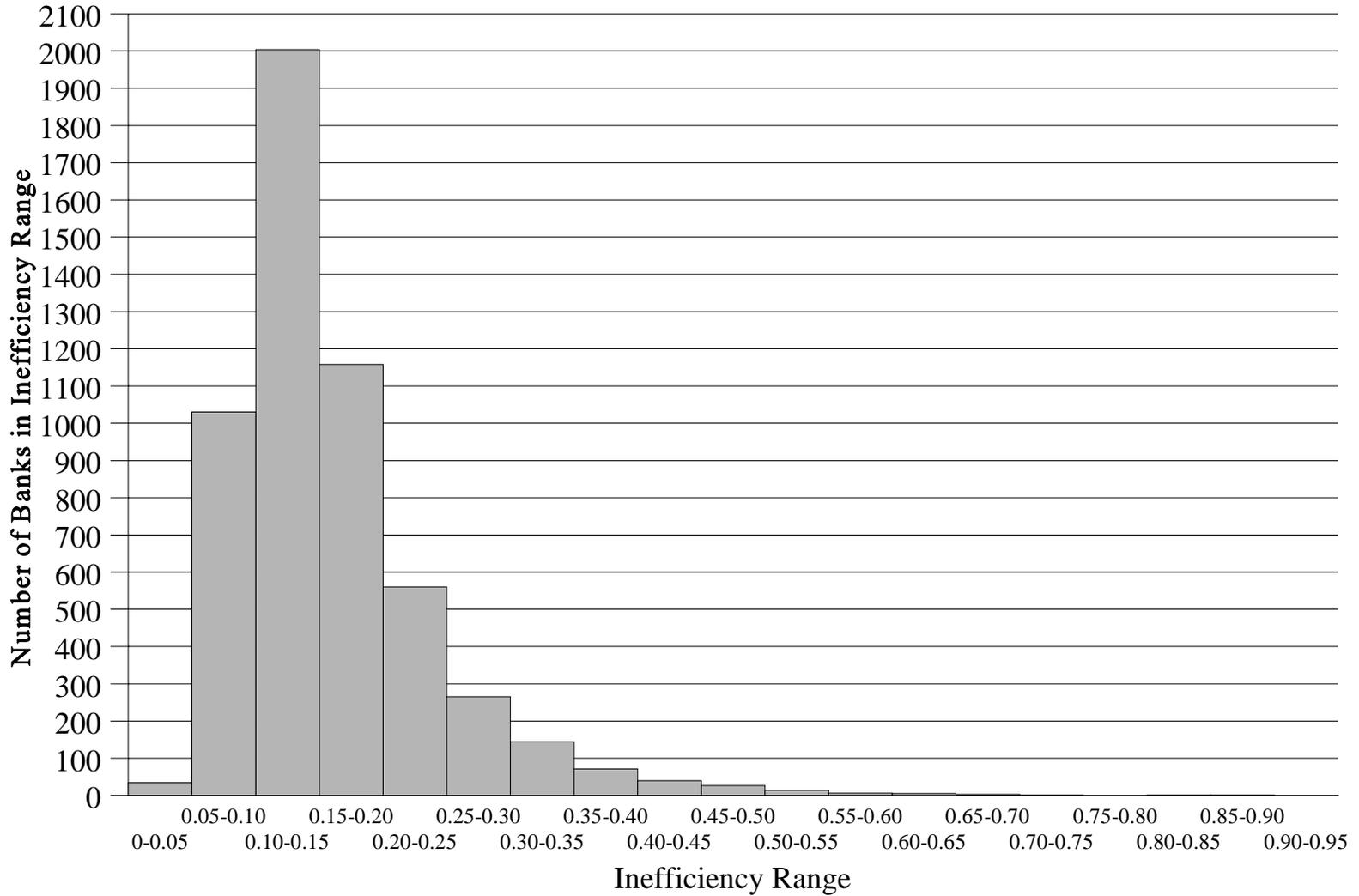
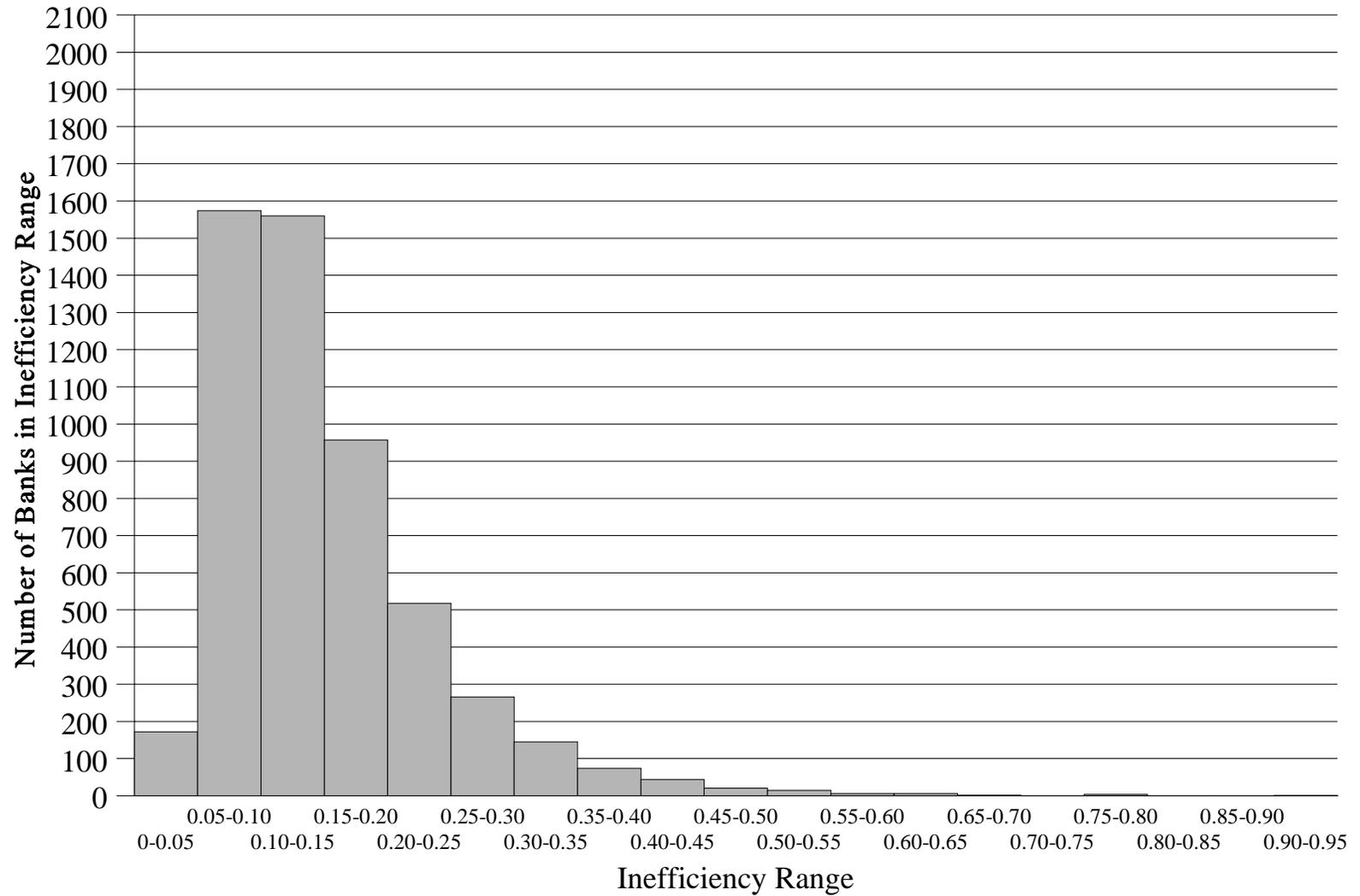


