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Explaining Educational Attainment across Countries and over Time

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Abstract

Consider the following facts. In 1950 the richest ten-percent of countries attained an average of 8.1 years of schooling whereas the poorest ten-percent of countries attained 1.3 years, a 6-fold difference. By 2005, the difference in schooling declined to 2-fold. The fact is that schooling has increased faster in poor than in rich countries. What explains educational attainment differences across countries and their evolution over time? We develop an otherwise standard model of human capital accumulation with two novel but important features: non-homotetic preferences and an operating labor supply margin. We use the model to assess the quantitative contribution of productivity and life expectancy differences across countries in explaining educational attainment. Calibrating the parameters of the model to reproduce the historical time-series data for the United States, we find that the model accounts for 96 percent of the difference in schooling levels between rich and poor countries in 1950 and 89 percent of the increase in schooling over time in poor countries. The model generates a faster increase in schooling in poor than in rich countries consistent with the data. These results highlight the role of development in education and thus have important implications for educational policy.

Keywords: schooling, productivity, life expectancy, education policy, labor supply.
JEL codes: O1, O4, E24, J22, J24.

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1 Introduction

Human capital accumulation is believed to play a crucial role in understanding income differences across countries, although a precise assessment of this importance has been hindered by the lack of empirical measures of human capital.[1] A key component of human capital formation is investment in schooling. Cross-country data on schooling indicates that although educational attainment is substantially lower in poor than in rich countries, over time poor countries have increased schooling faster than rich countries. The reduction in the dispersion of schooling levels is a fact that is difficult to account for using existing explanations of schooling differences across countries. To account for these facts, we develop an otherwise standard model of human capital accumulation extended to allow for non-homothetic preferences and an operating labor supply margin. We use the model to assess the quantitative significance of differences in productivity and life expectancy in explaining educational attainment across countries and over time. We find that the model accounts for 96 percent of the difference in schooling levels between rich and poor countries in 1950 and 89 percent of the increase in schooling over time in poor countries. The model generates a faster increase in schooling in poor than in rich economies. These implications of the model are consistent with cross-country data and have important implications for educational policy.

We combine education data from Barro and Lee (2010) and income per capita data from the Board (2010) to construct a panel dataset for 84 countries from 1950 to 2005. We emphasize three facts. First, there are large differences in schooling measures across countries, with average schooling being 8 years in rich countries and only 1 year in poor countries in 1950 (11 and 5 years in rich and poor countries in 2005). Second, average years of schooling have increased over time in all countries in our sample. Third, average years of schooling

[1] Important exceptions include Hendricks (2002) and Schoellman (2012). See also recent quantitative work by Manuelli and Seshadri (2006) and Erosa et al. (2010).

have increased more in poor than in rich countries. Hence, dispersion in schooling levels has decreased overtime. What can explain schooling differences across countries and their evolution over time?

We develop a model of human capital accumulation to account for these facts. The basic features of the model are standard and build closely from Bils and Klenow (2000).[2] There are two novel and noteworthy departures from the existing literature. First, the model features an income effect from non-homothetic preferences. Such preferences are central in theories of the allocation of labor across sectors associated with structural change and we argue are important in understanding the allocation of time between the production of goods and schooling across rich and poor countries.[3] Second, labor supply is endogenous. This feature is important because as the available data from the International Labor Organization show there are large cross-country differences in hours of work –average hours are lower in rich than in poor countries– thereby affecting schooling decisions. Broadly speaking, consider schooling as a time-allocation problem whereby a unit of time is allocated between the production of consumption goods, schooling, and leisure. Then, with non-homotetic preferences, increases in productivity lead to a reallocation of time away from the production of goods into schooling and leisure. Other things equal, abstracting from leisure over estimates the impact of increases in productivity on schooling (and hence over estimates the income elasticity of schooling). Other authors, such as Heckman (1976) and Blinder and Weiss (1976), have emphasized the importance of jointly modeling labor supply and human capital accumulation in bringing additional quantitative discipline to human capital models.

[2]The model builds broadly from the seminal works of Becker (1962), Ben-Porath (1967), and Mincer (1974). A recent quantitative version of these models applied to cross sectional inequality in the United States include for example Guvenen and Kuruscu (2010).

[3]For applications in development, see for instance Laitner (2000), Kongsamut et al. (2001), Gollin et al. (2002), and Duarte and Restuccia (2010). Other applications include the changing patterns of trade, e.g. Markusen (2010) and Fieler (2011); the study of broader transformations in an economy, e.g., Greenwood and Seshadri (2005) among others.

Our strategy to discipline the forces in the model is simple. We calibrate a benchmark economy to fit a long time series for schooling and hours in the United States. The calibration puts restrictions on the strength of the income effect in the model. In the data for the United States there is substantial variation over time in hours of work, schooling, and income so that our calibration strategy provides discipline to the income effect in the model.[4] We then perform a cross-country quantitative experiment to assess the ability of the model in explaining schooling levels across countries and their evolution over time. In our quantitative experiment, we allow the levels of productivity and life expectancy to vary over time and across countries. We discipline these elements by reproducing the cross-country distribution of GDP per capita in 1950 and 2005 as well as the cross-country relationship between life expectancy and income in 1950 and 2005. The main result is that the model is consistent with the three facts emphasized earlier in the cross-country and time-series data. In particular, the model generates substantial dispersion in schooling levels across countries: in 1950, the model accounts for 96 percent of the difference in schooling between rich and poor countries. In addition, the model implies a faster increase in schooling for poor countries than for rich countries and, therefore, is consistent with the convergence in schooling levels observed in the cross-country data.

Our paper relates to a large literature in macroeconomics and development addressing the disparities in schooling levels across countries. The main focus of this literature is on differences in schooling at a point in time and, as a result, most of the existing frameworks are not designed to address the evolution of schooling over time.[5] Within this literature, the closest paper to ours is Bils and Klenow (2000) who also emphasize differences in productivity and

[4]Over the period we analyze, GDP per capita in the United States increased by a factor of 10 and average years of schooling increased by a factor of 2. The factor difference between the richest and poorest ten-percent of countries in 1950 is 16-fold in GDP per capita and 6-fold in average years of schooling.

[5]One important exception is the study of Manuelli and Seshadri (2009) that looks at the evolution of education for a subset of Asian and Latin American countries. They find that the model cannot account for the substantial increase in schooling in Latin American economies that failed to catch up in income to the United States.

life expectancy. A key difference is that whereas Bils and Klenow (2000) mainly focus on the cross-sectional correlation of schooling and per-capita income growth found in the empirical literature –for instance Barro (1991), we focus on a broader dimension of the data, namely the level and time-series differences in schooling across countries. We also depart from Bils and Klenow (2000)'s framework by allowing for an income effect through non-homothetic preferences and for hours of work differences across countries. These departures are critical in understanding the convergence pattern in educational attainment across countries. In emphasizing the connection between life expectancy, education, and growth, our paper relates to the literature on life expectancy and human capital accumulation.[6] By modeling labor supply, our results rationalize the findings in Acemoglu and Johnson (2007) of a strong relationship between life expectancy and education across countries but a much weaker relationship with income per capita. Our paper relates to a recent literature in macroeconomics assessing the role of human capital in development, for instance Manuelli and Seshadri (2006) and Erosa et al. (2010). The focus of this literature is on the amplification effect of human capital in explaining income gaps across countries. Our framework abstracts from features that generate amplification effects since this is not our focus. Incorporating amplification effects in the model would reduce the size of productivity gaps needed to reproduce income differences across countries in the quantitative experiments without altering our main findings. Another related paper is Cordoba and Ripoll (forthcoming) that consider a model where fertility, mortality, credit constraints, and access to public education drive schooling differences across countries. We complement this work by emphasizing the time series dimension of the data. More importantly, we assess the contribution of productivity and life expectancy to schooling in a framework without frictions.

Ogaki (1992) studied the time-series and cross-sectional implications of Engel's law and hence is closely related to our work. While our paper focuses on the cross-country relationship

[6]See for instance Soares (2005), Jayachandran and Lleras-Muney (2009), and Hazan (2009).

across points in time using a representative agent framework, the mechanism emphasized in the model have cross-sectional implications as well. While we don't pursue these implications in this paper, Restuccia and Vandenbroucke (forthcoming, a) have studied a quantitative model very similar to the one studied here and show that the cross sectional implications of the model are consistent with the evidence on hours of work differences for individuals across the wage distribution and for hours and education differences across races in the United States. Finally, our paper is also related to a broad literature that studies the impact of particular educational policies.[7] Our results highlight the importance of productivity (and life expectancy) in explaining a large portion of schooling differences across countries and, as a result, have novel and substantial implications for educational policy. The results emphasize the need to address the factors driving low productivity in poor countries. Even though there is room for other factors to be relevant such as credit constraints for investment in education, restrictions on school infrastructure, aid and policy, among others; our results stress the need for greater focus on the productivity problem in poor countries.

The paper is organized as follows. In the next section, we present the facts from a panel dataset of 84 countries from 1950 to 2005 for a measure of educational attainment and income. Section 3 presents the model. In Section 4 we calibrate the model. Section 5 performs a quantitative analysis of cross-country differences productivity and life expectancy in explaining the patterns in the panel data. We conclude in Section 6.

2 Facts

Data We construct a panel dataset of schooling and income as follows. We obtain average years of schooling for the population aged 25 to 29 from Barro and Lee (2010). We restrict

[7] See, for instance, Duflo (2001).

the sample to the narrow age population to minimize the impact of demographics and other changes on schooling measures across time and space. It is also the definition that is best suited for the historical data on educational attainment we use for the United States and for the model we consider in Section 3. The schooling data is available for a large set of countries from 1950 to 2005 in 5 year intervals. We obtain gross domestic product (GDP) per capita from the Board (2010), Total Economy Database. We restrict the time frame of this data from 1950 to 2005. To abstract from short-run fluctuations in real GDP we filter it using the Hodrick and Prescott (1997) filter with $\lambda = 100$ for yearly observations and keep the trend component of these time series. When we merge these two sets of data, we end up with a sample of 84 countries that have available data for schooling and GDP per capita from 1950 to 2005.[8]

Facts We emphasize three sets of facts that arise from analyzing these data. First, schooling differences across countries are large at any point in time between 1950 to 2005. Second, schooling increases over time in all countries in our sample. Third, schooling differences across countries are smaller in 2005 than they were in 1950. The reduction in schooling differences across countries is systematic and occurs despite the fact that the income gap between rich and poor countries has not generally decreased. We now document these facts in detail.

