

# Can Stock Prices Reliably Predict Recessions?

*Leonard Mills\**

The stock market crash of October 19, 1987, has had its impacts on Wall Street—including congressional calls for more regulation of the financial markets, the New York Stock Exchange's proposals for limits on trading, and reduced volume and liquidity in the financial markets. The expected impact on Main Street, however, never seemed to materialize. Immediately following the crash, predictions of

recession—or, reminiscent of 1929, depression—were rampant. But overall the economy remained strong in the fourth quarter of 1987 and the first half of 1988, and fears of recession soon dissipated.

The economy's resilience in the wake of the crash has surprised many observers. Some have argued that the time between a decline in stock prices and a recession is so long that we have yet to see the upcoming recession. But others claim to be not surprised, arguing that stock prices have never been a reliable indicator of impending recessions. This view is summarized in the

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often-quoted quip by Paul Samuelson: "The stock market has predicted nine of the last five recessions!"<sup>1</sup>

To be fair, however, no indicator of future economic activity is infallible, and the October crash may be just one of those rare occasions when the stock market made an incorrect prediction. In short, the issue that the crash has resurrected is whether stock prices, by themselves, are a reliable leading indicator of recessions. Theory alone cannot provide the answer; it is an empirical issue. But analysts looking at the same set of numbers do not always reach the same conclusions. So we should first try to quantify objectively the stock market's performance as a leading indicator. One statistical technique, recently developed by Salih Neftci and one that has been applied to other economic indicators, can provide a helpful perspective when applied to stock prices. The results of the Neftci technique suggest that though stock prices alone offer some indication of the economy's future, broader indicators, such as the Index of Leading Indicators, are more reliable.

### DECLINING STOCK PRICES COULD SIGNAL RECESSIONS

There are sound economic reasons for thinking that a fall in stock prices would be a good leading indicator of recessions. One reason is that declining stock prices may have direct effects on consumer spending because falling stock prices lower the financial wealth of stockholders. This decline in wealth may induce them to decrease their spending on goods and services. Consumers who do not own stock also could be affected by falling stock prices because they may lose confidence in the economy and feel their own income prospects are dimmer. Hence, they may become more cautious in their current spending. For businesses, lower stock prices raise the cost of acquiring equity funds to pur-

chase new plant and equipment, so investment spending could be reduced when stock prices fall. And as investment and consumer spending decrease because of declining stock prices, the economy could grow at a slower rate and perhaps slip into a recession.

The 1987 stock crash, however, seemed to have only modest effects on consumer spending and investment. After slowing in the fourth quarter of 1987, consumption and business investment came back in the first half of 1988. In a recent study, Alan Garner concludes that "this relatively small effect is consistent with empirical studies showing that the stock market has only a modest impact on consumer spending."<sup>2</sup> Likewise, in an earlier study, Douglas Pearce finds that most empirical studies have concluded that decreases in stock prices lead to decreases in investment, but that the size of the effect is uncertain.<sup>3</sup>

Even if the direct impact of stock price declines is small, stock prices may still be a good leading indicator. The conventional view is that stock prices reflect firms' expected future earnings. According to this view, a general decline in stock prices means that market participants have lowered their expectations of firms' future earnings, presumably because they foresee a downturn in the economy. If the expected downturn actually occurs, the decline in stock prices will have preceded it. So to the extent that an economic downturn, whatever its cause, can be foreseen, its onset should be forewarned by the stock market.

An alternative view is that stock prices sometimes fluctuate for reasons unrelated to the economic fundamentals. In particular, the stock market may be subject to speculative bubbles. In a bubble, speculators bid up the current prices

<sup>2</sup>C. Alan Garner, "Has the Stock Market Crash Reduced Consumer Spending?" Federal Reserve Bank of Kansas City *Economic Review* (April 1988) pp. 3-16.

<sup>3</sup>Douglas K. Pearce, "Stock Prices and the Economy," Federal Reserve Bank of Kansas City *Economic Review* (November 1983) pp. 7-22.

<sup>1</sup>Paul A. Samuelson, "Science and Stocks," *Newsweek*, September 19, 1966.

of stocks simply because they expect to sell the stocks at still higher prices in the future, even though expectations of future earnings remain unchanged. For a while, the expectations of higher prices are self-fulfilling. A second group of buyers is willing to pay more than the first because it expects to get even higher prices from a third group, and so on. But at some point in time, people lose faith that prices will go any higher—an expectation that is likewise self-fulfilling. The bubble then bursts and stock prices come tumbling down. In this circumstance, a decline in stock prices is *not* the result of lowered expectations of future earnings. Some analysts have argued that the October crash, which followed a steep run-up in stock prices earlier in 1987, was just such an episode and that the crash did not mean that market participants had foreseen an economic downturn.<sup>4</sup>

Casual observation of stock prices over the postwar period reveals that they do seem to be a leading indicator of recessions, though an imperfect one. Since 1947 the S&P 500-stock index, shown in Figure 1 (p. 6), has often declined just before the onset of recessionary periods (depicted by the vertical bars). The recessions of 1959 and 1973 are examples. But stock prices do not seem to be completely reliable as a leading indicator. Sometimes, as in 1962, a bear market cried wolf: stock prices fell dramatically, but no recession followed. Ideally, a leading indicator would not generate these false signals. Other times, as in 1980, a recession started without a decline in stock prices: that is, the stock market gave no advance warning. An ideal leading indicator would anticipate all recessions.

