Credit Risk Modeling of Middle Markets

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Abstract

Proprietary and academic models of credit risk measurement are surveyed and compared. Emphasis is on the special challenges associated with estimating the credit risk exposure of middle market firms. A sample database of middle market obligations is used to contrast estimates across different model specifications.

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1. Introduction

Market risk exposure arises from unexpected security price fluctuations. Using long histories of daily price fluctuations we can distinguish between "typical" and "atypical" trading days in order to assess either expected losses (on a typical day) or unexpected losses (on an atypical day that occurs with a given likelihood). We don't have that luxury in measuring a loan's credit risk exposure. Since loans are not always traded, there is no history of daily price fluctuations available to build a loss distribution. Moreover, credit events such as default or rating downgrades are rare, often non-recurring events. Thus, we do not have enough statistical power to estimate daily measures of credit risk exposure. These data problems are exacerbated for middle market firms that may not be publicly traded. In this paper, we examine and compare both academic and proprietary models that measure credit risk exposure in the face of daunting data and methodological challenges. After a brief summary and critique of each of the most widely used models, we compare their credit risk estimates for a hypothetical portfolio of middle market credit obligations.[1]

Although our focus is on the more modern approaches to credit risk measurement, we begin with a brief survey of traditional models in Section 2. Structural models (such as KMV's Credit Manager and Moody's RiskCalc) that are based on the theoretical foundation of Merton's (1974) option pricing model are described in Section 3. A more

[1] For more comprehensive coverage of each of the models, see Saunders and Allen (2002).

recent strand of the literature covering intensity-based models (such as KPMG's Loan Analysis System and Kamakura's Risk Manager) models default as a point process with a random intensity rate. This literature is surveyed in Section 4. Value at risk models (such as CreditMetrics and Algorithmics Mark-to-Future) most closely parallel the technology used to measure market risk and are analyzed in Section 5. Mortality rate models (such as Credit Risk Plus) are covered in Section 6. The models' assumptions and empirical results are compared in Section 7 and the paper concludes in Section 8.

2. Traditional Approaches to Credit Risk Measurement

Traditional methods focus on estimating the probability of default (PD), rather than on the magnitude of potential losses in the event of default (so-called LGD, loss given default, also known as LIED, loss in the event of default). Moreover, traditional models typically specify "failure" to be bankruptcy filing, default, or liquidation, thereby ignoring consideration of the downgrades and upgrades in credit quality that are measured in mark to market models.[2] We consider three broad categories of traditional models used to estimate PD: (1) expert systems, including artificial neural networks; (2) rating systems; and (3) credit scoring models.

2.1 Expert Systems

Historically, bankers have relied on the 5 C's of expert systems to assess credit quality. They are character (reputation), capital (leverage), capacity (earnings volatility), collateral, and cycle (macroeconomic) conditions. Evaluation of the 5 C's is performed by human experts, who may be inconsistent and subjective in their assessments. Moreover, traditional expert systems specify no weighting scheme that would order the 5

[2] Default mode (DM) models estimate credit losses resulting from default events only, whereas mark to market (MTM) models classify any change in credit quality as a credit event.

C's in terms of their relative importance in forecasting PD. Thus, artificial neural networks have been introduced to evaluate expert systems more objectively and consistently. The neural network is "trained" using historical repayment experience and default data. Structural matches are found that coincide with defaulting firms and then used to determine a weighting scheme to forecast PD. Each time that the neural network evaluates the credit risk of a new loan opportunity, it updates its weighting scheme so that it continually "learns" from experience. Thus, neural networks are flexible, adaptable systems that can incorporate changing conditions into the decision making process.[3]

During "training" the neural network fits a system of weights to each financial variable included in a database consisting of historical repayment/default experiences. However, the network may be "overfit" to a particular database if excessive training has taken place, thereby resulting in poor out-of-sample estimates. Moreover, neural networks are costly to implement and maintain. Because of the large number of possible connections, the neural network can grow prohibitively large rather quickly. Finally, neural networks suffer from a lack of transparency. Since there is no economic interpretation attached to the hidden intermediate steps, the system cannot be checked for

[3] Kim and Scott (1991) use a supervised artificial neural network to predict bankruptcy in a sample of 190 Compustat firms. While the system performs well (87% prediction rate) during the year of bankruptcy, its accuracy declines markedly over time, showing only a 75%, 59%, and 47% prediction accuracy one-year prior, two-years prior, and three-years prior to bankruptcy, respectively. Altman, Marco and Varetto (1994) examine 1,000 Italian industrial firms from 1982-1992 and find that neural networks have about the same level of accuracy as do credit scoring models. Podding (1994), using data on 300 French firms collected over three years, claims that neural networks outperform credit scoring models in bankruptcy prediction. However, he finds that not all artificial neural systems are equal, noting that the multi-layer perception (or back propagation) network is best suited for bankruptcy prediction. Yang, et. al. (1999) uses a sample of oil and gas company debt to show that the back propagation neural network obtained the highest classification accuracy overall, when compared to the probabilistic neural network, and discriminant analysis. However, discriminant analysis outperforms all models of neural networks in minimizing type 2 classification errors, where a type 1 error misclassifies a bad loan as good and a type 2 error misclassifies a good loan as bad.

plausibility and accuracy. Structural errors will not be detected until PD estimates become noticeably inaccurate.

2.2 Rating Systems

External credit ratings provided by firms specializing in credit analysis were first offered in the U.S. by Moody's in 1909. White (2002) identifies 37 credit rating agencies with headquarters outside of the U.S. These firms offer bond investors access to low cost information about the creditworthiness of bond issuers. The usefulness of this information is not limited to bond investors. The Office of the Comptroller of the Currency (OCC) in the U.S. has long required banks to use internal ratings systems to rank the credit quality of loans in their portfolios. However, the rating system has been rather crude, with most loans rated as Pass/Performing and only a minority of loans differentiated according to the four non-performing classifications (listed in order of declining credit quality): other assets especially mentioned (OAEM), substandard, doubtful, and loss. Similarly, the National Association of Insurance Commissioners (NAIC) requires insurance companies to rank their assets using a rating schedule with six classifications corresponding to the following credit ratings: A and above, BBB, BB, B, below B, and default.