1. There are large differences in educational attainment across countries.

   Consider Table 1 which decomposes our sample into ten groups of countries according to the 1950 distribution of GDP per capita –i.e., the countries in each decile are the same in 1950 and in 2005. For each decile, the table reports the average GDP per capita

[8] Our sample of 84 countries comprises a fairly representative set of the world's income distribution. For instance, the factor difference in GDP per capita between the richest and poorest five-percent of countries is 25-fold which is comparable to many previous studies.

relative to the United States and the average years of schooling. In 1950 there is a 6-fold difference in schooling between the richest and poorest decile of the distribution. In 2005 there remains a noticeable 2-fold difference. These differences are not specific to the top and bottom decile and/or to the initial and end year of our sample. Figure 1 documents them across all countries, for selected years, and shows that there has been a significant dispersion of schooling across all levels of income and at all dates. To put the magnitude of cross-country differences in schooling in perspective, consider that in 1900 in the United States a 35-year old had completed about 7.4 years of schooling. Hence, a 25-29 year old in 2005 in the average poor country still had 2 years less of schooling than a 35-year old in the United Sates in 1900.[9]

2. Educational attainment increased over time in all countries.

   Schooling increased between 1950 and 2005 for every country in our sample. Table 1 conveys an aggregated view of this fact since average years of schooling increase for each decile of the distribution. The increase in schooling between 1950 and 2005 is 37 percent for countries in the top decile and 299 percent for countries in the bottom decile. We note that the increase in educational attainment is positive for all deciles of the income distribution regardless of the initial income level or subsequent income growth relative to the United States. We expand on this fact next.

3. Differences in educational attainment across countries have been reduced substantially over time.

   Poor countries exhibit a tendency to increase their schooling faster than rich countries. In Table 1 this is evidenced by the tendency of the 2005-to-1950 ratio of schooling (last column) to decrease with relative income. This is a remarkable finding given that for some deciles, such as the second and the fourth, relative income did not change between

[9]The figure 7.4 years of schooling for the average 35-year old is from Goldin and Katz (2008). Table 1 shows that the years of schooling for the average 25-29 year old in the average poor country is 5.07 in 2005.

1950 and 2005. For deciles such as the third or the fifth relative income increased and for the tenth decile relative income decreased. Yet, each group of countries experienced a substantial increase in schooling. A more complete and systematic documentation of the decline in schooling dispersion across countries is to report for each year the cross-sectional elasticity of schooling to income levels, as in Figure 2.[10] This elasticity decreases systematically over time. For instance, whereas two countries that differ in income per capita by one percent have in average a 0.6% difference in schooling in 1950, their schooling difference is reduced by half to 0.3% in 2005. The same declining pattern is observed for the time-series elasticity, that is the elasticity of income levels to schooling for each country over time, although with only 12 observations per country the pattern has more noise.

In summary, even though there are still large differences in educational attainment across countries, we find that these differences have been systematically reduced with poor countries increasing their educational attainment over time faster than rich countries. While we have reported these facts for individuals 25-29 years of age, we emphasize that the facts are robust to other age categories and for males and females. Moreover, the convergence pattern is also robust to a broader set of countries. The convergence pattern becomes even stronger if we consider all 146 countries in Barro and Lee (2010)'s data set. Table A.1 in the appendix reports average years of schooling for people 25-29 for countries by deciles of the schooling distribution in 1950 using the entire sample in Barro and Lee (2010). The countries with lowest schooling in 1950 (Decile 1) increased their educational attainment from 0.3 to 4.1 years whereas those countries with the highest schooling in 1950 (Decile 10) increased their schooling from 8.7 to 11.7 years. Hence, the factor difference in educational attainment between these groups of countries declined from a 31-fold in 1950 to less than 3-fold in 2005.

[10]In each year, we regress the log of average years of schooling (for people 25-29 years old) on a constant and the log of GDP per capita, and report the slope coefficient in Figure 2.

3 The Model

Time is continuous. The world comprises a set of closed economies and hence in what follows we focus on a single economy to describe the model. At every moment a generation of homogeneous individuals of size one is born and lives for an interval of time of length T_τ. The index τ denotes an individual's generation: the date at which the individual is of age 0. We use t to refer to calendar time. Individuals are endowed with one unit of productive time at each moment and no assets at birth. There is a worldwide rate of interest r, which we assume to be equal to the rate of time discount ρ and at which borrowing and lending can occur without constraint. The payment to a unit of human capital-hour is denoted by z_t at moment t. We generically refer to z_t as productivity and assume that it grows at a time invariant rate g.

Preferences

The preferences of an individual of generation τ are defined over lifetime sequences of consumption, $(c_{\tau,t})_{t=\tau}^{\tau+T_\tau}$, and leisure time, $(\ell_{\tau,t})_{t=\tau}^{\tau+T_\tau}$, as well as over time spent in school, s_τ. They are represented by

$$\int_{\tau}^{\tau+T_\tau} e^{-\rho t}\left[U\left(c_{\tau,t}\right)+\alpha V\left(\ell_{\tau,t}\right)\right]dt+\beta W\left(s_\tau\right),$$

where the functions U, V and W are concave and twice continuously differentiable. We abstract from life-cycle considerations by imposing that consumption and leisure time remain constant throughout the individual's life: $c_{\tau,t}=c_\tau$ and $\ell_{\tau,t}=\ell_\tau$. This assumption is not too restrictive since with separable utility in consumption and leisure and the additional assumption that the rate of time discount equals the rate of interest $(r=\rho)$, an individual optimally chooses a constant path of consumption over the lifecycle. However, these assumptions do not guarantee a constant path of leisure. We impose a constant leisure profile over the lifecycle for simplicity.[11]

The term $\beta W(s_\tau)$ is the utility derived from schooling measured in present value. Depending upon the sign of β, which is determined in the calibration of Section 4, time spent in school is either a good or a nuisance. Introducing a utility value of school time is common in models of schooling choices such as Heckman et al. (1998), Bils and Klenow (2000) or Wolpin and Lee (2010) to cite just a few, and is also consistent with findings from a large literature in development economics –see Schultz (1963). Furthermore, there exists estimates of a substantial "psychic" component of attending school. Heckman et al. (2006) and the references therein present and discuss such evidence. Overall, our representation of preferences is close to that of Bils and Klenow (2000) except for our addition of a taste for leisure time. Bils and Klenow model the utility from schooling as a constant flow of utils, ξ, derived from each moment spent in school, i.e. they assume $W(s) = \xi \int_0^s e^{-\rho t} dt$ which is also increasing and concave in s.

We chose the following functional forms for U, V and W:

$$U(c) = \ln(c - \bar{c}),\ V(\ell) = \frac{\ell^{1-\sigma} - 1}{1 - \sigma}, \quad \text{and} \quad W(s) = \ln(s),$$

where $\sigma > 0$ and where $\bar{c}$ introduces a non-homotheticity which has the standard interpretation of a subsistence level above which consumption must remain at every point in time. This feature of preferences plays an important role in both the theoretical and quantitative properties of the model. To illustrate this role, consider first the situation where income is

[11] In addition, we do not have detailed data on the lifecycle behavior of labor supply for generations dating back to the 19th century and the changes in lifecycle labor supply for recent cohorts are small in comparison with the variation in hours over time. Hence, we think there is little benefit of modeling labor supply over the lifecycle since our focus is on changes across countries and over time.

sufficiently large relative to $\bar{c}$ (or alternatively when $\bar{c} = 0$). Then the model displays the standard property of models with logarithmic preferences where labor supply is invariant to changes in productivity because income and substitution effects exactly offset each other. As we discuss below, it will transpire that our model has two margins of labor supply that are affected by this mechanism: the "intensive" choice of hours of work during the working lifecycle and the "extensive" choice of the length of working life (because of the choice of schooling years). In our specification, changes in productivity will leave each of these margins unaffected whenever income is high relative to $\bar{c}$. Consider now the situation where income is sufficiently low relative to $\bar{c}$, then changes in productivity affect the allocation of time at both margins: both leisure and schooling increase. In modeling non-homothetic preferences, we follow a tradition that is common to a broad literature: for example the literature emphasizing the shift in economic activity from agriculture to non-agriculture such as Gollin et al. (2002) and Duarte and Restuccia (2010); models of the allocation of hours such as Rogerson (2008), models of the dynamics of saving rates such as Christiano (1989), among many other applications. See Atkeson and Ogaki (1996) for empirical evidence from micro and macro data. See also Restuccia and Vandenbroucke (forthcoming, a) for evidence and a quantitative analysis of the cross-sectional implications of non-homothetic preferences for the allocation of hours across individuals in the wage distribution and hours and schooling differences among races in the United States.

Human Capital Technology

Individuals can acquire human capital by spending time in school and purchasing educational services. The human capital technology follows Bils and Klenow (2000) and is described by

$$H(s, x) = x^{\gamma} h(s) \equiv x^{\gamma} \exp\left(\frac{\theta}{1-\psi} s^{1-\psi}\right),$$

where x represents purchases of educational services whose relative price is denoted by q. These services are purchased up front. Hence, x is more appropriately described as the present value of educational services. The parameter $\gamma \in (0,1)$ measures the elasticity of human capital to educational services. At an optimum γ is the share of lifetime income spent by an individual in educational services.

Optimization

The optimization problem for an individual of generation τ is

$$\max_{c_{\tau,t},\ell_{\tau,t},x_\tau,s_\tau} \left\{ \int_\tau^{\tau+T_\tau} e^{-\rho t} \left[U\left(c_{\tau,t}\right) + \alpha V\left(\ell_{\tau,t}\right)\right] dt + \beta W\left(s_\tau\right) : \right.$$
$$\left. \int_\tau^{\tau+T_\tau} e^{-\rho t} c_{\tau,t} dt + x_\tau = z_\tau \int_{\tau+s_\tau}^{\tau+T_\tau} e^{(g-\rho)t}(1-\ell_{\tau,t})H(s_\tau,x_\tau)dt \right\}. \quad (1)$$

The left-hand side of the intertemporal budget constraint measures the expenditures in consumption and educational services in present value at date τ. On the right-hand side, the individual's lifetime income depends upon a number of variables. First, the level of productivity at the beginning of life, z_τ, which grows at the constant rate g. Second, the length of the individual's working life which extends from date $\tau+s_\tau$ until $\tau+T_\tau$. Working life is reduced by the choice to spend more time in school, therefore incurring an opportunity cost. Third, labor supply during the individual's working life, $(1-\ell_{\tau,t})$. Fourth, the individual's human capital stock acquired through s_τ years of school, $H(s_\tau, x_\tau)$.

