

# The Race Between Man and Machine: Implications of Technology for Growth, Factor Shares and Employment\*

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## Abstract

The advent of automation and the simultaneous decline in the labor share and employment among advanced economies raise concerns that labor will be marginalized and made redundant by new technologies. This paper examines this proposition in a task-based framework wherein tasks previously performed by labor are automated, more complex versions of existing tasks can be created, and in these new tasks labor tends to have a comparative advantage. We fully characterize the structure of equilibrium in this model, showing how the allocation of factors to tasks and factor prices are determined by the available technology and the endogenous choices of firms between capital and labor. We then demonstrate that although automation tends to reduce employment and the share of labor in national income, the creation of more complex tasks has the opposite effect, and both types of innovations contribute to economic growth.

Our full model endogenizes the direction of research and development towards automation and the creation of new complex tasks. We show that, under reasonable conditions, there exists a stable balanced growth path in which the two types of innovations go hand-in-hand. Consequently, an increase in automation reduces the wage to rental rate ratio, which discourages further automation and encourages greater creation of more labor-intensive tasks, restoring the share of labor in national income and the employment to population ratio back towards their initial values. Though the economy is self-correcting, the equilibrium allocation of research effort is not optimal: to the extent that wages reflect quasi-rents for workers, firms will engage in too much automation. Finally, we extend the model to include workers of different skills. We find that inequality increases during transitions, but the self-correcting forces also serve to limit the increase in inequality over longer periods.

Still in Progress. Comments Welcome.

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# 1 Introduction

The accelerated automation of tasks performed by labor raise concerns that these new technologies will make labor redundant (e.g., Brynjolfsson and McAfee, 2012, Akst, 2014). The recent declines in the share of labor in national income and the employment to population ratio in the US economy, shown in Figure 1,<sup>1</sup> are often interpreted to support the claims that as digital technologies, robotics and artificial intelligence penetrate the economy more deeply, workers will find it increasingly difficult to compete and their compensation will experience a relative or even absolute decline. Nevertheless, a comprehensive framework where such effects, as well as countervailing forces, are present remains to be developed. The need for such a framework stems not only from the importance of understanding how and when automation will have these transformative effects on the labor market, but also from the fact that similar claims have been made, and yet have not always come true, about previous waves of new technologies. Keynes (1930), for example, famously foresaw the steady increase in per capita income in the 20th century from the introduction of new technologies, but incorrectly predicted that this would create widespread technological unemployment as machines replaced men. Economic historian Robert Heilbroner confidently stated in 1965 that “as machines continue to invade society, duplicating greater and greater numbers of social tasks, it is human labor itself — at least, as we now think of ‘labor’ — that is gradually rendered redundant” (quoted in Akst, 2014), while another observer of mid-century automation, economist Ben Seligman, similarly predicted a future of work without men (Seligman, 1966). Though more understated, Wassily Leontief was equally pessimistic about the implications of new machines, drawing an analogy with the technologies of the early 20th century making horses redundant, and speculating “Labor will become less and less important... More and more workers will be replaced by machines. I do not see that new industries can employ everybody who wants a job” (Leontief, 1952).

This paper is a first step in developing a conceptual framework which both shows how machines replaced human labor and why this may or may not translate into disappearance of work and stagnant wages. Our main conceptual innovation is to introduce not only automation that replaces tasks previously performed by labor, but also the creation of new complex tasks where labor has a comparative advantage.<sup>2</sup> This point is well illustrated by the technological and organizational changes during the Second Industrial Revolution, which not only involved the replacement of the

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<sup>1</sup>Figure 1 presents the estimate trends in the employment to population ratio for potential workers aged 25-64, nonfarm business sector labor share and productivity. The trends are computed using the Hodrick-Prescott filter with parameter 6.25. See Karabarbounis and Neiman (2014), Piketty and Zucman (2014), and Oberfield and Raval (2014) for more detailed evidence on the decline of the share of labor in national income.

<sup>2</sup>And herein lies our answer to Leontief’s analogy: the difference between human labor and horse labor is that humans have a comparative advantage in more complex, new tasks. Horses did not.

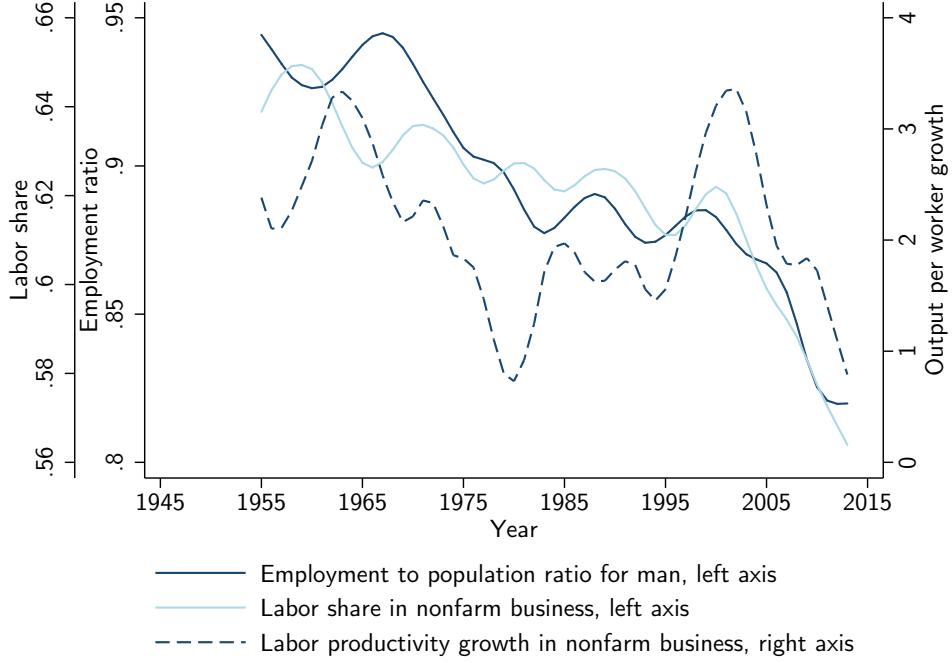


Figure 1: Trends in the employment to population ratio among men between 25-54 years, and the labor share in the nonfarm business sector in the United States.

stagecoach by the railroad, sailboats by steamboats, and of manual dock workers by cranes, but also the creation of new labor-intensive tasks — including a new class of engineers, machinists, repairmen, and conductors as well as modern managers and financiers involved with the introduction and operation of these new technologies (e.g., Landes, 1969, Chandler, 1977, and Mokyr, 1990).

Similarly today, while digital technologies and computer-controlled machines replace labor, we are witnessing the emergence of new tasks ranging from engineering and programming functions, to new professional jobs including audio-visual specialists, executive secretaries, data administrators and analysts, meeting planners or computer support specialists. In U.S. labor markets, the creation and expansion of these new tasks played a central role in generating employment. To document this fact, we use data on “task novelty” from Lin (2011), which measures the share of jobs and tasks in an occupation for which there were no previous job titles and which are considered by employers as different from existing ones. For instance, in 2000, about 70% of the tasks performed by computer software developers (an occupational group employing 1 million people at the time) did not appear in the 1990 Index of Occupations and are considered new. Similarly, radiology technologies are considered new in 1990 and management analysts are considered new in 1980. Figure 2 shows that each decade starting in 1980, 1990 and 2000, employment growth has been faster in occupations with more novel jobs and tasks. The regression line shows the empirical relationship, which implies

that occupational groups with 10 percentage point more novel jobs at the beginning of each decade grow 5.2% faster (standard error= 1.3%). From 1980 to 2007, employment grew by 17.1%, out of which about half (9%) is explained by the additional growth in occupations with more novel tasks and jobs—relative to a benchmark category with no new tasks.<sup>3</sup>

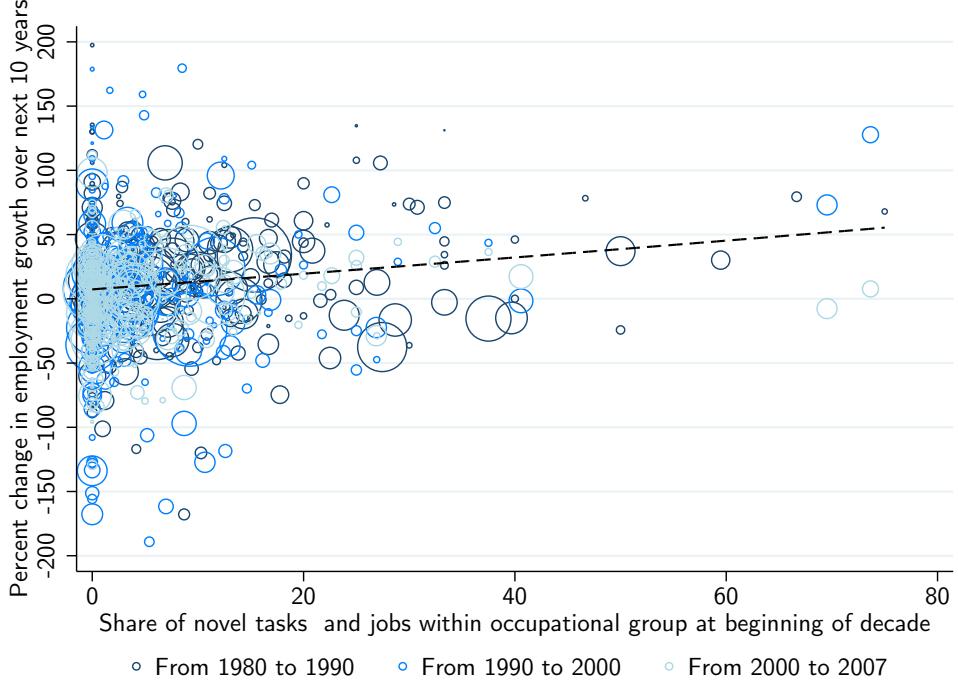


Figure 2: Scatter plot of employment growth and the share of novel jobs at the beginning of the decade across 330 occupational groups. Data from 1980 to 1990 (in dark blue), 1990 to 2000 (in blue) and 2000 to 2007 (in light blue, scaled). See the Appendix for sources and data construction details.

This paper develops a tractable but rich framework to study how automation and the creation of new tasks performed by labor impact factor prices, factor shares in national income and employment. In contrast to the more commonly-used models featuring factor-augmenting technological change, in this task-based framework new technologies that facilitate automation not only reduces the share of labor in national income, but may also reduce wages and employment. Conversely, the creation of new labor-intensive tasks increases wages, employment and the share of labor in national income, and may reduce the rate of return to capital. These comparative statics follow because factor prices are determined by the range of tasks performed by capital and labor (see also Acemoglu and Autor, 2011).

We then embed this framework in a dynamic economy in which capital accumulation is endoge-

<sup>3</sup>The data from 1980, 1990 and 2000 are from the U.S. Census. The data for 2007 are from the American Community Survey. Additional information on data and samples are provided in the Appendix, where we also present regression evidence to further document the relationship depicted in Figure 2 and its robustness.

nous. We characterize restrictions under which the model delivers balanced growth — which we take to be a good approximation to economic growth in the United States and the United Kingdom over the last two centuries. The key restriction is that there is exponential productivity growth from the creation of new tasks and that the two types of technological changes — automation and creation of new labor-intensive tasks — ought to advance at equal paces.

Our full model endogenizes the rate of improvement of these two types of technologies by marrying our task-based framework with a canonical directed technological setup. This full version of the model remains tractable and, under natural assumptions, generates asymptotically stable balanced growth: in the long run, there is equal advancement of the two types of technologies, and if one type of technology runs ahead of the other, market forces induce advances in the other type of technology. The economics of these self-correcting forces are instructive and highlight a crucial new force: increased automation pushes wages down relative to the rental rate of capital, and when technology is endogenous, encourages the creation of new tasks.<sup>4</sup> Even though there is an indirect market size effect due to an induced capital accumulation response, the (factor) price effect dominates and makes it more profitable to use the now cheaper labor, thus triggering the creation of new labor-intensive tasks and a powerful force towards restoring employment and the labor share to their values before the increase in automation. Put differently, in our model where new technologies replace tasks, relative factor prices emerge as the key object regulating the future path of technological change.<sup>5</sup>

The most important implication of the stability of the balanced growth path is that, in this model economy, periods in which automation runs ahead of the creation of new more complex tasks will tend to self-correct. Thus, contrary to the increasingly widespread concerns discussed above, our model raises the (theoretical) possibility that rapid automation may not signal the demise of labor, but may be a prelude to a new phase of new technologies favoring labor.<sup>6</sup>

The final major implication of our framework concerns the efficiency of equilibrium. In addition to the standard and well-known inefficiencies due to monopoly markups and appropriability problems in endogenous technological change models, our analysis identifies a new source of inefficiency

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<sup>4</sup>Our analysis also reveals another (partially) self-correcting economic force, a *productivity effect*: automation substitutes the cheaper capital for labor, thus increasing productivity and the demand for all factors. This effect is present in our model throughout, and does not change the fact that automation reduces the share of labor in national income (and may even reduce the wage rate). It becomes more powerful in the long run, however, when (and if) the interest rate is constant, e.g., due to capital accumulation, as we show below.

<sup>5</sup>The role of technologies replacing tasks in this result can be seen by noting that with factor-augmenting technological changes, the impact on relative factor prices is ambiguous (depending on the elasticity of substitution between factors), and the incentives determining the direction of innovation may be dominated by a strong market size effect.

<sup>6</sup>Of course, in the model, there are other types of structural changes which may have different long-run consequences. For example, if the developments we observe are triggered by a change in the innovation possibilities frontier (the technology of creating technologies) making it easier than before to invent automation technologies, then the economy may undergo an extended period of automation and ultimately settling for new balance growth path with a greater share of tasks performed by capital and a lower share of labor in national income.

in the direction of technological advance, pushing towards too much automation and too little creation of new tasks. This is because the market economy responds to factor prices, and thus when wages are high, automation becomes profitable as it enables firms to economize on wages; but when some of the wage payments accruing to workers are rents (as highlighted by our quasi-labor supply), these do not represent cost savings, implying that firms are engaging in too much automation. In contrast, the social planner's incentives to automate a task are determined by the opportunity cost of labor. hence The, the social planner automates less jobs, and conversely, her incentives for the creation of new tasks are always greater.

We consider two extensions of our model. In our baseline framework, all workers have the same skill level. In our first extension, we introduce heterogeneity in skills, and assume that skilled labor has a comparative advantage in newer tasks, which we deem as a natural assumption (in particular in view of the evidence presented in the next section).<sup>7</sup> Automation then tends to increase inequality by taking jobs from unskilled labor. The creation of new complex tasks also increases inequality at first, since skilled workers have comparative advantage in such tasks, but reduces it over longer periods as unskilled workers familiarize with the new technologies or tasks are standardized. This extension formalizes claims in the literature suggesting that both automation and new, more complex tasks, increase inequality, but also pointing out that short-run dynamics following such technological changes might be quite different — especially from their medium-term implications in the case of new labor-intensive tasks. Our second extension shows that under different assumptions on patents and the resulting creative destruction effects, there are similar qualitative forces, but the model might generate multiple and/or unstable steady-state equilibria.

Our paper relates to several literatures. It can be viewed as a combination of task-based models of the labor market with directed technological change models.<sup>8</sup> Task-based models have been developed both in the economic growth and labor literatures, dating back at least to Roy's seminal work (1955). The first important recent contribution is Zeira (1998), which proposed a model of economic growth based on capital-labor substitution and constitutes a special case of our model when technology (both automation and the set of tasks) are held fixed. Acemoglu and Zilibotti (2000) developed a simple task-based model with endogenous technology and applied it to the study of productivity differences across countries resulting from mismatch between new technologies and the skills of developing economies (see also Zeira, 2006, Acemoglu, 2010). Autor, Levy and Murnane (2003) suggested that the increase in inequality in the US labor market reflects the replacement of routine, labor-intensive tasks by technology.<sup>9</sup> The static, exogenous-technology part of our

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<sup>7</sup>This assumption builds on Schultz (1965) (see also Greenwood and Yorukoglu, 1997, Caselli, 1999, Galor and Moav, 2000, Acemoglu, Gancia and Zilibotti, 2010, and Beaudry, Green and Sand, 2013).

<sup>8</sup>On directed technological change and related models, see Acemoglu (1998, 2002, 2003a,b, 2007), Kiley (1999), Caselli and Coleman (2006), Gancia (2003), Thoenig and Verdier (2003) and Gancia and Zilibotti (2010).

<sup>9</sup>Acemoglu and Autor (2011), Autor and Dorn (2011), Jaimovich and Siu (2014), Foote and Ryan (2014), Burstein

model is most similar to Acemoglu and Autor's (2011) framework formalizing this notion. Our full model extends this framework not only because of the dynamic equilibrium incorporating directed technological change, but also because tasks are combined with a general elasticity of substitution (a feature that turns out to be important) and because the equilibrium allocation of tasks depends both on factor prices and the state of technology. Acemoglu and Autor's model, like ours, is one in which a discrete number of labor types are allocated to a continuum of tasks. Costinot and Vogel (2011) develop a complementary model in which skills and tasks form continuum sets.<sup>10</sup>

Three papers from the economic growth literature that are particularly related to our work are Acemoglu (2003a), Jones (2005), and Hemous and Olson (2014). The first two develop growth models in which the aggregate production function is endogenous and, in the long run, adapts to make balanced growth possible. In Jones (2005), this occurs because of endogenous choices about different combinations of activities/technologies being used. In Acemoglu (2003a), which is more closely related, this long-run behavior is a consequence of directed technological change. However, in contrast to the framework here, the two types of technologies that advance endogenously are both factor augmenting. The task-based framework developed here not only enables us to address questions related to automation and creation of new more complex tasks, which are our main focus here, but as already noted, also provides a more robust economic force ensuring the stability of a balanced growth path. As a result of these differences, in Acemoglu (2003a), a balanced growth path involving purely labor-augmenting technological change requires both somewhat more restrictive assumptions on the nature of the innovation possibilities frontier, and crucially also an elasticity of substitution between capital and labor that is less than 1. This is because, with factor-augmenting technologies, an elasticity of substitution greater than 1 implies that the factor that becomes more abundant commands a greater share of national income, triggering further factor-augmenting improvements favoring the more abundant factor. In a task-based framework, in contrast, further automation increases the relative price of capital to labor, directly exerting a stabilizing force. Hemous and Olson (2014) develop a model of automation and horizontal innovation with endogenous technology and use it to study the income inequality consequences of different types of technologies. In their model too, high wages (but this time for low-skill workers) encourage automation, but they also show how this depresses growth in the short run and may be countered by horizontal innovation in the long run.

The rest of the paper is organized as follows. Section 2 presents our basic task-based framework in the context of a static economy. Section 3 introduces capital accumulation and clarifies

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and Vogel (2012), and Burstein, Morales and Vogel (2014) provide various pieces of empirical evidence and quantitative evaluations on the importance of the endogenous allocation of tasks to factors in recent labor market dynamics.

<sup>10</sup>See also the recent paper by Hawkins, Ryan and Oh (2015), which shows how a task-based model is more successful than standard models in matching the co-movement of investment and employment at the firm level.

the requisite structure of task productivity that is necessary for balanced growth in this economy. Section 4 introduces our full model with endogenous technology and establishes, under some weak conditions, the existence and stability of a balanced growth path with two types of technologies advancing simultaneously. Section 5 compares the equilibrium composition of new technologies to the social planner’s allocation, establishing that the equilibrium will tend to have too much automation and too little creation of new labor-intensive tasks. Section 6 considers the two extensions mentioned above. Section 7 concludes. The Appendix contains the omitted proofs and the details of the empirical analysis described above.

## 2 Static Model

We start with a static environment with exogenous technology, which will enable us to introduce our main setup in the simplest fashion and characterize the impact of different types of technological change.

### 2.1 Environment

The economy contains a unique final good  $Y$ , produced by combining a continuum of tasks  $y(i)$  with an elasticity of substitution  $\sigma \in (0, \infty)$ . Namely,

$$Y = \left( \int_{N-1}^N y(i)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}. \quad (1)$$

The final good and each task is produced competitively.

The new feature in the aggregate production function (1) is that the index of tasks runs from  $N - 1$  to  $N$ , guaranteeing that the total measure of tasks performed at any point in time is 1. As described in the Introduction, the economy will feature creation of new more complex tasks, represented here by an increase in  $N$ . By assuming that the range of tasks is between  $N - 1$  and  $N$  we are imposing that the creation of new tasks always corresponds to the destruction of the lowest-index task, capturing the replacement or upgrading of an existing task — a feature we model explicitly below.<sup>11</sup>

Each task is produced combining labor or capital with a task-specific intermediate  $q(i)$ , which embeds the technology used both for production and for the possible automation of tasks. In preparation for our full model in Section 4, we assume that property rights to each intermediate is held by a technology monopolist which can produce it at the marginal cost  $\mu\psi$  in terms of the final good, where  $\mu \in (0, 1)$  and  $\psi > 0$ . The technology for each intermediate can be copied by a fringe

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<sup>11</sup>This formulation imposes that once a new task is created at  $N$ , it will automatically be utilized and as a consequence also replace the lowest available task, at  $N - 1$ . In Section 3, we provide conditions under which firms will indeed prefer to do so.

of competitive firms, which can produce each at a higher marginal cost of  $\psi$ . We assume that  $\mu$  is such that the unconstrained monopoly price of an intermediate would be greater than  $\psi$ , ensuring that the unique equilibrium price in the presence of the competitive fringe will be a limit price at  $\psi$  for all types of intermediates.

All tasks can be produced by labor. We model the technological constraints on automation by assuming that there exists  $I \in [N - 1, N]$  such that tasks  $i \leq I$  are *technologically automated* in the sense that it is technologically feasible for them to be produced by capital as well. Conversely, tasks  $i > I$  are not technologically automated, so cannot be produced by capital. Though tasks  $i < I$  are technologically automated, the equilibrium may not involve all of those being produced by capital depending on factor prices as we will next describe.

Let us next describe the production function of tasks in greater detail. For tasks  $i > I$ , which are not technologically automated, the production function takes the form

$$y(i) = B \left[ \eta q(i)^{\frac{\zeta-1}{\zeta}} + (1 - \eta) (\gamma(i)l(i))^{\frac{\zeta-1}{\zeta}} \right]^{\frac{\zeta}{\zeta-1}}, \quad (2)$$

where  $\gamma(i)$  denotes the productivity of labor in task  $i$ ,  $\zeta \in (0, \infty)$  is the elasticity of substitution between intermediates and labor,  $\eta \in (0, 1)$  is the distribution parameter of this constant elasticity of substitution production function, and finally,  $B$  is a normalizing constant, set equal to  $B \equiv (1 - \eta)^{\zeta/(1-\zeta)}$  to simplify the algebra.

In contrast, tasks  $i \leq I$  can be produced using labor or capital, and their production function takes the form

$$y(i) = B \left[ \eta q(i)^{\frac{\zeta-1}{\zeta}} + (1 - \eta) (k(i) + \gamma(i)l(i))^{\frac{\zeta-1}{\zeta}} \right]^{\frac{\zeta}{\zeta-1}}. \quad (3)$$

All of the parameters are thus common between the production function of tasks above and below the threshold  $I$ , with the only difference that those below  $I$  can be produced by capital as well as labor. This feature is embedded in (3) via the assumption that capital and labor are perfect substitutes — so that capital can fully replace labor at the task level.<sup>12</sup> One simplifying feature of (3) is that capital has the same productivity in all tasks — while labor has different productivity. This is a very convenient simplifying assumption, and could be relaxed, though at the cost of additional complexity.