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<sup>4</sup>For a discussion of experimental evidence suggesting that bubbles are possible, see Herbert Taylor, "Experimental Economics: Putting Markets Under the Microscope," this *Business Review* (March/April 1988) pp. 15-25, and the references therein. However, evidence that the run-up in stock prices in 1986-87 was not the result of a speculative bubble is presented in Gary Santoni, "The Great Bull Markets 1924-29 and 1982-87: Speculative Bubbles or Economic Fundamentals?" Federal Reserve Bank of St. Louis *Review* (November 1987) pp. 16-30.

In practice, there is no leading indicator that meets the ideal standard of emitting no false signals and anticipating every recession. And a precise answer to how many errors are acceptable for an indicator depends on the costs of these errors. Nevertheless, the number of false signals and the number of unanticipated recessions are useful quantitative measures in assessing a leading indicator's reliability.

### THE NEFTCI RULE HELPS EXTRACT THE SIGNALS

Evaluating an indicator's success or failure as a predictor requires some method of determining when the indicator is signaling recession. One method is the x-month rule. If the indicator decreases for x consecutive months, then it is said to be predicting that a recession is imminent. This kind of rule has been applied primarily to the Index of Leading Indicators by analysts who say that three consecutive monthly declines in this index presage a recession. Another method is the x-percent rule. In this rule, if the indicator declines by x percent, then it is said to be signaling a recession. This kind of rule has often been applied to stock prices by analysts who say that a 10 percent decline in stock prices, for example, signals a recession.<sup>5</sup> But any x-month or x-percent rule is somewhat arbitrary and may not take full advantage of the information provided by the indicator. An alternative approach for extracting a turning-point signal is to apply a more sophisticated statistical rule called Neftci's optimal prediction rule.

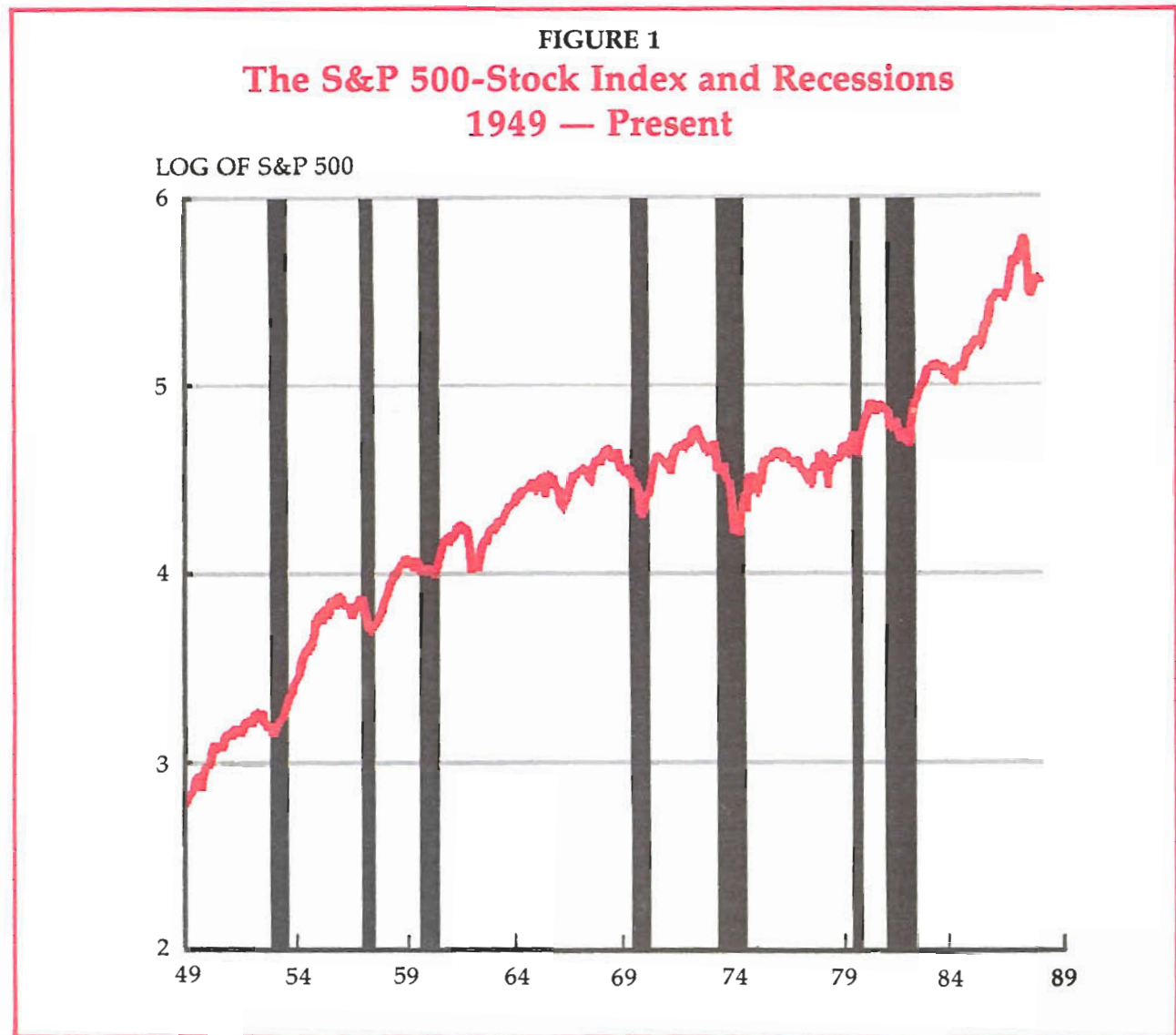
**How the Neftci Rule Works.** The Neftci rule starts with the same assumption underlying the more popular rules: that a substantial downturn in an indicator presages an upcoming recession. After each new reading of the indicator, an

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<sup>5</sup>For applications of the x-percent rule to stock prices as a leading indicator, see Bryon Higgins, "Is a Recession Inevitable This Year?" Federal Reserve Bank of Kansas City *Economic Review* (January 1988) pp. 3-16, and Alfred Malabre, "As Economy Goes, So Goes Stock Market," *Wall Street Journal*, February 9, 1987.