Problem (1) makes the two margins of time allocation explicit. At an "extensive" margin the individual chooses the length of working life by choosing how much time to spend in school, s_τ. At an "intensive" margin the individual chooses how much time to spend working and how much leisure to enjoy at any point in time, $\ell_{\tau,t}$. After imposing that $c_{\tau,t} = c_\tau$ and

$\ell_{\tau,t} = \ell_\tau$, the first oder condition for optimization can be written as

$$
\begin{aligned}
c_\tau &: \ 0 = U'(c_\tau) - \lambda, \\
x_\tau &: \ 0 = 1 - \gamma z_\tau x_\tau^{\gamma-1}(1-\ell)h(s)d_\tau(s_\tau), \\
\ell_\tau &: \ 0 = \alpha a_\tau V'(\ell_\tau) - \lambda z_\tau x^\gamma h(s_\tau)d_\tau(s_\tau), \\
s_\tau &: \ 0 = \beta W'(s_\tau) + \lambda z_\tau x^\gamma (1-\ell)[h'(s_\tau)d_\tau(s_\tau) + h(s_\tau)d_\tau'(s_\tau)],
\end{aligned}
$$

where λ is the Lagrange multiplier associated with the lifetime budget constraint and where $a_\tau = \int_\tau^{\tau+T_\tau} e^{-\rho t} dt$ and $d_\tau(s_\tau) = \int_{\tau+s_\tau}^{\tau+T_\tau} e^{(g-\rho)t} dt$ are discounting terms. Using the first order conditions with respect to c_τ and x_τ the first order conditions with respect to years of schooling and leisure time are

$$
\beta(1-\gamma)W'(s_\tau) = -a_\tau U'(c_\tau)c_\tau \left[\frac{h'(s_\tau)}{h(s_\tau)} + \frac{d_\tau'(s_\tau)}{d_\tau(s_\tau)} \right]. \tag{2}
$$

and

$$
\alpha(1-\gamma)V'(\ell_\tau)(1-\ell_\tau) = U'(c_\tau)c_\tau, \tag{3}
$$

The role of our assumptions about preferences: (i) the presence of a taste for schooling; and (ii) the non-homotheticity of preferences, transpire in Equations (2) and (3). First, suppose that $\beta = 0$ so that there are no utility benefits (or cost) of attending school. Then, the optimal level of schooling becomes the solution of

$$
\frac{h'(s_\tau)}{h(s_\tau)} + \frac{d_\tau'(s_\tau)}{d_\tau(s_\tau)} = 0,
$$

which is income maximizing. In this case, changes in productivity and/or the level of income do not matter for the level of schooling. Only differences in life expectancy affect schooling decisions through the discounting term $d_\tau(s_\tau)$. Hence, for productivity to matter for schooling decisions it is necessary that $\beta \neq 0$. In the calibration of Section 4 we use time-series data on life expectancy and productivity in the United States to impose discipline on the parameters of the model, and in particular on β. Second, when $\beta \neq 0$, equation (2) reveals that years of school are a function of the level of consumption, c_τ, and consequently the level of income, whenever the term $U'(c_\tau)c_\tau$ is a function of the level of consumption. Third, when the level of income is sufficiently large relative to $\bar{c}$ (or alternatively when $\bar{c} = 0$), our functional form assumption for U implies that $U'(c_\tau)c_\tau$ is independent of c_τ. Formally $\lim_{c\to\infty} U'(c)c = 1$ so that, with logarithmic preferences or for large enough levels of income years of school are independent of the level of productivity. This property, combined with the fact that $d'_\tau(s)/d_\tau(s) \to g - \rho$ as $T_\tau \to \infty$ imply that years of school converge to a constant $\tilde{s}$ in the long-run:

$$\beta(1-\gamma)W'(\tilde{s}) = -\frac{1}{\rho}\left[\theta\tilde{s}^{-\psi} + g - \rho\right]. \tag{4}$$

Fourth, at low levels of income the term $U'(c)c$ is decreasing, implying changes in the optimal level of schooling. Formally, we can show that when $\beta > 0$ years of school are increasing in productivity z_τ.[12] That is, different generations of individuals indexed by the level of productivity at the start of their lives would choose different levels of schooling. Similarly, individuals of the same generation, but living in different countries indexed by their level of productivity would also choose different levels of schooling. Furthermore, the same rate of growth in productivity would have different implications for the rate of change in schooling, depending upon the initial level of productivity. The highest the initial level, the lower the effect of productivity growth. In the limit, as mentioned above, changes in productivity do not matter for years of school. These observations suggest that, qualitatively at least, the model is equipped to address the key patterns of years of schooling observed across countries and over time and described in Section 2. We note that many utility functions share the property that $U'(c)c$ is decreasing. The function $U(c) = c^{1-\eta}/(1-\eta)$ with $\eta > 1$ is one example. Thus, the critical aspect of preferences that is needed for schooling to increase with

[12] The formal proof is available upon request.

productivity is that the marginal utility of consumption decreases faster than consumption increases. A non-homothetic term in preferences, such as the one we use is a convenient and easy-to-interpret device to generate such pattern. Finally, the first order condition for leisure time, that is equation (3) has a standard interpretation that is common to models with a consumption-leisure tradeoff. Observe, however, that changes in life expectancy do not directly matter for the optimal choice of leisure. They enter the leisure decision indirectly through the term $U'(c_\tau)c_\tau$. In the long-run leisure is constant at $\tilde{\ell}$:

$$\alpha(1-\gamma)V'(\tilde{\ell})(1-\tilde{\ell}) = 1. \tag{5}$$

We define y_τ as the period income of an individual of generation τ at age 35:

$$y_\tau = z_\tau e^{35g}(1-\ell_\tau)H(s_\tau, x_\tau).$$

Since we use this measure in our quantitative analysis, we emphasize how increases in productivity affect income. First, an increase in productivity raises income through three channels: a direct effect through z_τ; an indirect effect through increases in schooling s; and another indirect effect through increases in expenditures in education x and therefore human capital. Second, an increase in productivity induces an increase in leisure time and therefore reduce labor income. The increase in leisure hinders the incentive to acquire education. We note this as an important property of the model to keep in mind when analyzing the results in light of the data since the model will imply large changes in schooling for poor countries relative to rich countries with relatively minor catch up in income per capita. The catch up in schooling does produce convergence in human capital but will also reduce labor hours much more rapidly in poor countries tempering the effect of this catch up on per capita income. It is not surprising then that there is somewhat weak evidence on the effect on schooling on growth in income per-capita across countries (e.g. Benhabib and Spiegel (1994)).

4 Calibration

We calibrate a benchmark economy with exogenous variation in productivity z and life expectancy T to time-series data for the United States. The motivation for this strategy is that the United States has experienced a well-documented long-run increase in schooling followed by a flattening of the trend towards the end of the 20th century and a long-run decline in weekly hours followed by a flattening of the trend. Our calibration procedure exploits these trends to discipline the strength of the income effect that is central to the quantitative implications of the model across countries.[13]

The schooling data were provided by Claudia Goldin and Larry Katz and serve as the basis of Figures 1.4 to 1.6 in their book.[14] The data is years of schooling by birth cohort, completed at age 35 for white people starting in 1876 until 1975. We HP-filtered the time series and used, for calibration purposes, cohorts from 1880 to 1915. These cohorts of people are of age 35 in years 1915 through 1950. As expected, the data revels a substantial increase in schooling years. A 35-year-old person in 1915 had completed about 8 years of schooling while the same-age person in 1950 had completed close to 10.5 years.[15] We note that in this calibration we explicitly avoid using data on years of schooling past 1950. The reason is that there is a large literature emphasizing the role of skill-biased technological progress for the increase in education and wage inequality in the second half of the twentieth century.[16]

The hours data that we use are built from various sources. For the period 1830 to 1880 we use

[13] Restuccia and Vandenbroucke (forthcoming, a) show that changes in income and life expectancy account for most of the decline in hours of work and the increase in schooling observed in the United States in the last century.

[14] See Goldin and Katz (2008, Figures 1.4–1.6).

[15] Since the cross-country data we use for average years of schooling is from Barro and Lee (2010), we verified that the Goldin and Katz data used for calibration is consistent with the Barro and Lee data for the United States for the overlapping time period.

[16] See for instance Acemoglu (2002) and Violante (2008) for surveys of the vast literature and Restuccia and Vandenbroucke (forthcoming, b) for a quantitative assessment.

data from Whaples (1990, Table 2.1), for the period 1890 to 1940 we use data from Kendrick (1961, Table A-IX), and for the period 1950 to 2000 we use data reported by McGrattan and Rogerson (2004, Table 1).[17] We HP-filtered the data and linearly interpolate between census dates to build a time series of hours from 1830 through 2000. The trend shows a decline from close to 72 hours per week in 1830 to 40 hours in 2000. Importantly, the rate of this decline is non-constant. There is a moderate decline in hours from 1830 to about 1910, followed by a sharp decline until about 1980, and a substantial flattening after 1980. The fact that the workweek declined significantly in the United States has been recognized elsewhere. Rogerson (2006), for example, uses data from Whaples (1990) and proposes to rationalize the decline in the workweek using non-homothetic preferences similar to our specification. Maddison (1987, Table A-9) and Huberman and Minns (2007, Table 1) show patterns of hours over time for countries such as Belgium, Denmark, France, Germany, Ireland, Italy, Japan, the Netherlands, Spain, Sweden, Switzerland and the U.K. between 1870 and 1990 that are similar to the pattern in the United States.

The hours data are available in calendar time while the model predicts hours by generation. We choose to associate the 1830 hours data with the 1795 generation from the model, i.e. the generation that is 35 years old in 1830. That is, when we compare the model's predictions to the U.S. data we compare the hours chosen by the 1795 generation in the model with the 1830 data on hours. We associate subsequent data points and generations in the same way.

