Value at Risk: A New Methodology For Measuring Portfolio Risk

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Commercial banks, investment banks, insurance companies, nonfinancial firms, and pension funds hold portfolios of assets that may include stocks, bonds, currencies, and derivatives. Each institution needs to quantify the amount of risk its portfolio may incur in the course of a day, week, month, or year.

For example, a bank needs to assess its potential losses in order to set aside enough capital to cover them. Similarly, a company needs to track the value of its assets and any cash flows resulting from losses in its portfolio. An investment fund may want to understand potential losses on its portfolio, not only to allocate its assets better but also to fulfill its obligation to make set payments to investors. In addition, credit-rating and regulatory agencies must be able to assess likely losses on portfolios as well, since they need to set capital requirements and issue credit ratings.

How can these institutions judge the likelihood and magnitude of potential losses on their portfolios? A new methodology called value at risk (VAR or VaR) can be used to estimate these losses. This article describes the various methods used to calculate VAR, paying special attention to VAR's weaknesses.

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WHAT IS VALUE AT RISK?

Value at risk is an estimate of the largest loss that a portfolio is likely to suffer during all but truly exceptional periods. More precisely, the VAR is the maximum loss that an institution can be confident it would lose a certain fraction of the time over a particular period. Consider a bank with a portfolio of assets that would like to characterize its potential losses using VAR. For example, the bank could specify a horizon of one day and set the frequency of maximum loss to 98 percent. In that case, a VAR calculation might reveal that the maximum loss is $1 million. Thus, on average, in 98 trading days out of 100, the loss on the portfolio will not exceed $1 million over a one-day horizon. But on two trading days in 100, losses will, on average, exceed $1 million.

VAR can be used to assess the potential loss on a portfolio of assets generally. The user can specify any horizon and frequency of loss that fits his particular circumstances. But the method of calculating VAR depends not only on the horizon chosen but also on the kinds of assets in the portfolio. One method may yield good results with portfolios consisting of stocks, bonds, and currencies over a short horizon, but the same method may not work well over longer horizons such as a month or a year. If the portfolio contains derivatives, methods that differ from those used to analyze portfolios of stocks, bonds, or currencies may be needed.

VAR FOR A SINGLE SHARE OF STOCK

Ultimately, we want to calculate VAR for a general portfolio of different assets, such as stocks, bonds, currencies, and options.[1] Let's focus on the simplest case first: a single stock. A portfolio consisting of one asset will allow us to consider the different methods for assessing VAR in a simple context. Then, we can generalize the discussion by considering how the calculation changes when the institution has a portfolio of many stocks, bonds, or currencies. Finally, we will consider how the inclusion of derivatives in the portfolio can dramatically change the methodology for calculating VAR.

Randomness in the Stock Market. Let's consider a portfolio consisting of a single share of stock worth $1 at the beginning of trading today. We want to find the VAR over a one-day horizon at a 98 percent confidence level, that is, the largest one-day price drop we are likely to see on 98 out of every 100 trading days. Since VAR is essentially a statement about the likelihood of losses on a stock, we need to characterize the unpredictability of daily changes in our stock's price.

One way to picture the unpredictability of our stock's return over one day is to imagine the stock market spinning a roulette wheel. Of course, this is a fiction, but a useful one: economists have found that stock returns have a random component.

Suppose there are 100 equally likely outcomes on the wheel, with each outcome corresponding to a specific percentage daily price change or daily return for our stock.[2] In general, positive and negative returns are included on the wheel. To determine the return over one day, the stock market spins the roulette wheel. If the wheel comes up with a return of 25 percent, our stock would be worth $1.25 at the end of the day. Alternatively, a spin of the wheel may generate a return of minus 25 percent, in which case our stock would be worth $0.75 at the end of the day. We can't say for sure what the daily return will be, but we know that it will be one of the outcomes on the wheel.

[1] An option is a derivative security, i.e., its value is derived from the value of some other asset.

[2] In reality, when economists imagine stock returns on a wheel, they think of the wheel as having an infinite number of outcomes so that all possible returns are represented. To simplify the discussion, I have used 100 outcomes on the wheel as an approximation to an infinite-outcome wheel.