**FIGURE 1**  
**The S&P 500-Stock Index and Recessions**  
**1949 — Present**



analyst using the Neftci rule would assess the probability that the indicator has gone into a "down" phase. When this probability climbs above a critical value prespecified by the analyst, the indicator is interpreted to be signaling a coming recession. Taking stock prices as an example, each month the Neftci procedure will calculate a probability that the stock market has entered a "bear" market. If that probability is higher than the critical value, the analyst will interpret this to mean that the stock market is calling for a recession.

The critical probability value that must be reached before a recession is signaled also

determines the probability of false signals that the analyst is willing to accept. For instance, suppose an analyst—call her Denice D'spair—sets her critical probability at 75 percent. When the probability that the indicator has entered a down phase increases to .75 or higher, Denice warns that a recession is imminent. At this critical probability value, Denice is willing to accept the 25 percent probability that the indicator has not entered a down phase and hence that the prediction of recession is wrong.

In general, choosing the critical value involves a trade-off between the number of false signals and the number of unanticipated recessions that

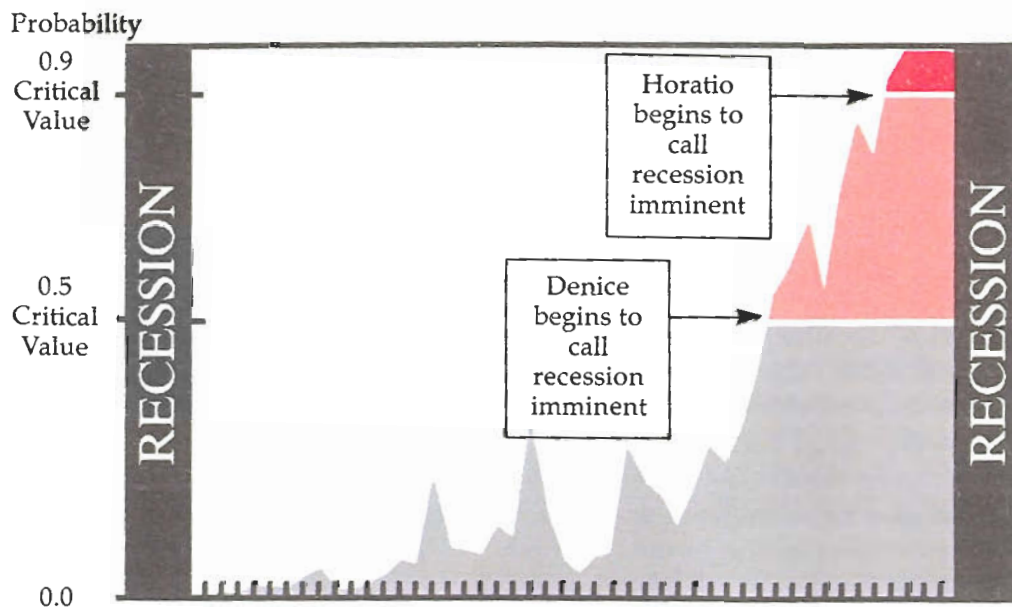
arise. The higher the critical value, the smaller the number of false signals but the larger the number of unanticipated recessions. The appropriate critical value depends on the relative costs of these errors to the analyst. As an example, suppose Denice's boss tells her that an unanticipated recession is very costly to the firm because it would leave the company with large inventories of unsold goods. Denice might then decide to lower her critical value to, say, .50. So when the probability that a downturn in stock prices has occurred is .50 or higher, Denice warns that a recession is coming. That is, Denice will predict a recession every time a recession is at least a 50-50 proposition. Thus, while it is unlikely that Denice's company will be caught by a surprise recession, there is also a good chance that a false signal of recession will be given.

For contrast, consider an analyst with a different company—call him Horatio Hope. Horatio's company is more concerned with preserving its market share and does not want to lose any

customers because of orders going unfilled. False signals of recession are more costly to Horatio's employer than are unanticipated recessions. Consequently, he chooses a high critical value, say .90. Horatio calls for a recession only when the Neftci probability value climbs above .90, implying only a 10 percent chance that a signal is false. (See Figure 2.)

To estimate the actual probability of a downturn, the analyst uses each new reading of the indicator to update the probability of recession by applying Neftci's rule. (See Appendix for a technical description of calculating the probability of recession.) For example, suppose times have been good so that Denice begins with a recession probability of 10 percent. Then she observes a large fall in stock prices, say a 7 percent monthly decline. Using this new information, Denice would then recompute the probability of recession, which would show an increase to perhaps 30 percent. This new figure then serves as her probability of a downturn

**FIGURE 2**  
**Effects of the Critical Value on the Forecaster's Decision to Call for a Recession**





until her next observation of the indicator. So suppose in the next month Denise observes a 15 percent increase in stock prices. Using the Neftci rule to combine the previous estimate of a 30 percent probability of a downturn with this new observation would produce a lower probability of recession, say 12 percent. Thus, as new information on stock prices becomes available, Denise's assessment of the probability of recession is revised based on both current and past movements in stock prices.