To calibrate the model lifespan T we note that its empirical counterpart is not life expectancy per se but rather the sum of years spent in school and on the labor market for a generation. Hazan (2009) reports market years for cohorts born in $1840, 1850, \ldots, 1930$. We add this data to Goldin and Katz's figures for years of schooling achieved by these generations to obtain a

[17] The Whaples data are weekly hours worked collected from two surveys of manufacturing hours taken by the federal government in the context of the 1880 Census. The Kendrick data are average weekly hours in the private non-farm sector, finally the McGrattan and Rogerson data are average weekly hours worked for all workers.

measure of T_τ for cohorts born in $1840, 1850, \ldots, 1930$. We then estimate a linear time trend for T_τ and obtain $T_\tau = 0.1716 \times \tau - 279.38$, and use it to compute T_τ for all cohorts of the model.[18]

We now discuss the specification of the non-homothetic preference term $\bar{c}$. This term is critical for the calibration of hours over time and as a result for the implications of the model for poor countries. While it is necessary in our model to have non-homothetic preferences ($\bar{c} > 0$) for schooling to vary over time, the parameters of the human capital technology allow for an excellent fit of the schooling data regardless of the specification of $\bar{c}$. However, we find that when we specify $\bar{c}$ as a constant our model implies an income elasticity of hours that falls exponentially as income rises. With constant income growth this implies that hours fall exponentially over time as well. This pattern of hours in the model can fit the U.S. hours data from 1910 onwards but not the pattern before 1910. Since the United States was poorer before 1910 than nowadays, fitting the income elasticity of hours at low levels of income has implications for the model's predictions about poor countries. Hence, allowing $\bar{c}$ to vary over time is critical to reproducing the historical time series of hours of work in the United States and to analyzing poor economies. In particular, for the model to fit the hours data in the United States before 1910, $\bar{c}$ must be lower before 1910. This implies that the model generates a lower income elasticity of hours (and therefore schooling) than with a constant $\bar{c}$. To allow for a flexible specification of $\bar{c}$ over time that we can estimate with data while still retaining the long-run implications of the model with a constant $\bar{c}$, we specify a transformed version of the logistic function as follows:

$$\bar{c}(z) = \frac{\mu}{1 + \exp(-\omega z + \kappa)},$$

where μ, ω and κ are parameters to be determined. With constant productivity growth,

[18] The t-statistics for the slope and intercept in this regression are 18 and 15, respectively. The regression's r^2 is 0.98.

$\bar{c}(z)$ is asymptotically constant and hence the long-run properties of the model remain as discussed previously.[19]

While this feature of our model is motivated by the behavior of the hours data, we propose in Appendix B a structural interpretation of this time-varying $\bar{c}$ in the context of a standard model of time allocation between market and household production. The interpretation behind the time varying $\bar{c}$ is then as follows. Suppose that there is a fixed level of minimum consumption, say $\bar{m}$, that can be satisfied with either home or market goods. Then our model of household production implies that the time varying $\bar{c}$ is the difference between the fixed level of minimum consumption and household production: i.e. the $\bar{c}$ corresponds to the part of minimum consumption that is satisfied with market goods. When home hours are substituted with market hours more market goods are used to provide minimum consumption, implying an increase in $\bar{c}$. The pace of substitution between home and market hours depends on preference and technology parameters such as the rates of growth of market and home productivity. To the extent that we cannot empirically pin down these growth rates, we opted for a reduced-form approach. We explore a quantitative version of such home production model in the Appendix.

We now describe the details of the calibration procedure. We start the calibration of the model by normalizing the productivity parameter $z_{1795} = 1$. We set the discount factor to 5 percent, i.e., $\rho = 0.05$ and, following Bils and Klenow (2000), we choose $\gamma = 0.1$ and $\psi = 0.3$.[20] We pick the rest of the parameters in order to minimize a measure of distance between the model's predictions and relevant U.S. data. Specifically, let λ be the vector of parameters to calibrate:

$$\lambda = (\alpha, \beta, \mu, \omega, \kappa, \sigma, \theta, g)',$$

[19] In Section 5.3, we explicitly estimate and explore the implications of the model assuming a constant $\bar{c}$ over time.

[20] Bils and Klenow (2000) report estimates for ψ between 0 and 0.6. Our choice of 0.3 is the middle of that range.

and let $\hat{s}_t(\lambda)$ and $\hat{n}_t(\lambda)$ represent optimal schooling and work time of generation τ. Let $\mathbf{s}_\tau$ and $\mathbf{n}_\tau$ be their empirical counterpart: $\mathbf{s}_\tau$ is years of schooling for generation τ in the U.S. data and $\mathbf{n}_\tau$ is the workweek at date $\tau + 35$ in the U.S. data. The mapping of hours between the model and the data is done by assuming that there are 112 hours of discretionary time per week.[21] Hence, a 40-hour workweek corresponds to 40/112 units of work time in the model. Finally, our calibration procedure also targets a growth rate in income per capita of 2 percent per year. Thus, we find λ by solving the following minimization problem:

$$\min_{\lambda} \left\{ \sum_{\tau=1880}^{1915} \left(\frac{\hat{s}_\tau(\lambda)}{\mathbf{s}_\tau} - 1 \right)^2 + \sum_{\tau=1795}^{1965} \left(\frac{\hat{n}_\tau(\lambda)}{\mathbf{n}_\tau/112} - 1 \right)^2 + \left(\frac{\hat{y}_{1965}(\lambda)/\hat{y}_{1795}(\lambda)}{\exp(0.02 \times 171)} - 1 \right)^2 \right\},$$

Table 2 shows the values of the calibrated parameters.

Figures 3 and 4 show the model's fit to the U.S. data on schooling and hours. In particular, Figure 3 reports the time series of years of schooling in the model and the U.S. data. The dashed line separates the period of U.S. data used in the calibration (the period between 1915 and 1950) with the period that is not used for calibration (after 1950). As the figure shows, the model is able to capture the trend in schooling very well for the calibrated period until 1950 and under predicts the increase in schooling during the second half of the twentieth century. As discussed earlier, this increase is best attributed to other forces not in the model such as skill-biased technical change and hence it is important that this increase in schooling is not attributed to parameters pertaining to the income effect in the model. We also note the excellent fit of the model to the hours data. Figure 4 illustrates how the time series of hours implied by the model fits the changing pattern of the rate of change in actual hours. The calibrated function of $\bar{c}(z)$ permits this fit since as we will discuss when $\bar{c}(z)$ is constant, the best fit the model can produce for the time series of hours is a strictly convex pattern that fails to fit the changing pattern in hours over time prior to 1910.

[21] Assuming that a person needs 8 hours for sleep and other necessities, there are $(24 - 8) \times 7 = 112$ hours of discretionary time in a week.

Given our calibration strategy it should not be surprising that the model implies that, as in the data reported by Hazan (2009), the number of years spent on the labor market increases while total lifetime hours decreases across generations. Nevertheless, for the sake of comparison with the data we report the implications of the model for the 1840 and 1930 cohorts. We find that time in the labor market in the model is 36 years for the 1840 cohort (v. 34 years in the data) whereas for the 1930 cohort it is 51 years (v. 40 years in the data). In terms of lifetime hours the model implies 101,631 hours for the 1840 cohort and 86,320 hours for the 1930 cohort. The data for these cohorts is 103,324 and 77,502.[22]

The calibration implies that $\bar{c}(z)$ reaches its long-term value, given by μ, around 1920. We compute the ratio of μ to the income of the last generation in the benchmark economy and find it to be 3%. It is not obvious how to compare this value with data. One possibility is to relate it to the final expenditures on food relative to GDP.[23] For the United States, the expenditure share of food is 5.2% in 1996. Another possibility is to compare the incomes of countries far back in time. Maddison (2009) reports that GDP per capita in Western Europe between 1 and 1500 was between 450 and 771 at constant 1990 dollars, representing a range of 2 to 4.5 percent of the 1970 GDP per capita. Since measured income is likely to be lower due to non market production, we conclude that the value of $\bar{c}$ relative to income of 3% is reasonable in light of the related evidence.

Finally, we note that the rate of growth of productivity, g, needed to achieve a 2 percent annual growth rate of income, is almost 2 percent. This is because our framework abstracts from amplification effects such as those emphasized in Manuelli and Seshadri (2006) and Erosa et al. (2010). The downward trend in hours magnifies the lack of amplification since increases in productivity would give rise to larger increases in income if hours remained

[22] We compute the number of years in the labor market of generation τ as $T_\tau - s_\tau$ and total lifetime hours of work as $(T_\tau - s_\tau)(1 - \ell_\tau) \times 52$.

[23] The data is for the 1996 Benchmark study of the International Comparison Program.

constant instead of decreasing. We emphasize, however, that abstracting from these amplification effects does not affect our quantitative results. For the purpose of calibrating the model, larger amplification effects would imply a smaller value of g and, in the cross-country experiments that follow, they would reduce the size of the productivity gaps needed to reproduce the calibrated income differences across countries leaving the impact of income on schooling the same.

5 Cross-Country Experiments

We conduct quantitative experiments using the calibrated model to assess the importance of productivity and life expectancy in explaining educational attainment across countries and over time.

5.1 Baseline Experiment

We use the calibration of the benchmark economy and assume that countries are identical except in terms of productivity and life expectancy. In particular, we assume that countries differ in the initial level of productivity z and its growth rate g as well as in the level and rate of change of life expectancy. We discipline our choice of life expectancy across countries by estimating two cross-sectional relationships between life expectancy and GDP per capita for 1950 and 2005. We then search for 10 combinations of z and g that match the relative income gaps in 1950 and 2005, as described in Table 1, while imposing that life expectancy in 1950 and 2005 be as described by the estimated cross-sectional relationships. A detailed description of this procedure is in the appendix. Table 3 displays the results of our baseline experiments. The implied values of z, g, and T for the first and last generations for each economy are reported in the first four columns.

