## The Forward Guidance Puzzle

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#### Abstract

With short-term interest rates at the zero lower bound, forward guidance has become a key tool for central bankers and yet we know little about its effectiveness. This paper first documents the impact of forward guidance announcements on 1) a broad cross section of financial markets data, and 2) on the panel of Blue Chip forecasts. We find that this effect has been very heterogeneous across announcements and relate this heterogeneity to the type of forward guidance, whether it conveys news about the economy (*Delphic*) or a commitment on the part of policymakers (*Odyssean*). We then show that standard medium-scale DSGE models tend to grossly overestimate the impact of forward guidance on the macroeconomy, a phenomenon we call the "forward guidance puzzle," and explain why this is the case. We propose a tentative resolution to the puzzle based on the fact that life is finite.

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## 1 Introduction

For decades, macroeconomists have attempted to quantify the effects of monetary policy actions on the economy. By now, a very large number of papers has documented the transmission mechanism of surprise changes in short-term interest rates onto the economy, using either VARs or DSGE models (e.g., Sims (1980), Christiano et al. (1999), Christiano et al. (2005)). While we arguably have some understanding of the effects of short-term interest rates, these have been constrained by the zero lower bound (ZLB) for a few years in most developed economies, so that for the time being they are no longer part of the policymakers' toolkit. Instead, many central banks have used other tools such as announcements about the future path of the policy rate ("forward guidance"), or "quantitative easing" measures involving a change in the size and especially the composition of the central bank balance sheet. Forward guidance has been used extensively and explicitly by the Federal Reserve since the FOMC meeting of December 16, 2008, so as to affect long-term bond yields and stimulate aggregate expenditures (see Woodford (2012) and Campbell et al. (2012a)).<sup>1</sup> Moreover, Woodford (2012), building on results by Krishnamurthy and Vissing-Jorgensen (2011) and Bauer and Rudebusch (2011), emphasizes the "signaling channel" of the Fed's asset purchases – that is, he argues that quantitative easing itself can at least in part be interpreted as implicit forward guidance.

While the literature has provided strong theoretical justifications for the use of such forward guidance (e.g., Eggertsson and Woodford (2003)), the evidence on the quantitative effects of such a policy tool on the macroeconomy is still limited. This may not be too surprising in light of the fact that the identification problem that needs to be surmounted in the case of contemporaneous policy shocks may be even more challenging in the case of shocks that are anticipated. In fact, an announcement by policymakers that they will maintain the policy rate at the ZLB for longer than initially anticipated by market participants may have two types of effects. On the one hand, it could be interpreted as more monetary stimulus: it

<sup>&</sup>lt;sup>1</sup>At that meeting, the FOMC's statement mentioned that economic conditions "are likely to warrant exceptionally low levels of the federal funds rate for some time." Three months later, the FOMC reinforced its forward guidance by stating that the exceptionally low levels of the federal funds rate would likely be warranted "for an extended period." This sentence was reiterated in each subsequent FOMC statement until August 9, 2011, when the FOMC argued that economic conditions "are likely to warrant exceptionally low levels of the federal funds rate at least through mid-2013." That sentence was maintained in subsequent statements until January 25, 2012, when the date was pushed forward to "late 2014."

should lower the market's expectation of future federal funds rate (FFR), which contributes to lower longer term yields, hence stimulates economic activity and puts upward pressure on inflation. On the other hand, such an announcement could be interpreted by market participants as revealing negative news about the state of the economy, if they believe that the FOMC has access to information not shared by market participants. In this case, such an announcement would be associated with lower long-term yields and lower projections of economic activity. The interpretation chosen by the market participants surely depends in very subtle ways on the FOMC communication.<sup>2</sup>

Empirically, Gürkaynak et al. (2005b) and more recently Campbell et al. (2012a) find strong evidence that FOMC announcements move asset prices. Yet when Campbell et al. (2012a) try to assess the impact of exogenous anticipated changes in monetary policy on the macroeconomy, they find that this has the opposite sign than expected, highlighting these identification challenges. Moreover, even if it was possible to identify the impact of, say, four quarters-ahead forward guidance, its effect would not necessarily be the same as, say, that of eight-quarters ahead forward guidance (Campbell et al. (2012a) consider one through four quarters ahead forward guidance; forward guidance communicated in 2012 was going through the end of 2014, and hence amounted to approximately eight quarters). Given that policymakers have seldom experimented with forward guidance this far in the future, there is little data to guide them.

New Keynesian DSGE models following the work of Christiano et al. (2005) and Smets and Wouters (2007) are in principle well suited to study the effects of forward guidance. Such models have been found to fit the data reasonably well and to provide a good forecasting performance relative to reduced form models such as VARs, private forecasters, or the Greenbook (see Smets and Wouters (2007), Del Negro et al. (2007), Edge and Gürkaynak (2010), and Del Negro and Schorfheide (2013)). Most importantly, being laboratory economies, they can be used to study the impact of policy experiments never performed before. As shown by Laseen and Svensson (2011), forward guidance can be captured in DSGE models using anticipated policy shocks. Such shocks reflect deviations of the short-term interest rate from the historical policy rule that are anticipated by the public. They can be affected by policymakers' announcements about their intentions regarding the future path of the policy rate.

<sup>&</sup>lt;sup>2</sup>Woodford (2012) argues that several recent announcements about the future path of policy rates have not indicated a clear commitment to maintaining short-term rates low, so that they run the risk of being interpreted as reflecting a deteriorating forecast for output and or inflation.

Milani and Treadwell (2011) study the impulse responses to anticipated policy shocks using a simple three-equations New Keynesian DSGE model. Campbell et al. (2012b) go quite a few steps further. They investigate the impact of forward guidance on the macroeconomy by estimating a medium scale DSGE model broadly similar to the one in this paper using data on market expectations for the federal funds rate, in addition to a standard set of macro variables, for the sample 1987-2007. They find that forward guidance explains about 9 percent of output and hours fluctuations at the business cycle frequency, and more than 50 percent of the movements in the federal funds rate. Their results indicate that even in the pre-Great Recession period forward guidance played a large role in monetary policy – a finding that echoes that of Gürkaynak et al. (2005b) – and a significant role in terms of business cycle fluctuations.

The problem with DSGE models, however, is that they appear to deliver unreasonably large responses of key macroeconomic variables to central bank announcements about future interest rates – a phenomenon we can call the "forward guidance puzzle". Carlstrom et al. (2012) show that the Smets and Wouters model would predict an explosive inflation and output if the short-term interest rate were pegged at the ZLB between eight and nine quarters. This is an unsettling finding given that the horizon of forward guidance by the FOMC has at times been of at least eight quarters.

This paper makes three contributions. First, we document empirically the response of financial market variables and forecasts of key macroeconomic variables to actual FOMC communications involving changes in forward guidance. Second, we characterize the quantitative implications of forward guidance in a setting that is arguably more realistic than that adopted by Carlstrom et al. (2012). In their experiment, these authors assess the impact of fixing the interest rate to the zero lower bound relative to the steady state baseline. Given the current state of the economy, we view the assumption that interest rates would be at steady state in absence of forward guidance as unrealistic. We instead incorporate current market expectations for the short rate in our baseline forecast using the approach described in Del Negro and Schorfheide (2013). Specifically, we use the FFR expected path through mid-2015 implied by OIS rates as of August 28, 2012. Doing so allows us to incorporate valuable information for the estimation of the state of the economy. We then investigate the effect of extending the forward guidance by two quarters, from the end of 2014 to mid-2015. Using the FRBNY-DSGE model we show that even for this much more modest (relative to

Carlstrom et al. (2012)) experiment, these authors' findings is confirmed: the model-implied response of macroeconomic variables is unrealistically large.

The third contribution of the paper is to point to the source of the problem and suggest a solution. Credible announcements about future short-term policy rates should affect the current long-term bond yields, and these in turn affect economic activity and inflation. However, the model predicts an excessive response of the long-term bond yield to policy announcements, compared to what is observed in the data. For instance, the relatively modest (two quarters) change in forward guidance delivers in the model a 25 basis points drop in the 10-year nominal yield. In comparison, the January 25, 2012, change in forward guidance, which shifted the announced lift-off date by more than four quarters (mid-2013 to end of 2014), produced a drop in the same rate by only 7 basis points. Why this excessive response of the long rate in the model relative to the data? Interestingly, the model tends to underestimate the response of bond yields with maturities of 1 to 5 years. Instead, it predicts excessive responses in the maturities much farther in the future.

We view this response to forward guidance of the expected short term rates beyond 5 years, which leads to overestimate the impact of forward guidance, as an incredible feature of this model: it appears unlikely that policymakers are able to affect FFR expectations farther than 5 years by announcements regarding the short term rate in the next two years. In order to address this issue, we propose to introduce in the model a realistic feature, namely the fact that life ends at some point. More specifically, we adopt Blanchard (1985)'s and Yaari (1965)'s perpetual youth model, which has been incorporated recently in simple New Keynesian models by Nisticò (2012) and Piergallini (2006). We assume as in these papers that agents face each period a constant probability of dying and being replaced before the next period begins. This feature induces agents to discount the future more heavily, in the aggregate, and so implies that announcements of policy changes far in the future generate smaller effects on current aggregate variables than is the case in models with infinitely lived agents. Our proposal bears some similarities with the mechanism proposed by McKay et al. (2014). In the latter paper, current conditions also matter more than conditions far into the future. The mechanism is however different, as these authors emphasize the role of precautionary savings and borrowing constraints, whereas we focus on stronger discounting in households' intertemporal consumption decisions, in firms' optimal price setting, and in determination of asset prices.

The paper proceeds as follows. Section 2 provides some empirical evidence on the effects of forward guidance in the US. It documents the responses of financial market data as well as private sector forecasts of key macroeconomic variables to FOMC announcements. Section 3 briefly describes the DSGE model used, its estimation, how we formalize the introduction of a fixed interest-rate path. It then presents the implications of interest rate announcements in this model, and reports that the model generates an excessive response to announcements about changes in the expected future interest rate path, a phenomenon we call the forward guidance puzzle. Section 4 proposes a potential solution to this puzzle, introducing positive death rates in the model. Section 5 concludes.

# 2 Some empirical evidence on the effects of forward guidance

In this section, we first document the impact of forward guidance announcements on a broad cross section of financial markets data, following the approach of Krishnamurthy and Vissing-Jorgensen (2011).<sup>3</sup> Next, we complement this evidence by documenting the impact of announcements on Blue Chip forecasters' expectations, after controlling for news about the state of the economy. We find that the effect of forward guidance has been very heterogeneous across announcements. In terms of the asset price responses, real long term rate always decreased following the announcements, but nominal long term rates and safety risk premia moved in the opposite direction. In terms of changes in the forecasts, some announcements resulted in negative forecast revisions for output growth, while others were followed by significantly positive and large revisions. We will relate this heterogeneity to the type of forward guidance, whether *delphic* (news about the economy) or *odyssean*, where the latter can amount to announced deviations from the policy rule (anticipated shocks), or possible changes in the rule itself (as in Engen et al. (2014)).

In order to more easily relate empirical evidence and theory, we focus on three FOMC announcements all featuring changes in calendar based forward guidance: August 2011, January 2012, and September 2012. While all three announcements were also accompanied

 $<sup>^{3}</sup>$ A subset of these data has already been analyzed by Filardo and Hoffman (2014), who limit their analysis to forward rates and nominal long term Treasuries, and Femia et al. (2013).

by information about long term asset purchases (henceforth, QE), arguably for all three the change in forward guidance was the most significant component of the statement.<sup>4</sup> Tables 1 and 2 below report relevant portions of the FOMC statements for the three forward guidance announcements, which, as we will see, possibly played a key role in determining its impact.

Table 1: Forward Guidance FOMC Statements Language – Economic Conditions

August 2011:

Information received ... indicates that economic growth so far this year has been considerably slower than the Committee had expected. ... Inflation picked up earlier in the year, .... More recently, inflation has moderated as prices of energy and some commodities have declined from their earlier peaks. The Committee now expects a somewhat slower pace of recovery over coming quarters than it did at the time of the previous meeting ... Moreover, downside risks to the economic outlook have increased...

January 2012:

Information received ... suggests that the economy has been expanding moderately, notwithstanding some slowing in global growth ... The Committee expects economic growth over coming quarters to be modest ... The Committee also anticipates that over coming quarters, inflation will run at levels at or below those consistent with the Committee's dual mandate.

September 2012:

Information received ... suggests that economic activity has continued to expand at a moderate pace in recent months. ... Inflation has been subdued, although the prices of some key commodities have increased recently.

#### 2.1 Evidence from financial markets

Table 3 reports changes in several asset prices (measured in basis points, unless otherwise noted) in the two day window following the event, as in Krishnamurthy and Vissing-Jorgensen (2011). Most of these asset prices coincide with those reported in Krishnamurthy and Vissing-Jorgensen (2011), and are constructed using the same methodology and data sources.

The top panel of Table 3 reports changes in Treasury, Agency, and Agency MBS yields, and shows that nominal rates declined substantially at most horizons following the January

<sup>&</sup>lt;sup>4</sup>This argument is more easily made for August 2011 and January 2012 than for September 2012, since the latter FOMC meeting marked the start of QE3. We will argue however that the financial markets reaction to the September 2012 annoucement was quite different from that documented in, e.g., Krishnamurthy and Vissing-Jorgensen (2011).

#### Table 2: Forward Guidance FOMC Statements Language - Policy

#### August 2011:

...The Committee currently anticipates that economic conditions—including low rates of resource utilization and a subdued outlook for inflation over the medium run—are likely to warrant exceptionally low levels for the federal funds rate at least through mid-2013. ...

#### *January 2012*:

...the Committee expects to maintain a highly accommodative stance for monetary policy. In particular, the Committee decided today to keep the target range for the federal funds rate at 0 to 1/4percent and currently anticipates that economic conditions-including low rates of resource utilization and a subdued outlook for inflation over the medium run-are likely to warrant exceptionally low levels for the federal funds rate at least through late 2014. ... The Committee also decided to continue its program to extend the average maturity of its holdings of securities as announced in September.

#### September 2012:

The Committee is concerned that, without further policy accommodation, economic growth might not be strong enough to generate sustained improvement in labor market conditions. ... the Committee agreed today to increase policy accommodation by purchasing additional agency mortgagebacked securities at a pace of \$40 billion per month... The Committee also will continue through the end of the year its program to extend the average maturity of its holdings of securities ... These actions ... will increase the ... holdings of longer-term securities by about \$85 billion each month through the end of the year, should put downward pressure on longer-term interest rates, ... and help to make broader financial conditions more accommodative ... If the outlook for the labor market does not improve substantially, the Committee will continue its purchases of agency mortgage-backed securities, undertake additional asset purchases, and employ its other policy tools as appropriate until such improvement is achieved in a context of price stability ...

