Methodology for the Aruoba-Diebold-Scotti Business Conditions Index

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1. Broad Overview

The Aruoba-Diebold-Scotti (ADS) business conditions index is designed to track real business conditions at high observation frequency. Its underlying seasonally adjusted economic components blend high-frequency and low-frequency data. The index is updated in real time as new or revised data on the index's components are released by U.S. government statistical agencies. (These statistical agencies are the U.S. Bureau of Economic Analysis [BEA], the U.S. Bureau of Labor Statistics [BLS], the U.S. Census Bureau [CB], the U.S. Department of Labor [DOL], and the Federal Reserve Board of Governors [FRB]). At the time of any ADS update, the index is based upon all information on all components available at that time. The index is updated about eight times per month. The components of the index are:

- Weekly Initial Unemployment Claims (Source: DOL);
- Monthly Payroll Employment (Source: BLS);
- Monthly Industrial Production (Source: FRB);
- Monthly Real Personal Income Less Transfer Payments (Source: BEA);
- Monthly Real Manufacturing and Trade Sales (Source: BEA, CB);
- Quarterly Real GDP (BEA).

The theoretical average value of the ADS index is zero because the underlying mathematical model imposes that value. However, over any particular time span of observations, the average ADS might deviate quite far from zero. Progressively larger positive ADS values over time indicate progressively better-than-average business conditions. Progressively more negative values indicate progressively worse-than-average business conditions.

The ADS index may be used to compare business conditions at different points in time. A value of -3.0, for example, would indicate business conditions noticeably worse than at any time in either the 1990-91 recession or the 2001 recession, during which the ADS index never dropped below -2.0.

2. Index Components and Data Transformations

Raw Data. The ADS index is based upon six underlying component series, also called the source data. We collect our source data from U.S. government statistical agencies via Haver Analytics. Note that, in some cases, we adjust the data in levels prior to computing any additional transformations such as growth rates.

Table 1 lists the component series on which the ADS index is based, as well as any additional source data we need to construct the component series. We also show our alternative names for each variable because these names appear below in the equations of the mathematical model that produces the ADS index.

Table 1. Source Data for the ADS Index

Long Name Description [Source Agency]	**Haver Name**	**Alternative Name**
Weekly Observations		
Initial Unemployment Claims, State Programs (SA, 000s) [Source: DOL]	LIC@WEEKLY	IJC = LIC/100

Monthly Observations		
Nonfarm Payroll Employment, Total (SA, 000s) [Source: BLS]	LANAGRA@USECON	EMP
Industrial Production Index, Total (SA, base year value = 100) [Source: FRB]	IP@IP	IP
Real Manufacturing and Trade Sales, All Industries (SA, chain weight, $M) [Source: CB and BEA]	TSTH@USECON	RSALES
Nominal Personal Income (SAAR, $B) [Source: BEA]	YPM@USECON	NINC
Nominal Personal Current Transfer Receipts (SAAR, $B) [Source: BEA]	YPTPM@USECON	TRR
PCE Chain Price Index (SA, base year value = 100) [Source: BEA]	JCBM@USECON	PPCE
Real Personal Income ex. Transfers [Constructed]	Constructed by Philadelphia Fed	RINC = (NINC-TRR) / (PPCE/100)
Quarterly Observations		
Real Gross Domestic Product (SAAR, chain weight, $B) [BEA]	GDPH@USECON	RGDP

Some noteworthy points characterize our collection of the source data for the ADS index:

- **Raw data.** In our production procedures for ADS, we always collect the data from Haver in raw, levels form. Any required transformations of the data are performed within our code that produces the ADS index.
- **Real Manufacturing and Trade Sales (TSTH@USECON / RSALES).** Prior to the observation for January 1997, the unofficial NAICS data are calculated by Haver Analytics using growth rates of sales of SIC manufacturing and trade industries. Starting with the observation for January 1997, the data are the U.S. government's official NAICS data.