There are two sets of results that we emphasize from Table 3: the cross-sectional implications of the model relative to the data in 1950 and the time-series behavior across countries relative to the data. We start with the cross-sectional implications in 1950. The model implies that poor countries in 1950 attain very few years of schooling compared to rich countries. Figure 5 illustrates the implications of the model for the deciles considered against the cross country data. The strong positive association between schooling and per capita income in the data is captured by the model. To summarize our findings, we note that the model accounts for 96 percent of the difference in schooling between countries in the 1st decile and the United States. To understand how we obtain this number, note that for countries in the poorest decile of income in 1950, the model implies 1.6 years of schooling whereas the data is 1.27 years (see the schooling data in Table 1). In 1950, the United States has 10.3 years of schooling which is closely reproduced by our calibrated benchmark economy. Hence, the model accounts for (10.3-1.6)/(10.3-1.27)=96% of the difference. The "Cross Section" column of Table 3 shows a similar calculation for each decile. It transpires that the model accounts for a lower percentage of schooling differences for countries in higher deciles of income. For example, the model accounts for 80% of the difference in schooling with the U.S. for the 5th decile and 27% for the 10th decile. This systematic tendency for the model to account for lower fractions of the schooling data as we consider richer countries results from the mechanisms emphasized in our theory: the quantitative importance of non-homotheticity in preferences tends to vanish at high levels of income to eventually play no role. For rich countries, factors other than income levels have first-order importance in the determination of schooling, e.g., skill-biased technical change, public policy towards education, labor market institutions that compress wages, among many others. In poor countries, however, increases in productivity and income allow individuals to move farther away from subsistence consumption having a first-order effect on the allocation of time in schooling.

We now turn to the time-series implications of the model for year of schooling across countries. Our first observation is that the model is consistent with the facts that (i) schooling increased in all countries; and (ii) this increase is faster in poor countries relative to rich countries. In Table 3 this transpires from the "s_{05}/s_{50}" column reporting the ratio of years of schooling in 2005 to 1950, and showing it to be larger at the bottom of the income distribution than at the top. Using the cross-country data generated by the model we estimate the income elasticity of schooling across countries just as we did for the data in Figure 2. We find an elasticity of 0.53 in 1950 (v. 0.6 in the data) and 0.31 (v. 0.27 in the data) in 2005. Again, the decrease of this elasticity is evidence of the reduced dispersion in years of schooling across countries in 2005 and, therefore, of the faster increase in schooling in poorer countries. Our second observation is that the model accounts for 89 percent of the increase in schooling in poor countries. We compute this statistic as follows. For the economy in the 1st decile, years of school increase from 1.6 in 1950 to 5.5 in 2005, a $\ln(5.5/1.6)/55 = 2.2\%$ annual rate of increase. This compares with a 2.5% annual rate of increase in the data. Thus, for this economy, the model accounts for 2.2/2.5=89% of the increase in years of school. The "Time Series" column of Table 3 shows a similar calculation for each decile. At the fifth and tenth decile the corresponding figures are 61 and 89%.

In terms of hours there is limited data that can be brought to bear on the implications of the model. Nevertheless, we use the available hours data from the Board (2010). They report yearly hours per worker and we plot the hours data for 1950 and 2005 against GDP per capita in Figure 6. We note that not only hours of work decline with income in 1950 and 2005, but also hours decrease as income rises for each country and hours fall faster for the poor than the rich countries. In the data in 1950, hours of work in poor countries relative to rich is about 1.4, while the same ratio drops to 1.2 in 2005. In the model, the ratio of hours in the poorest economy relative to the benchmark is 2.3 in 1950 and drops to 1.7 in 2005. The ratio of hours between an economy in the 5th decile and the benchmark is 1.6 in 1950 and drops to 1.3 in 2005. While the comparison here is very crude since hours data is missing for the poorest countries, the rough comparison suggests that the hours implications of the model are broadly in line with the data in terms of both the magnitude of hours differences in 1950 and the faster decline in hours over time in poor countries.

5.2 Equal Growth Rates across Countries

To illustrate the importance of differences in productivity growth and changes in life expectancy across countries, we conduct two additional experiments. First, we conduct an experiment similar to the Baseline except that we assume that the rate of growth of productivity, g, is the same in all countries at the value in the benchmark economy. In a second experiment we assume that in addition the change in life expectancy over time is also the same across countries at the value in the benchmark economy. Essentially, we show with these experiments that the implications of the model for the cross-country differences in schooling in 1950 are not substantially affected by the differential growth components. The levels of income per capita and life expectancy are critical for the time allocation in 1950. We also show that even when abstracting from cross-country differences in productivity growth and changes in life expectancy, the model still accounts for a substantial portion of the changes in schooling over time across countries.

In the first experiment, we follow the Baseline except that we assume equal productivity growth g across countries. Results are reported in Table 4. In the 1950 cross-section the model accounts for 90 percent of the difference in schooling between countries in the 1st decile and the United States. This compares to 96 percent in the baseline. At the 5th and

10th deciles the numbers are 76 and 30 percent (80 and 27 in the baseline). These figures are reported in the "Cross Section" column of Table 4 for all deciles. Turning to the time-series implications, the model with equal g across economies implies that in the 1st decile, the model accounts for 66 percent of the increase in schooling versus 89% in the baseline experiment. At the 5th and 10th deciles the numbers are 52 and 91% (61 and 89% in the baseline). These figures are reported in the "Time Series" column of Table 4 for all deciles. Just as in the baseline, the model with equal g predicts a narrowing of the schooling gap relative to income as observed in the data: the income elasticity of schooling across countries, estimated on the model-generated data, is 0.48 in 1950 and 0.3 in 2005.

In the second experiment, we follow the previous experiment and assume in addition that the change in life expectancy for each country is the same as in the benchmark economy. That is, relative to the previous experiment, life expectancy in 1950 is unchanged but the life expectancy in 2005 is 9.3 years above that of 1950 for all countries, which is the years increase in life expectancy in the benchmark economy between 1950 and 2005. The results of this experiment are reported in Table 5. We first note that the cross-sectional implications in 1950 of the model in this experiment and the previous experiment are identical. This is due to the fact that, by design, the experiment implies that the level of life expectancy and income are the same in 1950. The only difference between the two experiments is the level of life expectancy in 2005. In terms of the time series, abstracting from differences in rate of change of T reduces the ability of the model to account for the increase in schooling. Nevertheless, even when abstracting from differences in productivity growth and changes in life expectancy, the model accounts for 56% of the growth rate of schooling in the 1st decile. At the 5th and 10th deciles the corresponding figure are 39 and 88%. These figures are reported in the "Time Series" column of Table 5 for all deciles. The model also implies a faster growth in schooling in poor than in rich countries: the income elasticity of schooling measured across countries falls from 0.48 in 1950 to 0.35 in 2005.

The model's ability to replicate some aspects the schooling data across countries and over time, even when productivity growth and changes in life expectancy are the same across countries, results from the non-homotheticity of preferences. To understand this, remember that in the last experiment the initial level of productivity is country specific, and that it is chosen in order to match the distribution of income across countries in 1950. Given these differences in the level of income across countries in 1950, the non-homotheticity of preferences implies different rates of growth in schooling and hours of work, even in the absence of heterogeneity in exogenous driving variables across countries. The countries that are initially rich in 1950 tend to increase their schooling at a slower pace than those who are initially poorer. Quantitatively, this mechanism is important enough to generate the results just described.

5.3 Importance of $\bar{c}(z)$

We assess the quantitative importance of the assumption that $\bar{c}$ is time-varying. We conduct an experiment where we assume a constant $\bar{c}$. We calibrate the benchmark economy to U.S. data as in Section 4. The list of parameters to be determined by the minimization program is now $(\alpha, \beta, \sigma, \bar{c}, \theta, g)'$. Figure 7 shows the fit of this version of the model against the U.S. data. While this version of the model fits the time path for schooling as well as in the baseline, the implied time path of hours exhibits a convex shape that fails to fit the time series of hours, particularly in the earlier period. A consequence of this convex shape is that hours increase fast as productivity declines, potentially leading to unreasonable predictions about hours of work for poor countries.

We conduct the same cross-country experiment as described in Section 5.1 using the calibrated parameters of the benchmark economy with a constant $\bar{c}$. Table 6 reports the results.

The model generates such large effects on hours and schooling for poor countries that it is not possible to compute economies corresponding to levels of income below that of decile 9. In Table 6 we report the poorest economy that we are able to compute in decile 9: an economy with an income that is 61% of that of the U.S. in 1950, while the average economy in this decile has, in fact, an income that is 59% of that the U.S. in 1950. Individuals in this economy work 111.9 hours per week, out of the total of 112, and complete 0.1 years of school. Thus, even at this level of relative income the labor supply elasticity is very high (as suggested by Figure 7) and the predictions for schooling are magnified as a result.

6 Conclusions

We developed a model of human capital accumulation to quantitatively assess the importance of productivity and life expectancy in explaining differences in educational attainment across countries and over time. We calibrated a benchmark economy to reproduce the historical evolution of schooling and hours in the United States. We found that the model accounts for 96 percent of the difference in schooling between rich and poor countries in 1950. The model accounts for 89 percent of the increase in schooling levels over time in poor countries. The model generates a faster increase in schooling levels in poor than in rich countries. Hence, the model explains the convergence in cross-country schooling levels observed in the data. Our results emphasize the importance of productivity (and life expectancy) in explaining the bulk of differences in educational attainment across countries and their evolution over time. These results have important implications for educational policy as they shift the focus of attention from frictions and market imperfections to the determinants of low productivity in poor countries. Nevertheless, we think that extending our framework to incorporate complementary factors such as credit market frictions and public education (such as school infrastructure) can yield additional insights. We leave these important extensions of the model for future work.

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A Schooling Data

Table A.1 reports average years of schooling for people 25 to 29 years of age for countries by deciles of the schooling distribution in 1950. The data is from Barro and Lee (2010) and includes the entire sample of 147 countries. Compared to the dispersion in schooling between rich and poor countries in our restricted sample in Table 1, the larger sample in Table A.1 shows that the pattern of convergence in schooling across countries over time is even stronger, with the schooling gap between countries in the tenth and first deciles being a factor of 31-fold in 1950 and less than 3-fold in 2005.

Table A.1: Average Years of Schooling across Countries

Decile	s_{50}	s_{05}	s_{05}/s_{50}
1	0.28	4.06	14.60
2	0.60	6.11	10.26
3	1.07	7.02	6.57
4	1.58	7.34	4.66
5	2.41	8.63	3.58
6	3.39	9.64	2.85
7	4.40	10.11	2.30
8	5.28	10.74	2.03
9	6.85	11.26	1.64
10	8.73	11.69	1.34
$R_{10/1}$	31.41	2.88	–

Note: s is average years of schooling of the 25-29 year old population. Numbers reported are the average of each decile. The countries in each decile are the same in each year and represent the 1950 distribution of schooling.

