

Fundamental Disagreement*

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Abstract

We study the term structure of disagreement of professional forecasters for key macroeconomic variables. We document a novel set of facts: (1) forecasters disagree at all horizons including the very long run; (2) the term structure of disagreement differs markedly across variables: it is downward sloping for real output growth, relatively flat for CPI inflation, and upward sloping for the federal funds rate; (3) disagreement is time varying at all horizons including the very long run. We evaluate the ability of benchmark models of informational frictions to match these stylized facts. We show that these models require two additional ingredients. First, agents must decompose signals into temporary factors and low-frequency changes in fundamentals. Second, agents must take into account the dynamic interactions between variables when forming forecasts. The documented disagreement across forecasters is informative about how agents perceive structural macroeconomic relationships. In particular, the monetary policy rule perceived by professional forecasters features a high degree of interest-rate smoothing and time variation in the intercept.

Keywords: Expectations, survey forecasts, imperfect information, term structure of disagreement

JEL Classification: D83, D84, E37

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1 Introduction

People, even informed specialists, disagree about unknown economic outcomes. Surveys of expectations taken from consumers, firms, professional forecasters, financial analysts or FOMC members show that individuals have heterogeneous beliefs about the same economic variable. Recent research incorporating heterogeneous beliefs in macroeconomic and finance theories has shown promise in answering empirical questions that have proven to be challenging for a representative agent framework. In particular, sources of disagreement can lead to inertia in price dynamics (Mankiw and Reis, 2002; Woodford, 2003; Maćkowiak and Wiederholt, 2009), non-fundamental driven business cycle fluctuations (Lorenzoni, 2009; Angeletos and La’O, 2013; Rondina and Walker, 2012), speculative bubbles and boom and bust dynamics in asset prices (Scheinkman and Xiong, 2003; Burnside, Eichenbaum, and Rebelo, 2013), or deviations from the expectations hypothesis of the term structure of interest rates (Nimark, 2012).¹ However, while the heterogeneity of beliefs is a key ingredient in all of these theoretical contributions, relatively little is known about the empirical properties of disagreement.

The first contribution of this paper is to document some new facts about disagreement among professional forecasters that should help to discipline the design of imperfect information models with heterogeneous beliefs. More precisely, we use the Blue Chip Financial Forecast (BCFF) survey to characterize the properties of disagreement for US real output growth, consumer price index (CPI) inflation, and the federal funds rate from 1986 to 2013. The second contribution is to establish under what conditions two popular models of information frictions can replicate these facts. In addition to providing guidance for the modeling of expectations, these conditions are also informative about how forecasters think about macroeconomic dynamics. Our third contribution is to illustrate that disagreement across forecasters is informative about how agents perceive structural macroeconomic relationships and, in particular, the reaction function of the central bank.

The new stylized facts about disagreement can be illustrated by the two graphs below. The left panel of Figure 1 shows our measure of average disagreement across time for a set of different forecast horizons ranging from one quarter to 6-to-11 years ahead. Throughout the paper, we define disagreement as the average forecast of the highest ten responses minus that of the lowest ten responses of survey participants for a given variable and forecast horizon.² A first regularity that stands out from this figure is that, for each of the three

¹See, for example, Hansen (2007), Sargent (2008), and Mankiw and Reis (2010) for general discussions.

²As discussed in Section 2.1, this measure is strongly correlated with alternative measures of disagreement commonly used in the literature.

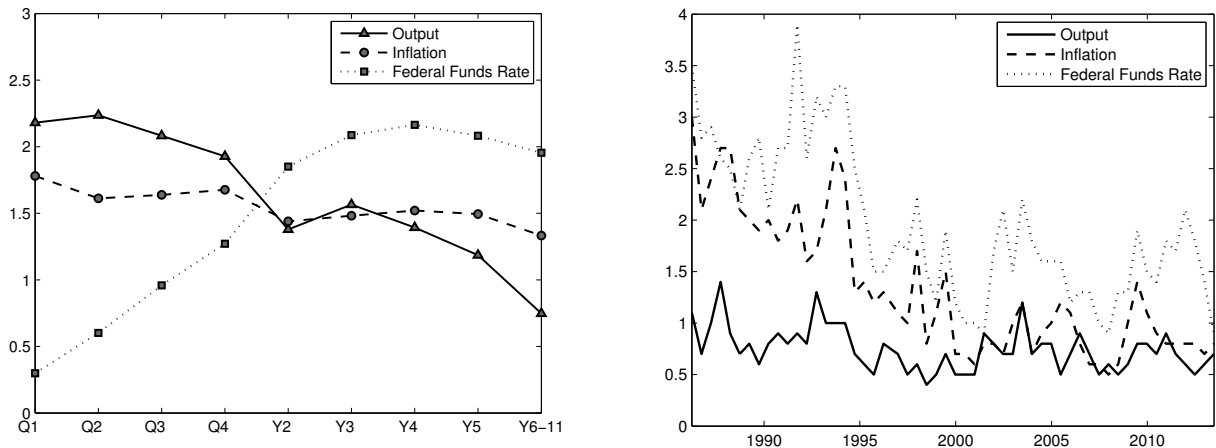


Figure 1: This figure shows selected statistics for forecaster disagreement from the Blue Chip Financial Forecasts survey. Disagreement is defined as the average forecast of the highest ten responses minus that of the lowest ten responses of survey participants. The left panel shows the term structure of disagreement averaged across time for real output growth, CPI inflation, and the federal funds rate for various forecast horizons. The longest horizon captures the average forecast for horizons from 6-to-11 years ahead. The right panel displays the time series of the 6-to-11 years ahead forecast disagreement for the three variables.

variables we consider, the disagreement is non-zero even for long horizons. We refer to this persistent disagreement among forecasters as *fundamental disagreement*, since it likely captures different views about low-frequency changes in the fundamentals of the economy. A second striking fact is that the shape of the average *term structure of disagreement* varies across variables. It is downward sloping for real output growth, almost flat for CPI inflation, and upward sloping for the federal funds rate. Hence, the dispersion of opinions about the long term can differ substantially from the disagreement in the short term. Finally, a third fact is shown in the right panel of Figure 1 which reports the time series of the long-run forecasts for the three variables from 1986 through 2013. It underlines that in addition to being non-zero, fundamental disagreement is not constant over time and covaries between variables.

These facts suggest that convincing theories in macroeconomics and finance should be able to generate persistent and time-varying disagreement. In this paper, we consider a simple model environment where informational frictions can deliver patterns of disagreement consistent with the facts described above. The model captures three important challenges that economic agents face. The first one is that they are not fully informed at all times about the true state of the economy. The second challenge is that when facing fluctuations in economic conditions, agents need to distinguish in real time between temporary and

permanent factors. The latter capture low-frequency shifts in the structure of the economy, as for example, changes in potential GDP growth, the long-term mean of inflation or the natural rate of interest. The third challenge is that the nature of economic fluctuations is inherently multidimensional and consequently agents must take into account the dynamic interactions across variables when forming expectations. We address the first challenge by modeling agents' expectations formation process subject to information frictions. We consider two widely-used specifications: the noisy information model of Sims (2003) and Woodford (2003) and the sticky information model of Mankiw and Reis (2002). We address the second challenge by augmenting these models with the assumption that the imperfectly observed state is the sum of two unobserved components: a transitory one which captures short-lived economic fluctuations, and a permanent one which captures fundamental changes to the economy. This decomposition follows a long tradition in macroeconomics that goes back at least to Kydland and Prescott (1982). Decompositions into persistent and transitory components also play an important role in finance, in particular the literature on long-run risk models (e.g., Bansal and Yaron, 2004). Finally, we address the third challenge by using a multivariate model. This is in contrast to the majority of the existing literature on imperfect information models.

We calibrate both specifications of this imperfect information model to the data using a maximum penalized likelihood approach and evaluate their ability to replicate the documented three facts about disagreement. To this end, we choose the models' parameters to match, as closely as possible, the empirical properties of realized real output growth, CPI inflation and the federal funds rate jointly with selected sample moments from the BCFF survey forecasts. Specifically, we penalize the model using the volatility of average (i.e., consensus) forecasts from the BCFF survey at different forecasting horizons and *only* the one-quarter ahead disagreement. The calibration exercise reveals that the two usual specifications of information frictions we consider are able to replicate the shapes of the term structure of disagreement that we observe in the data (see Figure 1) and imply very similar patterns of forecasts and forecast disagreement. Accordingly, the particular nature of the information friction is not important in generating these results. However, we show that both the presence of a slow-moving drift in each variable and the dynamic interaction between variables, as captured in a multivariate framework, are critical to matching the data. The unobserved slow-moving drift component in the model contributes to the time variation in forecast disagreement at all horizons, and produces a positive level of disagreement even in the very long run. The multivariate setup of the model is required to generate the different shapes of the term structures of disagreement that we observe in the data. Most importantly, a univariate version of our model cannot generate upward-sloping disagreement for reasonable

parameter values. The multivariate dimension is also needed to generate disagreement about variables that are perfectly observed such as the federal funds rate.

The term structure of disagreement thus reveals that forecasters consider slow-moving fundamentals as well as multivariate interactions as essential to predicting macroeconomic variables. These results are obtained without assuming any structural model behind agents' forecasts. We go one step further by giving a structural interpretation to forecasters' reduced-form model. We focus on the monetary policy reaction function and show that our reduced-form model parameters are consistent with a policy rule with plausible coefficients. We also evaluate through counterfactuals the role of the different components in the monetary policy rule in explaining the observed dispersion of federal funds rate forecasts at various horizons. Our results indicate that the monetary policy rule perceived by professional forecasters features both a high degree of interest-rate smoothing as well as time variation in the intercept. Moreover, we show that disagreement about the intercept term is largely, but not exclusively, driven by disagreement about the inflation target and the growth rate of potential output.

Perhaps surprisingly, our model's modest departure from the homogeneous full information setup proves to be sufficient to explain short- and long-term disagreement. In our generalized imperfect information model no agent is systematically endowed with "better" information than any other agent and they all know the true data-generating process (DGP). This stands in contrast to models where agents observe more informative signals either because they have more precise priors or higher signal-to-noise ratios. It also contrasts with models which feature persistent disagreement about the true DGP, either because agents can never fully learn about the true DGP or have immutable priors. We see the symmetry of agents as an appealing property as it is consistent with the well-documented fact that the consensus forecast is difficult to beat, i.e., that no individual forecaster has systematically better forecast performance (e.g., Bauer, Eisenbeis, Waggoner, and Zha, 2003; Stock and Watson, 2004). Moreover, Coibion and Gorodnichenko (2012a,b) provide evidence, by studying the conditional response of consensus forecast errors to shocks, that models with the aforementioned asymmetries are not consistent with the data. It is also important to point out that in our modeling setup, agents forecast an exogenous data generating process. That is, we abstract from any feedback from forecasts to outcomes in general equilibrium as well as from strategic interactions or other forms of endogenous information acquisition. While we believe that these effects may also be important, our focus is to keep the model environment as simple as possible.

There is a large literature in macroeconomics which studies survey data. The properties

of consensus or median survey forecasts have been widely documented. In particular, numerous papers have discussed the bias and the efficiency of consensus forecasts (see, for example, Pesaran and Weale, 2006 for a survey) or have used consensus forecasts in model evaluation and estimation (e.g., Roberts, 1995; Adam and Padula, 2011; Del Negro and Eusepi, 2011). More recently, the focus has shifted towards the cross-section of forecasters and in particular the evolution of short-term disagreement as well as forecast uncertainty at the individual level. Mankiw, Reis, and Wolfers (2003) emphasize that disagreement about short-term inflation forecasts in different surveys of the US economy is time varying and somewhat correlated with changes in macroeconomic variables such as inflation and output growth. Lahiri and Sheng (2008) and Patton and Timmermann (2010) study disagreement up to two years ahead using the Consensus Economics survey. This survey reports fixed-target forecasts. In contrast, we use fixed-horizon forecasts.³ Accordingly, it is important to highlight that our definition of the term structure of disagreement is different from the one used in these papers. Dovern, Fritsche, and Slacalek (2012) also rely on the Consensus Economics survey. However, they construct approximate fixed-horizon forecast series based on the reported fixed-target forecasts and document that the properties of near-term disagreement about real output growth, inflation, and short-term interest rates differ across variables and across G7 countries. A different strand of the literature discusses the relationship between disagreement about inflation and measures of inflation uncertainty as implied by density forecasts, see for example Rich and Tracy (2010). Moreover, Wright (2011) documents that disagreement of one-year ahead inflation forecasts from the Consensus Economics survey is correlated with nominal term premia in a number of countries. All of these papers have in common that they investigate the properties of forecast disagreement only up to horizons of at most two years.

Our paper is also linked to the growing literature that uses survey data to understand the formation of expectations. Mankiw, Reis, and Wolfers (2003) relate the properties of a sticky inflation model to the observed forecast disagreement about future inflation. Carroll (2003) uses consensus forecasts from households and professional forecasters to validate an epidemiological model of expectations. Branch (2004), Coibion and Gorodnichenko (2012a,b) and Andrade and Le Bihan (2013) use survey data to discriminate among various models of expectations including sticky and noisy information models. All these papers confront the implications of the various models with the properties of short-term survey forecasts only. In addition, the existing literature has mostly relied on univariate models (see, for example, the comprehensive study by Coibion and Gorodnichenko, 2012b) with Andrade and Le Bihan

³Fixed-horizon refers to forecasts at multiple horizons all based on the same information set. Fixed-target refers to forecasts of the same calendar period surveyed at different times.

(2013) an exception.

Our paper is organized as follows. Section 2 provides a detailed description of the BCFF data and our new set of facts. In Section 3 we introduce our model, discuss its properties and describe how we calibrate it to the data. Our main results are presented in Section 4. In Section 5, we use the observed and model-implied disagreement to discriminate between different monetary policy rules perceived by forecasters. Section 6 concludes.

2 Stylized Facts about Disagreement

2.1 Data

We study a unique collection of individual forecasts of real output growth, CPI inflation, and the federal funds rate from the Blue Chip Financial Forecasts (BCFF) survey. This survey, conducted monthly since 1982, asks participants ranging from broker-dealers to economic consulting firms to provide forecasts of the quarterly average of a variety of economic and financial variables for specific calendar quarters as far as six quarters in the future. Importantly, since 1986, this survey has also been collecting information on professional forecasts from as far as 6-to-11 years ahead.

The survey is typically released on the first day of the month, and is based on participants' responses that have been collected during the last week of the previous month. Interest rate forecasts are reported as the average over the target period at an annual rate. Real output and CPI targets are period-over-period percent changes at an annual rate. Real output forecasts are measured with respect to forecasts for real GNP prior to April 1992 and with respect to real GDP thereafter. Since its inception in November 1982, each monthly survey compiles individual forecasts for horizons of one quarter ahead to at least five quarters ahead. We collect the one- through four-quarters ahead forecasts as the four-quarters ahead forecast is the longest horizon forecast available in every month. Beginning in 1986, twice a year, participants were also surveyed on their longer-term forecasts for a selected set of financial and macroeconomic variables for upcoming calendar years between two and five years ahead along with an average value for a six-to-ten-years ahead horizon. Because the longer-term forecasts refer to specific calendar years and are collected biannually, the forecast horizons vary somewhat across surveys. For example, the horizon we refer to as two-years ahead (2Y) is either six or eight quarters ahead depending on whether we are using the survey taken later in the year or earlier in the year, respectively. When we calibrate the model we mimic

this sampling scheme to ensure we are consistent with the survey data. Between March 1986 and March 1996 long-run forecasts are provided in the March and October surveys. From December 1996 onward, long-run forecasts are provided in the June and December releases.⁴ The longest horizon 5-year-average forecasts sometimes shifts between horizons of 6-to-10 years ahead to 7-to-11 years ahead. We combine these time series for our analysis to approximately double the number of observations and label the series as the “6-11 years ahead” (6-11Y) forecast for simplicity.

Unfortunately, individual long-run forecasts are not available. Instead the BCFF survey reports the top-10 average long-run forecast and the bottom-10 average long-run forecast. This data limitation also guides our choice of calibration technique discussed in Section 3.3. Consequently, at all horizons we use the difference between the average forecast of the highest ten responses and the average forecast of the lowest ten responses as our measure of disagreement. For the shorter term forecasts up to five quarters ahead for which we observe individual forecasts, this measure of disagreement is almost perfectly correlated with the cross-sectional standard deviation of forecasts and highly correlated with the interquartile range of individual forecasts which have both been used as measures of disagreement in the literature.

