Money as a Unit of Account*

Matthias Doepke
Northwestern University

Martin Schneider
Stanford University

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Abstract

We develop a theory that rationalizes the use of a dominant unit of account in an economy. Agents enter into non-contingent contracts with a variety of business partners. Trade unfolds sequentially in credit chains and is subject to random matching. By using a dominant unit of account, agents can lower their exposure to relative price risk, avoid costly default, and create more total surplus. We discuss conditions under which it is optimal to adopt circulating government paper as the dominant unit of account, and the optimal choice of "currency areas" when there is variation in the intensity of trade within and across regions.

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1 Introduction

Classical economists pointed out money's various functions in society. Since then, large literatures have rationalized the use of money as a store of value and as a medium of exchange.[1] In contrast, the use of money as a unit of account for future payments has received little attention.[2] This fact is surprising given the widespread use of money-denominated long-term contracts (such as bonds and mortgages) in modern economies. The use of money as a unit of account implies that inflation has redistribution effects, which lie at the heart of Irving Fisher's debt-deflation theory of depressions (Fisher 1933) and which are just as relevant today.[3]

The goal of this paper is to explain the role of money as a unit of account for future payments. At first sight, the use of money as a unit of account might appear to be a matter of convenience only. If future payments will be settled in money anyway (since it is the medium of exchange), isn't it practical to specify the value of the payments in terms of money as well (as the unit of account)? While such an explanation may seem straightforward in modern economies where the same money serves both functions (such as in the United States, where the dollar is both the dominant medium of exchange and the dominant unit of account), monetary history offers numerous examples where the medium-of-exchange and unit-of-account functions do not coincide.

Indeed, in medieval Europe a separation of the different functions of money was the rule rather than the exception.[4] In France, for example, the *livre tournois* served as unit of account for centuries during the medieval and early modern periods, even when the corresponding coin was no longer in circulation.[5] In

[1] Two seminal contributions are Samuelson (1958) on money as a store of value in an overlapping-generations model, and Kiyotaki and Wright (1989) on money as a medium of exchange in a model with search frictions.

[2] Some authors consider separately the use of money as a "standard of deferred payment" that is used to settle contracts. We view settlement as a buyback of assets by way of a medium of exchange. The standard of deferred payment therefore need not be the same as the unit of account.

[3] See Doepke and Schneider (2006b) for a quantitative assessment of redistribution effects of inflation for the modern U.S. economy.

[4] See Spufford (1988) and Kindleberger (1993) for overviews of European monetary history.

[5] Moreover, it was also common that the value of a coin used as an accounting currency was

Germany, specific coins (such as the *Vereinsthaler*) were used as an accounting currency across large areas, even though different media of exchange circulated in the various sovereign states of Germany. Also common was the use of natural units in contracts (such as bushels of wheat) and of bundles (such as a combination of a natural unit and a monetary unit). A practice that is still common today is the use of foreign currency as a unit of account, such as the use of Italian *Florin* in medieval Europe, the modern use of the U.S. dollar in trade relationships not involving the United States, and mortgage borrowing denominated in euros or Swiss francs in Eastern Europe.

In light of these observations, we address two separate questions on the role of money as a unit of account. First, why do economic agents often find it useful to coordinate on a dominant unit of account? Second, what should a useful unit of account look like? The answer to the first question should also address the limits of coordination: why do different groups of people sometimes use different units of accounts, for example by forming currency areas? The answer to the second question should explain in particular the emergence of government-issued fiat money: Why is it often a dominant unit of account in modern times, but was less so in earlier times? And why are the different functions of money not always linked (as in medieval Europe, and in modern countries where private contracts are dollarized)?

Our theory is based on four features shared by most economies. First, agents enter into contracts that involve payment promises that are later costly to break or renegotiate. Second, there are multiple widely traded goods with fluctuating prices in which promised payments can, in principle, be denominated.[6] In this

different from the value of the actual coin of the same name in circulation, a phenomenon referred to as "ghost money" by Cipolla (1956) and "imaginary money" by Einaudi (1937, 1953). And even where actual coin used as unit of account was in circulation, usually its use in contracts was solely as a specification of value: "In many instances where the debt was stipulated only and explicitly in an amount of, let us say, *solidi* (shillings), it was tacitly assumed that the payment could be settled with any other commodity of equivalent value. A debt stipulated in 20 *solidi* in a French document of November 1107 was, we know from a later document, settled with a horse" (Cipolla 1956, p. 5). A modern example of a unit of account that is not also a medium of exchange is the *ECU* (European Currency Unit), which was based on a basket of European currencies and served as a unit of account in European Trade before the introduction of the euro.

[6]The model is set up so these widely traded goods can be interpreted broadly, for example as currencies, precious metals, or government paper.

setting, a contract between a lender and a borrower has to specify the unit of account, i.e., the good in which the value of future payments is specified. The cost of breaking promises along with price risk implies that borrowers can gain from using the same unit of account on both sides of the balance sheet. Specifically, if the price realization of a good that denominates a large part of a borrower's income is low, the borrower may have difficulty meeting his own promises. This risk can be hedged by denominating outgoing payments in the same good that denominates borrower income.[7]

To give an example of the balance-sheet risk that we have in mind, consider an economic agent (such as a household, a firm, or a bank) who holds assets that are denominated in U.S. dollars. In other words, the agent expects to receive future payments, the value of which is fixed in terms of dollars. Now suppose that the agent wants also to incur liabilities, such as by borrowing in order to invest in a business or buy a house. If these liabilities are denominated in a unit of account other than the U.S. dollar (say, euros), the agent faces the risk that the relative price of the units of account for assets and liabilities will change until future payments are due. Here, the risk is that the price of euros will rise relative to dollars. If there is a big change in the relative price, the value of the assets (the future payments in terms of dollars) may be too low to repay the liabilities (in terms of euros), resulting in costly default. By using the same unit of account for both assets and liabilities, the agent can avoid this relative-price risk and thereby lower the probability of default.

Hence, the first two elements of our theory, the cost of breaking promises and price risk, explain the demand for specific units of account. The third element of our theory is that efficient production requires an entire network of borrowing and lending relationships. As a result, a typical agent is both a borrower and a lender—he is a member of a credit chain. Credit chains arise naturally in modern economies not only in the organization of production (for example, raw materials, intermediates, and final-goods producers), but also in commerce (producer,

[7]We focus on the unit-of-account function of money for future payments precisely because the delay between making the promise and the actual payment implies the possibility of relative-price changes. In contrast, the unit-of-account function of money for quoting current prices is not subject to the same price risk. However, the unit of account for current prices may still matter if additional frictions are present, such as a cost of changing prices (from which we abstract here).

wholesaler, retailer) and finance (borrower, intermediary, investor). The presence of credit chains explains the propagation of units of accounts beyond bilateral relationships. In a credit chain, what is a promise for one agent is income for the next, thus leading to demand for a common unit of account in the entire chain.

The final element of our theory is that the formation of credit chains has a random-matching component that is not contractible. When a borrower and a lender meet, they typically cannot condition payment on the identity of their future business partners, let alone those partners' partners and so on. Balance sheet risk therefore comes from two sources. In addition to variation in relative prices there is matching uncertainty, as agents do not know what credit chain they will ultimately be part of. It then becomes advantageous to adopt a unit of account that is likely to be compatible with many future potential trading partners, leading to the optimality of an economywide dominant unit of account.

The nature of the efficiency gain from adopting a dominant unit of account depends on how costly it is to break promises. We consider the two extremes. On the one hand, if breaking promises is infinitely costly, then default never occurs. Borrowers lower debt ex ante to avoid default, which leads to inefficiently low production. Use of a dominant unit allows more borrowing and thereby more production. On the other hand, if breaking promises carries only a small cost, then borrowers will produce at the efficient scale and default if necessary. Use of a dominant unit of account then lowers average ex-post default costs.

The argument we have outlined so far explains why agents coordinate on a common unit of account, but leaves open the question of exactly what should be the unit of account. Given that in our theory a key role of the unit of account is to minimize balance-sheet risk, it follows that choosing a unit of account that already denominates the income of major borrowers would be useful. Here we arrive at the tight link between the use of government-issued fiat money as unit of account and the issuance of government debt. Consider a government that issues nominal bonds to be held by households, firms, and banks. The payments promised in the bonds are part of these agents' future income. If the same agents now incur future liabilities, they can reduce their balance-sheet risk by denominating these liabilities in fiat money also. Thus, the government's use of fiat

money for its own borrowing propagates to private contracts and leads to fiat money being the dominant unit of account.

Notice that our argument explaining the role of fiat money as the unit of account relies solely on the role of money as denominating government debt, but not on the medium of exchange role of fiat money.[8] This implication is consistent with the observation that, historically, units of account and media of exchange often used to be distinct, but became unified in modern economies characterized by the widespread use of government-issued nominal bonds.

In our theory, there are additional factors (other than the presence of government debt) that determine the optimal unit of account. For example, it is useful for a unit of account to be stable in value relative to other goods traded in the economy. This feature explains why if the value of fiat money is too volatile (i.e., volatile inflation), local currency may fail to be used as a unit of account even if nominal government debt is present. Such a scenario is akin to the dollarization of private contracts that is often observed in countries grappling with inflation. In addition, different regions or countries may have different dominant income sources. This scenario leads to a tradeoff between a unified unit of account versus multiple units that may be better suited to local conditions i.e., a theory of optimal currency areas.

The paper is structured as follows. In the following section, we relate our work to the existing literature. In Section 3 we describe the model environment. In Section 4, we consider a setting with large default costs (implying that contracts are non-contingent) and demonstrate the optimality of a dominant unit of account in this economy. In Section 5 and we discuss conditions under which government-issued paper (such as fiat money) may arise as the optimal unit of account. In Section 6, we consider the case of small default costs and apply the model to the issue of optimal currency areas. Section 7 concludes.

[8] Indeed, our theoretical model features a centralized spot market in which there is no need to a specialized medium of exchange.

2 Related Literature

Our paper is related to existing work on balance sheet effects of price changes. The basic idea that mismatched units of account on a balance sheet can create problems is familiar from the banking literature, and currency mismatch has played an important role in banking and financial crises (see for example Schneider and Tornell 2004 and Burnside, Eichenbaum, and Rebelo 2006). In this paper, we go beyond individual balance sheets and find conditions under which a dominant unit of account will be adopted in an entire economy. Relative to the banking literature, the key features that lead to this result are that production takes place in chains of credit (modeled as in Kiyotaki and Moore 1997) and that contracting is non-synchronized.

Our work is also related to a small literature on the optimality of nominal contracts. Jovanovic and Ueda (1997) consider a static moral hazard problem in which nominal output is observed before the price level (and therefore real output) is revealed. In addition, contracts are not renegotiation-proof, so that principal and agent have an incentive to renegotiate after nominal output is observed. In the optimal renegotiation-proof solution, the principal offers full insurance to the agent once nominal output is known. This implies that the real wage depends on nominal output, so that the contract can be interpreted as a nominal contract. Meh, Quadrini, and Terajima (2010) integrate a similar mechanism in a model with firm heterogeneity and financial constraints, and study how different monetary regimes affect the degree of indexation. The mechanism in these papers operates within a relatively short time horizon, namely the lag between the realization of a nominal variable and the observation of the corresponding price level. In contrast, the balance-sheet effects in our theory are equally relevant for long-term assets such as bonds and mortgages, which account for the major part of redistribution effects of inflation.