The updating aspect of the Neftci procedure takes the advantages of the  $x$ -month and  $x$ -percent rules and improves upon them. Like the  $x$ -month rule, the Neftci rule includes information from previous movements in stock prices. Like the  $x$ -percent rule, the Neftci rule also uses the information revealed by the magnitude of the change in stock prices. That is, a large decline in stock prices will raise the probability of recession more than a small decline will. But the  $x$ -month and  $x$ -percent rules allow only the crude statements that a recession is either likely or unlikely. There will always be some uncertainty in any economic forecast, but the popular rules do not quantify the degree of uncertainty. The Neftci rule improves on the popular rules because it produces probability statements, such as "the recent decline in the stock market implies that the probability of a coming recession is 67 percent," thus indicating the analyst's degree of uncertainty.

**Using the Neftci Rule to Count Errors.** Any leading indicator can be evaluated by comparing its signals of recession with the dates of actual recessions. To define what he means by a correct signal, however, the analyst must define an acceptable lead time. The lead time is the number of months between the time the indicator flashes the signal and the onset of the recession. For our purposes a lead time of 12 months or less may be considered acceptable. The shortest expansion in the postwar period lasted 12 months. Since we will compute the probability of a recession only while we are in an expansion, 12 months is the longest lead time

possible for all of the expansions.<sup>6</sup>

With the prespecified critical probability and a lead time that is considered acceptable, we can label every signal of recession as either correct or false and every recession as either anticipated or unanticipated. If the indicator gave a probability of recession above the critical value sometime within the 12 months prior to the recession, then the indicator correctly anticipated the recession. The top panel in Figure 3 gives an example of a correct signal. In contrast, if the indicator switched on, and then off, more than 12 months prior to the recession, then it gave a false alarm, as shown in the middle panel. Finally, if the signal was never on within the 12 months prior to the recession, then the indicator failed to anticipate the recession. The bottom panel of Figure 3 illustrates this type of error.

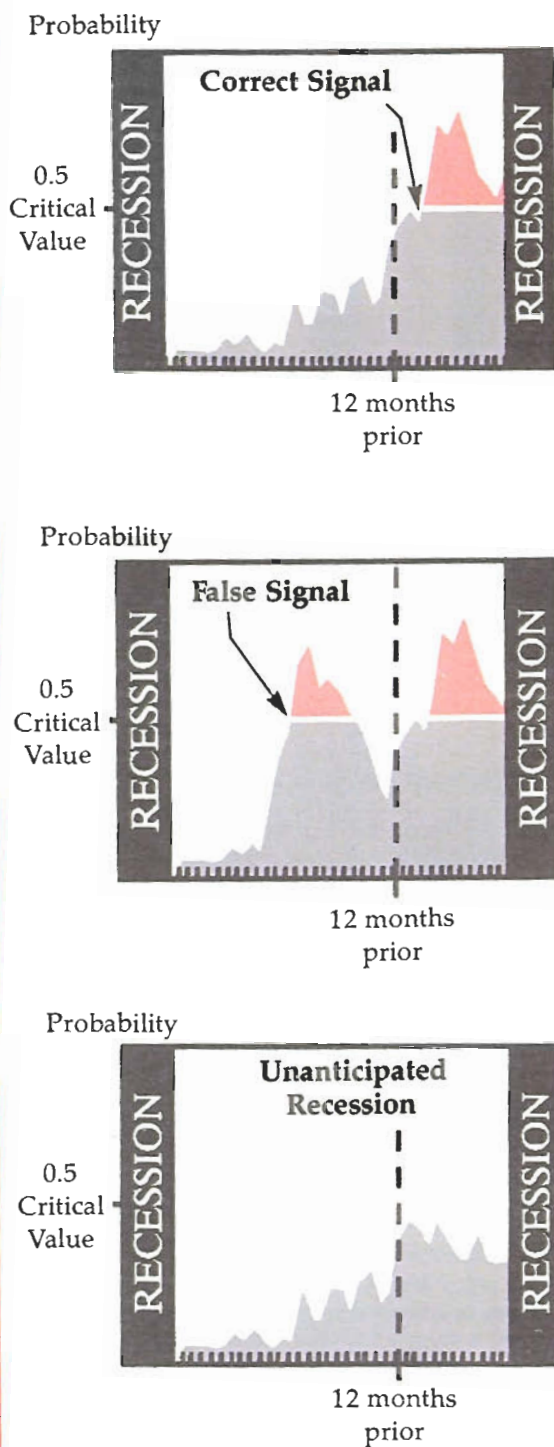
#### HOW RELIABLE ARE SIGNALS FROM STOCK PRICES?

**The Historical Record.** Applying the Neftci rule to the monthly growth rate in the S&P 500-stock index during the 1947-82 period produced the probabilities of recession shown in Figure 4 (p. 10). Clearly, stock prices contain some information about the economy's future direction; the probability of recession climbed before each of the seven recessions that occurred between 1949 and 1982 (represented by the vertical bars). Before five of the seven recessions, the probabilities based on the stock market rose above the 50 percent critical value within 12 months of the economic downturn. These five successful predictions of recession all occurred before 1975. The lead times of the five correct predictions ranged from one to 11 months. Although all of the recessions were anticipated during this period, stock prices did emit some false signals. Using the 50 percent critical value resulted in four false signals before the December 1969 peak and one false signal before

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<sup>6</sup>A lead time of zero months—or no lead time—is considered to be a useful signal because it usually takes several months to recognize that a recession has in fact occurred.

**FIGURE 3**  
**Types of Signals**



the November 1973 peak.<sup>7</sup> Thus, of the 10 recession signals given before 1975, five were false and five were correct.