Although the survey begins in late 1982, our data sample starts with the March 1986 survey and ends with the July 2013 survey, which guarantees we have no missing observations for consensus forecasts or disagreement at all horizons. All data are quarterly where we choose the January, April, July, and October surveys for the short-horizon forecasts matched with the nearest monthly survey which includes long-run forecasts.⁵ This results in 110 observations for nine reported forecast horizons (Q1, Q2, Q3, Q4, Y2, Y3, Y4, Y5, Y6-11).

2.2 Three Novel Facts about Forecaster Disagreement

We use the dataset to establish a novel set of stylized facts about forecasters’ disagreement. Figure 2 shows the time series of forecaster disagreement for real output growth (upper panel), CPI inflation (middle panel), and the federal funds rate (lower panel) for two forecast horizons: the very short term (one quarter ahead) and the very long term (6-to-11 years ahead). The time series of long term forecast disagreement was already shown in the right

⁴There is one exception to this rule. Long-run forecasts were provided in the January 2003 survey instead of the December 2002 survey.

⁵Recall that surveys are taken at the end of the month previous to the publication date. We choose these survey months as they are based on the maximum amount of information about the current quarter available to survey participants.

panel of Figure 1, but we contrast it here with the equivalent time series for short-term disagreement.

The charts along with Figure 1 document three novel facts about forecaster disagreement. First, forecasters disagree both about the short term but also the medium- and long-run prospects of the economy. Second, the disagreement among forecasters is time varying, even for long-term forecasts. Third, the shape of the term structure of disagreement differs markedly across variables. While disagreement at both short- and long-horizons is time varying for all three variables, the ordering of the level of disagreement across horizons differs for each variable. While the professional forecasters in the Blue Chip survey have disagreed more about output growth in the near term than in the long term over the entire sample from 1986 through 2013, the opposite is true about their forecasts of the federal funds rate. Indeed, while there is typically little disagreement about the federal funds rate in the next quarter, forecasters disagree substantially about the level of short term interest rates in the very long run. Interestingly, for CPI inflation disagreement about the short and long term was at similar levels in the late 1980s and the 1990s, but forecasters started to disagree more about near-term than long-term inflation since around the year 2000. While we only show the time series of disagreement for two different forecast horizons here for simplicity, the left panel of Figure 1 documents the term structures of average disagreement across all forecast horizons. In summary, our data show striking differences across variables: the term structure of disagreement is downward sloping for real output growth, relatively flat for inflation, and upward sloping for the federal funds rate.

At first sight, the results for real output growth and inflation appear to be at odds with the findings of Lahiri and Sheng (2008) and Patton and Timmermann (2010) who have studied forecast disagreement up to two years into the future using the Consensus Economics survey. These authors argue that disagreement increases with the forecast horizon for both variables. In order to understand the difference between their findings and the ones reported here, it is important to highlight the differences between the two sources of survey data. In the Consensus Economics survey the forecast target, i.e. the value of a variable in a particular calendar year, is held fixed across twenty four consecutive monthly forecasts. This implies that in this survey the forecast horizon is shrinking while time passes. In other words, the information set available to forecasters decreases with the forecast horizon. This is in contrast to the Blue Chip survey that we study which asks participants for forecasts at constant horizons. Hence, when interpreting the empirical findings of Lahiri and Sheng (2008) and Patton and Timmermann (2010), it is important to keep in mind that in these two studies, by the nature of the survey they are based on, the information available to

forecasters is not the same across forecast horizons. In contrast, in this paper we take the more conventional view that the information set available to forecasters is fixed in any given period and that based on this same information set forecasts at various horizons into the future are made.

In addition to Lahiri and Sheng (2008) and Patton and Timmermann (2010), a few other papers have studied certain aspects of the disagreement among forecasters. Mankiw, Reis, and Wolfers (2003) document that the disagreement about US inflation expectations up to 17 months ahead from various surveys of consumers and professional forecasters (not including the Blue Chip survey) is time varying. They also study the correlation of inflation disagreement with changes in macroeconomic variables such as inflation and GDP growth and find weak evidence of such correlations. Doovern, Fritsche, and Slacalek (2012) study the behavior of forecasts for real GDP growth, inflation, and short-term interest rates over the next year for the G7 countries. Their analysis is based on the Consensus Economics survey of professional forecasters which is also employed by Lahiri and Sheng (2008) and Patton and Timmermann (2010) for the US. Since that survey does not provide fixed-horizon forecasts for real GDP and inflation, Doovern, Fritsche, and Slacalek (2012) approximate these using the reported fixed-target forecasts. Based on their constructed series, they conclude that short-term disagreement differs across the three variables and across G7 countries. Wright (2011) documents that disagreement of one-year ahead inflation forecasts from the Consensus Economics survey is correlated with nominal term premia in a number of countries. He measures disagreement as the cross-sectional standard deviation of individual inflation forecasts and argues that this variable captures inflation uncertainty. Using data on individual point as well as density forecasts from the US Survey of Professional Forecasters, Rich and Tracy (2010) show that disagreement about US inflation is not systematically related to measures of inflation uncertainty. Boero, Smith, and Wallis (2008) study the relationship between forecast uncertainty and disagreement up to two years into the future for a UK survey of professional forecasts and find a sustained reduction of inflation uncertainty after the introduction of a formal inflation targeting regime by the Bank of England.

One common thread among the papers cited above is that they all study disagreement at horizons of at most two years into the future. To the best of our knowledge, this paper is the first documenting facts about disagreement in the very long term. As we will argue in the next section, these new facts about very long-term forecasts provide information that is important for differentiating between various models of expectations formation.

3 Modeling Disagreement

3.1 Generalized Imperfect Information Models

The true state of the macroeconomy is captured by the random vector $z_t = (g_t, \pi_t, i_t)'$ representing real output growth, g_t , inflation, π_t , and the central-bank policy rate i_t . The data generating process for these state variables is,

$$z_t = (I_3 - \Phi) \mu_t + \Phi z_{t-1} + v_t^z, \quad (3.1)$$

$$\mu_t = \mu_{t-1} + v_t^\mu, \quad (3.2)$$

with initial conditions z_0 and μ_0 . We define the elements of μ_t as $\mu_t = (\bar{g}_t, \bar{\pi}_t, \bar{i}_t)'$. We assume all of the eigenvalues of the matrix Φ are inside the unit circle and v_t^z and v_t^μ are *i.i.d.* Gaussian innovations which are mutually independent with variance-covariance matrices Σ^z and Σ^μ , respectively. Consequently, the variable μ_t plays the role of the “long-run” component in the sense that $\lim_{h \rightarrow \infty} \mathbb{E}[z_{t+h} | z_t, \mu_t, z_{t-1}, \mu_{t-1}, \dots] = \mu_t$. In the following sections we will compare our model to that of one without shifting endpoints (i.e., equation (3.1) with $\mu_t = \mu \forall t$).

The unobserved data can then be written in the compact form,

$$X_t = F X_{t-1} + \epsilon_t, \quad (3.3)$$

where $X_t = (z_t', \mu_t')'$, and ϵ_t are *i.i.d.* Gaussian innovations with variance matrix Σ^ϵ and

$$F = \begin{bmatrix} \Phi & (I_3 - \Phi) \\ 0 & I_3 \end{bmatrix}, \quad \Sigma^\epsilon = \begin{bmatrix} I_3 & (I_3 - \Phi) \\ 0 & I_3 \end{bmatrix} \begin{bmatrix} \Sigma^z & 0 \\ 0 & \Sigma^\mu \end{bmatrix} \begin{bmatrix} I_3 & (I_3 - \Phi) \\ 0 & I_3 \end{bmatrix}'. \quad (3.4)$$

There are N agents in our model. Each agent j observes the data $\{y_{jt} : t = 1, \dots, T\}$ where

$$y_{jt} = z_t + \eta_{jt} = H' X_t + \eta_{jt}, \quad (3.5)$$

where $H = [I_3 \ ; \ 0_{3 \times 3}]'$ and η_{jt} are *i.i.d.* Gaussian observation noise with diagonal variance matrix Σ^η . In words, each agent receives a noisy signal about z_t and uses the current and past history of y_{jt} to construct forecasts of each variable. In particular, we assume each agent has full knowledge of the parameters defined in equation (3.4) and produces forecasts for z_{t+h} , $h \in \mathbb{Z}_+$, conditional on $\Omega_{jt} = \{y_{jt}, y_{j(t-1)}, \dots\}$ based on the Kalman filter,

$$\mathbb{E}[z_{t+h} | \Omega_{jt}] = H' F^h X_{t|jt}, \quad (3.6)$$

where

$$X_{t|jt} = X_{t|j(t-1)} + P_{t|j(t-1)}H_{jt} (H'_{jt}P_{t|j(t-1)}H_{jt} + \Sigma^\eta)^{-1} (y_{jt} - H'_{jt}X_{t|j(t-1)}), \quad (3.7)$$

$$P_{t|jt} = P_{t|j(t-1)} - P_{t|j(t-1)}H_{jt} (H'_{jt}P_{t|j(t-1)}H_{jt} + \Sigma^\eta)^{-1} H'_{jt}P_{t|j(t-1)}, \quad (3.8)$$

$$P_{t|j(t-1)} = FP_{(t-1)|j(t-1)}F' + \Sigma^\epsilon, \quad (3.9)$$

with initial conditions $X_{0|j0}$ and $P_{0|j0}$.

This model nests two widely-used specifications of information frictions: the noisy information model of Sims (2003) and Woodford (2003) and the sticky information model of Mankiw and Reis (2002). We will discuss each in turn.

Noisy Information Model: The noisy information specification is characterized by equation (3.5) where y_{jt} is always observed. Under this specification, the recursive formulas make clear that at any point in time, disagreement about the current and future states of the macroeconomy depends only on the current realized observation error η_{jt} and all past realizations through the difference between the realized state X_t and the agents' previous period forecast $X_{t|j(t-1)}$.⁶ Each agent shares the same model and initial condition, $P_{0|0}$, and receives noisy signals drawn from an identical distribution. As a result, each agent has the same (optimal) Kalman gain and no agent will produce systematically better forecasts than any other. That is, $H_{jt} = H$, $P_{t|jt} = P_{t|t}$ and $P_{t|j(t-1)} = P_{t|(t-1)}$ for all j and t . Moreover, at each point in time, all agents update their estimate of the true state of the macroeconomy, which requires disentangling the “short-term” component from the “long-term” component.

Sticky Information Model: The sticky information specification is characterized by equation (3.5) where $\eta_{jt} = 0$ for all j and t but the k th element of y_{jt} is only observed with a fixed probability λ_k . Note that previous formulations of the sticky information model have only used a univariate censoring variable. We allow for this generality so that both the noisy and sticky information specifications are equally flexible. Then at each point in time for the k th variable, $\lfloor n\lambda_k \rfloor$ agents are randomly chosen from a discrete uniform distribution where $\lfloor \cdot \rfloor$ represents the integer part of the expression. These selected agents use the (perfectly observed) current value of the k th element of z_t when updating their forecasts. Thus, at any point in time, only a fraction of agents will observe a particular element of z_t . Each agent then updates its Kalman filter estimate of the states (equations (3.7) - (3.9)). When agents do not observe the full vector z_t they use the Kalman filter with missing observations (e.g.,

⁶One could consider alternative implementations of the noisy information model where agents also received a common noisy signal and/or could perfectly observe past states or past forecasts of other agents. We conjecture that our main results would be robust to these generalizations potentially requiring larger variation in the idiosyncratic noisy signals to match the behavior of the observed data.

Harvey (1989)). This implies that in our implementation of the sticky information model, when agents observe an element of z_t they do not necessarily observe all past observations of the same variable.⁷ Under this specification, disagreement about the current and future states of the macroeconomy depends on both the current and past history of all agents’ observations of z_t . In more detail, the dimension of H_{jt} changes across agents and across time depending on which elements of z_t are observed by forecaster j . If no element of z_t is observed then $X_{t|jt} = X_{t|j(t-1)}$ and $P_{t|jt} = P_{t|j(t-1)}$.

3.2 Model Properties and Predictions

3.2.1 Discussion of the Model

The noisy- and sticky-information models considered in this paper incorporate three important informational constraints that forecasters face. First, agents in the model do not perfectly observe at all times the “true state” of the economy, as represented by the vector X_t in the previous section. Second, agents have to infer to what extent changes in the observed variables are due to transitory shocks, as represented by the innovation v_t^z , or reflect changes to the slow-moving permanent components, as captured by the innovation v_t^μ . Third, agents must take into account the dynamic interactions across variables when forming expectations.

The first informational constraint is easily motivated by observing that economic variables such as real GDP and CPI inflation are released with a delay and feature sizable and significant future revisions (at least in the case of GDP). Hence, the macroeconomic releases that agents observe about these variables in real time are noisy signals of the true state of the economy that economic agents have to process. In the noisy information model, the noisy measures y_{jt} that agents observe can be interpreted as signals about the state of the economy, obtained by using private information or reflecting different weights given to available public information. This information friction induces disperse beliefs in a rational expectations framework and has been widely used in many macroeconomic and finance models, see, among others, Morris and Shin (2002), Sims (2003), Woodford (2003), Lorenzoni (2009), Maćkowiak and Wiederholt (2009), and Nimark (2012). The sticky information model captures the costs of processing the information available to produce a forecast update. A

⁷This differs from previous implementations in the literature where a fraction of agents are able to observe the full time series up to the current date. In a multivariate model with different values of λ_k for each series, calibrating the model using that approach would be computationally infeasible. We discuss the robustness of our results to our choice of formulation in Section 4.4.

narrow interpretation of the sticky information model, as a model of infrequent acquisition of information, best characterizes households' behavior, but not of professional forecasters which have access to a constant flow of information – see for example Carroll (2003). Here we follow Mankiw and Reis (2002) and interpret the model as capturing the fact that even professional forecasters have limited resources for *processing* newly acquired information into forecast updates or must pay a cost to do so. So, despite the availability of new information about the current state of the economy, forecasters may not incorporate it into their forecasts and report forecasts similar to what they would have reported had they not updated their information set. Last, most forecasters in the Blue Chip survey have broader business goals than just providing a forecast. As such, the sticky-information model might be viewed as a simple model of “rational inattention”, where the infrequent forecast update reflects resource allocation within firms.

The second constraint implies that agents optimally use different components of the signals they observe for short-term versus long-term forecasts. In particular, they need to filter from the observed data the highly volatile temporary factors from the slow-moving permanent components of the variables of interest. This decomposition into permanent and transitory elements has a long and widespread tradition in theoretical and empirical macroeconomic research. For instance, the seminal real-business cycle model in Kydland and Prescott (1982) considers such a decomposition of productivity growth. More recently, Sbordone and Cogley (2008) model inflation as having a permanent and a transitory component. Gürkaynak, Sack, and Swanson (2005) study the consequences of such a specification of inflation for the term structure of interest rates. Moreover, several studies show that a time-varying drift captures well both the dynamic properties of variables such as real GDP growth (Stock and Watson, 1989; Cogley and Sargent, 2005; Laubach and Williams, 2003), the inflation rate (Stock and Watson, 2007; Cogley, Primiceri, and Sargent, 2010), and the federal funds rate (Kozicki and Tinsley, 2001; Gürkaynak, Sack, and Swanson, 2005) as well as the slow movements of their consensus long-term expectations (Edge, Laubach, and Williams, 2007; Kozicki and Tinsley, 2012). Finally, to address the third constraint we utilize a multivariate model of expectations formation.

An important aspect of our model is that no agent has informational advantages over any other: in the noisy information model each forecaster draws from the same distribution of noisy signals; in the sticky information model, every agent faces the same probability of updating. In addition, in our model agents agree on the model of the economy and they do not have different priors about the model's parameters. As a result, our model implies that in a long enough sample no agent will systematically forecast better than other agents. We

think that these are appealing properties in light of the widely documented result that it is difficult to beat consensus forecasts of both survey participants and econometric models (see e.g. Bauer, Eisenbeis, Waggoner, and Zha (2003) or Stock and Watson (2004)).

3.2.2 Predictions of the Model about Forecaster Disagreement

We now review the main properties of the disagreement that the model presented in the previous section generates. To simplify the discussion, we assume here that the economy is populated by a continuum of forecasters and that at date t each agent had access to an infinite sequence of observations.