Figure 2**District Model Inefficiency Distribution**

(Districts 3, 4, 6, 7, 8, 9, 10, 11)



Notes

1. The United States is divided into 12 Federal Reserve Districts, with a Reserve Bank located in each District. These Reserve Banks implement many functions of the Federal Reserve System, including coin and currency distribution, commercial bank examinations, and fiscal-agency functions for the U.S. Treasury. (See *Board of Governors*, 1994).

2. See Hughes and Mester (1993) for further discussion.

3. Berger and DeYoung (1995) discuss the implications of using nonperforming loans to measure loan quality, as is done here and in other papers, including Mester (forthcoming), Hughes and Mester (1993), and two papers written subsequently to this one [Hughes, Lang, Mester and Moon (1995, forthcoming)].

4. Other common methodologies are data envelopment analysis, "thick frontier" analysis, and the "distribution-free" approach. Mester (forthcoming) and Berger, Hunter, and Timme (1993) discuss the pros and cons of these approaches.

5. See Berger and DeYoung (1995) for further discussion about controlling for nonperforming loans when measuring inefficiency. In particular, a portion of loan quality is endogenous; they propose a statistical test for determining whether nonperforming loans should be included in the cost function.

6. This discussion of the inefficiency measures and confidence intervals follows Mester (forthcoming).

7. This can be seen by adapting for the cost function the equation for the production function derived in Jondrow et al. (1982).

8. Note that in comparing the average inefficiency for two Districts based on the National Model, described below, the covariances between these average measures must also be taken into account; since the District Model, described below, estimates separate cost functions and error terms for the District, these covariances are zero.

9. To my knowledge, only Jondrow et al. (1982) and Mester (forthcoming) report confidence bands.

10. Henceforth, I will use quotation marks around the term confidence interval as a reminder of this

fact.

11. Smaller or other geographic or nongeographic delineations than Federal Reserve Districts might better define markets, but there would have been too few banks to estimate the models if a finer delineation had been used. We show below, however, that the differences in inefficiency across Districts reflect more than just differences in the geographic size of the Districts.

12. Since branching restrictions in some states had only relatively recently been eased (e.g., in Pennsylvania, branching throughout the state became totally unrestricted only on March 4, 1990), more years of data were not included in the study.