Freeman and Tabellini (1998) consider an overlapping-generations economy with spatially separated agents in which fiat money serves as a medium of exchange. They provide conditions for fiat money to serve also as a unit of account. In contrast, in our theory exchange takes place in a frictionless centralized market,

without a need for a medium of exchange. Rather, in our theory the occasional coincidence between the unit of account and the medium of exchange happens only in the presence of government debt denominated in fiat currency. Our approach has the advantage that it can explain why, in modern economies with widespread use of government debt, it is common for fiat money to serve both functions, whereas in earlier times distinct monetary units where used as unit of account and medium of exchange.

Cooper (1990) and Acemoglu (1995) also consider environments in which each agent writes several several contracts within a fixed network. They assume uncertainty about the value of money and provide conditions for the coexistence of multiple equilibria with indexed or nominal contracts. For example, both equilibria exist if the network of contracts is such that coordination on a single unit of account provides perfect hedging. Our setup has multiple goods (and hence multiple sources of price risk), so that a fixed network does not give rise to a dominant unit of account. Instead, random matching is critical both for making a dominant unit of account optimal and for determining what that unit looks like.

Random matching of agents also plays an important role in micro-founded theories of the medium-of-exchange function of money, such as Kiyotaki and Wright (1989). While the specific mechanisms are quite different, the similarity is that in both theories random matching induces a desire of compatibility with many possible trading partners. Within this literature, Lagos and Wright (2005) introduced a structure in which both decentralized and centralized markets play a role. However, in Lagos and Wright (2005) exchange takes place in both markets, and the centralized market mainly serves to offset the heterogeneity that is generated by random matching. In contrast, in our theory random matching affects only the contracting stage, and all contracts are ultimately settled in the centralized market.

Further, our work relates to the literature on the redistribution effects of inflation. Most of this literature focuses on a particular aspect of redistribution, namely the revaluation of government debt (see for example Bohn 1988, 1990, Persson, Persson, and Svensson 1998, Sims 2001). Government debt plays an important role in our model also, in a mechanism that renders fiat money an attractive choice for

the unit of account. Redistribution effects among private agents were recently considered by Doepke and Schneider (2006a), Doepke and Schneider (2006b), and Meh, Ríos-Rull, and Terajima (2010).

3 A Model of an Optimal Unit of Account

3.1 Environment: Dates, People, and Preferences

The model economy extends over three dates, 0, 1, and 2. The economy is populated by two groups of people, farmers and artisans. There are two types of farmers, A and B. There is a continuum of each type of farmer $i \in \{A, B\}$, where the mass of type i is denoted m_i. The total mass of farmers is $m_A + m_B = 1$, and we label types such that $m_A \geq m_B$, that is, type A is more numerous. There are N types of artisans, with a continuum of mass one of each type.

Every agent is endowed with one unit of time at date 1 and has a technology that uses date 1 labor to make goods that become available at date 2. Farmers can make farm goods that are specific to their types. In particular, a farmer of type i can use one unit of labor to produce $1 + \lambda$ units of farm good i, where $\lambda > 0$. Artisans make artisanal goods that are produced one-for-one from labor. Labor supply h is constrained to lie in a set H. Below we consider either indivisible labor supply, $H = \{0, 1\}$, or divisible labor supply, $H = [0, 1]$.

The key difference between farmers and artisans is in how their respective goods are sold. Farm goods are sold in a spot market that opens at date 2. In the spot market, farm good i can be exchanged into the consumption good C at relative price p_i. Let $\mathbf{p}$ denote the vector of farm-good prices. When quoting prices, we will use good C as the numeraire (i.e., p_i is the amount of good C that can be exchanged into one unit of farm good i). However, results do not depend on how prices are quoted in the spot market: only relative prices matter.

Consumption good C is produced by neither the farmers nor the artisans that we model; it can only be acquired in the date-2 spot market. The prices p_i are

stochastic, and we assume $E(p_i) = 1$ for all i. Price realizations are independent of any decisions taken by farmers or artisans. The economy can therefore be interpreted as a small open economy, where farm goods A and B are exported and the consumption good C is imported, and the economy is subject to fluctuations in world market prices. An equivalent closed-economy interpretation is that there is a competitive final-goods sector that combines farm goods A and B to produce the consumption good C, with a linear technology subject to productivity shocks. In this interpretation, p_i represents the amount of good C that can be produced per unit of farm good i. All our results hold in this alternative interpretation; the features that matter for our results are (i) that a spot market operates at date 2 and (ii) that prices in this spot market are subject to random fluctuations.

Unlike farmers, artisans produce customized goods for specific customers. At date 0 artisans are matched to potential customers. If the artisan agrees to make and supply an artisanal product to a matched customer at date 2, no other agent can obtain utility from that product at date 2. An artisan cannot produce artisanal products for himself, and artisanal goods are not traded in the spot market.

The utility of an individual of type i, where $i \in \{A, B, 1, 2, \ldots, N\}$, is given by

$$u = c + (1 + \lambda)x - h - s. \tag{1}$$

Here c is consumption of good C, x is consumption of the customized artisanal good produced for the individual by the matched supplier, h is labor supply, and $s \in \{0, \kappa\}$ is a time cost for settling contracts that will be discussed below. Notice that the parameter $\lambda > 0$, which measures the surplus generated by labor, also appears in the production function of farmers.

To complete the description of the environment, we next specify how artisans meet their customers.

3.2 The Highway: Locations and Matching

People are spatially separated in the economy. Specifically, agents are located along a highway. All farmers are located at the western end of the highway, in location 0. Artisans of type $i = 1$ are located immediately to the east of the farmers at location 1, next are artisans of type $i = 2$ at location 2, and so on until we reach the eastern end of the highway, which is populated by artisans of type $i = N$.

An artisan of type i who lives in location i can find a customer only in location $i - 1$, immediately to the west of where he lives. In particular, a type-1 artisan can find customers only among the farmers, and an artisan of type $i > 2$ can find customers only among the artisans of type $i - 1$. Correspondingly, an agent can find a supplier of artisanal goods only in location $i + 1$ immediately to the east of where he lives. As an example, for $N = 2$ the highway can be depicted as follows:

$$\begin{pmatrix} A \\ B \end{pmatrix} \longleftarrow 1 \longleftarrow 2.$$

Here the arrow means "can produce for."

In order to trade, people have to travel to match up with other agents up and down the highway (their customers and suppliers). The matching process takes place at date 0 and has two stages. In the first stage (morning), artisans with odd i travel east and artisans with even i travel west, so that type-1 artisans meet potential suppliers among type-2 artisans, type-3 artisans meet potential supplies among type-4 artisans, and so on. In the second stage (night), people move in the opposite direction, so that farmers of type A and B meet potential suppliers among type-1 artisans, type-2 artisans meet potential suppliers among type-3 artisans, and so on.[9]

At each stage, each potential supplier is matched up with exactly one customer (recall that the mass of farmers and each type of artisans is one). An individual artisan is thus equally likely to match with any individual customer drawn from

[9]Our results generalize to more general matching processes; this particular two-stage process is adopted to simplify the exposition.

the type living to his west. In particular, for all type-1 artisans the probability of matching with a farmer of type i is given by that farmer's type's population share m_i. Both stages of matching are completed by the end of date 0, before production takes place at date 1.

As a result of the two-stage matching process, every individual agent ends up as part of a chain of agents, where each chain is headed by a farmer of a specific type, followed by one of each type of artisan. Each artisan in the chain is the matched supplier of the agent to his west in the chain. The artisan of type N has a potential customer, but no supplier. Since matching is i.i.d., the probability that an individual artisan ends up in a chain headed by a farmer of type i is again m_i.

Formally, the matches formed at stage τ are described by a binary relation $\mathcal{N}_\tau$. Here $(i, j) \in \mathcal{N}_\tau$ means that i and j were matched at stage τ and that j is the potential supplier of i. The full network after both matching stages is given by $\mathcal{N} = \mathcal{N}_1 \cup \mathcal{N}_2$. Our assumption that at each stage all possible matches are equally likely implies a probability distribution over possible networks $\mathcal{N}$. The realization of uncertainty in the model is summarized by the pair $(\mathcal{N}, \boldsymbol{p})$, where $\mathcal{N}$ is the network realized by the end of date 0, and $\boldsymbol{p}$ is the vector of farm good prices realized at date 2.

3.3 Contracts: Promises and the Cost of Breaking Them

We now turn to contracting. When a potential customer and a supplier meet in one of the matching stages, a need for credit arises. An artisan must work at date 1 if he is to deliver the customized good at date 2. However, when meeting the supplier at date 0, the customer does not have any tradable goods that could be used to pay for the customized good up front. Rather, any payments have to take place at date 2, after uncertainty regarding the trading network and price realization has been resolved.

We would like to capture that it is costly to have payments depend on the realization $(\mathcal{N}, \boldsymbol{p})$ in complicated ways. In particular, we assume that contracts involve simple promises that can be changed later only at a cost. To this end, future payments are specified in two parts. The first component is a non-contingent

promised payment, namely a vector of farm-good quantities $\boldsymbol{\pi}_{i,j} = (\pi^A_{i,j},\ \pi^B_{i,j})'$, where i denotes the customer (who is making the promise) and j the supplier. By promising $\boldsymbol{\pi}_{i,j}$ to supplier j, customer i commits to delivering goods $\boldsymbol{\pi}_{i,j}$ to j at date 2. Since all farm goods can be freely exchanged in the spot market, the commitment is effectively to the value $\boldsymbol{p}'\boldsymbol{\pi}_{i,j}$. There is no need to settle the contract in the goods in which it is specified. The reason that only farm goods can serve as payment promises is precisely that only these have quoted market prices. Artisanal goods, in contrast, have value only for the matched customer; therefore they are not traded and do not have quoted prices.[10]

Given uncertainty over matches and prices, a customer may not always be able to deliver on a payment promise $\boldsymbol{\pi}_{i,j}$. For example, consider a farmer of type A who makes a promise in terms of good B, i.e, $\pi^A_{i,j} = 0$ and $\pi^B_{i,j} > 0$. The income of this farmer is in terms of good A. If now the price realization of good A is low relative to good B, the farmer may not have sufficient resources at date 2 to pay $\pi^B_{i,j}$. To deal with this possibility, the second component of the contracted payment consists of a fully contingent payment $v_{i,j}(\mathcal{N}, \boldsymbol{p})$, expressed in terms of the numeraire good C, where $v_{i,j}(\mathcal{N}, \boldsymbol{p}) \leq \boldsymbol{p}'\boldsymbol{\pi}_{i,j}$, i.e., the value of the alternative payment is no greater than the original promise. The actual payment that the customer has to make in state $(\mathcal{N}, \boldsymbol{p})$ is the smaller of the promised and the alternative payment:

$$\min\left\{\boldsymbol{p}'\boldsymbol{\pi}_{i,j}, v_{i,j}(\mathcal{N}, \boldsymbol{p})\right\}.$$

Given that $v_{i,j}(\mathcal{N}, \boldsymbol{p}) \leq \boldsymbol{p}'\boldsymbol{\pi}_{i,j}$, the actual payment is in fact always equal to $v_{i,j}(\mathcal{N}, \boldsymbol{p})$. This actual payment is fully enforced. The full contract between customer i and supplier j specifies the labor h to be exerted by the supplier at date 1, the artisanal goods $x = h$ to be delivered to the customer at date 2, and the payment promise $\boldsymbol{\pi}_{i,j}$ and actual payments $v_{i,j}(\mathcal{N}, \boldsymbol{p})$.

