

The Cyclical Volatility of Interest Rates

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The variability of short-term and long-term interest rates is a prominent feature of the economy. Interest rates change in response to a variety of economic events, such as changes in Fed policy, crises in domestic and international financial markets, and changes in the prospects for long-term economic growth and inflation. However, economic events such as these tend to be irregular. There is a more regular variability of interest rates associated

with the business cycle, the expansions and contractions that the economy experiences over time. For example, short-term interest rates rise in expansions and fall in recessions. Long-term interest rates do not appear to co-vary much with the level of economic output.

The term *cyclical volatility of interest rates* refers to the variability of interest rates over periods that correspond to the length of the typical business cycle. In this article, we will examine some facts and theory about the cyclical volatility of short-term and long-term interest rates. Why should we care about interest rate volatility? How do short-term and long-

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term interest rates behave over the business cycle? What determines the cyclical volatility of interest rates associated with different maturities of government bonds? These questions are important to ask and answer as we seek a fuller understanding of the dynamics of the business cycle in market economies.

WHY DOES INTEREST RATE VOLATILITY MATTER?

The variability of interest rates affects decisions about how to save and invest. Investors differ in their willingness to hold risky assets such as stocks and bonds. When the returns to holding stocks and bonds are highly volatile, investors who rely on these assets to provide for their consumption face a relatively large chance of having low consumption at any given time. For example, before retirement, people receive a steady stream of income that helps to buffer the changes in wealth associated with changes in the returns on their investment portfolios. This steady return from working helps them maintain a relatively steady level of consumption. After retirement, people no longer have the steady stream of income from working (though it will, in part, be replaced by pension income and Social Security), so a less volatile investment portfolio is called for. The lower volatility of investment returns allows retirees to maintain a relatively even level of consumption over time. Young investors, who are saving for retirement, are better able to absorb the risks of holding assets with highly volatile prices and returns. They can weight their portfolio more heavily toward risky stocks and bonds because they are receiving a steady return from working. For holding these riskier assets, the young investor will be rewarded with a higher average return on the investment.

Just as individuals care about managing risk in their investment portfolios, so do firms. To manage risk, firms must pay attention to interest rate volatility and the composition of

their portfolios. Many business firms hold portfolios containing large numbers of assets and, thus, are interested in quantifying the risk of losing large sums of money. As risks in the economy change, the expected gains and losses from the investment portfolio change. Measuring this risk involves knowing how volatile prices of and returns on assets are, as well as how the returns on different assets change together over time. The volatility of interest rates is likely to be an important component in quantifying risk and guiding the investment decisions of these institutions.

Interest rate volatility also has implications for how the prices of certain types of assets are determined. Options are assets that give investors the right, but not the obligation, to buy (call options) or sell (put options) other assets (such as stocks or bonds) at a prespecified price at or before some prespecified time in the future. For options purchased on interest-bearing securities, modern finance theory demonstrates that the option price depends on the volatility of returns on the underlying asset. The volatility of interest rates is related to the volatility of returns on these assets.

Thus, interest rates and their volatility have important implications for how both individuals and firms make investment decisions. These investment decisions are part of the process whereby resources are allocated in the economy. To begin, we'll briefly discuss how bond prices, interest rates, and maturities of bonds are related and how interest rates can be determined from bond prices.

INTEREST RATES, BOND PRICES, AND THE TERM STRUCTURE

There is a very close connection between bond prices and interest rates. We will focus on interest rates calculated from prices of traded U.S. government securities and show how the interest rate on a particularly simple type of security can be derived solely from its price. We focus on yields derived from U.S. govern-

ment securities because these assets are backed by the full faith and credit of the government and, therefore, have virtually no default risk.

The U.S. government issues securities of many different maturities: the maturity is the length of time until the final payment on the security is made by the issuer. Treasury bonds are fixed-coupon securities with initial maturities of more than 10 years. Treasury notes are fixed-coupon securities with initial maturities of from two to 10 years. Treasury bills are securities that are sold at a discount from face value and have initial maturities of a year or less.