Though all of our main results apply with the task production functions (2) and (3), we sometimes illustrate our results with one of two special cases, which lead to easier-to-interpret and particularly insightful expressions (without sacrificing any of the qualitative effects in the model): either  $\eta \rightarrow 0$  (so that the share of revenues going to intermediates is very low) or  $\zeta \rightarrow 1$  (so that the production functions for tasks become Cobb-Douglas between factors and intermediates).

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<sup>12</sup>The assumption implicit in writing this expression, that the same intermediate can be used regardless of whether this task is being produced by capital or labor, is for simplicity, and our results remain entirely unchanged if we have separate labor- and capital-specific intermediates.

The key assumption we make throughout this paper is that  $\gamma(i)$  is strictly increasing, so that labor has a *comparative advantage* in higher-indexed tasks. In the next section, we will strengthen this assumption by imposing a parametric form for  $\gamma(i)$ , which will ensure that productivity gains from the creation of new tasks is consistent with balanced growth (see in particular, equation (12)), but this functional form assumption plays no role in the analysis in this section. The important implication of strict comparative advantage is that, in equilibrium, there will exist some threshold task  $I^* \leq I$  such that all tasks  $i \leq I^*$  are produced using capital, while all tasks  $i > I^*$  use labor (see Acemoglu and Zilibotti, 2001, and Acemoglu and Autor, 2011).<sup>13</sup> The argument for the existence of such a threshold in our model is provided in the next subsection.

Figure 1 diagrammatically represents the allocation of tasks to factors and also how the creation of new tasks replaces existing tasks from the bottom of the distribution, which was described above.

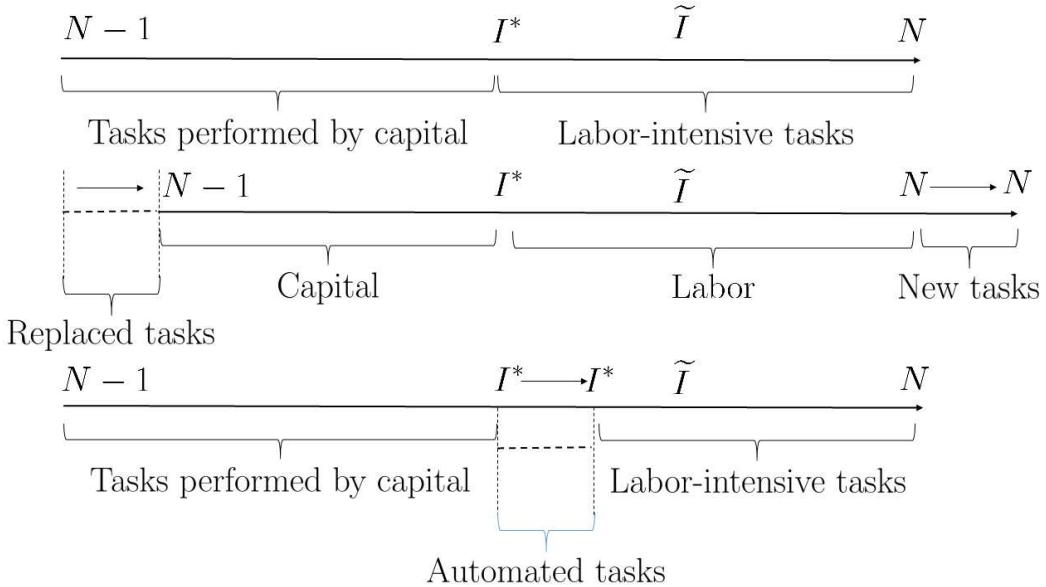


Figure 3: Task space, automation of existing tasks and introduction of new-complex tasks in which labor holds comparative advantage.

In the static model, we take the capital stock to be fixed at  $K$  (which will be endogenized via household decisions in Section 3). In addition, since we wish to study the impact of new technologies not just on factor prices but also on employment, we assume that the employment level is given by a quasi-labor supply taken to be an increasing function of the wage rate  $W$  relative to capital payments  $rK$ , i.e.,  $L^s(\frac{W}{rK})$ . This quasi-labor supply curve thus implies that as the wage rate increases relative to payments to capital, the employment level increases as well. Though we impose this as a reduced-form in the text, it is straightforward to derive it from various micro

<sup>13</sup>We impose without loss of any generality that when indifferent, firms use capital. This explains our convention of writing that all tasks  $i \leq \tilde{I}$  (rather than  $i < \tilde{I}$ ) are produced using capital.

foundations. In the Appendix, we show how an efficiency wage model generates this relationship, while in our companion paper, Acemoglu and Restrepo (2015), we derive this relationship from a search-matching model in a task-based framework. With this specification of the supply side, capital and labor market clearing can be written as

$$\begin{aligned}\int_{N-1}^N k(i)di &= K \\ \int_{N-1}^N l(i)di &= L^s\left(\frac{W}{rK}\right).\end{aligned}$$

We assume that  $L^s(0) > 0$ , so that labor never disappears from the economy entirely.

## 2.2 Equilibrium in the Static Model

We now characterize the equilibrium in this static economy. As noted above, all intermediates will be priced at  $\psi$ , and strict comparative advantage ensures that there will exist some threshold task  $I^*$  below which all tasks will be produced using capital. Given these intermediate prices and the threshold structure, an equilibrium can be represented as a function of the wage rate,  $W$ , the rental rate,  $r$ , and the equilibrium threshold  $I^*$ .

It is most convenient to proceed by characterizing the unit cost of producing tasks as a function of factor prices and the automation technology represented by  $I$ . Since tasks are produced competitively, their prices will be equal to these units costs. Thus

$$p(i) = \begin{cases} c^u \left( \min \left\{ r, \frac{W}{\gamma(i)} \right\} \right) \equiv \left[ \left( \frac{\eta}{1-\eta} \right)^\zeta \psi^{1-\zeta} + \min \left\{ r, \frac{W}{\gamma(i)} \right\}^{1-\zeta} \right]^{\frac{1}{1-\zeta}} & \text{if } i \leq I, \\ c^u \left( \psi, \frac{W}{\gamma(i)} \right) \equiv \left[ \left( \frac{\eta}{1-\eta} \right)^\zeta \psi^{1-\zeta} + \left( \frac{W}{\gamma(i)} \right)^{1-\zeta} \right]^{\frac{1}{1-\zeta}} & \text{if } i > I, \end{cases} \quad (4)$$

Here  $c^u$  is the constant unit cost of production of task  $i$  derived from the task production functions, (2) and (3). This unit cost also depends on the price of intermediates,  $\psi$ , but we suppress this dependence to simplify notation. The reason why the unit cost for tasks  $i \leq I$  is written as a function of  $\min \left\{ r, \frac{W}{\gamma(i)} \right\}$  is simply that, given perfect substitution between capital and labor, firms will choose whichever factor has a lower effective cost — where effective cost for labor is  $W/\gamma(i)$  in view of the fact that the productivity of labor in task  $i$  is  $\gamma(i)$ . Notice also that this expression distinguishes between  $i \leq I$  and  $i > I$  (and not  $i \leq I^*$  and  $i > I^*$ , since it refers to what is *technologically feasible*, not to the equilibrium allocation of tasks to capital and labor).

We choose the final good as the numeraire, which from (1) implies that the demand for task  $i$  is given by

$$y(i) = Y p(i)^{-\sigma}. \quad (5)$$

From equations (4) and (5), equilibrium levels of task production can be written as

$$y(i) = \begin{cases} Yc^u \left( \min \left\{ r, \frac{W}{\gamma(i)} \right\} \right)^{-\sigma} & \text{if } i \leq I, \\ Yc^u \left( \frac{W}{\gamma(i)} \right)^{-\sigma} & \text{if } i > I. \end{cases}$$

The result that, because of the strict comparative advantage, there will exist a threshold  $\tilde{I}$  such that tasks below  $I^* \equiv \min\{I, \tilde{I}\}$  will be produced with capital and the remaining more complex tasks with labor, can now be derived as a consequence of this expression. In particular, whenever  $\min \left\{ r, \frac{W}{\gamma(i)} \right\}$  picks  $r$ , the relevant task is produced by capital, and whenever it picks  $W/\gamma(i)$ , it is produced by labor.<sup>14</sup> Since  $\gamma(i)$  is strictly increasing, this implies that there exists a threshold  $\tilde{I}$  at which, conditional on technological feasibility, firms are indifferent between using capital and labor. Namely, at task  $\tilde{I}$ , we have that  $r = W/\gamma(\tilde{I})$ , or that

$$\frac{W}{r} = \gamma(\tilde{I}). \quad (6)$$

Put differently, this condition determines the cost-minimizing allocation of tasks between capital and labor. However, if  $\tilde{I} > I$ , firms will not be able to use capital all the way up to task  $\tilde{I}$  and achieve this cost-minimizing allocation because of the constraints imposed by the available automation technology. For this reason, the equilibrium threshold below which tasks are produced using capital is given by

$$I^* = \min\{I, \tilde{I}\},$$

meaning that  $I^* = \tilde{I}$  when this is technologically feasible, and  $I^* = I$  otherwise.

To fully characterize the static equilibrium, we next need to derive the quantities of tasks produced, given that equilibrium threshold  $I^*$ . Factor demands from each intermediate task can be derived from (2) and (3) as

$$k(i) = \begin{cases} Yc^u(r)^{\zeta-\sigma} r^{-\zeta} & \text{if } i \leq I^*, \\ 0 & \text{if } i > I^*. \end{cases}$$

and

$$l(i) = \begin{cases} 0 & \text{if } i \leq I^*, \\ \gamma(i)^{\zeta-1} Yc^u \left( \frac{W}{\gamma(i)} \right)^{\zeta-\sigma} W^{-\zeta} & \text{if } i > I^*. \end{cases}$$

Capital and labor market clearing conditions then yield the following equilibrium conditions,

$$Y(\min\{I, \tilde{I}\} - N + 1)c^u(r)^{\zeta-\sigma} r^{-\zeta} = K, \quad (7)$$

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<sup>14</sup>This discussion reveals an asymmetry in our treatment of automation and new labor-intensive technologies, because we have assumed that the latter type of technology is always used when it is created (and hence we have not distinguished  $N$ ,  $N^*$  and  $\tilde{N}$ ). This is because, as we show in Proposition 3, in the interesting part of the parameter space, where the interest rate is not too small (which in turn results from the discount rate in our full model,  $\rho$ , being at least some  $\bar{\rho}$ ), all new labor-intensive technologies will be used immediately, whereas all new automation technologies may or may not be depending on the relative state of the two types of technologies.

and

$$Y \int_{\min\{I, \tilde{I}\}}^N \gamma(i)^{\zeta-1} c^u \left( \frac{W}{\gamma(i)} \right)^{\zeta-\sigma} W^{-\zeta} di = L^s \left( \frac{W}{rK} \right). \quad (8)$$

The following proposition summarizes our characterization of the equilibrium.

**Proposition 1 (Equilibrium in the static model)** *For any range of tasks  $[N-1, N]$ , automation technology  $I \in (N-1, N]$ , and capital stock  $K$ , there exists a unique equilibrium characterized by factor prices,  $W$  and  $r$ , and threshold tasks,  $\tilde{I}$  and  $I^*$ , such that: (i)  $\tilde{I}$  is determined by equation (6) and  $I^* = \min\{I, \tilde{I}\}$ ; (ii) all tasks  $i \leq I^*$  are produced using capital and all tasks  $i > I^*$  are produced using labor; (iii) capital and labor market clearing conditions, equations (7) and (8), are satisfied; and (iii) factor prices satisfy:*

$$(I^* - N + 1)c^u(r)^{1-\sigma} + \int_{I^*}^N c^u \left( \frac{W}{\gamma(i)} \right)^{1-\sigma} di = 1. \quad (9)$$

**Proof.** All of the expressions in this proposition follow from the preceding derivations, and the full uniqueness proof is provided in the Appendix. ■

The equilibrium characterized in Proposition 1 is illustrated in Figure 4. The equilibrium is represented by the intersection of an upward and downward-sloping curve determining  $\omega \equiv \frac{W}{rK}$ . The downward-sloping curve,  $\omega(I^*, N, K)$ , corresponds to the relative demand for labor, which is obtained by combining the market clearing conditions for capital and labor, (7) and (8) together with the expression for the levels of factor prices, which is derived from the ideal price index, equation (9). The upward-sloping curve represents the cost-minimizing allocation of tasks to capital and labor, as represented by equation (6), with the constraint that the equilibrium level of automation can never exceed  $I$  (explaining the vertical portion).

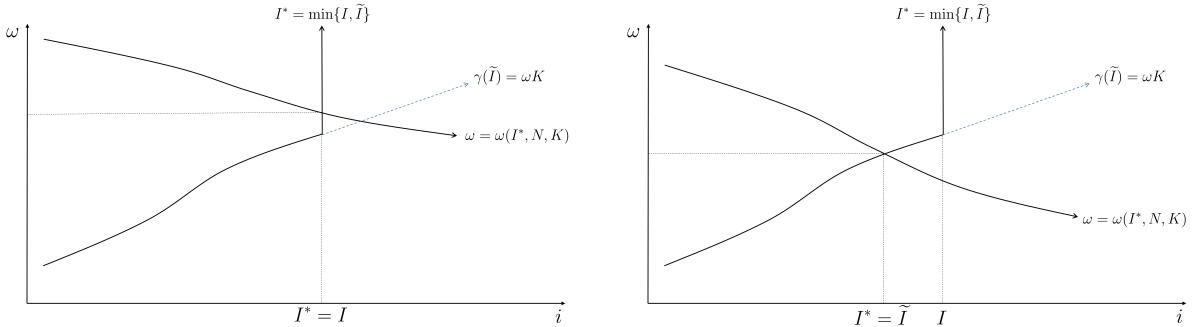


Figure 4: Static equilibrium of our model in the case in which  $I^* = I$  and the allocation of factors is constrained by technology (left panel) and for the case in which  $I^* = \tilde{I}$  (right panel).

The figure distinguishes between the two cases already highlighted above. In the top panel, we have the case where  $I^* = I < \tilde{I}$  and the allocation of factors is constrained by technology, while the bottom panel plots the case where  $I^* = \tilde{I} < I$  and where the cost-minimizing allocation can be achieved. An immediate implication of our characterization and of Figure 4 is that an increase in

$N$  (the creation of new, more complex tasks) always expands the set of tasks performed by labor and contracts those performed by capital, and an increase in  $I$  (greater technological automation) expands the set of tasks performed by capital and contracts those performed by labor provided that  $I < \tilde{I}$ . We will see the implications of these results in the comparative statics we present next.

The following proposition gives a complete characterization of comparative statics.

**Proposition 2 (Comparative statics)** *Let  $\omega \equiv \frac{W}{rK}$  be the ratio of wages to capital payments, and  $\varepsilon \equiv \frac{d \ln \gamma(I)}{dI} > 0$  be the semi-elasticity of the comparative advantage schedule. Then:*

- If  $I^* = I < \tilde{I}$  — so that the allocation of tasks to factors is constrained by technology, we have:

$$\frac{d \ln \omega}{dI} = \frac{d \ln(W/r)}{dI} = \frac{\partial \ln(W/r)}{\partial I^*} < 0, \quad \frac{d \ln \omega}{dN} = \frac{d \ln(W/r)}{dN} = \frac{\partial \ln(W/r)}{\partial N} > 0$$

and

$$\frac{d \ln \omega}{d \ln K} + 1 = \frac{d \ln(W/r)}{d \ln K} = \frac{1}{\sigma_{SR}} > 0.$$

Here,  $\sigma_{SR} \in [0, \infty)$  is the short-run elasticity of substitution between labor and capital holding the allocation of factors to tasks fixed, which in this model is a weighted average of  $\sigma$  and  $\zeta$ .

Moreover, if  $\sigma_{SR}$  is sufficiently large,  $\frac{d \ln W}{dI} < 0$ , and  $\frac{d \ln W}{dI} > 0$  otherwise.

- If  $I^* = \tilde{I} < I$  — so that tasks are allocated to factors in the unconstrained cost minimizing fashion, we have

$$dI^* = \frac{1}{\varepsilon} d \ln(W/r).$$

The resulting impact on factor prices and shares is given by

$$\begin{aligned} \frac{d \ln \omega}{dI} &= \frac{d \ln(W/r)}{dI} = 0, \quad \frac{d \ln \omega}{dN} = \frac{d \ln(W/r)}{dN} = \frac{\frac{\partial \ln(W/r)}{\partial N}}{1 - \frac{1}{\varepsilon} \frac{\partial \ln(W/r)}{\partial I^*}} > 0 \text{ and} \\ \frac{d \ln \omega}{d \ln K} + 1 &= \frac{d \ln(W/r)}{d \ln K} = \frac{\frac{1}{\sigma_{SR}}}{1 - \frac{1}{\varepsilon} \frac{\partial \ln(W/r)}{\partial I^*}} > 0. \end{aligned}$$

Thus, when the allocation of tasks to factors is unconstrained, the aggregate elasticity of substitution between capital and labor becomes

$$\sigma_{MR} = \sigma_{SR} \left( 1 - \frac{1}{\varepsilon} \frac{\partial \ln(W/r)}{\partial I^*} \right) > \sigma_{SR}.$$

Moreover, if the medium-run elasticity of substitution between labor and capital,  $\sigma_{MR}$ , is sufficiently large,  $\frac{d \ln r}{dN} < 0$ , and  $\frac{d \ln W}{dN} > 0$  otherwise.

- Finally, in both parts of the proposition, the labor share and employment move in the same direction as  $\omega$ .

**Proof.** These results follow directly from differentiating the equilibrium conditions, and the details are given in the Appendix. ■

The most important implication of Proposition 2 is that the two types of technological changes — automation and creation of new, more complex tasks — have polar implications. Automation, corresponding to an increase in  $I$ , tends to reduce  $W/r$ , the labor share and employment (unless firms were deciding not to use capital in all of the tasks that were technologically automated), while the creation of new tasks, corresponding to an increase in  $N$ , increase  $W/r$ , labor share and employment.

It is also useful to note that these comparative static results can be derived using Figure 4: automation moves us along the relative labor demand curve in the technology-constrained case shown in the top panel (and has no impact in the bottom panel), while the creation of new tasks, shifts out the relative labor demand curve.

Another important implication of Proposition 2 is that, when  $I^* = I$ , automation — an increase in  $I$  — can reduce wages. For example, automation expands the range of tasks performed by capital and pushes labor into a fewer set of tasks, where the diminishing returns to the quantity of a task puts downward pressure on the wage, counteracted by a positive effect coming from the fact that tasks are ( $q$ -)complements in the aggregate production function (1). This positive effect is weaker when  $\sigma$  is greater, explaining why the overall impact of automation on the wage rate is negative when  $\sigma$  is large.<sup>15</sup> Similarly, again when  $\sigma$  is large, the creation of new tasks — that is, an increase in  $N$  — can reduce the rental rate on capital. Even more important is that automation is *always* capital-biased (that is, it reduces  $W/r$ ), while the creation of new tasks is *always* labor-biased (that is, it increases  $W/r$ ). Both of these are major consequences of the task-based framework developed here. With factor-augmenting technologies, technological improvements always increase the price of both factors, but this is no longer the case when technological change alters the range of tasks performed by the two factors (see Acemoglu and Autor, 2011).<sup>16</sup> Also, as is well known, with factor-augmenting technologies, whether different types of technological changes are biased towards one factor or the other depends on the elasticity of substitution, but this too is different in our task-based framework — again because different types of technological changes directly alter the range of tasks performed by the two factors. This last feature will play a critical role in our full model in Section 4.

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<sup>15</sup>This negative impact does not require  $\sigma$  to be unrealistically large. For example, if  $\sigma = 1$ , automation reduces the marginal product of labor if  $K/Y < 2.7182$ .

<sup>16</sup>For instance, an increase in capital-augmenting technology, from the viewpoint of other factors, is equivalent to an increase in the effective amount of capital and it increases the marginal product of labor because factors are  $q$ -complements in any production function with constant returns to scale and two factors. To see this, let  $F(A_K K, A_L L)$  be such a production function. Then  $W = F_L$ , and  $\frac{dW}{dA_K} = KF_{LK} = -LF_{LL} > 0$  because of constant returns to scale, establishing the claim.

A final implication of Proposition 2 is the difference between the short-run and the “medium-run” elasticities of substitution between capital and labor. The short-run elasticity,  $\sigma_{SR}$  is obtained when the range of tasks allocated to capital and labor is fixed (as in the case where  $I^* = I$ ), and the medium-run elasticity,  $\sigma_{MR}$ , applies when the range of tasks responds to changes in factor prices (as in the case where  $I^* = \tilde{I}$ ).<sup>17</sup>

Though Proposition 2 provides a complete characterization of the responses of relative factor prices, factor shares and employment to automation and creation of new tasks, the results are qualitative and the explicit expressions are complicated; this is because imperfect substitution between factors and intermediates (the  $q(i)$ ’s) implies that as technology changes, the profits of intermediate producers change. As noted above, two special cases simplify this impact on profits and illustrate the workings of our model and the comparative statics more transparently. The first is when  $\eta \rightarrow 0$ , where these profits go to zero, and the second is when  $\zeta \rightarrow 1$ , where they become a constant fraction of revenue. We next provide the explicit expressions in these two special cases. We also simplify this illustration by taking  $L(\omega) = L$ , so that the quasi-labor supply coincides with the inelastic labor supply in the economy.

In both of these special cases we obtain a particularly revealing expression for aggregate output (or a “derived aggregate production function”):

$$Y = \left[ (I^* - N + 1)^{\frac{1}{\hat{\sigma}}} K^{\frac{\hat{\sigma}-1}{\hat{\sigma}}} + \left( \int_{I^*}^N \gamma(i)^{\hat{\sigma}-1} di \right)^{\frac{1}{\hat{\sigma}}} L^{\frac{\hat{\sigma}-1}{\hat{\sigma}}} \right]^{\frac{\hat{\sigma}}{\hat{\sigma}-1}}, \quad (10)$$

where  $\hat{\sigma} \equiv \eta + (1 - \eta)\sigma$  (which also implies that when  $\eta \rightarrow 0$ , we have the particularly simple case with  $\hat{\sigma} = \sigma$ ).

This expression emphasizes that aggregate output is a constant elasticity of substitution aggregate of capital and labor (with the short-run elasticity of substitution between capital and labor,  $\sigma_{SR}$ , simply being  $\hat{\sigma}$ ), but the distribution parameters are endogenous and depend on the state of the two types of technologies in the economy. In particular, automation increases the importance of capital and reduces the importance of labor in the (derived) aggregate production function, while the creation of new, more complex tasks does the opposite.

Relative factor demands are also straightforward to derive since, simple differentiation of (10), implies

$$\ln \omega = \left( \frac{1}{\hat{\sigma}} - 1 \right) \ln K + \frac{1}{\hat{\sigma}} \ln \left( \frac{\int_{I^*}^N \gamma(i)^{\hat{\sigma}-1} di}{I^* - N + 1} \right). \quad (11)$$

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<sup>17</sup>Another observation about the elasticity of substitution following from this proposition is that a long-run negative association between capital accumulation and the labor share is not sufficient to conclude that  $\sigma$  — the elasticity of substitution between labor and capital — is above 1 (as argued by Karabarbounis and Neiman, 2014). This reasoning would be valid in the special case when technology only takes a factor-augmenting form, but not in our task framework. For a stark counterexample, take  $\sigma = 1$  in our model with  $\eta \rightarrow 0$ . Then, factor shares depend only on technology and are not informative about  $\sigma$ .