Many banks have instituted internal ratings systems in preparation for the BIS New Capital Accords scheduled for implementation in 2005. The architecture of the internal rating system can be one-dimensional, in which an overall rating is assigned to each loan based on the probability of default (PD), or two-dimensional, in which each borrower's PD is assessed separately from the loss severity of the individual loan (LGD). Treacy and Carey (2000) estimate that 60 percent of the financial institutions in their survey had one-dimensional rating systems, although they recommend a two-dimensional system. Moreover, the BIS (2000) found that banks were better able to assess their borrowers' PD than their LGD.[4]

Treacy and Carey (2000) in their survey of the 50 largest US bank holding companies, and the BIS (2000) in their survey of 30 financial institutions across the G-10 countries found considerable diversity in internal ratings models. Although all used similar financial risk factors, there were differences across financial institutions with regard to the relative importance of each of the factors. Treacy and Carey (2000) found that qualitative factors played more of a role in determining the ratings of loans to small and medium-sized firms, with the loan officer chiefly responsible for the ratings, in contrast with loans to large firms in which the credit staff primarily set the ratings using quantitative methods such as credit-scoring models. Typically, ratings were set with a one year time horizon, although loan repayment behavior data were often available for 3-5 years.[5]

2.3 Credit Scoring Models

The most commonly used traditional credit risk measurement methodology is the multiple discriminant credit scoring analysis pioneered by Altman (1968). Mester (1997) documents the widespread use of credit scoring models: 97 percent of banks use credit scoring to approve credit card applications, whereas 70 percent of the banks use credit

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[4] In order to adopt the Internal-Ratings Based Advanced Approach in the new Basel Capital Accord, banks must adopt a risk rating system that assesses the borrower's credit risk exposure (LGD) separately from that of the transaction.

[5] A short time horizon may be appropriate in a mark to market model, in which downgrades of credit quality are considered, whereas a longer time horizon may be necessary for a default mode that considers only the default event. See Hirtle, et. al. (2001).

scoring in their small business lending.[6] There are four methodological forms of multivariate credit scoring models: (1) the linear probability model, (2) the logit model, (3) the probit model, and (4) the multiple discriminant analysis model. All of these models identify financial variables that have statistical explanatory power in differentiating defaulting firms from non-defaulting firms. Once the model's parameters are obtained, loan applicants are assigned a Z-score assessing their classification as good or bad. The Z-score itself can be converted into a PD.

Credit scoring models are relatively inexpensive to implement and do not suffer from the subjectivity and inconsistency of expert systems. Table 1 shows the spread of these models throughout the world, as surveyed by Altman and Narayanan (1997). What is striking is not so much the models' differences across countries of diverse sizes and in various stages of development, but rather their similarities. Most studies found that financial ratios measuring profitability, leverage, and liquidity had the most statistical power in differentiating defaulted from non-defaulted firms.

Shortcomings of credit scoring models are data limitations and the assumption of linearity. Discriminant analysis fits a linear function of explanatory variables to the historical data on default. Moreover, as shown in Table 1, the explanatory variables are predominately limited to balance sheet data. These data are updated infrequently and are determined by accounting procedures that rely on book, rather than market valuation. Finally, there is often limited economic theory as to why a particular financial ratio

[6] However, Mester (1997) reports that only 8% of banks with up to $5 billion in assets used scoring for small business loans. In March 1995, in order to make credit scoring of small business loans available to small banks, Fair, Isaac introduced its Small Business Scoring Service, based on 5 years of data on small business loans collected from 17 banks.

would be useful in forecasting default. In contrast, modern credit risk measurement models are more firmly grounded in financial theory.

INSERT TABLE 1 AROUND HERE

3. Structural Models of Credit Risk Measurement

Modern methods of credit risk measurement can be traced to two alternative branches in the asset pricing literature of academic finance: an *options-theoretic structural approach* pioneered by Merton (1974) and a *reduced form approach* utilizing intensity-based models to estimate stochastic hazard rates, following a literature pioneered by Jarrow and Turnbull (1995), Jarrow, Lando, and Turnbull (1997), and Duffie and Singleton (1998, 1999). These two schools of thought offer differing methodologies to accomplish the central task of all credit risk measurement models – estimation of default probabilities. The structural approach models the economic process of default, whereas reduced form models decompose risky debt prices in order to estimate the random intensity process underlying default.[7]

INSERT FIGURE 1 AROUND HERE

Merton (1974) models equity in a levered firm as a call option on the firm's assets with a strike price equal to the debt repayment amount (denoted B in Figure 1). If at expiration (coinciding to the maturity of the firm's liabilities, assumed to be comprised of pure discount debt instruments) the market value of the firm's assets (denoted A in Figure 1) exceeds the value of its debt, then the firm's shareholders will exercise the option to "repurchase" the company's assets by repaying the debt. However, if the market value of the firm's assets falls below the value of its debt ($A<B$), then the option will expire

[7] The two approaches can be reconciled if asset values follow a random intensity-based process, with shocks that may not be fully observed because of imperfect accounting disclosures. See Duffie and Lando (2001) and Zhou (1997, 2001).

unexercised and the firm's shareholders will default.[8] Thus, the PD until expiration (set equal to the maturity date of the firm's pure discount debt, typically assumed to be one year)[9] is equal to the likelihood that the option will expire out of the money. To determine the PD we value the call option.[10] We use an iterative method to estimate the unobserved variables that determine the value of the equity call option; in particular, A (the market value of assets) and σ_A (the volatility of assets). These values for A and σ_A are combined with the amount of debt liabilities B that have to be repaid at a given credit horizon in order to calculate the firm's Distance to Default (defined to be $\frac{A-B}{\sigma_A}$ or the number of standard deviations between current asset values and the debt repayment amount). The higher the Distance to Default (denoted DD), the lower the PD. To convert the DD into a PD estimate, Merton (1974) assumes that asset values are log normally distributed. Since this distributional assumption is often violated in practice, proprietary structural models use alternative approaches to map the DD into a PD estimate. For example, KMV estimates an empirical PD using historical default experience.[11]

3.1 KMV's Credit Manager

The DD is converted into a PD by determining the likelihood that the firm's assets will traverse the DD during the credit horizon period. KMV uses a historical database of

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[8] Assuming that shareholders are protected by limited liability, there are no costs of default, and that absolute priority rules are strictly observed, then the shareholders' payoff in the default region is zero.