Finding the VAR for our $1 stock is particularly simple if we know the returns on the roulette wheel. Suppose we look at the outcomes on our roulette wheel and see that 98 of them involve returns bigger than minus 30 percent while two outcomes have returns smaller than minus 30 percent. Then we have found the VAR for our $1 stock: the VAR is $0.30 at a 98 percent confidence level. We can be confident that 98 days out of 100 our daily stock loss will be no bigger than $0.30. But two days out of 100, the daily loss may indeed exceed $0.30.

Summary Measures of Randomness. To find the VAR for our stock, we needed to know the 100 returns on the wheel. But how do we know what they are? Imagine that, every day, the market is spinning the wheel behind a curtain. We can't see the outcomes on the wheel, but we do know which daily returns were selected in the past—we can look them up in the newspaper. By categorizing past daily returns, we should be able to infer the outcomes on the wheel. For example, if we saw that daily returns of 10 percent occurred on five trading days in 100, on average, we can assume that five outcomes on the wheel involve a 10 percent return. Similarly, if changes of minus 5 percent occurred on 10 trading days in 100, on average, a return of minus 5 percent must correspond to 10 outcomes on the wheel. By continuing this analysis, we can associate price changes with all outcomes on the wheel. Then we will have reconstructed the wheel that the economy spins daily. Using our reconstructed wheel, we can easily find the VAR.

A simpler way to do this reconstruction is to summarize the 100 returns on the wheel by using two numbers: the average return (mean) and the volatility (variance) of the returns. Elementary statistics teaches that if the returns follow a certain pattern, called the normal, or bell-shaped, distribution, all the outcomes on the wheel can be summarized by these two numbers.

We can estimate the average return as an equally weighted average of past daily returns selected by the roulette wheel, returns that, again, could be looked up in the newspaper. For technical reasons, analysts often don't perform this calculation but assume instead that the average return is zero.[3] The second number, the volatility, tells us how much the return is likely to deviate from its average value for any particular spin. The volatility, then, measures the capacity of the roulette wheel to generate extreme returns, whether positive or negative, with respect to the average value of zero. The higher the volatility of the roulette wheel, the more it tends to select large returns. We can estimate the volatility as an equally weighted average of past squared returns. We could use the same returns we looked up in the newspaper; we only need to square each change.

Armed with the average return of zero and the volatility of our stock's returns, we can find the VAR over a one-day horizon at the 98 percent confidence level by following a simple procedure. To calculate VAR for our stock, we need only multiply today's stock price of $1 times the square root of the volatility times a number corresponding to the 98 percent confidence level, called the confidence factor. The confidence factor is derived from the properties of the normal distribution. At the 98 percent confidence level, it equals 2.054.[4]

This procedure can be done on any day in

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[3]Since the average return is estimated very imprecisely, it may pay to set it to zero to avoid corrupting the rest of the VAR analysis. For more discussion on setting the average return equal to zero, see the article by Steven Figlewski and the 1995 article by David Hsieh.

[4]From elementary statistics, 2.054 standard deviations leave 2 percent of the normal distribution in its left tail, which corresponds to stock losses occurring 2 percent of the time. If the confidence level were 95 percent, the confidence factor would be 1.65, because 1.65 standard deviations leave 5 percent of the normal distribution in the left tail.

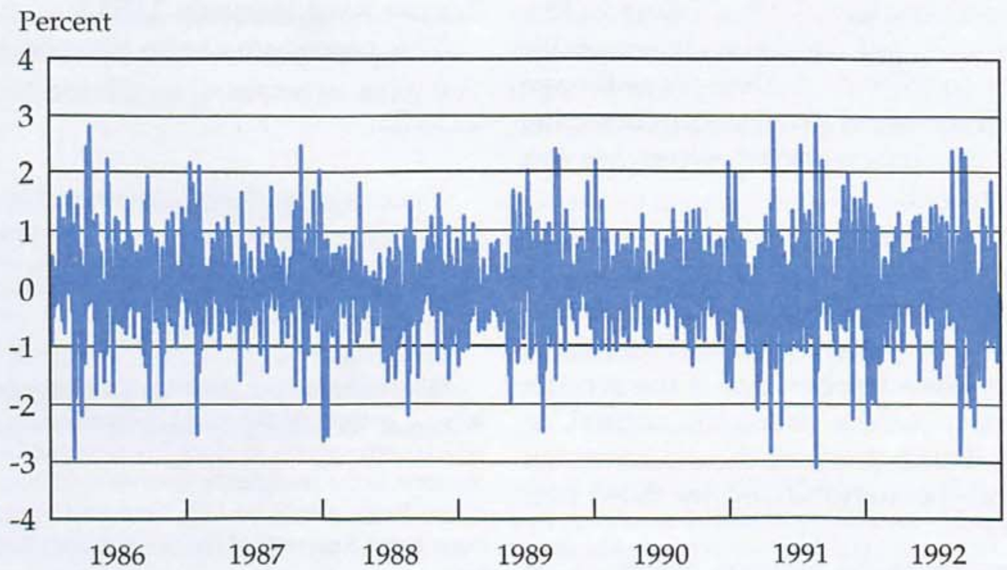
the future as well. Let's assume that it's now tomorrow and the stock price is $0.95. If we wanted to calculate VAR, we would follow the same procedure as before but use a stock price of $0.95. We don't need to change the volatility or the confidence number: they don't vary from day to day. When VAR is calculated in this fashion, we are using a constant volatility method.