In fact, equation (11) gives us an explicit expression for the relative labor demand plotted in Figure 2.

The next corollary provides a more explicit characterization of the comparative statics derived in Proposition 2 in the special cases.

**Corollary 1** *Suppose  $\eta \rightarrow 0$  or  $\zeta \rightarrow 1$ . Then:*

- If  $I < \tilde{I}$ :

$$\begin{aligned}\hat{\sigma} d \ln \omega &= (1 - \hat{\sigma}) d \ln K - \left[ \frac{\gamma(I)^{\hat{\sigma}-1}}{\int_I^N \gamma(i)^{\hat{\sigma}-1} di} + \frac{1}{I - N + 1} \right] dI \\ &\quad + \left[ \frac{\gamma(N)^{\hat{\sigma}-1}}{\int_I^N \gamma(i)^{\hat{\sigma}-1} di} + \frac{1}{I - N + 1} \right] dN.\end{aligned}$$

- If  $\tilde{I} < I$ :

$$\begin{aligned}(\hat{\sigma} + \Lambda/\varepsilon) d \ln \omega &= (1 - \hat{\sigma} - \Lambda/\varepsilon) d \ln K \\ &\quad + \left[ \frac{\gamma(N)^{\hat{\sigma}-1}}{\int_I^N \gamma(i)^{\hat{\sigma}-1} di} + \frac{1}{I - N + 1} \right] d \ln N,\end{aligned}$$

where

$$\Lambda \equiv \frac{\gamma(\tilde{I})^{\hat{\sigma}-1}}{\int_{\tilde{I}}^N \gamma(i)^{\hat{\sigma}-1} di} + \frac{1}{\tilde{I} - N + 1} > 0,$$

and  $\hat{\sigma} \equiv \eta + (1 - \eta)\sigma$ .

*The labor share and employment move in the same direction as  $\omega$ .*

In this corollary, the difference between the short-run and the medium-run elasticity of substitution can be seen quite clearly:  $\sigma_{SR} = \hat{\sigma}$ , and  $\sigma_{MR} = \hat{\sigma} + \Lambda/\varepsilon$ .

### 3 Dynamic Economy, Balanced Growth and the Productivity Effect

In this section, we extend our model to a dynamic economy in which the evolution of the capital stock is determined by households' saving decisions. We then investigate the conditions under which the economy admits a balanced growth path, where output, the capital stock and wages grow at a constant rate. We conclude this section by discussing the effect of automation on wages in the long run (when the interest rate is constant as in the balanced growth path), which highlights an important "productivity effect," creating a force from automation towards higher wages.

### 3.1 Balanced Growth

The most important assumption in this section will be to parametrize the comparative advantage schedule to ensure balanced growth. In particular since, as usual, balanced growth will be driven by technology, and in this model technological change comes in part from the creation of new tasks, exponential growth will require productivity improvements from new tasks to be exponential. In other words, we require

$$\gamma(i) = e^{Ai} \text{ with } A > 0, \quad (12)$$

which we impose in the remainder of the paper.<sup>18</sup>

Let also  $\{K(t), N(t), I(t)\}_{t=0}^{\infty}$  denote the path of technology and capital. These are the state variables of our model. Also, let  $\{r(t), W(t), Y(t)\}_{t=0}^{\infty}$  denote the path of factor prices and equilibrium output at each period. We start by assuming exogenous technological change, and define

$$n(t) \equiv N(t) - I(t)$$

as a summary measure of the state of technology. A higher  $n$  corresponds to the state of technology favoring new tasks more than automation. Clearly, as automation increases,  $n$  declines, and conversely, as there are new tasks being created,  $n$  increases. We further simplify the discussion and notation by assuming that  $I^*(t) = I(t)$ . As noted in the next section, with endogenous technology, this is the relevant region, since  $I^*(t) < I(t)$  would imply that there are resources spent on automating tasks that will not be immediately produced with capital. We discuss conditions that ensure  $I^*(t) = I(t)$  below.

The economy is assumed to admit a representative household. This representative household's preferences over consumption paths,  $\{C(t)\}_{t=0}^{\infty}$ , are given by

$$\int_0^{\infty} e^{-\rho t} \frac{C(t)^{1-\theta} - 1}{1-\theta} dt.$$

and the resource constraint faced by the household takes the form

$$\dot{K}(t) = Y(t) - C(t) - \delta K(t) - \psi \mu \int_{N-1}^N q(i, t) di,$$

where  $Y(t)$  continues to be given by (1), and  $\delta$  is the depreciation rate of capital. In addition,  $\psi \mu$ , with  $\mu \in [0, 1]$ , parametrizes the marginal cost of producing intermediates. Thus, we allow for intermediaries to sell their products at a markup  $1 - \mu \geq 0$ . This markup does not play any role

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<sup>18</sup>As usual we could impose this functional form only asymptotically, but simplify the analysis and exposition by imposing it throughout its range.

Notice also that the productivity of all tasks that are automated continues to be constant in this dynamic economy. This does not, however, imply that any of the previously automated tasks can be used regardless of  $N$ . As  $N$  increases, as emphasized by equation (1), the set of feasible tasks shifts to the right, and only tasks above  $N - 1$  can be combined with those currently in use.

in this section, and these profits will only be important when we turn to the case with endogenous technology.

We characterize the equilibrium by defining the normalized variables  $y(t) \equiv Y(t)/\gamma(I(t))$ ,  $k(t) \equiv K(t)/\gamma(I(t))$ ,  $c(t) \equiv C(t)/\gamma(I(t))$ , and  $w(t) \equiv W(t)/\gamma(I(t))$ .

At each point in time, technology and capital,  $n(t)$  and  $k(t)$ , fully determine output,  $y(t)$ , and factor prices  $w(t)$  and  $r(t)$  as in the static equilibrium (where, for consistency with our static analysis,  $r(t)$ , is taken to be the rental rate of capital, so that the interest rate is  $r(t) - \delta$ ). Specifically, the market clearing conditions for capital and labor, (7) and (8), and the ideal price index condition, (9), give the following equilibrium conditions in this case:

$$\begin{aligned} k(t) &= y(t)(1 - n(t))c^u(r(t))^{\zeta-\sigma}r(t)^{-\zeta}, \\ L^s \left( \frac{w(t)}{r(t)k(t)} \right) &= y(t) \int_0^{n(t)} \gamma(i)^{\zeta-1} c^u \left( \frac{w(t)}{\gamma(i)} \right)^{\zeta-\sigma} w^{-\zeta} di, \\ 1 &= (1 - n(t))c^u(r(t))^{1-\sigma} + \int_0^{n(t)} c^u \left( \frac{w(t)}{\gamma(i)} \right)^{1-\sigma} di \end{aligned}$$

The implied values for normalized output and factor prices can be written as  $y(t) = y^E(n(t), k(t))$ ,  $w(t) = w^E(n(t), k(t))$  and  $r^E(t)(n(t), k(t))$ , which are uniquely defined from Proposition 1. Importantly, we also have that  $w^E(n, k) \geq r$ , because the endogenous allocation of tasks to factors implies  $W/\gamma(I^*) \geq r$  (or  $\tilde{I} \geq I^*$ ). We also denote by  $f^E(n(t), k(t))$  the output net of intermediate costs.

Using this notation, we can describe the dynamic equilibrium of our model as a path for  $c(t)$  and  $k(t)$  satisfying the Euler equation

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta}(r^E(n(t), k(t)) - \delta - \rho) - g \quad (13)$$

coupled with the household's transversality condition

$$\lim_{t \rightarrow \infty} k(t)e^{-\int_0^t (\rho - (1-\theta)g) ds} = 0, \quad (14)$$

and the resource constraint

$$\dot{k}(t) = f^E(n(t), k(t)) - c(t) - (\delta + g)k(t). \quad (15)$$

Figure 5 presents the phase diagram for this system for  $n(t) \rightarrow n$ . The structure of the above system is similar to the standard neoclassical growth model, with the slight exception that technology monopolists' markups create a wedge between  $r^E$  and  $f_k^E$ .

We define a balanced growth path as an allocation in which  $Y, C, K$  and  $w$  grow at a constant rate and  $r$  is constant. The next proposition characterizes the conditions under which the asymptotic behavior of this economy will converge to a balanced growth path and also establishes that this involves both types of technological change.

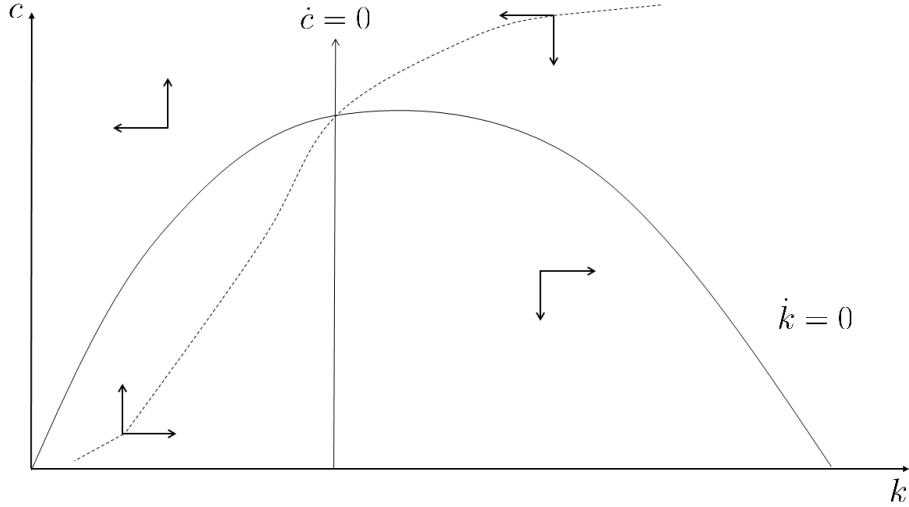


Figure 5: Steady state and dynamics for our model with exogenous technological change and  $n(t) \rightarrow n$ .

**Proposition 3 (Dynamic equilibrium with exogenous technological change)** Suppose that technology evolves exogenously. There exists a threshold  $\bar{\rho}$  such that, for  $\rho > \bar{\rho}$  we have:

1. There exists  $\bar{n}$  such that for  $n(t) < \bar{n}$ , we have  $I^* < I$ , while for  $n(t) \geq \bar{n}$ ,  $I^* = I$ . In both cases we have that all new labor-intensive (new complex) tasks are utilized immediately.
2. If (and only if) asymptotically  $\dot{N} = \dot{I} = \Delta$  and  $\lim_{t \rightarrow \infty} n(t) = n \in [\bar{n}, 1]$ , a unique balanced growth path with  $I^* = I$  exists. In this balanced growth path  $Y, C, K$  and  $w$  grow at a constant rate  $A\Delta$  and  $r$  is constant. In contrast, if  $\lim_{t \rightarrow \infty} n(t) \leq \bar{n}$ , there exists a balanced growth path in which  $I^* < I$ .
3. Moreover, given such a path of technological change (with  $\lim_{t \rightarrow \infty} n(t) = n \in [\bar{n}, 1]$ , or  $n(t) \leq \bar{n}$  for all  $t \geq T$ ), the dynamic equilibrium is unique starting from any initial level of capital stock and converges to the balanced growth path.

**Proof.** The condition  $\rho > \bar{\rho}$  guarantees the existence of the threshold  $\bar{n}$ , which is derived in Lemma A2 in the Appendix. Part 1 of the proposition follows as a corollary of this lemma.

For part 2 of the proposition, suppose that  $n(t) \rightarrow n \in [\bar{n}, 1]$ . Since in this steady state the normalized variables converge to a unique equilibrium, the aggregate variables grow at the same rate as  $\gamma(I)$ . If  $\lim_{t \rightarrow \infty} n(t) \leq \bar{n}$ , the economy converges to the same allocation that would obtain in the case in which  $n(t) = \bar{n}$ , since automated tasks do only use capital at this point. These observations establish the “if” direction of part 2.

We prove the “only if” part in the Appendix.

In addition, for any initial value of  $k(0)$ , the economy converges to its unique steady state, which depends only on  $n = \lim_{t \rightarrow \infty} n(t)$  (or  $\bar{n}$  if  $\lim_{t \rightarrow \infty} n(t) \leq \bar{n}$ ). This result establishes part 3 of

the proposition, and can be proved straightforwardly by noting that equations (13), (14) and (15) are essentially identical to the two equations characterizing dynamics in the canonical neoclassical growth model (see, for example, Proposition 8.5 and 8.6 in Acemoglu (2009)). The requirement on  $\rho$  also guarantees that  $\rho > A(1 - \theta)\Delta$ , which ensures the transversality condition holds. ■

The most important implication of Proposition 3 is that balanced growth can emerge from the simultaneous process of automation and development of new tasks. But it also highlights that this process needs to be balanced itself: the race between machine and man cannot be dominated by either. This implies, in particular, that the two types of technologies need to advance at the same rate, and moreover that if  $\lim_{t \rightarrow \infty} n(t) \geq \bar{n}$  (otherwise, not all available automation technologies will be used). We will see in the next section that the threshold  $\bar{n}$  also plays an important role when we endogenize technology.<sup>19</sup>

The additional requirement in Proposition 3,  $\rho > \bar{\rho}$ , ensures that the long-run equilibrium interest rate is not too close to 0. As we will see further in the next section, this is the interesting range of parameters for our focus (and this is the requirement that ensures that  $\bar{n}$  is well-defined). The Appendix also shows how the analysis needs to be modified in the case where  $\rho < \bar{\rho}$ .

Combining this proposition together with Proposition 2, we also see that when automation runs ahead of the creation of new tasks, i.e.,  $\dot{I} > \dot{N}$ , so that  $n(t)$  decreases, we will not only move away from balanced growth (presuming that we started at or near balanced growth), but also that this will reduce the share of labor in national income and employment. In light of this result, the patterns shown in Figure 1 in the Introduction can be interpreted as a consequence of automation outpacing the creation of new labor-intensive tasks over the last two decades.

Proposition 3 can also be further illustrated and strengthened in the two special cases considered in the previous section, where  $\eta \rightarrow 0$  or  $\zeta \rightarrow 1$ . Supposing also that  $\dot{N} = \dot{I} = \Delta$ , the aggregate production function can be simplified to  $Y(t) = f(K(t), A(t)L)$  as given in equation (10). We also have that

$$A(t) = \left( \int_{I(t)}^{N(t)} \gamma(i)^{\hat{\sigma}-1} di \right)^{\frac{1}{\hat{\sigma}-1}} = e^{AI(t)} \left( \frac{e^{A(\hat{\sigma}-1)n(t)} - 1}{A\hat{\sigma} - 1} \right)^{\frac{1}{\hat{\sigma}-1}},$$

so that  $A(t)$  grows at a rate  $A\Delta$ . In this case, technology is purely labor augmenting on net because labor and capital perform a fixed share of tasks; while labor is used on tasks in which it is more productive over time. This provides a direct connection between our model and Uzawa's Theorem, which implies that balanced growth requires purely labor-augmenting technological change (e.g., Acemoglu, 2009). The condition  $\dot{N} = \dot{I}$  ensures this in our economy.

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<sup>19</sup>We should also note that  $\rho > \bar{\rho}$  and  $\lim_{t \rightarrow \infty} n(t) \geq \bar{n}$  are not a very restrictive condition. For example, focusing on a standard annual parametrization of our model with  $\theta = 1$ ,  $\delta = 0.06$ ,  $g = 0.016$ ,  $\sigma = 0.5$ ,  $\zeta = 0.2$  (so that the elasticity of substitution between capital and labor lies between 0.5 and 0.2),  $A = 2$ ,  $\eta = 0.5$  and  $\psi = 0.9$ , we obtain  $\bar{\rho} = 0.012$ , so that the standard value of the discount rate,  $\rho = 0.05$  as required, and these parameters also imply  $\bar{n} = 0.56$ .

### 3.2 The Productivity Effect

We now study the dynamic implications of automation running ahead of the creation of new tasks. Though many of the insights from our static model apply in this case, the dynamic economy also highlights another economic force, which we will call the *productivity effect*: automation, by enabling the substitution of the cheaper capital for labor, increases productivity and thus the demand for labor.<sup>20</sup> The productivity effect is implicitly present in our analysis so far. But it becomes more powerful in the balanced growth path because the interest rate is constant (Proposition 3) as we show next.

We continue to assume that  $\rho > \bar{\rho}$  as in Proposition 3, and also focus in the case where  $n(t) \rightarrow n \in (\bar{n}, 1)$ , so that the balance growth path involves simultaneous advancement and use of both types of technologies. In this balanced growth path, capital adjusts to make  $r = \rho + \delta + \theta g$ , and the long-run normalized wage as a function of the state of technology is

$$w^{LR}(n) = w^E(n, k(n)).$$

Given our normalization (where the wage,  $W$ , is divided by  $\gamma(I) = \gamma(I^*)$ ), this is the wage per effective unit of labor paid in the least complex tasks performed by labor. The wage per effective unit of labor in the most complex task can then be written as  $w^{LR}(n)/\gamma(n)$ .

Proposition 3 implies that  $I^*(t) = I(t)$  and that all new labor-intensive (new complex) tasks are utilized immediately. In terms of the notation we have just introduced, this is equivalent to:

$$w^{LR}(n)/\gamma(n) \leq r \leq w^{LR}(n). \quad (16)$$

The long-run productivity effect can now be seen from the ideal price condition, (9):

$$(I - N - 1)c^u(\rho + \delta + \theta g)^{1-\sigma} + \int_I^N c^u \left( \frac{W}{\gamma(i)} \right)^{1-\sigma} = 1. \quad (17)$$

With the notation we have just introduced, this ideal price condition can also be written as

$$(1 - n)c^u(\rho + \delta + \theta g) + \int_0^n c^u \left( \frac{w^{LR}(n)}{\gamma(i)} \right)^{1-\sigma} = 1. \quad (18)$$

There are three important implications from the ideal price condition. First, automation cannot reduce wages in the long run. This claim follows from (17) when we use (16). In particular, straightforward differentiation gives

$$\frac{dW}{dI^*} \propto \frac{1}{\sigma - 1} [c^u(r)^{1-\sigma} - c^u(w^{LR}(n))^{1-\sigma}] \geq 0.$$

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<sup>20</sup>This is similar to the productivity or efficiency effect in models of offshoring such as Grossman and Rossi-Hansberg (2008), Rodriguez-Clare (2010) and Acemoglu, Gancia and Zilibotti (2015), which results from the substitution of cheaper foreign labor for domestic labor in certain tasks.

Intuitively, because the interest rate is constant in the long run, automation also increases the amount of capital used in production. Labor, which is the inelastic factor, earns the productivity gains in the form of higher wages.

Second, this time from (18), we also have that

$$\frac{dw^{LR}(n)}{dn} \propto \frac{dW}{dN} \propto -\frac{1}{\sigma-1} \left[ c^u(r)^{1-\sigma} - c^u \left( \frac{w^{LR}(n)}{\gamma(n)} \right)^{1-\sigma} \right] \geq 0,$$

so that automation (corresponding to a decrease in  $n$ ) reduces the wage per effective unit of labor in the least complex tasks; while the creation of new tasks increases it.

Finally, once again using (17), we also have

$$\frac{dw^{LR}(n)/\gamma(n)}{dn} \propto \frac{1}{\sigma-1} [c^u(w^{LR}(n))^{1-\sigma} - c^u(r)^{1-\sigma}] \leq 0,$$

so that the creation of new tasks reduces the wage per effective unit of labor in the most complex tasks; while automation increases it.

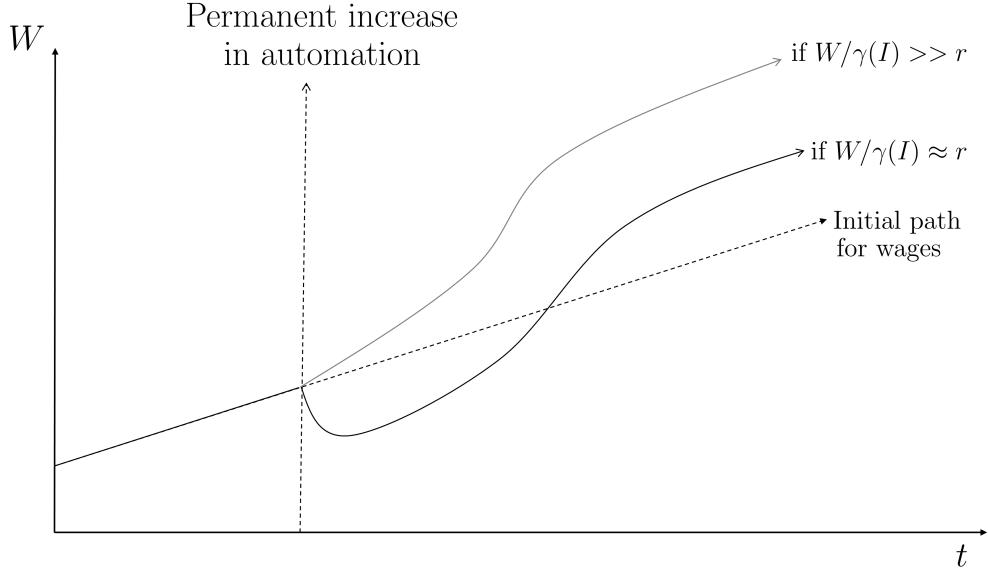


Figure 6: The evolution of equilibrium wage following a permanent increase in automation.

These observations thus establish that the majority of the results from the static model continue to apply, but because of the productivity effect, the potential negative impact of automation on the equilibrium wage level disappears in the long run. This is illustrated in Figure 6, which plots the behavior of the wage level,  $W$ , following a permanent, one-time decline in  $n$  due to additional automation (and continuation of the same technological paths thereafter): wages may fall in the short run following this increase in automation, but they necessarily increase in the long run because of the productivity effect.

## 4 Full Model: Tasks and Endogenous Technologies

The analysis in the previous section established the existence of a balanced growth path under the assumption that  $\dot{N} = \dot{I}$ . But why should these two types of technologies advance at the same rate? This is the question at the center of our paper, and we now develop our full model, which endogenizes the pace at which automation and creation of new tasks proceeds.

### 4.1 Endogenous and Directed Technological Change

We endogenize technological change by allowing new intermediates to be introduced by technology monopolists. New firms can introduce either technologies automating previously non-automated tasks or create new tasks. We assume that successful innovations always achieve automation or the creation of new tasks in the order of the intermediate indices,  $i \in [0, \infty)$ , so that lower-indexed tasks will always be automated before higher-index tasks, and a new labor-intensive task will always correspond to the lowest-indexed task that has not been created yet (and the lower index of integration at  $N - 1$  in the aggregate production function, (1), already imposes that new tasks replace the lowest-indexed task currently in use). As a consequence, the two types of endogenous technological changes will correspond to an increase in  $I$  and to an increase in  $N$ , respectively. We continue to assume that all intermediates, including those that have just been invented, can be produced at the fixed marginal cost of  $\mu\psi$ , and that the fringe of competitive firms is still present, forcing the technology monopolists to price at  $\psi$ , which of course implies a per-unit profit of  $(1 - \mu)\psi$ .