B Structural Model of Time Varying $\bar{c}$

We describe a structural interpretation of the time-varying $\bar{c}$ in our baseline model and show its empirical plausibility. Since the issue of time-varying $\bar{c}$ pertains to the model's ability to fit the labor supply data in the time series, we abstract from life-cycle and schooling decisions in the following discussion.

Suppose that an individual lives for one period and has preferences represented by

$$\ln(C(c_m, c_n) - \bar{m}) + \alpha \ln(\ell),$$

where c_m and c_n are consumption of a market and a non-market good. The variable ℓ represents leisure time and $\bar{m}$ is a constant. The function C aggregates the consumption of the market and home good. Assume that $C(c_m, c_n) = \phi c_m + (1-\phi)c_n$. The home good is produced with time, in line with the technology $c_n = z_n h^\mu$ where $\mu \in (0,1)$, h is the time devoted to home production and z_n is productivity in the home technology. The individual's budget constraint is $c_m = z_m(1-\ell-h)$, where z_m stands in for market productivity. The individual's optimization problem is then

$$\max_{c_m,h,\ell} \left\{ \ln(\phi c_m + (1-\phi) z_n h^\mu - \bar{m}) + \alpha \ln(\ell) : c_m + z_m(h+\ell) = z_m \right\}.$$

The solutions for home hours and leisure time are

$$h = \left(\frac{1-\phi}{\phi} \mu \frac{z_n}{z_m} \right)^{\frac{1}{1-\mu}} \quad \text{and} \quad \ell = \frac{\alpha}{1+\alpha} \left(1 - \delta \left(\frac{z_n}{z_m} \right)^{\frac{1}{1-\mu}} - \frac{\bar{m}/\phi}{z_m} \right),$$

where

$$\delta = \left(\frac{1-\phi}{\phi} \mu \right)^{1/(1-\mu)} - \frac{1-\phi}{\phi} \left(\frac{1-\phi}{\phi} \mu \right)^{\mu/(1-\mu)}.$$

Define labor supply as $n = 1 - \ell - h$ and income as $y = z_m n$. There are three points worth mentioning at this stage,

1. The utility derived from consumption can be written as $\ln(\phi c_m - \bar{c}(z_m, z_n))$ where

$$\bar{c}(z_m, z_n) = \bar{m} - (1-\phi) z_n h^{\mu}$$

is analogous to the time-varying $\bar{c}$ in our baseline model. Suppose now that z_m grows at rate g_m and that z_n grows at rate g_n. Then $\bar{c}(z_m, z_n)$ increases through time whenever $g_n < \mu g_m$, that is whenever market productivity grows fast enough relative to home productivity. Hence this model can deliver the feature discussed in Section 4 that the time-varying $\bar{c}$ must increase with productivity.

2. When market productivity grows faster than home productivity home hours decline and leisure time increases.

3. The interpretation of $\bar{c}(z_m, z_n)$ is that it the part of "subsistence consumption," $\bar{m}$, that is satisfied by market goods. Thus, the interpretation of $\bar{c}(z_m, z_n)$ being increasing through time is that the fraction of "subsistence consumption" that is satisfied using market goods instead of home produced goods increases as a country becomes richer.

We now investigate the ability of this model to fit the hours data. Let z_m and z_n grow at constant rates: $z_{m,\tau} = z_{m,1830} e^{g_m(\tau-1830)}$ and $z_{n,\tau} = z_{n,1830} e^{g_n(\tau-1830)}$. Normalize the level of market productivity $z_{m,1830} = 1$ and define the following vector of 7 parameters to be determined:

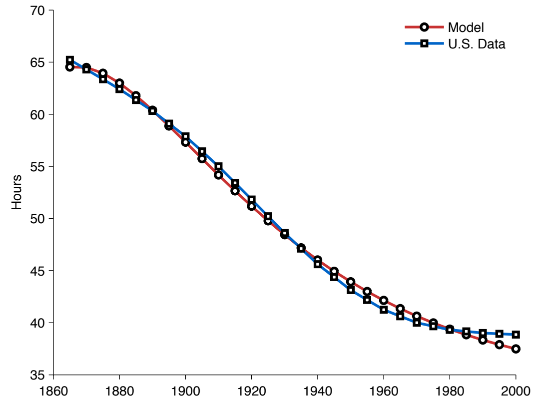
$$\lambda = (g_m, g_n, \alpha, \bar{m}, \phi, \mu, z_{n,1830})'$$

and define $\hat{n}_\tau(\lambda)$ and $\hat{y}_\tau(\lambda)$ as hours and income at date τ and let $\mathbf{n}_\tau$ be actual hours. In the same spirit as in the baseline calibration, we find λ by solving

$$\min_{\lambda} \left\{ \sum_{\tau=1830}^{2000} \left(\frac{\hat{n}_\tau(\lambda)}{\mathbf{n}_\tau/112} - 1 \right)^2 + \left(\frac{\hat{y}_{2000}(\lambda)/\hat{y}_{1830}(\lambda)}{\exp(0.02 \times 171)} - 1 \right)^2 \right\}.$$

We find $\alpha = 2.3$, $\phi = 0.72$, $\bar{m} = 0.47$ and $\mu = 0.38$. The rate of growth of market and home productivity are $g_m = 0.024$ and $g_n = 0.004$. The behavior of market hours is represented in Figure

Figure B.1: Work Hours – Model with Household Production and Constant Subsistence



B.1. The model predicts that home and market hours are declining over time and leisure time is increasing. Note that we assumed perfect substitutability between home and market consumption for the sake of simplicity. Allowing for a more flexible functional form such as, for example, a CES aggregator of home and market consumption would enhance the model's ability to fit the time series of work hours.

C Cross-Country Experiments

We describe in detail our strategy to restrict the four parameters varying across countries in the cross-country experiments in Section 5. These parameters are the level and growth rate of productivity (z, g) and the life expectancy of the 1915 generation (reaching 35 in 1950) and the 1970 generation (reaching 35 in 2005).

The empirical measure of T used for the benchmark economy that is best suited for the model is the sum of years spent in school and on the labor market for a generation. The same data is not readily available for the time period and set of countries that we analyze. Hence, our approach to calibrating T across countries and time is described in two steps:

1. We estimate an empirical relationship observed across countries between life expectancy at birth and income per capita as follows:

$$\text{Life Expectancy} = \text{slope} \times \ln(\text{GDP per capita}) + \text{constant} + \text{error}.$$

   We estimate this relationship for two time periods. We start with life expectancy of the 1915 generation. There does not exists a wealth of data to estimate this relationship. Thus, we use the data from Preston (1975) pertaining to the 1930s.[24] The estimated relationship fits the data very well with an adjusted $r^2 = 0.82$. The estimated slope coefficient is 9.481. We estimate the same relationship for life expectancy of the 1970's generation using data from the World Bank Development Indicators and Penn World Tables. The fit of the data is also very good with an $r^2 = 0.52$ and an estimated slope coefficient of 7.1684. The assumed empirical relationship implies that the difference in life expectancy between any two economies at a point in time is given by the slope coefficient times the log of the factor difference in income per capita between the two economies:

$$T_t - T_t^{us} = \text{slope}_t \times \ln\left(\frac{y_t}{y_t^{us}}\right).$$

   We use this relationship for 1915 and 1970 and the life expectancy for the benchmark economy to estimate the implied life expectancy for all economies in our cross-country experiments.

2. For each of the 10 economies we consider we search for an initial level and a growth rate of productivity, i.e. a pair (z, g), such that the model matches the income per capita of the economy relative to the benchmark economy as described in Table 1 for 1950 and 2005. Life expectancy of the generations reaching 35 in 1950 and 2005 are dictated by the relationships estimated in step 1.

[24] Preston also offers data for the 1900s but this data contains only 10 countries. The 1930s data report life expectancy at birth and real income per capita for 38 countries.

Table 1: GDP per Capita and Schooling across Countries

	1950		2005		
Decile	y_{50}	s_{50}	y_{05}	s_{05}	s_{05}/s_{50}
1	0.05	1.27	0.06	5.07	3.99
2	0.07	1.65	0.07	6.79	4.12
3	0.10	2.82	0.18	8.47	3.00
4	0.13	1.88	0.13	8.01	4.26
5	0.18	3.55	0.31	10.29	2.90
6	0.21	3.34	0.24	9.16	2.74
7	0.24	4.23	0.33	10.36	2.45
8	0.37	5.30	0.57	10.56	1.99
9	0.59	6.73	0.71	11.74	1.74
10	0.83	8.11	0.78	11.08	1.37
$R_{10/1}$	16.60	6.39	13.00	2.19	-
$R_{9/1}$	11.80	5.30	11.83	2.32	-

Note: y is real GDP per capita relative to that of the United States and s is average years of schooling of the 25-29 year old population. Numbers reported are the average of each decile. The countries in each decile are the same in each year and represent the 1950 distribution of GDP per capita.

Table 2: Calibration

Preferences	$\rho = 0.05$, $\alpha = 2.87$, $\beta = 5.91$, $\sigma = 0.55$
	$\mu = 1.71$, $\omega = 0.91$, $\kappa = 1.85$
Technology	$\gamma = 0.10$
	$\theta = 0.04$, $\psi = 0.30$
Productivity	$g = 2.07\%$, $z_{1795} = 1.00$
Demography	$T_\tau = 0.1716 \times \tau - 279.38$

Table 3: Model's Implications – Baseline Experiment

	Calibrated Parameters				1950 Results			2005 Results				Accounting	
	z_{1795}	g (%)	T_{1915}	T_{1970}	y	s	$1-\ell$	y	s	$1-\ell$	s_{05}/s_{50}	Cross Section	Time Series
1	0.02	2.65	21.4	36.1	0.05	1.6	101.8	0.06	5.5	65.8	3.43	96	89
2	0.05	2.18	24.1	39.4	0.07	3.0	82.2	0.07	6.1	63.4	2.05	84	51
3	0.01	3.25	27.4	42.7	0.10	3.3	81.6	0.18	7.0	58.7	2.11	93	68
4	0.11	2.09	29.9	45.3	0.13	4.5	68.8	0.13	7.6	56.8	1.68	68	36
5	0.03	3.18	33.0	49.9	0.18	4.9	68.6	0.31	9.4	48.8	1.92	80	61
6	0.10	2.47	34.5	49.1	0.21	5.3	65.4	0.24	9.3	48.7	1.74	71	55
7	0.06	2.84	35.7	50.8	0.24	5.5	65.1	0.33	10.0	46.1	1.81	79	66
8	0.06	3.13	39.8	53.6	0.37	6.2	62.6	0.57	11.3	41.8	1.81	81	86
9	0.22	2.62	44.3	56.1	0.59	7.8	54.5	0.71	12.2	39.4	1.57	70	81
10	0.92	2.00	47.5	57.6	0.83	9.7	45.3	0.78	12.8	38.2	1.32	27	89
U.S.	1.00	2.07	49.3	58.7	1.00	10.3	43.6	1.00	13.1	37.6	1.28		

Note: y is output per capita relative to the U.S., s is average years of schooling, and $1-\ell$ is hours worked. In this experiment countries differ by the level and rate of growth of productivity (z and g) and life expectancy in 1950 and 2005.
The "Cross-Section' columns report the fraction of the difference in years of school, in 1950, between a decile and the U.S. that is accounted for by the model. The "Time Series' columns report the ratio between the growth rate of years of school in the model and the data.