[9] Delianedis and Geske (1998) consider a more complex structure of liabilities.

[10] Using put-call parity, Merton (1974) values risky debt as a put option on the firm's assets giving the shareholders the right, not the obligation, to sell the firm's assets to the bondholders at the value of the debt outstanding. The default region then corresponds to the region in which the shareholders exercise the put option. The model uses equity volatility to estimate asset volatility since both the market value of firm assets and asset volatility are unobservable. See Ronn and Verma (1986).

[11] The Moody's approach uses a neural network to analyze historical experience and current financial data On February 11, 2002, Moody's announced that it was acquiring KMV for more than $200 million in cash.

default rates to determine an empirical estimate of the PD, denoted Expected Default Frequency (EDF). For example, historical evidence shows that firms with DD equal to 4 have an average historical default rate of 1%. Thus, KMV assigns an EDF of 1% to firms with DD equal to 4. If DD>4 (DD<4), then the KMV EDF is less (more) than 1%.[12] EDFs are calibrated on a scale of 0% to 20%.

INSERT FIGURE 2 AROUND HERE

Because KMV EDF scores are obtained from equity prices, they are more sensitive to changing financial circumstances than external credit ratings that rely predominately on accounting data (see critique of credit scoring models in Section 2.3). Figure 2 illustrates this for the case of Enron Corporation. On December 2, 2001, Enron Corporation filed for Chapter 11 bankruptcy protection. At an asset value of $49.53 billion, this was the largest bankruptcy filing in U.S. history. For months prior to the bankruptcy filing, a steadily declining stock price reflected negative information about the firm's financial condition, potential undisclosed conflicts of interest, and dwindling prospects for a merger with Dynegy Inc. However, as Figure 2 shows, the S&P rating stayed constant throughout the period from the end of 1996 until November 28, 2001, when Enron's debt was downgraded to "junk" status just days before the bankruptcy filing. In contrast, KMV EDF scores provided early warning of deteriorating credit quality as early as January 2000, with a marked increase in EDF after January 2001, eleven months prior to the bankruptcy filing.

[12] The complete mapping of KMV EDF scores to DD is proprietary.

3.2 Estimating KMV EDF Scores for Private Firms

Privately held firms do not have a series of equity prices that can be used to estimate asset values. Therefore, KMV's Private Firm Model requires four additional steps that precede the estimation of the firm's DD. They are:

Step 1: Calculate the Earnings Before Interest, Taxes, Depreciation, and Amortization (EBITDA) for the private firm *j* in industry *i*.

Step 2: Calculate the average equity multiple for industry *i* by dividing the industry average market value of equity by the industry average EBITDA.

Step 3: Obtain an estimate of the market value of equity for the private firm *j* by multiplying the industry equity multiple from Step 2 by firm *j*'s EBITDA.

Step 4: Firm *j*'s assets equal the Step 3 estimate of the market value of equity plus the book value of firm *j*'s debt. Once the private firm's asset values can be estimated, then the public firm model can be utilized to evaluate the call option of the firm's equity and obtain the KMV EDF score.

4. Reduced Form or Intensity-Based Models of Credit Risk Measurement

Default occurs after ample early warning in Merton's structural model. That is, default occurs only after a gradual descent (diffusion) in asset values to the default point (equal to the debt level). This process implies that the PD steadily approaches zero as the time to maturity declines, something not observed in empirical term structures of credit spreads. More realistic credit spreads are obtained from reduced form or intensity-based models. That is, whereas structural models view default as the outcome of a gradual process of deterioration in asset values,[13] intensity-based models view default as a

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[13] Exceptions are the jump-diffusion models of Zhou (2001) and Collin-Dufresne and Goldstein (2001) who allow leverage ratios to fluctuate over time.

sudden, unexpected event, thereby generating PD estimates that are more consistent with empirical observations.

In contrast to structural models, intensity-based models do not specify the economic process leading to default. Default is modeled as a point process. Defaults occur randomly with a probability determined by the intensity or "hazard" function. Intensity-based models decompose observed credit spreads on defaultable debt to ascertain both the PD (conditional on there being no default prior to time *t*) and the LGD (which is 1 minus the recovery rate). Thus, intensity-based models are fundamentally empirical, using observable risky debt prices (and credit spreads) in order to ascertain the stochastic jump process governing default.

Because the observed credit spread (defined as the spread over the risk-free rate) can be viewed as a measure of the expected cost of default, we can express it as follows:

$$CS = PD \times LGD \qquad (1)$$

where CS = the credit spread on risky debt = risky debt yield minus the risk-free rate,

PD = the probability of default,

LGD = the loss given default = 1 – the recovery rate.