Noisy Information Model: In the noisy information version of the model, rewriting equation (3.7), we see that the h -step ahead optimal forecast of agent i is given by

$$z_{t+h|jt} = H'F^h X_{t|j(t-1)} + H'F^h P_{t|(t-1)} H (H'P_{t|(t-1)}H + \Sigma^\eta)^{-1} (H'(X_t - X_{t|j(t-1)}) + \eta_{jt}). \quad (3.10)$$

Then the steady-state disagreement, captured by the cross-sectional variance of forecasts in the model can be described by this simple expression:

$$V_h^z = H'F^h [(I - GH') \mathbf{V}_1^X (I - GH')' + G\Sigma^\eta G'] (H'F^h)', \quad (3.11)$$

where $G = F^{-1}\bar{K}$ with $\bar{K} = F\bar{P}H (H'\bar{P}H + \Sigma^\eta)^{-1}$ denotes the steady-state Kalman gain and \bar{P} the steady-state mean squared error matrix, and where $\mathbf{V}_1^X = V(X_{t|jt-1}|t)$ stands for the cross-sectional variance of agents' predictions in $t - 1$ for the state vector at date t , $X_{t|j(t-1)}$.

We now review how the model can potentially explain the observed term structures of disagreement. We discuss the implications for the time variation of disagreement at the end of this section. Let us start from the simplest possible model and progressively add features as needed to explain the facts. Consider a simple univariate model without shifting endpoints, so that all the terms in equation (3.11) are scalars and $|F| < 1$. Then, it is immediate to see that: (i) for $h \rightarrow \infty$, disagreement tends to zero and (ii) the term structure of disagreement is monotonically decreasing with the forecasting horizon ($F^{2h} \downarrow 0$). If we add shifting endpoints, the maximum eigenvalue of F is now equal to one. From equation (3.11) it is easy to see that disagreement in the long-run is positive. However, it can be shown that this model can only generate an upward sloping term structure of disagreement for unreasonably large values of the variance of the innovation to the long-run component. Instead, it appears

a more natural restriction to assume that the diagonal elements of Σ^z are much larger than those of Σ^μ in a pointwise comparison, since the long-term component is meant to capture a slow moving trend. Thus, a univariate model would not be able to generate the different observed shapes of the term structure under these reasonable assumptions.

Consider instead a multivariate model without shifting endpoints. As apparent already from the discussion above this model cannot generate long-term disagreement. However, specific choices for F and Σ^ϵ can deliver any shape of disagreement in the short-run. Intuitively, as the forecasting horizon h increases, some of the off-diagonal elements of F may increase or decrease, generating different patterns of disagreement for different variables. Finally, augmenting this model with shifting endpoints would then inherit these properties along with generating positive fundamental disagreement. One remarkable feature of a multivariate framework is that it does not require idiosyncratic observation noise for all variables in the system in order to generate disagreement for all the variables. This is an appealing feature because for some economic variables such as interest rates and stock prices, it is difficult to argue that they are imperfectly observed by economic agents.

Finally, we discuss the implications for the time variation in disagreement. Equation (3.11) shows that in this model, taking the limit as the number of forecasters grows, disagreement is constant over time. This is inconsistent with the evidence from the BCFF survey and points to a possible limitation of the model. However, the stylized facts we introduce are, of course, derived for a finite number of forecasters. In this case the model does predict some time variation in disagreement and correspondingly some potential correlation in the disagreement measures across different variables. The next section evaluates to what extent the fixed number of forecasters considered here is sufficient to match the second moments of disagreement observed in the data.

Sticky Information Model: Under sticky information, the h -step ahead optimal forecast of agent j derived from equation (3.7) is

$$z_{t+h|jt} = H' F^h [X_{t|j(t-1)} + B_{jt}(X_t - X_{t|j(t-1)})] = H' F^h Z_{jt}, \quad (3.12)$$

with $B_{jt} = P_{t|j(t-1)} H_{jt} (H'_{jt} P_{t|j(t-1)} H_{jt})^{-1} H'_{jt}$. Using $V(Z_{jt}) = V[\mathbb{E}(Z_{jt}|B_{jt})] + \mathbb{E}[V(Z_{jt}|B_{jt})]$, the cross-sectional variance of forecasts can be decomposed into

$$\begin{aligned} V_{ht}^z &= H' F^h (X_t - \bar{X}_{t|(t-1)}) \mathbf{V}^B (X_t - \bar{X}_{t|(t-1)})' (H' F^h)' \\ &+ H' F^h (I - \bar{B}) \mathbf{V}_{1t}^X (I - \bar{B})' (H' F^h)', \end{aligned} \quad (3.13)$$

where $\bar{X}_{t|(t-1)} = \mathbb{E}(X_{t|j(t-1)}|t)$, $\bar{B} = \mathbb{E}(B_{jt}|t)$, and where $\mathbf{V}_{1t}^X = V(X_{t|j(t-1)}|t)$ stands for the

cross-sectional variance of agents' predictions in $t - 1$ for the state vector at date t , $X_{t|j(t-1)}$ and $\mathbf{V}^B = V(B_{jt}|t)$ denotes the cross-sectional heterogeneity of agents' updating matrices in t , B_{jt} , which is constant when one assumes that at date t each agent had access to an infinite sequence of observations $\{z_{jt}, z_{j(t-1)}, \dots\}$.

The expression makes clear that, like in the noisy information version of the model, fundamental disagreement requires shifting endpoints and the associated unit roots in F . If all the eigenvalues of F are smaller than 1 in absolute value, then disagreement goes to zero as the forecast horizon grows. Moreover, as in the noisy information setup, an upward sloping term structure of disagreement requires cross-variable linkages under reasonable assumptions about the relative variability of the short-run and long-run components. In the univariate case under a reasonable parameterization of the variance of the innovation to the long-run component, disagreement under sticky information is, at each date t , a decreasing function of the forecast horizon. Finally, an important difference with the noisy information version of the model is that sticky information generates some time variance of disagreement, even for an infinite number of forecasters. This comes from two sources: (i) the average gap between state realizations and forecasters' state predictions, $(X_t - \bar{X}_{t|(t-1)})$, with bigger underlying shocks at date t increasing disagreement at all horizons; and (ii) the induced time varying cross-sectional dispersion of agents' predictions in $t - 1$ for the state vector at date t , \mathbf{V}_{1t}^X .

3.3 Calibration

The generalized imperfect information model introduced in the previous section appears to have the ability to replicate the key features of our new set of facts. However, it is important to assess the performance of the model in reproducing these facts when parameter values are "reasonable" in the sense of being consistent with the properties of our data. In order to do so, define $\theta_1 = (\Phi, \Sigma^z, \Sigma^\mu)$ and consider the following criterion function,

$$\mathcal{C}(\theta_1, \theta_2, \tilde{\Sigma}^\eta; \alpha) = \mathcal{L}(\theta_1, \tilde{\Sigma}^\eta; \mathcal{Y}_1, \dots, \mathcal{Y}_T) + \alpha \cdot \mathcal{P}(\theta_1, \theta_2; \mathcal{S}_1, \dots, \mathcal{S}_T),$$

where $\theta_2 = \Sigma^\eta$ in the noisy information specification and $\theta_2 = \lambda$ in the sticky information specification.⁸ \mathcal{Y}_t are the actual output, inflation and interest rate data and \mathcal{S}_t are the BCFF

⁸Recall that under our assumptions θ_2 is of the same dimension (three) in both cases.

survey data at time t and \mathcal{L} is the negative of the Gaussian likelihood function,

$$\mathcal{L}(\theta_1, \tilde{\Sigma}^\eta; \mathcal{Y}_1, \dots, \mathcal{Y}_T) = -(2\pi)^{-3/2} \left| H' \tilde{P}_{t|(t-1)} H + \tilde{\Sigma}^\eta \right|^{-1/2} \times \exp \left\{ -\frac{1}{2} \left(\mathcal{Y}_t - H' \tilde{X}_{t|(t-1)} \right)' \left(H' \tilde{P}_{t|(t-1)} H + \tilde{\Sigma}^\eta \right)^{-1} \left(\mathcal{Y}_t - H' \tilde{X}_{t|(t-1)} \right) \right\},$$

where a tilde denotes a variable pertaining to the econometrician (as opposed to the agents in the model). Specifically, we allow for a difference in information available to the econometrician versus the agents via the variable $\tilde{\Sigma}^\eta$. Our interpretation of the model is that neither the econometrician nor the agents perfectly observe the “true” state of the macroeconomy at all points in time. However, as we discuss in Section 4.4 the results are robust to imposing $\tilde{\Sigma}^\eta = 0$.

The second term in the criterion function is a penalization term of the observed moments from the survey forecasts relative to the corresponding model-implied moments,

$$\mathcal{P}(\theta_1, \theta_2; \mathcal{S}_1, \dots, \mathcal{S}_T) = (g(\theta_1, \theta_2) - g_S(\mathcal{S}_1, \dots, \mathcal{S}_T))' W (g(\theta_1, \theta_2) - g_S(\mathcal{S}_1, \dots, \mathcal{S}_T)),$$

where W is a positive semi-definite weighting matrix and $g_S(\mathcal{S}_1, \dots, \mathcal{S}_T)$ is a collection of moments from the data:

- We use real GNP and GDP data as provided by the Bureau of Economic Analysis, headline CPI from the Bureau of Labor Statistics and the federal funds rate from the H.15 data provided by the Board of Governors of the Federal Reserve. The data are quarterly from 1955Q1-2013Q2.
- We use 15 sample moments from the BCFF survey (5 sample moments for each of the three variables). The data are quarterly from 1986Q1-2013Q2.
 - Our disagreement measure for the one-quarter ahead forecast only;
 - The standard deviation of consensus forecast for one- and four-quarters ahead along with two-years ahead and six-to-eleven years ahead.

The corresponding model-implied statistics are constructed by the function $g(\theta_1, \theta_2)$ via a simulation approach:

- We simulate the model using $T = 120$ (approximately the length of the survey data sample) and choose $N = 50$ (consistent with number of participants in the survey) across 100 simulations in our optimization procedure;

- We choose a diagonal weighting matrix which places a weight of 1 on the disagreement related moments and a weight of 0.1 on standard deviation related moments.

The weight matrix is selected so as to choose parameter values such that, as closely as possible, the level of the model-implied one-quarter ahead disagreement is consistent with the data without generating excessively volatile consensus forecasts. We can then evaluate the performance of the model to match the term structure of disagreement using the least amount of disagreement data to do so. Very loosely, we are “normalizing” the model so that the shortest-horizon forecast disagreement is approximately correct.

We would then like to solve $\min_{\theta_1, \theta_2, \tilde{\Sigma}^\eta} \mathcal{C}(\theta_1, \theta_2, \tilde{\Sigma}^\eta)$. The final input necessary to the model calibration is the choice of initial conditions. We have the advantage that we can observe, at least partially, the dispersion in agents’ expectations at the beginning of the sample. In order to calibrate the model we use information from the March 1986 BCFF survey to provide initial conditions for both $z_{t|jt}$ and $\mu_{t|jt}$. For the former, we use the forecasts for the first quarter of 1986 from the March 1986 BCFF survey as a “nowcast”. For the latter, we do not observe individual long-term forecasts, so instead we scale the initial conditions from the nowcast to replicate the 6-to-11 years ahead disagreement measured by the top-10 average minus the bottom-10 average in the same survey.⁹

We want to emphasize that we do not interpret the variation in initial conditions as a reflection in different priors about the structural parameters of the economy, but rather as a result of past observation errors that occurred prior to our sample period. Regardless, in the next section we show that removing the influence of the initial conditions does not alter the main conclusions implied by the model.

4 Results

In this section we use our calibrated parameters to assess the ability of both model specifications to reproduce the new stylized facts from Section 2. We start by discussing the values of the calibrated parameters in both specifications. In Section 4.2 we discuss each model’s implications for the term structure of disagreement (Facts 1 and 2). In Section 4.3 we present the corresponding results for the time variation and co-movement of disagreement across variables (Fact 3). Finally, Section 4.4 presents a number of robustness checks for each model.

⁹The March 1986 survey only includes forecasts for 45 participants. The additional 5 agents in our model are endowed with initial conditions equal to the median of the survey data.

4.1 Calibration Results

For both the noisy and sticky information specifications we discuss the calibrated parameters corresponding to a value of $\alpha = 50$. We view this as our “baseline” calibration as it ensures that the volatility of model-implied consensus forecasts matches the data well across horizons. We will show in Section 4.4 that model-implied disagreement is largely insensitive to variations in α .

The calibrated parameters for the noisy information model are provided in Table 1. Looking at the calibrated parameters we first note that the observation errors faced by the econometrician are smaller than those faced by the agents for real output growth and CPI inflation, whereas the observation error for the federal funds rate is essentially zero for both calibrations.¹⁰ This result is striking since the calibration puts, a priori, no restriction on the variance of the noise specific to the federal funds rate, while in reality, the policy rate can be perfectly observed in real time whereas output and inflation are subject to revisions and publication lags.¹¹ Another important finding is that the variance of the calibrated long-term component (Σ^μ) is substantially lower for all three variables relative to the corresponding variance of the short-term component (Σ^z). This accords with our interpretation of μ_t as capturing slow-moving drifts in the economy’s fundamentals.

Table 2 presents the calibrated parameters for the sticky information specification. Recall that the key difference between the two models is that in the sticky information specification agents do not receive a noisy signal about the state. Instead their ability to observe the elements of the state vector z_t depends on three independent random censoring processes. The results are very similar to those of the noisy information model for all parameters common to both specifications (Φ , Σ^μ , and Σ^z). As opposed to the variance of the observation noise, Σ^η , the λ parameters govern the degree of information stickiness for each variable in this model. The calibrated values for λ correspond to 13, 13 and 50 out of 50 agents observing the current realization of real output growth, CPI inflation and the federal funds rate, respectively, in each period. Thus, in our baseline version of the sticky information specification, as in the noisy information specification, all agents observe the federal funds rate perfectly. For output growth and inflation the calibrated λ implies an average frequency of updating for Blue Chip forecasters of slightly less than four quarters, in line with the results of Mankiw, Reis, and Wolfers (2003).

¹⁰All reported standard deviations are in annualized terms.

¹¹The calibrated variance for the observation error for the federal funds rate is very small for both the econometrician and the agents. Throughout the simulation experiments in the rest of the paper we set it equal to exactly zero so that the short-term interest rate is perfectly observed.

4.2 The Model-implied Term Structure of Disagreement

In Figure 3 we show the model-implied term structure of disagreement for both models along with the disagreement observed in the data. Both calibrated models do a remarkable job replicating the different shapes of the observed term structure of disagreement for the three variables. The term structure implied by both models is downward sloping for output growth, approximately flat for CPI inflation, and upward sloping for the federal funds rate. Moreover, the model-implied term structure of disagreement is strikingly similar across the two different specifications. The only noteworthy difference is that the sticky information model matches the term structure of disagreement for CPI inflation almost perfectly. It is important to emphasize that we only use one quarter ahead disagreement (designated by the open circle in each graph) in the calibration of the models. In sum, both models can reproduce Facts 1 and 2 for parameter values that are consistent with the actual and survey data.

As discussed in Section 3.2.2, the shape of the term structure of disagreement is determined jointly by all of the parameters of the model. Thus it is difficult to give a precise description of the underlying mechanism which generates the results. However, we can provide a heuristic explanation of how each is generated by revisiting the parameter values from Tables 1 and 2. The downward slope in the term structure for real output growth can be explained by the fact that, in both model specifications, it displays fairly volatile temporary shocks and is observed with a fairly high amount of noise or stickiness. This explains the high level of disagreement in the short term and the fact that forecasts at longer horizons respond less to changes in observed real output, delivering relatively low disagreement at medium- to long-term horizons. CPI inflation is also imperfectly observed but, at the same time, its long-term component is relatively more volatile than its temporary component when compared to output growth. Accordingly, this results in a relatively flat term structure of disagreement. Finally, the federal funds rate is perfectly observed and, given its estimated persistence, is predicted well using only its past value. As a result, there is minimal disagreement at short horizons. At longer horizons, though, disagreement about future real output growth, CPI inflation and the long-term level of the federal funds rate (i.e., \bar{i}_t) generate disagreement about the federal funds rate forecasts. In the left panels of Figure 4 and Figure 5 we add 95% confidence intervals around the term structure of disagreement. These plots show that even allowing for variation across simulations, the shapes of the term structure of disagreement mimic those in the data.

Of course, the results on the model-implied term structures of disagreement may come at the

cost of unrealistically volatile forecasts. We now compare the performance of the two models in terms of the variability of model-implied consensus forecasts which we also use in the calibration. The right panels of Figures 4 and 5 present the standard deviation of consensus forecasts from the BCFF survey along with the model-implied standard deviations for the two model specifications. Both specifications capture well the term structure of consensus forecast volatility for all three variables, as shown by the fact that the standard deviation of consensus forecasts falls within the 95% confidence bands indicated by the shaded area with the exception of the three-quarter ahead and four-quarter ahead forecasts of real output growth.