Though per-unit profits of technology monopolists are constant, their net present discounted value is a complex object for two reasons. First, the fact that  $I$  and  $N$  will grow at some fixed rate, for example in the balanced growth path as characterized in Proposition 3, implies that there will be a deterministic component to the length of time during which a monopolist will be able to enjoy profits from its technology. Despite this first complication, we will see that the dynamics of endogenous technology can be characterized, though this will involve somewhat different arguments than in the standard endogenous technological change models. Second, as in other models of quality improvements (e.g., Aghion and Howitt, 1992; Grossman and Helpman, 1991), new intermediates replace some existing ones. This generates the “creative destruction” of profits of existing producers by new firms, at least under the assumptions used in the literature, which is that new firms do not have to respect the intellectual property rights of the technology on which they are building. This assumption, however, creates more complex dynamics, especially coupled with the deterministic replacement of products in our model. For this reason, we adopt an alternative (and arguably equally plausible) structure of protection of intellectual property rights whereby building and replacing an existing technology is viewed as infringement of the patent of

that technology. This implies that the inventor of a new technology will have to buy this existing patent (or license the technology). We assume that this takes place with the inventor making a take-it-or-leave-it offer to the holder of the patent on the technology on which it is building. Consequently, a firm automating a task previously performed by labor will have to license or buy the relevant patent from an existing firm supplying the intermediate to this task, and a firm creating a new task, which is effectively creating a more complex, labor-intensive version of an existing task, will have to obtain the patent from an existing firm for the intermediate used in this task (which, in any equilibrium with automation, will be an automated task, since it is the lowest-indexed task currently in use). This game form ensures that each technology monopolist will receive the same flow of revenues regardless of whether its own product is replaced or not (either as profits when he is continuing to operate or as payments for its patent when it is replaced). We return to the analysis of how the results change when we allow for the creative destruction of profits in Section 6.

We are now in a position to describe the innovation possibilities frontier (the technology of creating new technologies). We assume that innovation requires scientists, and there is a fixed (inelastic) supply of  $S$  scientists in this economy.<sup>21</sup> At each point in time,  $S_I(t) \geq 0$  of these scientists are hired by monopolists at a competitive wage  $W^S$  for automation, and  $S_N(t) \geq 0$  of them are hired at the same wage for creating new tasks. The market clearing condition for scientists is

$$S_I(t) + S_N(t) \leq S,$$

with the wage  $W^S$  being equal to zero if this inequality is strict.

We assume that advances in automation and creation of new tasks follow the next two differential equations

$$\dot{I}(t) = \kappa_I S_I(t), \quad (19)$$

and

$$\dot{N}(t) = \kappa_N S_N(t), \quad (20)$$

where  $\kappa_I$  and  $\kappa_N$  are positive constants, representing the difficulty/ease of the corresponding type of technological change.

## 4.2 Equilibrium with Endogenous Technological Change

The key objects we need to compute to characterize the equilibrium with endogenous technological change are value functions determining the net present discounted value of new automation and

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<sup>21</sup>Focusing on an innovation possibilities frontier using just scientists, rather than variable factors such as in the lab-equipment specifications, is convenient because it enables us to focus on the direction of technological change — and not on the overall amount of technological change — especially when we turn to the welfare analysis in the next section.

labor-intensive innovations. We denote these by  $V_I(t)$  and  $V_N(t)$ . More specifically,  $V_I(t)$  is the value of a new technology automating the task at  $i = I(t)^+$  (i.e., the highest-indexed task that has not yet been automated, or more formally  $i = I(t) + \varepsilon$  for  $\varepsilon$  arbitrarily small and positive). Likewise,  $V_N(t)$  is the value of a new technology creating a more complex task at  $i = N(t)^+$ .

Given these value functions, an equilibrium with endogenous technology is given by paths  $\{K(t), N(t), I(t)\}_{t=0}^\infty$  for capital and technology (starting from an initial values  $K(0), N(0), I(0)$ ), paths  $\{r(t), W(t), W^S(t)\}_{t=0}^\infty$  for factor prices, paths  $\{V_N(t), V_I(t)\}_{t=0}^\infty$  for the value functions of technology monopolists, and paths  $\{S_N(t), S_I(t)\}_{t=0}^\infty$  for the allocation of scientists such that all markets clear, all firms, including prospective technology monopolists, maximize profits, the representative household maximizes its utility, and  $N(t)$  and  $I(t)$  evolve endogenously according to equations (19) and (20).

We start by characterizing the value functions for technology monopolists. Suppose also that in this equilibrium,  $n > \bar{n}$  so that  $I^* = I$  and new automated task starts being used immediately. Let us next compute the flow profits from automation, which naturally replaces a task previously performed by labor (i.e.,  $i > I(t)$ ) and can be written as

$$\pi_I(t, i) = (1 - \mu) \left( \frac{\eta}{1 - \eta} \right)^\zeta \psi^{1-\zeta} Y(t) c^u(r(t))^{\zeta-\sigma}.$$

Intuitively, these profits come from the ability of firms to produce task  $i$  using capital (which is necessarily profitable given our assumption that  $I^*(t) = I(t)$ ). Similarly, the flow profits of producing such task using labor are

$$\pi_N(t, i) = (1 - \mu) \left( \frac{\eta}{1 - \eta} \right)^\zeta \psi^{1-\zeta} Y(t) c^u \left( \frac{W(t)}{\gamma(i)} \right)^{\zeta-\sigma}.$$

It is then straightforward to compute the offer that a monopolist with a new technology automating task  $I$  at time  $t$  needs to make to the firm currently holding the patent for the (labor-intensive) technology of that intermediate. This offer will be given by the net present discounted value of the profit streams, discounted using the path of future interest rates, that the existing patent holder would obtain

$$(1 - \mu) \left( \frac{\eta}{1 - \eta} \right)^\zeta \psi^{1-\zeta} \int_t^\infty e^{-\int_0^\tau (r(s) - \delta) ds} Y(\tau) c^u \left( \frac{W(\tau)}{\gamma(I)} \right)^{\zeta-\sigma} d\tau.$$

Since this is a take-it-or-leave-it offer, the best response of the patent holder is to accept it.<sup>22</sup>

Similarly, the offer of the technology monopolist with a new technology for creating a new labor-intensive task while replacing task  $N - 1$  (which is necessarily automated in any equilibrium

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<sup>22</sup>This expression is written by assuming that the patent-holder will also turn down subsequent less generous offers in the future. Writing it in the value function form, using the one-step ahead deviation principle, leads to the same conclusion.

with automation) is given by

$$(1 - \mu) \left( \frac{\eta}{1 - \eta} \right)^\zeta \psi^{1-\zeta} \int_t^\infty e^{-\int_0^\tau (r(s) - \delta) ds} Y(\tau) c^u(r(\tau))^{\zeta-\sigma} d\tau.$$

Both of these offers will be accepted by the patent-holders with the current technologies. Incorporating this, we can then compute the values of firms that innovate (respectively with automation and creation of new tasks):

$$V_I(t) = (1 - \mu) \left( \frac{\eta}{1 - \eta} \right)^\zeta \psi^{1-\zeta} \int_t^\infty e^{-\int_t^\tau (r(s) - \delta) ds} Y(\tau) \left( c^u(r(\tau))^{\zeta-\sigma} - c^u \left( \frac{W(\tau)}{\gamma(I(t))} \right)^{\zeta-\sigma} \right) d\tau, \quad (21)$$

and

$$V_N(t) = (1 - \mu) \left( \frac{\eta}{1 - \eta} \right)^\zeta \psi^{1-\zeta} \int_t^\infty e^{-\int_t^\tau (r(s) - \delta) ds} Y(\tau) \left( c^u \left( \frac{W(\tau)}{\gamma(N(t))} \right)^{\zeta-\sigma} - c^u(r(\tau))^{\zeta-\sigma} \right) d\tau. \quad (22)$$

Both of these expressions have a common form: they subtract the lower cost of producing a task with the factor for which the new technology is designed from the higher cost of producing the same task with the other factor (working with the older technology). Note also that both of these expressions factor in the fact that, because of the same structure of offers that will be forthcoming in the future, the innovator will continue to earn a flow of revenues in the future, regardless of when its technology is replaced — and this is the reason why the time of replacement does not feature in these expressions. Observe also, for future reference, that these values are positive only when  $\sigma > \zeta$ . This can be seen from (21), by virtue of the fact that a task performed by labor is being automated,  $c^u(r(\tau)) < c^u \left( \frac{w(\tau)}{\gamma(I(t))} \right)$ . Thus if we had  $\zeta > \sigma$ , the profit stream would be negative. The same applies to (22). The intuitive reason for this is that, in the case where  $\zeta > \sigma$ , profits are lower when the firm is more productive, and thus when the holder of the new technology buys out the patent-holder of the less productive technology, it ends up with negative net profits.

Once these value functions are derived, the allocation of scientists to the two different types of technological change follows immediately by noting that the market wage of scientists will be equal to their value in the activity where their productivity is greater. Therefore,

$$\begin{aligned} S_N(t) &= S, & S_I(t) &= 0 && \text{if } \kappa_N V_N(t) > \kappa_I V_I(t) \\ S_N(t) &= 0, & S_I(t) &= S && \text{if } \kappa_N V_N(t) < \kappa_I V_I(t) \\ S_N(t) &\in [0, S] , & S_I(t) &= S - S_N(t) && \text{if } \kappa_N V_N(t) = \kappa_I V_I(t). \end{aligned}$$

Intuitively, whenever one of the two types of technologies (automation versus creation of new tasks) is more profitable, all scientists will be allocated to this activity, and their wage,  $W^S$ , will be equal to their value in this activity. But this also implies that this wage will exceed their value in the other technological activity, unless we are in the case where  $\kappa_N V_N(t) = \kappa_I V_I(t)$ .

These observations enable us to represent, using the same normalizations as in the previous section, the equilibrium path with endogenous technology by the time path of the tuple  $\{n(t), k(t), c(t), S_I(t)\}_{t=0}^\infty$  such that:

- The evolution of the state variables is given by

$$\begin{aligned}\dot{k}(t) &= f^E(k(t), n(t)) - c(t) - (\delta + A\kappa_I S_I(t))k(t) \\ \dot{n}(t) &= \kappa_N(S - S_I(t)) - \kappa_I S_I(t).\end{aligned}$$

- Consumption satisfies the Euler equation (13) coupled with the transversality condition in equation (14).
- The allocation of scientists satisfies:

$$S_I(t) = \begin{cases} 0 & \text{if } \kappa_I V_I(t) < \kappa_N V_N(t) \\ \in [0, S] & \text{if } \kappa_I V_I(t) = \kappa_N V_N(t) \\ S & \text{if } \kappa_I V_I(t) > \kappa_N V_N(t) \end{cases},$$

with  $V_N(t)$  and  $V_I(t)$  given by equations (21) and (22).

We next characterize the dynamic equilibrium with endogenous technology. A balanced growth path is defined as in Proposition 3, as an allocation in which normalized capital  $k(t)$  and the interest rate  $r(t)$  are constant, except that now  $n$  will be determined endogenously. The next proposition gives another one of the main results of the paper. It establishes conditions for the existence of a unique balance growth path in which there are both types of technological changes and also shows that, under the same set of conditions, it is (saddle-path) stable.

**Proposition 4 (Equilibrium with endogenous technological change)** *Suppose that  $\sigma > \zeta$ . There exist  $\bar{\rho}$  and  $\bar{S}$  such that for  $\rho > \bar{\rho}$  and  $S < \bar{S}$  (where  $\bar{\rho}$  is defined in Proposition 3 and  $\bar{S}$  is defined in the Appendix), the following are true:*

1. *There exists  $\bar{\kappa}$  such that for  $\frac{\kappa_I}{\kappa_N} > \bar{\kappa}$ , there is a unique balanced growth path, where  $\dot{N} = \dot{I} = \frac{\kappa_I \kappa_N}{\kappa_I + \kappa_N} S$ , and  $Y, C, K$  and  $W$  grow at the constant rate  $g = A \frac{\kappa_I \kappa_N}{\kappa_I + \kappa_N} S$ , and the interest rate,  $r$ , the labor share and employment are constant. Along this path, we have  $N(t) - I(t) = n^D$ , with  $n^D$  determined endogenously from the condition  $\kappa_N V_N = \kappa_I V_I$ , and satisfies  $n^D \in (\bar{n}, 1)$ , where  $\bar{n}$  is as defined in Proposition 3.*
2. *Moreover, the dynamic equilibrium is unique in the neighborhood of the balanced growth path and is locally (saddle-path) stable. When  $\theta \rightarrow 0$ , the dynamic equilibrium is globally stable.*

**Proof.** In the Appendix, we present a more general result from which the current proposition follows as a corollary. ■

The first important result contained in this proposition is the existence and uniqueness of the balanced growth path. The second critical result, established in the third part, is that this

balanced growth path is locally asymptotically stable and also globally asymptotically stable when  $\theta$  is small (so that preferences are approximately linear or equivalently have an infinite elasticity of intertemporal substitution). This result implies that there are powerful market forces pushing the economy towards the balance growth path.

These results are established under several conditions. First, we have imposed that  $\sigma > \zeta$ . This condition ensures that innovations are directed towards technologies using the cheaper factors.<sup>23</sup> Recall from Section 2 that more tasks are allocated to the factor that is cheaper. This creates a natural force that tends to push innovations to be directed towards the same cheaper factor. One way of understanding this effect is that as a factor becomes cheaper, the range of activities in which it is used expands. Holding the proportions at which this factor is combined with intermediates in the task production functions, (2) and (3), this implies that the quantity of the corresponding intermediate,  $q(i)$ , also increases. This makes technologies working with this factor more profitable, encouraging innovation beneficial for this factor. The extent of this positive force is regulated by the elasticity of substitution  $\sigma$ : the greater is  $\sigma$ , the more powerful is this effect directing innovation towards the cheaper factor. There is a countervailing effect as well, however: as a factor becomes cheaper, it is substituted for the intermediate it is combined with, so that the quantity of the corresponding intermediate declines holding the level of task production fixed. This creates a negative force, discouraging innovations directed towards the cheaper factor. Task production functions, (2) and (3), clarify that the extent of the substitution effect will depend on the elasticity of substitution between the factor in question and the intermediates,  $\zeta$ . The condition  $\sigma > \zeta$  guarantees the positive effect dominates so that innovations are directed towards the cheaper factor.<sup>24</sup>

We should note that this condition is quite plausible: in our model, when  $\sigma < \zeta$ , there will be no research at all since, as observed above, the net present discounted values from innovation will be negative (recall equations (21) and (22)). This is because, as explained above, somewhat pathologically profits are higher when the producer is less productive. Put differently, in this case, there is such a strong substitution effect allowing the substitution of the cheaper factor for intermediates that there is no incentive to innovate on intermediates working with cheaper factors. Because there will be no technological change with a negative net present discounted value from innovation, this condition is imposed even for the existence result in part 1 of the proposition. Moreover, the condition  $\sigma > \zeta$  is also empirically plausible. We expect the elasticity of substitution between factors and intermediates,  $\zeta$ , to be very low — in the limit, zero as in the Leontief case

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<sup>23</sup>By the term “innovation directed towards the cheaper factor”, we mean a comparative static statement: as the relative price of a factor declines, is innovation directed more or less towards this factor?

<sup>24</sup>Our assumption ruling out the creative destruction of profits of existing producers is also playing a role in the stability result as we discussed further Section 6, which shows that though the balanced growth path equilibrium is very similar, stability is no longer guaranteed.

since new technologies, for example enabling automation, are embedded in these intermediates.<sup>25</sup>

Second, the assumption  $S < \bar{S}$  is used for guaranteeing that the growth rate is not too high (and also ensures that the net present discounted value of the representative household is finite). If the growth rate is above the threshold implied by  $\bar{S}$ , the creation of new tasks is discouraged (even if current wages are low) because firms anticipate that wages will grow very rapidly, reducing the future profitability of these labor-intensive tasks.

Finally, as in Proposition 3,  $\rho > \bar{\rho}$ . As discussed in that context, this assumption ensures that the interest rate is not too close to 0, which in turn guarantees that there exists a well-defined threshold  $\bar{n}$ ; when  $n > \bar{n}$ , all technologically automated tasks will be immediately automated in equilibrium, and as also noted above,  $\rho > \bar{\rho}$  also guarantees that all new labor-intensive tasks will immediately start being used with labor. (The case in which  $\rho < \bar{\rho}$  is studied in the Appendix.) The proposition also shows that when  $\rho > \bar{\rho}$ , and the other conditions in the proposition are satisfied, the long-run equilibrium will indeed involve  $n > \bar{n}$ , so this latter condition does not need to be imposed, but follows as an implication.

Figure 7 draws the net present discounted value (normalized by output) of allocating scientists to creating new labor-intensive tasks or to automation as a function of  $n$  for  $\rho > \bar{\rho}$  and  $S < \bar{S}$ , denoted respectively by  $v_N(n)$  and  $v_I(n)$  (where  $v_I = V_I/Y$  and  $v_N = V_N/Y$ ). In the region where  $n > \bar{n}$ , the value of automation decreases as the economy automates more tasks ( $n$  decreases). This is the key economic force that generates stability in our model: greater automation increases wages per unit of production relative to the interest rate, and thus the relative value of creating new labor-intensive tasks increases with automation. Though there is also the productivity effect acting towards increasing the labor cost, the assumption that  $\rho > \bar{\rho}$  ensures that this effect is dominated by the direct substitution effect, and thus guarantees that the balanced growth path is asymptotically stable.

These forces are, however, not sufficient to guarantee that the curves for  $\kappa_I v_I(n)$  and  $\kappa_N v_N(n)$  intersect. Though the former always starts below the latter as shown in Figure 7, it could always remain below the latter. The condition in Proposition 4 that  $\kappa_I/\kappa_N$  is sufficiently large ensures that such an intersection takes place and thus there exists a unique “interior” balanced growth path. This discussion also clarifies that when this condition does not hold, the long-run equilibrium will be one in which only new tasks are developed, and there is no automation.

An important conclusion of this discussion is that the critical economic force highlighted by this result is that, differently from models with factor-augmenting technologies, it is factor prices that guide the direction of technological change, and there are stronger incentives to undertake the

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<sup>25</sup>Observe also that this condition does not impose any restrictions on the short-run elasticity of substitution between capital and labor, which can be less than one as in many of the studies reviewed in Acemoglu and Robinson (2015).

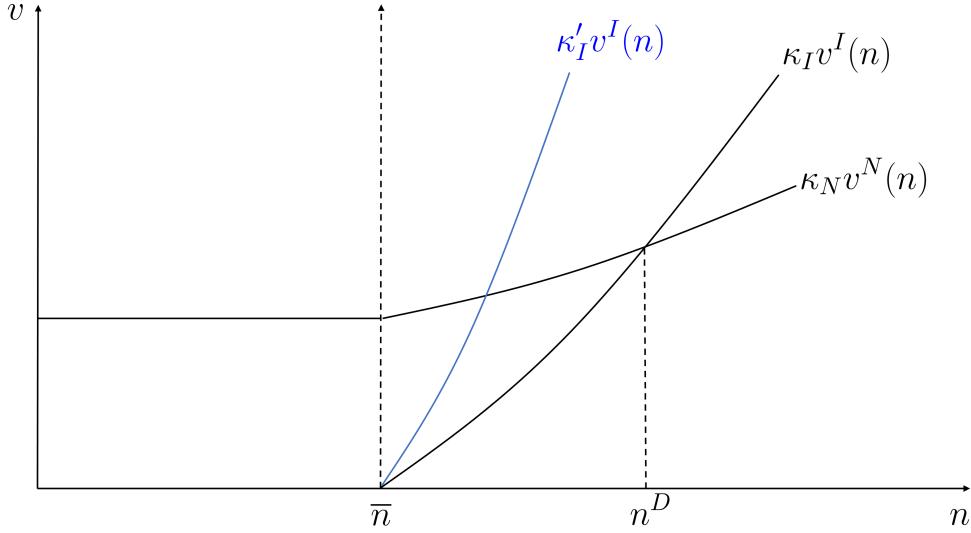


Figure 7: Determination of  $n^D$  in steady state.

type of innovation that will work with the factor that has a relatively cheaper user cost.

We can also observe that the long-run elasticity of substitution between capital and labor,  $\sigma_{LR}$ , which allows both for the endogeneity of technology and for capital accumulation, is equal to 1 because following a shock to technology or capital stock, the economy returns back to its balanced growth path, where the share of labor in national income is constant. This implies that, interestingly, the long run elasticity of substitution need not be larger than the medium-run and short-run elasticities  $\sigma_{MR}$  and  $\sigma_{SR}$  defined above. This is because it is not only technology but also the capital stock of the economy that adjusts in the long run (and thus bringing in the productivity effects discussed in the previous section).

Finally, the emphasis of our main result in this section, Proposition 4, has been to show that shocks to technology, for example in the form of a series of new automation technologies, will set in motion self-correcting forces, so that in the long run the economy returns back to its pre-shock balanced growth path with the same employment level and labor share in national income. This does not, however, imply that all changes will leave the long-run prospects of labor unchanged. The next corollary shows that if there is a change in the innovation possibilities frontier, making automation easier than before, then there will be a new balanced growth path with lower employment and lower share of labor in national income.

**Corollary 2** *Suppose that there is a one-time permanent increase in  $\kappa_I/\kappa_N$ . Then the economy converges to a new balanced growth path with lower  $n^D$ , lower employment and lower share of labor in national income.*

This corollary follows immediately because an increase in  $\kappa_I/\kappa_N$  shifts the intersection in Figure

7 to the left, leading to a lower value of  $n^D$  in the balanced growth path.

One implication of this corollary, in conjunction with Proposition 4, is that it clearly delineates the types of changes in technology that will set in motion self-correcting dynamics: those driven by faster than usual arrival of automation technologies. In contrast, those which changed the ability of the society to create new automation technologies will not create such self-correcting dynamics and will result in lower prospects for labor in the future.

## 5 Welfare

In this section we turn to an analysis of the efficiency of the equilibrium described in Proposition 4. Our main finding is that the presence of rents for workers, as captured by our quasi-labor supply, distorts the composition of equilibrium technology towards too much automation and too little creation of new, more complex (labor-intensive) technologies — and this is in addition to other distortions that exist in this class of models. We present two complementary results shedding light on this inefficiency. First, we characterize the constrained efficient allocation of a social planner who is subject to the same quasi-labor supply schedule, as well as to the constraint that wages have to be given by (possibly subsidized) marginal product of labor of firms and technologies evolve according to the same innovation possibilities frontier. We then show how this constrained efficient allocation can be decentralized by a set of taxes and subsidies. This exercise shows that, in addition to the usual wedges (taxes/subsidies) between the social planner’s allocation and the decentralized equilibrium, workers’ rents create an additional reason to subsidize the creation of new tasks relative to automation. Second, for a particular set of parameters that help us isolate this novel inefficiency, we show the decentralized equilibrium could be improved by altering the composition of R&D in the direction of the creation of new tasks.

We start by characterizing the constrained efficient allocation, which we will use in deriving both results. In this constrained efficient allocation. Let us denote by  $F^P(N, I, K, L)$  the *net* aggregate output (net of the costs of producing intermediates) when the level of employment is  $L$ , the capital stock is  $K$ , the state of technologies is represented by  $\{N, I\}$ , and intermediates are priced at their marginal cost (which is the relevant net aggregate output expression for the social planner, since she would always price all intermediates at marginal cost). Also, let  $W^P(N, I, K)$  and  $r^P(N, I, K)$  denote the resulting marginal products of labor and capital (corresponding to the wage and interest rates in the decentralized allocation) with the level of employment given by the quasi-labor supply schedule,  $L = L^s(\omega)$ . Finally, let  $\omega^P(N, I, K)$  denote the equilibrium value for  $W/rK$  in this case. It is straightforward to prove that these variables satisfy the same comparative statics described in Proposition 2.