To support continued progress toward maximum employment and price stability, the Committee expects that a highly accommodative stance of monetary policy will remain appropriate for a considerable time after the economic recovery strengthens. In particular, the Committee also decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that exceptionally low levels for the federal funds rate are likely to be warranted at least through mid-2015.

2012, and especially the August 2011, announcements. In the two day following the August 2011 meeting, constant maturity Treasury yields fell by 23 basis points. Figure 1 reports changes in federal funds rate futures, swap basis-adjusted Eurodollar futures, and forward rates extracted from the Treasury yield curve. The yield curve flattened substantially not only at the short horizons, consistently with the announcements, but also at longer horizons. The flattening of the yield curve at shorter horizons seems to indicate that explanations for the forward guidance puzzle based on imperfect credibility of the central bank (e.g.,

	<b>Treasury Yields</b> (constant maturity)					Agency Yields (Fannie/Freddie)					MBS Yields		
$\begin{array}{c} \text{Maturity} \\ \text{(years)} \end{array}$	30	10	5	3	1		30	10	5	3		30	15
8/9/2011	-14	-23	-18	-12	-3		-19	-23	-27	-25		-24	-26
1/25/2012	-5	-12	-15	-8	0		-10	-13	-18	-14		-16	-18
9/13/2012	17	11	2	2	0		10	5	0	1		-13	-11
		(const	TIPS	<b>5</b> turity)		I	mplie Vol.	d	SP 500	DJ IA	T	FX	IR
$\begin{array}{c} \text{Maturity} \\ \text{(years)} \end{array}$	30	20	10	7	5				(% cł	ange)	(*	% chang	e)
8/9/2011	-26	-16	-33	-52	-39		-8.11		0.12	-0.83		-0.01	
1/25/2012	-8	-11	-15	-18	-20		-4.21		0.29	0.46		0.56	
9/13/2012	-9	-8	-15	-19	-25		-1.13		2.03	1.95		1.78	
Breakevens			Inf	Inflation Swaps				TIF	S Spi	read			
Maturity (years)	20	10	5		30	20	10	5	1		20	10	5
8/9/2011	-7	10	21		8	9	14	13	-3		16	4	-8
1/25/2012	3	3	5		3	3	4	8	12		0	1	3
9/13/2012	24	26	27		26	27	21	28	23		3	-5	1
		Inte	ermed	iate te	erm	Corp	orate	Yields	5 Long	term			
	Aaa	Aa	A	Baa	Ba	В	Aaa	Aa	A	Baa	Ba	В	

Bodenstein et al. (2012)) are not consistent with the evidence.

Table 3: Evidence from Financial Markets

Intermediate term Corporate Yields Long term													
	Aaa	Aa	А	Baa	Ba	В	Aaa	Aa	Α	Baa	Ba	В	
8/9/2011	-8	-6	-8	-8	2	16	-11	-9	-5	-5	26	33	
1/25/2012	-10	-13	-11	-16	-9	-13	-12	-15	-17	-13	-16	-10	
9/13/2012	11	10	7	-2	-8	-15	0	-1	-1	5	-12	-18	

Notes: All figures are in basis points unless otherwise noted.

Agency and Agency MBS yields fell slightly more than Treasuries in August 2011, and TIPS yield fell even more, especially at shorter horizons. Hanson and Stein (2014) propose an explanation for the fall in real yields based on demand effects coming from "yield-oriented" investors, and indeed use August 2011 as the poster child for their theory. In August 2011

breakevens and inflation swaps increased, especially at horizons between 5 and 10 years.<sup>5</sup> The Dollar did not depreciate versus the Euro, however, and the stock market was also roughly unchanged, as measured by the SP500, and fell in terms of the Dow Jones Industrial Average. Most interestingly, corporate yields fell less than Treasuries for high credit quality bonds, but actually rise substantially for low credit quality bonds, suggesting an increase in the safety premium. Caballero and Farhi (2014) provide a model where forward guidance has no effect on economic activity but results in increased safety premia, and this evidence appears to be consistent with such theory. Evidence shown in the next section however shows that the August 2011 statement may have been interpreted by forecasters as conveying bad news about the economy, and many of the asset prices movements shown here (the decrease in real rate, the stall in the stock market in spite of significant lower real rates, and the increase in safety premia) are also consistent with this explanation.

Asset prices changes following the January 2012 announcement for government securities were in line with those of August 2011 in terms of the sign of the response, but were much more muted. The main difference between the two episodes is that the stock market rose a bit and the Dollar depreciated. Also, yields on corporate bonds fell roughly fell by the same amount as Treasuries, regardless of the credit rating, suggesting little variation in the safety premium following the announcement.

The response of financial markets to the September 2012 announcement was altogether different. Nominal yields on government securities rose, instead of falling. Agency debt yield also rose, but less than for corresponding Treasuries, while agency MBS yields fell. Real yields fell substantially. Note that the rise in nominal yields makes the yield search explanation for the fall in real yields arguably less plausible. Breakeven and inflation swap rates rose at both short and long horizons, and the Dollar depreciated. The stock market rose by about two percent. Most notably, while high credit quality corporate bond yields rose in line with Treasuries, the yield on low credit quality fell, indicating a sizable decrease in the safety premium.

One explanation for this evidence is related to the start of QE3: as shown in Table 2 the central bank announced that it intended to "increase policy accommodation by purchasing additional agency mortgage-backed securities at a pace of \$40 billion per month." According

<sup>&</sup>lt;sup>5</sup>Changes in the TIPS spread provide some measure of variations in the liquidity premium in these markets, following Fleckenstein et al. (2014).



Figure 1: Change in the Nominal Yield Curve following FOMC Announcements

*Notes:* Federal funds futures, swap basis-adjusted Eurodollar futures, and forward rates extracted from the Treasury yield curve are in purple, blue, and red, respectively. Pre- and post-announcement yield curves are solid and dotted, respectively.

to this explanation, the announced acquisition of agency MBS lowered safety premia, affecting the yield of low credit quality corporate bonds, while the increase in nominal Treasury was then the result of disappointed expectations that QE3 would also include long-term Treasuries. A complementary explanation is that this annoucement, together with other parts of the statement ("the Committee expects that a highly accommodative stance of monetary policy will remain appropriate for a considerable time after the economic recovery strengthens" and the emphasis on labor market conditions) also signaled that monetary policy was going to be accommodative than previously thought, resulting in a decline in real rates, and increased inflation and growth expectations. Evidence presented in the next section, based on the revision of Blue Chip forecasts, is certainly consistent with this explanation. The more optimistic outlook, in turn, resulted in a flattening of the safety premium.

### 2.2 Evidence from surveys of forecasts

In this section we document the evolution of expectations around forward guidance announcements using the panel data in the Blue Chip Financial Forecasts survey. Our approach borrows from the event study literature: if we had information on expectations for economic activity, inflation, and financial market variables the day before and two days after the event, we would simply report the two day-window change in expectations and attribute it to the news about monetary policy. Unfortunately, the survey is available only once a month. We do know however the collection dates for the survey, and we can therefore control for any macroeconomic news and changes in asset prices in between surveys. We can then compute the residual change in expectations during the (approximate) one-month window containing the forward guidance event (surveys are collected over a two-day period changing across months).<sup>6</sup>

Specifically, we run the following panel regression for each variable (k) and horizon (h)

$$\Delta f_{(k,h)_{i,t}} = \gamma_{(k,h)_0} + \gamma_{(k,h)_1'} \Delta M_t + \gamma_{(k,h)_2'} \Delta A P_t + \gamma_{(k,h)_3'} Z_{(k,h)_{t,i}} + \beta_{(k,h)^e} D_t^e + \varepsilon_{(k,h)_{i,t}}, \text{ for } t = 1, ..., T, \ i = 1, ..., n,$$
(1)

where  $\Delta f_{(k,h)_{i,t}}$  is the change in the *h*-quarters ahead forecast of participant *i* for variable *k* (e.g., GDP growth) between periods *t* and t-1,  $\Delta M_t$  and  $\Delta AP_t$  are vectors of macroeconomic surprises (e.g., payroll report) and changes in asset prices (e.g., stock prices) in the onemonth window, respectively,  $Z_{(k,h)_{t,i}}$  is a vector of participant-specific variables (e.g., the lagged change in the forecast), and  $D_t^e$  is the event dummy which is equal to one if the forward guidance event takes place in the one-month window (that is,  $t = t^e$ ), and is zero otherwise. The  $\gamma$  and  $\beta$  coefficients are indexed by (k,h) to stress the fact that these vary across variables *k* and forecast horizons *h*. They are therefore estimated running separate regressions for each *h* and k.<sup>7</sup>

The vector  $\Delta M_t$  is comprised of surprises in macroeconomic releases occurred between the time the t-1 and t surveys were collected, where we assume that forecasters incorporate news occurred in the two-day collection period.<sup>8</sup> These surprises, which are listed in Table 4, are computed as the difference between the actual release and the median in the Bloomberg survey, in the spirit of the literature computing the effect of economic news on financial variables (e.g., Fleming and Remolona (1999) and Gürkaynak et al. (2005a)), and

<sup>&</sup>lt;sup>6</sup>Altavilla and Giannone (2014) follow a related approach to assess the impact of unconventional monetary policy on forecasters' projections for bond yields. These authors use the Survey of Professional Forecasters however, which is available only once a quarter, which makes it harder to control for other developments in the economy.

<sup>&</sup>lt;sup>7</sup>Of course, this implies that we ignore the potential correlation of  $\varepsilon(k,h)_{i,t}$  across k and h.

<sup>&</sup>lt;sup>8</sup>News are usually released in the morning, so we assume that forecasters would adjust their Blue Chip submission in light of relevant information released on the last day of collection of the survey.

are expressed in the units of the release. The vector  $\Delta AP_t$  consists of changes in asset prices occurred between the time the t-1 and t surveys were collected.<sup>9</sup> These are a subset of the variables discussed in the previous section, one for each category. Table 4 lists these variables, along with the units in which the change is measures (basis points or log change). Of course, part of these changes may be due to the forward guidance announcement itself. In computing  $AP_t$  we therefore subtract to the overall change within the approximate onemonth window the change in asset prices in the two-day window after the forward guidance announcement, as reported in the previous section. Finally, the vector of participant-specific variables  $Z_{(k,h)_{t,i}}$  includes both the lagged value of the change in forecast  $\Delta f_{(k,h)_{i,t-1}}$  as well as the previous period's forecast itself  $f_{(k,h)_{i,t-1}}$  (and in some specifications also participant specific fixed-effects). These variables are included as we do not necessarily assume that forecasters are rational, and hence respond to current period news only (see Coibion and Gorodnichenko (2012)).

Roughly speaking, the estimate of  $\beta(k,h)$  can be understood as follows: First, we use the entire panel to obtain estimates of the  $\gamma$  coefficients. Then, we use these coefficients to extract the residual change in expectation in period  $t = t^e$ , and compute  $\beta(k,h)^e$  as its average across survey participants. If we were to assume that  $\varepsilon(k,h)_{i,t}$  were homoskedastic and uncorrelated across *i* (which we are not, as detailed later), we would assess the significance of the impact of the event on expectations by comparing  $\beta(k,h)$  with the average standard deviation of  $\varepsilon(k,h)_{i,t}$ . In fact, while we use OLS to obtain the estimates of all coefficients in the spirit of White (1982), we take into account both heteroskedasticity and correlation across *i* in computing the standard errors. Specifically, we assume a factor structure for  $\varepsilon(k,h)_{i,t}$ :

$$\varepsilon_{(k,h)_{i,t}} = e_{(k,h)_t} + \nu_{(k,h)_{i,t}}, \ E[e_{(k,h)_t}^2] = \sigma^2_{(k,h)_0}, \ E[\nu_{(k,h)_{i,t}}\nu_{(k,h)_{j,t}}] = \begin{cases} \sigma^2_{(k,h)_i} & \text{for } i = j \\ 0 & \text{otherwise}, \end{cases}$$
$$E[e_{(k,h)_t}e_{(k,h)_s}] = E[\nu_{(k,h)_{i,t}}\nu_{(k,h)_{i,s}}] = 0, \ \text{for } t \neq s.$$
(2)

We estimate  $\sigma^{2}(k,h)_{0}$  as the sample variance of  $\hat{e}(k,h)_{t} = \frac{1}{n} \sum_{i} \hat{\varepsilon}(k,h)_{i,t}$ , and  $\sigma^{2}(k,h)_{i}$  as the sample variance of  $\hat{\nu}(k,h)_{i,t} = \hat{\varepsilon}(k,h)_{i,t} - \hat{e}(k,h)_{t}$ . We then use the  $n \times n$  variance covariance matrix  $\hat{\Sigma}(k,h)$ , whose diagonal and off-diagonal elements are  $\hat{\sigma}^{2}(k,h)_{0} + \hat{\sigma}^{2}(k,h)_{i}$  and  $\hat{\sigma}^{2}(k,h)_{0}$ , respectively, to construct the White (1980)-robust standard errors.

<sup>&</sup>lt;sup>9</sup>Specifically, this window ranges from the day after the last collection day of the t - 1 release and the last collection day of the t release.