Unfortunately, the two recessions in the 1980s were cases in which the stock market failed to provide a useful signal. Before the recession that began in January 1980, the stock market sent a signal that was considerably earlier than our 12-month lead time. The probability of a downturn climbed above 50 percent in April 1977 and stayed there for the remaining 33 months of the expansion. This type of signal is considered correct, even though its lead time is greater than 12 months, because the probability never fell below the 50 percent critical value prior to the recession. But the severe prematurity of this signal, relative to the lead times of the previous correct signals, means that stock prices had little value in predicting the timing of the 1980 recession. Before the recession that began in July 1981, the stock market did not send any signal at all; stock prices failed to push the probability of recession above the 50 percent level. In short, using the .50 critical value for extracting recession signals from the stock market worked reasonably well through the 1970s, but the stock market's performance as an indicator seems to have deteriorated in the 1980s.

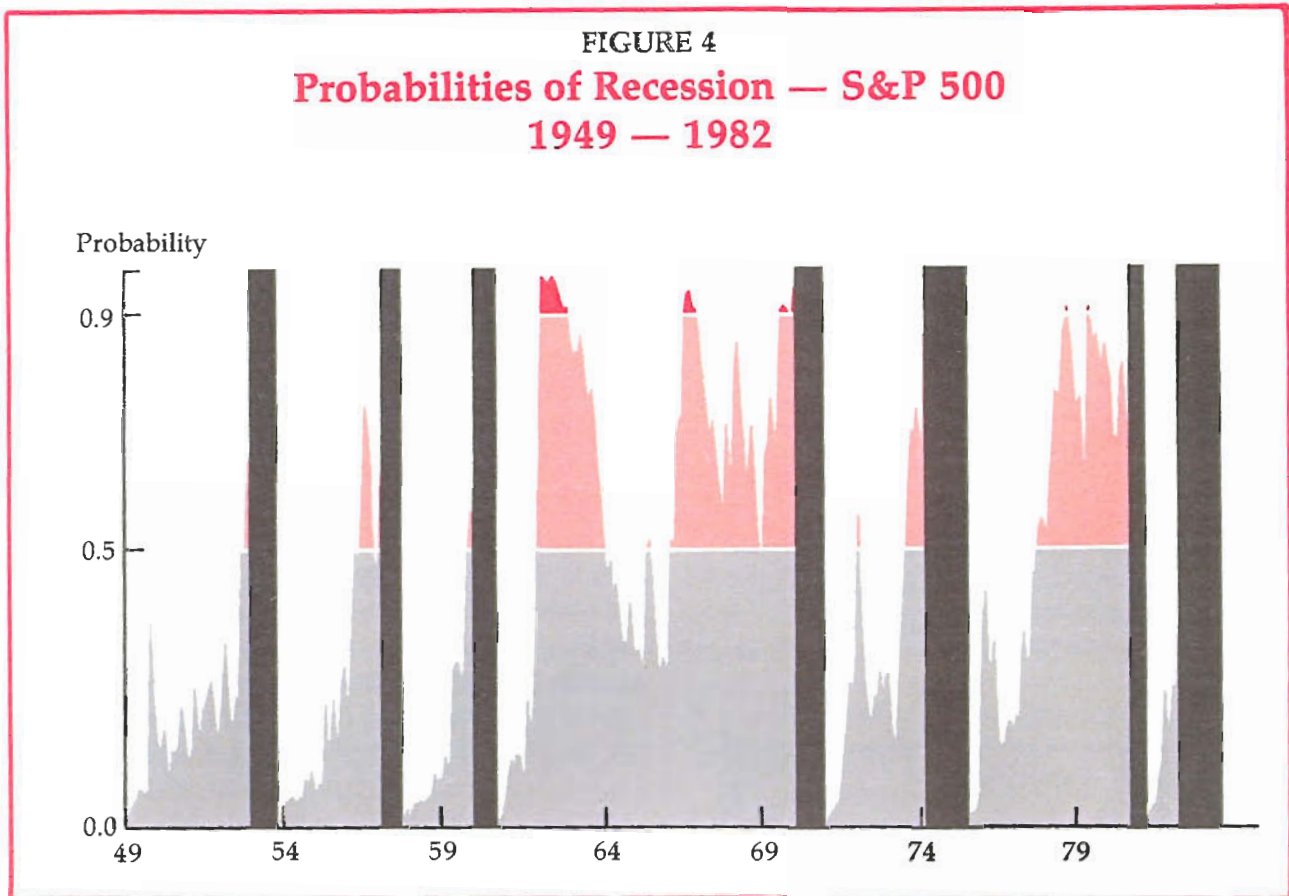
In Figure 4, raising the critical value for recession signals to .90 reduces the number of false signals slightly, from five to four over the entire postwar period. But the number of unanticipated recessions increases dramatically, from one to six. Apparently, we cannot presume that a high probability of a stock market downturn is associated with a high probability of recession. We should certainly be suspicious, then, of claims that stock prices have always been a reliable leading indicator.

**The Crash of 1987.** The stock market crash of

<sup>7</sup>In fairness to stock prices, some of these false signals were associated with pronounced economic slowdowns that were not quite severe enough to be labeled recessions; an example is 1966-67.