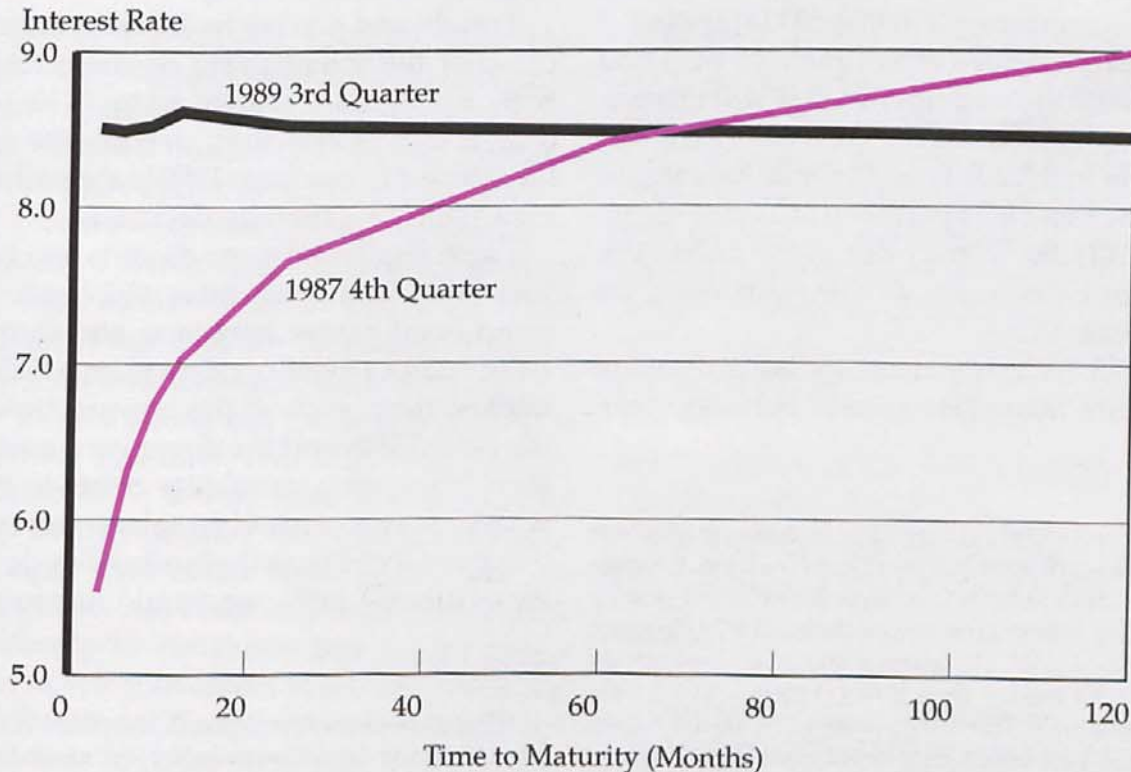
If we know a bond's current price and the payments that the bondholder will receive over the course of the bond's life, we can calculate the implied interest rate on the bond. This interest rate, called yield-to-maturity,

equates the current price of the bond to the present value of the bond's payment stream.¹ The relationship between the maturity of bonds and the interest rates implied by bond prices is called the term structure of interest rates. A plot of the relationship between interest rates and bond maturity, called the yield curve, can take a variety of shapes (Figure 1). Typically, interest rates on short-term bonds are lower than interest rates on long-term bonds, in which case the yield curve is upward sloping, as shown in the figure for the fourth quarter 1987. But sometimes the yield curve inverts, in which case interest rates on short-term bonds are

¹For more detail on how yield-to-maturity is calculated, see my article in the July/August 1994 *Business Review*.

FIGURE 1

Term Structure of Interest Rates



higher than interest rates on long-term bonds, as shown in the figure for the third quarter 1989.

The shape of the typical yield curve shows that interest rates often vary with maturity. We might also suspect that the volatility of interest rates varies with maturity. But before we turn to how volatility is measured and how volatility is related to maturity, let's clarify the relationship between interest rates and the price of a particularly simple type of bond.

Interest Rates and Bond Prices. Interest rates on certain types of bonds can be derived solely from the bonds' price and maturity. Let's look at a particular type of bond called a discount, or zero-coupon, bond. A discount bond sells at a discount from its face value and makes no interest payments over its lifetime. When the bond matures, the bondholder receives the bond's face value. For example, a one-year Treasury bill with a face value of \$10,000 is a discount bond that promises to pay the holder \$10,000 in one year's time. Such a bond may sell for a current price of \$9434, in which case the implied interest rate on the bond is 6 percent $((\$10,000 - \$9434) / \$9434 = .06)$. Clearly, as the current price of the bond changes, the implied interest rate will change. For example, suppose the current price of the bond falls to \$9009. Then the implied interest rate on the bond is 11 percent $((\$10,000 - \$9009) / \$9009 = .11)$. So, as the price of the bond falls, the interest rate rises; as the price rises, the interest rate falls.²

The U.S. Treasury does not issue discount bonds with maturities greater than one year.

However, financial market participants create pure discount bonds from long-term, coupon-paying Treasury bonds by "stripping" the coupon (semiannual interest) payments from the principal payment and selling the components as separate discount securities. In February 1985, the Treasury announced the STRIPS (separate trading of registered interest and principal of securities) program, which facilitated the "stripping" of long-term Treasury bonds. Under the STRIPS program, all newly issued Treasury bonds and notes with maturities of 10 years or longer are eligible for stripping. The prices of these pure discount bonds can be found in publications such as the *Wall Street Journal*.

Since there is a clearly defined relationship between interest rates and prices for discount bonds, we need to refer to only one of these elements, not both. When we consider discount bond prices, we can easily derive the implied interest rates. Similarly, when we talk about the volatility of discount bond prices, we will easily be able to make inferences about the volatility of interest rates.