The constrained efficient allocation solves the problem

$$\max_{\{C(t), L(t), S_N(t), S_I(t)\}} \int_0^\infty e^{-\rho t} \frac{C(t)^{1-\theta} - 1}{1-\theta} dt,$$

subject to the endogenous evolution of the state variables:

$$\begin{aligned}\dot{K}(t) &= F^P(N(t), I(t), K(t), L(t)) - C(t) - \delta K(t), \\ \dot{N}(t) &= \kappa_N S_N(t), \\ \dot{I}(t) &= \kappa_I S_I(t).\end{aligned}$$

In using the net aggregate production function,  $F^P$ , we have already incorporated that the planner will price all intermediates at marginal cost,  $\mu\psi$ . Furthermore, we have written the objective function of the social planner as just maximizing the net present discounted value of consumption streams, thus imposing that there is no disutility or opportunity cost of labor supply and all wages received by workers are “quasi rents” — which, as noted above, will be an additional source of deviation between the social planner’s allocation and the equilibrium. This formulation is justified by the microfoundation provided for the quasi-labor supply schedule in the Appendix, and we also note that the results are entirely analogous if there is a positive opportunity cost of labor lower than the market wage (and assuming that this opportunity cost is equal to zero is merely for notational simplicity). The most important implication of this structure is that, all else equal, the social planner would like to maximize employment as this increases net output and wage payments without any disutility cost.<sup>26</sup>

Because the planner faces the same quasi-labor supply schedule and labor demand relations, we also have:<sup>27</sup>

$$L(t) \leq L^s(\omega^P(N, I, K)).$$

The relationship imposes that the planner will take into account the impact of technology and capital accumulation on employment.

Let  $\mu_N$  and  $\mu_I$  denote the shadow values of the two types of technology, respectively, and  $\mu_L$  and  $\mu_K$  the shadow values of labor and capital. The maximum principle (see Acemoglu, 2009, Theorem 7.9) implies these satisfy the necessary conditions:

$$\begin{aligned}\rho\mu_N - \dot{\mu}_N &= \mu_K F_N^P + \mu_L L_\omega^s \omega_N^P, & \rho\mu_I - \dot{\mu}_I &= \mu_K F_I^P + \mu_L L_\omega^s \omega_I^P, \\ \rho\mu_K - \dot{\mu}_K &= \mu_K(r^P - \delta) + \mu_L L_\omega^s \omega_K^P, & \mu_L &= \mu_K W^P.\end{aligned}$$

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<sup>26</sup>This program also imposes that the threshold that the social planner will set for automation,  $I^*(t)$  in the terminology used in the previous section, is given by  $I(t)$ , that is, by the available automation technology. As in the previous Section, this will be the case for  $\rho > \bar{\rho}$ . As in the case with endogenous technology, the planner will always be in the region in which  $n > \bar{n}$ .

<sup>27</sup>Recall that  $W^P(N, I, K)$ ,  $r^P(N, I, K)$  and  $\omega^P(N, I, K)$  are defined in terms of the allocation  $\{N, I, K\}$ .

All the functions in the above equations are evaluated at their corresponding arguments at time  $t$ , and subscripts denote partial derivatives. Moreover, we show in the Appendix that the current value Hamiltonean associated with the planner's problem is concave, so these conditions (plus the Euler equation for consumption and the transversality condition) are sufficient for characterizing the constrained efficient allocation.

Let  $\Psi_N(t) \equiv \mu_N(t)/\mu_K(t)$  and  $\Psi_I \equiv \mu_I(t)/\mu_K(t)$  be the shadow discounted net present values of new technologies (in terms of additional net output they create). The optimal allocation of scientists to two different types of research then satisfies

$$\begin{aligned} S_N(t) &= S, & S_I(t) &= 0 && \text{if } \kappa_N \Psi_N(t) > \kappa_I \Psi_I(t) \\ S_N(t) &= 0, & S_I(t) &= S && \text{if } \kappa_N \Psi_N(t) < \kappa_I \Psi_I(t) \\ S_N(t) &\in [0, S] , & S_I(t) &= S - S_N(t) && \text{if } \kappa_N \Psi_N(t) = \kappa_I \Psi_I(t). \end{aligned}$$

Thus, intuitively,  $\Psi_N$  and  $\Psi_I$  play an analogous role to  $V_N$  and  $V_I$  in the decentralized allocation, and can be also written as integrals of future net benefits:

$$\begin{aligned} \Psi_N &= \int_t^\infty e^{-\int_0^\tau (r^P - \delta + W^P L_\omega^s \omega_K^P) ds} (F_N^P + W^P L_\omega^P \omega_N^P) d\tau, \\ \Psi_I &= \int_t^\infty e^{-\int_0^\tau (r^P - \delta + W^P L_\omega^s \omega_K^P) ds} (F_I^P + W^P L_\omega^P \omega_I^P) d\tau. \end{aligned}$$

These equations are clearly analogous to the expressions for  $V_N$  and  $V_I$  in the decentralized equilibrium given by equations (21) and (22).

To complete our characterization, let  $f^P(n, k) = F^P(N, I, K, L(\omega^P(N, I, K)))/\gamma(I)$  denote the normalized net output;  $w^P(n, k) = W^P(N, I, K)/\gamma(I)$  the normalized wages; and  $r^P(n, k)$  the normalized interest rate obtained when intermediates are priced at their marginal cost. These are defined in the same way as in the decentralized equilibrium as detailed in Section 4. Summarizing, the constrained efficient allocation can be represented as  $\{n(t), k(t), c(t), S_I(t)\}_{t=0}^\infty$  — where we normalize  $n(t) = N(t) - I(t)$ ,  $k(t) = K(t)/\gamma(I(t))$ ,  $c(t) = C(t)/\gamma(I(t))$ , such that:

- The evolution of the state variables is given by

$$\begin{aligned} \dot{k}(t) &= f^P(n(t), k(t)) - c(t) - (\delta + A\kappa_I S_I(t))k(t) \\ \dot{n}(t) &= \kappa_N(S - S_I(t)) - \kappa_I S_I(t). \end{aligned} \tag{25}$$

- Normalized consumption satisfies the Euler equation

$$\dot{c}(t) = c(t) \left( \frac{1}{\theta} (r^P(n(t), k(t)) \left( 1 - \omega^P L^s \frac{\partial \ln L^s}{\partial \ln \omega} \frac{\partial \ln \omega^P}{\partial \ln K} \right) - \delta - \rho) - \kappa_I S_I(t) \right). \tag{26}$$

- The allocation of scientists satisfies:

$$S_I(t) = \begin{cases} 0 & \text{if } \kappa_I \Psi_I(t) < \kappa_N \Psi_N(t) \\ \in [0, S] & \text{if } \kappa_I \Psi_I(t) = \kappa_N \Psi_N(t) \\ S & \text{if } \kappa_I \Psi_I(t) > \kappa_N \Psi_N(t) \end{cases}, \quad (27)$$

with  $S_N(t) = S - S_I(t)$ .

- The transversality condition holds, i.e.,

$$\lim \mu_k e^{-\rho t} = 0 \quad (28)$$

holds.

This characterization implies that the constrained efficient allocation has a similar structure to the equilibrium described in Proposition 4. The next proposition summarizes this result and shows that it also has the same asymptotic and stability properties. Most importantly, it also characterizes the set of taxes and subsidies that can be used in the equilibrium to decentralize this constrained efficient allocation.

**Proposition 5 (Constrained efficient allocation and decentralization)** *Suppose that  $\sigma > \zeta$  and  $\rho > \bar{\rho}$ . Then:*

- *The constrained efficient allocation is uniquely defined by the solution to (25)-(28). Moreover, under the same conditions derived in Proposition 4, this allocation locally converges to the unique constrained efficient balanced growth path, and if  $\theta \rightarrow 0$ , it globally converges to this efficient balance growth path.*
- *The constrained efficient allocation can be decentralized by using the following sets of taxes and subsidies:*
  1. *a proportional subsidy at the rate  $1 - \mu$  on intermediate prices to remove the monopoly markups;*
  2. *a proportional tax/subsidy of  $\tau_k = -\omega^P L^s \frac{\partial \ln L^s}{\partial \ln \omega} \frac{\partial \ln \omega^P}{\partial \ln K}$  on savings to correct for the impact of capital on employment (this expression is positive, i.e., tax, when  $\sigma_{SR} > 1$ , zero when  $\sigma_{SR} = 1$ , and a subsidy, i.e., negative, when  $\sigma_{SR} < 1$ );*
  3. *additive taxes/subsidies for successful innovators who entered the market at time  $t_0$ , which correct for the technological externality generated by the two different types of innovation;*
  4. *an additive subsidy  $W^P L^P \frac{\partial \ln L^s}{\partial \ln \omega} \frac{\partial \ln \omega^P}{\partial N} \geq 0$  for successful innovators of new more complex tasks, and an additive tax  $W^P L^P \frac{\partial \ln L^s}{\partial \ln \omega} \frac{\partial \ln \omega^P}{\partial I} \leq 0$  on successful innovators of new automation technologies; this tax and subsidy correct for the fact that technology monopolists do not take into account the effect of technologies on the level of equilibrium employment.*

**Proof.** We present explicit formulas for all of the taxes and subsidies in the appendix. ■

This proposition contains several important results. First, it characterizes the constrained efficient allocation, establishing that it has a similar structure to the equilibrium. Second, in contrast to neoclassical models of capital taxation (e.g., Chamley, 1986 and Judd, 1985, but also see Straub and Werning, 2014), the decentralization of the constrained efficient allocation requires taxing or subsidizing capital accumulation. This is because the capital stock affects wages and thus the level of employment through the quasi-labor supply schedule. For instance, if  $\sigma_{SR} < 1$ , capital increases employment in the short run (see Proposition 2), which is, as noted above, beneficial. Thus in this case, the social planner would set  $\tau_K < 0$ , further encouraging capital accumulation, while when  $\sigma_{SR} > 1$ , the opposite applies.

Third, the quality ladder structure in the creation of new labor-intensive complex tasks introduces a technological externality. By undertaking this type of innovation and thus increasing  $N$ , a technology monopolist also allows new entrants to create more productive new tasks (because  $\gamma(N)$  is increasing). The externality created by automation is somewhat more subtle. Because capital has the same productivity in all automated tasks, this direct technological externality is absent. But automation today forces future innovators to automate higher-indexed tasks, which are the ones where labor has a comparative advantage (because  $\gamma(I)$  is increasing), and this reduces the profits of future innovators. Though in different environments, some of these externalities could be internalized through a more sophisticated patent system, here we have focused on taxes and subsidies, which explains the wedges introduced in part 3 of the proposition.

Finally, the quasi-labor supply schedule creates an additional, and novel, distortion in the equilibrium relative to the constrained efficient allocation. Because firms do not internalize the quasi-rents received by workers, they automate tasks taking into account the wage rate. In contrast, the social planner internalizes these quasi-rents, and thus at the margin prefers to create more employment as we have already noted (or equivalently, at the margin she uses the opportunity cost of labor rather than the market wage in the automation decision). The resulting greater incentives of firms to automate tasks with given technology then translate into a stronger impetus for R&D directed towards automation and too little towards the creation of new, more complex tasks. For this reason, the social planner would like to encourage more R&D towards creation of labor-intensive new tasks and less automation, and she achieves this by using taxes on automation innovations and subsidies to innovations creating new labor-intensive tasks as outlined in part 4 of the proposition.

Proposition 5 outlined how the constrained efficient allocation can be decentralized. A key result, as we have just emphasized, is that conditional on the other taxes and subsidies necessary for dealing with markups and technological externalities, there needs to be an additional set of

taxes and subsidies to encourage less automation and more effort towards the creation of new, more complex tasks. The complementary question is whether starting from a decentralized allocation, and without this full set of subsidies, the social planner would still like to discourage automation. The next proposition answers this question (in the affirmative), focusing on the configuration where  $\zeta \rightarrow 1$  which, as we have already emphasized, is a particularly tractable special case of our model, and assuming that the proportional subsidy at the rate  $1 - \mu$  removing the main effect of monopoly markups is present.

**Proposition 6 (Excessive automation)** *Suppose that  $\rho > \bar{\rho}$  and  $S < \bar{S}$  as in Proposition 4, and that  $\sigma > \zeta \rightarrow 1$ . Moreover, suppose intermediate goods are subsidized and can be purchased at their marginal cost (or equivalently  $\mu \rightarrow 1$ ). Consider the decentralized equilibrium path starting from some initial level of capital,  $K(0)$ , and technologies,  $N(0)$  and  $I(0)$ , converging to the balanced growth path described in Proposition 4 (i.e.,  $n^D(t) = N(t) - I(t)$  converging to  $n^D$ ). Then there exists a feasible allocation satisfying  $n^P(t) \geq n^D(t)$  with  $\lim_{t \rightarrow \infty} n^P(t) > n^D$  that achieves strictly greater welfare than the decentralized equilibrium.*

**Proof.** The proof is constructive, and proceeds by showing that slightly reducing  $S_I(t)$  whenever  $S_I(t) \in (0, 1)$  produces a welfare improvement. All the details and derivations are presented in the appendix. ■

This proposition therefore establishes that even without the full set of other taxes and subsidies, departing from the equilibrium in the direction of discouraging automation and further encouraging the creation of new, more complex tasks will be welfare improving. The assumption that  $\zeta \rightarrow 1$  plays an important role in this result. Because in this special case, the production function for intermediates becomes Cobb-Douglas, and monopoly profits are proportional to revenues. This, coupled with the assumption that monopoly markups are removed, ensures that incentives to undertake different types of innovations, as summarized by the value functions  $V_N$  and  $V_I$ , are proportional to social values except for the distortion working through the quasi-labor supply schedule, and thus enables us to focus on this novel source of distortion in the composition of R&D and direction of technological change.<sup>28</sup>

## 6 Extensions

In this section, we discuss two extensions. First we introduce heterogeneous skills, enabling us to analyze the impact of the two types of technological changes we have studied in this paper on inequality between different skill types. Second, we reintroduce the creative destruction of profits, and show how similar balanced growth path results continue to apply in this case, though there may also exist other balance growth path or steady states.

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<sup>28</sup>The same result can be established without these assumptions if the quasi-labor supply curve is sufficiently elastic, so that the benefits from small increases in wages (in terms of expanding employment) outweigh costs that may come from other nonlinear effects that are not removed by taxes and subsidies in this case.

## 6.1 Automation, New Tasks and Inequality

In this subsection, we introduce heterogeneous skills and study how automation and creation of new tasks impact inequality.

This extension is motivated by the observation that, since new tasks are more complex, its creation favors high-skill workers who may have a comparative advantage in new and complex tasks. This natural assumption receives support from the data. As the left panel of Figure 8 shows, each decade starting in 1980, 1990 and 2000, employment growth over subsequent years has been faster in occupations with more skill requirements—as measured by the average years of education among employees at the start of each decade.

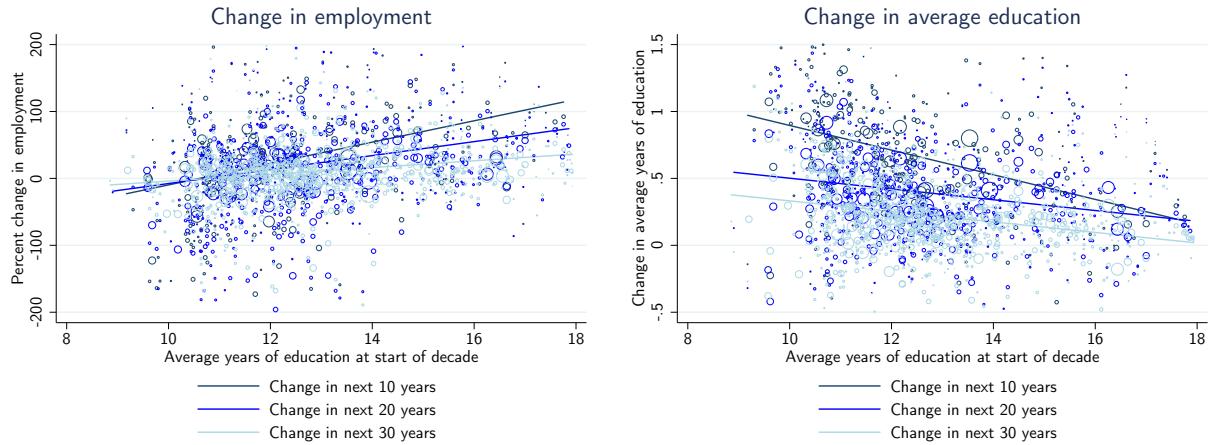


Figure 8: Scatter plots of employment growth within each occupational group (left panel) or the change in average skills among employees (right panel), and its skill requirements at the beginning of the decade. Employment growth or the change in average skills over the next 10 years are plotted in dark blue, over the next 20 years in blue, and over the next 30 years in light blue. Employment counts and skill requirements for 1980, 1990 and 2000 are from the U.S. Census; while data for 2007 is from the American Community survey.

Though the left panel of Figure 8 confirms that it is the skilled occupations that grow faster, its right panel shows a pattern of strong “mean reversion” in skill requirements, whereby average education declines over subsequent years in these high skill requirement occupations as they become more open to lower-skill workers.

We incorporate these features into our model by assuming there are two types of workers: skilled and unskilled. The pattern of comparative advantage is slightly more complicated and reflects our interpretation of the patterns in the data. We first assume that high-skilled labor has productivity analogous to what we have assumed so far for labor overall:

$$\gamma_H(i) = e^{A_H i}.$$

For low-skilled workers, we assume

$$\gamma_L(i, t) = e^{A_L i + (A_H - A_L)\Delta(t - t_0(i))},$$

where  $A_L < A_H$  and  $t$  is calendar time and  $t_0(i)$  the date at which task  $i$  was introduced. This structure thus implies that the productivity of low-skill labor increases as time passes from the initial date at which a task was first invented/introduced. This assumption captures the feature that new technologies and tasks are standardized over time (e.g., Acemoglu, Gancia and Zilibotti, 2010) or that low-skill workers may not be good at adapting to a changing environment or new technologies (e.g., Schultz, 1965, Nelson and Phelps, 1966, Greenwood and Yorukoglu, 1997, Caselli, 1999, Galor and Moav, 2000, and Beaudry, Green and Sand, 2013).

The implication of this assumption for our setup is that while capital has a comparative advantage in low-indexed tasks that have been automated, high-skill labor will have a comparative advantage in high-indexed tasks that have recently been introduced; while low-skill labor will perform intermediate-indexed tasks. In particular, it follows straightforwardly that there exists a threshold task  $M$  such that high-skill labor performs tasks in  $(M, N]$ , low-skill labor performs tasks in  $(I, M]$ , and tasks in  $[N - 1, M]$  are performed by capital.

In addition, we assume there is a quasi-labor supply of low-skill labor given by  $L^s(\frac{w_L}{rK})$ , and a quasi-labor supply of high-skill labor given by  $H_s(\frac{w_H}{rK})$ . The respective wages of these two types of labor are denoted by  $w_L$  and  $w_H$ . For simplicity, we focus on the dynamic economy with exogenous technology.

The main implications of this model with heterogeneous labor are summarized in the next proposition.

**Proposition 7 (Automation, new tasks and inequality)** *Suppose technology evolves exogenously:*

1. *Then a balanced growth path exists if and only if asymptotically  $\dot{N} = \dot{I} = \Delta$  (and  $A_H(1-\theta)\Delta < \rho$  so that net present discounted value of household income is finite). In this balanced growth path  $Y, C, K, w_H$  and  $w_L$  grow at a constant rate  $A_H\Delta$  and  $r$  is constant. Moreover, the wage ratio between high-skilled and low-skilled workers ( $w_H/w_L$ ) is constant but depends on  $n = N - I$*
2. *Given such a path of technological change, the dynamic equilibrium is unique starting from any initial condition and converges to the balanced growth path.*
3. *The immediate effect of increases in both  $I$  and  $N$  is to increase  $w_H/w_L$ . But the medium-run impact of an increase in  $N$  is to reduce inequality.*

**Proof.** As explained above, high-skill workers will perform tasks in  $(M, N]$ ; while low-skilled workers perform tasks in  $(I, M]$ .

This threshold is given by

$$\frac{e^{A_H M(t)}}{w_H(t)} = \frac{e^{A_L M(t) + (A_H - A_L)\Delta(t - t_0(M))}}{w_L(t)},$$

which grows at a rate  $\Delta$  over time as well, along the balanced growth path.

Likewise, the threshold for automation is defined as

$$\frac{1}{r(t)} = \frac{e^{A_L \tilde{I}(t) + (A_H - A_L)\Delta(t - t_0(I))}}{w_L(t)},$$

which also grows at the rate  $\Delta$  over time maintaining the economy balanced.

Notice that if learning took place at a speed below  $(A_H - A_L)\Delta$ , low-skilled workers would get squeezed over time and perform a decreasing fraction of tasks at lower wages.

Finally, for the comparative statics notice that a temporal shock to  $I$  reduces the amount of tasks performed by low-skill workers. Since they haven't had enough time to learn how to perform more complex tasks,  $M$  grows by less than  $I$  and their wages fall. Importantly, their wages also fall relative to high-skill workers, and inequality increases. Thus, in this case, we would observe a declining labor share coinciding with more inequality, but this would eventually revert in steady state.

On the other hand, a temporal shock to  $N$  increases the range of tasks performed by high-skill workers, raising their wages. However,  $N$  only increases mildly and  $w_H/w_L$  falls because low-skill workers haven't had enough time to gain comparative advantage in complex tasks. Thus, in this case, we would observe an increasing labor share coinciding also with more inequality, but this would revert in steady state. ■

A number of features are worth noting. First, this extended model generates not only an endogenous distribution of income between capital and labor, but also inequality between high-skill and low-skill workers. Moreover, this latter inequality also reflects comparative advantage — now the comparative advantage of high-skill workers relative to their low-skill brethren. This comparative advantage structure also implies that automation, by squeezing out tasks previously performed by low-skill labor increases inequality between the two types of skills. Interestingly, however, the creation of new tasks also tends to increase inequality at first because it is high-skilled labor that has a comparative advantage in the higher-index tasks (i.e., new tasks). However, given our standardization assumption, that as tasks become standardized (as more time passes from their introduction), the productivity of low-skill workers increases, the medium-term implications of automation and creation of new tasks are very different. The first, just like in the short-run, tends to increase inequality in the medium-run also. In contrast, the creation of new tasks increase inequality in the short run, but not in the medium run. In fact, low-skill workers gain relative to capital in the medium run from the creation of new tasks.

Interestingly, inequality may be particularly high following a period of adjustment in which the labor share first declines — due to increases in automation— and then recovers — due to the introduction of new complex tasks. Inequality may remain large for a while, until learning by low-skilled workers pushes their wages up.

## 6.2 Creative Destruction of Profits

In this subsection, we modify the assumption we have made on the structure of intellectual property rights, reverting to the assumption that new technologies destroy the rents/profits of existing technologies. We will show that this has little effect on the balanced growth path in our model, but makes dynamics and stability more complicated.

Formally, we follow the standard models of quality improvements such as Aghion and Howitt (1992) and Grossman and Helpman (1991), and assume that a new innovation building on the previous technology directly replaces the previous technology without making any licensing nor patent payments.