Name	Description	Unit		
Economic Ne	ews ( $\Delta M$ )			
GDP	real GDP QoQ: Advanced, Preliminary, Final	% change		
NAPMPMI	ISM Manufacturing PMI SA	level		
NFP	US Employees on Nonfarm Payrolls Total MoM Net Change SA	level		
USURTOT	U-3 US Unemployment Rate			
	Total in Labor Force Seasonally Adjusted	level		
RSTAXMOM	Adjusted Retail Sales Less Autos SA Monthly % Change	% change		
CPI	US CPI Urban Consumers MoM SA	% change		
CPTI	US Capacity Utilization $\%$ of Total Capacity SA	level		
IP	US Industrial Production MoM 2007 $=100$ SA	% change		
CPUP	US CPI Urban Consumers Less Food & Energy MoM SA	% change		
DGNO	US Durable Goods New Orders Industries MoM SA	% change		
Asset Prices ( $\Delta AP$ )				
BBOX	Barclays Swaption Volatility	log change		
MBS30	Average of Mortgage Backed Securities			
	(GNMA, FMNMA, GOLD), 30 Year Constant Maturity	bp difference		
SP500	S&P 500 Index	log change		
FXEDUS	EURUSD Spot Exchange Rate - Price of 1 EUR in USD	log change		
DGS10	US Treasury Yield Curve Rate			
	T Note Constant Maturity 10 Year	bp difference		
TIPS10	S&P 10 Year US TIPS Index Average Yield	bp difference		

Table 4: List of Economic News and Asset Prices Regressors in Equation (1)

We can properly recover the effect of the announcement on expectations under the following assumptions: 1) the  $\gamma$  coefficients are stable over time and are consistently estimated using the panel regression, 2) the regressors capture all public (common) news over the one-month event window, 3) the effect of the announcement on financial markets is fully captured by the two-day window change in asset prices. Assumption (1) is standard, and we use only post-2008 data (that is, data that are mostly in the zero lower bound regime)



Notes: The panel in Figure 2 shows the estimates of  $\beta(k,h)^e$  for three different events, August 2011, January 2012, and September 2012, and four different variables, GDP growth, CPI inflation, the 3-month TBill, and the 10-year Treasury rate. Variables and events correspond to rows and columns in the panel, respectively, while the horizon h is in the horizontal axis of each plot.For each triplet (e, k, h) we report the OLS estimate of  $\beta(k, h)^e$  (solid black) and the 68 and 90 percent bands (dash-and-dotted and dotted lines, respectively) computed using heteroskedasticity-robust standard errors. The sample for each regression is t = 2008.06, ..., 2015.02.

to account for the fact that the  $\gamma$ s may have been different in the pre-zero lower bound period. Assumption (2) states that the average residual  $\varepsilon_{(k,h)_{i,t^e}}$  only captures the reaction to the event. This assumption can be defended by including all possible sources of common information, especially asset price movements. To the extent that the latter span all possible common knowledge, this assumption ought to be met. Finally, assumption (3) is taken from the event-study literature (Krishnamurthy and Vissing-Jorgensen (2011), Krishnamurthy and Vissing-Jorgensen (2013), Gagnon et al. (2010)). If it is violated because the impact of the event on asset prices extends beyond the two-day window, then some of the movements in  $\Delta AP_{t^e}$  are attributable to the event. Vice versa, if the movements in the twoday window reflect information other than the event, then this information is not adequately captured in  $\Delta AP_{t^e}$ . Under our assumptions, the mapping between horizon h and  $\beta_{(k,h)}^e$  can be interpreted as an impulse response, as it shows the change in the projection for variable k following the forward guidance announcement,  $E[f(h,k)|\Omega_{t-1} \cup e] - E[f(h,k)|\Omega_{t-1}]$ , over horizon h, where  $\Omega_{t-1}$  represents the time t-1 information set of survey forecasters. Of course this interpretation is somewhat heroic, as it presumes we are able to perfectly control for all other factors affecting the forecast, but for the sake of exposition we will follow it in the remainder of this section.

The panel in Figure 2 shows the estimates of  $\beta_{(k,h)}^e$  for three different events, August 2011, January 2012, and September 2012, and four different variables, GDP growth, CPI inflation, the 3-month TBill, and the 10-year Treasury rate. The sample for each regression is t = 2008.06, ..., 2015.02. Variables and events correspond to rows and columns in the panel, respectively, while the horizon h is in the horizontal axis of each plot.<sup>10</sup> For each triplet (e, k, h) we report the OLS estimate of  $\beta_{(k,h)}^e$  (solid black) and the 68 and 90 percent bands (dash-and-dotted and dotted lines, respectively) computed using heteroskedasticity-robust standard errors. For the results in Figure 2 we run separate regressions for each event (that is, we use one dummy at the time), but the results are nearly identical if we use all dummies at the same time.

The panel shows that the estimated impulse-responses are very different across events. The August 2011 event lowers the GDP growth nowcast by about 0.7 percent, although

<sup>&</sup>lt;sup>10</sup>Note that for August 2011 and January 2012 we report the change in the nowcast (h = 0), that is, the forecast for the quarter in which the event takes place, for September 2012 this information is not available given that the first survey after the event is the October survey, for which the nowcast is the first quarter after the event. Hence the estimate for  $\beta(k, h)^e$  begin with the first quarter (h = 1).

the estimates are quite uncertain and the 90 percent bands include zero. One possible interpretation of this finding is that the forward guidance was interpreted as "delphic" in the terminology of Campbell et al. (2012a). That is, the wording "economic conditions ... are likely to warrant exceptionally low levels of the FFR at least through mid-2013" could have been interpreted as conveying bad news about the economy. The bad news on the current state of the economy have a decreasing effect on projections for longer horizons, which is intuitive, but the effect is nonetheless quite persistent as the 68 percent bands are in negative territory through h = 4. Of course, the "delphic" interpretation of our finding needs some caveat, given that August 2011 followed the peak of the European crisis, with the stock market declining substantially over the month and volatility rising. While we control for stock market and other asset price movements, it may well be that we that our regression does not fully account for these developments.

The response of CPI inflation to the August 2011 announcement is slightly negative, but overall insignificant.<sup>11</sup> The response of the expected 3-month TBill is muted at short horizons, since rates were already expected to stay at the zero lower bound, but is very strong at longer horizon, with the expected TBill about one percent lower for h = 5. This response has the same sign and pattern as those for future short term rates extracted from financial markets, although the magnitude is larger in terms of the point estimates.<sup>12</sup> Similarly, the response of the nominal 10-year Treasury rate is significantly negative and very persistent.

The January 2012 announcement seems to produce little significant movement in expectations, both for macro and financial variables. Expected growth and inflation both increase with a point estimate of about 20 basis points, but the uncertainty around the estimates is much larger. Expectations for the 3-month TBill and the 10-year Treasury rates are essentially unchanged following the announcement. This is slightly at odds with the evidence on forward rates and nominal long term rates presented in the previous section, which show

<sup>&</sup>lt;sup>11</sup>The August 2011 statement emphasized negative news about economic activity ("Information received since the Federal Open Market Committee met in June indicates that economic growth so far this year has been considerably slower than the Committee had expected") but was more nuanced about inflation ("Inflation picked up earlier in the year, mainly reflecting higher prices for some commodities ... More recently, inflation has moderated as prices of energy and some commodities have declined from their earlier peaks.").

<sup>&</sup>lt;sup>12</sup>We should note that the actual forecasts revisions  $\Delta f(k,h)_{i,t}$ , 5-quarters ahead, are larger than the adjusted revisions  $\beta(k,h)^e + \varepsilon(k,h)_{i,t}$ . As we would expect, the controls attribute some of the downward revisions to factors other than the forward guidance announcement.

both as declining, although the extent of this decline is indeed much smaller than in August 2011.

The impact of the September 2012 announcement is very different from that in the previous two episodes. Both output growth and inflation expectations increase by about one percent, and in both cases the increase is significantly different from zero. The largest expected increase pertains to the first quarter (h = 1) after the announcement (recall that we do not have information about the change in the nowcast from September to October), but the change in expectations is quite persistent, especially for output growth. This implies that the impulse response on the *level* of output is hump-shaped, with a peak at least 5 quarters after the shock, in line with the VAR evidence on the impact of policy shocks. Expectations for the short-term rate increase by a small amount (less than 20 basis points) while those for the 10-year rate increase more substantially, in line with the reaction of financial markets to the announcement, but the change in expectations is quite imprecisely estimated.

Why the large difference in the reaction of expectations to the announcements? One possible explanation has to do with the language used in the FOMC statements. Specifically, Table 2 shows a substantial change in the language associated with the interest rate announcement. Instead of "Committee currently anticipates that economic conditions ... are likely to warrant exceptionally low levels for the federal funds rate," in September 2012 the FOMC introduced the additional policy accommodation by stating: "[T]o support continued progress toward maximum employment and price stability, the Committee expects that a highly accommodative stance of monetary policy will remain appropriate for a considerable time after the economic recovery strengthens. In particular, the Committee ... currently anticipates that exceptionally low levels for the federal funds rate are likely to be warranted at least through mid-2015." Of course, September 2012 also included language about QE3 ("the Committee agreed today to increase policy accommodation by purchasing additional agency mortgage-backed securities at a pace of \$40 billion per month.") but as we discussed in the previous section, the reaction of financial markets to that language was quite different from that reported for other QE episodes.

Even if one agrees that the change in expectations following the September 2012 announcement is due to policy accommodation, the fact that both short and long-term nominal rates *increase* may seem puzzling: How can policy accommodation lead to an increase in rates? This response is – at least qualitatively – consistent with the one implied by DSGE models, if instead of having anticipated deviations from the policy rule (anticipated policy shocks) the accommodation takes the form of changes in the coefficients of the reaction function toward more accommodation (e.g., a higher response to the output gap coefficient). This is because, in the DSGE model, the announced change in the reaction function increases inflationary expectations more than nominal rates increase, thereby reducing the real rate and stimulating the economy. The increase in nominal rates is then simply the equilibrium response to a much improved state of the economy.<sup>13</sup> Did the FOMC announce a change in the reaction function in the September 2012 statement? Certainly not explicitly, but the statement did put particular emphasis on labor market conditions ("The Committee is concerned that, without further policy accommodation, economic growth might not be strong enough to generate sustained improvement in labor market conditions. ... If the outlook for the labor market does not improve substantially..."), so it could be interpreted as a clarification of the reaction function stressing the weight given to the labor market.

## 3 The macroeconomic implications of interest rate announcements

We now proceed with an evaluation of the effects of extending the forward guidance focusing on the stimulative effects of policy, and abstracting from the possible effects of information conveyed by the FOMC regarding the assessment the state of the economy. In this section, we first briefly describe the DSGE model, its estimation, and the baseline forecasts. In particular we discuss the modification of the standard feedback rule describing monetary policy to allow for anticipated policy shocks, and how we incorporate current FFR market expectations into the forecast. Next, we describe the algorithm used for conditioning the forecast on a specific interest-rate path. We show that it produces results that are hardly credible and explain why this is the case.

<sup>&</sup>lt;sup>13</sup>In rational expectations DSGE models the expectational channel is very strong, and may well exaggerate the impact of the change in the rule.

#### 3.1 Model and baseline forecasts

The FRBNY DSGE model is a medium-scale, one-sector, dynamic stochastic general equilibrium model. It builds on the neoclassical growth model by adding nominal wage and price rigidities, variable capital utilization, costs of adjusting investment, and habit formation in consumption. The model follows the work of Christiano et al. (2005) and Smets and Wouters (2007), but also includes credit frictions, as in the financial accelerator model developed by Bernanke et al. (1999). The actual implementation of the credit frictions closely follows Christiano et al. (2009). Detailed information about the equilibrium, the data, and the priors used in the Bayesian estimation of this model are contained in Del Negro et al. (2013). The appendix to this paper also includes the list of log linearized equilibrium conditions, as well as the priors and posteriors for the estimated parameters. In this section we focus on the features of the model that are needed to properly describe this exercise. In particular, we discuss: i) the state-space representation of the linearized DSGE model, ii) anticipated policy shocks, iii) incorporating market's FFR expectations into the baseline forecast.

The solution to the log-linear approximation of the model's equilibrium conditions around the deterministic steady state (obtained using the method in Sims (2002)) yields the following transition equation:

$$s_t = \Phi_1(\theta)s_{t-1} + \Phi_\epsilon(\theta)\epsilon_t \tag{3}$$

where  $s_t$  is the model's vector of "state" variables, the matrices  $\Phi_1$  and  $\Phi_{\epsilon}$  are functions of the vector of all model parameters  $\theta$ , and  $\epsilon_t$  is the vector of structural shocks. The vector of observables  $y_t$  described below is in turn related to the states according to the system of measurement equations:

$$y_t = \Psi_1(\theta) + \Psi_2(\theta)s_t. \tag{4}$$

The variables included in  $y_t$  are: 1) annualized real GDP per capita growth, where the real gross domestic product is computed as the ratio of nominal GDP (SAAR) to the chain-type price index from the BEA;<sup>14</sup> 2) the log of labor hours, measured as per capita hours in non-farm payroll; 3) the log of labor share, computed as the ratio of compensation of employees to nominal GDP, from the BEA; 4) the annualized rate of change of the core PCE deflator (PCE excluding food and energy, but including purchased meals and beverages), seasonally

<sup>&</sup>lt;sup>14</sup>Per capita variables are obtained by dividing through the civilian non-institutionalized population over 16. We HP-filter the population series in order to smooth out the impact of Census revisions.

adjusted; 5) the effective federal funds rate, percent annualized, computed from daily data; and 6) the spread between the Baa rate and the rate on 10 year Treasuries. We estimate the vector of model parameters  $\theta$  using data from 1984Q1 to 2012Q3 using Bayesian methods as described in Del Negro and Schorfheide (2010), applied to the state-space representation of the linearized DSGE model provided by equations (3) and (4).

Starting in 2008Q3 (one period before the implementation of the zero lower bound) we incorporate FFR market expectations, as measured by OIS rates, into our outlook following the approach described in Section 5.4 of Del Negro and Schorfheide (2013). Specifically, we take FFR expectations up to K quarters ahead into account by augmenting the measurement equation (4) with the expectations for the policy rate:

$$FFR_{t,t+k}^{e} = 400 \left( I\!\!E_{t}\widehat{R}_{t+k} + \ln R_{*} \right)$$
  
= 400  $\left( \Psi_{R,2}(\theta)\Phi_{1}(\theta)^{k}s_{t} + \Psi_{R,1}(\theta) \right), \quad k = 1, ..., K$  (5)

where  $FFR_{t,t+k}^e$  are the market's expectations for the FFR k quarters ahead,  $\Psi_{R,2}(\theta)$  and  $\Psi_{R,1}(\theta)$  are the rows of  $\Psi_2(\theta)$  and  $\Psi_1(\theta)$ , respectively, corresponding to the interest rate, and  $R_*$  is the gross steady state nominal interest rate. This observation equation contains valuable information for the estimation of the state of the economy. The market expectations of continued low interest rates reflect both a relatively weak economy as well as an accommodative monetary policy.