Trends and Cycles in Interest Rates. We can plot the interest rate on discount bonds with a 10-year maturity from 1959 to 1990 (Figure 2).³ Notice that, overall, the interest rate tended to rise from 1959 to the early 1980s, after which it generally declined.

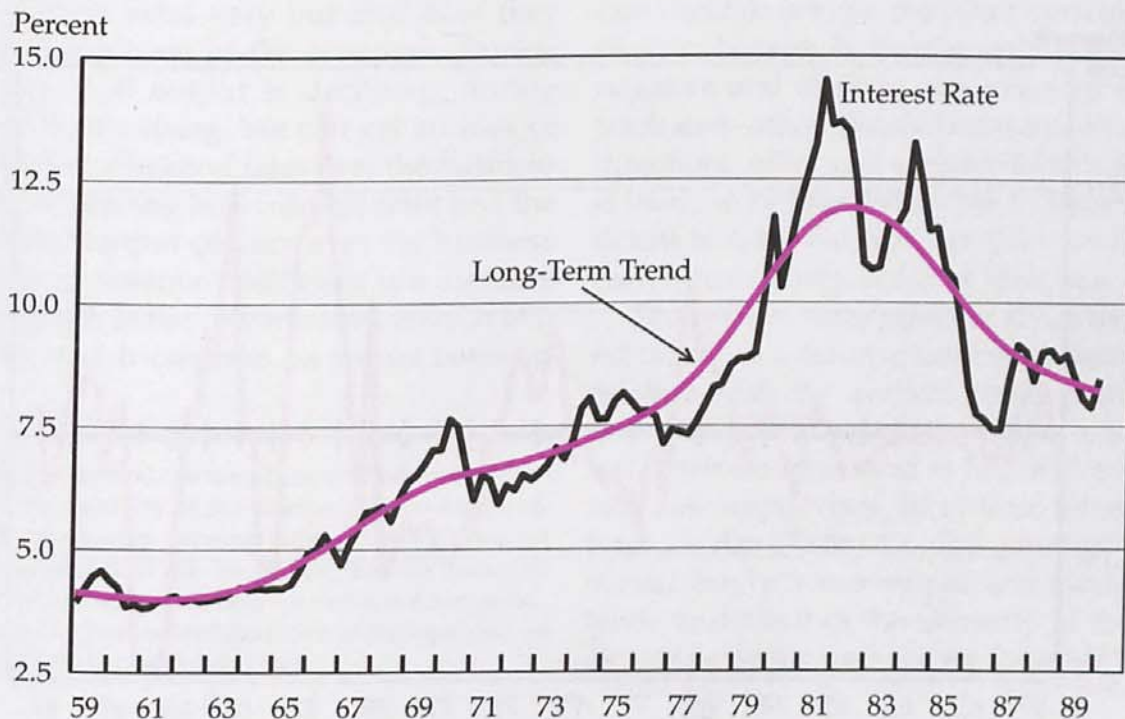
From Figure 2 we can discern two types of variability in interest rates and hence in discount bond prices: long term and short term. Long-term variability refers to broad trends in interest rates, such as the upward trend until the early 1980s and the downward trend since then. Short-term variability refers to how interest rates vary *around* these long-term swings. Since our focus is on the business-cycle volatility of interest rates, we would like to remove

²There is a simple relationship between the interest rate on a discount bond and the price of the bond. Suppose the price today of a bond that pays off \$100 in five years is \$75. The five-year interest rate on the bond is 33.3 percent $[(100 - 75) / 75 = .333]$. The average annual interest rate on the bond is 5.9 percent since $[\$75 \times (1.059)^5 = \$100]$. Generalizing this idea, if the interest rate is r on a bond paying \$1 in j years, the current price of the bond is $\$1 / (1+r)^j$.

³The data plotted in Figure 2 are yields on discount bonds from the dataset compiled by McCulloch and Kwon.

FIGURE 2

Ten-Year Interest Rate and Long-Term Trend



that part of interest rate volatility associated with swings of longer duration than the typical business cycle.

The National Bureau of Economic Research defines minor cycles as recurrent fluctuations lasting from two to four years and major cycles as recurrent fluctuations lasting about eight years. Figure 2 clearly shows long-term trends in interest rates that are of greater duration than typical business-cycle lengths. In Figure 2 we've plotted a long-run trend that is fitted to the interest rate data. This long-term trend is chosen in such a way that it removes the swings in interest rates associated with periods longer than about eight years.⁴ The remaining short-run variability then corresponds more closely to variability that is part of the business-cycle movement in interest rates. We will define the difference between the actual interest rate and the long-run trend as the

cyclical component of the interest rate (Figure 3). The long swings in interest rates have been taken out, and all the variability in interest rates is around zero because this figure plots deviations from the long-term trend. When the interest rate is zero in Figure 3, we are on the long-term trend line in Figure 2. Henceforth, when we refer to interest rate volatility, we will be referring to the variability of this short-run component.