Let us now compute the behavior of  $V_N(t)$  and  $V_I(t)$  under this assumption. To do so, let us first define  $V_N(t, i)$  and  $V_I(t, i)$  as the values at time  $t$  of having introduced different technologies for the production of task  $i$  (respectively, new labor-intensive tasks and automation). As before, flow profits from introducing new technologies are given by  $\pi_I(t, i)$  and  $\pi_N(t, i)$ , respectively for automation and creation of new tasks. Since firms need not purchase production rights as before, their value functions while producing satisfy the Bellman equations:

$$\begin{aligned} r(t)V_N(t, i) - \dot{V}_N(t, i) &= \pi_N(t, i) \\ r(t)V_I(t, i) - \dot{V}_I(t, i) &= \pi_I(t, i). \end{aligned}$$

For a firm creating a labor-intensive technology for task  $i$ , let  $T^N(i)$  denote the time at which it will be replaced by a technology allowing the automation of this task. Likewise, for a firm automating task  $i$  at time  $t$ , let  $T^I(i)$  denote the time at which it will be replaced by a more complex technology using labor. Given that firms anticipate these deterministic replacement dates, their value functions also satisfy the boundary conditions  $V_N(T^N(i), i) = 0$  and  $V_I(T^I(i), i) = 0$ .

Using the Bellman equations together with the boundary conditions derived above, we find the following formula for these value functions:

$$\begin{aligned} V_N(t) = V_N(N(t), t) &= \int_t^{T^N(N(t))} e^{-\int_t^\tau r(s)ds} \psi(1 - \mu) Y(\tau) B c^u \left( \frac{W(\tau)}{\gamma(N(t))} \right)^{\zeta-\sigma}, \\ V_I(t) = V_I(I(t), t) &= \int_t^{T^I(I(t))} e^{-\int_t^\tau r(s)ds} \psi(1 - \mu) \kappa Y(\tau) B c^u \left( \min \left\{ r(\tau), \frac{w(\tau)}{\gamma(I(t))} \right\} \right)^{\zeta-\sigma} d\tau. \end{aligned}$$

In addition, for reasons that will become readily clear, we modify the evolution of the technology frontier and assume that advances in automation take the form

$$\dot{I}(t) = \kappa_I \phi(n(t)) S_I(t). \quad (29)$$

Here, the function  $\phi(n(t))$  is included and assumed to be nondecreasing to capture the possibility that automating tasks closer to the frontier (the highest available task) may be more difficult. In particular, if  $n(t)$  is close to 0, then it will be the recently invented tasks that are being automated, which may be more difficult than the case in which  $n(t)$  is close to 1. It is straightforward to verify that Proposition 4 remains unaffected if we replace (19) with (29).

**Proposition 8 (Equilibrium with creative destruction)** *Suppose that  $\sigma > \zeta$ ,  $\rho > \bar{\rho}$  and  $S < \bar{S}$  (where  $\bar{\rho}$  and  $\bar{S}$  are defined as in Proposition 4). If there is creative destruction of profits of existing technologies. Then:*

1. *There exist  $\underline{\phi} < \bar{\phi}$  such that if  $\phi(0) < \underline{\phi}$  and  $\phi(1) > \bar{\phi}$ , there exists at least one stable balanced growth path with both automation and creation of new tasks. In this balanced growth path, we have  $N(t) - I(t) = n^{DR}$ ,  $\kappa_N V_N(t) = \kappa_I \phi(n^{DR}) V_I(t)$  and  $\dot{N} = \dot{I} = \frac{\kappa_I \kappa_N \phi(n^{DR})}{\kappa_I \phi(n^{DR}) + \kappa_N} S$ . Also,  $Y, C, K$  and  $w$  grow at the constant rate  $g = A \frac{\kappa_I \kappa_N \phi(n^{DR})}{\kappa_I \phi(n^{DR}) + \kappa_N} S$ ,  $r$  is constant, and the labor share and employment are constant.*
2. *There may also exist other steady states or balanced growth paths.*

The first part of the proposition follows using analogous lines of argument to the proof of Proposition 4, with the only difference that, because of the presence of the  $\phi(n)$  in (29) in this case, the key condition determining a balanced growth path becomes  $\kappa_I \phi(n) V_I(n) = \kappa_N V_N(n)$ . In addition, in a balanced growth path, we have  $T^N(N(t)) - t = \frac{n^{DR}}{\Delta}$ , and  $T^I(I(t)) - t = \frac{1-n^{DR}}{\Delta}$ . Thus, both types of innovations are replaced at a fixed length of time, ensuring that the creative destruction of rents does not change the balance of the incentives for innovation.

The major difference with our previous analysis is that, in the presence of the creative destruction of rents,  $V_N > 0$  is increasing in  $n$  and  $V_I > 0$  is constant. Thus, if  $\phi(n)$  were constant, the intersection between these  $\kappa_N V_N(n)$  and  $\kappa_I V_I(n)$  would give an unstable balanced growth path. Economically, this is a consequence of the productivity effect combined with the fact that the creative destruction of rents implies that, in contrast to the socially planned economy and to our baseline model, when introducing new innovations, firms do not take into account the *incremental value* of automation or new labor-intensive tasks (which is to save on the more costly alternative input), but simply that the value of profits created. For example, the net present discounted value of introducing new labor-intensive tasks is increasing in  $n$  in this case, because wages always increase from both types of innovations due to the productivity effect and the opportunity cost in terms of the cost of capital is ignored by firms. Thus it is the level of wages not the difference between (ratio

of) wages to the interest rate that guide the direction of innovation, causing instability. Stability is then guaranteed by the presence of the  $\phi(n)$  function, and in particular, the condition  $\phi(0) < \underline{\phi}$  and  $\phi(1) > \bar{\phi}$  guarantees that intersections of these two curves where  $\kappa_I\phi(n)V_I(n)$  is more steep  $\kappa_NV_N(n)$  are asymptotically stable.

Therefore, this proposition thus shows that most of the qualitative results concerning the nature of the balanced growth path applied, but additional forces need to be introduced to guarantee stability.

## 7 Conclusion

As the pace of new technological advances automating tasks previously performed by labor has accelerated, concerns that these new technologies will make labor increasingly redundant have also intensified. This paper has attempted to develop a comprehensive framework in which these forces can be analyzed and contrasted with countervailing effects. At the center of our model is a task-based framework in which an endogenous set of tasks are allocated between capital and labor. Automation is modeled as (endogenous) expansion of the set of tasks that can be performed by capital, thus replacing labor in tasks that it previously controlled. The main new feature of our framework is that in addition to automation, there is another type of technological change enabling the creation of new, more complex versions of existing tasks, and it is labor that tends to have a comparative advantage in these new tasks. We fully characterize the structure of equilibrium in such a model, showing how the allocation of tasks between capital and labor is determined both by available technology and the endogenous choices of firms between capital and labor given factor prices. One attractive feature of task-based models is the link they highlight between factor prices and the range of tasks allocated to the two factors. More generally, as the equilibrium range of tasks allocated to capital increases (for example as a result of automation), the wage relative to the rental rate of capital and the share of labor in national income decline, and the equilibrium wage rate may also decline. Conversely, as the equilibrium range of tasks allocated to labor increases, the opposite result obtains. In our model, we also make the supply of labor potentially elastic by introducing a quasi-labor supply curve (which also implies that equilibrium wage may be greater than the opportunity cost of labor). Given this relationship, automation also tends to reduce employment, while the creation of new tasks increase employment. These results highlight that, while both types of technological changes underpin economic growth, they have very different implications for the factor distribution of income and also for employment.

Our full model endogenizes the direction of research and development towards automation and the creation of new complex tasks, showing how this framework naturally leads to a (unique) balanced growth path in which both types of innovations go hand-in-hand. Moreover, the dynamic

equilibrium is also unique and, starting from any initial conditions, converges to the (unique) balanced growth path. Underpinning this global stability result is the impact of relative factor prices on the direction of technological change. The task-based framework (differently from the standard models of directed technological change which are based on factor-augmenting technologies) implies that as a factor becomes cheaper, this not only expands the range of tasks allocated to this factor in equilibrium, but also generates stronger incentives for the type of technological change working this factor. These economic incentives then imply that automation, by reducing wages relative to the rental rate of capital, and encourages the creation of new labor-intensive tasks, generating a powerful self-correcting force towards stability.

Though market forces ensure the stability of the balanced growth path, they do not necessarily generate the efficient composition of technology. In particular, the presence of the quasi-labor supply, by creating a wedge between the market wage and the opportunity cost of labor, creates an equilibrium distortion in the type of new technologies that are created. Firms tend to have an excessive bias for automation, because they derive profits by replacing labor by cheaper capital. The social planner, on the other hand, recognizes that part of the wage is rent captured by workers, and has weaker incentives to replace labor by capital. Put differently, the social planner prefers to choose a different composition of technologies (specifically, biased towards the creation of new tasks and away from automation) because she would like to expand employment, which generates greater rents for workers along the equilibrium path.

In addition to claims about automation leading to the demise of labor, several commentators are concerned about the inequality implications of automation the new technologies. In one of our extensions, we have studied this question by introducing a distinction between low-skilled and high-skilled labor, where the latter has a comparative advantage in producing with newer technologies. This structure implies that both automation, which squeezes out tasks previously performed by low-skill labor, and the creation of new tasks, which directly benefits high-skill labor, will increase inequality between the two labor types. Nevertheless, we show that the medium-term implications of creation of new tasks could be very different, because these tasks are later standardized and used by low-skill labor. As a result of this effect, we show that there exists a unique balance growth path in which not only the factor distribution of income (between capital and labor) but also inequality between the two skill types is constant.

Our second extension reintroduces the creative destruction of profits of existing technologies by new innovations, which was eliminated by assuming that new technologies have to buy the patents from the technologies on which they are building. This extension shows that the presence of this creative destruction effect has little impact on the balance growth path, but complicates and enriches dynamics.

We consider our paper to be a first step towards a systematic investigation of different types of technological changes that impact capital and labor differentially. Several areas of research appear fruitful based on this first step. First, rather than the reduced-form quasi-labor supply curve, a richer model of the labor market based on search and matching can be introduced and combined with this task-based framework. Such a model is developed in our companion paper, Acemoglu and Restrepo (2015). Second, there may be major differences in the ability of technology to automate and also to create new tasks across industries (e.g., Polanyi, 1966, Autor, Levy and Murnane, 2003). An interesting step is to construct realistic models in which the sectoral composition of the allocation of capital and labor and technological change evolve endogenously and subject to industry-level ecological and automation constraints. Third and perhaps most importantly, our model highlights the need for additional empirical evidence on how automation takes place and incentives for automation and creation of new tasks respond to incentives and policies. One interesting direction would be to construct measures of automation and creation of new tasks, potentially at the industry level, and then exploit industry-level variation in wages and institutional restrictions on capital-labor substitution on technology choices and innovation.

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## Appendix A: Details of the Empirical Analysis

In this part of the Appendix, we provide information about the data and samples used in constructing Figure 2 and provide a regression analysis documenting the robustness of the pattern illustrated in that figure.

**Data:** We use demographic and employment data from the U.S. Censuses for 1980, 1990 and 2000 and the American Community Survey for 2007, and aggregate all data to the 330 consistently defined occupational groups proposed by David Dorn (see <http://www.ddorn.net/data.htm>).

The measure of new tasks and jobs is from Lin (2011), who uses new occupational titles added to new waves of the Dictionary of Occupational Titles to create measures of new jobs in each census occupational group for 1980 and 1990. He also compares the 1990 Census Index of Occupations with its 2000 counterpart, as well as technical documentation provided by the Census to determine the share of new job classes in each occupational category of the 2000 Census. The data are available from his website <https://sites.google.com/site/jeffrlin/newwork>.

**Analysis:** To document the role of novel tasks and jobs, we estimate the regression

$$\ln E_{it+10} - \ln E_{it} = \beta N_{it} + \delta_t + \Gamma_t X_{it} + \varepsilon_{it}. \quad (\text{A1})$$

Here, the dependent variable is the percent change in employment from year  $t$  to  $t + 10$ , in each occupational category  $i$ . We stack this model using data for  $t = 1980, 1990, 2000$ . For  $t = 2000$ , we use the change from 2000 to 2007 as dependent variable and scale it to match a 10-year change. In all regressions we include a full set of decadal effects  $\delta_t$ , and in some models we also control for differential decadal trends by occupational characteristics,  $X_{it}$ . These characteristics include the share of employees in 5-year age brackets, from different races (black, hispanic) and origins (foreigner), and that are male. These flexibly control for demographic changes that may affect labor supply and for potential differential sectors of demand.  $\varepsilon_{it}$  is an error term that may be correlated over time within occupational groups.

The coefficient of interest is  $\beta$ , which represents the additional employment growth in occupations with more novel tasks and jobs,  $N_{it}$ . Throughout, all standard errors are robust against arbitrary heteroscedasticity and serial correlation.

Panel A in Table A1 presents estimates of equation A1. Column 1 contains no additional covariates (the number of observations in this column is 328, including all occupational groups for which we have data from Lin). Our estimates indicate that occupational groups with 10 percentage point more novel jobs at the beginning of each decade grow 5.2% faster (standard error= 1.3%). If occupational groups with more novel jobs did not grow any faster than the benchmark category with no novel jobs, employment growth for each decade from 1980 to 2007 would have been, on

average, 2.7% instead of the actual 5.7%, implying that approximately 3% of the 5.7% growth is accounted for by noble jobs and tasks as reported at the bottom.

Table A1: Differential employment growth in occupational groups with more new jobs and tasks

	(1)	(2)	(3)	(4)	(5)	(6)	Weighted by size
Panel A: Stacked differences over decades.							
Share of novel tasks and jobs at start of decade	0.522*** (0.131)	0.584*** (0.130)	0.495*** (0.140)	0.381*** (0.144)	0.199 (0.176)	0.441*** (0.140)	
log of employment at start of decade		-0.035*** (0.012)	-0.048*** (0.014)	-0.044*** (0.013)	-0.000 (0.011)	0.004 (0.010)	
Average years of education at start of decade				10.574*** (1.841)	8.494*** (1.828)	8.356*** (1.795)	
R-squared	0.03	0.04	0.11	0.15	0.13	0.15	
Observations	986	986	986	986	986	977	
Employment growth by decade in p.p.	5.7	5.7	5.7	5.7	5.7	5.7	
Contribution of novel tasks and jobs	3.0	3.3	2.8	2.2	1.1	2.5	
Panel B: Long differences from 1980-2007.							
Share of novel tasks and jobs in 1980	1.241*** (0.391)	1.401*** (0.343)	1.452*** (0.347)	1.153*** (0.322)	0.056 (0.492)	0.516 (0.407)	
log of employment in 1980		-0.155*** (0.031)	-0.193*** (0.035)	-0.164*** (0.034)	-0.048 (0.031)	-0.038 (0.030)	
Average years of education in 1980				23.943*** (4.128)	16.357*** (4.390)	16.067*** (4.354)	
R-squared	0.02	0.08	0.17	0.26	0.17	0.17	
Observations	328	328	328	328	328	325	
Employment growth from 1980-2007 in p.p.	15.8	15.8	15.8	15.8	15.8	15.8	
Contribution of novel tasks and jobs	7.0	7.9	8.2	6.5	0.3	2.9	
<i>Covariates:</i>							
Decade fixed effects	✓	✓	✓	✓	✓	✓	
Demographics × decade effects			✓	✓	✓	✓	

*Notes:* The table presents 10-years stacked-differences estimates (Panel A) and long-differences estimates (Panel B) of the share of novel tasks and jobs in an occupational group on subsequent employment growth. The bottom row in each panel reports the observed growth and the share explained by growth in occupations with more novel tasks and jobs. Additional covariates that are not reported are indicated in the bottom of the table. In Column 5 we re-weight the data using the share of employment in each occupation, and in Column 6 we exclude three large employment categories that are outliers in the model of column 5. These include office supervisors, office clerks, and production supervisors. Standard errors robust to heteroskedasticity and serial correlation within occupations are presented in parentheses.

In column 2 we control for the log of employment at the beginning of the decade (year  $t$ ). The coefficient of interest increases slightly to 0.584 and continues to be precisely estimated. The log of employment at year  $t$  appears with a negative coefficient, which indicates that smaller occupations tend to grow more over time and suggests that employment growth is not driven by already well-established occupations employing a large share of the population. In any case, the quantitative

contribution of new tasks and jobs remains very similar to column 1, increasing slightly to 3.3%.

In column 3 we control for the demographic covariates described above, with little effect on the qualitative or quantitative pattern of results. In column 4, we also control for the average education of employees at the beginning the decade. Consistent with the patterns documented in Figure 8, occupations with more skilled/educated workers tend to grow more rapidly. This control also reduces the quantitative magnitudes of the share of novel tasks and jobs, which nevertheless remains highly significant. The contribution of these novel tasks and jobs is now estimated at 2.2% out of the 5.7% average decadal growth between 1980 and 2007.

Column 5 repeats the specification of column 4, but this time using the share of employment in each occupation as weights. This weakens the relationship of interest, and the share of novel tasks and jobs is no longer statistically significant. However, this lack of significance is driven by a few large occupations that are outliers in the estimated relationship. (In contrast, there are no major outliers in the unweighted regressions reported in columns 1-4). These outliers include office supervisors, office clerks, and production supervisors, three occupational groups which combined employed about 4 million workers in 1980 and have been on the decline since then. Though these occupational groups introduced a significant number of novel jobs and tasks in 1980, they shed a large amount of workers from 1980 to 1990. In column 6, we exclude these occupational groups from our analysis, and obtain a similar pattern to column 4.

Finally, in Panel B, we present regressions that focus on long differences between 1980 and 2007. The overall patterns are very similar, and now the contribution of novel tasks and jobs to the 15.8% growth in employment between 1980 and 2007 is between 6.5 and 8.2%.

## Appendix B: Omitted Proofs and Additional Results

### Proof of Proposition 1

We proceed in three steps: first, we show that a given  $I^*$ ,  $N$  and  $K$  uniquely determine  $r, W, Y$ . This allows us to define the function  $\omega(I^*, N, K)$  introduced in the text as the relative demand for labor. Second, we show that  $\omega(I^*, N, K)$  is decreasing in  $I^*$  near any equilibrium. Third, we show that  $\min\{I, \tilde{I}\}$  is weakly increasing in  $\omega$  and conclude that there is a unique pair  $\{\omega^*, I^*\}$  such that  $I^* = \min\{I, \tilde{I}\}$  and  $\omega^* = \omega(I^*, N, K)$ . This pair uniquely determines the equilibrium.

Before proceeding with the proof, we establish a useful lemma, which guarantees factor demands are downward sloping for a given output.

**Lemma A1** *For all  $x > 0$ , we have that  $c^u(x)^{\zeta-\sigma}x^{-\zeta}$  is decreasing in  $x$  and converges to 0 when  $x \rightarrow \infty$  and to  $\infty$  when  $x \rightarrow 0$ .*

**Proof.** Differentiating the expression, we find that the elasticity of  $c^u(x)^{\zeta-\sigma}x^{-\zeta}$  with respect to  $x$

is:

$$\frac{x^{1-\zeta}}{x^{1-\zeta} + \left(\frac{\eta}{1-\eta}\right)^\zeta \psi^{1-\zeta}} (\zeta - \sigma) - \zeta < 0,$$

thus establishing the first part of the result.

The fact that this elasticity is negative implies that  $c^u(x)^{\zeta-\sigma} x^{-\zeta}$  is decreasing in  $x$  and converges to 0 when  $x \rightarrow \infty$  and to  $\infty$  when  $x \rightarrow 0$ . ■

We now present the three steps mentioned above in detail.

- **Step 1:** Take  $I^*, N, K$  as given (with  $I^* \in (N-1, N)$ ). Then,  $r, W, Y$  satisfy the system of equations given by the capital and labor demand (equations (7) and (8) in the main text) and the ideal price index (equation (9) in the main text).

Dividing the labor and capital demand equations yields:

$$\frac{\int_{I^*}^N \gamma(i)^{\zeta-1} c^u \left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma} W^{-\zeta} di}{L^s \left(\frac{W}{rK}\right) (I^* - N + 1) c^u(r)^{\zeta-\sigma} r^{-\zeta}} = \frac{1}{K} \quad (\text{A2})$$

This equation gives an upward sloping locus between  $r$  and  $W$ , since lemma A1 implies the left-hand side is decreasing in  $W$  and increasing in  $r$ .

On the other hand, the ideal price index condition (equation (9)) gives a downward sloping locus between  $r$  and  $W$ .

These observations imply there is at most one intersection between the locus determined by equation (A2) and the ideal price index condition in the  $(W, r)$  space. Thus, there is at most one equilibrium presented diagrammatically in Figure A1.

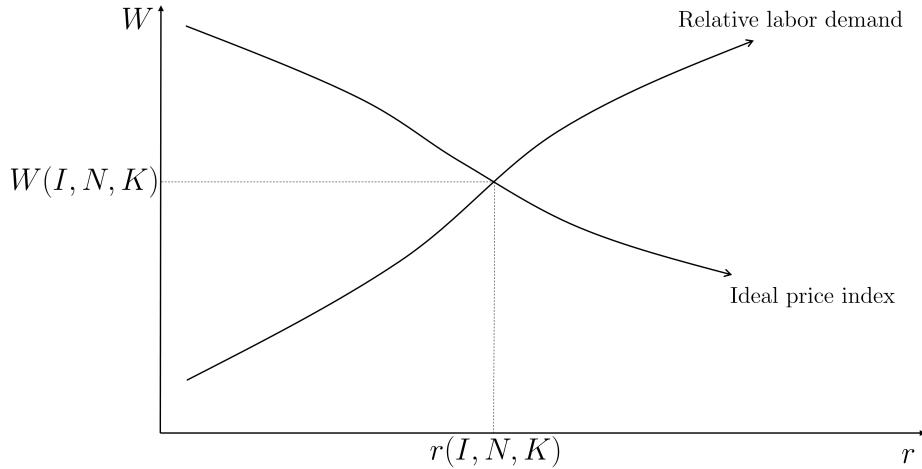


Figure A1: Construction of function  $\omega(I^*, N, K)$ .

To prove the existence, note that as  $W \rightarrow 0$ , the ideal price index condition requires a bounded interest rate  $r_0 > 0$  to be valid. However, equation (A2) requires  $r \rightarrow 0$  to hold, since the demand for labor grows without bound (as stated in Lemma A1). Moreover, as  $W \rightarrow \infty$ , equation (A2) requires  $r \rightarrow \infty$  as well, since the demand for labor becomes arbitrarily small (see again Lemma A1). This implies that the relative demand curve and the ideal price index condition intercept at a unique point  $(W, r)$  by the intermediate value theorem.

This also implies the equilibrium object  $\omega(I^*, N, K) = \frac{W}{rK}$  is well defined.

- **Step 2:** Differentiating the ideal price index condition and equation (A2), we obtain:

$$d \ln \left( \frac{W}{r} \right) \propto -dI \left( \frac{\gamma(I)^{\zeta-1} c^u \left( \frac{W}{\gamma(I)} \right)^{\zeta-\sigma}}{\int_{I^*}^N \gamma(i)^{\zeta-1} c^u \left( \frac{W}{\gamma(i)} \right)^{\zeta-\sigma} di} + \frac{1}{(I^* - N + 1)} + \frac{\sigma - \zeta}{1 - \sigma} \left( c^u \left( \frac{W}{\gamma(I)} \right)^{1-\sigma} - c^u(r)^{1-\sigma} \right) \right).$$

Since  $\frac{W}{\gamma(I)} \geq r$  near any equilibrium, automation reduces  $W/r$ , and hence  $\omega$  as wanted.