Differing assumptions are used to disentangle the PD from the LGD in the observed credit spread. Das and Tufano (1996) obtain PD using a deterministic intensity function and assume that LGD is correlated with the default risk-free spot rate. Longstaff and Schwartz (1995) utilize a two factor model that specifies a negative relationship between the stochastic processes determining credit spreads and default-free interest rates. Jarrow and Turnbull (1995) assume that the recovery rate is a known fraction of the bond's face value at maturity date, whereas Duffie and Singleton (1998) assume that the recovery rate is a known fraction of the bond's value just prior to default. In Duffie and Singleton (1999), both PD and LGD are modeled as a function of economic state variables. Madan and Unal (1998) and Unal et. al. (2001) model the differential recovery rates on junior and senior debt. Kamakura, in its proprietary model which is based on Jarrow (2001), uses equity as well as debt prices in order to disentangle the PD from the LGD.

In the intensity-based approach, default probability is modeled as a Poisson process with intensity h such that the probability of default over the next short time period, Δ, is approximately Δh and the expected time to default is $1/h$; therefore, in continuous time, the probability of survival without default for t years is:

$$1\text{-PD}(t) = e^{-ht} \qquad (2)$$

Thus, if an A rated firm has an h=.001, it is expected to default once in 1,000 years; using equation (2) to compute the probability of survival over the next year we obtain 99.9 percent. Thus, the firm's PD over a one year horizon is 0.1 percent. Alternatively, if a B rated firm has an h=.05, it is expected to default once in 20 years and substituting into equation (2), we find that the probability of survival, 1 – PD(t), over the next year is 95 percent and the PD is 5 percent.[14] If a portfolio consists of 1,000 loans to A rated firms and 100 loans to B rated firms, then there are 6 defaults expected per year.[15] A hazard rate (or alternatively, the arrival rate of default at time t) can be defined as the arrival time of default, i.e., $-p'(t)/p(t)$ where $p(t)$ is the probability of survival to time t and $p'(t)$ is the first derivative of the survival probability function (assumed to be differentiable with

[14] Using equation (2) to calculate the PD over a five year time horizon, we obtain a PD of 0.5 percent for the A rated firm and 22.12 percent for the B rated firm.

[15] The intensity of the sum of independent Poisson processes is just the sum of the individual processes' intensities; therefore, the portfolio's total intensity is: 1,000*.001 + 100*.05 = 6 defaults per year.

respect to t). Since the probability of survival depends on the intensity h, the terms hazard rate and intensity are often used interchangeably.[16]

Since intensity-based models use observed risky debt prices, they are better able to reflect complex term structures of default than are structural models. However, although the bond market is several times the size of US equity markets,[17] it is not nearly as transparent.[18] One reason is that less than 2 percent of the volume of corporate bond trading occurs on the NYSE or AMEX exchanges. The rest of the trades are conducted over the counter by bond dealers. Saunders, Srinivasan, and Walter (2002) show that this inter-dealer market is not very competitive. It is characterized by large spreads and infrequent trades. Pricing data are often inaccurate, consisting of matrix prices that use simplistic algorithms to price infrequently traded bonds. Even the commercially available pricing services are often unreliable. Hancock and Kwast (2001) find significant discrepancies between commercial bond pricing services, Bloomberg and Interactive Data Corporation, in all but the most liquid bond issues. Bohn (1999) finds that there is more noise in senior issues than in subordinated debt prices. Corporate bond price performance is particularly erratic for maturities of less than one year. The sparsity of trading makes it difficult to obtain anything more frequent than monthly pricing data; see Warga (1999). A study by Schwartz (1998) indicates that even for monthly bond data, the number of outliers (measured relative to similar debt issues) is significant. One can attribute these outliers to the illiquidity in the market.

[16] Indeed, with constant intensity, the two terms are synonymous.

[17] In 2000, there was a total of \$17.7 trillion in domestic (traded and untraded) debt outstanding; see Basak and Shapiro (2000).

[18] As of 1998, about \$350 billion of bonds traded each day in the US as compared to \$50 billion of stocks that are exchanged; see Bohn (1999).

The considerable noise in bond prices, as well as investors' preferences for liquidity, suggest that there is a liquidity premium built into bond spreads. Thus, if risky bond yields are decomposed into the risk-free rate plus the credit spread only, the estimate of credit risk exposure will be biased upward. The proprietary model Kamakura Risk Manager explicitly adjusts for liquidity effects. However, noise from embedded options and other structural anomalies in the default risk-free market further distorts risky debt prices, thereby impacting the results of intensity-based models. Other proprietary models control for some of these biases in credit spreads. For example, KPMG's Loan Analysis System adjusts for embedded options and Kamakura includes a stock market bubble factor [see Saunders and Allen (2002)].

5. Proprietary VaR Models of Credit Risk Measurement

Once the default probability for each asset is computed (using either the structural or intensity-based approach),[19] each loan in the portfolio can be valued (using either analytical solutions or Monte Carlo simulation) so as to derive a probability distribution of portfolio values. A loss distribution can then be calculated permitting the computation of Value at Risk (VaR) measures of unexpected losses by specifying the minimum losses that will be exceeded with a certain probability. That is, a 99 percentile VaR of, say, \$100 million denotes that there is a 99 percent probability that unexpected losses will be less than \$100 million and only a one percent probability that unexpected losses will exceed \$100 million. We now turn to two proprietary VaR models for credit risk measurement: CreditMetrics and Algorithmics Mark-to-Future.

[19] Of course, for mark-to-market models, the entire matrix of credit transition probabilities must be computed in addition to the default probability for default mode models.

5.1 CreditMetrics

CreditMetrics models default probabilities using the historical default experience of comparable borrowing firms. That is, the CreditMetrics model is built around a credit migration matrix that measures the probability that the credit rating of any given debt security will change over the course of the credit horizon (usually one year).[20] The credit migration matrix considers the entire range of credit events, including upgrades and downgrades as well as actual default. Thus, CreditMetrics is a mark-to-market (MTM), rather than default mode (DM) model. Since loan prices and volatilities are generally unobservable, CreditMetrics uses migration probabilities to estimate each loan's loss distribution. We describe the model for the individual loan case using transition matrices based on external credit ratings.