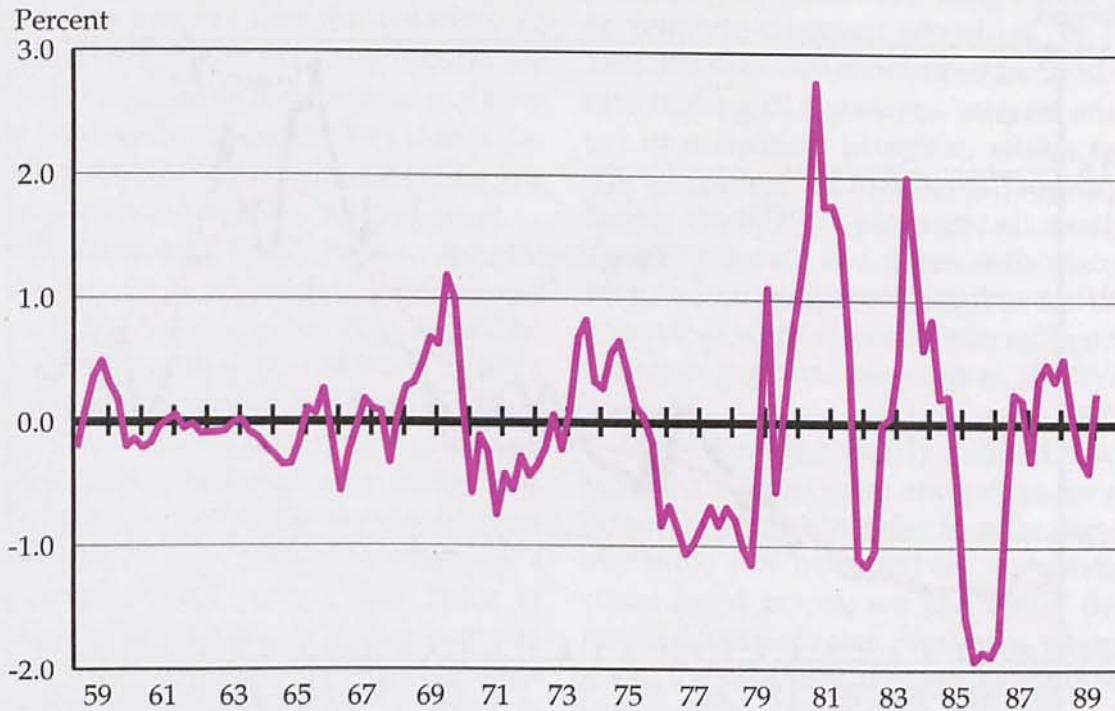
MEASURING THE VOLATILITY OF INTEREST RATES

We will measure interest rate volatility using a statistic called the standard deviation.

⁴The fitted trend is calculated using the Hodrick-Prescott filter. More details on how this filter works are provided in the paper by Robert King and Sergio Rebelo.

FIGURE 3

Cyclical Component of Ten-Year Interest Rate



The standard deviation measures how dispersed a variable is around its average value. If the standard deviation is high, observations on a variable tend to be far away from the variable's average value. If the standard deviation is low, observations on the variable tend to be clustered around the average value. Therefore, as the standard deviation increases, there is a greater chance that we will see large changes in the value of the variable.⁵

The volatility of interest rates can be calcu-

lated over the entire term structure of interest rates: we simply use historical data to calculate the standard deviation of interest rates for each maturity. Table 1 presents the relationship between interest rate volatility and the maturity of bonds as well as the standard deviation of the associated prices for discount bonds.⁶ The table shows that short-term interest rates are more volatile than long-term interest rates and that long-term discount bond

⁵The standard deviation is calculated as the square root of the variance of a variable. Suppose we have n observations on a variable $\{X_1, X_2, \dots, X_n\}$. Denote the average value of X by \bar{X} . The variance of X is then the average of the squared deviations of X from its mean: $V(x) = [(X_1 - \bar{X})^2 + (X_2 - \bar{X})^2 + \dots + (X_n - \bar{X})^2] / n$. The standard deviation of X is the positive square root of $V(x)$.