Given that the actual payment is fully contingent, the two-part payment specifi-

[10] In contrast, the consumption good C does have a market price. However, we view good C as a stand-in for an entire consumption basket, the price of which would be difficult to determine in practice. Using good C as a unit of account would therefore violate the requirement that a promise should be simple and have an immediately verifiable market price. Nevertheless, in Appendix A.2, we discuss how our findings are modified if good C can denominate promises as well.

cation as such does not constitute a deviation from complete markets. However, we assume that making a payment that is different from the initial promise is costly. If the promise is met, the customer's cost for settling the contract is zero, $s = 0$. In contrast, whenever we have $v_{i,j}(\mathcal{N}, \boldsymbol{p}) < \boldsymbol{p}'\boldsymbol{\pi}_{i,j}$, the customer faces a fixed cost $s = \kappa \geq 0$ in terms of time at date 2. The interpretation is that enforcing the contract and executing the alternative payment in the case of a broken promise involves a legal cost. For different values of κ, this setup captures the usual complete-market setting ($\kappa = 0$), fully non-contingent contracts ($\kappa = \infty$), and settings where the contracting friction affects outcomes, but is not sufficiently strong to reduce to the non-contingent case.

A *system of contracts* is a specification of contracts for all possible meetings between a customer i and a supplier j given the probability distribution of network realizations. A system of contracts is said to be *feasible* if every agent can in fact make the contract payments at date 2. In particular, given that artisans start without endowments, for every artisan i the contract payment to his supplier j must be covered by a contract payment from his customer h:

$$v_{i,j}(\mathcal{N}, \boldsymbol{p}) \leq v_{h,i}(\mathcal{N}, \boldsymbol{p}) \tag{2}$$

for all $(\mathcal{N}, \boldsymbol{p})$. A farmer does not receive contract payments, but instead receives proceeds from selling farm goods. The system of contracts must thus also satisfy, for every farmer i,

$$v_{i,j}(\mathcal{N}, \boldsymbol{p}) \leq p_i(1 + \lambda)h_i(\mathcal{N}), \tag{3}$$

for all $(\mathcal{N}, \boldsymbol{p})$.[11] Here $h_i(\mathcal{N})$ is the labor supply of farmer i, which can depend on the network realization (i.e., the farmer may decide how much to work based on whom he met during the matching process).

3.4 The Social Planning Problem and the Unit of Account

The assumptions on contracting now allow us to discuss the use of units of account in our economy. In any customer-supplier relationship, the unit of ac-

[11] In a slight abuse of notation, here we use i both to identify a particular farmer and this farmer's type.

count used in contracting is given by the bundle of farm goods that denominates the payment promise $\boldsymbol{\pi}_{i,j}$. For example, whenever in a given contract we have $\pi^A_{i,j} > 0$ and $\pi^B_{i,j} = 0$, we say that farm good A serves as the unit of account. In this case, the value of the promise is specified in terms of units of good A, just as in the U.S. economy most future payments are in terms of U.S. dollars. It is also possible that a non-degenerate bundle serves as the unit of account, $\pi^A_{i,j} > 0$ and $\pi^B_{i,j} > 0$. In the real world, this case would correspond to a contract that specifies payments in two different currencies or commodities, such as U.S. dollars and euros.

To formalize the notion of a unit of account, we parameterize the payment promise $\boldsymbol{\pi}_{i,j}$ using the *amount* of the payment $q_{i,j}$ and the *unit of account* of the payment $u_{i,j}$, where $0 \leq u_{i,j} \leq 1$. The promised amounts of goods A and B are then given by $\pi^A_{i,j} = u_{i,j}q_{i,j}$ and $\pi^B_{i,j} = (1 - u_{i,j})q_{i,j}$, so that the payment vector is:

$$\boldsymbol{\pi}_{i,j} = q_{i,j}\ (u_{i,j},\ 1 - u_{i,j})'.$$

When $u_{i,j} = 1$, good A serves as the unit of account, $u_{i,j} = 0$ means that good B is the unit of account, and for $0 < u_{i,j} < 1$, the unit is a bundle.

We would like to examine the implications of our theory for the optimal use of units of accounts. In particular, we would like to find conditions under which it is optimal to use a dominant unit of account for most transactions in the economy (as observed in many actual economies). Moreover, if a dominant unit of account arises, we want to know what good, or bundle of goods, should serve as the unit of account.

We approach these questions by formulating a social planning problem. The planner chooses the real allocation (production, consumption, and labor supply) as well as the system of payments (including the choice of units of account) subject to budget constraints, resource constraints, payment feasibility constraints, and participation constraints for all agents in the economy. The participation constraints require that no agent is worse off by participating in exchange, and corresponds to individual optimization in a decentralized equilibrium. We use a planning approach because, in order to define a decentralized equilibrium, we would have to make additional assumptions on bargaining power in bilateral re-

lationships. The planning approach can be interpreted as searching jointly over the set of possible distributions of relative bargaining power and the associated equilibria. Later on, we will show how the outcome of the planning problem can be decentralized under a specific bargaining protocol.

Since utilities are linear in the consumption good, Pareto-optimal allocations can be computed by maximizing the sum of agents' utility. Let μ denote the (exogenous) probability over network and price realizations. The planner's objective can then be written as:

$$\int_{\mathcal{N},\boldsymbol{p}} \int_i \left[\lambda h_i(\mathcal{N}) - \kappa I(\boldsymbol{p}'\boldsymbol{\pi}_{i,j} > v_{i,j}(\mathcal{N},\boldsymbol{p}))\right] di \; d\mu(\mathcal{N},\boldsymbol{p}) \tag{4}$$

Here the inner integral sums over all agents i in the economy, the outer integral is the expectation over networks and prices, and $I(\cdot)$ is the indicator function. In the second term, j is understood to be the (unique) supplier assigned to i by network $\mathcal{N}$, i.e., $j = \{j|(i,j) \in \mathcal{N}\}$. Notice that, instead of as a sum of utilities, the planner's objective is in expressed in terms of the surplus generated by labor. We can do so because our assumptions imply that each unit of labor supply (farm labor or artisanal labor) generates a fixed surplus of λ. The planner's objective therefore is to maximize the difference between the surplus generated by labor and the costs s for settling contracts that result from broken promises, where $s = \kappa$ if a promise is broken and $s = 0$ otherwise.

The participation constraints state that all agents have to be at least as well off by participating in exchange as they would be on their own. For a farmer of type i, the best alternative option is to work at date 1 and sell the produced farm goods at date 2 in the spot market at price p_i. Also, given that farmers meet their artisan suppliers in the second stage of matching, the full network realization $\mathcal{N}$ is already known when farmers decide whether to participate in exchange. For a given $\mathcal{N}$, the participation constraint for farmer i with matched supplier j is given by:

$$E\left[c_i + (1+\lambda)\, x_i - h_i - \kappa I(\boldsymbol{p}'\boldsymbol{\pi}_{i,j} > v_{i,j}(\mathcal{N},\boldsymbol{p}))|\mathcal{N}\right] \geq (1+\lambda)E[p_i] - 1 = \lambda. \tag{5}$$

Here the expectations on both sides are over the realization of the price vector $\mathbf{p}$,

and a separate constraint is imposed for all feasible network realizations $\mathcal{N}$.

The participation constraints for artisans are more complicated, because artisans enter into different contracts at both stages of matching. First, artisans have to be no worse off in expectation from participating in all proposed exchanges. By remaining in autarky, an artisan receives a utility of zero. The ex-ante participation constraint for artisan i can be written as:

$$E\left[c_i + (1+\lambda)\, x_i - h_i - \kappa I(\boldsymbol{p}'\boldsymbol{\pi}_{i,j} > v_{i,j}(\mathcal{N}, \boldsymbol{p}))|\mathcal{N}_1\right] \geq 0. \tag{6}$$

Here the expectations on both sides are over the realization of the price vector $\mathbf{p}$ and the second-stage matching outcome $\mathcal{N}_2$, and a separate constraint is imposed for all feasible first-stage matching outcomes $\mathcal{N}_1$. Next, consider an artisan i who met a customer h in the first stage, and now reaches the second stage. This artisan has to prefer entering into the prescribed contract with the second meeting partner over only carrying out the first contract. The resulting participation constraint can be written as:

$$E\left[c_i + (1+\lambda)\, x_i - h_i - \kappa I(\boldsymbol{p}'\boldsymbol{\pi}_{i,j} > v_{i,j}(\mathcal{N}, \boldsymbol{p}))|\mathcal{N}\right] \geq E\left[v_{h,i}(\mathcal{N}, \boldsymbol{p}) - h_i|\mathcal{N}\right]. \tag{7}$$

Here the expectations on both sides are over the realization of the price vector $\mathbf{p}$, and a separate constraint is imposed for all feasible network realizations $\mathcal{N}$. Notice that if the artisan only carries out the first contract, he simply uses the payment received from the customer to consume the consumption good C. If the artisan does not enter into a contract with a supplier, he does not make any payments and thus never faces the cost of breaking a promise. In principle, a similar constraint needs to be imposed for artisans who first meet suppliers. However, those constraints are never binding, because the artisans also have to make sure that they are able to make the contracted payments to their suppliers (recall that while the promise $\boldsymbol{\pi}_{i,j}$ can be broken at a cost, the actual payment $v_{i,j}(\mathcal{N}, \boldsymbol{p})$ is enforced). Thus, artisans who have already committed to payments have no choice but to enter into contracts in the second stage, so that only the ex-ante constraint applies to them.

In addition to participation constraints, the allocation and system of payments

chosen by the planner must also satisfy budget constraints, resource constraints, payment feasibility constraints, and timing constraints. Specifically, for agents who meet at the first stage of matching (morning), payment promises and contracted labor supply can only depend on the first-stage matching outcome. The following definition summarizes the definition of the planning problem.

Definition 1 (Planning Problem) *The planning problem is to maximize the objective* (4) *by choosing a system of payment promises $\boldsymbol{\pi}_{i,j}(\mathcal{N})$, actual payments $v_{i,j}(\mathcal{N}, \boldsymbol{p})$, a labor allocation $h_i(\mathcal{N})$, and a consumption allocation $(c_i(\mathcal{N}, \boldsymbol{p}), x_i(\mathcal{N}))$ subject to the payment feasibility constraints* (2)–(3)*, the participation constraints* (5)–(7)*, budget constraints for farmers and artisans (where h is the customer and j the supplier of i):*

$$
\begin{aligned}
c_i(\mathcal{N}, \boldsymbol{p}) &= p_i(1+\lambda)h_i(\mathcal{N}) - v_{i,j}(\mathcal{N}, \boldsymbol{p}), \\
c_i(\mathcal{N}, \boldsymbol{p}) &= v_{h,i}(\mathcal{N}, \boldsymbol{p}) - v_{i,j}(\mathcal{N}, \boldsymbol{p}),
\end{aligned}
$$

resource constraints for artisanal goods:

$$
x_i(\mathcal{N}) = h_j(\mathcal{N}),
$$

and timing constraints that require that payment promises, consumption of artisanal goods, and contracted labor supply in first-stage meetings can only depend on the first-stage matching outcome, i.e., for all matches where the customer i is an odd-type artisan, we have:

$$
\begin{aligned}
\boldsymbol{\pi}_{i,j}(\mathcal{N}) &= \boldsymbol{\pi}_{i,j}(\tilde{\mathcal{N}}), \\
x_i(\mathcal{N}) &= x_i(\tilde{\mathcal{N}}), \\
h_j(\mathcal{N}) &= h_j(\tilde{\mathcal{N}}),
\end{aligned}
$$

for all $\mathcal{N}_1$, $\mathcal{N}_2$, and $\tilde{\mathcal{N}}_2$, where $\mathcal{N} = \mathcal{N}_1 \cup \mathcal{N}_2$ and $\tilde{\mathcal{N}} = \mathcal{N}_1 \cup \tilde{\mathcal{N}}_2$.