FIGURE 4  
**Probabilities of Recession — S&P 500**  
**1949 — 1982**



1987 can now be interpreted with the benefit of this historical perspective. Figure 5 shows the probabilities of recession given by stock prices since the beginning of the current expansion in December 1982. Stock prices generated false signals early in this expansion: at the .50 critical value, they flashed a recession warning between February and December of 1984. Thereafter, generally rising stock prices reduced the probability of recession to very low levels. The low point was achieved at the stock market peak, in August 1987, when the probability of recession was only about 5 percent. After climbing to 14 percent in September, the probability of recession shot up to 88 percent after the crash in October. Thus, the probability of recession as determined by stock prices was certainly increased by the crash. Since stock prices subsequently fell further and have yet to fully recover, the current probability of recession is

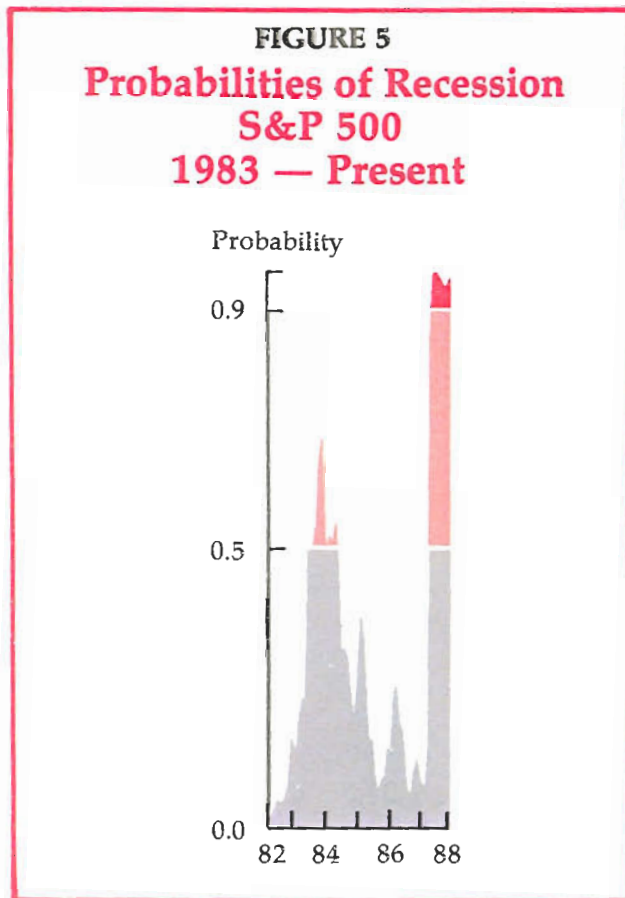
even higher, 98 percent as of May 1988. But as we have seen, probabilities of a downturn exceeding 90 percent have turned out, more often than not, to be false signals of recessions. Thus, while it may be too early to tell whether stock prices will accurately predict the next recession, it would not be too surprising if even this strong signal turned out to be false.<sup>8</sup>

#### A BROADER INDICATOR SENDS A CLEARER SIGNAL

Because stock prices may move for reasons

<sup>8</sup>In a recent article, Joe Peek and Eric S. Rosengren, "The Stock Market and Economic Activity," Federal Reserve Bank of Boston *New England Economic Review* (May/June 1988) pp. 39-50, suggest that real stock prices are more reliable than nominal prices in predicting economic slowdowns. Applying the Neftci rule to real stock prices (measured as the S&P 500 divided by the CPI), however, did not improve the stock market's recession predictions.





unrelated to the economic fundamentals, the information that these prices provide may be noisy. Our statistical analysis certainly indicates this. But many other economic statistics are subject to the same criticism. For example, measures of the money stock may exhibit this noise problem as well. Dramatic changes in the public's demand for money, such as those that occurred during the period of deposit deregulation in the early 1980s, can distort the signal the money supply provides about the economy's future direction.<sup>9</sup>

Because any single economic statistic is subject to idiosyncratic movements that may not

<sup>9</sup>See Herbert Taylor, "What Has Happened to M1?" this *Business Review* (September/October 1986) pp. 3-14, for a discussion of the deterioration of the relation between M1 and future GNP movements.

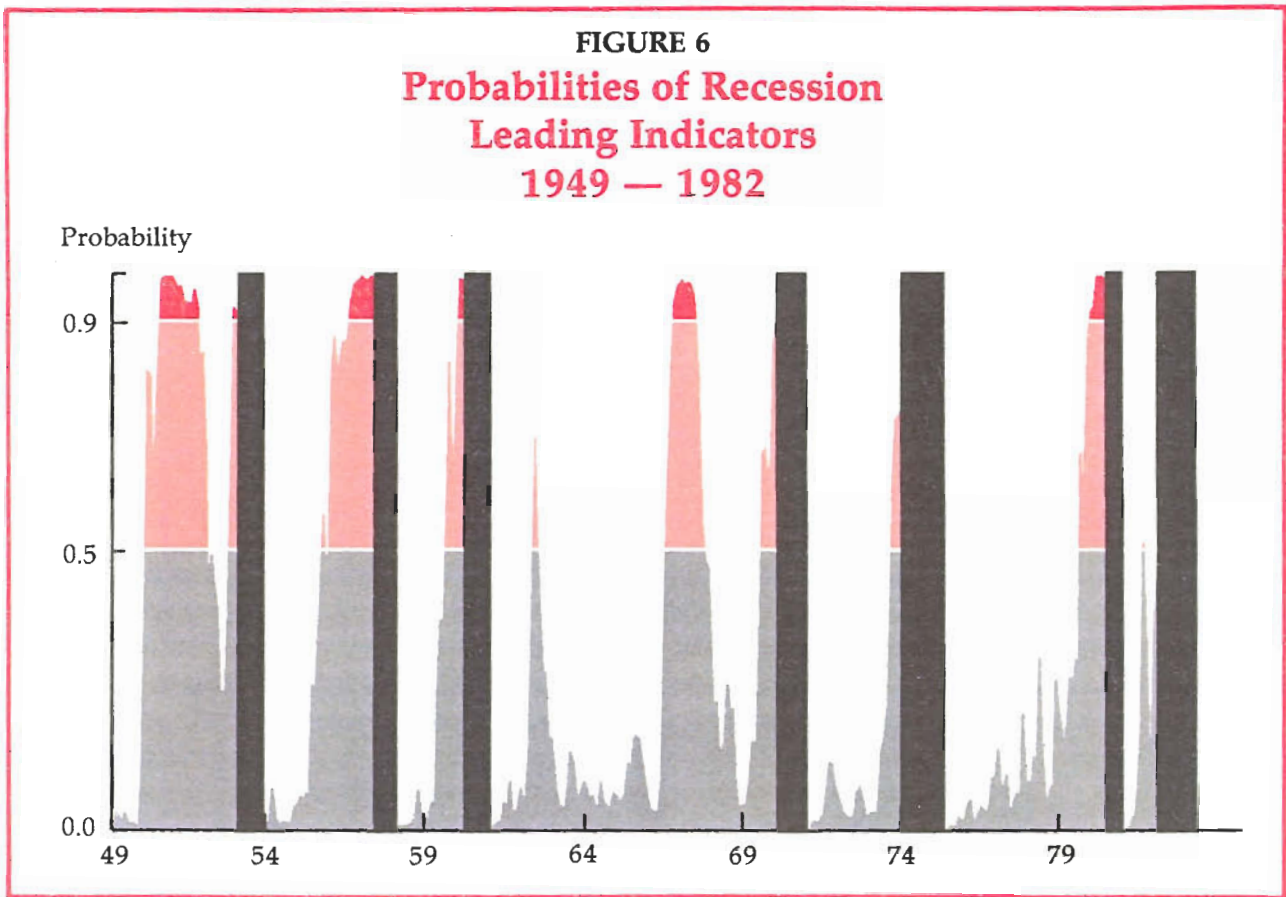
have broad economic implications, analysts usually look at several variables that convey information about the economy. It is in this spirit that the Index of Leading Indicators (ILI), computed by the Department of Commerce, was designed. This index is an average of many variables that seem to lead the business cycle, and it includes stock prices.<sup>10</sup> The hope is that this index averages out the disturbances specific to each statistic and retains the information that each statistic provides about the overall economy.