⁶More specifically, we calculate the standard deviation of detrended yields and logarithms of bond prices using quarterly data over the period 1959:Q1 to 1990:Q1. We use the logarithm of the discount bond price because it is proportional to the yield-to-maturity of the bond, with the factor of proportionality equal to the maturity of the bond. The interest rates and bond prices are detrended using the Hodrick-Prescott filter.

prices are more volatile than short-term discount bond prices.⁷

In describing the cyclical volatility of interest rates we would like to know not just how much interest rates vary but also how they vary with the state of the economy. During recessions, real output is declining; during expansions, it's rising. We can get an idea of the behavior of interest rates over the business cycle by evaluating how interest rates and the level of real output co-vary over the business cycle. The correlation coefficient is a measure of the strength of the co-variation between two variables, and it can take on values between

minus one and one. When the correlation coefficient between two variables is positive and close to one, the two variables track each other closely and move in the same direction: when one variable is high, the other variable is very likely to be high. If the correlation coefficient is negative and close to one, the two variables track each other closely but move in opposite directions: when one variable is high, the other is likely to be low. When the correlation coefficient is zero, the two variables do not track each other closely in either direction.

The cyclical component of short-term interest rates has a positive contemporaneous correlation with the cyclical component of real output. So when current output falls, short-term interest rates tend to fall, and when current real output rises, short-term interest rates tend to rise (Table 2). The strength of this correlation between output and interest rates tends to decline as the maturity of the bonds increases. By the time we get to bonds with 10-

⁷There is a direct relationship between bond-price volatility and the volatility of the interest rate on the bond. Using the relationship between bond prices and interest rates in footnote 2, it can be shown that the standard deviation of the interest rate on a j-period bond is approximately equal to the standard deviation of the logarithm of the j-period bond price divided by j.

TABLE 1

Interest Rate and Bond-Price Volatility 1959:Q1 - 1990:Q1

Time to Maturity	Standard Deviation of Detrended Interest Rate	Standard Deviation of Detrended Discount Bond Price
1 quarter	.0032	.0032
2 quarters	.0031	.0063
3 quarters	.0030	.0092
1 year	.0029	.0117
2 years	.0025	.0205
5 years	.0021	.0426
10 years	.0018	.0735

Standard deviations are calculated from the term structure data in McCulloch and Kwon (1993). Standard deviation is of the logarithm of discount bond prices (see footnote 5).

TABLE 2

Correlations of Interest Rates and Real GDP 1959:Q1 - 1990:Q1

Time to Maturity	Correlation of Detrended Interest Rate in Period t with Detrended Output in Period:		
	t-1	t	t+1
1 quarter	.42	.35	.11
2 quarters	.41	.34	.09
3 quarters	.40	.32	.08
1 year	.37	.29	.05
2 years	.27	.20	-.02
5 years	.11	.04	-.15
10 years	.02	-.05	-.21

Output is measured as the logarithm of real GDP. Interest rate data are from the term structure data in McCulloch and Kwon (1993).

year maturity, the contemporaneous correlation is negative, though quite small. This implies that there is little co-variation between the cyclical movements in current real output and the cyclical movements in long-term interest rates. These facts can be expressed by saying that short-term interest rates are procyclical and long-term interest rates are acyclical. The results in Table 2 suggest significant business-cycle variability in short-term interest rates but relatively little business-cycle variability in long-term interest rates.

The last column of Table 2 shows that the correlation between current interest rates and real output one quarter into the future is positive for short-term and negative for long-term interest rates. This fact suggests that upward movements in short-term interest rates are associated with upward movements in near-term output, but that higher long-term interest rates forecast lower near-term output. The

first column of Table 2 shows the correlation between current interest rates and the level of real output one quarter in the past. These correlations suggest that increases in current output are associated with increases in future interest rates.

We can also make some deductions about the shape of the yield curve over the business cycle using the data in Table 2. We have seen that short-term interest rates tend to move up when output moves up but that the correlation tends to decline as the maturity of the bond increases. Thus, when current output rises, the yield curve tends to flatten, since short-term interest rates tend to rise and long-term interest rates move relatively little. Similarly, when current output declines, the yield curve tends to steepen, since short-term interest rates tend to fall with output and long-term interest rates tend to remain about the same.

We have seen how the volatility of interest

rates changes with maturity and how interest rates move in relation to real output over the business cycle. But how are interest rates on bonds of different maturities related to each other? In general, interest rates on bonds of different maturities are highly correlated with each other, with the highest correlations occurring between bonds of similar maturities (Table 3). Let's take the case of the interest rate on a security with one-quarter maturity. We see that the one-quarter interest rate is most highly correlated with the interest rate on a bond with two-quarter maturity, and that the correlation declines, though remains strong, as we compare bonds with increasingly different maturities. These correlations suggest a tendency for the entire yield curve to shift up and down, while allowing for the possibility that the shape of the yield curve can change.

Finally, if we re-examine Figure 3, we might suspect that the measured volatility of interest rates depends on the period we're looking at. Since the late 1970s, long-term interest rates appear to have shown more short-run variabil-

ity, and the deviations of the interest rate on 10-year bonds from the trend line have been large and persistent.