As discussed in Section 3.2.2, disagreement in the medium to long-run depends on agents' disagreement about the decomposition into temporary and long-term factors for all three variables. To highlight this last point, the left panel of Figure 4 and Figure 5 also shows term structures of disagreement for the model without shifting endpoints, shown by the green line. We calibrate this model using the same method as for the model with shifting endpoints. We use the same moments of disagreement (one-quarter ahead only for each variable) but only the one-quarter and four-quarter ahead standard deviations of consensus forecasts, as this model cannot generate variability in long-term forecasts. As before, the circles indicate the moments that are used in the calibration.¹² The model without shifting endpoints clearly falls short at explaining disagreement for all but the shortest horizons. As expected, for long horizons the disagreement implied by this model approaches zero for all variables. Of note, the model with shifting endpoints provides a better fit to disagreement at horizons above one year without compromising the fit of short-term disagreement. This improved performance is not entirely obvious as the model with shifting endpoints has six more parameters, but the calibration imposes six additional restrictions which discipline the volatility of the model-implied longer-term consensus forecasts. In terms of the volatility of consensus forecasts, the model without shifting endpoints has a comparable fit at short to intermediate horizons. The exception is CPI inflation in Figure 4 where the volatility is much higher than in the data. Furthermore, the model without shifting endpoints consistently implies a volatility of consensus forecasts which is too low at longer horizons. These results confirm our analysis in Section 3.2.2. In summary, the introduction of shifting endpoints results in a dramatic improvement of the fit of the term structure of disagreement, especially for horizons above one year. We conclude that the presence of a slow-moving, low-frequency component in the DGP is vital to replicating the first two facts about forecast disagreement that we document.

¹²The open white circles highlight the moments used in the calibration for both models, and the light blue circles single out the moments used only in the calibration of the model with shifting endpoints.

4.3 The Model-implied Second Moments of Disagreement

We next turn to a discussion of the third fact related to the time variance and correlation of disagreement. The left panels of Figure 6 show the standard deviation (hereafter, volatility) of disagreement from the BCFF data and its model-implied counterpart for the noisy information specification. Although the noisy information specification qualitatively matches the different shapes of the term structure of the volatility of disagreement, it is not capable of explaining the levels observed in the data. This is not surprising as we showed in Section 3.2.2 that the time variance of disagreement goes to zero when the number of forecasters goes to infinity in this model. In the right column of the figure we show the pairwise time series correlations of disagreement for different horizons for both the model and the data. We start by briefly discussing the properties of these correlations in the data. First, note that in the survey there is a substantial degree of correlation among the three time series of disagreement at various horizons. An interesting exception is the correlation between disagreement about CPI inflation and the federal funds rate at the one-quarter ahead horizon which almost exactly equals zero. In contrast, the same correlation between the federal funds rate and output growth disagreements equals 60 percent in the data. Consequently, the small amount of short-term disagreement observed for the federal funds rate appears to be to a large extent driven by disagreement about near term-growth prospects. At long horizons, the correlation between disagreement about the federal funds rate and CPI inflation (real output growth) forecasts is more than 80 (60) percent correlated in the data. Hence, long term disagreement about the federal funds rate is clearly driven by disagreement about the determinants of interest rate policy. We will provide a further discussion of this in Section 5. Finally, note that the time series of disagreement about real output growth and inflation are positively correlated at all forecast horizons including the long term. For all three pairs of variables, the noisy information specification tends to match the observed correlations well.

In Figure 7 we show the corresponding second moment properties of the model-implied disagreement for the calibrated sticky information specification. One argument frequently used in favor of the sticky information model is that, even with a continuum of agents, the model will produce time variation of disagreement. The left panels of the figure show the model-implied volatility of disagreement for each variable. While the model-implied volatility of disagreement is similar to that for the noisy information model at long forecast horizons, it is considerably more elevated at short horizons for real output growth and CPI inflation. In particular, the sticky information specification captures well the volatility of disagreement about near-term inflation forecasts. However, the volatility of disagreement about the federal funds rate is almost exactly the same as in the noisy information model (and both are too

low compared to the data). More generally, although the sticky information model produces at least as much time variation of disagreement as the noisy information model, it still fails to fully match the behavior of the survey data. The term structure of correlations of disagreement across the three variables, shown in the right column of Figure 7, is also broadly similar to the one implied by the noisy information model, and both match the correlations observed in the data well. This is all the more remarkable given that the survey moments used in the calibration are only marginal moments of disagreement and consensus forecasts. These results provide an additional motivation for adopting a multivariate framework to study the evolution of survey forecasts as the correlations of disagreement across variables are clearly non-zero in general.

4.4 Robustness and Additional Results

In this section, we assess the robustness of our findings relative to a number of dimensions: (1) the role of the initial conditions; (2) the choice of the penalty parameter; (3) the role of observation error for the econometrician; (4) the disagreement about short-term real interest rates implied by the two models; and (5) the results for the sticky information model relative to an implementation where agents observe the full history of a variable when they update. We discuss these qualitatively in turn, and relegate some of the corresponding tables and figures to the supplementary appendix in order to conserve space.

As discussed in Section 3.3, we choose initial conditions to match the observed short-term and long-term forecaster disagreement at the beginning of our sample. However, since we only use average one-quarter-ahead disagreement in our objective function, it is useful to examine the time series path of model-implied long-term disagreement which we show in Figure 8. We show the average path across simulations along with a shaded area which represents the 95% confidence interval. Both models closely mimic the time series path of observed long-term disagreement including the gradual decline in disagreement for CPI and the federal funds rate seen in the first part of the sample.

While our choice of initial conditions better represents the actual survey data it is important to emphasize that the main results are not driven by this initialization. To show this we simulate both models for 240 periods (twice the original simulation sample size) and discard the first 120 observations of the simulated paths. The results are shown in Figures 9 and 10. In both cases, the long-run disagreement implied by these paths is slightly lower for all three variables but the shapes of the term structure of disagreement and the standard deviation of consensus forecasts are virtually unchanged.

As noted above, we have chosen $\alpha = 50$ as the penalty parameter in our baseline specification. This choice guarantees that our target moments are matched reasonably well by the respective model. At the same time, a higher value of α might compromise the ability of the model to fit the actual data, as measured by the likelihood. To address this issue, we also calibrate the model for values of $\alpha = 1$ and $\alpha = 10$. For these values, the calibrated parameters are similar to those where $\alpha = 50$. More importantly, they imply term structures of disagreement that are close to the baseline in both models. However, not surprisingly, small penalty parameters imply standard deviations of consensus forecasts that do not match the survey data as well as when $\alpha = 50$.

Recall that our calibration allows for measurement error in the observation equation for the econometrician which only affects the likelihood component of the objective function in Section 3.3. One implication of this choice is that the filtered states z_t could be much smoother than the actual observed variables y_t if the observation error is large. In order to evaluate the role of this additional degree of freedom for our main results, we calibrate versions of both models assuming that the econometrician observes y_t perfectly. We find that the results are very similar to the baseline calibration for both models.

As an additional robustness check we exploit the fact that we observe individual forecasts at shorter horizons to construct forecasts and disagreement about real short-term interest rates, defined as the federal funds rate less CPI inflation. Because the disagreement about these two series is correlated at longer than one-quarter-ahead horizons, it is not clear that the model-implied disagreement about real rates should match that observed in the data. That said, as shown in Figure 11 both models produce levels of disagreement and standard deviations of consensus forecasts that are near the actual data (available only out to four quarters). Interestingly, the model-implied term structures of disagreement about real rates are “U-shaped”, declining at short horizons before rising at longer horizons. This shape is distinct from that of output, inflation or the federal funds rate. As the real rate affects intertemporal consumption choices, this pattern suggests that the future consumption uncertainty results from the interplay of uncertainty about future inflation, real output growth and interest rates.

We discussed in Section 3.1 that our formulation of the sticky information model differs slightly from that in the literature. Agents in our model, when allowed to update, only observe the current value for an element of z_t rather than the entire history of that variable up to time t . The standard formulation, where agents observe the full history of a variable when they are allowed to update, is computationally infeasible to calibrate if no restrictions on the λ vector are made. One class of restrictions which alleviates the computational

difficulties is to set $\lambda = 1$ for the federal funds rate and to introduce a common censoring process for both real output growth and CPI inflation. In other words, when an agent updates they observe the entire history of all three variables up to that point in time. This special case coincides with the calibrated values of λ presented in Table 2. In unreported results, we confirm that the term structure of disagreement and other implied moments of the forecast distribution are essentially the same in this specification when agents observe the full history of a variable when they update.

5 Monetary Policy Rules

We have thus far presented a reduced-form model used by the agents to produce forecasts. In this section we provide a structural interpretation of these forecasts in terms of simple monetary policy rules.¹³ We consider the following class of monetary policy rules:

$$i_t = \rho \cdot i_{t-1} + (1 - \rho) \cdot i_t^* + \varepsilon_t, \quad (5.14)$$

$$i_t^* = \bar{i}_t + \varphi_\pi \cdot (\pi_t - \bar{\pi}_t) + \varphi_g \cdot (g_t - \bar{g}_t) \quad (5.15)$$

This is a fairly general class of rules which embeds many popular specifications suggested in the literature. Here, ρ plays the role of determining the degree of interest-rate smoothing. The rule has a time-varying intercept, $(1 - \rho) \cdot \bar{i}_t$, reflecting low-frequency movements in the interest rate. Finally, this class of rules embeds a time-varying inflation target, as measured by $\bar{\pi}_t$, and similarly a long-run equilibrium growth rate given by \bar{g}_t . Consistent with the underlying assumption that we have made throughout the paper, agents agree about the coefficients of the policy rule $(\rho, \varphi_\pi, \varphi_g)$. Consequently, disagreement about the path of i_t will depend only on disagreement about the paths of z_t and μ_t .¹⁴

In this section we consider three different exercises. First, we will discuss a monetary policy rule with coefficients similar to those found in the empirical literature (e.g., Clarida, Galí, and Gertler (2000)) and investigate the implications for forecaster disagreement. Second, we will show that our reduced-form model, to a high degree of approximation, is consistent with the rule in equations (5.14) and (5.15) with similarly “reasonable” coefficients of the policy rule. Third, we will evaluate the role of the different components in the monetary policy

¹³This is potentially in line with Carvalho and Nechio (2014) who show that professional forecasters and at least some subgroups of households survey participants form their expectations up to one year about the future path of interest rates, inflation, and unemployment in a way that is consistent with simple monetary policy rules.

¹⁴It could also depend on disagreement about future deviations from this rule, ε_{t+h} . Here we focus on sources of disagreement that we can observe with the BCFE survey.

rule in explaining the observed dispersion of federal funds rate forecasts at various horizons.

Each agent j in our model forms interest-rate forecasts based on a linear combination of the elements of $z_{t|jt}$ and $\mu_{t|jt}$ governed by (powers of) the matrix F in equation (3.4). An alternative class of interest-rate forecasts are based on the rule above. Given the calibrated value of F we can solve for the choices of $(\rho, \varphi_\pi, \varphi_g)$ which provide the best approximation to the reduced-form forecasts of a structural forecast. Both the reduced form interest rate forecasts and those based on the policy rule are linear combinations of z_t and μ_t . However, the policy rule constrains this linear combination to be a function of only three parameters. This is an over-identified system of equations which we solve via a minimum distance approach.

In the top chart of Figure 12, we show the model-implied forecaster disagreement based on different policy rules. In all simulations, the individual forecasts for output growth, inflation and $\mu_{t|jt}$, needed to compute individual forecast path for the federal funds rate, are computed using the reduced-form model. The solid black line shows the implied disagreement using a rule with coefficients $\rho = 0.90$, $\varphi_\pi = 2.0$, and $\varphi_g = 0.5$. This rule broadly follows the actual disagreement especially at short to medium horizons but overshoots at the longest horizon. As we discussed in Section 4, the high degree of persistence in the federal funds rate is an important factor in explaining the upward slope in the term structure of disagreement. To further illustrate this point we show the same rule with ρ set equal to zero as the dashed black line. This rule implies a downward sloping term structure of disagreement with very high levels at short to medium horizons. Hence, the observed term structure of disagreement for the federal funds rate implies that forecasters perceive a high degree of interest-rate smoothing in the policy function. The blue line is our model-implied structural rule. Despite the fact that we calibrate F from the data based on a reduced-form model, the interest-rate forecasting rule used by our agents is perfectly approximated by a monetary policy rule with “reasonable” parameters (see Figure 3).¹⁵ The corresponding coefficients are

$$\tilde{\rho} = 0.98, \quad (1 - \tilde{\rho}) \cdot \tilde{\varphi}_\pi = 0.26, \quad (1 - \tilde{\rho}) \cdot \tilde{\varphi}_g = 0.30.$$

In the third exercise we evaluate which features of our our model-implied structural rule explain the observed disagreement about the federal funds rate. To that end, we present different term structures of disagreement based on counterfactual interest rate forecasts obtained by adjusting the policy rule. In particular, we would like to evaluate the role of the time-varying intercept. In the bottom chart of Figure 12, we show three alternative

¹⁵The norm of the minimum distance criterion function based on one-quarter ahead forecasts is of the order 10^{-10} . Differences in model-implied disagreement between the reduced-form and structural rules are, at most, of the order 10^{-3} .

specifications for \bar{i}_t . As in the previous graph, the blue line shows our model-implied disagreement where \bar{i}_t is time varying according to the model and parameters in equations (3.1) and (3.2) and Table 1, respectively. The dotted black line corresponds to a constant intercept where $\bar{i}_t = \bar{i}$.¹⁶ This rule is able to generate an upward-sloping term structure of disagreement, but the level of disagreement is fairly small compared to the data at medium to long horizons. We then proceed by generalizing first by assuming that $\bar{i}_t = \bar{r} + \bar{\pi}_t$ where r is the constant real rate and next $\bar{i}_t = -400 \cdot \log(\beta) + \bar{g}_t + \bar{\pi}_t$. The latter case corresponds to a time-varying real interest rate where time variation arises solely from variations in long-term real output growth. Here $\log(\beta)$ describes a constant discount rate. The dashed and solid black line show the results of these two specifications. Although disagreement about long-term inflation and real rates helps to bridge the gap between a constant intercept and the data, some distance still remains. This could potentially reflect time variation in economic agents' preferences (i.e. the discount factor β) or other factors. The key takeaways from these exercises are that interest rate smoothing and the presence of a time-varying intercept in the policy rule are both important factors driving the observed term structure of disagreement about the future path of the federal funds rate, as emphasized for instance in Coibion and Gorodnichenko (2011).

6 Conclusion

This paper documents a novel set of facts about disagreement among professional forecasters: (1) forecasters disagree at all horizons including the very long run; (2) the term structure of disagreement differs markedly across variables: the term structure is downward sloping for real output growth, relatively flat for CPI inflation, and upward sloping for the federal funds rate; (3) disagreement is time varying at all horizons including the very long run. We present two specifications of a general imperfect information model of expectation formation. Both specifications produce similar results and are able to replicate most of our new facts. Hence, the particular nature of the information friction is not important in generating these results. Our analysis instead shows that what is important is an economic environment with the following features: first, the state of the economy is comprised of unobserved transitory and persistent components which agents must disentangle; second, agents must also take into account the dynamic interaction between variables. Explicitly incorporating these elements in the expectation formation process are critical to explaining the cross-section and time series of survey forecasts.

¹⁶Note that since agents know the model parameters, disagreement is invariant to any constant parameter.

An important aspect of our model is that no agent has informational advantages over any other and agents have rational expectations and full knowledge about the structure of the economy. While our model captures the main features of the term structure of disagreement well it does not consistently generate enough time variation in disagreement compared to the survey data. Several extensions to our model could be introduced to overcome this limitation. One approach might be to relax our strict assumptions and assume that agents do not have full knowledge of the DGP, for example, if they must learn about the parameters. An alternative approach would be to endow the model with endogenously generated time variation in the precision of signals that depends on the state of the economy as in Van Nieuwerburgh and Veldkamp (2006). Finally, having shown that our proposed model of expectation formation matches various important features of survey forecasts, one avenue for future research would be to embed the model in a general equilibrium setup.

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Table 1: **Results of Calibration for $\alpha = 50$**
Noisy Information Model

This table provides the calibrated parameters for $\alpha = 50$ as discussed in Section 4.1 of the paper. $|\cdot|$ designates the modulus of a complex number. Results are based on 5,000 simulations.