- **Step 3:** We have that  $\gamma(\tilde{I}) = \omega K$ . Thus,  $\tilde{I}$  is increasing in  $\omega$ , and so is  $\min\{I, \tilde{I}\}$ .

Therefore, there is at most a pair  $(\omega, I^*)$  solving  $\omega = \omega(I^*, N, K)$  and  $I^* = \min\{I, \tilde{I}\}$ , as plotted in Figure 4, because  $\omega(I^*, N, K)$  is decreasing at any interception.

To prove such pair exists, take  $I^* \rightarrow N - 1$ . Then, the locus for  $I^* = \min\{I, \tilde{I}\}$ , gives  $\omega \rightarrow \gamma(N - 1)/K$ , while the locus for  $\omega = \omega(I^*, N, K)$  gives  $\omega = \infty$  since  $r \rightarrow 0$ . Likewise, take  $I^* \rightarrow N$ . Then, the locus for  $I^* = \min\{I, \tilde{I}\}$ , gives  $\omega \rightarrow \gamma(N)/K$ , while the locus for  $\omega = \omega(I^*, N, K)$  gives  $\omega = 0$ , since wages go to zero. Thus, both curves must cross at some unique point by the intermediate value theorem, establishing Proposition 1.

## Proof of Proposition 2

We proceed in four steps: First we prove the comparative statics results for  $I$ , then for  $N$  and finally for  $K$ . In the last step we prove that all the above changes move total employment and the labor share in the same direction.

- **Comparative statics for  $I$ :** Clearly  $I$  is only binding when  $I^* = I$ . In this case, an increase in  $I$  shifts the curve  $I^* = \min\{I, \tilde{I}\}$  to the right in Figure 4, increasing  $I^*$  in the same amount and reducing  $\omega$  as stated in the proposition.
- **Comparative statics for  $N$ :** The same argument used in Step 2 in the proof of Proposition 1 establishes that  $\omega(I^*, N, K)$  increases with  $N$  near the equilibrium. Thus,  $N$  shifts the locus for  $\omega = \omega(I^*, N, K)$  upwards in Figure 4, weakly increasing  $I^*$  and always increasing  $\omega$ .

It also follows that  $N$  increases  $W/r$  as stated in the proposition.

When  $I^* = I$ , a change in  $N$  only has a direct effect on  $\omega$  since it does not change the allocation of factors. Thus

$$\frac{d \ln \omega}{dN} = \frac{d \ln(W/r)}{dN} = \frac{\partial \ln(W/r)}{\partial N} > 0,$$

and its total effect equal its partial effect.

When  $I^* = \tilde{I}$ , a change in  $N$  also increases  $I^*$  by  $\frac{1}{\varepsilon} d \ln \omega$  (from equation (6)). Therefore, the total change in  $\omega$  is given by:

$$d \ln \omega = \frac{\partial \ln \omega}{\partial N} dN + \frac{\partial \ln \omega}{\partial I^*} \frac{1}{\varepsilon} d \ln \omega.$$

Solving for  $d \ln \omega$  yields:

$$\frac{d \ln \omega}{dN} = \frac{\frac{\partial \ln \omega}{\partial N}}{1 - \frac{1}{\varepsilon} \frac{\partial \ln \omega}{\partial I^*}}.$$

- **Comparative statics for  $K$ :** The definition of  $\omega$  gives the identity:

$$\frac{d \ln \omega}{d \ln K} = \frac{d \ln(W/r)}{d \ln K} - 1.$$

Consider an increase in  $K$  holding  $\omega$  fixed — so that we are computing the effect of  $K$  on  $W/r$ . The increase shifts the locus of points  $(r, W)$  satisfying equation (A2) upwards, so that a given  $r$  requires a higher wage to be consistent with market clearing. Therefore,  $K$  reduces  $r$  and increases  $W$ , and

$$\frac{\partial \ln(W/r)}{\partial \ln K} = \frac{1}{\sigma_{SR}} > 0.$$

When  $I^* = I$ , the partial effect of  $K$  equals the total effect since it does not affect  $I^*$ . However, when  $I^* = \tilde{I}$ , we have

$$d \ln(W/r) = \frac{\partial \ln(W/r)}{\partial \ln K} d \ln K + \frac{\partial \ln(W/r)}{\partial \ln K} \frac{1}{\varepsilon} d \ln(W/r).$$

Solving for  $d \ln W/r$  yields:

$$\frac{d \ln(W/r)}{d \ln K} = \frac{\frac{\partial \ln(W/r)}{\partial \ln K}}{1 - \frac{1}{\varepsilon} \frac{\partial \ln(W/r)}{\partial I^*}},$$

as stated in the text.

### Proof of Proposition 3

To prove this proposition, we start with a lemma. Let  $w^N(n) = W/\gamma(N)$  and  $w^I(n) = W/\gamma(I^*)$ . In the long run, these are defined as a function of  $n = N - I$  since capital adjusts to keep the interest rate constant.

**Lemma A2 (Asymptotic behavior of wages  $w^N, w^I$ )** Assume that for some  $x \in [0, 1]$  we have:

$$w^I(x) > \rho + \delta + \theta g > w^N(x). \quad (\text{A3})$$

Then, there exists a positive threshold  $\bar{\rho}$  such that:

1. For  $\rho > \bar{\rho}$ , there exists  $\bar{n} \in (0, 1)$  such that:

- for  $n \geq \bar{n}$ , we have  $I^* = I$ , and for  $n < \bar{n}$ , we have  $I^* < I$ .
- for  $n \geq \bar{n}$ ,  $w^N(n)$  is increasing and  $w^I(n)$  decreasing in  $n$ . Both wages are constant for  $n < \bar{n}$ .

2. For  $\rho \leq \bar{\rho}$ , there exists a different threshold  $\tilde{n} \in [0, 1)$  such that:

- for  $n \geq \tilde{n}$ , both technologies are used, while for  $n < \tilde{n}$ , firms do not create or use new tasks (because labor is not productive or cheap enough compared to capital).
- for  $n \geq \tilde{n}$ ,  $w^N(n)$  is increasing and  $w^I(n)$  decreasing in  $n$ . Both wages are decreasing in  $n$  for  $n < \tilde{n}$ .

**Proof.** We proceed in several steps. First, we show that for  $n \geq x$ , we have that  $w^I(n)$  is increasing and  $w^N(n)$  decreasing in  $n$ .

This claim follows from our discussion of the productivity effect in Subsection 3.2, and the comparative statics outlined there.

Second, we establish the existence of  $\bar{n}$  or  $\tilde{n}$ . To do so, consider the behavior of  $w^I(n)$  is increasing and  $w^N(n)$  for  $n < x$ . So long as

$$w^I(n) > \rho + \delta + \theta g > w^N(n), \quad (\text{A4})$$

we have that  $w^I(n)$  is increasing and  $w^N(n)$  decreasing in  $n$ .

However, equation (A4) cannot hold for all  $n \in [0, x]$ . If it did, we would have

$$w^I(0) > \rho + \delta + \theta g > w^N(0), \quad (\text{A5})$$

which creates a contradiction, since in this case  $w^I(0) = \frac{W}{\gamma(0)} = w^N(0)$ .

Thus, there exists  $\bar{n}$  such that

$$w^I(\bar{n}) = \rho + \delta + \theta g, \quad (\text{A6})$$

or there exists  $\tilde{n}$  such that

$$w^N(\tilde{n}) = \rho + \delta + \theta g. \quad (\text{A7})$$

Now, we show that only one of these cases may occur, and that  $\rho$  determines which case it is.

First, suppose that as we move from  $x$  to its left, we reach  $\bar{n}$  first. At this point, firms will not use capital in newly automated tasks and  $I^* < I$ . Further increases in  $I$ —or reductions in  $n$ —do

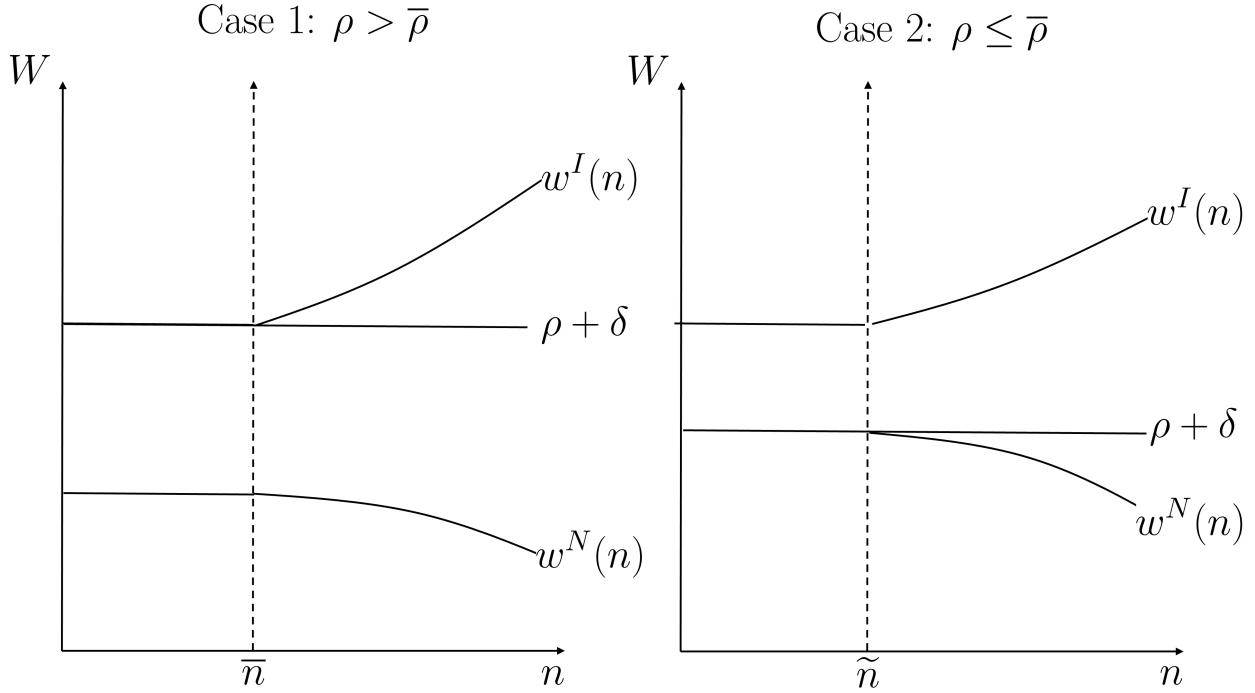


Figure A2: Behavior of unit costs of labor with respect to changes in  $n = N - I$  in steady state.

not change the equilibrium allocation. Thus, for  $n < \bar{n}$ ,  $w^I(\bar{n})$  and  $w^N(n)$  are constant. Thus, in this case, wages behave as shown in the left panel of Figure A2.

Now, suppose that as we move from  $x$  to its left, we reach  $\tilde{n}$  first. For smaller  $n$ , firms will stop using labor in newly created tasks, since labor productivity is not large enough. For small  $n$ , an increase in  $N$ —or in  $n$ —actually causes a decline in productivity, since capital is cheaper than labor, even for the most complex tasks. Thus, for  $n < \tilde{n}$ ,  $w^I(\bar{n})$  and  $w^N(n)$  are decreasing functions. In this case, wages behave as shown in the right panel of Figure A2.

These observations imply that we always have exactly one of these cases. Note that as  $\rho$  increases, so does the interest rate in steady state and we eventually move from the second case to the first one. The value at which the switching occurs defines the threshold  $\bar{\rho}$ , which satisfies:

$$w^I(0) = \bar{\rho} + \delta + \theta g. \quad (\text{A8})$$

This observation concludes the proof. ■

The lemma establishes the first part of Proposition 3 as a corollary.

For the “only if” part of numeral 2, recall that in a balanced growth path,  $Y, C, K$  and  $w$  grow at a constant rate  $g$ . Therefore,  $y, c, k$  and  $w$  also grow at some constant rate  $\tilde{g}$  and  $r$  is constant. We will show that  $\tilde{g} = 0$ .

First, suppose by way of contradiction that  $\tilde{g} < 0$ . Then  $w$  is eventually below  $r$ , contradicting the fact that  $w \geq r$  (since otherwise, task  $I^*$  would be cheaper to produce with labor, contradicting

the definition of  $I^*$ ).

Second, suppose by way of contradiction that  $\tilde{g} > 0$ . This implies that, eventually,  $\frac{w(t)}{\gamma(n(t))} \geq \frac{w(t)}{\gamma(1)} > r$  (recall  $r$  is fixed). When  $\frac{w(t)}{\gamma(n(t))} > r$  and  $r$  is fixed, the ideal price index condition requires  $n(t)$  to decrease over time in order to keep  $w(t)$  increasing. However, since  $n(t) \geq 0$  this cannot go on indefinitely and  $n(t)$  must reach zero. At this point, all tasks are automated and use capital, so the economy converges to an  $AK$  economy. Thus, labor is not used along the equilibrium and  $w = 0$ . However, our assumption that  $L(0) > 0$  rules out this equilibrium, contradicting our initial assumption that  $\tilde{g} > 0$ .

These contradictions imply  $\tilde{g} = 0$ , as wanted. In this case,  $w$  is constant, and the ideal price index condition implies  $n(t) = n \in (0, 1)$  is fixed. Here,  $n(t) = 0$  is ruled out as above by noting that  $L(0) > 0$ . Also,  $n(t) = 1$  requires  $r(t) = 0$ , but  $r(t) = \rho + \delta + \theta g > 0$ . Finally, since  $n(t) = n \in (0, 1)$ ,  $\dot{N} = \dot{I}$  as wanted, thus establishing the “only if” direction.

## Proof of Proposition 4

We start with a more general version of Proposition 4

**Proposition A1** *There exists thresholds  $\bar{S}$ ,  $\bar{\rho}$  and  $\underline{\kappa} \geq \underline{\kappa}$  such that for  $S < \bar{S}$ , the following are true:*

1. For  $\rho > \bar{\rho}$ :

- If  $\frac{\kappa_I}{\kappa_N} > \bar{\kappa}$ , there is a unique balanced growth path, where both technologies are immediately used (so that  $I^* = I$ ) and  $\kappa_N V_N = \kappa_I V_I > 0$ . Moreover this balanced growth path is locally asymptotically stable and globally stable when  $\theta \rightarrow 0$
- If  $\frac{\kappa_I}{\kappa_N} < \underline{\kappa}$ , we have that  $\kappa_N V_N > \kappa_I V_I$  and all scientists are allocated to create new labor-intensive tasks, and thus  $n \rightarrow 1$  and the economy converges asymptotically to an  $A(t)L$  economy which only uses labor to produce the final good and labor productivity increases at a constant rate over time.
- For  $\frac{\kappa_I}{\kappa_N} \in [\underline{\kappa}, \bar{\kappa}]$ , there may be multiple balanced growth paths, and the balance growth path with the lowest  $n$  is locally asymptotically stable.

2. For  $\rho < \bar{\rho}$ , starting from a small  $n$ , the economy admits a unique equilibrium path in which  $\kappa_N V_N < \kappa_I V_I$  and it converges to an  $AK$  economy, where the economy grows at a constant rate by accumulating more capital.

To prove this proposition, we will use Lemma A2 as well as the next result:

**Lemma A3 (Asymptotic behavior of value functions  $V_N, V_I$ )** *Suppose  $\sigma > \zeta$ ,  $S < \bar{S}$  and that the conditions required in Lemma A2 hold.*

*Let  $V_N(n)$  and  $V_I(n)$  be the value functions for the creation of new tasks and the automation of existing tasks.*

1. For  $\rho > \bar{\rho}$ :

- for  $n < \bar{n}$ , we have  $V_N > 0$  and  $V_I = 0$ .
- for  $n \geq \bar{n}$ ,  $V_N$  and  $V_I$  are strictly increasing.

2. For  $\rho \leq \bar{\rho}$ :

- for  $n < \tilde{n}$ , we have  $V_I > 0$  and  $V_N \leq 0$ .
- for  $n \geq \tilde{n}$ , both  $V_N$  and  $V_I$  are strictly increasing.

**Proof.** When  $S < \bar{S}$  is small,  $g$  is small, enabling us to write:

$$v_N(n) = \frac{V_N(n)}{Y(n)} = \frac{1-\mu}{\rho} \left( \frac{\eta}{1-\eta} \right)^\zeta \left[ c^u (w^N(n))^{\zeta-\sigma} - c^u (\rho + \delta)^{\zeta-\sigma} \right],$$

$$v_I(n) = \frac{V_I(n)}{Y(n)} = \frac{1-\mu}{\rho} \left( \frac{\eta}{1-\eta} \right)^\zeta \left[ c^u (\rho + \delta)^{\zeta-\sigma} - c^u (w^I(n))^{\zeta-\sigma} \right].$$

Thus, the value functions only depend on the unit cost of labor  $w^N(n)$  and  $w^I(n)$ , and on the wage which is pinned down by  $\rho + \delta$ .

The comparative statics now follow as a corollary of Lemma A2. The implied behavior of the value functions is depicted in Figure A3.

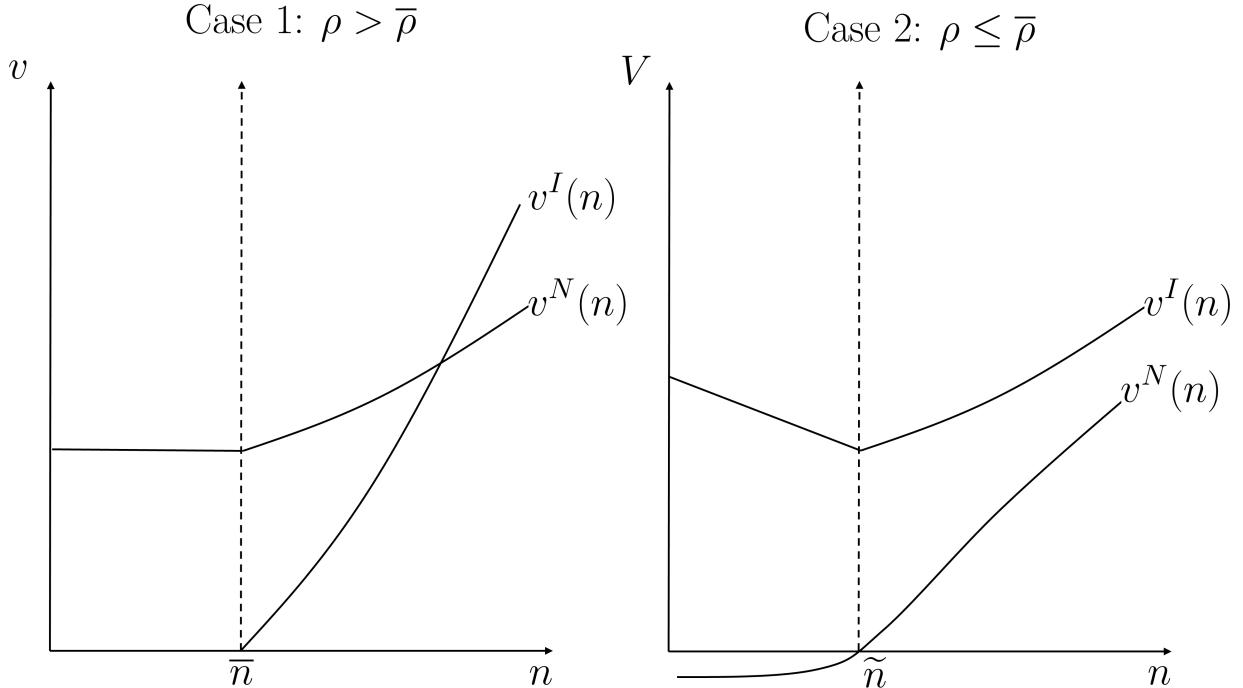


Figure A3: Behavior of value functions in steady state with respect to changes in  $n = N - I$ . ■

Using these lemmas, we are in a position to prove Proposition A1. We start by characterizing the existence of a BGP in all of the cases described in the Proposition.

The existence of a balanced growth path follows by noting that, as stated in Proposition 3, it emerges if and only if  $\dot{I} = \dot{N}$ , and  $n(t) = n^D$ . Since all scientists are allocated to developing one of the two available technologies, we must have:

$$S_I = \frac{\kappa_N}{\kappa_I + \kappa_N} S \quad S_N = \frac{\kappa_I}{\kappa_I + \kappa_N} S,$$

and the growth rate of the economy is therefore  $g = \frac{\kappa_N \kappa_I}{\kappa_I + \kappa_N} S$ .

In the balanced growth path, the Euler equation, (13), implies the interest rate equals  $r = \rho + \delta + \theta g$ , and wages are then given by  $w^N(n)$  and  $w^I(n)$ . Moreover, when  $\rho > A \frac{\kappa_I \kappa_N}{\kappa_I + \kappa_N} S(1 - \theta)$ , the transversality condition will be automatically satisfied.

The key equilibrium condition is for research and both types of technologies to be profitable (so that  $S_I > 0$  and  $S_N > 0$ ):

$$\kappa_I v_I(n) = \kappa_N v_N(n),$$

The existence of a BGP boils down to a solution  $n^D$  to this equation.

If  $\rho > \bar{\rho}$ , Lemma A3 implies that at  $n = \bar{n}$ , we always have

$$\kappa_I v_I(\bar{n}) < \kappa_N v_N(\bar{n}).$$

Intuitively, once many tasks are automated, firms would start using labor instead of capital and will make further automation unprofitable.

Since  $v_I(n), v_N(n) > 0$  for  $n > \bar{n}$ , as the ratio  $\frac{\kappa_I}{\kappa_N}$  increases—starting from zero, the curves  $\kappa_I v_I(n) = \kappa_N v_N(n)$  eventually cut each other at an interior point  $n^D \in (\bar{n}, 1)$ . This first interception defines the threshold  $\underline{\kappa}$ . As  $\frac{\kappa_I}{\kappa_N}$  keeps increasing, it reaches a point at which  $\kappa_I v_I(n) = \kappa_N v_N(n)$  cut each other at a *unique* point. This defines the threshold  $\bar{\kappa}$ . In between  $\underline{\kappa}$  and  $\bar{\kappa}$ , there may be several interceptions, each of which defines a different BGP.

For  $\kappa < \underline{\kappa}$ , we have that  $\kappa_I v_I < \kappa_N v_N$  throughout, and the economy only creates new tasks and does not automate. In the long-run, the economy converges to an  $A(t)L$  economy in which  $A(t)$  grows over time due to the more complex varieties introduced.

If  $\rho \leq \bar{\rho}$ , Lemma A3 implies that at  $n = \tilde{n}$ , we always have

$$\kappa_I v_I(\bar{n}) > \kappa_N v_N(\bar{n}).$$

In this case, capital is too cheap. The next task new task that could be created is not as productive as to make its production with labor more profitable than with capital. Thus, firms do not have incentives to create new tasks.

In this case, independently of the value of  $\frac{\kappa_I}{\kappa_N}$ , there is always a stable equilibrium in which  $\kappa_I v_I > \kappa_N v_N$  throughout. In this equilibrium, the economy automates all jobs. Labor is only used in the production of the most complex task, and its wage is fully pinned down by the rental rate

of capital which determines the price of all the tasks. In the long-run, the economy converges to an  $AK$  economy and does not introduce additional innovations.