In fact, the results in Table 4 show that interest rates at all maturities may have been more variable since that time. The table shows the standard deviation of interest rates using the same data, but the sample is divided into two subsamples: from first quarter 1959 to first quarter 1979 and from second quarter 1979 to first quarter 1990. We see that interest rates at all maturities have been more volatile since 1979. This result suggests the possibility that some structural change in the economy has affected the variability of interest rates and bond prices.⁸

⁸For short-term interest rates in particular, higher volatility after 1979 may reflect a change in the way that the Federal Reserve implements monetary policy. After late 1979, and especially between late 1979 and late 1982, monetary policy placed less emphasis on smoothing short-term interest rates. Thus, after 1979, short-term interest rates were more likely to reflect changes in the state of the economy.

TABLE 3

Cross Correlations of Detrended Interest Rates 1959:Q1 - 1990:Q1

Maturity	1 qtr	2 qtr	3 qtr	1 year	2 years	5 years	10 years
1 quarter	1.0						
2 quarters	.99	1.0					
3 quarters	.97	.99	1.0				
1 year	.96	.99	.99	1.0			
2 years	.90	.94	.96	.98	1.0		
5 years	.79	.84	.87	.90	.96	1.0	
10 years	.69	.75	.78	.81	.90	.98	1.0

Interest rate data are from McCulloch and Kwon (1993).

TABLE 4

Interest Rate and Bond-Price Volatility

Time to Maturity	Standard Deviation of Detrended Interest Rate		Standard Deviation of Detrended Discount Bond Price	
	59:1-79:1	79:2-90:1	59:1-79:1	79:2-90:1
1 quarter	.0026	.0040	.0026	.0040
2 quarters	.0026	.0040	.0051	.0079
3 quarters	.0025	.0039	.0075	.0116
1 year	.0024	.0037	.0094	.0148
2 years	.0019	.0034	.0151	.0275
5 years	.0014	.0030	.0278	.0608
10 years	.0011	.0027	.0442	.1080

Standard deviations are calculated from the term structure data in McCulloch and Kwon (1993). Logarithms of discount bond prices are taken before the standard deviation is calculated (see footnote 5).

WHAT DETERMINES INTEREST RATE VOLATILITY?

The postwar data imply that prices of long-term discount bonds are more variable than those of short-term discount bonds and that long-term interest rates, measured by yield-to-maturity, are less volatile than short-term rates. In addition, we find that short-term interest rates are procyclical, while long-term interest rates vary little with current output. What economic factors influence interest rate variability? If we can isolate some economic determinants of the levels of interest rates and bond prices, we will be well on our way to finding determinants of this variability.

Determination of Short-Term Interest Rates. A standard economic model will help us think about how the interest rate on short-term discount bonds is determined. Let's consider the case of a discount bond that will pay off \$100 with certainty in one year. Suppose a prospective bond buyer expects her real in-

come over the coming year to be higher than usual (real income refers to income adjusted for any change in the general level of prices over time). In that case, she has less of an incentive to increase her savings by purchasing a bond today. In fact, she may well decide to borrow against some of her expected increase in income. If all prospective bond purchasers expect higher real income over the coming year, demand for current one-year bonds will fall, and their prices will fall as well, which means that the one-year interest rate will rise. On the other hand, investors may decide to hedge against the risk of lower future income by purchasing bonds today that provide a guaranteed future payoff.

If current real output (and thus aggregate real income) is low, investors may expect future output to be low, because there is some persistence to output movements. Hence, a downward movement in current output is consistent with a downward movement in

current short-term interest rates if people expect output and income in the near future to be low as well. This theory is consistent with procyclical movement in short-term interest rates.

The yield curve tends to flatten when output is high and tends to steepen when output is low. Suppose we are currently in a boom, but people expect a recession in one year. Investors may buy one-year bonds to hedge the risk of low future income, and they may pay for these bonds, in part, by cashing in their shorter-term assets. This portfolio reallocation tends to lower one-year interest rates and raise shorter-term interest rates, thus leading to a flatter yield curve. Empirical studies have found that the shape of the yield curve does help predict recessions and expansions.⁹

Expected inflation is also a determinant of interest rates. Consider again the case of a discount bond that pays \$100 with certainty in one year. Suppose now that prospective bond purchasers expect inflation to rise over the coming year. When inflation rises, the current price of one-year bonds will fall because investors realize that their dollars buy less when prices rise. For example, if the price of a cup of coffee one year from now is \$1, bondholders can buy 100 cups of coffee with the \$100 that the bond pays off. But if the price of a cup of coffee is expected to rise to \$1.05, bondholders will be able to buy only 95 cups of coffee. To be compensated for the loss in purchasing power, investors must get a higher dollar return on their investments. Thus, bond prices will fall and interest rates will rise when expected inflation rises.

This model suggests that when expected income or expected inflation rises, bond prices will fall. This fall in bond prices translates into higher interest rates. So, when we think about how short-term interest rates are determined,

we want to think about people's forecasts for real income growth and inflation. Any current economic variables that help to predict real income growth and inflation will help to determine current short-term bond prices and interest rates.