Table 4: Model's Implications – The Effect of Equal Productivity Growth across Countries

	Calibrated Parameters				1950 Results			2005 Results				Accounting	
	z_{1795}	g (%)	T_{1915}	T_{1970}	y	s	$1-\ell$	y	s	$1-\ell$	s_{05}/s_{50}	Cross Section	Time Series
1	0.04	2.07	21.4	36.1	0.05	2.2	92.1	0.06	5.5	66.1	2.50	90	66
2	0.06	2.07	24.1	39.4	0.07	3.1	81.1	0.07	6.1	63.5	2.00	84	49
3	0.09	2.07	27.4	42.7	0.10	4.0	72.5	0.18	6.8	60.3	1.72	85	49
4	0.12	2.07	29.9	45.3	0.13	4.5	68.8	0.13	7.6	56.8	1.67	68	36
5	0.16	2.07	33.0	49.9	0.18	5.1	65.8	0.31	8.9	51.7	1.74	76	52
6	0.19	2.07	34.5	49.1	0.21	5.4	64.7	0.24	9.1	49.7	1.69	70	52
7	0.21	2.07	35.7	50.8	0.24	5.6	63.7	0.33	9.7	47.9	1.71	77	60
8	0.33	2.07	39.8	53.6	0.37	6.6	59.2	0.57	11.0	43.2	1.66	73	73
9	0.56	2.07	44.3	56.1	0.59	8.3	50.8	0.71	12.2	39.7	1.46	56	69
10	0.82	2.07	47.5	57.6	0.83	9.6	45.6	0.78	12.8	38.2	1.33	30	91
U.S.	1.00	2.07	49.3	58.7	1.00	10.3	43.6	1.00	13.1	37.6	1.28		

Note: y is output per capita relative to the U.S., s is average years of schooling, and $1-\ell$ is hours worked. In this experiment countries differ by the level of productivity (z) and life expectancy in 1950 and 2005. Productivity growth (g) is assumed the same across countries as in the benchmark economy.

The "Cross-Section' columns report the fraction of the difference in years of school, in 1950, between a decile and the U.S. that is accounted for by the model. The "Time Series' columns report the ratio between the growth rate of years of school in the model and the data.

Table 5: Model's Implications – The Effect of Equal Productivity Growth and Change in Life Expectancy across Countries

	Calibrated Parameters				1950 Results			2005 Results				Accounting	
	z_{1795}	g (%)	T_{1915}	T_{1970}	y	s	$1-\ell$	y	s	$1-\ell$	s_{05}/s_{50}	Cross Section	Time Series
1	0.04	2.07	21.4	30.7	0.05	2.2	92.1	0.06	4.7	67.6	2.16	90	56
2	0.06	2.07	24.1	33.4	0.07	3.1	81.1	0.07	5.3	65.0	1.72	84	38
3	0.09	2.07	27.4	36.7	0.10	4.0	72.5	0.18	6.0	61.5	1.51	85	37
4	0.12	2.07	29.9	39.2	0.13	4.5	68.8	0.13	6.7	57.8	1.48	68	27
5	0.16	2.07	33.0	42.3	0.18	5.1	65.8	0.31	7.8	52.6	1.52	76	39
6	0.19	2.07	34.5	43.8	0.21	5.4	64.7	0.24	8.3	50.2	1.54	70	43
7	0.21	2.07	35.7	45.0	0.24	5.6	63.7	0.33	8.8	48.4	1.56	77	49
8	0.33	2.07	39.8	49.1	0.37	6.6	59.2	0.57	10.3	43.4	1.55	73	64
9	0.56	2.07	44.3	53.6	0.59	8.3	50.8	0.71	11.8	39.8	1.42	56	63
10	0.82	2.07	47.5	56.8	0.83	9.6	45.6	0.78	12.7	38.2	1.32	30	88
U.S.	1.00	2.07	49.3	58.6	1.00	10.3	43.6	1.00	13.1	37.6	1.28		

Note: y is output per capita relative to the U.S., s is average years of schooling, and $1-\ell$ is hours worked. In this experiment countries differ by the level of productivity (z) and life expectancy in 1950 and 2005. Productivity growth (g) and the change in life expectancy between 1950 and 2005 are assumed the same across countries as in the benchmark economy.

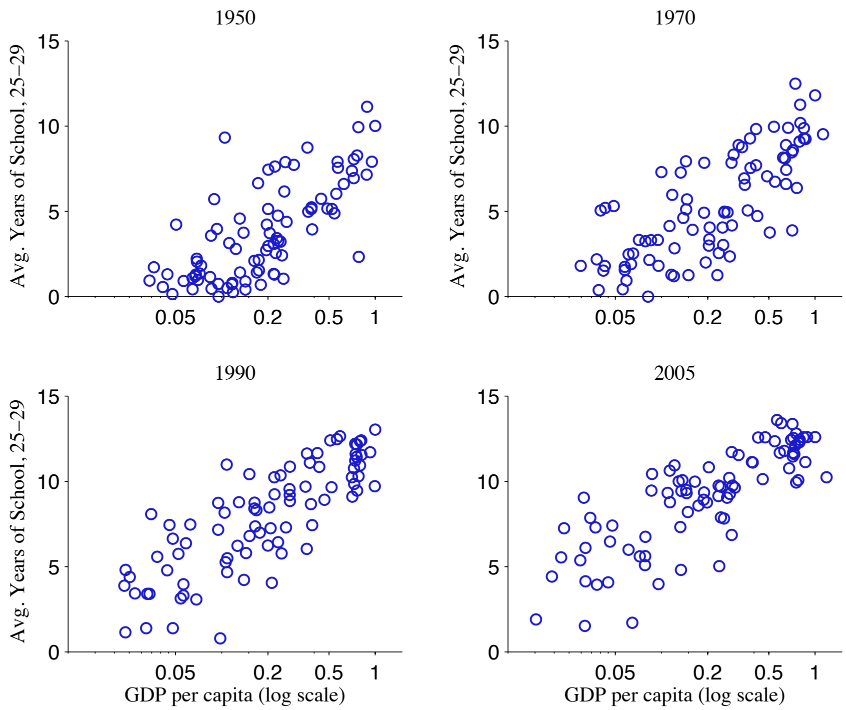
The "Cross-Section' columns report the fraction of the difference in years of school, in 1950, between a decile and the U.S. that is accounted for by the model. The "Time Series' columns report the ratio between the growth rate of years of school in the model and the data.

Table 6: Model's Implications with $\bar{c}$ Constant – Baseline Experiment

	Calibrated Parameters				1950 Results			2005 Results			
	z_{1795}	g (%)	T_{1915}	T_{1970}	y	s	$1-\ell$	y	s	$1-\ell$	s_{05}/s_{50}
1	-	-	-	-	-	-	-	-	-	-	-
2	-	-	-	-	-	-	-	-	-	-	-
3	-	-	-	-	-	-	-	-	-	-	-
4	-	-	-	-	-	-	-	-	-	-	-
5	-	-	-	-	-	-	-	-	-	-	-
6	-	-	-	-	-	-	-	-	-	-	-
7	-	-	-	-	-	-	-	-	-	-	-
8	-	-	-	-	-	-	-	-	-	-	-
9	0.07	1.72	44.6	56.1	0.61	0.1	111.9	0.71	6.4	60.2	118.92
10	0.93	0.68	47.5	57.6	0.83	8.7	49.6	0.78	12.0	41.4	1.39
U.S.	1.00	0.80	49.3	58.7	1.00	10.3	44.7	1.00	14.2	36.9	1.38

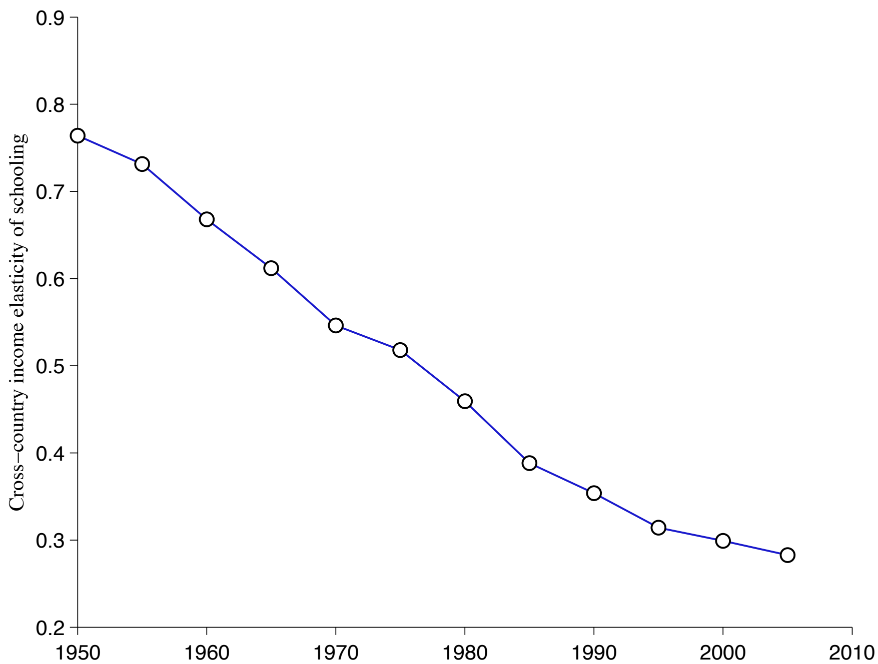
Note: y is output per capita relative to the U.S., s is average years of schooling, and $1-\ell$ is hours worked. In this experiment countries differ by the level and rate of growth of productivity (z and g) and life expectancy in 1950 and 2005.