Growth Rates. With the exception of initial unemployment claims (IJC), we covert all component series into period-over-period growth rates, using the formula for continuous compounding and express the growth rate in annualized percentage points. These growth rates are the variables we use in the mathematical model that produces the ADS index values. We do not transform IJC in the model for ADS. The formulas for growth rates (indicated below as DLxxx) make use of “m” as an index for the monthly observation date and “q” as an index for the quarterly observation date; “log” denotes a natural logarithm.

Monthly Observations

$$DLemp_m \equiv 1200 \log\left(\frac{EMP_m}{EMP_{m-1}}\right)$$

$$DLip_m \equiv 1200 \log\left(\frac{IP_m}{IP_{m-1}}\right)$$

$$DLrsales_m \equiv 1200 \log\left(\frac{RSALES_m}{RSALES_{m-1}}\right)$$

$$DLrinc_m \equiv 1200 \log\left(\frac{RINC_m}{RINC_{m-1}}\right)$$

Quarterly Observations

$$DLrgdp_q \equiv 400 \log\left(\frac{RGDP_q}{RGDP_{q-1}}\right)$$

3. Timing of Releases

We publish an updated ADS index whenever U.S. government statistical agencies publish new data for one or more of the components on which we base the index. Thus, the timing of our ADS updates depends on the timing of the releases for the source data. We update the ADS seven to eight times monthly, following any new release of component source data. A new release of component source data could reflect:

- An additional observation for one or more components of the ADS index, and/or
- Revisions to observations previously released.

Table 2 gives the approximate (or exact) timing of the U.S. government's release of the ADS's component source data and, hence, the approximate (or exact) timing of our updates to the ADS index.

Table 2. Timing of Source Data Releases for the ADS Index

Variable	**Timing of Release**
IJC: Initial Unemployment Claims	Every Thursday
EMP: Nonfarm Payroll Employment	First Friday of the month (almost always)
IP: Industrial Production Index	Third week of the month
RSALES: Real Manufacturing and Trade Sales **RINC:** Real Personal Income	Last week of the month (usually a couple days after RGDP is released)
RGDP: Real GDP	Last week of the month

4. ADS Econometric Model

Preliminary Discussion of Mixed-Observation Frequency Data

We now give the mathematical model that we use to construct the ADS index. Because the ADS model uses mixed-observation frequency data (weekly, monthly, quarterly) and we produce daily observations for the ADS index itself, we must be precise in how we denote units of time (" t "). In the ADS mathematical model, the subscript "t" indexes a daily observation date. The model uses no daily source data, so we must arrange the source data with missing values on any daily date that is not the last day of the period covered by the input source observation.

Based on the economic interpretation and statistical construction of the input source data, the ADS mathematical model imposes the following rules on the source variables:

- **Initial unemployment claims (IJC):** IJC is a weekly flow that begins on Sunday and ends on Saturday. Thus, the ADS model treats IJC as missing on any day other than Saturday.

- **Month-over-month growth in employment (DLemp), industrial production (DLip), real sales (DLrsales), and real personal income (DLrinc):** These monthly variables represent continuous flows of economic activity beginning on the first day of the month and ending on the last day of the month. Accordingly, the ADS model treats these growth rates as missing on any day other than the last day of the month.

- **Quarter-over-quarter growth in real GDP (DLrgdp):** DLrgdp is a quarterly flow beginning on the first day of the quarter and ending on the last day of the quarter. Accordingly, the ADS model treats this variable as missing on any day other than the last day of the quarter.