Time-Varying Volatility. The problem with the constant volatility method is that substantial empirical evidence shows volatility is not constant from day to day but rather varies over time.[5] A look at a graph of the daily dollar return on the deutsche mark shows that volatility tends to cluster together (Figure 1). Notice that highly volatile times, characterized by large up-and-down swings in the exchange rate, tend to follow one another, while quiet periods, characterized by smaller up-and-down swings, tend to follow each other as well. For example, volatility seems to have been higher in 1991 than in 1990. A graph of the daily return on the S&P 500 confirms this impression for stock prices (Figure 2). The increase in volatility is particularly apparent after the stock market crash in 1987. Time-varying volatility seems to be a general feature of asset prices that is seen not only in currencies but also in stocks. Consequently, using the constant volatility method to calculate VAR could be very misleading.

What does time-varying volatility mean for our roulette wheel analogy? When the average return and the volatility don't vary from day to day, the returns on the wheel don't vary either. Thus, the market is spinning the same roulette wheel every day. But if the volatility is changing from day to day (time-varying volatility), the returns on the wheel must also be changing; therefore the market is spinning a

[5]The evidence suggests that volatility is time-varying for short horizons such as up to a week or 10 days. For longer horizons, the evidence for time-varying volatility is weaker. If a firm is interested in calculating VAR over a much longer horizon, the time-varying volatility issue may not be so important.

FIGURE 1
Daily Percent Dollar Return on Deutsche Mark



different wheel each day.

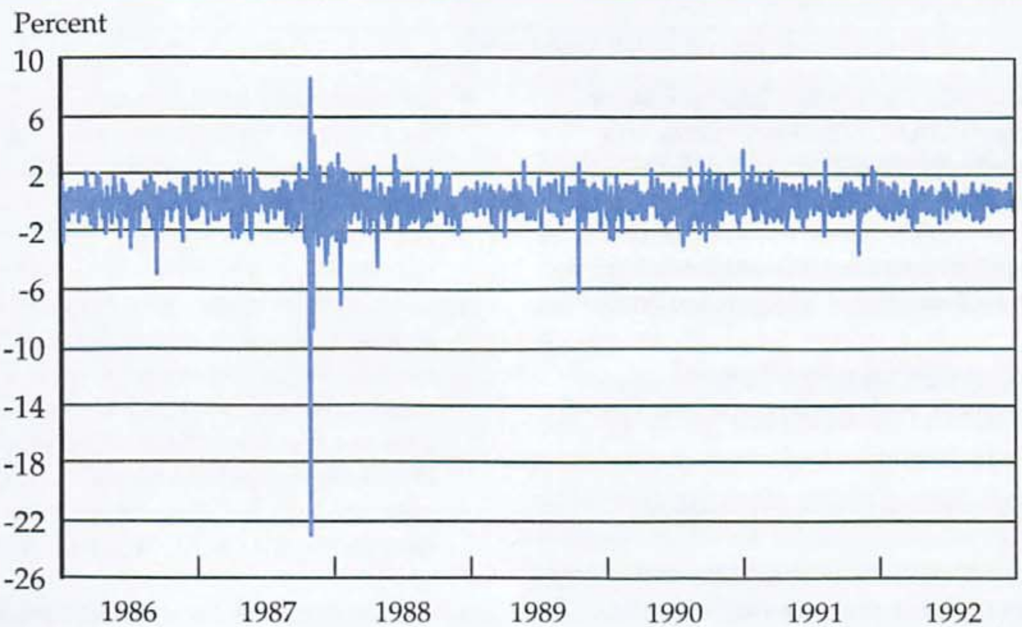
If the market spins a different roulette wheel every day, VAR becomes more complicated. How do we know which returns will be on the wheel today? Equivalently, how do we know today's volatility? The most common solution to this problem was introduced in 1986 by economist Tim Bollerslev, who generalized work done by economist Robert Engle in 1982. Bollerslev's time-varying volatility technique, called the GARCH method, allows us to base our knowledge of today's roulette wheel on yesterday's wheel.