These market expectations are assumed to be driven by the policy rule that the Cental Bank is expected to follow as well as on the deviations from that rule that the Central Bank has already communicated in its forward guidance. Specifically, we assume that the Central Bank sets the short-term interest rate according to the following feedback rule

$$\hat{R}_{t} = \rho_{R}\hat{R}_{t-1} + (1 - \rho_{R})\left(\psi_{\pi}\sum_{j=0}^{3}\hat{\pi}_{t-j} + \psi_{y}\sum_{j=0}^{3}(\hat{y}_{t-j} - \hat{y}_{t-j-1} + \hat{z}_{t-j})\right) + \epsilon_{t}^{R} + \sum_{k=1}^{K}\epsilon_{k,t-k}^{R}, \quad (6)$$

where  $\sum_{j=0}^{3} \hat{\pi}_{t-j}$  is 4-quarter inflation expressed in deviation from the Central Bank's objective  $\pi_*$  (which corresponds to steady state inflation),  $\sum_{j=0}^{3} (\hat{y}_{t-j} - \hat{y}_{t-j-1} + \hat{z}_{t-j})$  is 4-quarter growth rate in real GDP expressed in deviation from steady state growth, and  $\epsilon_{R,t}$  is the standard

contemporaneous policy shock, where  $\epsilon_t^R \sim N(0, \sigma_r^2)$ , *i.i.d.*<sup>15</sup> The last term captures forward guidance following Laseen and Svensson (2011), where  $\epsilon_{k,t-k}^R$  is a policy shock that is known to agents at time t - k, but affects the policy rule k periods later, that is, at time t. We assume that  $\epsilon_{k,t-k}^R \sim N(0, \sigma_{k,r}^2)$ , *i.i.d.* We express the anticipated shocks in recursive form by augmenting the state vector  $s_t$  with K additional states  $\nu_t^R, \ldots, \nu_{t-K}^R$  whose law of motion follows<sup>16</sup>

$$\nu_{1,t}^{R} = \nu_{2,t-1}^{R} + \epsilon_{1,t}^{R} \\
\nu_{2,t}^{R} = \nu_{3,t-1}^{R} + \epsilon_{2,t}^{R} \\
\vdots \\
\nu_{K,t}^{R} = \epsilon_{K,t}^{R}.$$
(7)

We also augment the vector of shocks  $\epsilon_t$  in equation (3) with the anticipated shocks  $[\epsilon_{1,t}^R, ..., \epsilon_{K,t}^R]'$ and resolve the model to compute the matrices  $\Phi_1(\theta)$  and  $\Phi_{\epsilon}(\theta)$  appropriately. Note that we make the – arguably counterfactual – assumption that the anticipated shocks are independent from one another. Campbell et al. (2012b) forcefully argue, based on their own findings as well as Gürkaynak et al. (2005b)'s, that anticipated shocks follow a factor structure. It would be important to relax the independence assumption if we were to estimate the model with forward guidance shocks. However, this assumption bears no implications in the policy exercise described in sections 3.2 and 3.3.<sup>17</sup>

For simplicity we estimate the model parameters assuming no forward guidance – that is setting the last term in (6) to zero –, and without adding (5) to the system of measurement equations. Implicitly we are assuming that forward guidance has little impact on the estimated model parameters. We are however recognizing that it has a potentially large impact on our inference about the state of the economy  $s_t$  in the 2008Q3-2012Q3 period (conditional on the estimated parameters), and hence on the model's forecasts. We are therefore re-estimating  $s_t$  during this period in light of the information provided by (5).<sup>18</sup> Our baseline

<sup>&</sup>lt;sup>15</sup>The economy displays a stochastic trend, so if  $\hat{y}_{t-j}$  is output in deviation from this trend and  $\hat{z}_t$  corresponds to the growth rate of technology in deviations from steady state, then the growth rate of output in period t is  $\hat{y}_t - \hat{y}_{t-1} + \hat{z}_t$ .

<sup>&</sup>lt;sup>16</sup>It is easy to verify that  $\nu_{1,t-1}^R = \sum_{k=1}^K \epsilon_{k,t-k}^R$ , that is,  $\nu_{1,t-1}^R$  is a "bin" that collects all anticipated shocks that affect the policy rule in period t.

<sup>&</sup>lt;sup>17</sup> In this log-linearized model the variance-covariance matrix of the shocks does not affect the equilibrium conditions.

<sup>&</sup>lt;sup>18</sup>The only extra parameters introduced by the forward guidance are the standard deviations  $\sigma_{k,r}$  of the

	2012	2013	2014	2015	
	(Q4/Q4)	(Q4/Q4)	(Q4/Q4)	(Q4/Q4)	
GDP growth					
Baseline	1.9	2.2	1.7	1.3	
FFR at 25bp	3.5	4.9	1.5	0.3	
Forward guidance with					
constrained 10y yield	2.4	3.0	1.5	0.9	
Core PCE inflation					
Baseline	1.6	1.2	1.5	1.6	
FFR at 25bp	1.8	1.9	1.8	1.7	
Forward guidance with					
constrained 10y yield	1.6	1.4	1.5	1.6	
Federal funds rate					
Baseline	0.10	0.20	0.40	1.32	
FFR at 25bp	0.25	0.25	0.25	1.09	
Forward guidance with					
constrained 10y yield	0.08	0.08	0.13	1.32	

Table 5: The macroeconomic consequences of forward guidance

*Notes:* The table reports the model's predictions conditional on alternative assumptions regarding the federal funds rate: the baseline forecast, a counterfactual policy experiment in which the federal funds rate is maintained at 25 basis points until 2015Q2, and a counterfactual policy experiment in which more forward guidance is provided about the federal funds rate such that the 10-year bond yield falls by 10 basis points.

forecast, which is described in Table 5 and Figure 3, is therefore obtained using data released through 2012Q2 augmented for 2012Q3 with observations on the federal funds rate and the Baa corporate bond spread, and with market's FFR expectations through mid-2015 (hence K = 11 in equation (5)) as measured by OIS rates on August 28, 2012.<sup>19</sup>

Figure 3 shows the model's predictions for real GDP growth, core PCE inflation and the federal funds rate, conditional on alternative assumptions regarding the federal funds rate. These forecasts are obtained using the mode of the posterior distribution for  $\theta$  and  $s_t$ , although these modal forecasts in the baseline case essentially coincide with the mean of the forecast distribution obtained by drawing from the full posterior of  $\theta$  and  $s_t$ . The black solid

anticipated shocks. Since we do not have estimates for these parameters, we assume that these shocks have the same standard deviation as the contemporaneous shock:  $\sigma_{k,r} = \sigma_r$ . Importantly, note that the parameters  $\sigma_{k,r}$  do not enter *any* of the policy experiments described below.

 $<sup>^{19}</sup>$ As 2012Q3 observations for the FFR and the Baa corporate bond spread we are using the average of daily rates during the quarter up to this date.



Figure 3: The macroeconomic consequences of forward guidance

*Notes:* The figure shows the model's predictions conditional on alternative assumptions regarding the federal funds rate. The black solid lines show the historical data. The dashed red lines show the FRBNY DSGE model's baseline forecast. The solid blue lines show in turn the model's predictions in a counterfactual policy experiment in which the federal funds rate is set to 0.25 percent until 2015Q2. The solid red lines show the model's predictions in a counterfactual policy experiment in which more forward guidance is provided about the federal funds rate such that the 10-year bond yield falls by 10 basis points.

lines show the historical data. The dashed red lines show the FRBNY DSGE model's baseline forecast. In this forecast, GDP growth is 1.9 percent in 2012 (Q4/Q4), rises to 2.2 percent in 2013 but remains mostly below 2 percent throughout the rest of the forecast horizon (see

the first row in each of the three panels of Table 5). Core PCE inflation is predicted to be at 1.6 percent in 2012 and is also expected to remain below 2 percent throughout the forecast horizon.

#### **3.2** Using anticipated shocks to condition on an interest-rate path

We now proceed with our counterfactual policy experiment in which the federal funds rate is set to 25 basis points (the current rate paid on excess reserves held at the central bank, or IOR) until 2015Q2, and that it follows the historical policy rule after that.<sup>20</sup> We first summarize the procedure used to condition the model's predictions on a given interest-rate path, which is taken from section 6.3 of Del Negro and Schorfheide (2013), and then describe the outcome of the experiment.

Suppose that at the end of period T, after time T shocks are realized, the central bank announces its intention to commit to a given interest-rate path:  $\bar{R}_{T+1}, \ldots, \bar{R}_{T+\bar{H}}$ . For the agents, the announcement is a one-time surprise in period T + 1. This corresponds to the realization of a single unanticipated monetary policy shock  $\epsilon_{T+1}^R$  and a sequence of anticipated shocks  $\{\epsilon_{1,T+1}^R, \epsilon_{2,T+1}^R, \ldots, \epsilon_{K,T+1}^R\}$  where  $K = \bar{H} - 1$ . Notice that all policy shocks that are used to implement the interest rate path are dated T + 1. We denote by  $\epsilon_t$  the vector that collects the innovations of the unanticipated shocks (both policy and non policy shocks), and by  $\epsilon_{1:K,t}^R$  the vector of anticipated policy shocks. The following algorithm determines the time T + 1 monetary policy shocks as a function of the desired interest rate sequence  $\bar{R}_{T+1}, \ldots, \bar{R}_{T+\bar{H}}$  to generate predictions conditional on an announced interest rate path. The announced interest rate path will be attained in expectation.

#### Algorithm 1. Drawing Counterfactual Forecasts via Anticipated Shocks.<sup>21</sup>

1. Use the Kalman filter to compute the mean  $s_{T|T}$  of the distribution  $p(s_T|\theta, Y_{1:T})$ .

<sup>&</sup>lt;sup>20</sup>At the time we wrote the paper, this was one policy option discussed by market commentary for the upcoming FOMC meeting, see the September 10, 2012 WSJ "MarketBeat" Blog at blogs.wsj.com/marketbeat//qe3-what-everybody-that-matters-on-wall-street-expects/. We chose 25 basis points for simplicity as it coincides with the IOR, but of course choosing any lower rate would make the results even stronger as the policy would be even more accommodative.

<sup>&</sup>lt;sup>21</sup>The algorithm in Del Negro and Schorfheide (2013) describes how to draw from the entire counterfactual predictive distribution, conditional on draws of  $\theta$  from the posterior density. Here we focus on the mode of the posterior density for  $\theta$ .

2. Consider the following system of equations, omitting the  $\theta$  argument of the system matrices:

$$\bar{R}_{T+1} = \Psi_{R,1} + \Psi_{R,2}\Phi_{1}s_{T} + \Psi_{R,2}\Phi_{\epsilon}[\bar{\epsilon}_{T+1}^{R}, 0, \dots, 0, \bar{\epsilon}_{1:K,T+1}^{R'}]'$$

$$\bar{R}_{T+2} = \Psi_{R,1} + \Psi_{R,2}(\Phi_{1})^{2}s_{T} + \Psi_{R,2}\Phi_{1}\Phi_{\epsilon}[\bar{\epsilon}_{T+1}^{R}, 0, \dots, 0, \bar{\epsilon}_{1:K,T+1}^{R'}]'$$

$$\vdots$$

$$\bar{R}_{T+\bar{H}} = \Psi_{R,1} + \Psi_{R,2}(\Phi_{1})^{\bar{H}}s_{T} + \Psi_{R,2}(\Phi_{1})^{\bar{H}-1}\Phi_{\epsilon}[\bar{\epsilon}_{T+1}^{R}, 0, \dots, 0, \bar{\epsilon}_{1:K,T+1}^{R'}]'$$
(8)

This linear system of  $\overline{H}$  equations with  $\overline{H}$  unknowns can be solved for for the vector of policy shocks  $\overline{\epsilon}^R = [\overline{\epsilon}_{T+1}^R, \overline{\epsilon}_{1:K,T+1}^{R'}]'$ . Specifically, rewrite the system (8) as

$$b = M_{\bar{H}}\bar{\epsilon}^R,\tag{9}$$

where

$$b = [\bar{R}_{T+1}, ..., \bar{R}_{T+\bar{H}}]' - [\Psi_{R,1} + \Psi_{R,2}\Phi_1 s_T, ..., \Psi_{R,1} + \Psi_{R,2}(\Phi_1)^{\bar{H}} s_T]',$$
  

$$M_{\bar{H}} = [\Psi_{R,2}, \Psi_{R,2}\Phi_1, ..., \Psi_{R,2}(\Phi_1)^{\bar{H}-1}]\Phi_{\epsilon,R},$$
(10)

and  $\Phi_{\epsilon,R}$  collects the columns of the matrix  $\Phi_{\epsilon}$  corresponding to the vector of policy shocks  $\bar{\epsilon}^{R}$ . The solution of (8) is then

$$\bar{\epsilon}^R = M_{\bar{\mu}}^{-1}b. \tag{11}$$

3. Starting from  $s_{T|T}$ , iterate the state transition equation (3) forward to obtain a sequence  $s_{T+1:T+H|T}$ :

$$s_{t|T} = \Phi_1(\theta^{(j)}) s_{t-1|T} + \Phi_\epsilon(\theta^{(j)}) [\epsilon_t^R, 0, \dots, 0, \epsilon_{1:K,t}^R]', \quad t = T+1, \dots, T+H_t$$

where (i)  $\epsilon_{T+1}^R = \bar{\epsilon}_{T+1}^R$  and  $\epsilon_t^R = 0$  for  $t = T+2, \ldots, T+\bar{H}$ ; (ii)  $\epsilon_{1:K,T+1}^R = \bar{\epsilon}_{1:K,T+1}^R$ and  $\epsilon_{1:K,t}^R = 0$  for  $t = T+2, \ldots, T+\bar{H}$  (that is, in both cases use solved-for values in period T+1 and zeros thereafter).

4. Use the measurement equation (4) to compute  $y_{T+1:T+H}$  based on  $s_{T+1:T+H|T}$ .

The solid blue lines show in turn the model's predictions in our counterfactual policy experiment. Such a policy change would imply a reduction in the expected federal funds rate of 15 basis points at the end of 2014 compared to the baseline forecast. According to the model, this alternative policy assumption generates a massive stimulus in 2012 and 2013. Indeed, in this alternative scenario, real GDP growth is forecast to jump to 3.5 percent in 2012 (Q4/Q4), and to 4.9 percent in 2013. GDP growth is however lower than under the baseline scenario in 2014 and 2015, as the effects of the policy stimulus fade over time and the GDP *level* returns to the level it would have had without the stimulus (see the second row in each of the three panels of Table 5). The stimulative effect of policy also raises inflation in 2012 and 2013 to respectively 1.8 percent (Q4/Q4) and 1.9 percent, but inflation is also forecast to remain below 2 percent in 2014 and 2015. The model seems to be generating an implausibly large response of real GDP growth and inflation to an apparently small change in the federal funds rate. What is responsible for this?

#### 3.3 What is the excessive response due to?

To understand this, consider a simplified version of the FRBNY DSGE model in which there is no habit persistence and no shocks other than monetary policy shocks. In this case, the consumption Euler equation reduces to the conventional expression

$$\hat{c}_t = I\!\!E_t[\hat{c}_{t+1}] - (\hat{R}_t - I\!\!E_t[\hat{\pi}_{t+1}]), \qquad (12)$$

where  $\hat{c}_t$  denotes consumption deviations from steady state. Iterating this equation forward to eliminate expected future consumption, we obtain

$$\hat{c}_t = -\sum_{j=0}^{\infty} I\!\!E_t [\hat{R}_{t+j} - \hat{\pi}_{t+1+j}], \qquad (13)$$

so that contemporaneous consumption is directly negatively related to the long-term real interest rate (at infinite maturity), which is given by  $\hat{r}_t^L = \sum_{j=0}^{\infty} I\!\!E_t [\hat{R}_{t+j} - \hat{\pi}_{t+1+j}]$ . It follows that anticipated changes in the short-term real rate in the future affect consumption both today and in the future. To see this, suppose for now that prices are fixed so that the nominal and real interest rates move by the same amount, and suppose that the short-term rates declines by  $\hat{R}_t = -\Delta$  contemporaneously but reverts to steady state  $\hat{R}_{t+j} = 0$  after that. In this case, the long run real rate would also decline by  $\Delta$  in period t and revert to steady state after that, so that consumption would increase temporarily by  $\Delta$  in period t. Consider now an the announcement of a temporary decline in the short-term rate at date

t+j of  $\hat{R}_{t+j} = -\Delta$ . In that case, the long-term real rate  $\hat{r}_t^L$  would decline from periods t to t+j, before reverting to steady state from t+j+1 onward. This implies that consumption would increase by  $\Delta$  in period t and remain at that level until t+j, before reverting to steady state.