13. As in Hunter, Timme, and Yang (1990), Mester (1992), and Hughes and Mester (1993), and Mester (forthcoming), the interest expense was weighted by total output/total earning assets to reflect the interest expense that can be allocated to the bank's loan output.

14. To put this number into perspective, consider that the average annual cost of output production at banks in the sample was about \$5-1/4 million, so a 16 percent reduction in cost could potentially add about \$840,000 to bank profits. Since the average bank's size in the sample was around \$144 million in assets, this constitutes a potential increase of 0.6 percent in before-tax return on assets, or nearly 0.4 percent in after-tax ROA, a significant increase.

15. In only two Districts, 6 and 11, is the average width of the "confidence intervals" based on the District Model larger (and only slightly so) than the average width of the "confidence intervals" based on the National Model. This might suggest that the Sixth and Eleventh Districts have substantial within-District heterogeneity.

16. Dropping Districts 1, 2, 5, and 12, the value of the likelihood ratio test statistic is 1441.9577, and the number of restrictions is 266. The data also reject the more flexible model obtained by allowing the constant terms to differ across Districts in the National model, but constraining the rest of the cost function and error term parameters to be the same across Districts.

17. The results are qualitatively similar if instead I specify a linear relationship in step 1.

18. The results of these estimations are available from the author.

19. When a linear relationship instead of a logistic relationship is estimated in step 1, the null hypothesis is still strongly rejected: the values of the likelihood ratio test statistics are 320.56 for the National Model and 70.02 for the District Model.

Means of the Variables

Variable		Mean Value at U.S. Banks (6630 banks)
	Total assets (thousands \$)	143,990
y_1	Real estate loans (thousands \$)	42,513
y_2	Commercial and industrial and other loans (thousands \$)	22,932
y_3	Loans to individuals (thousands \$)	14,435
w_1	Price of labor (thousands \$ per employee)	30.4
w_2	Price of physical capital (thousands \$ per thousand \$)	0.42
w_3	Price of deposits and other borrowed money (thousands \$ per thousand \$)	0.038
k	Financial capital (thousands \$)	11,095
q	Nonperforming loans (thousands \$)	3,525
C	Total cost (thousands \$)	5,288

National Model: Five Most Efficient and Five Least Efficient Banks

	Location (District)	$\hat{E}(u_i \varepsilon_i)$ (Approx. Std. Err.)	90 percent “confidence interval”	95 percent “confidence interval”
Five Most Efficient Banks	3	0.0224* (3.56×10^{-4})	[0.00123, 0.0685]	[0.000609, 0.0835]
	3	0.0304* (4.84×10^{-4})	[0.00165, 0.0886]	[0.000816, 0.107]
	12	0.0312* (4.97×10^{-4})	[0.00170, 0.0908]	[0.000840, 0.110]
	1	0.0312* (4.98×10^{-4})	[0.00170, 0.0910]	[0.000842, 0.110]
	5	0.0321* (5.11×10^{-4})	[0.00175, 0.0931]	[0.000866, 0.113]
Five Least Efficient Banks	10	0.892* (1.42×10^{-2})	[0.691, 1.09]	[0.653, 1.13]
	2	0.815* (1.30×10^{-2})	[0.614, 1.02]	[0.575, 1.05]
	11	0.811* (1.29×10^{-2})	[0.610, 1.01]	[0.571, 1.05]
	6	0.717* (1.14×10^{-2})	[0.516, 0.917]	[0.477, 0.956]
	10	0.691* (1.09×10^{-2})	[0.491, 0.892]	[0.452, 0.931]

*Significantly different from zero at the 5 percent level, two-tailed test.

District Model: Five Most Efficient and Five Least Efficient Banks

	Location (District)	$\hat{E}(u_i \varepsilon_i)$ (Approx. Std. Err.)	90 percent “confidence interval”	95 percent “confidence interval”
Five Most Efficient Banks	7	0.0263* (1.27×10 ⁻³)	[0.00148, 0.0751]	[0.000731, 0.0901]
	4	0.0266* (2.71×10 ⁻³)	[0.00149, 0.0759]	[0.000737, 0.0911]
	7	0.0269* (1.29×10 ⁻³)	[0.00152, 0.0766]	[0.000749, 0.0918]
	9	0.0279* (2.03×10 ⁻³)	[0.00157, 0.0796]	[0.000776, 0.0954]
	8	0.0293* (1.90×10 ⁻³)	[0.00166, 0.0834]	[0.000820, 0.0999]
Five Least Efficient Banks	10	0.926* (2.89×10 ⁻²)	[0.740, 1.11]	[0.704, 1.15]
	6	0.773* (4.87×10 ⁻²)	[0.570, 0.977]	[0.531, 1.02]
	7	0.773* (3.72×10 ⁻²)	[0.639, 0.906]	[0.614, 0.931]
	10	0.767* (2.39×10 ⁻²)	[0.580, 0.953]	[0.545, 0.988]
	7	0.761* (3.66×10 ⁻²)	[0.627, 0.894]	[0.602, 0.920]

*Significantly different from zero at the 5 percent level, two-tailed test.