CreditMetrics evaluates each loan's cash flows under eight possible credit migration assumptions, corresponding to each of eight credit ratings: AAA, AA, A, BBB, BB, B, CCC, and default.[21] For example, suppose that a loan is initially rated BBB. The loan's value over the upcoming year is calculated under different possible scenarios over the succeeding year, e.g., the rating improves to AAA, AA, A, or deteriorates in credit quality or possibly defaults, as well as under the most likely scenario that the loan's credit rating remains the unchanged. Historical data on publicly traded bonds are used to estimate the probability of each of these credit migration scenarios.[22] Putting together the loan valuations under each possible credit migration scenario and their likelihood of

[20] CreditMetrics transition matrices can be obtained using historical data based on ratings agency migration probabilities or KMV EDFs; see Bucay and Rosen (1999).
[21] If the +/- modifiers ("notches") are utilized, there are 22 different rating categories, see Bahar and Nagpal (2000).
[22] Kreinin and Sidelnikova (2001) desribe algorithms for constructing transition matrices.

occurrence, we obtain the distribution of the loan's value. At this point, standard VaR technology may be utilized.

Consider the following example of a five-year fixed-rate BBB rated loan of $100 million made at 6 percent annual (fixed) interest.[23] Based on historical data on publicly-traded bonds (or preferably loans), the probability that a BBB borrower will stay at BBB over the next year is estimated at 86.93 percent. There is also some probability that the borrower will be upgraded (e.g., to A) or will be downgraded (e.g., to CCC or even to default, D). Indeed, eight transitions are possible for the borrower during the next year. Table 2 shows the estimated probabilities of these credit migration transitions. The migration process is modeled as a finite Markov chain, which assumes that the credit rating changes from one rating to another with a certain constant probability at each time interval.

INSERT TABLE 2 AROUND HERE

The effect of rating upgrades and downgrades is to impact the required credit risk spreads or premiums on the loan's remaining cash flows, and, thus, the implied market (or present) value of the loan. If a loan is downgraded, the required credit spread premium should rise (remember that the contractual loan rate in our example is assumed fixed at 6 percent) so that the present value of the loan should fall. A credit rating upgrade has the opposite effect. Technically, because we are revaluing the five-year, $100 million, 6 percent loan at the end of the first year (the end of the credit event horizon), after a "credit-event" has occurred during that year, then (measured in millions of dollars):[24]

[23] This example is based on the one used in Gupton, et. al., CreditMetrics-Technical Document (1997).
[24] Technically, from a valuation perspective the credit-event occurs (by assumption) at the very end of the first year. Currently, CreditMetrics is expanding to allow the credit event "window" to be as short as 3 months or as long as 5 years.

$$P = 6 + \frac{6}{(1+r_1+s_1)} + \frac{6}{(1+r_2+s_2)^2} + \frac{6}{(1+r_3+s_3)^3} + \frac{106}{(1+r_4+s_4)^4} \qquad (3)$$

where r_i are the risk-free rates (the forward risk-free rates) on zero-coupon US Treasury bonds *expected* to exist one year into the future. Further, the series of s_i is the annual credit spread on (zero coupon) loans of a particular rating class of one-year, two-year, three-year, and four-year maturities (derived from observed spreads in the corporate bond market over Treasuries). In the above example, the first year's coupon or interest payment of $6 million (to be received on the valuation date at the end of the first year) is undiscounted and can be regarded as equivalent to accrued interest earned on a bond or a loan.

In CreditMetrics, interest rates are assumed to be deterministic.[25] Thus, the risk-free rates, r_i, are obtained by decomposing the current spot yield curve in order to obtain the one-year forward zero curve and then adding fixed credit spreads to the forward zero coupon Treasury yield curve. That is, the risk-free spot yield curve is first derived using U.S. Treasury yields. Pure discount yields for all maturities can be obtained using yields on coupon-bearing U.S. Treasury securities. Once the risk-free spot yield curve is obtained, the forward yield curve can be derived using the expectations hypothesis. The values of r_i are read off this forward yield curve. For example, if today's risk-free spot rate were 3.01 percent p.a. for 1 year maturity pure discount U.S. Treasury securities and 3.25 percent for 2 year maturities, then we can calculate r_1, the forward risk-free rate

[25] The assumption that interest rates are deterministic is particularly unsatisfying for credit derivatives because fluctuations in risk-free rates may cause the counterparty to default as the derivative moves in or out of the money. Thus, the portfolio VAR, as well as VAR for credit derivatives, (see for example CIBC's CreditVaR II) assume a stochastic interest rate process that allows the entire risk-free term structure to shift over time. See Crouhy, et. al. (2000).

expected one year from now on 1-year maturity U.S. Treasury securities using the expectations hypothesis as follows:[26]

$$(1 + .0325)^2 = (1 + .0301)(1+r_1) \qquad (4)$$

Thereby solving for r_1 = 3.5 percent p.a. This procedure can be repeated for the 2-year maturity risk-free rate expected in one year r_2, and continuing for as many rates as required to value the multiyear loan (until r_4 for the five year loan in this example).

CreditMetrics obtains fixed credit spreads s_i for different credit ratings from commercial firms such as Bridge Information Systems. For example, if during the year a credit event occurred so that the five year loan in our example was upgraded to an A rating (from a BBB), then the value of the credit spread for an A rated bond would be added to the risk-free forward rate for each maturity; suppose that the credit spread s_1 was 22 basis points in the first year. Evaluating the first coupon payment after the credit horizon is reached in one year, the risk-free forward rate of 3.5 percent p.a. would be added to the one year credit spread for A rated bonds of 22 basis points to obtain a risk-adjusted rate of 3.72 percent p.a. Using different credit spreads s_i for each loan payment date and the forward rates r_i we can solve for the end of year value of a \$100 million five year 6 percent coupon loan that is upgraded from a BBB rating to an A rating within the next year such that:

$$P = 6 + \frac{6}{(1.0372)} + \frac{6}{(1.0432)^2} + \frac{6}{(1.0493)^3} + \frac{106}{(1.0532)^4} = \$108.66 \qquad (5)$$

[26] In this simplified example, we annualize semi-annual rates corresponding to coupon payment dates on U.S. Treasury securities. For a more precise explanation of this methodology, see Appendix 6.1 in Saunders and Allen (2002).