Now letting prices adjust, a New Keynesian Phillips curve, would imply that inflation also increases on impact as demand for goods and services is expected to surpass the potential output of the economy from periods t to t + j. The persistent increase in inflation induces the real interest to fall by more than the nominal interest rate, which reinforces the effect of the initial policy shock. Countervailing this force however is the effect of monetary policy which tends to raise the real rate as inflation increases. All these effects are playing out in the medium scale model.

While we have focused here on the response of consumption, real investment is also related to the long-term real interest rate in the medium scale model. A natural question is then whether the strong response of economic activity in the model to changes in the nearterm path of the short-term interest rate is due to too strong a response of consumption and inflation to given changes in the long-term interest rate, or alternatively to too strong a response of the long-term interest rate.

Looking at the model's interest-rate projections farther into the future provides valuable insights. Figure 4 shows the paths of short-term interest rates under the baseline projection (red dashed lines), and the counterfactual policy (blue solid line) until 2027Q4. This figure reveals that while the expected short-term rate is only 15 basis points lower in the counterfactual than in the baseline at the end of 2014, the difference between the two interest-rate paths is expected to be much larger farther in the future, in particular between 5 and 10 years following the current policy announcement. These large drop far in the future of the expected future short-term rate compared to the baseline path is in turn resulting in a large drop of the long-term interest rate.

To see this more clearly, we compute the long-run interest rate response, proceeding as follows. At the end of period T, after the realization of all period-T shocks, the preintervention interest rate with maturity L at date T+1 is computed as the average of future short-term rate over the relevant horizon, and is given by the following expression:

$$R_{T+1}^{L} = \frac{1}{L} \sum_{j=1}^{L} \mathbb{I}_{T}[R_{T+j}]$$

$$= \Psi_{R,1} + \frac{1}{L} \Psi_{R,2} (I - \Phi_{1})^{-1} (I - \Phi_{1}^{L}) \Phi_{1} s_{T}.$$
(14)

The post-intervention 10-year rate  $R_T^{L*}$ , i.e., the rate obtained after the announcement of period-T + 1 policy shocks  $(\bar{\epsilon}_{T+1}^R, \bar{\epsilon}_{1:K,T+1}^{R'})$  is given by:

$$R_{T+1}^{L,*} = \frac{1}{L} \sum_{j=1}^{L} \mathbb{I}_{T} [R_{T+j} | \bar{\epsilon}_{T+1}^{R}, \bar{\epsilon}_{1:K,T+1}^{R'}]$$

$$= R_{T+1}^{L} + \Psi_{R,2} \frac{1}{L} (I - \Phi_{1})^{-1} (I - \Phi_{1}^{L}) \Phi_{\epsilon,R} [\bar{\epsilon}_{T+1}^{R}, \bar{\epsilon}_{1:K,T+1}^{R'}]'.$$
(15)

Call  $\Delta R_{T+1}^L = R_{T+1}^{L,*} - R_{T+1}^L$  the impact of the intervention on rate with maturity L. It satisfies:

$$\Delta R_{T+1}^L = N_L[\bar{\epsilon}_{T+1}^R, \bar{\epsilon}_{1:K,T+1}^{R'}]'$$
(16)

where

$$N_L = \Psi_{R,2} \frac{1}{L} (I - \Phi_1)^{-1} (I - \Phi_1^L) \Phi_{\epsilon,R}.$$
 (17)

In the counterfactual experiment, the 5-year yield falls by 16 basis points upon the announcement, compared to the baseline scenario, and the 10-year yield falls by as much as 25 basis points. The fact that the 10-year yields falls by more than the 5-year yield is simply a reflection, again, that the short-term interest rate is expected to deviate more from the baseline at long horizons than in the near term, as shown in Figure 4.

The model-implied responses for the long-term rate do not seem to match the 5 and 10year yield responses observed in the data, however. Following the January 25, 2012 FOMC meeting, for instance, the statement reinforced the forward guidance about the federal funds rate by announcing an extension of the first liftoff date. This resulted in a reduction in 5 and 10-year yields of 8 and 7 basis points, respectively.<sup>22</sup>

<sup>&</sup>lt;sup>22</sup>As mentioned in the introduction, Carlstrom et al. (2012) assess the impact of pegging the policy rate in three variants of the New Keynesian model. They start from the steady-state equilibrium and analyze the effect of lowering the policy rate to the ZLB for K quarters. In their calibration, this amounts to lowering the policy rate by 4 percentage points for K quarters. In the simple version of the New Keynesian model, it can be shown that the response of inflation or output is directly proportional the distance between the steady



Figure 4: Interest-rate projections farther into the future

*Notes:* The figure shows the model's predictions for the federal funds rate farther into the future. The black solid line shows the historical data. The dashed red line shows the FRBNY DSGE model's baseline forecast. The solid blue line shows the model's predictions in a counterfactual policy experiment in which the federal funds rate is set to 0.25 percent until 2015Q2. The solid red line shows the model's predictions in a counterfactual policy experiment in which the 10-year bond yield falls by 10 basis points.

Figure 5 shows the impulse response functions to contemporaneous and anticipated policy shocks in order to provide some more intuition for what is happening. Specifically, the figure shows the response of the short term interest rate, the 10-year nominal rate, the level of output, and inflation to expansionary 50 basis points shocks. The difference between the three columns in Figure 5 is that this 50 basis points shock is either contemporaneous (left column) or anticipated 4 and 8 periods ahead (middle and right columns, respectively). We want to highlight four features of Figure 5: i) Since the anticipated expansionary shock leads to higher inflation and output *before the shock takes place*, and since the policy authorities are bound to follow the rule before that date, the interest rate follows a zig-zag pattern, where it first rises and then falls. If this pattern of interest rates appears awkward, bear in

state interest rate and the level at the ZLB, and the response of inflation and output grows exponentially with the number of periods that the policy rate is expected to be maintained at the ZLB. Since the shortterm rate is assumed to return back to steady state K + 1 periods after the announcement, the 10-year long-run nominal rate is assumed to fall by 400K/40 = 10K basis points in their experiment. Concretely, their experiment assumes that a forward guidance of 8 quarters would imply a drop of 80 basis points in the 10-year bond yield.

mind that we are unlikely to see an eight periods-ahead shock *in isolation* (e.g., Campbell et al. (2012b)). ii) The response of the 10-year rate, quite understandably, reaches its lowest point at the time the shock takes place, and the trough decreases monotonically with the anticipation horizon. iii) The peaks in the response of output roughly coincide (with a slight delay) with the peaks in the response of the 10-year rate, in agreement with equation (13). In addition, the effect on output increases monotonically with the horizon. The delay is due to features like habit persistence. iv) The impact on inflation also increases monotonically with the horizon – not a surprising finding given the output responses.

The responses in Figure 5 provide some economic intuition behind the finding of Carlstrom et al. (2012) that the response of macroeconomic variables to an interest rate peg is a convex function of the horizon of the peg. Imagine the policymakers want to lower interest rates by 50 basis points for 7 periods. This can be implemented with a sequence of contemporaneous and anticipated shocks up to 7 periods ahead. Now imagine they decide to extend the peg one extra period. Because of the zig-zag feature of the 8th period impulse response, that decision will tend to lift the short-term rate in quarters 0 to 7 and so requires a cascade of shocks over that period to push the interest rate back down. In light of these impulse responses (and the related impact on the long rate) it is not surprising that even a modest amount of forward guidance produces large effects, as long as it extends far enough into the future.

## 4 A Proposed Resolution

We propose here a potential resolution to the Forward Guidance Puzzle described above, by introducing in our model the fact that life is finite. More specifically, we adopt Blanchard (1985)'s and Yaari (1965)'s perpetual youth model, which has been incorporated recently in simple New Keynesian models by Nisticò (2012) and Piergallini (2006). We assume as in these papers that the economy is populated by an indefinite number of cohorts that face each period a constant probability  $\gamma$  of dying and being replaced before the next period begins. This feature induces agents to discount future consumption more heavily in the aggregate. Indeed, in the aggregate linearized consumption Euler equation, consumption depends on discounted future consumption. Compared to standard models, a positive death probability also induces higher discounting of future expected dividends in a linearized asset pricing



Figure 5: Impulse response functions to contemporaneous and anticipated policy shocks

*Notes:* The figure shows the percent change over a 12 quarter horizon of the short term interest rate, the 10-year nominal rate, the level of output, and Core PCE inflation in response to a contemporaneous, 4 quarter and 8 quarter ahead negative 50 basis points policy shock.

condition, and leads price setters to discount more future marginal costs, in setting their prices. An implication of this is that announcements of policy changes far in the future tend to generate smaller effects on current aggregate variables than is the case in models with infinitely lived agents.

To explore in details how a positive death probability alters the effects of forward guidance, we first perform our analysis in a calibrated version of the basic New Keynesian model. To assess the quantitative relevance of this mechanism in a more realistic model, we proceed with the analysis of an estimated medium-scale DSGE model that allows for positive death probability, in subsection 4.3.

#### 4.1 Microeconomic Foundations

#### 4.1.1 Demand side

As in Nisticò (2012), we assume that the representative agent born at date  $j \leq 0$  chooses consumption,  $C_{j,t}$ , and hours worked,  $N_{j,t}$ , to maximize discounted sum of utility flows

$$E_0 \sum_{t=0}^{\infty} \beta^t (1-\gamma)^t e^{v_t} \left[ \log C_{j,t} + \varphi \log (1-N_{j,t}) \right], \qquad \beta \in (0,1), \ \varphi > 0$$

subject to its budget constraint. The discounting involves in addition to the rate of time preference  $\beta$  the probability of survival from one period to the next,  $1 - \gamma$ . We allow the period utility to be perturbed by aggregate, exogenous and mean-zero preference shocks  $v_t$ . Agents may hold two types of financial assets: one-period state-contingent nominal bonds, which pay off  $B_{j,t+1}$  in t+1, and equity shares issued by each monopolistic competitive firm,  $Z_{t+1}(i)$ , and whose real price is  $Q_t$  in period t. In addition, as in Blanchard (1985), we assume that there exist life-insurance companies which offer a contract paying each period and to each agent a fraction  $\gamma$  of its wealth as long as it lives, in exchange for receiving the agent's entire wealth when it dies. Free entry implies zero profits for these insurance companies. Agent j's budget constraint is

$$P_t C_{j,t} + E_t \left( \mathcal{F}_{t,t+1} B_{j,t+1} \right) + P_t \int_0^1 Q_t \left( i \right) Z_{j,t+1} \left( i \right) di = W_t N_{j,t} + \Omega_{j,t}$$
(18)

$$\Omega_{j,t} \equiv \frac{1}{1-\gamma} \left[ B_{j,t} + P_t \int_0^1 \left( Q_t \left( i \right) + D_t \left( i \right) \right) Z_{j,t} \left( i \right) di \right].$$
(19)

The wealth carried over from the previous period gets multiplied by  $\frac{1}{1-\gamma}$  due to the life insurance contract.<sup>23</sup> This implies individuals do not receive any life-insurance transfer when they are born, as they start life with zero financial wealth.

The first-order conditions involve the intra-temporal condition with respect to consumption and leisure

$$\varphi \frac{C_{j,t}}{1 - N_{j,t}} = \frac{W_t}{P_t},\tag{20}$$

and the two inter-temporal optimality conditions with respect to both financial assets

$$\mathcal{F}_{t,t+1} = \beta \frac{P_t C_{j,t}}{P_{t+1} C_{j,t+1}} e^{v_{t+1} - v_t}$$
(21)

$$P_t Q_t (i) = E_t \left\{ \mathcal{F}_{t,t+1} P_{t+1} \left[ Q_{t+1} (i) + D_{t+1} (i) \right] \right\}.$$
(22)

The gross nominal riskless return  $R_t$  relates to the stochastic discount factor according to

$$E_t (\mathcal{F}_{t,t+1}) = (R_t)^{-1}.$$
 (23)

Combining with (21), we obtain the standard consumption Euler equation

$$\frac{1}{C_{j,t}} = E_t \left[ \beta \left( R_t \right) \frac{P_t}{P_{t+1}} \frac{1}{C_{j,t+1}} e^{v_{t+1} - v_t} \right]$$
(24)

for each cohort j. This equation indicates, that in each cohort, agents are choosing the intertemporal allocation of consumption by equating their marginal utility of consumption in period t with their discounted expected future marginal utility of consumption.

Combining (24), (22), and the definition of wealth (19) with the budget constraint (18), we obtain an expression for the evolution of financial wealth

$$P_t C_{j,t} + E_t \left[ \mathcal{F}_{t,t+1} \left( 1 - \gamma \right) \Omega_{j,t+1} \right] = W_t N_{j,t} + \Omega_{j,t}.$$
(25)

<sup>23</sup>To understand why, note that  $\frac{1}{1-\gamma} = 1 + \frac{\gamma}{1-\gamma}$  where for any dollar of financial wealth carried over from the previous period, agent *j* receives in addition  $\frac{\gamma}{1-\gamma}$  dollars from the life insurance which collects a fraction  $\gamma$  of the financial wealth each cohort (as a fraction  $\gamma$  of the agents in each cohort dies) and divides it up among the  $(1-\gamma)$  remaining agents. We furthermore define human wealth for cohort j as the discounted sum of expected future labor incomes:

$$h_{j,t} \equiv E_t \left[ \sum_{k=0}^{\infty} \mathcal{F}_{t,t+k} \left( 1 - \gamma \right)^k W_{t+k} N_{j,t+k} \right]$$
(26)

Solving (25) forward for  $\Omega_{j,t}$ , and using (21), (26) and a no-Ponzi game condition allows us to obtain the consumption function

$$P_t C_{j,t} = \frac{1}{\Sigma_t} \left( \Omega_{j,t} + h_{j,t} \right) \tag{27}$$

where  $\Sigma_t \equiv E_t \left[ \sum_{k=0}^{\infty} \beta^k (1-\gamma)^k e^{v_{t+1}-v_t} \right]$  is the inverse of the marginal propensity to consume out of financial and human wealth.<sup>24</sup> Since the generation born in period t starts with no financial wealth ( $\Omega_{t,t} = 0$ ), it consumes only out of its human wealth, i.e.,  $P_t C_{t,t} = \frac{1}{\Sigma_t} h_{j,t}$ .