4 Large Default Costs and the Optimality of a Dominant Unit of Account

We start our analysis of the planning problem by focusing on the case of large default costs, $\kappa = \infty$, implying that promised payments $\boldsymbol{\pi}_{i,j}$ cannot be reneged upon. We also assume that the set of feasible labor supply levels is given by $H = [0, 1]$ (that is, labor supply can vary continuously between zero and one), and that the distributions of prices satisfy the following assumption:

Assumption 1 *The distributions of prices of farm goods are independent and have the same support, $p_A, p_B \in [\underline{p}, \overline{p}]$ with $0 < \underline{p} < 1$ and $E(p_i) = 1$. In addition, the parameters $\underline{p}$ and $\overline{p}$ satisfy:*

$$\frac{\overline{p}}{\underline{p}} > 1 + 2\lambda. \tag{8}$$

The assumptions on prices are for ease of exposition; the main results are unchanged for more general price distributions. Inequality (8) is imposed to ensure that prices are sufficiently variable to affect allocations.

We would like to characterize the solution to the planning problem in Definition 1 for this setting. The attractive feature of the large-default-cost case is that given the objective function (4), maximizing social welfare is equivalent to maximizing labor supply, i.e., the planner would like to ensure that all agents work and produce as much as possible. This feature considerably simplifies the characterization of the social optimum and the associated system of units of account.

4.1 The Optimal Unit of Account for $N = 2$

Consider first the case of $N = 2$, when there are only two types of artisans. There are two types of meetings between agents in this economy. In the morning, artisans of type 1 meet artisans of type 2, and may agree to a contract. At night, type-1 artisans meet farmers, and may agree to another contract. The fundamental friction in this setting is that type-1 artisans need to receive a payment from a

farmer in order to be able to pay an artisan of type 2. However, when 1 meets 2, he does not yet know which type of farmer (A or B) he will meet at night. This makes if difficult to decide what the unit of account for the promised payment from 1 to 2 should be.

The optimal system of contracts consists of a promise $\boldsymbol{\pi}_{1,2}$ from type-1 to type-2 artisans, a delivery of artisanal goods $x_1 = h_2$ from 2 to 1 in exchange for this promise, payment promises $\boldsymbol{\pi}_{A,1}$ and $\boldsymbol{\pi}_{B,1}$ resulting from meetings of each type of farmer with type-1 artisans, and artisanal goods $x_A = x_B = h_1$ in exchange for these payments. In principle, it would be possible to assign different contracts in different meetings of a given type, but this turns out not to be optimal. We would like to know which choice of contracts solves the planning problem in Definition 1, and in particular which units of accounts will be used to specify payments. Recall that maximizing social welfare is equivalent to maximizing production in this setting. We start by showing that in meetings involving a farmer, the farmer's good will serve as the unit of account.

Proposition 1 *In meetings between a type-1 artisan and a farmer of type $i \in \{A, B\}$, the optimal labor supply of the two agents is $h_i = h_1 = 1$, the optimal unit of account is given by farm good i, and the optimal amount of the payment is $q_{i,j} = 1 + \lambda$. That is, for type-A farmers we have $\boldsymbol{\pi}_{A,1} = (\pi^A_{A,1},\ \pi^B_{A,1})' = (1 + \lambda)\ (1,\ 0)'$ and for type-B farmers $\boldsymbol{\pi}_{B,1} = (1 + \lambda)\ (0,\ 1)'$.*

Proof: We first note that these choices satisfy all applicable constraints. The participation constraint (5) of farmer i is:

$$E\left[c_i + (1 + \lambda)\, x_i - h_i\right] \geq \lambda.$$

The proposed contract features $c_i = 0$ (the entire harvest is used as a payment to 1), $x_i = 1$, and $h_1 = 1$, so that the constraint reads $\lambda \geq \lambda$ and is satisfied as an equality. The payment feasibility constraint of the farmer (3) in this case reduces to:

$$p_i q_{i,j} = p_i(1 + \lambda) \leq p_i(1 + \lambda),$$

which is again satisfied as an equality. Finally, the artisan receives goods with expected value of $1 + \lambda$ and exerts an effort of $h_1 = 1$, so that the artisan's participation constraint is satisfied as well. The full participation constraint of 1 also involves the contract with 2, but regardless of the other contract, 1 is always better off by accepting the contract with farmer i.

Next, to be part of the overall optimum this contract should also maximize the potential surplus that can be generated from other contracts in the economy. The only way in which the contract between farmer i and 1 affects other contracts is through the payment feasibility constraint (2) that 1 faces when contracting with 2. The constraint is loosened when 1 has more resources. Since in the proposed contract farmer i turns over his entire income from production to artisan 1, the proposed contract entails the maximum possible payment to 1 and thus allows for the highest possible surplus to be realized in the contract between 1 and 2. □

Note that if the variability of prices is small, this contract may not be the only optimum. For example, in the case where there is no price variability at all, contracting in either unit of account is equivalent, so that the choice of unit does not matter. However, when there is sufficient variability for payment-feasibility constraints to bind in the optimum, the optimum is unique.

The proposition demonstrates one of the main forces driving the choice of a unit of account: it is useful to have the promised payment covary with the income of borrowers. Here, specifying the payment in terms of the income of the farmer ensures that the farmer is able to make the payment regardless of price realizations. To see how things would change if a different unit of account were chosen, consider a meeting between farmer A and artisan 1 in which good B is chosen as the unit of account, so that $\boldsymbol{\pi}_{A,1} = q_{A,1}\ (0,\ 1)'$. In this case, the payment feasibility constraint (3) for the farmer would read:

$$p_B q_{A,1} \leq p_A(1 + \lambda).$$

This constraint would have to be satisfied for all price realizations. The binding constraint would be the one for the highest price realization for good B, $p_B = \overline{p}$,

and the lowest realization for A, $p_A = \underline{B}$, leading to the constraint:

$$q_{A,1} \leq \frac{p}{\overline{p}}(1+\lambda),$$

that is, the payment promise would have to be scaled down by the ratio of the lowest to the highest price realization. The lower payment promise, in turn, would imply that artisan 1 would work less (to satisfy his participation constraint), and that the payment feasibility constraint for the promise from 1 to 2 would become more binding, leading to lower production as well. By choosing his own good as the unit of account, the farmer can insure against relative price risk, and as a result a higher level of production becomes possible.

We now turn to the second set of contracts in this economy, those between artisans of type 1 and type 2. Recall that 1 and 2 meet in the morning, and at this time they don't yet know which type of farmer 1 will meet at night. If they did, the optimal solution to the contracting problem would be straightforward. To see this, consider the case $m_B = 0$, i.e., there are no type-B farmers and hence 1 is sure to meet a type-A farmer. Then 1 and 2 know that, under the optimal contract, 1 will receive a payment of $1+\lambda$ in units of good A (which is equivalent to $p_A(1+\lambda)$ in terms of the consumption good). In this knowledge, 1 and 2 have to specify a payment $\boldsymbol{\pi}_{1,2}$ and artisanal goods $x_1 = h_2$ to be delivered to 1. What turns out to be optimal is to specify the payment $\boldsymbol{\pi}_{1,2}$ in units of good A, too; that is, the unit of account is passed from the A-1 matches to the 1-2 matches.

Proposition 2 *Let $m_B = 0$, so that artisans at location 1 are sure to meet type-A farmers at night. The solution of the planning problem in Definition 1 is given by contracts between type-A farmers and type-1 artisans as specified in Proposition 1, and contracts between type-1 and type-2 artisans that specify $x_1 = h_2 = 1$ and $\boldsymbol{\pi}_{1,2} = q_{1,2}\ (1,0)'$ with $1 \leq q_{1,2} \leq 1+\lambda$, that is, good A serves as the unit of account in the 1-2 contract also.*

Proof: The constraints that govern the choices $x_1 = h_2$ and $\boldsymbol{\pi}_{1,2}$ are the payment feasibility constraint (2) for 1, and participation constraints for 1 and 2, which can be written as (taking as given the optimal contract between A and 1 from

Proposition 1):

$$\boldsymbol{p}'\boldsymbol{\pi}_{1,2} \leq p_A(1+\lambda) \quad \forall \boldsymbol{p}, \tag{9}$$

$$E\left[p_A(1+\lambda) - \boldsymbol{p}'\boldsymbol{\pi}_{1,2}\right] + (1+\lambda)\, x_1 - 1 \geq 0, \tag{10}$$

$$E\left[\boldsymbol{p}'\boldsymbol{\pi}_{1,2}\right] - x_1 \geq 0. \tag{11}$$

When good A is chosen as the unit of account and we set production to the optimal level $x_1 = h_2 = 1$, only (9) and (11) can bind and simplify to:

$$q_{1,2} \leq 1 + \lambda,$$
$$q_{1,2} \geq 1.$$

Hence, the optimum can be implemented with any payment that satisfies the constraint $1 \leq q_{1,2} \leq 1 + \lambda$. □

If a unit of account other than good A were used, the expected value of the payment would have to be lowered to satisfy (9) for all possible price realizations, which in turn would require lower production to satisfy constraint (11).

Proposition 2 demonstrates a second feature of the optimal use of units of account, namely that units of account are passed along in credit chains. Notice that 1 and 2 neither produce nor consume good A. Yet, when they meet in the morning, it is still optimal for them to use good A as the unit of account, because they know that by nightfall they will end up in a credit chain headed by a farmer who does produce good A.

Of course, in our general setting 1 and 2 do not yet know in the morning which type of farmer 1 will meet at night. Thus, when $m_B > 0$, they should choose a unit of account that is as compatible as possible for meetings with both type-A and type-B farmers. The following Proposition characterizes the optimal unit of account for this case.

Proposition 3 *Let $m_B > 0$, so that artisans at location 1 meet both type-A and type-B farmers with positive probability. The solution of the planning problem in Definition 1 is given by a contract between type-i farmers and type-1 artisans as specified in Proposi-*

tion 1 and contracts between type-1 and type-2 artisans that specify:

$$x_1 = h_2 = q_{1,2} = \frac{2(1+\lambda)}{\frac{\overline{p}}{\underline{p}} + 1} < 1$$

and $u_{1,2} = 0.5$*, so that:*

$$\boldsymbol{\pi}_{1,2} = q_{1,2}\ (u_{1,2},\ 1 - u_{1,2})' = \frac{2(1+\lambda)}{\frac{\overline{p}}{\underline{p}} + 1}\ (0.5,\ 0.5)'.$$

That is, the unit of account is an equally weighted bundle of farm goods A and B, and the scale of production is reduced relative to the first best without contracting frictions.