Bollerslev's GARCH technique estimates the volatility of today's roulette wheel using yesterday's estimate of volatility and the squared value of yesterday's return. If yesterday's return was large, in either a positive or negative direction, and yesterday's volatility was high, today's roulette wheel will tend to have a high volatility. Thus, today's spin of the wheel will tend to produce large returns as well. In this way, large returns, positive or negative, would tend to follow one another, leading to periods of high and low volatility as we saw in Figures 1 and 2.

How can we estimate today's volatility and find the VAR using Bollerslev's GARCH method? The daily volatility using GARCH turns out to be a weighted average of past squared returns, just as it was in the constant volatility case. The difference is that the constant volatility method weights past squared returns equally while Bollerslev's GARCH method weights recent squared returns more heavily than distant returns.

It is easy to calculate volatility using the constant volatility method. Bollerslev's GARCH method is much harder to implement: to find the right weight for each past squared return, we must employ a complicated, computer-intensive procedure. Once we have found today's volatility, we can multiply the confidence factor times the square root of today's volatility times today's stock price to find today's VAR. When we use Bollerslev's GARCH method, the

FIGURE 2
Daily Percent Dollar Return on S&P500



confidence factor is the only number that does not change daily.

RiskMetrics™. Bollerslev's GARCH method has found widespread empirical support among financial economists, but the difficulty in estimating daily volatilities has slowed its adoption by many institutions engaged in risk management. To make the calculations easier, J.P. Morgan introduced RiskMetrics™, a risk management system that includes techniques to approximate GARCH volatilities (see *Pros and Cons of Using RiskMetrics™ as a Risk-Management Tool*). Like Bollerslev's method, the RiskMetrics™ estimate of daily volatility involves a weighted average of past squared returns, with recent squared returns weighted more heavily. The RiskMetrics™ weights are chosen to produce daily volatility estimates similar to GARCH volatilities. The set of weights calculated by the RiskMetrics™ method is easier to compute and can be used for any asset in the portfolio. For example, the analyst would use the same set of weights to calculate volatilities of stocks, bonds, and currencies. Bollerslev's GARCH method, in contrast, requires the computation of different weights for each volatility calculation, and each set of weights is harder to calculate than it would be using the RiskMetrics™ method.[6]

Other Methods. Two other methods of calculating volatility are sometimes used. The first method relies on recognizing that pricing methods for options require the user to specify his estimate of the future volatility of an asset. For example, if a user wants to price an option on a stock using a method such as the popular Black-Scholes method, he must specify an estimate of the volatility of the stock over the life of the option.[7] Since option prices are observable in

[6]Under the RiskMetrics™ method, a different set of weights is calculated for each of a series of over 400 assets. The weights are then combined to yield a single composite set of weights that can be used for any asset in the portfolio.

Pros and Cons of Using RiskMetrics™ as a Risk-Management Tool

Pros

- Computationally convenient approximation to Bollerslev's GARCH method. Thus, will require relatively smaller investment in research and information systems.
- Not a proprietary system The methodology is explained in detail in J.P. Morgan publications.
- J.P. Morgan publishes volatilities and correlations on a wide variety of assets free of charge.
- Substantial third-party software support.

Cons

- Commits user to a one-size-fits-all method: the GARCH method. This may be misleading for stocks, especially following large changes in stock prices. GARCH may also not describe covariances well.
- There is no consensus on how well GARCH models forecast volatility. Even if GARCH models forecast volatility well in a statistical sense, that is, make small forecast errors, they may not forecast well in an economic sense. For example, the RiskMetrics™ volatility estimate may not maximize profits even if it does forecast volatility well in a statistical sense.
- VAR may be the wrong methodology for the firm.

the marketplace, the market's view of volatility can be backed out of the option price using the Black-Scholes formula. Volatility estimates inferred from option prices in this way are called implied volatilities.