**4.1.1.1 Aggregation** Aggregating across cohorts, with weights given by cohort sizes, we have  $X_t \equiv \sum_{j=-\infty}^{t} \gamma (1-\gamma)^{t-j} X_{j,t}$  for each variable X = C, N, B, h, Z(i). The initial size of each cohort is assumed to be  $\gamma$ . The resulting aggregate conditions are given by (20) and (26), where we drop the index j. In addition, the aggregate budget constraint can be expressed as

$$P_t C_t + E_t \left[ \mathcal{F}_{t,t+1} \Omega_{t+1} \right] = W_t N_t + \Omega_t.$$
(28)

where we define aggregate (nominal) financial wealth as

$$\Omega_t \equiv \left[ B_t + P_t \int_0^1 \left( Q_t \left( i \right) + D_t \left( i \right) \right) Z_t \left( i \right) di \right].$$

The aggregate consumption function (27) is thus given by

$$P_t C_t = \frac{1}{\Sigma_t} \left( \frac{1}{1 - \gamma} \Omega_t + h_t \right).$$
<sup>(29)</sup>

Combining the aggregate budget constraint (28) and the aggregate consumption function (29), and using the aggregate version of (26), we obtain the dynamic equation for consumption

$$(\Sigma_t - 1) C_t = \gamma E_t \left[ \mathcal{F}_{t,t+1} \Omega_{t+1} P_{t+1} / P_t \right] + (1 - \gamma) E_t \left[ \mathcal{F}_{t,t+1} \Sigma_{t+1} C_{t+1} P_{t+1} / P_t \right].$$

<sup>24</sup>In steady state, the marginal propensity to consume out of wealth is given by  $\Sigma^{-1} \equiv \left[\sum_{k=0}^{\infty} \beta^k \left(1-\gamma\right)^k\right]^{-1} = 1 - \beta \left(1-\gamma\right).$ 

This shows that aggregate consumption depends not only on discounted future consumption, weighted by the survival probability  $(1 - \gamma)$ , but also on aggregate financial wealth. Another way to get some intuition for this is by aggregating equation (21) and using (23), so as to obtain the aggregate consumption Euler equation

$$\frac{1}{C_t} = E_t \left[ \beta \left( R_t \right) \frac{P_t}{P_{t+1}} \frac{(1-\gamma)}{C_{t+1} - \gamma C_{t+1,t+1}} e^{v_{t+1} - v_t} \right].$$
(30)

This equation reveals that the optimal allocation of consumption involves equating the marginal utility of aggregate consumption at date t with the discounted marginal utility of next period's consumption of the currently living agents, i.e., aggregate consumption minus the consumption of the agents born next period,  $C_{t+1,t+1}$ . Since aggregate consumption depends on financial and human wealth, whereas the consumption of these born next period will depend on their human wealth only, by subtracting the consumption of the agents born next period, which depends on their human wealth, we reinforce the weight attributed to financial wealth in today's consumption decision.

#### 4.1.2 Supply side

The supply side is standard. A perfectly competitive retail sector produces a final consumption good  $Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\mu}} di\right]^{1+\mu}$ ,  $\mu > 0$ , using a continuum of differentiated intermediate goods  $Y_t(i)$ . The demand for each intermediate good is given by  $Y_t(i) = (P_t(i)/P_t)^{-(1+\mu)/\mu} Y_t$ , where the price index satisfies  $P_t = \left[\int_0^1 P_t(i)^{-1/\mu} di\right]^{-\mu}$ . The latter goods are produced using labor  $N_t(i)$  hired on a competitive labor market, and exogenous productivity  $A_t$ , according to the production function  $Y_t(i) = A_t N_t(i)$ . As in Calvo's (1983) model, the intermediate producers may reset their prices with a probability  $1 - \zeta_p$ . Whenever they get to change their price, firms maximize the expected stream of future dividends, and hence the value of their outstanding shares, realizing that they won't be able to re-optimize their price until period t + k with probability  $\zeta_p^k$ . Specifically, a firm i which resets its price at date t solves:

$$\max_{P_t^*(i)} E_t \left\{ \sum_{k=0}^{\infty} \zeta_p^k \mathcal{F}_{t,t+k} Y_{t+k} \left( i \right) \left[ P_t^* \left( i \right) - P_{t+k} m c_{t+k} \right] \right\}$$

subject to the demand for its good, where  $mc_t = W_t/(A_tP_t)$  denotes real marginal costs, which are the same for all firms. The first-order condition to this problem can be written as

$$E_t \left\{ \sum_{k=0}^{\infty} \zeta_p^k \mathcal{F}_{t,t+k} Y_{t+k} P_{t+k}^{(1+\mu)/\mu} \left[ P_t^* - (1+\mu) P_{t+k} m c_{t+k} \right] \right\} = 0.$$
(31)

This condition determines the optimal reset price  $P_t^*$  chosen by all firms which get to reoptimize their price at date t. Given the definition of the aggregate price index, the evolution of the aggregate price level is given by

$$P_t = \left[\zeta_p P_{t-1}^{-1/\mu} + (1 - \zeta_p) \left(P_t^*\right)^{-1/\mu}\right]^{-\mu}.$$
(32)

#### 4.1.3 Equilibrium

In equilibrium, the state contingent bonds are in zero net supply so that  $B_t = 0$ . We normalize the total amount of issued shares to one for all intermediate firms,  $Z_t(i) = 1$ , so that the discounted value of future financial nominal wealth satisfies

$$E_t\left(\mathcal{F}_{t,t+1}\Omega_{t+1}\right) = P_tQ_t$$

where  $Q_t \equiv \int_0^1 Q_t(i) di$  is the aggregate real stock price index.

We then have the resource constraints

$$Y_t = C_t$$
$$P_t Y_t = W_t N_t + P_t D_t$$

where  $D_t \equiv \int_0^1 D_t(i) \, di$  denotes total real dividend payments, the labor supply

$$\varphi \frac{C_t}{1 - N_t} = \frac{W_t}{P_t},$$

the aggregate consumption Euler equation

$$(\Sigma_t - 1) C_t = \gamma Q_t + (1 - \gamma) E_t [\mathcal{F}_{t,t+1} \Sigma_{t+1} C_{t+1} P_{t+1} / P_t],$$

and the asset price condition

$$Q_{t} = E_{t} \left[ \mathcal{F}_{t,t+1} \left( Q_{t+1} + D_{t+1} \right) P_{t+1} / P_{t} \right]$$

The evolution of goods prices is then given by (31) and (32).

We close the model with (23) and an equation characterizing the behavior of monetary policy. We do so by assuming a policy rule of the form described below.

#### 4.1.4 Linearized equilibrium conditions

We consider a deterministic steady state with zero inflation and constant output. In this steady state, we have

$$(1+r)^{-1} = \beta\eta$$

where r is the steady-state (net) real interest rate,

$$\eta \equiv \frac{1}{1 + \frac{\Omega}{PC} \gamma \left(\frac{1}{1 - \gamma} - \beta\right)},$$

and  $\frac{\Omega}{PC}$  is the steady-state wealth to consumption ratio. In the conventional case that  $\gamma = 0$ , we have  $\eta = 1$ . When  $\gamma > 0$ , and household net wealth is positive in steady state  $(\frac{\Omega}{PC} > 0)$ , we have  $\eta < 1$ , so that the gross real rate of interest is higher than  $\beta^{-1}$ .

Log-linearizing the equilibrium conditions around this steady state, and combining the conditions, we obtain:

$$y_t = \eta E_t y_{t+1} + (1 - \eta) q_t - \eta \left( \hat{R}_t - E_t \pi_{t+1} \right) + \tilde{\nu}_t$$
(33)

$$q_{t} = \eta \beta E_{t} q_{t+1} + (1 - \eta \beta) \left( \lambda E_{t} a_{t+1} - (\lambda - 1) E_{t} y_{t+1} \right) - \left( \hat{R}_{t} - E_{t} \pi_{t+1} \right)$$
(34)

$$\pi_t = \eta \beta E_t \pi_{t+1} + \kappa \left( y_t - a_t \right) \tag{35}$$

where  $x_t = \log (X_t/X)$  for X = Y, Q, A, and  $\pi_t = \log (P_t/P_{t-1})$ ,  $\hat{R}_t = R_t - R$ . The exogenous shock  $\tilde{\nu}_t$  summarizes the preference shocks  $E_t \Delta \nu_{t+1}$ . The parameter  $\lambda \equiv \frac{1 + \varphi^{-1}}{\mu} > 1$ depends on the degree of monopoly power in the intermediate goods sector,  $\mu$ , and the Frisch elasticity of labor supply  $\varphi = (1 - N)/N$ , and  $\kappa \equiv \frac{(1 - \zeta_p)(1 - \zeta_p \eta \beta)}{\zeta_p} (1 + \varphi^{-1})$ .

Equation (33) is the log-linearized output Euler equation or intertemporal IS equation. It reveals that because of intertemporal consumption smoothing by agents, output depends positively on expected future output and negatively on the real interest rate, with weight  $\eta$ . Demand for output depends also positively on the real value of stock prices,  $q_t$ , as well as on preference shocks  $\tilde{\nu}_t$ , which is proportional to  $E_t \Delta \nu_{t+1}$ . Equation (34) shows that real stock prices depend positively on expected future stock prices and on expected future productivity  $E_t a_{t+1}$ , but vary inversely with real interest rates and expected future output. Equation (35) is the New Keynesian Phillips curve which states that inflation depends on the gap between output and potential output, here summarized by total factor productivity  $a_t$ , and on discounted expectation of future inflation, where the discounting involves the time preference parameter  $\beta$  as well as  $\eta$ , which summarizes the survival probability  $(1 - \gamma)$  and the financial wealth to consumption ratio.

We assume that monetary policy is set according to a simple interest rate rule

$$\hat{R}_{t} = \rho_{R}\hat{R}_{t-1} + (1 - \rho_{i})\left[\psi_{\pi}\pi_{t} + \psi_{y}\left(y_{t} - a_{t}\right)\right] + \varepsilon_{0,t} + \sum_{h=1}^{H} \varepsilon_{h,t-h}$$

where  $\varepsilon_{0,t}$  denotes unanticipated monetary policy shocks, and where  $\varepsilon_{h,t-h}$  represent anticipated shocks announced at date t-h and which affect the interest rate at date t. We assume that all monetary policy shocks are iid. Finally, we assume that the exogenous variables  $a_t$ and  $\nu_t$  are driven by some stochastic processes.

#### 4.2 Analysis of the Simple NK Model with Finite Life

In the case of infinite life,  $\gamma = 0$ , we have  $\eta = 1$  so that the model reduces to the conventional output Euler equation  $y_t = E_t y_{t+1} - (\hat{R}_t - E_t \pi_{t+1}) + E_t \Delta \nu_{t+1}$  and the standard New Keynesian Phillips curve  $\pi_t = \beta E_t \pi_{t+1} + \kappa (y_t - a_t)$ . In the case of a positive probability of death,  $\gamma > 0$  (and hence  $\eta < 1$ ), the output Euler equation reveals that current output depends less than one for one on expected future output and the real interest rate; it responds however directly on the real value of the stock market. To understand this, it is useful to integrate forward equations (33)–(35), to obtain:

$$y_t = E_t \sum_{j=0}^{\infty} \eta^j \left[ (1-\eta) q_{t+j} - \eta \left( \hat{R}_{t+j} - \pi_{t+1+j} \right) + \tilde{\nu}_{t+j} \right]$$
(36)

$$q_t = E_t \sum_{j=0}^{\infty} (\eta \beta)^j \left[ (1 - \eta \beta) (\lambda a_{t+j+1} - (\lambda - 1) y_{t+j+1}) - (\hat{R}_{t+j} - \pi_{t+j+1}) \right]$$
(37)

$$\pi_t = E_t \sum_{j=0}^{\infty} (\eta \beta)^j \kappa (y_{t+j} - a_{t+j}).$$
(38)

The integrated Euler equation (36) shows that output depends negatively on all future real interest rates  $(\hat{R}_{t+j} - \pi_{t+1+j})$ , however with weights  $\eta^j$  that are declining with the horizon. This will be the key mechanism to address the forward guidance puzzle. In the infinite horizon case, i.e., when  $\gamma = 0$ , these weights are constant at  $\eta = 1$ , so that current output depends on the sum of all future deviations of the short-term real interest rate  $(\hat{R}_{t+j} - \pi_{t+1+j})$  from its steady state. In the hypothetical case that the real interest rate would be brought permanently below its steady state, output would jump to  $+\infty$ . When  $\gamma > 0$  instead, the response of output to a similar permanent drop in the real interest rate is finite. It is given by

$$\frac{dy}{d\left(\hat{R}-\pi\right)} = -\frac{1-\beta\eta^2}{(1-\eta)\left(1-\beta\eta\right)}\lambda < 0$$

While we have focused on the output response, we note that wealth effects matter too. Expected interest rate declines tend to boost equities as shown in (37). However, as output surpasses its efficient level (given here by total factor productivity  $a_t$ ), profits tend to decline which reduces stock prices and hence mitigates the stimulus to consumption and output. Stock prices also depend less on expected future stock prices, when the death probability is positive; in effect, agents discount the future cash flows more rapidly in this case. Indeed the weights  $(\eta\beta)^j$  in (37) decline faster with the horizon, when  $\eta < 1$ .

Turning to the Phillips curve, expression (38) shows that inflation is proportional to the discounted sum of expected future output gaps  $(y_{t+j} - a_{t+j})$ . Again, when the death probability is positive,  $\eta < 1$  which results is a more rapid discounting of future output gaps, and thus to less inflation reaction to shocks that affect that output gap far in the future.

#### 4.2.1 Calibration

To get a sense of the quantitative implications of the mechanism proposed here, we calibrate the model parameters as follows. We set the time preference parameter to the conventional value  $\beta = 0.99$ . To calibrate the ratio of financial wealth to consumption, we compute the ratio of household net worth (including nonprofit organizations) to final (quarterly) consumption expenditures, from the Integrated Macroeconomic Accounts (Table S.3.a). Over the period 1960-2013, this ratio has fluctuated between 22 and 28; we set  $\Omega/C$  to its sample average of 24.4.

We now turn to our death probability  $\gamma$ . We use the actuarial life table from the Social Security Administration to construct the death probability for individuals of ages 20 and above, weighting by the population at each age. This yields an average probability of dying before the next quarter of  $\gamma = 0.0042$ . These calculations treat the wealth to consumption ratio and the probability of death as independent from each other. In the data, however, both the probability of death and wealth are rising with age. As an alternative calibration, we weight the probability of death at each age by both the fraction of population at each age and by the share of wealth in each age category.<sup>25</sup> The resulting weighted average probability of dying before the next quarter is  $\gamma = 0.0076$ .