INSERT TABLE 3 AROUND HERE

Table 3 shows the loan's value at the credit horizon for all possible credit migrations. To obtain the distribution of loan values, we discount each of the loan's cash flows at the appropriate risk-adjusted forward rate. As shown in equation (5), if the loan's credit quality is upgraded from BBB to A, then the loan's value will increase to \$108.66 million. However, Table 3 shows that if the loan's credit quality deteriorates to CCC, then the loan's value will fall to \$83.64 million. Moreover, if the loan defaults, its value will fall to its recovery value, shown in Table 3 to be \$51.13 million.[27]

INSERT FIGURE 3 AROUND HERE

The distribution of loan values on the one year credit horizon date can be drawn using the transition probabilities in Table 2 and the loan valuations in Table 3. Figure 3 shows that the distribution of loan values is not normal. CreditMetrics can estimate a VaR measure based on the actual distribution as well as on an approximation using a normal distribution of loan values.[28] The mean of the value distribution shown in Figure 3 is \$107.09 million. If the loan had retained its BBB rating, then the loan's value would have been \$107.55 million at the end of the credit horizon. Thus, the expected losses on this loan are \$460,000 (=\$107.55 minus \$107.09 million). However, unexpected losses (to be covered by economic capital) are determined by the probable losses over and above expected losses. We measure unexpected losses using the credit VaR to calculate the minimum possible losses that will occur at a certain confidence level. Figure 3 shows that the 1 percent loan cut-off value is \$100.12 million; that is, there is only a 1 percent chance that loan values will be lower than \$100.12 million. Thus, the 99 percentile VaR

[27] CreditMetrics models recovery values as beta distributed, although the simple model assumes a deterministic recovery value set equal to the mean of the distribution.
[28] For a discussion of the calculation of VaR using the actual distribution, see Saunders and Allen (2002).

for unexpected losses total $6.97 million ($107.09 minus $100.12).[29] CreditMetrics estimates that the loan's unexpected losses would exceed $6.97 million in only one year out of a 100.[30]

This example illustrates the CreditMetrics approach for individual loans. However, if asset returns are not perfectly correlated, then the credit risk of the portfolio may be less than the sum of individual asset risks because of the benefits of diversification. CreditMetrics solves for correlations by first regressing equity returns on industry indices. Then the correlation between any pair of equity returns is calculated using the correlations across the industry indices. Once we obtain equity correlations, we can solve for joint migration probabilities to estimate the likelihood that the joint credit quality of the loans in the portfolio will be either upgraded or downgraded. Portfolio loss distributions are then obtained by calculating individual asset values for each possible joint migration scenario.

5.2 Algorithmics' Mark-to-Future

Although the portfolio version of CreditMetrics incorporates stochastic interest rates, we have seen that the basic CreditMetrics model focuses on credit risk measurement while incorporating a rather static view of interest rate risk. Algorithmics Mark-to-Future (MtF) attempts to link market risk, credit risk and liquidity risk in a scenario-based framework [see Iscoe, et. al. (1999)]. That is, whereas the fundamental risk driver in CreditMetrics is the credit migration matrix, Algorithmics simulates

[29] We obtained the 1 percent maximum loan value assuming that loan values were normally distributed with a standard deviation of loan value of $2.99 million; thus, the 1 percent VaR is $6.97 million (equal to 2.33 standard deviations, or 2.33 x $2.99 million).

[30] If the actual distribution is used rather than assuming that the loan's value is normally distributed, the 1 percent VaR in this example is $14.8 million.

portfolio valuations using hundreds of different risk factors.[31] Table 4 summarizes these risk factors. Scenarios are defined by states of the world over time and are comprised of both market factors (interest rates, foreign exchange rates, equity prices and commodity prices) as well as credit drivers (systemic and macroeconomic factors). As its name suggests, the MtF model is a mark-to-market model. Each asset in the portfolio is revalued as scenario-driven credit or market events occur, thereby causing credit spreads to fluctuate over time. MtF differs from other credit risk measurement models in that it views market risk and credit risk as inseparable.[32] Stress tests show that credit risk measures are quite sensitive to market risk factors.[33] Indeed, it is the systemic market risk parameters that drive creditworthiness in MtF.

INSERT TABLE 4 AROUND HERE

Dembo, et. al. (2000) offer an example of this simulation analysis using a BB rated swap obligation. The firm's credit risk is estimated using a Merton options-theoretic model of default; that is, MtF defines a creditworthiness index (CWI) that specifies the distance to default as the distance between the value of the firm's assets and a (nonconstant) default boundary.[34] Figure 4 shows the scenario simulation of the CWI, illustrating two possible scenarios of firm asset values: (Scenario 1) the firm defaults in

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[31] However, unconditional credit migration matrices and non-stochastic yield curves (similar to those used in CreditMetrics) are fundamental inputs into the MtF model. Nevertheless, Algorithmics permits scenario-driven shifts in these static migration probabilities and yield curves.

[32] Finger (2000a) proposes an extension of CreditMetrics that would incorporate the correlation between market risk factors and credit exposure size. This is particularly relevant for the measurement of counterparty credit risk on derivatives instruments because the derivative can move in or out of the money as market factors fluctuate. In June 1999, the Counterparty Risk Management Policy Group called for the development of stress tests to estimate "wrong-way credit exposure" such as experienced by US banks during the Asian currency crises; i.e., credit exposure to Asian counterparties increased just as the foreign currency declines caused FX losses on derivatives positions.