Determination of Long-Term Interest Rates. Long-term interest rates can be linked to short-term interest rates by the expectations theory of the term structure. This theory says that long-term interest rates are equal to an average of expected short-term interest rates plus a risk premium.¹⁰ The risk premium accounts for the co-variation over time of variables like income growth and inflation that could influence the level of interest rates.

The logic of the expectations theory of bond prices is most clearly seen in an example in which we ignore the risk premium. Take the case of an investor who has a two-year investment horizon. The investor can purchase a two-year bond, or he can purchase a one-year bond today and, when that bond matures, purchase another one-year bond. The expected return on these alternative investment strategies should be equal. Since there is a direct relationship between interest rates on bonds and bond prices, the expectations theory also links long-term discount bond prices to expected short-term discount bond prices over the life of the long-term bond.

In terms of expected future short-term bond prices, the same variables that affect short-term bond prices basically determine long-term bond prices and interest rates. Thus, expected future income growth and expected inflation are also determinants of long-term bond prices, but now the forecasts of income growth and inflation are for further in the future. It is still the case that if, over the life of

¹⁰For more detail on the expectations theory and risk premiums, see my article in the July/August 1994 *Business Review*.

⁹See the article by Campbell R. Harvey.

the bond, expected future income growth or expected future inflation rises, long-term interest rates will rise. Including a risk premium does not alter these basic conclusions about the determinants of interest rates. However, the risk premium can be an additional source of variability for interest rates because it picks up some indirect effects of income growth and inflation on interest rates, as well as other risk factors.

This model helps us think about why long-term interest rates co-vary less with current output than do short-term interest rates. Current movements in real output are much more closely correlated with output movements in the near future than they are with output movements in the far future. Since the payment stream on a long-term bond extends further out into the future than that on a short-term bond, long-term interest rates are less likely to have a strong co-variation with current output movements.

Determinants of Interest Rate Volatility. The same basic economic factors that determine interest rates and the prices of bonds also determine the volatilities of interest rates and bond prices. This economic model suggests that expected real income growth and expected inflation determine bond prices and interest rates. It follows then that the volatility of expected real income growth and the volatility of expected inflation, as well as the correlation between the two, determine the volatility of interest rates and bond prices.

The reasoning behind this conclusion is straightforward. Take the case of real income growth. We saw above that if real income growth is expected to be high, current bond prices will fall and interest rates will rise. The higher real income growth is expected to be, the higher interest rates will be. Thus, large changes in expected real income growth are associated with large changes in interest rates. When real income growth has high volatility, large changes in real income growth occur

more frequently, and hence large changes in current bond prices and interest rates occur more frequently. When large changes in interest rates occur more often, interest rates are more volatile. Similar reasoning holds for the case of inflation. When large changes in expected inflation occur, large changes in current bond prices and interest rates occur also. So, more volatile inflation translates into more volatile bond prices and interest rates.¹¹

What determines how volatile income growth and inflation will be? One factor is monetary policy. Take the case of monetary policy and inflation. Economists generally believe that a persistent inflation has its root causes in monetary policy, in particular, how fast the money supply grows relative to real income growth. If growth of the money supply is excessive, inflation is likely to be high. If we take growth of the money supply as the primary determinant of inflation, highly volatile growth in the money supply can lead to volatile inflation. This does not mean that every change in the money supply necessarily leads to a change in inflation. Rather, if, on average, money supply growth becomes more volatile, inflation can become more volatile as well. As we have seen, the model then suggests that bond prices and interest rates will also be more volatile.

Monetary policy could also have an effect on real income, although economists disagree on the mechanism by which this occurs. One theory is that workers write contracts with their employers that fix a nominal wage rate over the contract period. Workers and firms

¹¹Higher volatility of income growth and inflation suggests that price volatilities for both short-term and long-term bonds will increase. Long-term volatility remains higher than short-term volatility because investors who buy long-term bonds have to make forecasts about future variables that are not relevant for determining the prices of short-term bonds.

negotiate the contracted wage based, in part, on their expectations of what inflation will be over the contract period. Since monetary policy affects inflation, this requires workers and firms to forecast what monetary policy will be over this same period. If monetary policy and the price level turn out to be different from what workers and firms expected when they wrote the contract, employment and output could be affected because firms' demand for workers depends on the real wage rate that must be paid. If nominal wages are fixed by a contract and prices rise unexpectedly, real wages fall, and firms demand more workers and produce more output. If prices fall unexpectedly, real wages rise, firms lay off workers, and output falls. Thus, variability of the money supply, through its impact on prices, could have an impact on the variability of real income.

We can point to many other factors, besides monetary policy, as potentially influencing the volatility of output and inflation. For example, variability in weather can affect agricultural output as well as production in the economy. Changes in productivity due to the introduction of new technologies can influence the variability of output and inflation as well. A whole class of economic models, called real-business-cycle models, attempts to account for output volatility over the business cycle. These models assume that shocks to productivity are the main cause of business cycles.¹² Shocks to current productivity affect peoples' forecasts of the future course of the economy and thereby affect their expectations about economic variables like real income growth and inflation. The more persistent productivity shocks are, the greater their effect on long-term interest rates will be, since output and inflation far into the future will be affected.