Does a broad-based approach, as summarized by the Index of Leading Indicators, perform any better by the Neftci standard? The answer seems to be yes. Figure 6 (p. 12) is comparable to Figure 4, except that the ILI replaces the S&P 500 in computing the probabilities of recession. The results are encouraging. Just by comparing Figures 4 and 6, we see that the peaks in the probabilities provided by the ILI are sharper than those provided by stock prices.<sup>11</sup> A closer look at the results reveals that a clearer signal is provided by such an index that combines several indicators. With a .50 critical probability, the ILI anticipated all seven of the postwar recessions and sent only four false signals. Choosing the higher .90 critical value reduced the number of false signals to two, but at the cost of two unanticipated recessions. Finally, the range of lead times seems narrow enough for the ILI to provide a useful signal; lead times ranged from two to 15 months with the .50 critical value and from zero to eight months with the .90 cutoff.

"Sure," someone might say, "but what has this

<sup>10</sup>Gary Gorton, "Forecasting With the Index of Leading Indicators," this *Business Review* (November/December 1982) pp. 15-27, describes the Index of Leading Indicators in more detail and discusses its usefulness.

<sup>11</sup>Stock prices are clearly a superior leading indicator with respect to timeliness. In particular, stock prices are observed instantaneously and are not subject to revision. The Index of Leading Indicators, like several other indicators, is observed with a one-month lag and is subject to several revisions.



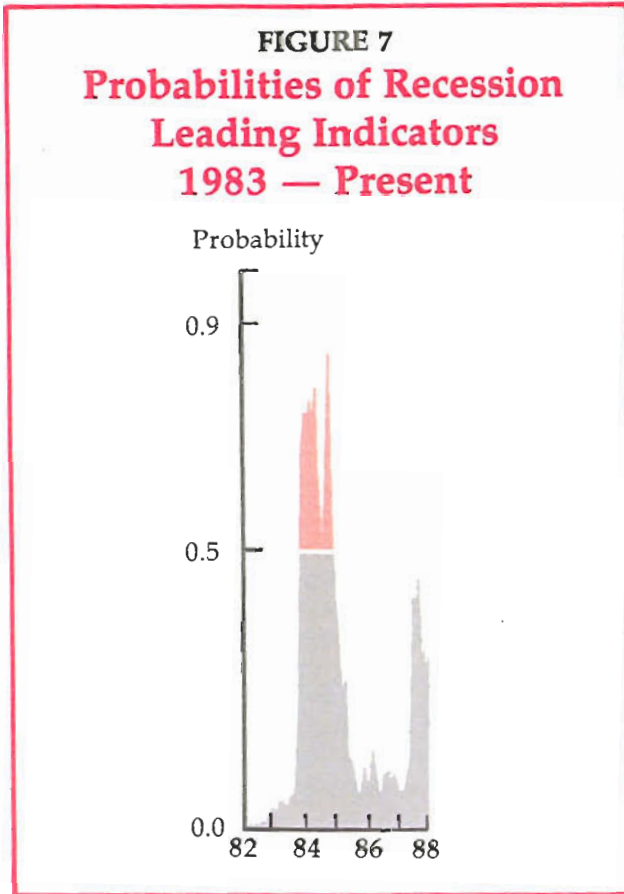
index done for me lately?" Figure 7 shows the probabilities of recession generated by the ILI since December 1982. Like stock prices, the ILI generated some false recession signals early on in the current expansion. But the important difference between the ILI and stock prices is in the recent behavior of both. While the likelihood of a recession is almost certain if we use stock prices as a leading indicator, the likelihood is much smaller using the ILI. Although the probability of a recession based on the ILI increased after October 1987, it remained below even the .50 critical value. The probability based on the ILI peaked at 45 percent in January 1988 and has since fallen to 31 percent in May.<sup>12</sup> In contrast, the probability based on stock prices soared above the .90 critical value and has remained

there. Given the relative performance of these two indicators in the past, the prediction from the Index of Leading Indicators seems more reliable.