Using our rules for constructing the ADS index, we can define the following variables:

$$IJC_t = \begin{cases} IJC_{w(t)}, & \text{t = Saturday} \\ \text{N/A}, & \text{t} \neq \text{Saturday} \end{cases}$$

$$DLemp_t = \begin{cases} DLemp_{m(t)}, & \text{t = last day of month} \\ \text{N/A}, & \text{t} \neq \text{last day of month} \end{cases}$$

$$DLip_t = \begin{cases} DLip_{m(t)}, & \text{t = last day of month} \\ \text{N/A}, & \text{t} \neq \text{last day of month} \end{cases}$$

$$DLrsales_t = \begin{cases} DLrsales_{m(t)}, & \text{t = last day of month} \\ \text{N/A}, & \text{t} \neq \text{last day of month} \end{cases}$$

$$DLrinc_t = \begin{cases} DLrinc_{m(t)}, & \text{t = last day of month} \\ \text{N/A}, & \text{t} \neq \text{last day of month} \end{cases}$$

$$DLrgdp_t = \begin{cases} DLrgdp_{q(t)}, & \text{t = last day of quarter} \\ \text{N/A}, & \text{t} \neq \text{last day of quarter} \end{cases}$$

where $w(t),\ m(t),\ q(t)$ respectively index the week, month, and quarter that contain the daily observation date t.

Mathematical Model

As above, let t index a daily observation date. Define $D_t^M\,(D_t^Q)$ as the number of calendar days in the month (quarter) in which daily observation t falls. The equations of the ADS model are given by

$$ADS_t = \rho ADS_{t-1} + \eta_t; \quad \eta_t \overset{iid}{\sim} N\left(0,\, 1-\rho^2\right) \tag{4.1}$$

$$IJC_t = k_1 + \beta_1 C_t^{(W)} + \gamma_1 IJC_{t-7} + e_{1t}, \quad e_{1t} \overset{iid}{\sim} N\left(0, \sigma_1^2\right) \tag{4.2}$$

$$DLemp_t = k_2 + \beta_2 C_t^{(M)} + \gamma_2 DLemp_{t-D_t^M} + e_{2t}, \quad e_{2t} \overset{iid}{\sim} N\left(0, \sigma_2^2\right) \tag{4.3}$$

$$DLip_t = k_3 + \beta_3 C_t^{(M)} + \gamma_3 DLip_{t-D_t^M} + e_{3t}, \quad e_{3t} \overset{iid}{\sim} N\left(0, \sigma_3^2\right) \tag{4.4}$$

$$DLrinc_t = k_4 + \beta_4 C_t^{(M)} + \gamma_4 DLrinc_{t-D_t^M} + e_{4t}, \quad e_{4t} \overset{iid}{\sim} N\left(0, \sigma_4^2\right) \tag{4.5}$$

$$DLrsales_t = k_5 + \beta_5 C_t^{(M)} + \gamma_5 DLrsales_{t-D_t^M} + e_{5t}, \quad e_{5t} \overset{iid}{\sim} N\left(0, \sigma_5^2\right) \tag{4.6}$$

$$DLrgdp_t = k_6 + \beta_6 C_t^{(Q)} + \gamma_6 DLrgdp_{t-D_t^Q} + e_{6t}, \quad e_{6t} \overset{iid}{\sim} N\left(0, \sigma_6^2\right) \tag{4.7}$$

where: $Ee_{it}e_{jt} = 0, \;\; \text{all } i \neq j$, $E\eta_t e_{it} = 0, \; i = 1,\ldots,6$, and

$$C_t^{(W)} = \xi_t^{(W)} C_{t-1}^{(W)} + ADS_t = \xi_t^{(W)} C_{t-1}^{(W)} + \rho ADS_{t-1} + \eta_t$$

$$C_t^{(M)} = \xi_t^{(M)} C_{t-1}^{(M)} + ADS_t = \xi_t^{(M)} C_{t-1}^{(M)} + \rho ADS_{t-1} + \eta_t \qquad (4.8)$$

$$C_t^{(Q)} = \xi_t^{(Q)} C_{t-1}^{(Q)} + ADS_t = \xi_t^{(Q)} C_{t-1}^{(Q)} + \rho ADS_{t-1} + \eta_t$$

$$\xi_t^{(W)} = \begin{cases} 0, & \text{t = Sunday} \\ 1, & \text{t} \neq \text{Sunday} \end{cases}$$

$$\xi_t^{(M)} = \begin{cases} 0, & \text{t = First day of month} \\ 1, & \text{t} \neq \text{First day of month} \end{cases} \qquad (4.9)$$

$$\xi_t^{(Q)} = \begin{cases} 0, & \text{t = First day of quarter} \\ 1, & \text{t} \neq \text{First day of quarter} \end{cases}$$