This method has two disadvantages that limit its appeal. First, options may not be traded on the particular asset of interest. Thus, implied volatility estimates may not be obtainable for some assets in the portfolio. Second, economists are unsure about whether implied volatility estimates are better than GARCH estimates of daily volatility.

The other method of estimating volatility is based on judgment. The user analyzes the economic environment and forecasts volatility based on his subjective views. This method has limited appeal as well, since testing the validity of a subjective view is difficult.

VAR FOR A PORTFOLIO OF ASSETS

Up to this point, we have considered only how to calculate the VAR of a portfolio consisting of a single stock. Now let's look at a portfolio of two stocks. The principles we are about to discuss apply generally to portfolios of many assets, but we will consider just two stocks to make the ideas clear.

As before, ultimately we want to find the volatility of the return on the portfolio. It's clear that the volatility of the portfolio should depend on the volatility of the return of each stock in the portfolio. So, we need to estimate the volatilities of the returns of both stocks. But stock returns may covary as well. For example, if the covariance between the stocks in a portfolio of two stocks is negative, then when one stock has a positive return, the other has a negative return, and vice versa. Thus, the two stocks dampen each other's swings in return, producing a portfolio whose volatility is lower than the volatility of each stock in the portfolio. Adding more stocks to the portfolio would reduce the volatility further, provided the additional stocks' returns are not highly positively correlated with the return of the initial portfolio. To account for this effect, we must also estimate the covariance between the stocks' returns. Once we know the stock returns' volatilities and covariances, we can calculate the volatility of the entire portfolio and find the VAR as before.

As an example of the calculation, suppose we have invested $1 in stocks 1, 2, and 3. Then by an elementary statistical formula, the daily volatility of the portfolio would be

volatility(portfolio) = volatility(stock 1) +
volatility(stock 2) + volatility(stock 3) +
2.0 x covariance(stock 1, stock 2) +
2.0 x covariance(stock 1, stock 3) +
2.0 x covariance(stock 2, stock 3)

Notice that if the covariance between the daily returns of stocks 1, 2, and 3 were zero, we could sum the volatilities of each stock to get the volatility of the portfolio. Thus, if covariances between all assets were zero, we could find the VAR of each asset separately and then sum them to get the VAR of the portfolio. But since covariances are, in general, not zero, we can't, in general, find the VAR of individual assets and sum them to get the VAR of the portfolio. Moreover, we can't find the VARs of asset classes such as stock and currency portfolios separately and sum them. We must account for the covariances between asset classes as well.

To calculate covariances between the assets' returns using the constant covariance method, we use an equally weighted average of the products of each stock's past daily returns. However, since economists have found evidence that covariances change over time, it may be advisable to estimate time-varying covariances using an extension of Bollerslev's

[7] For an explanation of this method, see the article by Fischer Black and Myron Scholes.

GARCH method or the RiskMetrics™ GARCH approximation.[8]

WHAT ABOUT DERIVATIVES?

Many portfolios have significant numbers of derivatives such as futures, options, and swaps, all of which are securities whose value is derived from the value of some other asset. Consider a derivative on our $1 stock. We know how to find the VAR of the stock over a one-day horizon at the 98 percent confidence level: we find the volatility of its return and multiply its square root by the product of today's stock price and the confidence factor. But how can we find the VAR of a derivative on this stock?

One method is to link the derivative to the underlying stock and use the standard VAR method. To do this, we use a derivative-pricing method, such as the Black-Scholes model, to calculate a number called delta, which gives us a way to translate the derivative portfolio into the stock portfolio. A derivative's delta tells us how the derivative's price changes when the stock price changes a small amount. For example, if the delta is 0.5, the derivative's price goes up half as much as the stock's price. For small price changes, a derivative with a delta of 0.5 behaves as if it is half a share of the $1 stock. So, using our estimate of the stock's volatility, we could calculate VAR as before: by multiplying $0.50 times the square root of the stock's volatility times the confidence factor.

A serious drawback to this method is that it works well only when stock price changes are small. For larger changes, delta itself can change dramatically, leading to inaccurate VAR estimates. In general, we need to account for how delta changes, considerably complicating the analysis.