Proof: The constraints that govern $x_1 = h_2$ and $\boldsymbol{\pi}_{1,2}$ are the payment feasibility constraint (2) for 1, and participation constraints for 1 and 2. Taking as the given the optimal contract between 1 and the farmer 1 from Proposition 1, the constraints can be written as:

$$\boldsymbol{p}'\boldsymbol{\pi}_{1,2} \leq p_i(1+\lambda) \quad \forall \boldsymbol{p}, \tag{12}$$

$$1 + \lambda - E\left[\boldsymbol{p}'\boldsymbol{\pi}_{1,2}\right] + (1+\lambda)\, x_1 - 1 \geq 0, \tag{13}$$

$$E\left[\boldsymbol{p}'\boldsymbol{\pi}_{1,2}\right] - x_1 \geq 0. \tag{14}$$

Recall that the objective is to maximize x_1. Given assumptions, constraint (14) can be written as $q_{1,2} \geq x_1$. The optimum then features $x_1 = q_{1,2}$, which also implies that (13) is not binding. $q_{1,2}$ and $u_{1,2}$, in turn, need to be chosen to maximize $q_{1,2}$ subject to constraint (12). The constraint can be rewritten as:

$$q_{1,2} \leq \frac{p_i(1+\lambda)}{p_A u_{1,2} + p_B(1-u_{1,2})} \quad \forall \boldsymbol{p}. \tag{15}$$

Given that the constraint has to be satisfied for all $\mathbf{p}$, the unit of account should be chosen to maximize the minimum of the right-hand side, i.e.:

$$u_{1,2} = \underset{u}{\operatorname{argmax}} \left\{ \min_{i,\boldsymbol{p}} \left\{ \frac{p_i(1+\lambda)}{p_A u + p_B(1-u)} \right\} \right\}.$$

Intuitively, what is being maximized is the minimum of the value of 1's income

(numerator) relative to the value of the unit of account (denominator). Given Assumption 1, we have:

$$\min_{i,\boldsymbol{p}} \left\{ \frac{p_i(1+\lambda)}{p_A u + p_B(1-u)} \right\} = \frac{\underline{p}(1+\lambda)}{\overline{p}\max\{u, 1-u\} + \underline{p}\min\{u, 1-u\}},$$

so that the optimal unit of account is $u_{1,2} = 0.5$. Plugging this back into (15) and taking the minimum with respect to $\mathbf{p}$ on the right-hand side, we get the optimal amount of the payment:

$$q_{1,2} = \frac{2\underline{p}(1+\lambda)}{\overline{p} + \underline{p}} = \frac{2(1+\lambda)}{\frac{\overline{p}}{\underline{p}} + 1},$$

as stated. We also have $q_{1,2} < 1$ because of equation (8) in Assumption 1. □

In the setting illustrated by Proposition 3, the unconstrained first best can no longer be achieved, because the uncertainty about which farmer Artisan 1 will meet leaves Artisan 1 unable to commit to a sufficiently large payment to Artisan 2. Instead, Artisan 1 commits to the largest expected payment that he can fulfill for all price realizations, and production is reduced so satisfy Artisan 2's participation constraint. The reduction of production by Artisan 2 (and the resulting reduction in surplus) represents the social loss due to contracting frictions in this economy.

The proposition demonstrates another important feature of optimal units of account: if there is uncertainty over future trading partners, the chosen unit of account should be one that minimizes the variability of the value of the promised payment relative to the income of the possible trading partners.

4.2 The Optimality of a Dominant Unit of Account for $N > 2$

The results in the previous section illustrate a number of the forces that drive the optimal use of units of accounts in our model economy. In particular, there is a force that favors units of accounts that are correlated with (or, ideally, identical to) the units that denominate the income of important borrowers in the economy (Proposition 1); in a credit chain, units of account should be passed along

(Proposition 2); and if there is uncertainty over future trading partners, units of account should be chosen to minimize the variability of their value relative to the income of the possible trading partners (Proposition 3). We now use these same insights to show how a dominant unit of account emerges in an economy with longer chains of credit. To this end, consider the same setting as above with more types of artisans, $N > 2$. There will now be additional types of meetings, such as between type-2 and type-3 artisans, type-3 and type-4 artisans, and so on. Applying the insights from the results so far, we can see that the optimal unit of account in those additional meetings is the same as in meetings of type-1 and type-2 artisans. All artisans end up in credit chains that begin with a link from a farmer to a type-1 artisan, and continue with a link between type-1 and type-2 artisan. By using the same unit of account as in the 1-2 link further down the chain, all artisans apart from type 1 (who meet farmers) do not face any relative price risk, in the sense that their income (from the contract with their customer) and their liability (from the contract with their supplier) are denominated in the same unit. The following proposition summarizes this result.

Proposition 4 *Let $m_B > 0$, so that artisans at location 1 meet both type-A and type-B farmers with positive probability, and $N > 2$. The solution of the planning problem in Definition 1 is given by a contract between type-i farmers and type-1 artisans as specified in Proposition 1, and a contract between artisans of type $N-1$ and N as specified for types 1 and 2 in Proposition 3, that is:*

$$x_{N-1} = h_N = q_{N-1,N} = \frac{2(1+\lambda)}{\frac{\overline{p}}{\underline{p}} + 1} < 1$$

and $u_{N-1,N} = 0.5$, so that:

$$\boldsymbol{\pi}_{N-1,N} = q_{N-1,N}\ (u_{N-1,N},\ 1 - u_{N-1,N})' = \frac{2(1+\lambda)}{\frac{\overline{p}}{\underline{p}} + 1}\ (0.5,\ 0.5)'\,.$$

For matches of i and $i+1$ with $i < N-1$, we have:

$$q_{i,i+1} = q_{N-1,N} = \frac{2(1+\lambda)}{\frac{\bar{p}}{\underline{p}} + 1},$$
$$u_{i,i+1} = u_{N-1,N} = 0.5,$$
$$x_i = h_{i+1} = min\{(1+\lambda)x_{i+1}, 1\}.$$

That is, with the exception of matches between farmers and type-1 artisans, all matches in the economy use the same unit of account, given by an equally weighted bundle of farm goods A and B. The scale of production is declining within each chain, and production is at full scale at the beginning of the chain if chains are sufficiently long.

Proof: In terms of the choice of the optimal unit of account, the objective is to maximize the amount of resources that can be passed down the chain in terms of payments. The analysis in Proposition 3 of the optimal unit in the 1-2 match given the payment feasibility constraint and Proposition 1 still applies, leading to the same optimal payment and unit of account in the 1-2 matches in the case considered here. To maximize payments further down the chain, it is optimal to use the same unit of account and the same amount of payment in all additional matches. This implies that from artisan 2 onward all artisans pass down the entire payment they receive, which ultimately ends up with artisan N. Given these results, the quantities x_{N-1}, h_N produced and consumed in the match between $N-1$ and N are governed by the participation constraint of N, which is identical to the participation constraint of 2 in Proposition 3, so that the result from that proposition applies here, too. Finally, quantities x_i, h_{i+1} for $i < N-1$ are pinned down by the participation constraint of $i+1$. Given that in the proposed contract the payment is passed down, we have $c_{i+1} = 0$ and the participation constraint is:

$$(1+\lambda)x_{i+1} - h_{i+1} \geq 0,$$

which implies that the maximum possible production is:

$$x_i = h_{i+1} = \min\{(1+\lambda)x_{i+1}, 1\}$$

as stated, where the second term in the minimum on the right hand side is explained by h_{i+1} having to lie in the interval $H = [0, 1]$. □

The reason why the scale of production increases from the end of the chain is that the surplus generated by the production of artisanal goods can be used to elicit higher labor supply. Artisan $N - 1$ consumes artisanal goods in the amount of x_{N-1}, but the utility derived from this is $(1+\lambda)x_{N-1}$. Artisan $N-1$ is thus willing to work up to $(1+\lambda)x_{N-1}$ in exchange for this consumption, implying that artisan $N - 2$ can be provided with higher consumption. This effect propagates along the chain until the full scale of production is reached.

4.3 Decentralizing the Social Optimum as an Equilibrium

So far, we have focused on a planning problem subject to contracting constraints. Implicitly, the outcome can be envisioned as the planner making a contract proposal to all agents who meet, with the agents being able to either accept or reject the proposal. In this section, we show that our main results still go through in a decentralized setting without a planner, where all contracts are decided through bilateral bargaining under specific assumptions on the bargaining process. Specifically, the social optimum is an equilibrium if, in any meeting between a farmer and an artisan, the artisan can make a take-it-or-leave-it offer; and if in meetings between artisans, the customer can make a take-it-or-leave-it offer.

For bargaining to be well defined, we need to take a stand on what happens when offers are made that are off the equilibrium path. For example, consider a night meeting between artisans of type i and $i + 1$. In the morning meeting between $i + 1$ and $i + 2$, $i + 1$ committed to some non-contingent payment $\pi_{i+1,i+2}$. What happens if i makes an offer to $i + 1$ that leaves $i + 1$ unable to fulfill the contract with $i + 2$, because the payment from i to $i + 1$ is too small? Our assumption is that that the legal system seizes all assets of agents that do not fulfill a contract; that is, if $i + 1$ does not pay the contracted amount to $i + 2$, the consumption of this agent will be $c_{i+1} = x_{i+1} = 0$. This implies that all such offers will be rejected. The following definition formalizes the notion of equilibrium.

Definition 2 (Take-it-or-leave-it Equilibrium) *An equilibrium consists of a system of payment promises $\boldsymbol{\pi}_{i,j}(\mathcal{N})$, a labor allocation $h_i(\mathcal{N})$, and a consumption allocation $(c_i(\mathcal{N}, \boldsymbol{p}), x_i(\mathcal{N}))$ such that:*

1. *In each meeting between two artisans of type i and $i+1$, the contract $x_i(\mathcal{N}) = h_{i+1}(\mathcal{N})$, $\boldsymbol{\pi}_{i,j}(\mathcal{N})$ is the offer that maximizes the utility of i subject to:*

   - *The payment feasibility and budget constraints of i, given by:*

$$\begin{aligned} \boldsymbol{p}'\boldsymbol{\pi}_{i,j}(\mathcal{N}) &\leq \boldsymbol{p}'\boldsymbol{\pi}_{h,i}(\mathcal{N}), \\ c_i(\mathcal{N}, \boldsymbol{p}) &= \boldsymbol{p}'\boldsymbol{\pi}_{h,i}(\mathcal{N}) - \boldsymbol{p}'\boldsymbol{\pi}_{i,j}(\mathcal{N}). \end{aligned}$$

   - *The timing constraint requiring that the contract in morning meetings cannot depend on the night matching outcome.*
   - *The requirement that $i+1$ is willing to accept the offer, i.e., the offer leaves $i+1$ at least as well off as rejecting the offer.*

2. *In each meeting between a farmer of type i and an artisan of type 1, the contract $x_i(\mathcal{N}) = h_1(\mathcal{N})$, $\pi_{i,1}(\mathcal{N})$ is the offer that maximizes the utility of artisan 1 subject to:*

   - *The payment feasibility constraint of i and the budget constraint of 1, given by:*

$$\begin{aligned} \boldsymbol{p}'\boldsymbol{\pi}_{i,1}(\mathcal{N}) &\leq p_i(1+\lambda)h_i(\mathcal{N}) \quad \forall \boldsymbol{p}, \\ c_1(\mathcal{N}, \boldsymbol{p}) &= \boldsymbol{p}'\boldsymbol{\pi}_{i,1}(\mathcal{N}) - \boldsymbol{p}'\boldsymbol{\pi}_{1,2}(\mathcal{N}). \end{aligned}$$

   - *The requirement that farmer i is willing to accept the offer, i.e., the offer leaves i at least as well off as rejecting the offer.*

We can now establish our decentralization result.

Proposition 5 *The optimal allocation and contract characterized in Proposition 4 is an equilibrium in the sense of Definition 2.*

The proof for the proposition is provided in the mathematical appendix. Notice that while the proposition shows that the social optimum can be decentralized, it does not state that the optimum is the unique equilibrium. Indeed, there are many other equilibria. One source of multiplicity is precisely that the choice of a unit of account amounts to an economy-wide coordination problem. Consider, for example, an alternative equilibrium in which all artisan-artisan matches coordinate on using only farm good A as the unit of account. Given the expectation that their other meeting partners will be using this unit, it is not profitable for an individual artisan-artisan match to deviate and use the socially optimal bundle instead (indeed, for the same output they would no longer be able to satisfy the payment feasibility constraint for all prices). Thus, there are equilibria in which good A is the dominant unit of account, but these are dominated in terms of social welfare by the optimal equilibrium.

Also notice that the specific bargaining protocol that is adopted here is necessary to generate the maximum social surplus, but it is not crucial for the results regarding the optimal unit of account. Regardless of the distribution of bargaining power, it is always useful to minimize the impact of price fluctuations on agents' ability to meet their obligations. Thus, equilibria under a different distribution of bargaining power would not achieve the social optimum, but the best equilibrium given the bargaining protocol would still be characterized by the same optimal unit of account.