While our model refers formally to the probability of dying, such an event amounts to re-setting the initial wealth of the household to zero. In this light, one can think of death in the model as representing also default on the part of households. Based on the Quarterly Report on Household Debt and Credit from the Federal Reserve Bank of New York, the fraction of consumers with new bankruptcies in a given quarter has been on average 0.2 percent from 2003 to 2014. Focusing on mortgage owners, the probability of transitioning from being current on mortgage payments to being 90+ days late on that payment within a quarter has been on average 0.0034.<sup>26</sup> Taking into account both the probability of a new bankruptcy and the probability of being 90+ days late on a mortgage payment, and noting that the two events can be correlated, we set the probability of defaulting in a given quarter to at least 0.005. Adding this to the probability of dying mentioned above, suggests that our calibrated value for  $\gamma$  should be at least 0.0126.

As an alternative case, we raise the value of  $\gamma$  to 0.03 to include other forms of "re-setting" the agent's initial wealth that we might have omitted. Based on a Bayesian estimation using macroeconomic data, Castelnuovo and Nistico (2010) find a substantially higher posterior mean of  $\gamma = 0.1292$ .

The calibration of the Frisch elasticity of labor supply is still controversial with microeconomic studies suggesting that  $\varphi$  should be close to 0.5 while macroeconomic studies find values larger than 1. We set  $\varphi = 1$ . Regarding the degree of monopoly power, we set the net markup  $\mu = 0.1$ , as is common in the literature. For the degree of price rigidities, we set  $\zeta_p = 0.74$ , the value estimated by Castelnuovo and Nistico (2010). This value implies that prices are re-optimized on average every 1/(1 - 0.74) = 3.84 quarters. Finally, to clarify the

<sup>&</sup>lt;sup>25</sup>For wealth, we use data from the Federal Reserve's Survey of Consumer Finances for 2010, and use median wealth by age category.

<sup>&</sup>lt;sup>26</sup>In addition, the probability of transitioning from being current to being 30-60 days late on the mortgage payment in substantially larger, averaging 1.7 percent per quarter. This provides a signal of incoming default as the probability of transitioning form this category to being 90+ days late on a mortgage payment is 24 percent. It follows that the probability in any given quarter of being 90+ days late on the mortgage payment within the next 6 months is 0.0034 + 0.017 \* 0.24 = 0.00748.

effects of the model features, we keep the monetary policy rule as simple as possible, setting the interest rate response to inflation at  $\psi_{\pi} = 1.5$ , and leaving the other policy coefficients equal to 0. Later on, we will also discuss how the response to anticipated policy shocks vary with alternative policy coefficients. These parameters are summarized in Table 1.

Time preference	β	0.99
Wealth to consumption ratio		24.4
Death probability	$\gamma$	$\{0.0126, 0.03, 0.1292\}$
Frisch elasticity of labor supply	$\varphi$	1
Net markup	$\mu$	0.1
Degree of price rigidities	$\zeta_p$	0.74
Monetary policy rule:		
Response to inflation	$\psi_{\pi}$	1.5
Response to output gap	$\psi_y$	0
Response to lagged interest rate	$ ho_R$	0

Table 1. Model Calibration

#### 4.2.2 Results

Table 2 shows how different assumptions about the death probability  $\gamma$  affect some of the key parameters. The implied value for the discounting coefficient entering equations (33)–(35) varies significantly with changes in  $\gamma$ . Our lower bound on  $\gamma (= 0.126)$  implies a discounting coefficient of  $\eta = 0.99$ . This coefficient is lowered to 0.667 when  $\gamma$  is set at our high value. The slope of the Phillips curve  $\kappa$  is also affected, rising with the death probability  $\gamma$ . Finally, while output would surge to  $+\infty$  in response to a permanent drop in the real interest rate when  $\gamma = 0$ , the output response to such a change in the interest rate would be considerably smaller with positive death probabilities.

I					
Death probability	$\gamma$	0	0.0126	0.03	0.1292
Implied coefficients					
Discounting	$\eta$	1	0.993	0.971	0.667
Slope of Phillips Curve	$\kappa$	0.188	0.192	0.203	0.359
Output response to permanent change in real interest rate	$\frac{dy}{d\left(\hat{R}-\pi\right)}$	$-\infty$	$-200.9\lambda$	$-59.2\lambda$	$-4.9\lambda$

Table 2. Implied Coefficients for Alternative Death Probabilities

To understand how the dynamic response of key economic variables is affected by the death probability, we show in the following figures the response of the short-term nominal and real interest rates, inflation and output to the announcement of a 25 basis points (annualized) drop in the short-term interest rate, H quarters ahead, for H varying from 1 quarter to 8 quarters.

4.2.2.1 Completely sticky prices We start by assuming that prices are completely fixed ( $\zeta_p = 1$ ) so that the Phillips curve is flat ( $\kappa = 0$ ). It follows that the responses of the nominal and real interest rates are identical. Figure 6 shows the standard case, when the death probability  $\gamma = 0$ . In response to the announcement, the (annualized) long-term real interest rates falls by 25 basis points until period H, at which point it returns to steady state. As a result, output jumps on impact and stays at that level until period H, when it returns to the initial steady state.

With a death probability at  $\gamma = 0.0126$ , and hence with a discount factor  $\eta < 1$ , the output response is somewhat mitigated as show in Figure 7. Indeed, as discussed above, we see from (36) that the announcement of an interest rate change at some horizon H is discounted by  $\eta^{H}$ .

With a death probability  $\gamma = 0.03$ , the output response is much more mitigated, as shown in Figure 8.

Figure 9 shows that output response on impact as a function of the horizon H, for all three cases just discussed. Clearly, the output response on impact gets smaller as the horizon



Figure 6: Anticipated monetary policy shocks when  $\gamma = 0$ 

Notes: The figure shows impulse response functions to anticipated monetary shocks at horizons H = 1...8, for  $\gamma = 0$ , when prices are assumed to be completely fixed ( $\zeta_p = 1.$ )

Figure 7: Anticipated monetary policy shocks when  $\gamma = 0.0126$ 



Notes: The figure shows impulse response functions to anticipated monetary shocks at horizons H = 1...8, for  $\gamma = 0.0126$ , when prices are assumed to be completely fixed ( $\zeta_p = 1.$ )

grows, so that discounting due to death probability mitigates any effect of forward guidance on output.



Figure 8: Anticipated monetary policy shocks when  $\gamma = 0.03$ 

Notes: The figure shows impulse response functions to anticipated monetary shocks at horizons H = 1...8, for  $\gamma = 0.03$ , when prices are assumed to be completely fixed ( $\zeta_p = 1.$ )

Figure 9: Impact response of output to anticipated monetary shocks at horizons H = 1, ...8, for various values of  $\gamma$ .



Notes: The figure shows the impact response of output to anticipated monetary shocks at horizons H = 1...8, for various values of  $\gamma$ , when prices are assumed to be completely fixed ( $\zeta_p = 1.$ )

4.2.2.2 Moderate price stickiness When we impose a more realistic degree of price rigidities ( $\zeta_p = 0.74$ ), inflation adjusts according to the Phillips curve, so that the responses of the real and nominal interest rates differ. Figures 10–8 reproduce the responses to anticipated shocks with this moderate degree of price rigidities, for different values of  $\gamma$ . Again the

impact response of output tends to be less sensitive to anticipated shocks the higher the death probability. However, in this case more effects contribute to the overall response.



Figure 10: Anticipated monetary policy shocks when  $\gamma = 0$ 

Notes: The figure shows impulse response functions to anticipated monetary shocks at horizons H = 1...8, for  $\gamma = 0$ , when prices are moderately flexible ( $\zeta_p = 0.74$ .)



Figure 11: Anticipated monetary policy shocks when  $\gamma = 0.0126$ 

Notes: The figure shows impulse response functions to anticipated monetary shocks at horizons H = 1...8, for  $\gamma = 0.0126$ , when prices are moderately flexible ( $\zeta_p = 0.74$ .)

On the one hand, as inflation responds positively to the stimulus, the real interest rate is actually declining more than the nominal rate. So, for the announcement of a given change



Figure 12: Anticipated monetary policy shocks when  $\gamma = 0.03$ 

Notes: The figure shows impulse response functions to anticipated monetary shocks at horizons H = 1...8, for  $\gamma = 0.03$ , when prices are moderately flexible ( $\zeta_p = 0.74$ .)

in the nominal rate, the real interest rate responds more when prices can adjust. On the other hand, as inflation increases in response to the announcement, the policy rule requires that the interest rate be raised as well. The real rate is increasing in turn since the policy rule coefficient  $\psi_{\pi} > 1$ . This tends to slow down the economy. In all, for our calibration, output contracts on impact as the latter effect dominates, before rising several quarters later. Inflation increases on impact given expectations of future increases in output.

The inflation response tends to be more muted with higher values of  $\gamma$ , as price-setters discount more heavily future changes in output, as shown in Figure 13. This occurs despite the fact that the slope of the Phillips curve  $\kappa$  steepens with higher values of  $\gamma$ . Figure 13 indicates that the inflation response to the announcement of a short-term interest rate drop 8 quarters ahead falls by half when the death probability is raised from  $\gamma = 0$  to  $\gamma =$ 0.03. Accounting for a positive death probability has therefore the potential to mitigate the excessively strong inflation and output responses to forward guidance obtained in standard monetary DSGE models.





Notes: The figure shows the impact response of inflation to anticipated monetary shocks at horizons H = 1...8, for various values of  $\gamma$ , when prices are moderately flexible ( $\zeta_p = 0.74$ .)

#### 4.3 Quantitative Assessment in an Estimated DSGE Model

To assess the quantitative relevance of the mechanism just discussed, we finally turn to an estimated medium-scale DSGE model that allows for a positive death probability  $\gamma$ . The model considered has been developed in Castelnuovo and Nistico (2010). It corresponds to the simple model described in section 4.1 above, but augmented with other frictions such as habit formation in consumption, wage stickiness, and indexation of price and wage inflation to past inflation. (In a future version of this paper, we intend to perform this analysis in an estimated version of the FRBNY DSGE model described in Section 3, generalized to allow for a positive death probability.) The model has been estimated via Bayesian methods using US quarterly data from 1954Q3 to 2007Q2. We use Castelnuovo and Nistico's posterior mean estimates for the model parameters except for those that determine  $\eta$  and thus that govern the extent to which agents discount the future. To determine the effect that positive death probabilities exert on the model dynamics, we set as above  $\beta$  to 0.99 and  $\Omega/(PC)$  to 24.4, and consider different values for the death probability  $\gamma$  ranging from 0 to Castelnuovo and Nistico's estimate 0.1292.

Figure 14 shows the effect of an unanticipated announcement at date 0 that the FFR we be lowered by 25 basis points for the following 8 quarters. When  $\gamma = 0$  (shown in blue), inflation surges on impact by about 0.8 percentage point and then gradually returns to steady state. The sharp rise in inflation results in a decline in the real interest rate by more than a percentage point, which stimulates aggregate demand and thus output growth. With a death probability  $\gamma$  raised to 0.03, as indicated by the red lines, the responses of these variables are cut in half. This confirms that with realistic values for  $\gamma$ , the model implies much more reasonable responses of inflation, output and real interest rates to forward guidance announcements. Finally, when  $\gamma$  is raised to the 0.1292 (green lines), the economy barely responds to forward guidance announcements.





Notes: The figure shows the effect of an unanticipated announcement at date 0 that the FFR we be lowered by 25 basis points for the following 8 quarters, for various values of  $\gamma$ .

## 5 Conclusion

Forward guidance has become an essential tool for monetary policy in many industrial economies confronted to the effective lower bound on interest rates. Yet, little is know about the quantitative effects of forward guidance announcements on the economy. In this paper, we document the impact of forward guidance announcements on broad cross section of financial markets data, and on forecasts of key economic variables reported in the panel of Blue Chip forecasts. We find that these impacts depend in very subtle ways on the type forward guidance announcements. While the September 2012 announcement is associated with more policy stimulus and an upward revision in the forecasts of economic activity, the August 2011 announcement appears more *delphic*, in the sense that it conveyed in part bad news about the economy.

The paper analyzes the effect of forward guidance announcements in the context of a fairly standard medium scale DSGE model, and shows that this model tends to grossly overestimate the impact of forward guidance on the macroeconomy, a phenomenon called the "forward guidance puzzle." We argue that the model's excess response to announcements about future policy changes is likely due to the lack of discounting of future economic outcomes. We propose a tentative resolution to this puzzle based on the fact that life is finite. We show that this feature induces agents to discount the future more heavily, in households' intertemporal consumption decisions, in firms' optimal price setting, and in determination of asset prices. This implies that announcements of policy changes far in the future generate smaller effects on current aggregate variables than is the case in models with infinitely lived agents.

While the paper has focused on effects of forward guidance announcements, it illustrates the fact that conventional DSGE models tend to generate more generally excessively strong responses of key macroeconomic variables to news about conditions far in the future. We have argued that incorporating a Blanchard-Yaari perpetual youth structure in medium-scale DSGE models is desirable, as it is feasible – given its tractability – and likely to generate more realistic model dynamics.

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## A Appendix

#### A.1 Model Description

In this appendix, we summarize the log-linear equations that characterize the FRBNY DSGE model. The microeconomic foundations of the model are described in Del Negro et al. (2013). Because the model has a source of non-stationarity in the process for technology  $Z_t$ , to solve the model we first rewrite its equilibrium conditions in terms of stationary variables, and then solve for the non-stochastic steady state of the transformed model. Finally we take a log-linear approximation of the transformed model around its steady state. This approximation generates a set of log-linear equations, which we solve to obtain the model's state-space representation, using the method of Sims (2002). We then use the state-space representation procedure.

Below we list the log-linear equations of the model. We follow the usual convention of denoting log-deviations from steady state with hatted variables: for any stationary variable  $x_t$ ,  $\hat{x}_t \equiv \log(x_t/x_*)$ , where  $x_*$  denotes its steady state value. The steady state itself is a function of the model's parameters. Equations describing the mapping between parameters and steady state variables are available upon request.

The *Consumption Euler Equation* that characterizes the optimal allocation of consumption over time is given by

$$\hat{\xi}_t = \hat{R}_t + I\!\!\!E_t[\hat{\xi}_{t+1}] - I\!\!\!E_t[\hat{z}_{t+1}] - I\!\!\!E_t[\hat{\pi}_{t+1}], \qquad (39)$$

where  $\hat{R}_t$  is the gross nominal interest rate on government bonds, and  $\hat{\xi}_t$  is the marginal utility of consumption.

The Marginal Utility of Consumption  $\xi_t$  evolves according to

$$\begin{aligned} (e^{\gamma} - h\beta)(e^{\gamma} - h)\hat{\xi}_t &= -(e^{2\gamma} + \beta h^2)\hat{c}_t + he^{\gamma}\hat{c}_{t-1} - he^{\gamma}\hat{z}_t \\ &+ \beta he^{\gamma} \mathbb{E}_t[\hat{c}_{t+1}] + \beta he^{\gamma} \mathbb{E}_t[\hat{z}_{t+1}], \end{aligned}$$

where  $\hat{c}_t$  is consumption,  $e^{\gamma}$  is the steady-state (gross) growth rate of the economy and h captures habit persistence in consumption.