The last step in our analysis is to establish the stability of a balanced growth path in some of the cases covered above. The precise statement we prove is summarized in the following lemma:

**Lemma A4** Suppose  $S < \bar{S}$ . Consider a balanced growth path characterized by  $n^D \in (0, 1)$ . If  $\kappa_I v'_I(n^D) > \kappa_N v'_N(n^D)$ , the balanced growth path is locally saddle path stable. If in addition,  $\kappa_I v_I(n) - \kappa_N v_N(n)$  crosses zero only at  $n^D$  and  $\theta \rightarrow 0$ , the equilibrium is globally asymptotically stable.

**Proof.** We start by providing a proof for the global stability of the balanced growth path when  $\theta \rightarrow 0$ . We assume  $S \rightarrow 0$ . By continuity arguments, the results generalize to the case in which  $S < \bar{S}$ .

In this case, capital adjusts immediately and its equilibrium stock only depends on  $n$ , which becomes the unique state variable of the model. The rental rate of capital is fixed at  $r = \rho + \delta$  at each point in time, and wages are given by  $w^I(n)$  and  $w^N(n)$ .

Define  $v = \kappa_I v_I - \kappa_N v_N$ .

Starting from any  $n(0)$  the equilibrium path with endogenous technology is given by a tuple  $\{n(t), S_I(t)\}_{t=0}^\infty$  such that:

- The evolution of the state variable is given by

$$\dot{n}(t) = \kappa_N S - (\kappa_N + \kappa_I) S_I(t).$$

- The allocation of scientists satisfies:

$$S_I(t) = \begin{cases} 0 & \text{if } v < 0 \\ \in [0, S] & \text{if } v = 0 \\ S & \text{if } v > 0 \end{cases},$$

with  $v$  satisfying the forward looking differential equation:

$$\begin{aligned} rv - \dot{v} &= \frac{1-\mu}{\rho} \left( \frac{\eta}{1-\eta} \right)^\zeta \kappa_I \left( c^u (\rho + \delta)^{\zeta-\sigma} - c^u (w^I(n))^{\zeta-\sigma} \right) - \\ &\quad \frac{1-\mu}{\rho} \left( \frac{\eta}{1-\eta} \right)^\zeta \kappa_N \left( c^u (w^N(n))^{\zeta-\sigma} - c^u (\rho + \delta)^{\zeta-\sigma} \right). \end{aligned}$$

This expression is derived for the limit in which  $g \rightarrow 0$ , which is implied by our assumption that  $S \rightarrow 0$ .

Since,  $\kappa_I v_I(n) - \kappa_N v_N(n)$  crosses zero only at  $n^D$ , this is the unique BGP and the equilibrium can be represented graphically as we do in Figure 7. We now prove it is globally stable. Figure A4 presents the phase diagram of the system in  $(v, n)$ . Importantly, the locus for  $\dot{v} = 0$  crosses  $v = 0$

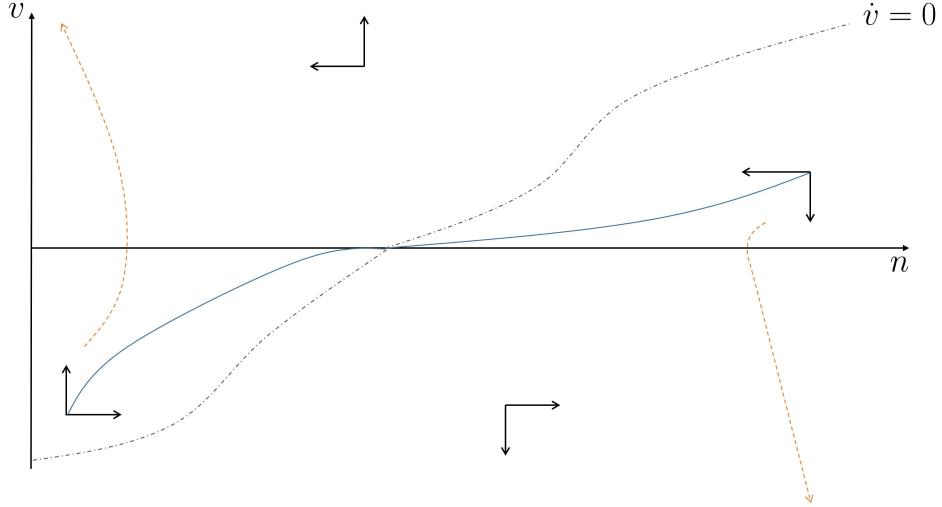


Figure A4: Phase diagram and global saddle path stability when  $\theta = 0$ .

at  $n^D$  from below only once. This follows from the fact that  $\kappa_I v'_I(D) > \kappa_N v'_N(D)$ . Thus,  $n^D$  is saddle path stable, and for each  $n(0)$  there is a unique  $v(0)$  in the stable arm of the system.

In order to show all equilibria must be along the stable arm, we need to rule out other paths. From the figure it is clear that either the equilibrium settles at  $n^D$ , or it reaches the region with  $\dot{v} > 0$  and  $\dot{n} < 0$ , or the region with  $\dot{v} < 0$  and  $\dot{n} > 0$ . In the first case,  $v$  is strictly increasing and  $n$  is strictly decreasing, and hence there are no interior limit points. Moreover,  $n$  cannot cross  $\bar{n}$  because in this region we have  $\dot{n} > 0$  (there are no incentives for automation). This implies  $v \rightarrow \infty$  along any such path, which violates the transversality condition for households entitled to profits from automation. In the second case,  $v \rightarrow -\infty$  and  $n \rightarrow 1$ ; which again violates the transversality condition for households entitled to profits from the creation of new tasks.

To finalize the proof of the Lemma, we establish the local saddle path stability in the general case with  $\theta > 0$ . As above, we work with the limit  $S \rightarrow 0$ , which generalizes to the case with  $S < \bar{S}$  by continuity.

Let

$$\begin{aligned}\pi_N(n, k) &= c^u \left( \frac{w^E(n, k)}{\gamma(n)} \right)^{\zeta-\sigma} - c^u (r^E(n, k))^{\zeta-\sigma} \\ \pi_I(n, k) &= c^u (r^E(n, k))^{\zeta-\sigma} - c^u (w^E(n, k))^{\zeta-\sigma}\end{aligned}$$

be the flow profits for innovators.

Also, define the partial derivatives

$$Q_k = \kappa_I \frac{\partial \pi_I}{\partial k} - \kappa_N \frac{\partial \pi_N}{\partial k}$$

$$Q_n = \kappa_I \frac{\partial \pi_I}{\partial n} - \kappa_N \frac{\partial \pi_N}{\partial n}$$

evaluated at their balanced growth path values.

Using this notation and applying a first-order Taylor expansion, the equilibrium conditions for  $v, c, k, n$  can be expressed around their balance growth path values as

$$\begin{aligned}\dot{n} &= -Q_v v \\ \dot{v} &= rv - Q_k[k(t) - k^D] - Q_n[n(t) - n^D], \\ \dot{c} &= \frac{\theta}{c^D} r_n^E [n(t) - n^D] + \frac{\theta}{c^D} r_k^E [k(t) - k^D] \\ \dot{k} &= f_n^E [n(t) - n^D] + (f_k^E - \delta)[k(t) - k^D] - c.\end{aligned}$$

Here,  $Q_v > 0$  is a constant capturing the fact that  $\dot{n}$  changes discontinuously at  $v = 0$ , with  $\dot{n} > 0$  if  $v < 0$  and  $\dot{n} < 0$  if  $v > 0$ . As  $Q_v \rightarrow \infty$ , the above system approximates the local behavior of the dynamic economy near the steady state.

The characteristic polynomial of this system of differential equations (with all derivatives still evaluated at their balance growth path values) can be written as

$$P(\lambda) = \left| \begin{pmatrix} -\lambda & -Q_v & 0 & 0 \\ -Q_n & r - \lambda & 0 & -Q_k \\ \frac{\theta}{c^D} r_n^E & 0 & -\lambda & \frac{\theta}{c^D} r_k^E \\ f_n^E & 0 & -1 & f_k^E - \delta - \lambda \end{pmatrix} \right|,$$

or expanding it:

$$\begin{aligned}P(\lambda) &= \lambda^4 - \lambda^3(f_k^E - \delta + r) + \lambda^2(-Q_v Q_n + \frac{\theta}{c^D} r_k^E + r(f_k^E - \delta)) \\ &\quad - \lambda(Q_v(f_n^E Q_k - (f_k^E - \delta) Q_n) + r \frac{\theta}{c^D} r_k^E) + Q_v(r_n^E Q_k - r_k^E Q_n).\end{aligned}$$

The assumption that  $\kappa_I v'_I^D > \kappa_N v'_N^D$  implies that  $r_n^E Q_k - r_k^E Q_n > 0$ , so that an increase in  $n$  reduces  $\kappa_N \pi_N - \kappa_I \pi_I$  near the steady state if capital adjusts immediately to keep the interest rate constant. The fact that  $r_n^E Q_k - r_k^E Q_n > 0$  is the same force exploited in the stability of the balance growth path in the case where  $\theta \rightarrow 0$ . In addition, the comparative statics for the static case imply that  $Q_n > 0$ . That is, as  $n$  increases—holding capital constant—the incentives to do automation increase.

Let  $\lambda_1, \lambda_2, \lambda_3, \lambda_4$  be the eigenvalues of the above system. Then  $\lambda_1 \lambda_2 \lambda_3 \lambda_4 = Q_v(r_n^E Q_k - r_k^E Q_n) > 0$ . This implies that either all eigenvalues are negative, or all are positive, or two are negative and two are positive.

All eigenvalues cannot be negative either, since their sum is  $f_k^E - \delta + r > 0$  (this is the trace of the system matrix). The last inequality follows by noting that  $r > \delta$  (since  $r = \rho + \delta$ ).

All eigenvalues cannot be positive. This follows by noting that,

$$\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4 = -Q_v Q_n + \frac{\theta}{c^D} r_k^E + r(f_k^E - \delta),$$

and the right hand side is negative for  $Q_v \rightarrow \infty$  (since  $Q_n > 0$ ).

Thus, the system has two positive and two negative eigenvalues, and since there are two state variables ( $k$  and  $n$ ), it is locally saddle path stable. ■

## Proof of Proposition 5

we start by providing formulas for  $F_N^P$  and  $F_I^P$ . These are given by

$$\begin{aligned} F_N^P &= \frac{1}{\sigma-1} Y c^u \left( \frac{w^P}{\gamma(n)} \right)^{\zeta-\sigma} \left( \sigma \left( \frac{w^P}{\gamma(n)} \right)^{1-\zeta} + (\mu\psi)^{1-\zeta} \left( \frac{\eta}{1-\eta} \right)^\zeta \right) \\ &\quad - \frac{1}{\sigma-1} Y c^u (r^P)^{\zeta-\sigma} \left( \sigma r^{P1-\zeta} + (\mu\psi)^{1-\zeta} \left( \frac{\eta}{1-\eta} \right)^\zeta \right) \\ F_I^P &= \frac{1}{\sigma-1} Y c^u (r^P)^{\zeta-\sigma} \left( \sigma r^{P1-\zeta} + (\mu\psi)^{1-\zeta} \left( \frac{\eta}{1-\eta} \right)^\zeta \right) \\ &\quad - \frac{1}{\sigma-1} Y c^u (w^P)^{\zeta-\sigma} \left( \sigma (w^P)^{1-\zeta} + (\mu\psi)^{1-\zeta} \left( \frac{\eta}{1-\eta} \right)^\zeta \right). \end{aligned}$$

Using these formulas, we establish the decentralization result by construction.

First, assume the planner subsidizes a fraction  $1 - \mu$  to the price of intermediate goods, and sets a tax/subsidy to capital savings of  $\tau_k = \omega^P L_s^s \frac{\partial \ln L_s^s}{\partial \ln \omega} \frac{\partial \ln \omega^P}{\partial \ln K}$ . This guarantees households discount future income at a rate  $r^P - \delta - r^P \omega^P L_s^s \omega_K^P$ , which coincides with the planner's discount rate.

Absent the flow subsidies/taxes for successful innovators, the value of automating jobs or creating new tasks are given by a small modification of equations (21) and (22), which take into account that firms sell intermediates at a price  $\psi$ , but buyers perceive a price  $\mu\psi$  because of the subsidy. These values also discount future profits as the same rate the planner does, because of the taxes/subsidies to capital accumulation. Thus:

$$\begin{aligned} V_I(t) &= (1 - \mu)\psi \left( \frac{\eta}{1-\eta} \right)^\zeta (\mu\psi)^{-\zeta} \int_t^\infty e^{-\int_t^\tau (r^P - \delta + W^P L_s^s \omega_K^P) ds} Y(\tau) \left( c^u (r^P(\tau))^{\zeta-\sigma} - c^u \left( w^P(\tau) \frac{\gamma(I(\tau))}{\gamma(I(t))} \right)^{\zeta-\sigma} \right) d\tau, \\ V_N(t) &= (1 - \mu)\psi \left( \frac{\eta}{1-\eta} \right)^\zeta (\mu\psi)^{-\zeta} \int_t^\infty e^{-\int_t^\tau (r^P - \delta + W^P L_s^s \omega_K^P) ds} Y(\tau) \left( c^u \left( \frac{w^P(\tau)}{\gamma(n(t))} \frac{\gamma(I(\tau))}{\gamma(I(t))} \right)^{\zeta-\sigma} - c^u (r^P(\tau))^{\zeta-\sigma} \right) d\tau. \end{aligned}$$

Now, we can define the flow subsidies/taxes for successful innovators as follows. First, we have

a component to adjust for the appropriability problems,  $\tau_N^A(t), \tau_I^A(t)$ . These are given by:

$$\begin{aligned}\tau_N^A(t) &= \frac{1}{\sigma-1} Y c^u \left( \frac{w^P}{\gamma(n)} \right)^{\zeta-\sigma} \left( \sigma \left( \frac{w^P}{\gamma(n)} \right)^{1-\zeta} + (\mu\psi)^{1-\zeta} \left( \frac{\eta}{1-\eta} \right)^\zeta \left( 1 - (\sigma-1) \frac{\mu}{1-\mu} \right) \right) \\ &\quad - \frac{1}{\sigma-1} Y c^u (r^P)^{\zeta-\sigma} \left( \sigma r^{P1-\zeta} + (\mu\psi)^{1-\zeta} \left( \frac{\eta}{1-\eta} \right)^\zeta \left( 1 - (\sigma-1) \frac{\mu}{1-\mu} \right) \right) \\ \tau_I^A &= \frac{1}{\sigma-1} Y c^u (r^P)^{\zeta-\sigma} \left( \sigma r^{P1-\zeta} + (\mu\psi)^{1-\zeta} \left( \frac{\eta}{1-\eta} \right)^\zeta \left( 1 - (\sigma-1) \frac{\mu}{1-\mu} \right) \right) \\ &\quad - \frac{1}{\sigma-1} Y c^u (w^P)^{\zeta-\sigma} \left( \sigma (w^P)^{1-\zeta} + (\mu\psi)^{1-\zeta} \left( \frac{\eta}{1-\eta} \right)^\zeta \left( 1 - (\sigma-1) \frac{\mu}{1-\mu} \right) \right).\end{aligned}$$

The wedge between the private and social values of innovation captured by  $\tau_N^A(t), \tau_I^A(t)$  is well known, and arises because monopolists cannot extract the full value of introducing new tasks in models of expanding varieties. Here, this is the case for monopolists automating jobs or creating new tasks, so the taxes/subsidies  $\tau_N^A(t), \tau_I^A(t)$  have ambiguous signs and orderings.

Second,  $\tau_N^T(t_0, t), \tau_I^T(t_0, t)$ , for  $t \geq t_0$ , are given by:

$$\begin{aligned}\tau_N^T(t_0, t) &= (1-\mu)\psi \left( \frac{\eta}{1-\eta} \right)^\zeta (\mu\psi)^{-\zeta} \left[ c^u \left( \frac{w^S(t)}{\gamma(n(t))} \right)^{\zeta-\sigma} - c^u \left( \frac{w^S(t)}{\gamma(n(t))} \frac{\gamma(I(t))}{\gamma(I(t_0))} \right)^{\zeta-\sigma} \right] \geq 0 \\ \tau_I^T(t_0, t) &= (1-\mu)\psi \left( \frac{\eta}{1-\eta} \right)^\zeta (\mu\psi)^{-\zeta} \left[ c^u \left( w^S(t) \frac{\gamma(I(t))}{\gamma(I(t_0))} \right)^{\zeta-\sigma} - c^u (w^S(t))^{\zeta-\sigma} \right] \leq 0.\end{aligned}$$

$\tau_N^T(t_0, t) \geq 0$  and  $\tau_I^T(t_0, t) \leq 0$  correct for a *technological externality*: by inventing new tasks and increasing  $N$ , monopolists improve the quality of intermediates that future entrants will develop. The opposite occurs for automation: by automating task  $I$ , new entrants will be forced to automate more complex tasks, receiving fewer profits. These taxes/subsidies, depend on the time at which a task was introduced  $t_0$ —since they are a compensation (or charge) for all technologies built on top of them.

Finally,  $\tau_N^W$  and  $\tau_I^W$  correct for the fact that technology monopolists do not take into account the effect of technologies on the quasi-supply of labor.

It is straightforward to verify that once we add these flow subsidies/taxes to the private profits from developing new technologies, we obtain these become  $\Psi_N$  and  $\Psi_I$ , establishing the decentralization result.

Notice that the scientist allocation can be decentralized in many ways. In particular, since there is a fixed supply of scientists, we only need to get the *relative* expected profits from each type of innovation right. The particular decentralization outlined here guarantees the level of innovators' profits also matches the social value of innovation. Even if both types of technology end up being subsidized in equilibrium, this does not matter because the money can be recovered by taxing scientists.

## Proof of Proposition 6

Let  $S_I^D(t)$  and  $S_N^D(t)$  denote the allocation of scientists, and consider the allocation obtained by a small deviation  $S_N^P(t) = \min\{S_N^D(t) + \epsilon, 0\}$  and  $S_I^P(t) = \max\{S_I^D(t) - \epsilon, 0\}$  if  $S_I^D < 1$ , and  $S_N^P(t) = S_N^D(t)$ ,  $S_I^P(t) = S_I^D(t)$  otherwise. We prove in the appendix that for a small  $\epsilon > 0$ , such deviation increases welfare and reduces the extent of automation.

Clearly, the new allocation satisfies  $n^P(t) \geq n^D(t)$  as wanted. For  $\epsilon$  small enough, we have that the above allocation changes welfare by  $\epsilon(\kappa_N \mu_N - \kappa_I \mu_I)$ , whenever  $S_I^D(t), S_N^D(t) \in (0, 1)$ . Moreover, in these cases  $\kappa_N V_N(t) = \kappa_I V_I(t)$ .

Thus, to prove our deviation increases welfare, it is enough to verify  $\kappa_N \mu_N - \kappa_I \mu_I > 0$  whenever  $\kappa_N V_N(t) = \kappa_I V_I(t)$ . In fact, we prove the stronger statement, that at all points in time  $\frac{\Psi_N(t)}{\Psi_I(t)} > \frac{V_N(t)}{V_I(t)}$ .

Notice that, as  $\epsilon \rightarrow 0$ , we are along the market allocation. Thus, we can compute  $\Psi_N$  and  $\Psi_I$  as:

$$\begin{aligned} \Psi_N(t) &= \int_t^\infty e^{-\int_0^\tau (r^P(n^D(s), k^D(s)) - \delta) ds} \left( \frac{\sigma + \frac{\eta}{1-\eta}}{\sigma - 1} Y(\tau) \left[ c^u \left( \frac{w^P(\tau)}{\gamma(n(\tau))} \right)^{\zeta-\sigma} - c^u (r^P(\tau))^{\zeta-\sigma} \right] + W^P L_\omega^S \omega_N^P \right) d\tau, \\ \Psi_I(t) &= \int_t^\infty e^{-\int_0^\tau (r^P(n^D(s), k^D(s)) - \delta) ds} \left( \frac{\sigma + \frac{\eta}{1-\eta}}{\sigma - 1} Y(\tau) \left[ c^u (r^P(\tau))^{\zeta-\sigma} - c^u (w^P(\tau))^{\zeta-\sigma} \right] + W^P L_\omega^S \omega_I^E \right) d\tau. \end{aligned}$$

However, this implies the inequalities:

$$\begin{aligned} \frac{\Psi_N(t)}{\Psi_I(t)} &= \frac{\int_t^\infty e^{-\int_0^\tau (r^P(n^D(s), k^D(s)) - \delta) ds} \left( \frac{\sigma + \frac{\eta}{1-\eta}}{\sigma - 1} Y(\tau) \left[ c^u \left( \frac{w^P(\tau)}{\gamma(n(\tau))} \right)^{\zeta-\sigma} - c^u (r^P(\tau))^{\zeta-\sigma} \right] + W^P L_\omega^S \omega_N^P \right) d\tau}{\int_t^\infty e^{-\int_0^\tau (r^P(n^D(s), k^D(s)) - \delta) ds} \left( \frac{\sigma + \frac{\eta}{1-\eta}}{\sigma - 1} Y(\tau) \left[ c^u (r^P(\tau))^{\zeta-\sigma} - c^u (w^P(\tau))^{\zeta-\sigma} \right] + W^P L_\omega^S \omega_I^E \right) d\tau} \\ &> \frac{\int_t^\infty e^{-\int_0^\tau (r^P(n^D(s), k^D(s)) - \delta) ds} \left( \frac{\sigma + \frac{\eta}{1-\eta}}{\sigma - 1} Y(\tau) \left[ c^u \left( \frac{w^P(\tau)}{\gamma(n(\tau))} \gamma(I(\tau-t)) \right)^{\zeta-\sigma} - c^u (r^P(\tau))^{\zeta-\sigma} \right] + W^P L_\omega^S \omega_N^P \right) d\tau}{\int_t^\infty e^{-\int_0^\tau (r^P(n^D(s), k^D(s)) - \delta) ds} \left( \frac{\sigma + \frac{\eta}{1-\eta}}{\sigma - 1} Y(\tau) \left[ c^u (r^P(\tau))^{\zeta-\sigma} - c^u (w^P(\tau) \gamma(I(\tau-t)))^{\zeta-\sigma} \right] + W^P L_\omega^S \omega_I^E \right) d\tau} \\ &\geq \frac{\int_t^\infty e^{-\int_0^\tau (r^P(n^D(s), k^D(s)) - \delta) ds} \frac{\sigma + \frac{\eta}{1-\eta}}{\sigma - 1} Y(\tau) \left[ c^u \left( \frac{w^P(\tau)}{\gamma(n(\tau)) \gamma(I(\tau-t))} \right)^{\zeta-\sigma} - c^u (r^P(\tau))^{\zeta-\sigma} \right] d\tau}{\int_t^\infty e^{-\int_0^\tau (r^P(n^D(s), k^D(s)) - \delta) ds} \frac{\sigma + \frac{\eta}{1-\eta}}{\sigma - 1} Y(\tau) \left[ c^u (r^P(\tau))^{\zeta-\sigma} - c^u (w^P(\tau) \gamma(I(\tau-t)))^{\zeta-\sigma} \right] d\tau} \\ &= \frac{V_N(t)}{V_I(t)}, \end{aligned}$$

as wanted.

The first inequality results from the technological externality; which as explained above pushes towards the underprovision of new tasks. The second inequality results from the novel inefficiency underscored in this paper: the fact that labor gets rents in equilibrium pushes towards the underprovision of new tasks and excessive automation. This inequality is strict whenever  $L_\omega^s > 0$ —that is, labor gets rents.