5 Government Debt and the Choice of an Optimal Unit of Account

The preceding analysis has shown how a dominant unit of account leads to better allocations in economies characterized by relative price risk, credit chains, and uncertainty about future trading partners. However, the optimal unit of account turned out to be a bundle of goods. In actual economies, in contrast, the dominant unit of account usually consists of government-issued fiat money. In this section, we explore how our theory can be extended to account for the prominent

role of government paper in real-world units of account.

We believe that government-issued fiat money is a common unit of account for the reason already articulated in Proposition 1: Money denominates a large fraction of the income of important borrowers in the economy. The reason for this is that the government, the largest player in the economy, chooses to denominate its own obligations in terms of the money that it controls. The most important (but not the only) example of such a government obligation is nominal government debt. Issuing debt in nominal terms has clear advantages for the government; nominal debt is implicitly state-contingent (through the government's control of inflation) and can therefore provide insurance for future government spending shocks.[12] If a lot of government debt is in circulation, private agents derive more of their income in nominal terms (through interest payments and principal repayment on government bonds), which makes money more attractive as a unit of account for private transactions, too.

To articulate this mechanism within the framework of our model, we introduce a new actor, the government. To focus on government debt as the optimal unit of account, the only role this government has is to issue IOUs (government debt) and repay them later on. Specifically, in period 1 (i.e., before price uncertainty has been realized) the government acquires a claim on g units of each farmer's output, where $0 < g < 1$, and in exchange issues g units of government IOUs to each farmer. A unit of IOU is defined as a claim on one unit of real government revenue T, where $E_0(T) = 1$. The revenue T (in terms of the consumption good C) is stochastic. We do not model the origin of the revenue (i.e., it derives from taxation or production activities unrelated to the agents in the model), although this could be generalized.

The bottom line is that a farmer of type i, rather than deriving all income from farm good i, now derives income partially from the farm good and partially from the government IOU. Since the expected tax revenue (and hence the expected value of an IOU) equals one, this does not change the farmer's expected income. However, the presence of government IOUs does change the optimal unit of ac-

[12]The insurance role of nominal government debt in a stochastic macroeconomic environment was first pointed out by Bohn (1988).

count in the economy.

The attractiveness of government paper as a unit of account depends not only on the amount of IOUs in circulation (measured by g), but also on the volatility of the price of IOUs. What matters here is the price at which IOUs trade in the spot market where all payments are settled. In period 1, before the spot market opens, news about government revenue arrives. Given that agents are not risk averse with respect to the consumption of good C, the expected value of revenues $E_1(T)$ pins down the price of IOUs in the spot market:

$$p_{IOU} = E_1(T).$$

The realization of government revenue T takes place at the end of period 2, after the spot market closes, but before consumption takes place.

To characterize sharply the optimal unit of account in the economy with government debt, we place the following assumptions on the distributions of farm-good prices and the price of IOUs:

Assumption 2 *In addition to the conditions in Assumption 1, the distribution of farm-good prices satisfies:*

$$\frac{\underline{p} + \overline{p}}{2} > 1. \tag{16}$$

The distribution of prices for government IOUs (i.e., the distribution of $E_1(T)$) is independent of farm-good prices and has support $p_{IOU} \in [\underline{p}_{IOU}, \overline{p}_{IOU}]$ with $0 < \underline{p}_{IOU} < 1$ and $E_0(p_{IOU}) = 1$.

The independence assumption is for simplicity, and we will discuss the role of condition (16) below. We are now ready to characterize the optimal unit of account in the economy with circulating government paper.

Proposition 6 *Let $m_B > 0$ and let the distributions of farm-good and IOU prices satisfy Assumption 2. Consider a version of the planning problem in Definition 1 for the economy with government IOUs in circulation where government IOUs can serve as a unit of account, so that payment promises are given by:*

$$\boldsymbol{\pi}_{i,j} = (\pi_{i,j}^{IOU},\ \pi_{i,j}^{A},\ \pi_{i,j}^{B})' = q_{i,j}\ (u_{i,j}^{IOU},\ u_{i,j}^{A},\ u_{i,j}^{B})'$$

with $u_{i,j}^{IOU} + u_{i,j}^{A} + u_{i,j}^{B} = 1$, $u_{i,j}^{IOU}, u_{i,j}^{A}, u_{i,j}^{B} \geq 0$. *We then have:*

- *If* $\overline{p}_{IOU} \leq \frac{\underline{p}+\overline{p}}{2}$, *the optimal unit of account in all artisan-artisan matches is given by government IOUs, i.e., we have* $u_{i,j}^{IOU} = 1$ *and payments can be written as:*

$$\boldsymbol{\pi}_{i,j} = q_{i,j}\ (1,\ 0,\ 0).'$$

- *If* $\overline{p}_{IOU} > \frac{\underline{p}+\overline{p}}{2}$, *the optimal unit of account in all artisan-artisan matches is given by:*

$$u_{i,j}^{IOU} = \frac{g}{g + (1 + \lambda - g)\frac{2\underline{p}}{\underline{p}+\overline{p}}} \equiv \tilde{u}^{IOU}, \tag{17}$$

$$u_{i,j}^{A} = u_{i,j}^{B} = \frac{1 - u_{i,j}^{IOU}}{2}. \tag{18}$$

That is, if real government revenue and hence the price of government paper is relatively stable, IOUs are the sole unit of account. If the price of IOUs is volatile, the optimal unit of account is a combination of IOUs and an equally-weighted bundle of farm goods, with the weight on IOUs increasing in the amount g *of IOUs in circulation.*

The proof for the proposition is provided in the mathematical appendix.

Of course, it might be the case that condition (16) does not hold, in which case it is possible that IOUs do not enter the optimal unit of account. However, if we generalize the model to allow for many farm goods, it is plausible that (16) will hold. The issue here is the value of an equally-weighted bundle of farm goods in the worst-case scenario in terms of meeting the payment feasibility constraint. If we maintain the assumption that all prices are independent and lie in an interval $[\underline{p}, \overline{p}]$, there is always a sufficient number of farm goods such that in the worst-case scenario the price of the bundle is above 1.

To gain intuition for the result in Proposition 6, consider the case in which there is no uncertainty about government revenue, so that $p_{IOU} = 1$ in all states, and compare the option of using either IOUs as a unit of account or an equally weighted bundle of farm goods A and B. Consider an artisan of type 1 who has met a

farmer of type A. For this artisan, the payment feasibility constraint is binding if the price of good A is at the minimum of $\underline{p}$ and the price of farm good B is at the maximum of $\overline{p}$. If we now have $(\underline{p}+\overline{p})/2 > 1$ (which is condition (16) in Assumption 2), in this state the value of a bundle of A and B is higher than the value of an IOU. Thus, if IOUs are the unit of account, the ratio of A's income to the value of unit of account is higher, and larger expected payments can be supported. Put differently, if condition (16) is satisfied, the average value of a bundle of A and B is lower than the worst-case value, which makes the bundle unattractive as a unit of account. More generally, IOUs are the optimal unit of account as long as (16) is satisfied and the volatility of p_{IOU} is low.

If we increase the volatility of p_{IOU}, ultimately a point is reached at which IOUs are no longer the optimal unit of account. From that point on, the unit of account is chosen such that the ratio of the income of artisan 1 to the value of the unit of account is independent of p_{IOU}. This requires that the weighting of IOUs in the unit of account is increasing in the quantity g of IOUs in circulation.

To translate these results into more familiar terms, we can refer to an IOU as a "euro." If the euro is the unit of account and the measured consumption basket consists only of good C (given that artisanal goods do not have quoted prices and farm goods are exported), the consumer price index is $CPI = (p_{IOU})^{-1}$. A high volatility of the price of euros then translates into a volatile CPI, i.e., volatile inflation. The worst-case scenario that drives the choice of the optimal unit of account is one of a high p_{IOU} and hence a low CPI. Intuitively, when the euro is the unit of account, artisan 1 promises a fixed number of euros to artisan 2. If now realized inflation is low or negative (deflation), the real value of that euro-denominated promise is high, possibly leading to a binding payment feasibility constraint (as in Fisher's debt-deflation theory). If inflation becomes too volatile, the euro ultimately is no longer the optimal unit of account. This is akin to the dollarization of an economy when the local currency becomes overly volatile, and alternative units of account (such as foreign currency) start to be used.[13]

[13]See Neumeyer (1998) for a general-equilibrium analysis of the breakdown of trade in nominal assets when inflation risk becomes too high.

6 Small Default Costs and Optimal Currency Areas

Up to this point, all our results were derived for an economy in which default costs are large, so that only non-contingent contracts can be used. We now examine the robustness of our findings by considering outcomes with small default costs, in the sense that the cost of breaking promises is sufficiently small for breaking promises to be optimal in some cases. In particular, we examine a setting in which default costs are sufficiently small for production to reach the first-best level. Even in this case, the unit of account still matters, because an appropriately chosen unit of account is necessary to minimize the cost of settling contracts in the economy.

For the most part, our results for large default costs in Section 4 carry over unchanged to the small-cost setting. The key differences arise when we consider what the optimal unit of account should look like. In the small-default-cost setting, the objective is to minimize the probability of default, but not to avoid default entirely. This implies that the probability of meeting different types of agents becomes an important determinant of the optimal unit of account. In Section 6.2, we exploit this feature to apply our model to the issue of optimal currency areas.

6.1 The Optimal Unit of Account with Small Default Costs

Consider our general setting under the restrictions $0 < \kappa < \lambda$ and $h \in H = \{0, 1\}$, i.e., labor is indivisible. These two assumptions jointly imply that production is always socially optimal: Given $\kappa < \lambda$, the surplus generated by a given agent's production exceeds the cost of settling the contract, even if the customer is in default with probability one. As a consequence, solving the planning problem in Definition 1 amounts to arranging contracts to minimize the probability of default. The following proposition shows that the solution to the planning problem inherits many features from the case of non-contingent contracts. In particular, in the optimum farmers promise their entire harvest to their customers, and a common unit of account is used in all artisan-artisan matches.

Proposition 7 *Let $0 < \kappa < \lambda$ and $h \in H = \{0, 1\}$. The solution to the planning problem in Definition 1 is such that:*

1. *All agents work, i.e., $h_i = 1$ for all agents i.*

2. *All farmers promise their entire harvest and then indeed pay their entire harvest. That is, for a farmer of type i we have:*

$$\boldsymbol{p}'\boldsymbol{\pi}_{i,1} = v_{i,1}(\mathcal{N}, \boldsymbol{p}) = p_i(1 + \lambda).$$

3. *In all matches between an artisan i and an artisan j, the same promise is made, given by:*

$$\boldsymbol{\pi}_{i,j} = \underset{\boldsymbol{\pi}}{\operatorname{argmax}} \left\{ \sum_{h=A,B} m_h \Pr\left[p_h\left(1 + \lambda\right) \geq \boldsymbol{p}'\boldsymbol{\pi}\right] \right\} \equiv \bar{\boldsymbol{\pi}},$$

   where the maximization is subject to:

$$E\left[\min\left\{p_h\left(1 + \lambda\right), \boldsymbol{p}'\boldsymbol{\pi}\right\}\right] \geq 1$$

   and the expectation is over price realizations and over the type of farmer h.

4. *The actual payment made by an artisan i who ends up in a chain headed by farmer of type h is given by:*

$$v_{i,j}(\mathcal{N}, \boldsymbol{p}) = \min\left\{\boldsymbol{p}'\bar{\boldsymbol{\pi}}, p_h\left(1 + \lambda\right)\right\}.$$

The proof for the proposition is provided in the mathematical appendix.