#### CONCLUSION

While economists are always looking for clues about the economy's future course, no indicator has proven infallible in its predictions. Sometimes an indicator will fail to signal an upcoming recession. Sometimes it will send false signals. Stock prices have proven to be a particularly unreliable leading indicator in recent years, and the stock market crash of 1987 may prove a telling example. Movements in stock prices do seem to offer some information about the economy's future. But our analysis suggests that a combination of various indicators, such as the Index of Leading Indicators, provides more reliable signals of future economic activity.

<sup>12</sup>Based on preliminary data released June 29, 1988.



## APPENDIX

### Estimating the Probability of Recession

The Neftci approach to estimating the probability of recession from observing a selected indicator, such as a stock price index, builds on two assumptions. The first is that the indicator is always operating under one of two regimes: an upturn regime, during which we are more likely to see increases in the indicator, or a downturn regime, during which we are more likely to see the indicator decline. The second assumption is that the probability of the indicator being in its downturn regime is related to the probability of the economy as a whole going into a recession.

The analyst begins computing the probability of an upcoming recession in the first month of the expansion. At that point the initial probability of a downturn in the indicator (and the economy) is equal to zero. Then, each month, as the analyst gets a new reading on the indicator, he revises his probability that the indicator (and hence the economy) is in its downturn regime by applying the Neftci rule:



$$\Pi_t = \{[\Pi_{t-1} + P(1-\Pi_{t-1})]p_t^d\} / \{[\Pi_{t-1} + P(1-\Pi_{t-1})]p_t^d + (1-\Pi_{t-1})p_t^u(1-P)\}$$

where  $\Pi_t$  = the conditional probability that the indicator is in the downturn regime;

$p_t^u, p_t^d$  = the probability that the observed movement in the indicator came from the upturn regime and downturn regime, respectively;

$P$  = the unconditional probability that a switch from an upturn regime to a downturn regime will occur in the current period.

This rule produces the best estimate of the probability that the indicator has entered its downturn regime. Salih Neftci shows that using this rule minimizes the average delay in signaling a downturn for a given critical value.<sup>a</sup> We use a procedure that is similar to that used by Francis Diebold and Glenn Rudebusch in implementing the Neftci rule.<sup>b</sup>

$p_t^u$  and  $p_t^d$  are the probability densities for the event that an observed change in the indicator variable was drawn from the upturn regime and downturn regime, respectively. Estimation of these densities requires some judgment on the dating of the downturn and upturn regimes for the indicator variable. Because the expansions have lasted about four times as long as recessions, we define a downturn regime as one year (the shortest expansion in the sample) prior to the business cycle peak through three months prior to the business cycle trough. This dating captures the major movements in both the S&P 500 and the Index of Leading Indicators. Alternative dates for the regimes using shorter lead times did not alter the results. The probability densities,  $p_t^u$  and  $p_t^d$ , were assumed to be normally distributed using the mean and standard deviation of the monthly growth rates estimated for each regime. These parameters were estimated from the 1948-82 period. Thus, the Neftci probabilities for the period 1949-82 are analogous to within-sample predictions, and the probabilities for the period 1983-88 are analogous to out-of-sample predictions.

$P$  is the unconditional transition probability, that is, the probability that a switch from a downturn regime to an upturn regime will occur in the current period given that it has not yet occurred.  $P$  is an unconditional probability because it is not based on the movement of the indicator variable. In Neftci's original application, this transition probability was determined by the length of the expansion because of an assumption that expansions "age" and become weaker. This assumption has recently been questioned and was not used in this application so that we might focus more sharply on the proposed indicators.<sup>c</sup> Instead, a constant transition probability of switching from the upturn to the downturn regime was used. This probability was estimated to be .029 by following the procedure outlined in J. Huston McCulloch.<sup>d</sup> Further, the hypothesis that this probability is a constant could not be rejected at usual significance levels.

<sup>a</sup>For derivations of the Neftci rule, see Salih Neftci, "Optimal Prediction of Cyclical Downturns," *Journal of Economic Dynamics and Control* 4 (1982) pp. 225-41, and Francis X. Diebold and Glenn D. Rudebusch, "Scoring the Leading Indicators," Federal Reserve Board *Special Studies Paper No. 206* (1987). Carl J. Palash and Lawrence J. Radecki, "Using Monetary and Financial Variables to Predict Cyclical Downturns," Federal Reserve Bank of New York *Quarterly Review* (Summer 1985) pp. 36-45, use the Neftci rule to evaluate the leading indicator properties of other financial variables.

<sup>b</sup>Francis X. Diebold and Glenn D. Rudebusch, "Scoring the Leading Indicators."

<sup>c</sup>Studies that find that business expansions do not become weaker with age include J. Huston McCulloch, "The Monte Carlo Cycle of Business Activity," *Economic Inquiry* 13 (September 1975) pp. 303-21, Francis X. Diebold and Glenn D. Rudebusch, "Does the Business Cycle Have Duration Memory," Federal Reserve Board *Special Studies Paper No. 223* (1987), and Victor Zarnowitz, "The Regularity of Business Cycles," National Bureau of Economic Research *Working Paper No. 2381* (1987). This issue has not been settled, however; see Frank de Leeuw, "Do Expansions Have Memory?" Bureau of Economic Analysis *Discussion Paper 16* (1987).

<sup>d</sup>J. Huston McCulloch, "The Monte Carlo Cycle of Business Activity."