The *Capital Stock* follows

$$\hat{\bar{k}}_t = -(1 - \frac{i_*}{k_*})\hat{z}_t + (1 - \frac{i_*}{k_*})\hat{\bar{k}}_{t-1} + \frac{i_*}{k_*}\hat{\mu}_t + \frac{i_*}{k_*}\hat{\imath}_t,$$
(40)

where  $\hat{k}_t$  is installed capital,  $\hat{z}_t$  is the growth rate of productivity,  $i_*$  and  $k_*$  are steady state investment and the level of capital, respectively, and  $\hat{\mu}_t$  is the exogenous process that affects the efficiency by which a foregone unit of consumption contributes to capital utilization.

The *Effective Capital*  $\hat{k}_t$  is in turn given by

$$\hat{k}_t = \hat{u}_t - \hat{z}_t + \bar{k}_{t-1},$$
(41)

where  $\hat{u}_t$  is the level of capital utilization.

Capital Utilization is given by

$$r_*^k \hat{r}_t^k = a''(u) \ \hat{u}_t, \tag{42}$$

where  $r_*^k$  is the steady state rental rate of capital and the function a(u) captures the utilization cost.

The Optimal Investment decision satisfies the Euler equation

$$\hat{i}_{t} = \frac{1}{1+\beta} \mathbb{I}_{t} [\hat{i}_{t-1} - \hat{z}_{t}] + \frac{\beta}{1+\beta} \mathbb{I}_{t} [\hat{i}_{t+1} + \hat{z}_{t+1}] + \frac{1}{(1+\beta)S''e^{2\gamma}} \hat{q}_{t}^{k} + \frac{1}{(1+\beta)S''e^{2\gamma}} \hat{\mu}_{t}, \quad (43)$$

where  $\hat{i}_t$  is investment, S(.) is the cost of adjusting capital, with S' and S'' > 0, and  $\hat{q}_t^k$  is the price of capital.

The *Realized Return on Capital* is given by:

$$\widehat{\tilde{R}}_{t}^{k} - \hat{\pi}_{t} = \frac{r_{*}^{k}}{r_{*}^{k} + (1-\delta)} \widehat{r}_{t}^{k} + \frac{(1-\delta)}{r_{*}^{k} + (1-\delta)} \widehat{q}_{t}^{k} - \widehat{q}_{t-1}^{k},$$
(44)

where  $\delta$  is the rate of capital depreciation,  $\pi_t$  is the inflation rate, whose evolution is described below,  $\hat{r}_t^k$  is the capital rental rate and  $\hat{q}_t^k$  is the price of capital.

The Expected Excess Return on Capital (or 'spread')

$$I\!\!E_t \left[ \widehat{\tilde{R}}_{t+1}^k - \hat{R}_t \right] = \zeta_{sp,b} \left( \hat{q}_t^k + \widehat{\bar{k}}_t - \hat{n}_t \right) + \tilde{\sigma}_{\omega,t}$$
(45)

can be expressed as a function of the entrepreneurs' leverage (i.e., the ratio of the value of capital to nominal net worth) and exogenous fluctuations in the volatility of the entrepreneurs' idiosyncratic productivity,  $\tilde{\sigma}_{\omega,t} \equiv \zeta_{sp,\sigma_{\omega}} \hat{\sigma}_{\omega,t}$ . The parameter  $\zeta_{sp,b}$  is the elasticity of the spread with respect to leverage, and  $\zeta_{sp,\sigma_{\omega}}$  is the elasticity of the spread with respect to the volatility of the spread shock. The Entrepreneurs' Net Worth,  $\hat{n}_t$ , evolves according to

$$\hat{n}_{t} = \zeta_{n,\tilde{R}^{k}} \left( \widehat{\tilde{R}}_{t}^{k} - \hat{\pi}_{t} \right) - \zeta_{n,R} \left( \hat{R}_{t-1} - \hat{\pi}_{t} \right) + \zeta_{n,qK} \left( \hat{q}_{t-1}^{k} + \widehat{\bar{k}}_{t-1} \right) + \zeta_{n,n} \hat{n}_{t-1} - \gamma^{e} \frac{v_{*}}{n_{*}} \hat{z}_{t} - \frac{\zeta_{n,\sigma_{\omega}}}{\zeta_{sp,\sigma_{\omega}}} \tilde{\sigma}_{\omega,t-1}, \quad (46)$$

where  $\zeta_{n,\tilde{R}^k}$ ,  $\zeta_{n,R}$ ,  $\zeta_{n,q^K}$ ,  $\zeta_{n,n}$ , and  $\zeta_{n,\sigma_{\omega}}$  are the elasticities of net worth to the return on capital, the nominal interest rate, the cost of capital, net worth itself and the volatility  $\sigma_{\omega}$ , respectively, and  $\gamma^e$  is the fraction of entrepreneurs who survive each period.

The evolution of the Aggregate Nominal Wage is then given by

$$\hat{w}_t = \hat{w}_{t-1} - \hat{\pi}_t + \frac{1 - \zeta_w}{\zeta_w} \hat{w}_t, \tag{47}$$

where  $\zeta_w$  is the fraction of workers who cannot adjust their wages in a given period and  $\hat{w}_t$  is the optimal wage chosen by workers that can freely set it, or optimal reset wage.

The Optimal Reset Wage follows

$$(1+\nu_l\frac{1+\lambda_w}{\lambda_w})\hat{\tilde{w}}_t + (1+\zeta_w\beta\nu_l(\frac{1+\lambda_w}{\lambda_w}))\hat{w}_t = \zeta_w\beta(1+\nu_l\frac{1+\lambda_w}{\lambda_w})\mathbb{E}_t[\hat{\tilde{w}}_{t+1}+\hat{w}_{t+1}] + \hat{\varphi}_t + (1-\zeta_w\beta)(\nu_l\hat{L}_t - \hat{\xi}_t) + \zeta_w\beta(1+\nu_l\frac{1+\lambda_w}{\lambda_w})\mathbb{E}_t[\hat{\pi}_{t+1}+\hat{z}_{t+1}],$$

where  $\hat{\varphi}_t$  is a stochastic preference shifter affecting the marginal utility of leisure and  $\lambda_w$  is the parameter that determines the elasticity of substitution between differentiated labor services.

The optimal price-setting decision yields a *Phillips Curve* equation

$$\hat{\pi}_t = \beta I\!\!E_t[\hat{\pi}_{t+1}] + \frac{(1-\zeta_p\beta)(1-\zeta_p)}{\zeta_p} \hat{mc}_t + \frac{1}{\zeta_p} \tilde{\lambda}_{f,t},$$
(48)

where  $\hat{\pi}_t$  is inflation,  $\hat{mc}_t$  is nominal marginal cost,  $\beta$  is the discount factor, and  $\zeta_p$  is the Calvo parameter, representing the fraction of firms that cannot adjust their prices each period.  $\tilde{\lambda}_{f,t}$  is the following re-parametrization of the cost-push shock  $\lambda_{f,t}$ :  $\tilde{\lambda}_{f,t} = [(1 - \zeta_p \beta)(1 - \zeta_p)\lambda_f/(1 + \lambda_f)]\lambda_{f,t}$ , where  $\lambda_f$  is the steady state value of the markup shock.

The Marginal Cost (or labor share) satisfies

$$\hat{mc}_t = (1 - \alpha)\hat{w}_t + \alpha \hat{r}_t^k, \tag{49}$$

where  $\alpha$  is the output elasticity to capital and  $\hat{r}_t^k$  is the capital rental rate.

The *Production Function* is given by

$$\hat{y}_t = \alpha \hat{k}_t + (1 - \alpha) \hat{L}_t, \tag{50}$$

where the *Capital-Labor Ratio* satisfies

$$\hat{k}_t = \hat{w}_t - \hat{r}_t^k + \hat{L}_t.$$
(51)

The Resource Constraint is

$$\hat{y}_t = \hat{g}_t + \frac{c_*}{c_* + i_*} \hat{c}_t + \frac{i_*}{c_* + i_*} \hat{i}_t + \frac{r_*^k k_*}{c_* + i_*} \hat{u}_t,$$
(52)

where  $\hat{y}_t$  is output and  $\hat{g}_t$  is government spending.

Finally, the *Policy Rule* is

$$\hat{R}_{t} = \rho_{R}\hat{R}_{t-1} + (1-\rho_{R})\left(\psi_{\pi}\sum_{j=0}^{3}\hat{\pi}_{t-j} + \psi_{y}\sum_{j=0}^{3}(\hat{y}_{t-j} - \hat{y}_{t-j-1} + \hat{z}_{t-j})\right) + \epsilon_{t}^{R} + \sum_{k=1}^{K}\epsilon_{k,t-k}^{R}, \quad (53)$$

where  $\sum_{j=0}^{3} \hat{\pi}_{t-j}$  is 4-quarter inflation expressed in deviation from the Central Bank's objective

 $\pi_*$  (which corresponds to steady state inflation),  $\sum_{j=0}^{3} (\hat{y}_{t-j} - \hat{y}_{t-j-1} + \hat{z}_{t-j})$  is the 4-quarter growth rate of real GDP expressed in deviation from steady state growth,  $\epsilon_t^R$  is the standard contemporaneous policy shock, and the terms  $\epsilon_{k,t-k}^R$  are anticipated policy shocks, known to agents at time t - k.

#### A.2 The Exogenous Processes

The exogenous processes  $\hat{z}_t$ ,  $\hat{\varphi}_t$ ,  $\tilde{\lambda}_{f,t}$ ,  $\hat{\mu}_t$ ,  $\tilde{\sigma}_{\omega,t}$  and  $\hat{g}_t$  are assumed to follow AR(1) processes with autocorrelation parameters denoted by  $\rho_z$ ,  $\rho_{\varphi}$ ,  $\rho_{\lambda_f}$ ,  $\rho_{\mu}$ ,  $\rho_{\sigma_{\omega}}$ , and  $\rho_g$ , respectively. The innovations to these processes are structural shocks driving the model dynamics. They are assumed to be normally distributed with mean zero and a standard deviation denoted by  $\sigma_z$ ,  $\sigma_{\varphi}$ ,  $\sigma_{\lambda_f}$ ,  $\sigma_{\mu}$ ,  $\sigma_{\sigma_{\omega}}$ , and  $\sigma_g$ , respectively. The remaining structural shocks are the monetary policy shocks, both unanticipated,  $\epsilon_t^R$ , and anticipated,  $\epsilon_{k,t-k}^R$ , all assumed *i.i.d.* 

## A.3 Measurement Equations

$400(\ln Y_t - \ln Y_{t-1})$	=	$400(\widehat{y}_t - \widehat{y}_{t-1} + z_t)$
$100 \ln L_t$	=	$100(L_t + \ln L^{adj})$
$100 \ln LS_t$	=	$100(\widehat{L}_t + \widehat{w}_t - \widehat{y}_t + \ln LS_*)$
$\pi_t^{Core}$	=	$400(\widehat{\pi}_t + \ln \pi_*)$
$FFR_t$	=	$400(\widehat{R}_t + \ln R_*),$
$SP_t$	=	$400(I\!\!E_t\left[\widehat{\tilde{R}}_{t+1}^k - \hat{R}_t\right] + SP_*),$
	$400(\ln Y_t - \ln Y_{t-1})$ $100 \ln L_t$ $100 \ln LS_t$ $\pi_t^{Core}$ $FFR_t$ $SP_t$	$400(\ln Y_t - \ln Y_{t-1}) =$ $100 \ln L_t =$ $100 \ln LS_t =$ $\pi_t^{Core} =$ $FFR_t =$ $SP_t =$

where the parameter  $L^{adj}$  captures the units of measured hours.

## A.4 Prior and Posterior

Parameters	Prior Mean	Prior Stdd	Post Mean	90% Lower Band	90% Upper Band
Policy Par	rameters				
$\psi_1$	2.000	0.250	2.016	1.782	2.257
$\psi_2$	0.200	0.100	0.273	0.175	0.369
$ ho_r$	0.500	0.200	0.762	0.721	0.804
$\sigma_r$	0.200	4.000	0.152	0.132	0.170
Nominal <b>H</b>	Rigidities Par	rameters			
$\zeta_p$	0.750	0.100	0.879	0.852	0.908
$\zeta_w$	0.750	0.100	0.904	0.872	0.938
Other "Er	ndogenous P	ropagation a	and Steady	State" Parame	eters
$\alpha$	0.330	0.020	0.350	0.345	0.355
$a^{\prime\prime}$	0.200	0.100	0.294	0.129	0.444
h	0.700	0.050	0.704	0.636	0.770
S''	4.000	1.500	3.121	2.247	3.994
$ u_l$	2.000	0.750	1.273	0.465	2.017
$r^*$	1.500	1.000	0.288	0.038	0.525
$\pi^*$	2.000	0.250	2.382	2.115	2.655
$\gamma$	2.750	0.500	1.687	1.307	2.063
$g^*$	0.300	0.100	0.195	0.090	0.300
$\zeta_{sp}$	0.050	0.020	0.070	0.041	0.100
$spr_*$	2.000	0.500	1.163	0.750	1.556
$ ho \mathbf{s} \ \mathbf{and} \ \sigma \mathbf{s}$					
$ ho_z$	0.400	0.250	0.487	0.369	0.605
$ ho_{\phi}$	0.750	0.150	0.284	0.165	0.397
$ ho_{\lambda_f}$	0.750	0.150	0.470	0.364	0.572
$ ho_{\mu}$	0.750	0.150	0.991	0.982	1.000
$ ho_g$	0.750	0.150	0.927	0.847	0.999
$ ho_{sigw}$	0.750	0.150	0.965	0.938	0.995
$\sigma_z$	0.300	4.000	0.788	0.695	0.877
$\sigma_{\phi}$	3.000	4.000	29.403	12.878	45.636
$\sigma_{\lambda_f}$	0.200	4.000	0.087	0.072	0.101
$\sigma_{\mu}$	0.750	4.000	0.358	0.271	0.450
$\sigma_g$	0.500	4.000	0.224	0.180	0.266
$\sigma_{sigw}$	0.050	4.000	0.085	0.076	0.095

Note: The following parameters are fixed:  $\delta = 0.025$ ,  $\nu_m = 2$ ,  $\lambda_w = 0.3$ ,  $\chi = 0.1$ ,  $\lambda_f = 0.15$ ,  $F(\omega) = 0.15$ ,  $\gamma_* = 0.99$ .  $L^{adj}$  has a prior mean of 253.500, with standard deviation at 5.