Φ	Σ^z	$\text{sqrt}(\text{diag}(\tilde{\Sigma}^\eta))$
$\begin{bmatrix} 0.378 & -0.503 & -0.153 \\ 0.125 & 0.974 & -0.033 \\ 0.147 & 0.104 & 0.924 \end{bmatrix}$	$\begin{bmatrix} 3.419 & -0.019 & 0.561 \\ -0.019 & 0.645 & 0.365 \\ 0.561 & 0.365 & 0.632 \end{bmatrix}$	$\begin{bmatrix} 2.592 \\ 1.429 \\ 0.000 \end{bmatrix}$
$ \text{eig}(\Phi) $	Σ^μ	$\text{sqrt}(\text{diag}(\Sigma^\eta))$
$\begin{bmatrix} 0.920 \\ 0.711 \\ 0.646 \end{bmatrix}$	$\begin{bmatrix} 0.008 & 0.014 & 0.026 \\ 0.014 & 0.024 & 0.045 \\ 0.026 & 0.045 & 0.085 \end{bmatrix}$	$\begin{bmatrix} 4.317 \\ 2.731 \\ 0.000 \end{bmatrix}$

Table 2: **Results of Calibration for $\alpha = 50$**
Sticky Information Model

This table provides the calibrated parameters for $\alpha = 50$ as discussed in Section 4.1 of the paper. $|\cdot|$ designates the modulus of a complex number. Results are based on 2,500 simulations.

Φ	Σ^z	$\text{sqrt}(\text{diag}(\tilde{\Sigma}^\eta))$
$\begin{bmatrix} 0.392 & -0.478 & -0.142 \\ 0.122 & 0.939 & -0.024 \\ 0.146 & 0.087 & 0.931 \end{bmatrix}$	$\begin{bmatrix} 3.736 & -0.065 & 0.564 \\ -0.065 & 0.911 & 0.347 \\ 0.564 & 0.347 & 0.635 \end{bmatrix}$	$\begin{bmatrix} 2.586 \\ 1.355 \\ 0.000 \end{bmatrix}$
$ \text{eig}(\Phi) $	Σ^μ	λ
$\begin{bmatrix} 0.920 \\ 0.674 \\ 0.674 \end{bmatrix}$	$\begin{bmatrix} 0.007 & 0.012 & 0.022 \\ 0.012 & 0.021 & 0.039 \\ 0.022 & 0.039 & 0.073 \end{bmatrix}$	$\begin{bmatrix} 0.260 \\ 0.260 \\ 1.000 \end{bmatrix}$

Figure 2: **Time Series of Disagreement**

This figure shows the time series of forecaster disagreement as measured by the average forecast of the highest ten responses minus that of the lowest ten responses for the shortest and longest forecast horizons from the Blue Chip Financial Forecasts survey. The sample period is from 1986Q1 - 2013Q2.

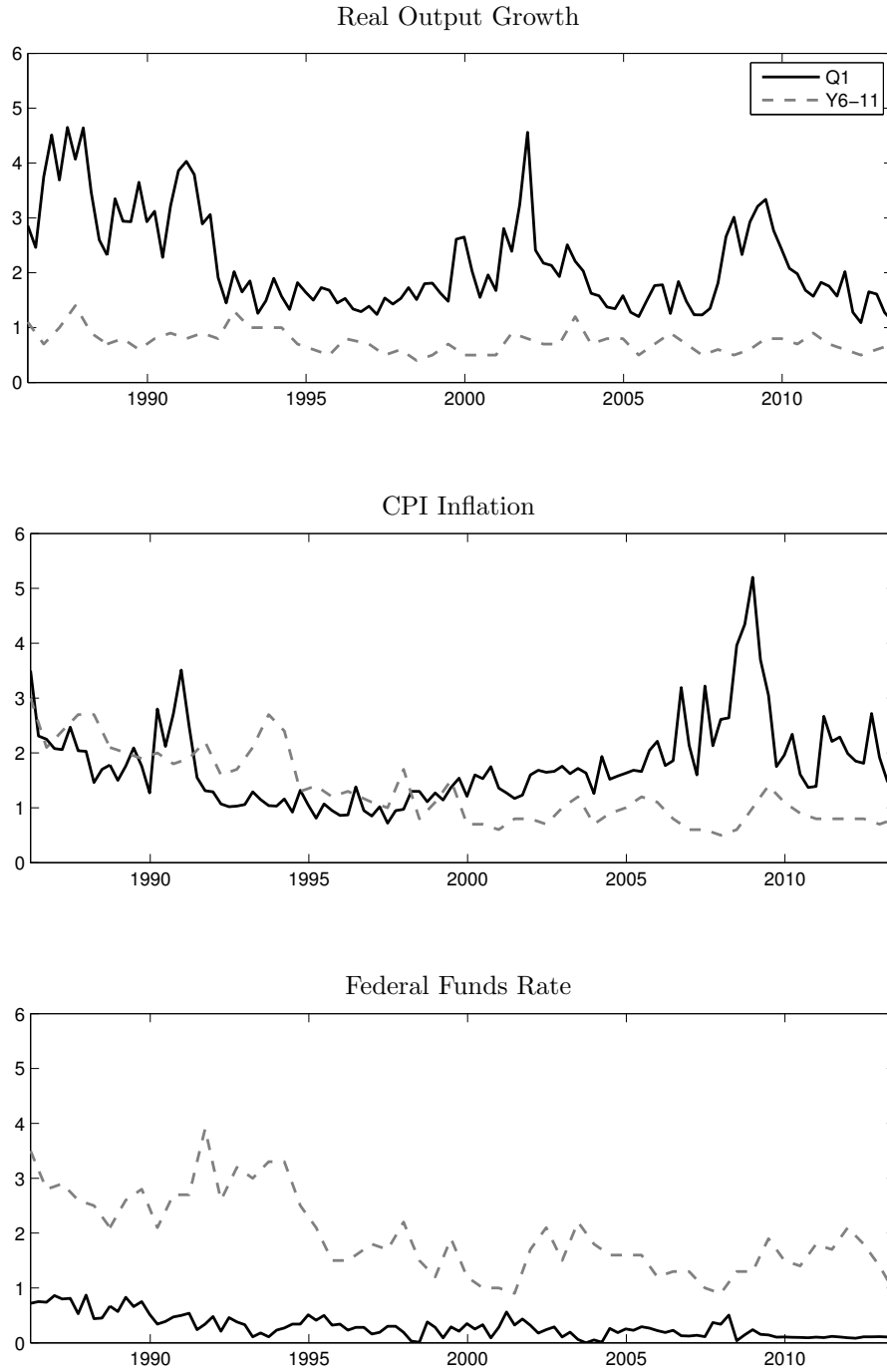


Figure 3: **Term Structure of Disagreement**
Noisy and Sticky Information Models

This figure displays the model-implied (time) average of disagreement across different horizons for the generalized noisy information model (dark blue) and the generalized sticky information model (light blue) calibrated with $\alpha = 50$ along with the Blue Chip Financial Forecasts survey (red). Open circles designate survey moments used to form the penalization term $\mathcal{P}(\theta_1, \theta_2; \mathcal{S}_1, \dots, \mathcal{S}_T)$. Results for the noisy and sticky information models are based on 5,000 and 2,500 simulations, respectively.

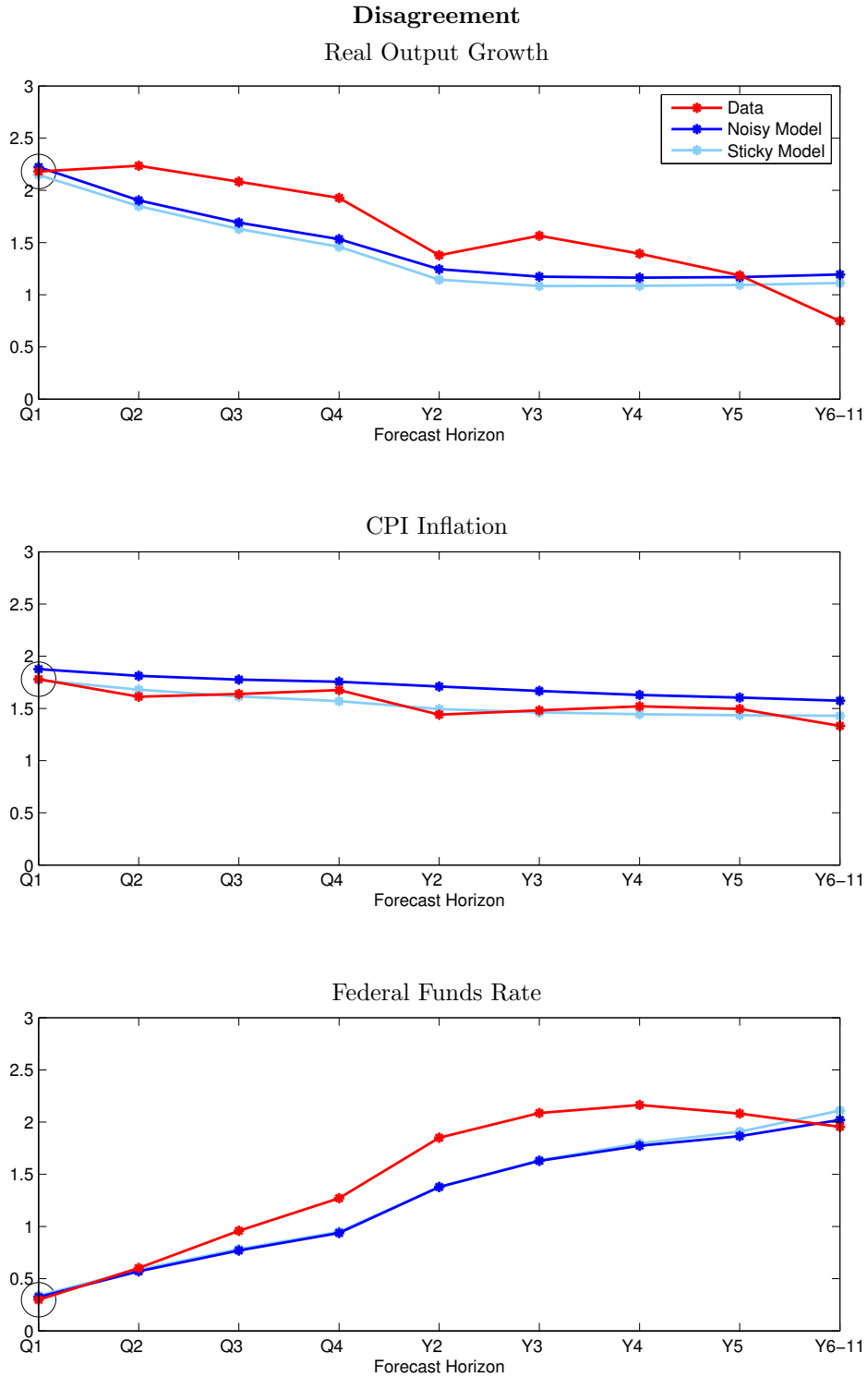


Figure 4: **Disagreement and Standard Deviation of Forecasts**
Noisy Information Model

The first column displays the model-implied disagreement for the generalized noisy information model (blue) and the noisy information model without shifting endpoints (green) calibrated with $\alpha = 50$ along with the Blue Chip Financial Forecasts survey (red). The second column displays the corresponding standard deviation of consensus forecasts. Open white circles designate survey moments used to form the penalization term $\mathcal{P}(\theta_1, \theta_2; \mathcal{S}_1, \dots, \mathcal{S}_T)$ for the model without shifting endpoints. Open white and light blue circles designate survey moments used to form the penalization term for the generalized noisy information model. Model-implied 95% confidence intervals for the model with and without shifting endpoints are designated by shaded regions and dotted lines, respectively. Results are based on 5,000 simulations.

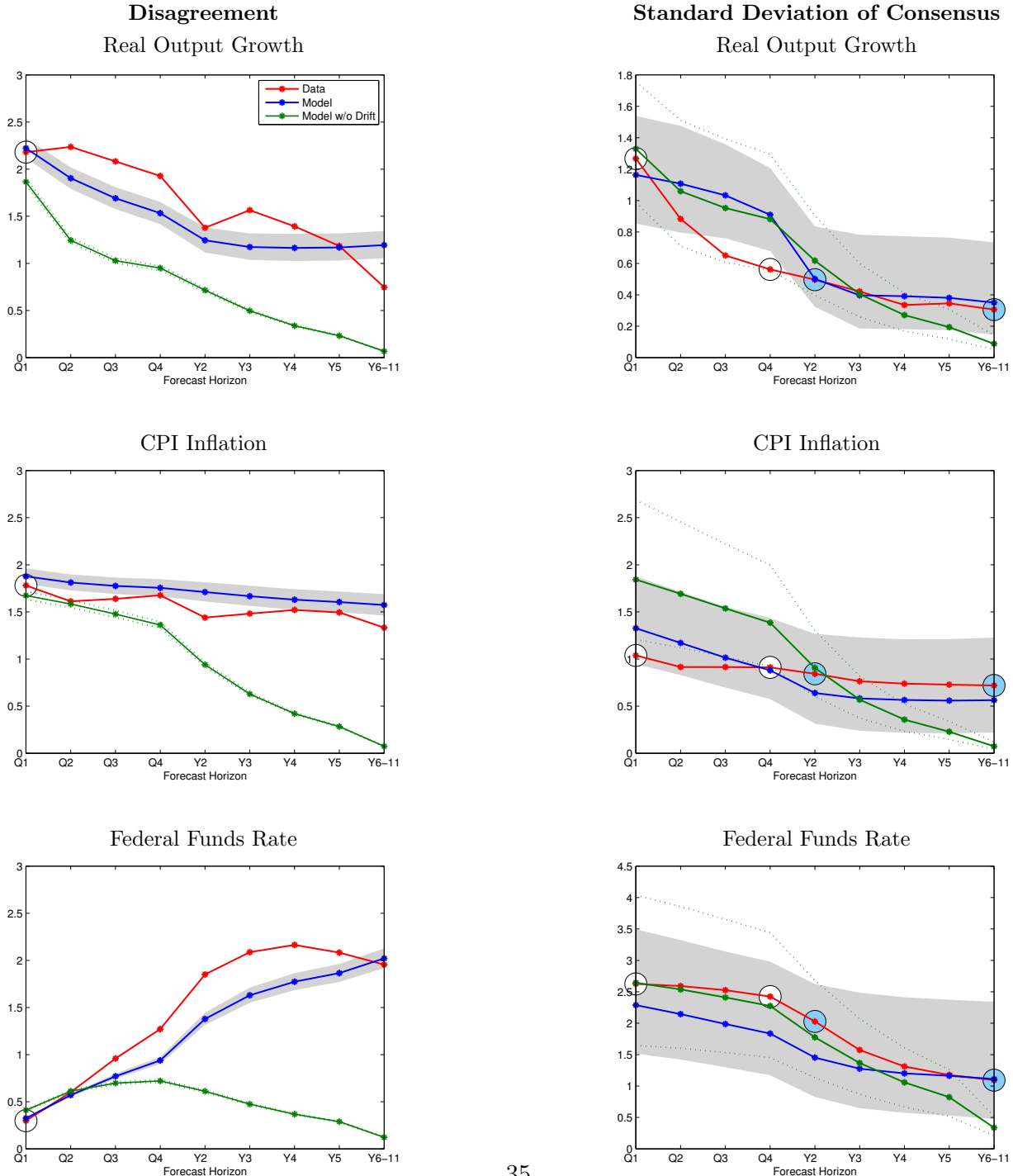


Figure 5: **Disagreement and Standard Deviation of Forecasts**
Sticky Information Model

The first column displays the model-implied disagreement for the generalized sticky information model (blue) and the sticky information model without shifting endpoints (green) calibrated with $\alpha = 50$ along with the Blue Chip Financial Forecasts survey (red). The second column displays the corresponding standard deviation of consensus forecasts. Open white circles designate survey moments used to form the penalization term $\mathcal{P}(\theta_1, \theta_2; \mathcal{S}_1, \dots, \mathcal{S}_T)$ for the model without shifting endpoints. Open white and light blue circles designate survey moments used to form the penalization term for the generalized sticky information model. Model-implied 95% confidence intervals for the model with and without shifting endpoints are designated by shaded regions and dotted lines, respectively. Results are based on 2,500 simulations.