Figure 1: Average Years of Schooling Population 25 to 29 – Selected Years



Note: The source of data is Barro and Lee (2010) for schooling and the Board (2010), Total Economy Database for GDP per capita. The horizontal axis measures GDP per capita relative to the United States. The vertical axis measures average years of schooling for the 25-29 population.

Figure 2: Income Elasticity of Schooling across Countries



Note: For each year, we regress the (natural) logarithm of average years of schooling on a constant and log real GDP per capita across countries in our sample. The slope coefficient is plotted for each year.

Figure 3: Years of School Completed at age 35 – Model and U.S. Data

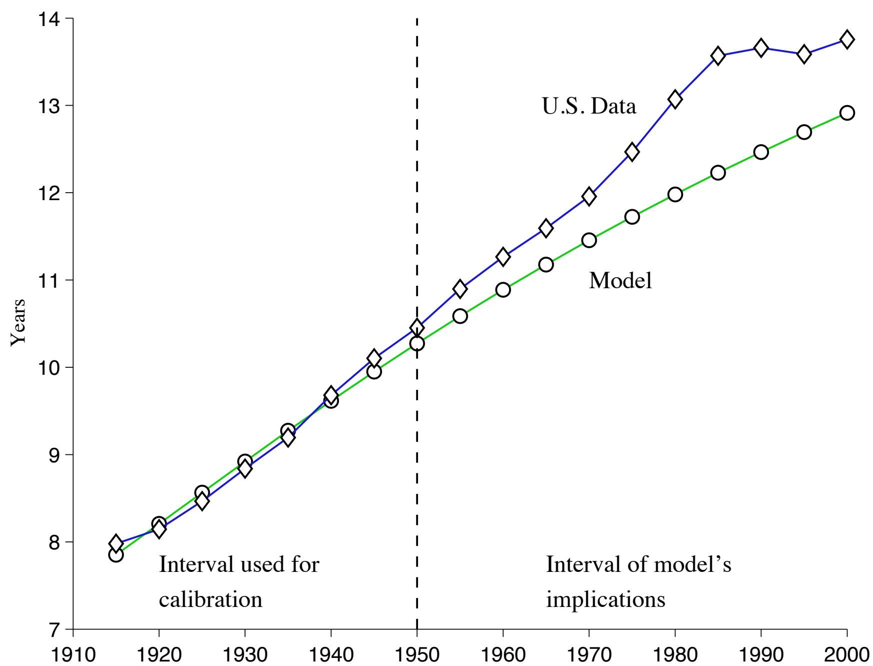


Figure 4: Work Hours – Model and U.S. Data

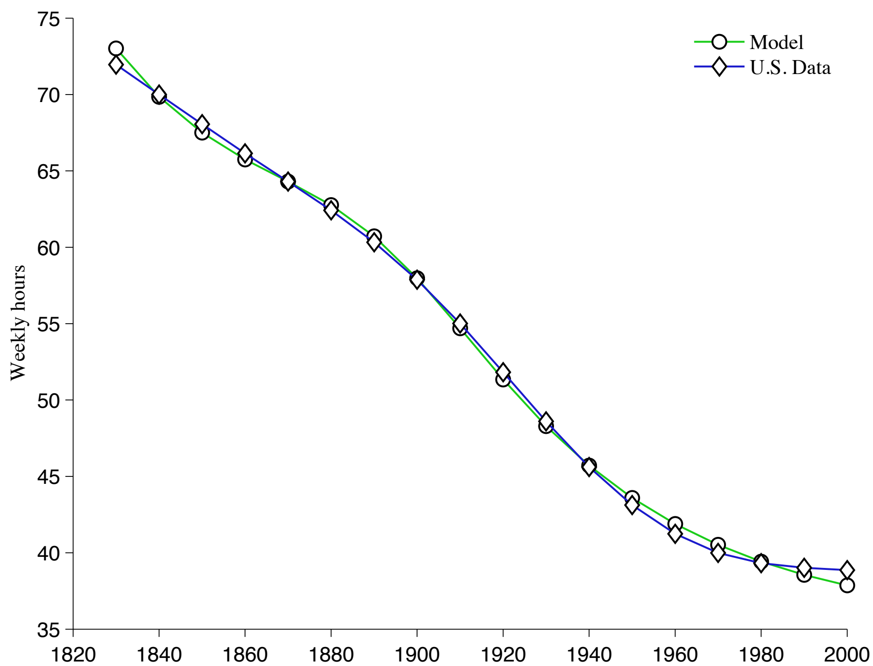


Figure 5: Years of Schooling across Countries – Model and Data

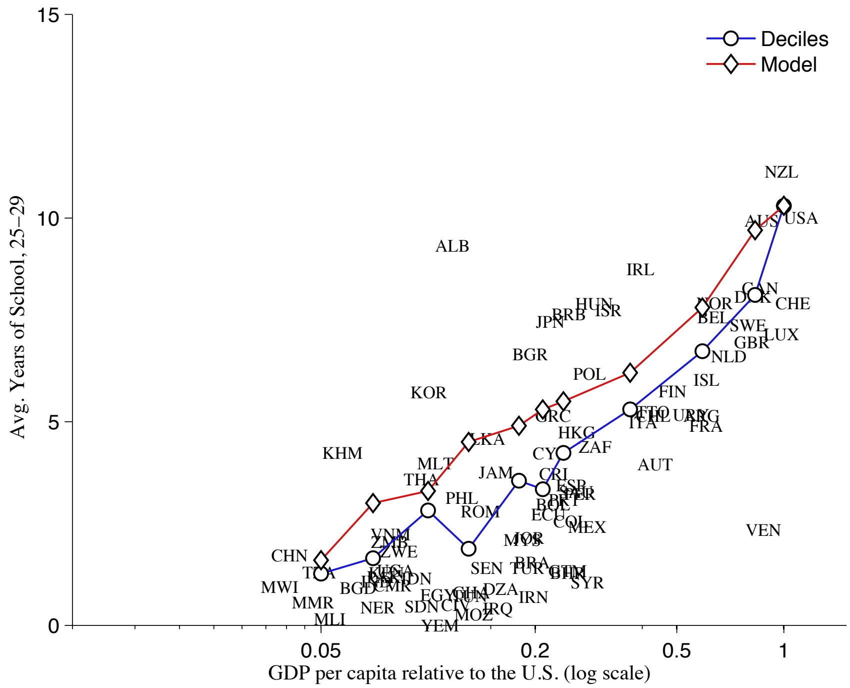
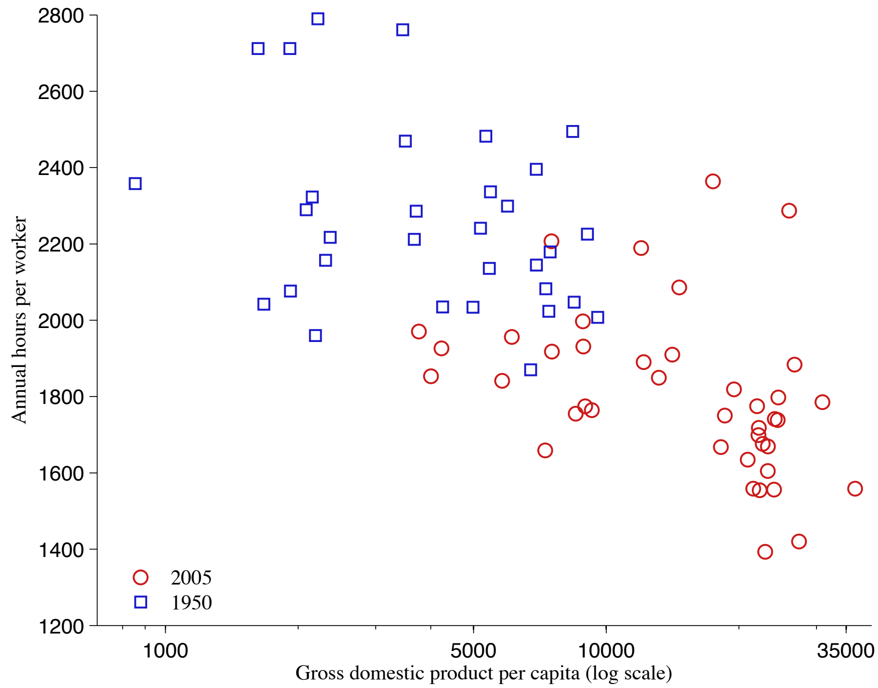
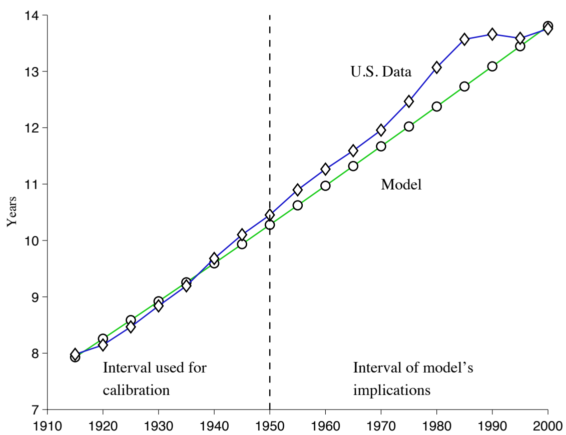


Figure 6: Work Hours across Countries

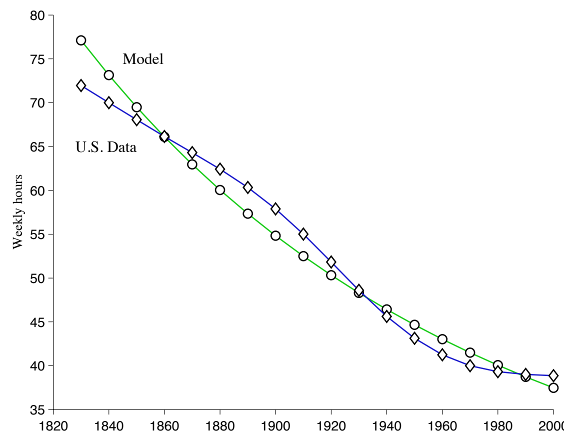


Note: Average annual hours per worker from the Board (2010), Total Economy Database.

Figure 7: Model with Constant $\bar{c}$ and U.S. Data



Years of School



Hours