[33] Fraser (2000) finds that a doubling of the spread between Baa rated bonds over US Treasury securities from 150 basis points to 300 basis points increases the 99 percent VAR measure from 1.77 percent to 3.25 percent for a Eurobond portfolio.

[34] Although the default boundary is not observable, it can be computed from the (unconditional) default probability term structure observed for BB rated firms.

year 3, and (Scenario 2) the firm remains solvent for the next 10 years. The default date under each scenario is represented by the point at which the firm's asset value first hits the default boundary.[35] MtF assumes that the CWI follows a geometric Brownian motion standardized to have a mean of zero and a variance of one. The basic building block of the CWI is the unconditional cumulative default probabilities for typical BB rated firms obtained using the Merton model (as discussed in Section 3). Using the unconditional default probabilities as a foundation, a conditional cumulative default probability distribution is generated for each scenario. That is, the sensitivity of the default probability to scenario risk factors is estimated for each scenario of systematic market risk factors – the risk driver. MtF estimates the historical sensitivity to the risk driver using a multifactor model that incorporates both systemic and idiosyncratic company specific factors. A return distribution can then be derived using the full range of possible scenarios and the conditional default probabilities.

INSERT FIGURE 4 AROUND HERE

Integrating different risk drivers is critical to obtaining more accurate VaR estimates for credit risk. For example, the Russian debt default in August 1998 was foreshadowed by the devaluation of the ruble. Incorporating data on foreign exchange rates as well as interest rates (during the first few days of August 1998, yields on US Treasury bonds reached historic lows) could have forecast the increased risk of default almost a week before the Russian government announced its debt restructuring plan. Dembo, et. al. (2000) show that if a "Russian scenario" were used in January 1999 during a similar crisis for Brazilian debt, the 95 percentile Credit VaR estimate would have

[35] Default is assumed to be an absorbing state, so Figure 4 shows that the curve representing the firm's asset value in scenario 1 coincides with the default boundary for all periods after year 3.

forecast a 57 percent decline in portfolio value over the two week crisis period. Thus, integrating the market risk drivers into a model of credit risk measurement can improve the quality of the VaR estimates, particularly during crisis periods. Therefore, dichotomizing credit risk and market risk undermines the accuracy of all risk measurement models.

The primary disadvantage of the scenario-based MtF is its computational complexity. The cost of implementing the model is directly related to the number of simulations that must be performed in order to estimate the portfolio's loss distribution. To reduce that cost, MtF separates the scenario analysis stage from the exposure analysis stage. Therefore, MtF loss estimates (denoted MtF Cubes) are calculated for each instrument independent of the actual counterparty exposures. Individual exposure loss distributions are then computed by combining the MtF Cubes across all scenarios using conditional obligor default/migration probabilities (assuming exposures are not scenario-dependent). Aggregation across the assets in the portfolio is simplified because once conditioned on a particular scenario, obligor credit events are independent.

6 Mortality Rate Models

Credit Risk Plus, a proprietary model developed by Credit Suisse Financial Products (CSFP) stands in direct contrast to VaR models in its objectives and its theoretical foundations. CreditMetrics and Algorithmics MtF seek to estimate the full VaR of a loan or loan portfolio by viewing rating upgrades and downgrades and the associated effects of spread changes in the discount rate as part of the VaR exposure of a loan. Credit Risk Plus views spread risk as part of market risk rather than credit risk. As a result, in any period, only two states of the world are considered - default and non-default - and the focus is on measuring expected and unexpected losses rather than expected value and unexpected changes in value (or VaR). Thus, Credit Risk Plus is a default mode (DM) model.

The second major difference is that, in CreditMetrics and Algorithmics MtF, the default probability in any year is discrete (as are the upgrade/ downgrade probabilities). In Credit Risk Plus, default is modeled as a continuous variable with a probability distribution. Thus, Credit Risk Plus is based on the theoretical underpinnings of intensity-based models (see discussion in Section 4). An analogy from property fire insurance is relevant. When a whole portfolio of homes is insured, there is a small probability that each house will burn down, and (in general) the probability that each house will burn down can be viewed as an independent event.[36] Similarly, many types of loans, such as mortgages and small business loans, can be thought of in the same way, with respect to their default risk. Thus, under Credit Risk Plus, each individual loan is regarded as having a small probability of default, and each loan's probability of default is independent of the default on other loans.[37] This assumption makes the distribution of the default probabilities of a loan portfolio resemble a Poisson distribution.[38]

Default rate uncertainty is only one type of uncertainty modeled in Credit Risk Plus. A second type of uncertainty surrounds the size or severity of the losses themselves. Borrowing again from the fire insurance analogy, when a house "catches fire," the degree

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[36] That is, there is a constant probability that any given house will burn down (or equivalently, a loan will default) within a predetermined time period. Credit Risk Plus has the flexibility to calculate default probabilities over a constant time horizon (say, one year) or over a hold-to-maturity horizon.

[37] Moreover, the probability of default is assumed to be constant over time. This is strictly true for only the simplest of the models in Credit Risk Plus. A more sophisticated version ties loan default probabilities to the systematically varying mean default rate of the "economy" or "sector" of interest.

[38] The continuous time extension of Credit Risk Plus is the intensity-based model of Duffie and Singleton (1998) which stipulates that over a given small time internal, the probability of default is independent across loans and proportional to a fixed default intensity function. The evolution of this intensity process follows a Poisson distribution as assumed in the discrete version Credit Risk Plus. See Finger (2000b).

of loss severity can vary from the loss of a roof to the complete destruction of the house. In Credit Risk Plus, the fact that severity rates are uncertain is acknowledged, but because of the difficulty of measuring severity on an individual loan-by-loan basis, loss severities or loan exposures are rounded and banded (for example, into discrete $20,000 severity or loss bands). The smaller the bands, the less the degree of inaccuracy that is built into the model as a result of banding.