The ADS variable is unobserved. Equation (4.1) says that although it is unobserved, we think ADS follows a daily first-order autoregressive process. The ADS equation also imposes a variance on the residual (η_t) equal to $1-\rho^2$. It is easy to show that this variance restriction on the residual imposes a theoretical unit variance restriction on the ADS index itself. The imposition of this variance restriction serves to define the scale of the ADS index. Note also that because the ADS equation includes no parameter for a constant, non-zero value of the index, the ADS has a theoretical average value of zero.

Equations (4.2) - (4.7) relate the values of the component source data to their own lagged values and to the ADS index itself. The ADS index appears indirectly through the weekly, monthly, and quarterly cumulation variables $(C_t^{(W)}, C_t^{(M)}, C_t^{(Q)})$, shown in equations (4.8). These cumulation variables represent the cumulative sum of the daily ADS values over the week, month, and quarter. Equations (4.9) define zero-one dummy variables that tell the cumulation equations when to begin and end the cumulation of daily ADS index values over the week, month, and quarter.

State-Space Representation

Estimation of the ADS model's parameters is difficult and requires special techniques grounded upon the so-called Kalman filter. To be precise, we must estimate not only the model's parameters $\left(\rho; k_1, \ldots k_6; \beta_1, \ldots, \beta_6; \gamma_1, \ldots, \gamma_6; \sigma_1^2, \ldots, \sigma_6^2\right)$ but also the daily ADS index values, one daily index value for each day over the years beginning with 1960. The special techniques use the Kalman filter. In using the Kalman filter, we transform the model's equations into their state-space representation, which is given by the "state" equation and the "observation" equation.

State Equation

$$\begin{pmatrix} ADS_t \\ C_t^{(W)} \\ C_t^{(M)} \\ C_t^{(Q)} \end{pmatrix} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ \rho & \xi_t^{(W)} & 0 & 0 \\ \rho & 0 & \xi_t^{(M)} & 0 \\ \rho & 0 & 0 & \xi_t^{(Q)} \end{pmatrix} \begin{pmatrix} ADS_{t-1} \\ C_{t-1}^{(W)} \\ C_{t-1}^{(M)} \\ C_{t-1}^{(Q)} \end{pmatrix} + \begin{pmatrix} \eta_t \\ \eta_t \\ \eta_t \\ \eta_t \end{pmatrix}$$

Observation Equation

$$\begin{pmatrix} IJC_t \\ DLemp_t \\ DLip_t \\ DLrinc_t \\ DLrsales_t \\ DLrgdp_t \end{pmatrix} = \begin{pmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \\ k_5 \\ k_6 \end{pmatrix} + \begin{pmatrix} \gamma_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma_6 \end{pmatrix} \begin{pmatrix} IJC_{t-7} \\ DLemp_{t-D_t^M} \\ DLip_{t-D_t^M} \\ DLrinc_{t-D_t^M} \\ DLrsales_{t-D_t^M} \\ \overline{DLrgdp_{t-D_t^M}} \end{pmatrix}$$

$$+ \begin{pmatrix} 0 & \beta_1 & 0 & 0 \\ 0 & 0 & \beta_2 & 0 \\ 0 & 0 & \beta_3 & 0 \\ 0 & 0 & \beta_4 & 0 \\ 0 & 0 & \beta_5 & 0 \\ 0 & 0 & 0 & \beta_6 \end{pmatrix} \begin{pmatrix} ADS_t \\ C_t^{(W)} \\ C_t^{(M)} \\ C_t^{(Q)} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \\ e_{4t} \\ e_{5t} \\ e_{6t} \end{pmatrix}$$

where:

$$\begin{pmatrix} \eta_t \\ e_{1t} \\ e_{2t} \\ e_{3t} \\ e_{4t} \\ e_{5t} \\ e_{6t} \end{pmatrix} \overset{iid}{\sim} N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1-\rho^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_1^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_2^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_3^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_4^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_5^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_6^2 \end{bmatrix} \right)$$