To avoid this complication, risk managers often use an alternative method called Monte Carlo analysis. Using the volatility and covariance estimates for the derivatives' underlying assets as well as a derivative pricing tool such as the Black-Scholes method, risk managers construct a new roulette wheel. The new wheel will still have 100 numbers, but each number will correspond to a potential change in the derivative's price. The computer can then look at the largest loss the derivative will sustain for 98 of the outcomes. Let's suppose this loss is $0.01. Then the VAR of the derivative over a one-day horizon at the 98 percent confidence level is $0.01. Since RiskMetrics™ yields volatility and covariance estimates, Monte Carlo evaluation of derivative portfolios can be done under J.P. Morgan's system as well.[9]

WEAKNESSES OF VAR

When properly used, VAR can give an institution an idea about the maximum losses it can expect to incur on its portfolio a certain fraction of the time, making VAR an important risk-management tool. Using VAR calculations, an institution can judge how it should reallocate the assets in its portfolio to achieve the risk level it desires. But VAR methodology is not without its weaknesses, and, improperly used, it may lead an institution to make poor risk-management decisions. This can happen for one of two reasons: either the VAR is incorrectly calculated or the VAR is correctly calculated but irrelevant to the institution's real risk-management goals.

What Is the Best Method for Estimating Volatility? Bollerslev's GARCH method works better for currencies than it does for stock prices. Financial economists have found that stock volatility goes up more as a result of a large negative return than it does as a result of a large

[8]For further discussion on covariance GARCH techniques, see the paper by Robert Engle and Kenneth Kroner and the 1990 paper by Tim Bollerslev.

[9]For more detail on this process, see the RiskMetrics™ technical document. For an example of a related methodology, see the 1993 articles by David Hsieh.

positive return. A weakness of Bollerslev's GARCH method is that GARCH volatility estimates don't depend on whether yesterday's return was positive or negative. Thus, this method can't allow for stock volatility's asymmetric response to past returns.

To account for this effect, financial economists have developed methods for estimating asymmetric volatilities.[10] These methods are important because they can give very different estimates of volatility for days following large stock returns than would the GARCH or RiskMetrics™ method. For small daily returns, Bollerslev's method, RiskMetrics™, and the asymmetric volatility method yield similar one-day-ahead volatility predictions, leading a user to think, perhaps, that one model is as good as the others for daily volatility predictions. But for large daily returns, the one-day-ahead volatility predictions of these methods can be substantially different. If an asymmetric volatility method is appropriate for stock prices, both Bollerslev's method and RiskMetrics™ may understate one-day-ahead volatility whenever a large drop in stock prices occurred the previous day, thus producing a potentially substantial underestimate of daily VAR. Similarly, the GARCH or RiskMetrics™ method could overestimate the VAR after a large increase in stock prices.

Robert Engle and Victor Ng have provided evidence that a particular asymmetric volatility method well describes the volatility of Japanese stock returns and that GARCH methods can substantially underpredict volatility following large negative returns. Thus, VAR estimates of stock portfolios produced by GARCH or the RiskMetrics™ GARCH approximation should be viewed with caution if the calculations are done on days with large stock returns.

Although having the right method for calculating the volatilities of assets is important, correctly calculating the covariances between the returns on assets is also important. Unfortunately, not as much work has been done by financial economists to identify the right method for calculating covariances. To date, many methods have been proposed, but no consensus has yet emerged. Thus, we don't yet know for sure how we should handle covariances in portfolios. This uncertainty introduces the risk that any method we use may substantially under- or overestimate VAR. In particular, RiskMetrics™ commits the user to a special case of Bollerslev's GARCH method. Since we don't yet know whether Bollerslev's GARCH method is adequate in describing covariances, we should use even more caution in interpreting results whenever we have used covariances in our VAR calculations.

In the long run, the volatility estimates produced by GARCH methods tend, in general, to approach the values that the constant volatility method would have calculated. Thus, for horizons much longer than one day, using the constant volatility method to calculate VAR may be warranted.[11]

Frequency of Large Returns. Using either Bollerslev's GARCH model or the constant volatility method, we could find the VAR by assuming that the returns on the wheel follow a normal distribution. However, a substantial amount of evidence indicates that the normal distribution is inadequate because large daily returns, positive or negative, occur more often in the market than a normal distribution would suggest. One remedy is to use a different distribution for the price changes, one that generates more frequent large returns.[12] Alternatively,

[10]The prototypical asymmetric volatility model is EGARCH. See the article by Daniel Nelson.