Economic Models and Interest Rate Volatility. This economic model for determining bond prices and interest rates suggests that investors' expectations of future real income growth and inflation are the primary determinants of current bond prices and interest rates. There are, of course, other determinants of interest rates and interest rate volatility in the economy. But we can try to assess how well this view of interest rate determination explains the interest rate volatility that we observe in the actual economy.

One approach to assessing how well a model performs is to use the model to simulate interest rates and then compare the properties of the simulated interest rates to the properties of actual interest rates. For example, we can set up models and use them to simulate price data on discount bonds of various maturities. We can then calculate the standard deviation of these simulated data and compare it to the standard deviation of discount bond prices implied from the interest rates we observe in the economy. We can also examine how the simulated bond prices and interest rates covary with simulated output and compare the correlations to the correlations we find in the actual data. In this way, we can assess the ability of the model to account for the cyclical volatility of interest rates.

SIMULATION RESULTS

In my 1994 working paper, I present an exercise similar to the one following. Briefly, in the model, expected real income growth and expected money growth determine current discount bond prices and yields. Expected money growth is assumed to be the primary determinant of inflation. The model also requires some input on investor characteristics, such as how willing investors are to undertake risky investments. Table 5 shows the variability of the bond prices and yields simulated by one particular version of the model and reproduces the variability of bond prices and yields

¹²See the article by Satyajit Chatterjee in the September/October 1995 *Business Review*.

TABLE 5

Yield and Bond-Price Volatility From Model Simulations

Time to Maturity	Standard Deviation of Detrended Interest Rate		Standard Deviation of Detrended Discount Bond Price	
	Simulated	Actual	Simulated	Actual
1 quarter	.00362	.00322	.00362	.00322
2 quarters	.00332	.00315	.00663	.00629
3 quarters	.00303	.00308	.00909	.00923
1 year	.00278	.00293	.01112	.01172
2 years	.00202	.00256	.01619	.02051
5 years	.00100	.00213	.02005	.04265
10 years	.00051	.00184	.02045	.07348

Standard deviations are of the yields and logarithms of discount bond prices (see footnote 5). Actual discount bond prices are calculated from the term structure data in McCulloch and Kwon (1993).

derived from actual interest rate data for comparison.¹³

The model generates data in which volatility of interest rates falls but bond price volatility rises with the maturity of the bond. Out to a maturity of about one year, the variability of the simulated bond yields and prices matches the variability of the data fairly closely. At a maturity of three months, the model overpredicts the volatility of bond prices and yields about 12 percent. At a maturity of one year, the model underpredicts the volatility of bond prices and yields about 5 percent. These results suggest that the variability of income

growth and money growth account for a substantial portion of the variability of short-term discount bond prices and hence of short-term interest rates.

For longer maturities, the variability of simulated bond prices and yields underpredicts the volatility of actual yields and implied prices of discount bonds by a progressively larger amount. When we look at the historical data, the variability of implied prices for a discount bond with 10-year maturity is about 23 times larger than the variability of short-term discount bond prices. But in the simulated data, the variability of 10-year discount bond prices is only about five times greater than the variability of short-term bond prices.

Many reasons might explain why the growth of the money supply and the growth of real income do not account for much of the vari-

¹³The model also replicates some of the correlations between discount bond prices and output as well as some features of the correlation patterns of bond prices.

ability of long-term bond prices and yields. The basic model is designed to highlight the business-cycle variability of interest rates, and as we have seen, long-term interest rates do not appear to have a large business-cycle component. In addition, the model is very simple, and so it is missing some important elements found in actual economies. For example, the model does not account for the fact that different people have different beliefs about the future course of the economy or that people are continually learning about the economic environment. Changes in fiscal and monetary policies may induce greater volatility in interest rates than the simple economic model accounts for. The expectations theory may be an inadequate model of the term structure of interest rates. Despite difficulties such as this, the model's implication that real income growth and money supply growth are factors that help to determine the volatilities of interest rates and discount bond prices does find some sup-

port, especially for shorter maturities, when we compare the model with actual data.

CONCLUSION

We have seen that the volatility of interest rates depends on the maturity of the underlying bond: long-term interest rates are less variable than short-term interest rates. Short-term interest rates are procyclical while long-term interest rates co-vary little with movements in output over the business cycle. Economic theory suggests that both the level and volatility of interest rates should be tied to economic variables such as income growth and inflation. Simulation results suggest that the volatility of both income growth and money growth accounts for a large portion of the volatility of short-term discount bond prices. However, these same economic variables by themselves are able to account for only a small fraction of the volatility of long-term discount bond prices.

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