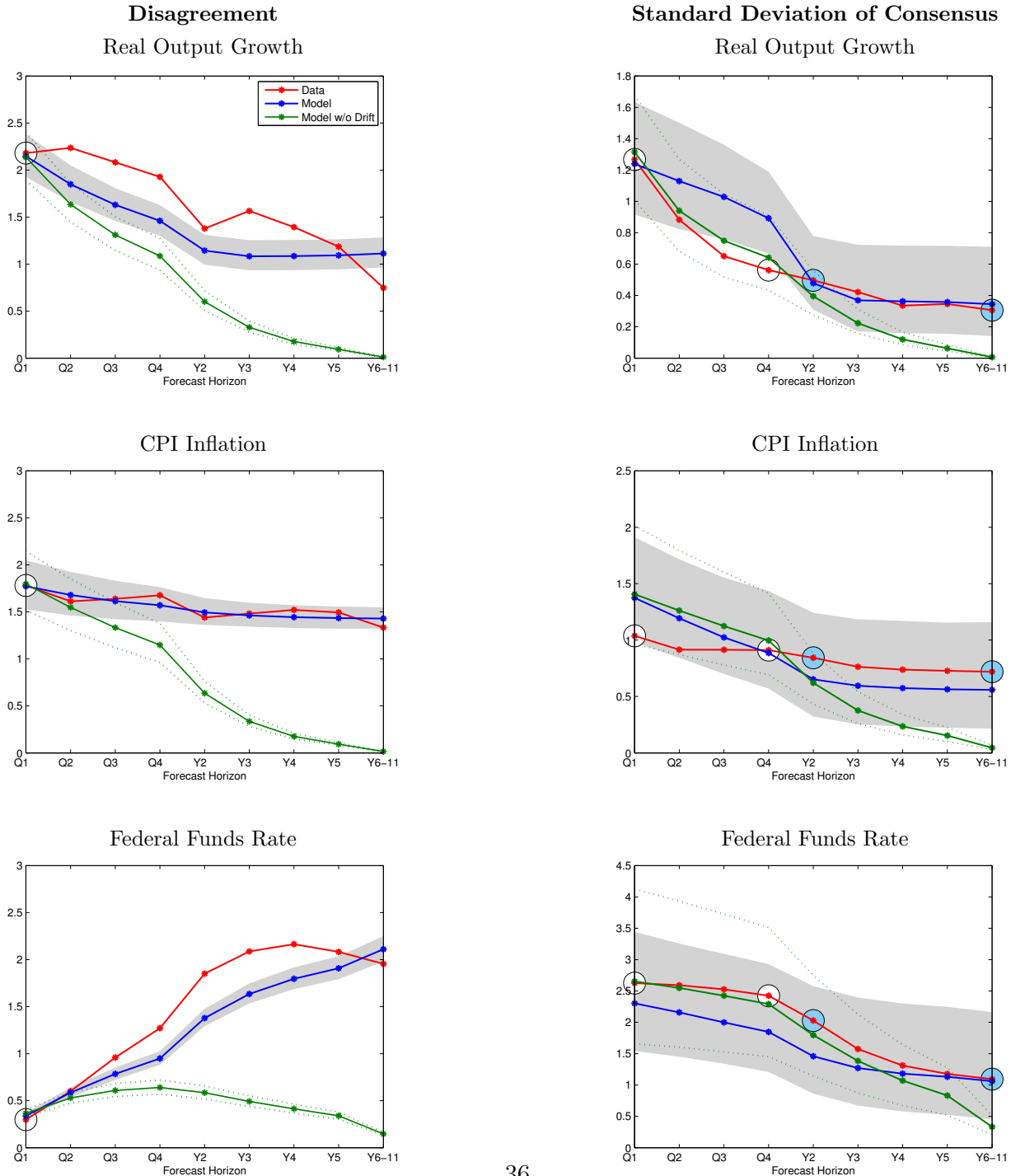


Figure 6: **Second Moments of Disagreement**
Noisy Information Model

The first column displays the model-implied (time) standard deviation of disagreement for the generalized noisy information model calibrated with $\alpha = 50$ (blue) along with the Blue Chip Financial Forecasts survey (red). The second column displays the corresponding correlation of disagreement between variables. Model-implied 95% confidence intervals are designated by shaded regions. Results are based on 5,000 simulations.

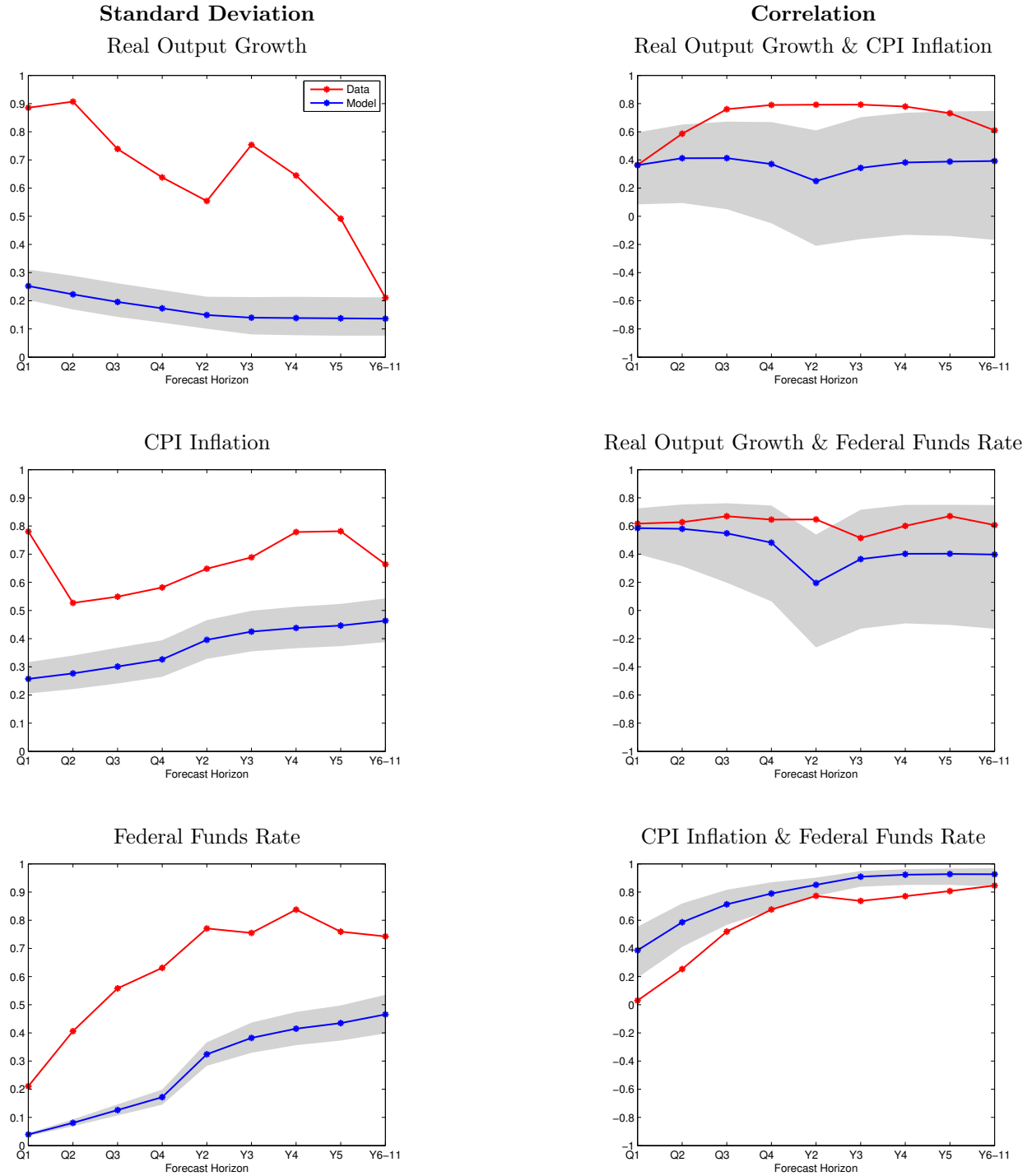


Figure 7: **Second Moments of Disagreement**
Sticky Information Model

The first column displays the model-implied (time) standard deviation of disagreement for the generalized sticky information model calibrated with $\alpha = 50$ (blue) along with the Blue Chip Financial Forecasts survey (red). The second column displays the corresponding correlation of disagreement between variables. Model-implied 95% confidence intervals are designated by shaded regions. Results are based on 2,500 simulations

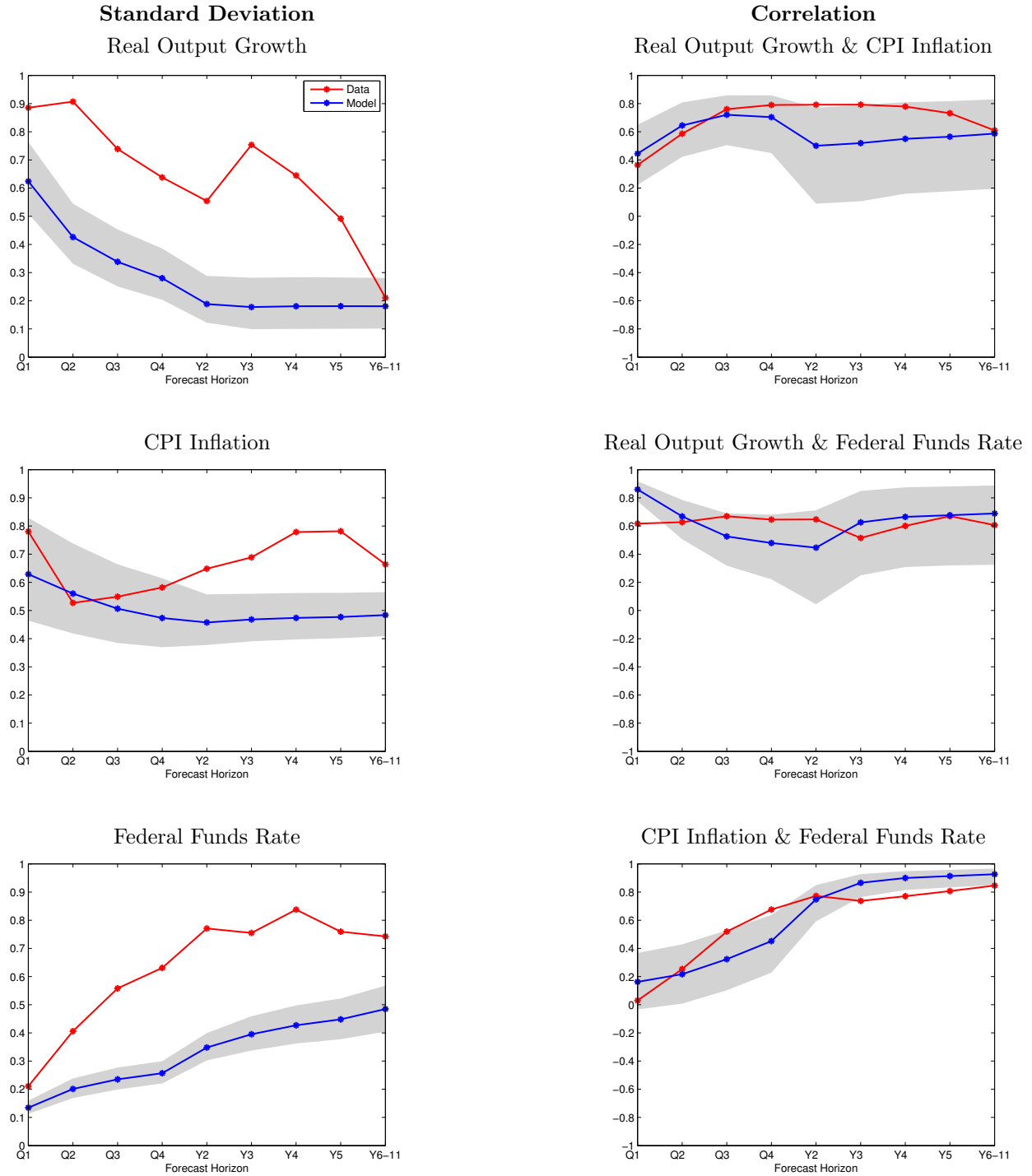


Figure 8: **Time Series of Disagreement**
Noisy and Sticky Information Models

This first column shows the model-implied time series of disagreement of the 6-to-11 years ahead forecast from the generalized noisy information model (blue) calibrated with $\alpha = 50$ and the Blue Chip Financial Forecasts survey (red). The second column displays the corresponding model-implied disagreement for the generalized sticky information model. The sample period is from 1986Q1 - 2013Q2. Results for the noisy and sticky information models are based on 5,000 and 2,500 simulations, respectively.

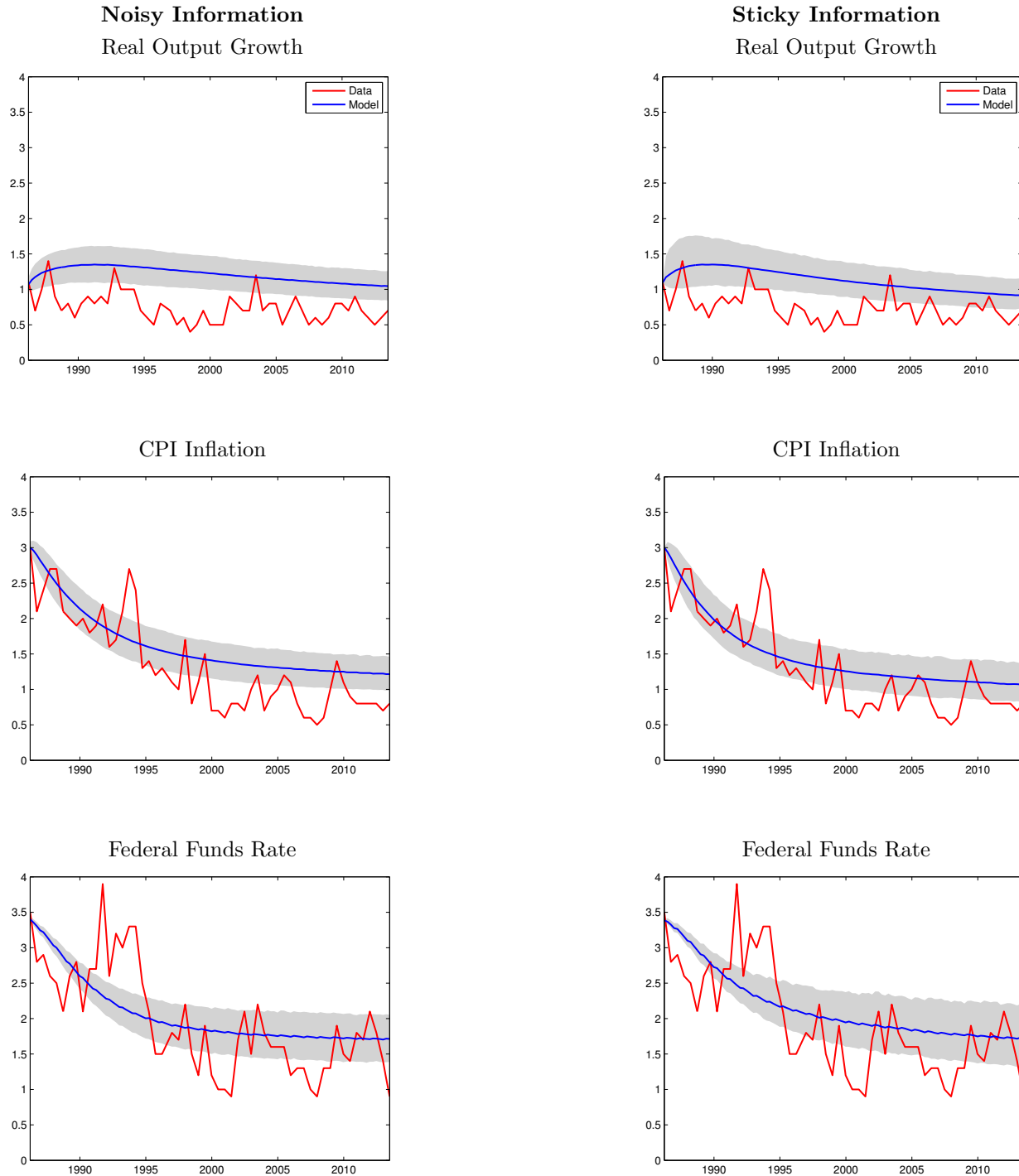


Figure 9: Comparison to Results with Burn In
Noisy Information Model

The first column displays the model-implied disagreement for the generalized noisy information model calibrated with $\alpha = 50$ with and without burn in of 120 observations (purple and blue, respectively) along with the Blue Chip Financial Forecasts survey (red). The second column displays the corresponding standard deviation of consensus forecasts. Open circles designate survey moments used to form the penalization term $\mathcal{P}(\theta_1, \theta_2; \mathcal{S}_1, \dots, \mathcal{S}_T)$. Model-implied 95% confidence intervals for the model with and without burn in are designated by dotted lines and shaded regions, respectively. Results are based on 5,000 simulations.