[11]See the article by David Hsieh (1993a) for a discussion about when the constant volatility model may be appropriate.

[12]For an example of this technique, see the article by Daniel Nelson.

we could use statistical methods that assume the returns follow the normal distribution, but which remain valid even if this assumption is mistaken.

Whichever method we use, we are essentially looking at the past frequencies and magnitudes of returns and attempting to represent them on a reconstructed wheel. Even if we account for the nonnormality of returns during this process, there is still a problem: we're going to put on the wheel only those returns we saw in the past with the frequency we saw in the past. So, if some potential negative returns are rare or have not yet occurred, we may underrepresent them on the wheel, implying that the VAR will be underestimated.

Structural Shifts in the Economy. VAR may be underestimated if the wheel the market is spinning suddenly changes in an unpredictable way because of a structural change in the underlying economy. For example, consider the European Exchange Rate Mechanism (ERM), which kept daily returns of major European currencies small. In 1993, in response to economic pressures, much larger returns were suddenly allowed. Thus, the volatility of the returns suddenly shot up faster than Bollerslev's GARCH method would have forecast based on past volatilities and returns. If we had calculated the VAR the day before the shift, we would have underestimated it because we would have used an estimate of the volatility that was too low. More subtly, since we never know when the economy may suddenly shift to higher or lower volatility as a result of a structural change, we will incorrectly estimate the VAR unless we explicitly account for this possibility.

Because of the problems caused by infrequent large returns and structural shifts in the economy, it seems prudent, then, to supplement statistical calculations of VAR with judgmental estimates. For example, an institution could have asked its economists to project the likely price effects if the ERM suddenly allowed larger price changes. These projections could be based on similar historical episodes, economic theory, and empirical experience. VAR estimates based on judgment could be generated for changes in central bank monetary regimes, political instability, structural economic changes, and other events that have either never happened or happen infrequently.

Liquidity of Assets. VAR measures the maximum loss that an institution can expect a certain fraction of the time over a specific horizon. Losses are measured by assuming that the assets can be sold at current market prices. However, if a firm has highly illiquid assets—meaning that they cannot quickly be resold—VAR may underestimate the true losses, since the assets may have to be sold at a discount.

Credit Risk. Another potential problem for VAR is that the methods used to evaluate the assets in the portfolio may not properly treat credit risk. Suppose a bank buys a portfolio of derivatives from many different firms. The derivatives are valuable to the bank because they impose obligations on the firms. For example, one of the derivatives may obligate a firm to sell foreign currency to the bank at a price below the current market price, yielding a profit to the bank under some conditions, but it may also obligate the bank to deliver foreign exchange at a below-market price under other conditions. Using the Black-Scholes method and a Monte Carlo simulation, which assume no derivative credit risk, the bank calculates a VAR of $5 million at a 98 percent confidence rate for a three-month horizon. But if some of the firms may default on their obligations, the true value of these derivatives is lower than would be estimated by the Black-Scholes method coupled with Monte Carlo analysis. Thus, the true value at risk is larger than $5 million. To account for this possibility when valuing derivatives, the bank should use a method that includes credit risk. For some applications, credit risk may be small enough to ignore, but, in general, users need to include credit risk analysis in their VAR methods.

Is VAR the Right Methodology? In many situations, VAR may not be the correct risk-management methodology. If we pick a specific loss such as $1 million, VAR allows us to estimate how often we can expect to experience this particular loss. For example, using VAR we might estimate that we will lose at least $1 million on one trading day in 20, on average. During some 20-day periods, we might lose less than $1 million. During other 20-day periods, we might lose more than $1 million on more than one day. VAR tells us how often we can expect to experience particular losses. It doesn't tell us how large those losses are likely to be. In particular, in any 20-day period, there is always one day on which the worst loss is experienced. If we want to know the size and frequency of the worst loss, VAR provides no guidance.

One way of handling this is to use worst-case-scenario analysis (WCSA), proposed by Jacob Boudoukh, Matthew Richardson, and Robert Whitlelaw. WCSA might show that on the day with the worst price change in a 20-day period, we can expect to lose at least $2.77 million 5 percent of the time, a number substantially bigger than $1 million. Thus, if a firm is interested in the size of a worst-case loss, VAR could underestimate it.