To summarize, most of the findings for the large-default-cost case in Proposition 1, Proposition 3, and Proposition 4 go through also for the small-cost case. In particular, in meetings between farmers and type-1 artisans it is optimal to specify the farmer's entire income as the payment by using the farmer's good as the unit of account, and it is optimal to use the same unit of account (and indeed the same exact payment) in all artisan-artisan matches. The main differences are that in the large-cost case the first-best level of production can be achieved, and

that different units of account may be optimal in the two cases. The exact implication of Proposition 7 for what the optimal unit of account should be depends on parameters. When prices satisfy Assumption 1 and there are equal numbers of the two types of farmers, $m_A = m_B = 1$, an equally-weighted bundle of the two farm goods is often optimal, as in the large-cost case. However, even in such a symmetric case it is possible that using just one of the goods as unit of account yields higher utility. When we move away from the symmetric case and increase the number of type-A farmers, there is always a threshold for m_A above which using farm-good A in all artisan-artisan matches is optimal.

We now continue to an extension of our model where the probability of meeting particular types of people plays a central role.

6.2 Optimal Currency Areas

So far, we have assumed that matching at each link is entirely random; meetings between an artisan of type i and any given artisan of type $i+1$ are equally likely. We now consider an extension of our model with an additional spatial structure, where agents live in two different regions, and are more likely to be matched to people within the region than to those outside it. In this setting, a tension arises between adopting a "global" unit versus adopting several "regional" units of account that are more suited to local conditions.[14] The analysis therefore leads to a theory of optimal currency areas, where the optimality of a common unit of account depends on the degree of specialization across countries and on the intensity of cross-border links.

We consider a simple example. Let $N = 2$, and let $p_A, p_B \in \{1-\theta, 1, 1+\theta\}$, where each state has probability one-third and price realizations for the two goods are independent. Let $m_A = m_B = 0.5$, $0 < \lambda < 0.4$, and let:

$$\theta = \lambda + \epsilon, \tag{19}$$

[14]Related issues arise in the search-theoretic models of Matsuyama, Kiyotaki, and Matsui (1993) and Wright and Trejos (2001), in which money is used as a medium of exchange.

with $\epsilon > 0$. Different from our original setup, assume that artisans are located in two regions, A and B, corresponding to farmers of type A and B. In the morning, artisans of type 1 and 2 meet within their region, i.e., type-1 artisans in region A meet type-2 artisans in region A, and the same for region B. Then, in the evening with probability $1 - x$, where $0 \leq x \leq 0.5$, an artisan of type 1 meets a farmer in his own region, and with probability x he meets a farmer from the other region. We can now show that the optimality of adopting a common unit of account for both regions depends on the intensity of cross-border trade x.

Proposition 8 *If $x = 0$ (no trade), for a sufficiently small ϵ the solution to the planning problem is such that all artisan-artisan matches in region A adopt farm good A as the unit of account, and in region B farm good B is adopted as the unit of account for artisan-artisan matches (i.e., separate currencies). There is a threshold $\bar{x}$ such that for $x \geq \bar{x}$, it is optimal to adopt an equally-weighted bundle of farm goods A and B as the unit of account for artisan-artisan matches across both regions (i.e., currency union).*

The proof for the proposition is provided in the mathematical appendix.

The result that the benefits of a currency union increase in the intensity of cross-border trade x is perhaps unsurprising. However, when we expand the analysis to the case $N > 2$ we get the additional implication that the benefits of a currency union are increasing also in the length of credit chains. Consider a simple extension of the setup in which the probability x of meeting someone from the other region applies, separately, to each level of the chain of artisans. This implies that as N increases, for a given x there is an increase in the probability that each credit chain contains at least one agent from the other region. Thus, for fixed x the benefits of a currency union are increasing in N.

7 Conclusions

The theoretical framework described in this paper provides a basic rationale for why adopting a dominant unit of account can be optimal. We have shown why circulating government paper (such as fiat money) can arise as an optimal unit

of account. We have also demonstrated that when the value of government paper is overly volatile (i.e., volatile inflation) agents may be better off adopting a different unit of account, as can occur when people in countries with unstable monetary policy adopt foreign currency as a unit of account. Our analysis, then, provides a new mechanism for the costs of monetary instability. In future work, we plan to examine quantitatively the implications of our theory for the welfare implications of different policy regimes.

In another application, we examined the implications of our theory for optimal currency areas, i.e., we have derived conditions under which two regions are better off under a common unit of account. Unlike in baseline models of optimal currency areas that take as exogenously given the economic benefits of a unified currency in terms of reducing trade costs (see for example Alesina and Barro 2002), our theory provides an explicit mechanism for how unification affects trade.

One could extend the theoretical framework described here in a number of directions. For example, some agents might have access to collateral that allows them to bear additional risk. The design of a unit of account could then be used to help allocate risk to those who can bear it, and to help economize on the overall need for collateral. Also, in real-world economies the balance-sheet risks that are central to our theory are highly concentrated among specific economics actors, such as banks. Extending the model to make the role of banks and other financial intermediaries explicit would be a promising avenue for future research on money as a unit of account.

A Mathematical Appendix

A.1 Proofs for Propositions

Proof of Proposition 5: We already know that the optimal allocation characterized in Proposition 4 satisfies budget constraints, payment feasibility constraints, and timing constraints, because these constraints are imposed also on the plan-

ning problem in Definition 1. We still need to show that each contract also maximizes the utility of the customer (who makes the take-it-or-leave-it offer) subject to the supplier being willing to accept the offer.

Consider first the meetings at the end of the chain between artisans of type $N-1$ and N. In the proposed equilibrium, artisan $N-1$ (who is making the offer) knows that he is to receive a payment characterized by:

$$q_{N-2,N-1} = \frac{2(1+\lambda)}{\frac{\bar{p}}{\underline{p}}+1} \equiv \bar{q},$$
$$u_{N-2,N-1} = u_{N-1,N} = 0.5$$

from an artisan of type $N-2$. When proposing a contract to N, it is optimal for $N=1$ to use the same unit of account $u_{N-1,N} = 0.5$, because N cares only about the amount of the payment, and choosing a different unit of account can only make it more difficult to for $N-1$ to satisfy his payment feasibility constraint. The contract that $N-1$ proposes is then characterized by $q_{N-1,N}$ and $x_{N-1} = h_N$ and is chosen to solve:

$$\max\{(1+\lambda)x_{N-1} - q_{N-1,N}\}$$

subject to:

$$q_{N-1,N} \leq q_{N-2,N-1} = \bar{q},$$
$$q_{N-1,N} \geq x_{N-1},$$

where the first constraint is the payment feasibility constraint and the second constraint states that N has to be willing to accept the contract. The fact that the objective is increasing in x_{N-1} together with the last constraint implies that the offer entails $q_{N-1,N} = x_{N-1}$, and substituting this in the objective implies that x_{N-1} and $q_{N-1,N}$ should be as large as possible, so that given the payment feasibility constraint we have:

$$x_{N-1} = q_{N-1,N} = q_{N-2,N-1} = \bar{q},$$

as required.

Consider, next, the meetings between artisans of type $N-2$ and $N-1$. The customer $N-2$ knows that he will receive a payment $\bar{q}$. If $N-2$ and $N-1$ meet at night, $N-1$ (as explained above) has already promised $\bar{q}$ to N. Thus, the only payment that is acceptable to $N-1$ is $\bar{q}$. This payment also satisfies the payment feasibility constraint. The quantity x_{N-1} that is specified in the offer by $N-2$ has to be such that $N-1$ is at least indifferent between accepting and refusing the offer (which would yield zero utility for $N-1$). Given the contract between $N-1$ and N, this is accomplished by setting:

$$x_{N-2} = (1+\lambda)x_{N-1},$$

as required. If, alternatively, $N-2$ and $N-1$ meet in the morning (which happens if N is odd), the optimal offer is the same, because this maximizes the surplus of $N-2$ subject to keeping $N-1$ indifferent. The same logic applies to matches closer to the beginning of the chain. Finally, in matches between farmers of type i and artisans of type 1 (which occur at night), the artisans (who make the offers) have already promised $\bar{q}$ to 2. Hence, they will make an offer that allows them to make this payment for all price realizations, which requires a payment of the entire harvest $p_i(1+\lambda)$. To make the farmer indifferent, 1 then has to promise to deliver $x_i = h_1 = 1$ units of artisanal goods to the farmer, as required. □

Proof of Proposition 6: Following the same reasoning as in the proof of Proposition 1, in matches between farmers and artisans of type 1 it is optimal to choose the unit of account such that the entire income of the farmer can be passed on to 1. Thus, if artisan 1 meets farmer i, artisan 1 will receive g units of government IOUs and $1+\lambda-g$ units of farm good i. For the reasons articulated in Proposition 4, it will also be optimal to use the same unit of account in all artisan-artisan matches. As in the proof of Proposition 3, this dominant unit of account should be chosen to maximize the payment that can be passed on from type-1 artisans to other artisans. The unit of account should therefore be given by:

$$\{u_{i,j}^{IOU},\ u_{i,j}^{A},\ u_{i,j}^{B}\} = \underset{u^A,\,u^B,\,u^{IOU}}{\operatorname{argmax}} \left\{ \min_{i,\boldsymbol{p},p_{IOU}} \left\{ \frac{gp_{IOU} + (1+\lambda-g)p_i}{u^{IOU}p_{IOU} + u^A p_A + u^B p_B} \right\} \right\}$$

subject to $u^{IOU}_{i,j} + u^{A}_{i,j} + u^{B}_{i,j} = 1$ and $u^{IOU}_{i,j}, u^{A}_{i,j}, u^{B}_{i,j} \geq 0$. As before, what is being maximized is the minimum of the value of 1's income (numerator) relative to the value of the unit of account (denominator). Given the symmetric price distribution for goods A and B (Assumption 1), it is optimal to set:

$$u^A = u^B = \frac{1 - u^{IOU}}{2}.$$

We then have:

$$\begin{aligned}
\min_{i,p,p_{IOU}} \left\{ \frac{g p_{IOU} + (1 + \lambda - g) p_i}{u^A p_A + u^B p_B + u^{IOU} p_{IOU}} \right\} &= \min_{p_{IOU}} \left\{ \frac{g p_{IOU} + (1 + \lambda - g)\underline{p}}{u^{IOU} p_{IOU} + (1 - u^{IOU})\frac{\underline{p}+\overline{p}}{2}} \right\} \\
&= \begin{cases} \frac{g\overline{p}_{IOU} + (1+\lambda-g)\underline{p}}{u^{IOU}\overline{p}_{IOU} + (1-u^{IOU})\frac{\underline{p}+\overline{p}}{2}} & \text{if} \quad u^{IOU} > \tilde{u}^{IOU} \\ g + (1+\lambda-g)\frac{2\underline{p}}{\underline{p}+\overline{p}} & \text{if} \quad u^{IOU} = \tilde{u}^{IOU} \\ \frac{g\underline{p}_{IOU} + (1+\lambda-g)\underline{p}}{u^{IOU}\underline{p}_{IOU} + (1-u^{IOU})\frac{\underline{p}+\overline{p}}{2}} & \text{if} \quad u^{IOU} < \tilde{u}^{IOU}. \end{cases}
\end{aligned}$$