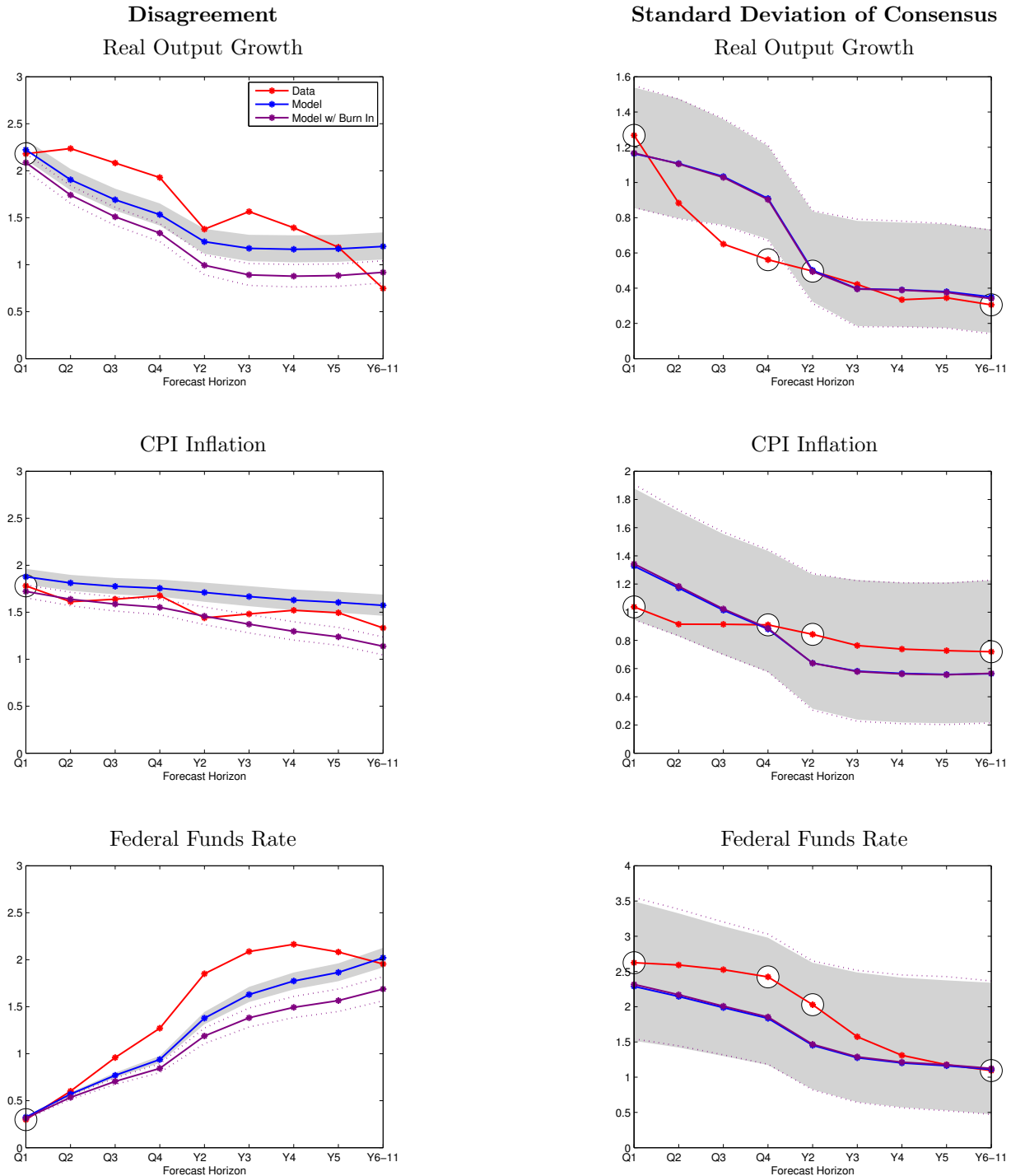


Figure 10: **Comparison to Results with Burn In**
Sticky Information Model

The first column displays the model-implied disagreement for the generalized sticky information model calibrated with $\alpha = 50$ with and without burn in of 120 observations (purple and blue, respectively) along with the Blue Chip Financial Forecasts survey (red). The second column displays the corresponding standard deviation of consensus forecasts. Open circles designate survey moments used to form the penalization term $\mathcal{P}(\theta_1, \theta_2; \mathcal{S}_1, \dots, \mathcal{S}_T)$. Model-implied 95% confidence intervals for the model with and without burn in are designated by dotted lines and shaded regions, respectively. Results are based on 1,500 simulations.

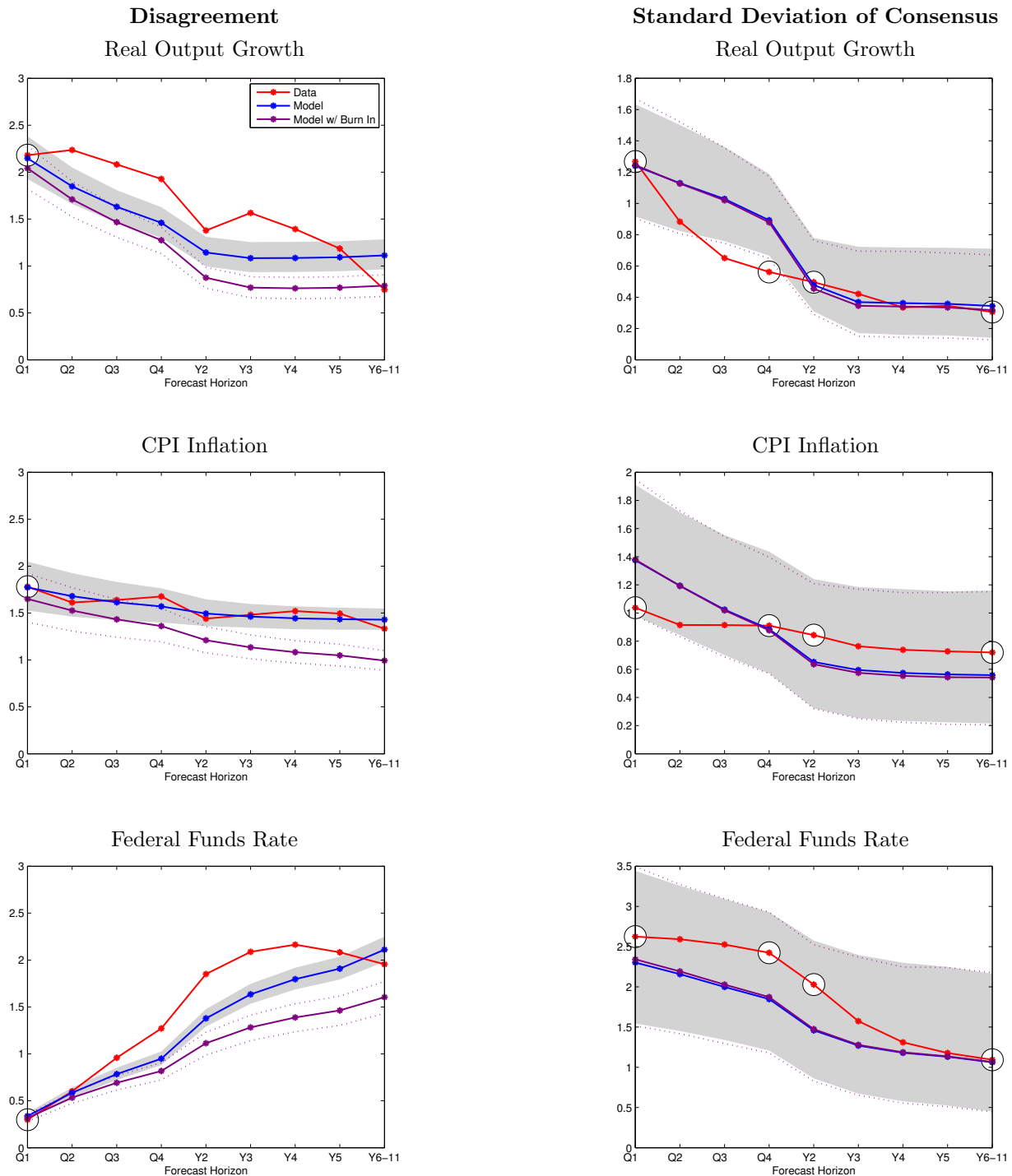


Figure 11: **Disagreement and Standard Deviation of Real Interest Rate Forecasts**
Noisy and Sticky Information Models

The first column displays the model implied disagreement for the generalized noisy information model (blue, top chart) and sticky information model (blue, bottom chart) calibrated with $\alpha = 50$ along with the Blue Chip Financial Forecasts survey (red). The second column displays the corresponding standard deviation of consensus forecasts. Model-implied 95% confidence intervals are designated by shaded regions. Results for the noisy and sticky information models are based on 5,000 and 2,500 simulations, respectively.

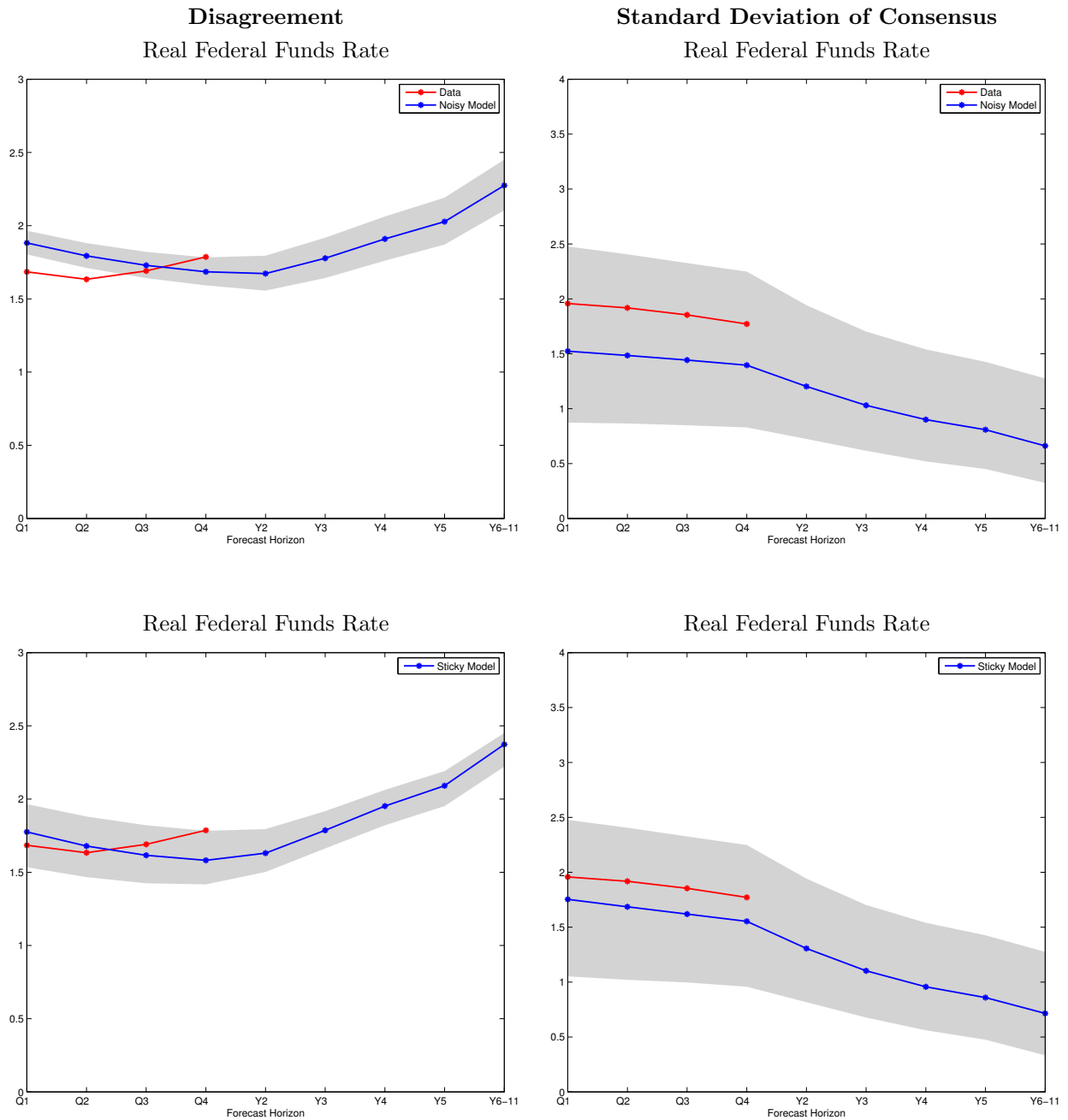
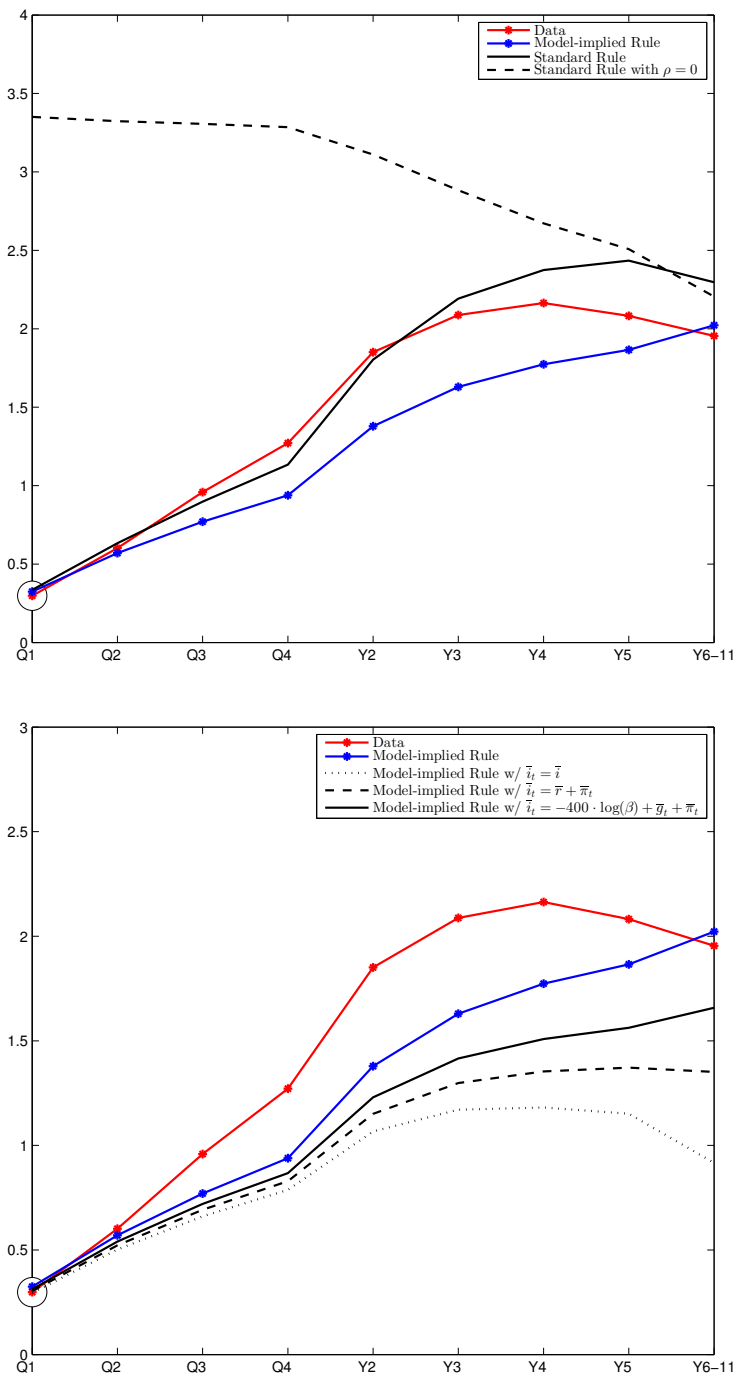


Figure 12: **Monetary Policy Rules**
Noisy Information Model

This figure shows the results of the analysis discussed in Section 5. The top chart displays the model-implied disagreement for different values of $(\rho, \varphi_\pi, \varphi_g)$ along with the Blue Chip Financial Forecasts survey (red). The “standard rule” is given by $(\rho, \varphi_\pi, \varphi_g) = (0.90, 2.0, 0.5)$. The bottom chart shows model-implied disagreement for different specifications of \bar{i}_t . Open circles designate survey moments used to form the penalization term $\mathcal{P}(\theta_1, \theta_2; \mathcal{S}_1, \dots, \mathcal{S}_T)$. Results are based on 5,000 simulations.