CONCLUSION

VAR is an important new concept in portfolio risk management. It gives the maximum loss that an institution can expect to lose with a certain frequency over a specific horizon, and it can be calculated by using a constant volatility or time-varying volatility method. There are, however, problems in implementation and interpretation. To implement VAR calculations, it is important to use the right method, especially under unusual circumstances such as stock market crashes. Although much progress has been made in describing how volatilities change through time, not as much progress has been made in the description of time-varying covariances. Thus, VAR numbers should be viewed with caution at this point.

Besides the problem of identifying the right method, VAR measures may mislead unless they properly account for liquidity risk, rare or unique events, and credit risk. In many situations, it may not be the right risk-management concept. An institution may want to investigate an alternative, such as worst-case-scenario analysis.

Despite the contribution that VAR can make to a firm's understanding of the risks in its portfolio, these risks can be misunderstood if they are not communicated effectively to a management that understands the value and limitations of sophisticated financial technology. Poor management practices, which could lead to unauthorized trades, may also contribute to this misunderstanding. Thus, a firm should use VAR in the context of a broader risk-management culture, fostered not only by the firm's risk managers but also by its senior management.

APPENDIX
VAR and Capital Requirements for Market Risk

In 1995, the Basle Committee on Banking Supervision at the Bank for International Settlements (Basle Committee) issued a proposal for comment entitled "Internal Model-Based Approach to Market Risk Capital Requirements." This proposal would establish a VAR-based method of measuring banks' portfolio risk. In January 1996, the Basle Committee approved an approach that would allow banks to use their own internal risk-management models or the Basle Committee's standard model. The internal risk-management models would be subject, however, to qualitative and quantitative restrictions. U.S. regulators are expected to implement this approach for nine or 10 of the largest U.S. banks. Some examples of the restrictions the Basle Committee would impose on internal models are:

Quantitative Criteria:

- VAR must be computed daily using a horizon of 10 trading days.
- The confidence level should be set to 99 percent.
- Models should account for changing delta when computing VAR. In addition, VAR models should account for the impact of time-varying volatility on option prices.
- Banks may use covariances within and across asset classes.

Qualitative Criteria:

- Banks should have independent risk-management units that report directly to senior management.
- VAR reports and analyses should be considered when setting trading limits.

REFERENCES

Academic Literature:

Black, Fischer, and Myron Scholes. "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, 81 (1973), pp. 637-59.

Bollerslev, Tim. "Generalized Autoregressive Conditional Heteroskedasticity," *Journal of Econometrics*, 31 (1986), pp. 307-27.

Bollerslev, Tim. "Modelling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalized ARCH Model," *Review of Economics and Statistics*, 78 (1990), pp. 498-505.

Engle, Robert F. "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation," *Econometrica*, 50 (1982), pp. 987-1008.

Engle, Robert F., and Victor K. Ng. "Measuring and Testing the Impact of News on Volatility," *Journal of Finance*, 48 (1993), pp. 1749-78.

Engle, Robert F., and Kenneth K. Kroner. "Multivariate Simultaneous Generalized Arch," University of California at San Diego mimeo (1993).

Figlewski, Steven. "Forecasting Volatility Using Historical Data," New York University Working Paper, S-94-13 (1994).

Hsieh, David A. "Implications of Nonlinear Dynamics for Financial Risk Management," *Journal of Financial and Quantitative Analysis*, 28 (1993a), pp. 41-64.

Nelson, Daniel B. "Conditional Heteroskedasticity in Asset Returns: A New Approach," *Econometrica*, 59 (1991), pp. 347-70.

Practitioner Literature:

Boudoukh, Jacob, Matthew Richardson, and Robert Whitelaw. "Expect the Worst," *Risk* (September 1995), pp. 100-01.

Hsieh, David A. "Assessing the Market and Credit Risks of Long-Term Interest Rate and Foreign Currency Products," *Financial Analysts Journal*, July-August (1993b), pp. 75-79.

Hsieh, David A. "Nonlinear Dynamics in Financial Markets: Evidence and Implications," *Financial Analysts Journal*, July-August (1995), pp. 55-62.

RiskMetrics™-Technical Document, Morgan Guaranty Trust Company, Global Research, New York, 1995.