Now notice that for p_{IOU} fixed, the expression on the right-hand side is monotonic in u^{IOU}. Hence, if it is optimal to set $u^{IOU} > \tilde{u}^{IOU}$, the best choice is $u^{IOU} = 1$, and similarly if $u^{IOU} < \tilde{u}^{IOU}$ is optimal the best choice is $u^{IOU} = 0$. Setting $u^{IOU} = 0$ would be better than $u^{IOU} = \tilde{u}^{IOU}$ if the inequality:

$$\frac{g\underline{p}_{IOU} + (1 + \lambda - g)\underline{p}}{\frac{\underline{p}+\overline{p}}{2}} > g + (1 + \lambda - g)\frac{2\underline{p}}{\underline{p} + \overline{p}}$$

held, which can be simplified to:

$$\underline{p}_{IOU} \geq \frac{\underline{p} + \overline{p}}{2},$$

which is ruled out by assumption Assumption 2 (recall that $\underline{p}_{IOU} \leq 1$). Thus, the possible optima consist of $u^{IOU} = 1$ and $u^{IOU} = \tilde{u}^{IOU}$. Setting $u^{IOU} = 1$ is better than $u^{IOU} = \tilde{u}^{IOU}$ if the inequality:

$$\frac{g\overline{p}_{IOU} + (1 + \lambda - g)\underline{p}}{\overline{p}_{IOU}} \geq g + (1 + \lambda - g)\frac{2\underline{p}}{\underline{p} + \overline{p}}$$

holds, which can be solved for:

$$\overline{p}_{IOU} \leq \frac{\underline{p} + \overline{p}}{2},$$

which is the condition stated in the proposition. □

Proof of Proposition 7: We first check that the proposed contract satisfies all constraints. First, notice that all payment feasibility constraints are satisfied. If in a chain headed by a farmer of type h we have $\boldsymbol{p}'\bar{\boldsymbol{\pi}} \geq p_h(1+\lambda)$, then all payments are equal to the harvest, $v_{i,j}(\mathcal{N}, \boldsymbol{p}) = p_h(1+\lambda)$. In contrast, if $\boldsymbol{p}'\bar{\boldsymbol{\pi}} < p_h(1+\lambda)$, then the farmer pays the harvest, and all artisans pay $\boldsymbol{p}'\bar{\boldsymbol{\pi}}$ which is strictly less than what artisan 1 receives.

Next, notice that farmers' participation constraints hold as an equality because they receive utility λ (they work and consume their artisanal good for sure), the expected utility of their outside option. Artisans i with $1 < i < N$ want to participate since they receive utility λ for sure (they work and consume the artisanal good), which is strictly better than their outside option of zero. Artisans of type N are indifferent to participation since they receive zero expected utility (they work and receive a payment that equals one in expectation), the same as their outside option of zero. Artisans of type 1 want to participate since they obtain utility:

$$\lambda + E\left[\max\left\{p_h(1+\lambda) - \boldsymbol{p}'\bar{\boldsymbol{\pi}}, 0\right\}\right],$$

that is, they work, consume the artisanal good, and receive a random payment that is positive in expectation, strictly better than the outside option of zero.

We now show that the proposed contract is optimal. Consider first the property that all agents work $h = 1$. Under the proposed contract, the type-1 artisans must work $h = 1$. Suppose to the contrary that there is an optimal contract such that a type-1 artisan does not work. From the farmer's participation constraint such an artisan does not receive a payment, and by payment feasibility the artisan cannot make any payment, so that artisan 2 does not work either. We can then change the contract by having artisan 1 work and the farmer work and pay the harvest $p_h(1+\lambda)$ to the artisan. This is feasible, satisfies the farmer's and artisan 1's participation constraints, and does not affect the artisan 2. It also increases surplus

by λ. Under the proposed contract, artisan 2 must also work $h = 1$. Suppose to the contrary that there is an optimal contract in which artisan 2 does not work. Then we can change the contract as sketched above and promise artisan 2 $\bar{\boldsymbol{\pi}}$. This is again feasible and increases surplus by at least $\lambda - \kappa > 0$. Under the optimal contract, all artisans of type $i > 2$ must also work. This follows by induction on the position of an artisan. We have already shown that artisan 2 must work. Suppose we know that all artisans up to i must work, but that there is an optimal contract in which artisan $i + 1$ does not work. Then we can change the contract as above for all artisans up to i. We can then offer $\bar{\boldsymbol{\pi}}$ to artisan $n + 1$ and realize at least additional surplus $\lambda - \kappa > 0$.

Now turn to the payments. It is not possible to have the farmer pay more than the harvest. There is no gain from lowering the farmer's payment, since this will lower the funds that artisan 1 has and thus make default more likely. Therefore it is (weakly) optimal to have the farmer promise and pay the entire harvest. There is no gain from making promised payments by artisans $n > 1$ different from the promise made by artisan 1. This can only increase the probability of default, and does not increase labor. Finally, suppose there is an optimal contract in which artisan 1 promises some payment $\boldsymbol{\pi} \neq \bar{\boldsymbol{\pi}}$. Since the contract is optimal, the payments by artisans $2, .., N - 1$ must also be equal to $\boldsymbol{\pi}$. Moreover, since artisan N must work under the optimal contract and payments must be feasible, the constraint in the statement of the proposition must hold. But then we cannot improve upon $\bar{\boldsymbol{\pi}}$ which already minimizes the probability of default. □

Proof of Proposition 8: When separate units of accounts are adopted in each region, payments are always feasible when an artisan of type 1 meets a farmer from the same region. When artisan 1 meets a farmer from the other region, under separate units of account condition (19) implies that a default occurs whenever the price realization of the foreign good is higher than that of the home good, and for sufficiently small ϵ no default occurs in other states. The overall probability of default when adopting the home as unit of account is then $\frac{x}{3}$. Conversely, when the equally weighted A-B bundle is adopted as unit of account, for sufficiently small ϵ default occurs only when artisan 1 meets the farmer whose farm good has the lowest price realization and the other good has the highest price realization

(the condition $\lambda < 0.4$ guarantees that ϵ can be chosen this way). The probability of default is therefore $\frac{1}{9}$. Hence, adopting the equally weighted bundle of A and B as the common unit of account is optimal whenever:

$$\frac{x}{3} \geq \frac{1}{9},$$

that is, when:

$$x \geq \bar{x} \equiv \frac{1}{3}.$$

□

A.2 Optimal Contracts when the Consumption Good Can Serve as Unit of Account

So far, we have restricted payment promises to consist of vectors of farm goods. However, in the centralized market at date 2, in addition to farm goods A and B the consumption good C also is traded. The value of this good is therefore well defined, so that good C could also serve as a unit of account. We now discuss how our results are modified if agents are able to make promises in terms of good C.

To start, notice that the results in Proposition 1, Proposition 2 go through unchanged even if good C is available, that is, in matches between artisans and farmers it is still optimal to use the farmer's good as the unit of account, and the same unit should be passed on in chains of credit if there are only type-A farmers in the economy. Moreover, analogous versions of Proposition 4 and Proposition 5 go through as well. That is, in long credit chains it is optimal to use the same unit of account in all matches between artisans, and the optimum can be decentralized through a system of take-it-or-leave-it offers. The only finding that depends on the available units of account is the result in Proposition 3 that the optimal unit of account is an equally-weighted bundle of farm goods A and B. If good C is available as well, the optimal unit turns out to consist either of an equally weighted bundle of A and B (as before), or entirely of good C. Given that Proposition 4 applies to this case as well, for simplicity we state the result for $N = 2$ (there are two types of artisans).

Proposition 9 *Let $m_B > 0$ and $N = 2$. Consider a variation of the model in which good C can serve as the unit of account. That is, payment promises consist of vectors:*

$$\boldsymbol{\pi}_{i,j} = \left(\pi^A_{i,j},\ \pi^B_{i,j},\ \pi^C_{i,j}\right)',$$

where $\pi^C_{i,j}$ is a promise in terms of consumption good C. If the condition:

$$\frac{\underline{p} + \overline{p}}{2} < 1 \tag{20}$$

is satisfied, the solution of the planning problem in Definition 1 is the one given in Proposition 3, i.e., the optimal unit of account for matches between type-1 and type-2 artisans is an equally-weighted bundle of farm goods A and B. If (20) *is violated, it is optimal to use only good C as the unit of the account in artisan-artisan matches, and the contract between 1 and 2 is given by:*

$$\begin{aligned} x_1 = h_2 = q_{1,2} &= \underline{p}(1+\lambda) < 1, \\ \boldsymbol{\pi}_{1,2} &= \underline{p}(1+\lambda)\ (0,\ 0,\ 1)'. \end{aligned}$$

Proof: The constraints that govern $x_1 = h_2$ and $\boldsymbol{\pi}_{1,2}$ are the payment feasibility constraint (2) for 1 and participation constraints for 1 and 2. The payment can be parameterized as:

$$\boldsymbol{\pi}_{1,2} = q_{1,2}\ \left(u^A,\ u^B,\ u^C\right)'$$

with $u^i \geq 0$ and $u^A + u^B + u^C = 1$. The participation constraint for 2 requires that $q_{1,2} \geq x_1$, and as in the proof of Proposition 3 the participation constraint for 1 is not binding in the optimum. The optimum then features $x_1 = q_{1,2}$, and the unit of account needs to be chosen to maximize $q_{1,2}$ subject to the payment feasibility constraint. The constraint can be written as:

$$q_{1,2} \leq \frac{p_i(1+\lambda)}{p_A u^A + p_B u^B + u^C} \quad \forall \boldsymbol{p}.$$

Given that the constraint has to be satisfied for all $\mathbf{p}$, the unit of account should

be chosen to maximize the minimum of the right-hand side, i.e.:

$$u_{1,2} = \underset{u}{\operatorname{argmax}} \left\{ \min_{i,\boldsymbol{p}} \left\{ \frac{p_i(1+\lambda)}{p_A u^A + p_B u^B + u^C} \right\} \right\}$$

As before, what is being maximized is the minimum of the value of 1's income (numerator) relative to the value of the unit of account (denominator). Given the symmetric price distribution for goods A and B (Assumption 1), it is optimal to set:

$$u^A = u^B = \frac{1-u^C}{2}.$$

We then have:

$$\min_{i,\boldsymbol{p}} \left\{ \frac{p_i(1+\lambda)}{p_A u^A + p_B u^B + u^C} \right\} = \frac{\underline{p}(1+\lambda)}{\frac{\underline{p}+\overline{p}}{2}(1-u^C) + u^C}.$$

Notice that u^C only appears in the denominator. To maximize the expression, the denominator should be minimized; it is therefore optimal to set $u^C = 0$ if (20) is satisfied, resulting in $u^A = u^B = 0.5$ as in Proposition 3, whereas $u^C = 1$ is optimal if the condition is violated, implying that good C serves as the unit of account. □

Intuitively, what drives the choice of unit of account is the value of the unit in the worst-case scenario when one of the farm goods has the lowest price realization $\underline{p}$ while the other has the highest possible price $\overline{p}$. The advantage of using the bundle of farm goods as the unit of account is that the inclusion of the farm good with the low price realization pulls the value of the unit of account down, too; the disadvantage is that the inclusion of the other farm good pulls the value in the opposite direction. If $(\underline{p}+\overline{p})/2 > 1$, the second effect dominates, and using good C (which does not fluctuate in price) is preferable to using the bundle of farm goods.

The intuition is parallel to the result in Proposition 6 that under the same condition on prices and low volatility of the price of IOUs, government paper is preferable to a bundle of farm goods as the unit of account. If both good C and government IOUs could serve as unit of account and the condition on prices is satisfied, the optimal unit of account is a bundle of C and IOUs with the weight

on IOUs increasing in the amount g of IOUs in circulation.

We focus on bundles of farm goods and on IOUs as possible units of account in the main analysis because the central idea behind our contracting setup is that promises should be simple, i.e., be in terms of at most a few goods. While contracting in terms of C is simple in the theory, we regard good C as a stand-in for a large consumption basket, the price of which would be difficult to determine.

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