Additional Restrictions on the Model

We impose additional restrictions on the ADS model (beyond the unit-variance and zero-mean restrictions discussed above). First, we impose dynamic stability on the coefficient estimates attached to the first-lagged right-hand-side variable in every equation. The restriction says the coefficient estimate must be between minus unity and positive unity. For example, in equation (1), we impose the restriction $-1<\hat{\rho}<1$. Second, because the econometric model specification does not identify the sign of the ADS index itself, we impose a sign restriction that says the ADS must move positively with $DLrgdp$, implying that the ADS moves pro-cyclically over time. To be more precise, we impose:

- **A Dynamic Stability Restriction on Coefficients.** In each equation, we require the coefficient estimate on the first-lagged dependent variable to lie inside the unit circle. For equation (1), we set $\rho \equiv (e^{2\phi}-1)/(e^{2\phi}+1)$ and estimate ϕ freely, without restrictions. Similarly, for equation (2), we set $\gamma_1 \equiv (e^{2\phi_1}-1)/(e^{2\phi_1}+1)$ and estimate ϕ_1 freely, without restrictions. Equations (3) – (7) are treated in the same way as equation (2).

- **A Sign Restriction on ADS.** Nothing in the ADS model identifies the sign of the ADS index. Indeed, the likelihood function is unaltered if we replace ADS_t with $-ADS_t$, $t=1,...,T$. In other words, from the perspective of model specification and the likelihood function, an ADS that rises in recessions is just as acceptable as an ADS that falls in recessions. Philosophically, however, we think of the ADS as procyclical, with negative values in recessions and positive values in expansions. Operationally, referencing equation (7), we impose procyclical movements on the ADS after estimation, according to the following rule, which depends on the sign of the estimated coefficient attached to the ADS variable in the observation equation for $DLrgdp$ $(\hat{\beta}_6)$

$$ADS_t = \begin{cases} ADS_t, & \hat{\beta}_6 > 0 \\ -ADS_t, & \hat{\beta}_6 < 0 \end{cases} \cdot$$
$$t = 1,\ldots,T$$

Maximum Likelihood Parameter Estimation and the Kalman Smoother Estimate for ADS

We estimate the parameters of the ADS model's state-space representation via maximum likelihood according to the BFGS algorithm. Using these estimated parameters, we compute the daily ADS index values as the mathematical expectations of $ADS_t, \quad t = 1, 2, \ldots, T$, conditional on the source data through the daily time period given by $t = T$, where T represents the most recent observation date in the set of last observation dates across all component variables. (Generally, the last observation date for each ADS update is the Saturday associated with the most recent observation date for initial unemployment claims.) Analytically, our published ADS index values are those given by the Kalman-smoother estimates defined by the following conditional expectation:

$$ADS_t =$$

$$E\left(ADS_t \mid IJC_1, \ldots IJC_T; DLemp_1, \ldots, DLemp_T; \cdots; DLrgdp_1, \ldots, DLrgdp_T\right),$$
$$t = 1,\ldots,T$$

Notice that this analytical expression for the estimated ADS index shows that the daily estimate depends upon the values of all component series over the full sample of observations, $t = 1,\ldots,T$.

COVID-19 Adjustment

As of the September 28th, 2023 vintage of the ADS, we adjust for COVID-19 related extreme values in the source data by setting these values to missing when estimating the parameters of the ADS model, but restoring the extreme values when calculating the ADS time series. Prior to the September 28th, 2023 vintage, we accounted for the COVID-19 related extreme values by restricting the sample period we used to estimate the parameters of the ADS model to the period before March 14th, 2020.