**Supplementary Appendix for “Fundamental
Disagreement”**

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Table 3: **Results of Calibration for $\alpha = 1$**
Noisy Information Model

This table provides the calibrated parameters for $\alpha = 1$ as discussed in Section 4.1 of the paper. $|\cdot|$ designates the modulus of a complex number. Results are based on 5,000 simulations.

Φ	Σ^z	$\text{sqrt}(\text{diag}(\tilde{\Sigma}^\eta))$
$\begin{bmatrix} 0.343 & -0.423 & -0.262 \\ 0.175 & 1.048 & -0.051 \\ 0.168 & 0.135 & 0.906 \end{bmatrix}$	$\begin{bmatrix} 2.726 & -0.074 & 0.562 \\ -0.074 & 0.546 & 0.329 \\ 0.562 & 0.329 & 0.631 \end{bmatrix}$	$\begin{bmatrix} 2.687 \\ 1.439 \\ 0.000 \end{bmatrix}$
$ \text{eig}(\Phi) $	Σ^μ	$\text{sqrt}(\text{diag}(\Sigma^\eta))$
$\begin{bmatrix} 0.902 \\ 0.714 \\ 0.681 \end{bmatrix}$	$\begin{bmatrix} 0.012 & 0.025 & 0.046 \\ 0.025 & 0.051 & 0.095 \\ 0.046 & 0.095 & 0.176 \end{bmatrix}$	$\begin{bmatrix} 4.787 \\ 2.112 \\ 0.000 \end{bmatrix}$

Table 4: **Results of Calibration for $\alpha = 1$**
Sticky Information Model

This table provides the calibrated parameters for $\alpha = 1$ as discussed in Section 4.1 of the paper. $|\cdot|$ designates the modulus of a complex number. Results are based on 2,500 simulations.

Φ	Σ^z	$\text{sqrt}(\text{diag}(\tilde{\Sigma}^\eta))$
$\begin{bmatrix} 0.350 & -0.422 & -0.267 \\ 0.172 & 1.044 & -0.051 \\ 0.167 & 0.133 & 0.906 \end{bmatrix}$	$\begin{bmatrix} 2.837 & -0.072 & 0.565 \\ -0.072 & 0.569 & 0.331 \\ 0.564 & 0.331 & 0.633 \end{bmatrix}$	$\begin{bmatrix} 2.767 \\ 1.434 \\ 0.000 \end{bmatrix}$
$ \text{eig}(\Phi) $	Σ^μ	λ
$\begin{bmatrix} 0.903 \\ 0.701 \\ 0.701 \end{bmatrix}$	$\begin{bmatrix} 0.013 & 0.025 & 0.047 \\ 0.025 & 0.051 & 0.094 \\ 0.047 & 0.094 & 0.175 \end{bmatrix}$	$\begin{bmatrix} 0.440 \\ 0.380 \\ 0.920 \end{bmatrix}$

Figure 13: **Results for Different Values of α**
Noisy Information Model

The first column displays the model-implied disagreement for the generalized noisy information model for different values of the penalty parameter α along with the Blue Chip Financial Forecasts survey (red). The second column displays the corresponding standard deviation of consensus forecasts. Open circles designate survey moments used to form the penalization term $\mathcal{P}(\theta_1, \theta_2; \mathcal{S}_1, \dots, \mathcal{S}_T)$. Results are based on 5,000 simulations.

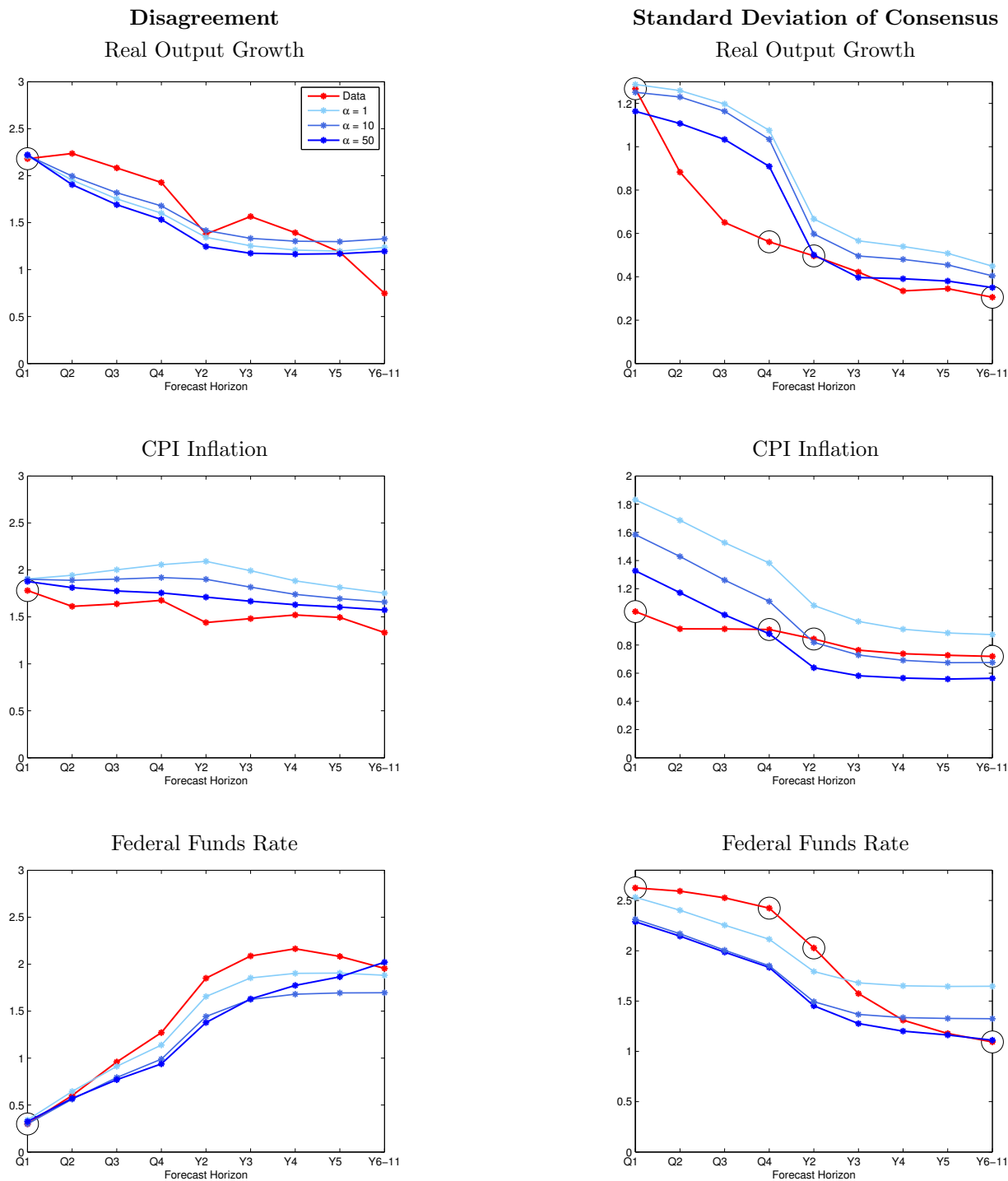


Figure 14: **Results for Different Values of α**
Sticky Information Model

The first column displays the model-implied disagreement for the generalized sticky information model for different values of the penalty parameter α along with the Blue Chip Financial Forecasts survey (red). The second column displays the corresponding standard deviation of consensus forecasts. Open circles designate survey moments used to form the penalization term $\mathcal{P}(\theta_1, \theta_2; \mathcal{S}_1, \dots, \mathcal{S}_T)$. Results are based on 2,500 simulations.

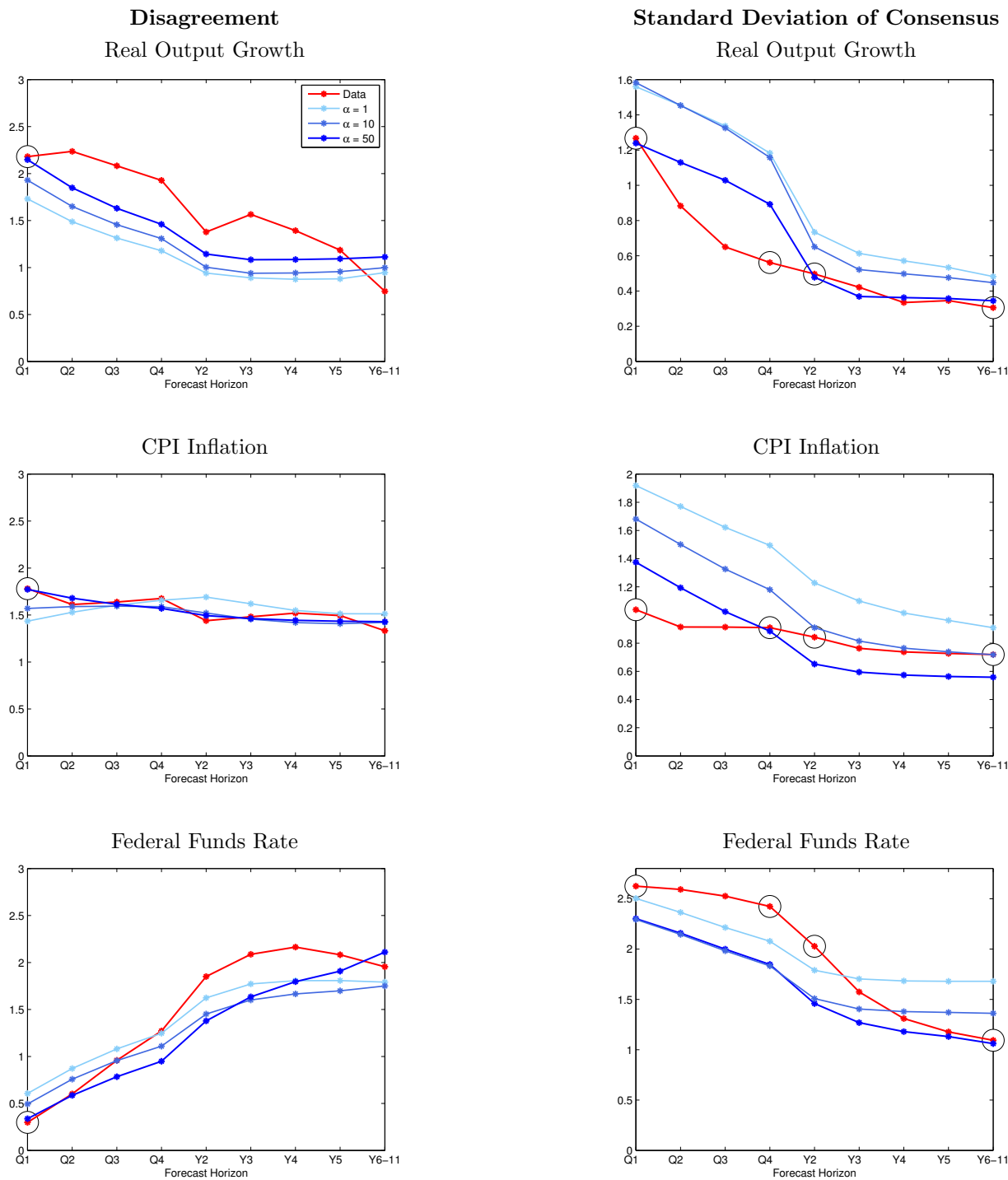


Figure 15: **Results with $\tilde{\Sigma}^\eta = 0$**
Noisy Information Model

The first column displays the model-implied disagreement for the generalized noisy information model with $\tilde{\Sigma}^\eta = 0$ for different values of the penalty parameter α along with the Blue Chip Financial Forecasts survey (red). The second column displays the corresponding standard deviation of consensus forecasts. Open circles designate survey moments used to form the penalization term $\mathcal{P}(\theta_1, \theta_2; \mathcal{S}_1, \dots, \mathcal{S}_T)$. Results are based on 5,000 simulations.

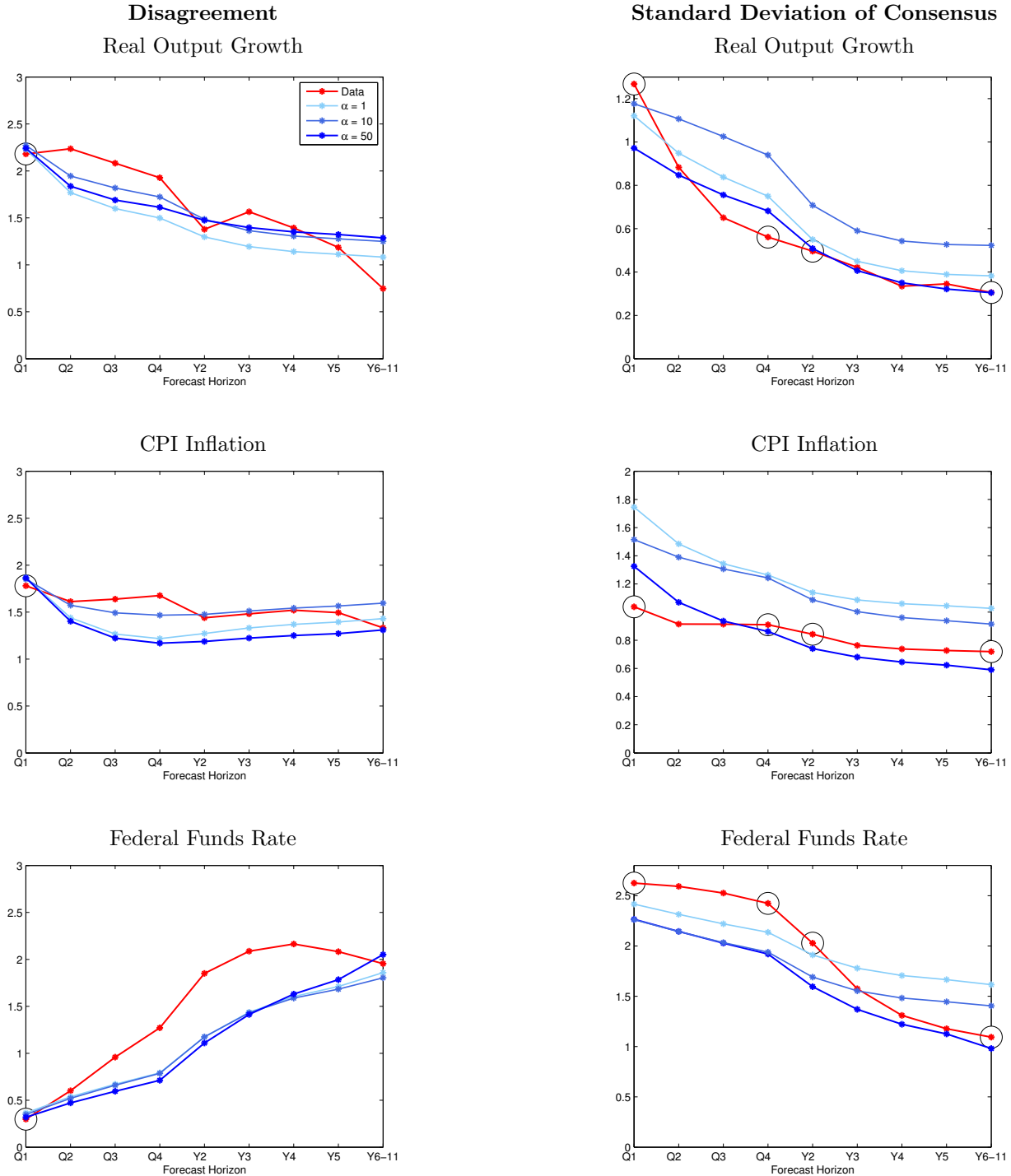


Figure 16: **Results with $\tilde{\Sigma}^\eta = 0$**
Sticky Information Model

The first column displays the model-implied disagreement for the generalized sticky information model with $\tilde{\Sigma}^\eta = 0$ for different values of the penalty parameter α along with the Blue Chip Financial Forecasts survey (red). The second column displays the corresponding standard deviation of consensus forecasts. Open circles designate survey moments used to form the penalization term $\mathcal{P}(\theta_1, \theta_2; \mathcal{S}_1, \dots, \mathcal{S}_T)$. Results are based on 2,500 simulations.