The two degrees of uncertainty - the frequency of defaults and the severity of losses - produce a distribution of losses for each exposure band. Summing (or accumulating) these losses across exposure bands produces a distribution of losses for the portfolio of loans. The great advantage of the Credit Risk Plus model is its parsimonious data requirements. The key data inputs are mean loss rates and loss severities, for various bands in the loan portfolio, both of which are potentially amenable to collection, either internally or externally.

The assumption of a default rate with a Poisson distribution implies that the mean default rate of a portfolio of loans should equal its variance. However, this does not hold in general, especially for lower quality credits. For B-rated bonds, Carty and Lieberman (1996) found the mean default rate was 7.62 percent and the square root of the mean was 2.76 percent, but the observed standard deviation was 5.1 percent, or almost twice as large as the square root of the mean. Thus, the Poisson distribution appears to underestimate the actual probability of default.

What extra degree of uncertainty might explain the higher variance (fatter tails) in observed loss distributions? The additional uncertainty modeled by Credit Risk Plus is that the mean default rate itself can vary over time (or over the business cycle). For example, in economic expansions, the mean default rate will be low; in economic contractions, it may rise significantly.[39] In the extended Credit Risk Plus model, there are three types of uncertainty: (1) the uncertainty of the default rate around any given mean default rate, (2) the uncertainty about the severity of loss, and (3) the uncertainty about the mean default rate itself [modeled as a gamma distribution by CSFB (1997)]. Credit Risk Plus derives a closed form solution for the loss distribution by assuming that these types of uncertainty are all independent.[40]

Appropriately modeled, a loss distribution can be generated along with expected losses and unexpected losses that exhibit observable fatter tails. The latter can then be used to calculate unexpected losses due to credit risk exposure. Note that this credit risk measure is not the same as the VaR measured under MTM models like CreditMetrics. Since Credit Risk Plus is a DM model, it does not consider non-default migrations in credit quality. Thus, the Credit Risk Plus credit risk measure is closer to a loss-of-earnings or book-value capital measure than a full market value of economic capital measure.

7 Comparison of Credit Risk Measurement Models

There are many dimensions along which to compare the modern models of credit risk measurement surveyed in Sections 3-6. Table 5 focuses on ten key dimensions of the following four models: (1) options pricing models such as KMV and Moody's (discussed in Section 3); (2) reduced form models such as KPMG and Kamakura Corporation (Section 4); (3) CreditMetrics (Section 5); and (4) Credit Risk Plus (Section

[39] The most speculative risk classifications' default probabilities are most sensitive to these shifts in macroeconomic conditions; See Crouhy, et. al. (2000).
[40] The assumption of independence may be violated if the volatility in mean default rates reflects the correlation of default events through interrlated macroeconomic factors.

6), Analytically and empirically, these models are not as different as they may first appear. Indeed, similar arguments stressing the structural similarities have been made by Gordy (2000), Koyluoglu and Hickman (1998), and Crouhy, et. al. (2000), using different model anatomies.

INSERT TABLE 5 AROUND HERE

Table 5 compares the different models. CreditMetrics and reduced form models are mark-to-market (MTM) models, in contrast with the default mode (DM) models of Credit Risk Plus and Merton options pricing models. At first sight, the key risk drivers of these models appear to be quite different. CreditMetrics, KMV, and Moody's have their analytic foundations in a Merton-type model; a firm's asset values and the volatility of asset values are the key drivers of default risk. In Credit Risk Plus, the risk driver is the mean level of default risk and its volatility; in reduced form models, it is the credit spreads obtained from risky debt yields.

Yet, if couched in terms of multifactor models, all four models can be viewed as having similar roots. Specifically, the variability of a firm's asset returns in CreditMetrics (as in KMV and Moody's) is modeled as being directly linked to the variability in a firm's stock returns. To the extent that multifactor asset pricing models drive all risky security prices, the results of reduced form models are driven by the same risk factors. In turn, in calculating correlations among firms' asset returns, the stocks of individual firms are viewed as being driven by a set of systematic risk factors (industry factors, country factors, and so on) and unsystematic risk factors. The systematic risk factors, along with correlations among systematic risk factors (and their weighted importance), drive the asset returns of individual firms and the default correlations among firms.[41] In particular, a set of systematic "country-wide" macro factors and unsystematic macroeconomic shocks drives default risk and the correlations of default risks among borrowers.[42] The key risk driver in Credit Risk Plus is the variable mean default rate in the economy. This mean default rate can also be viewed as being linked systematically to the "state of the macro economy;" when the macro economy deteriorates, the mean default rate is likely to rise, as are default losses. An improvement in economic conditions has the opposite effect. Thus, the risk drivers and correlations in all four models can be viewed as being linked, to some degree, to a set of macroeconomic and systematic risk factors that describe the evolution of economy-wide conditions.

Although the fundamental risk drivers may be similar, comparison across the models must be based on their relative performance. In February 2000, the International Swaps and Derivatives Association (ISDA) and the Institute of International Finance (IIF) published the results of an ambitious joint project to test credit risk measurement models in 25 commercial banks from 10 countries with varying sizes and specialties. In the report, hereinafter referred to as IIF/ISDA, four models (CreditMetrics, CreditPortfolio View, Credit Risk Plus, and KMV) are compared to internal models for standardized portfolios (without option elements) created to replicate four markets: corporate bonds and loans, middle markets, mortgages[43], and retail credits. The most important conclusions of the study are:[44]

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[41] For example, Froot and Stein (1998) examine a two factor model in a RAROC framework.
[42] CreditPortfolio View is a VaR-type model that explicitly models this macroeconomic risk factor by estimating transition matrices that are conditional of economic conditions. See Wilson (1997a,b).
[43] Because mortgages are more sensitive to specific local economic conditions than are other debt instruments, the study's participants could not agree on a meaningful common base case portfolio to be used to compare publicly available credit risk models for the mortgage portfolio. Differences of opinion among the bank participants in the survey dealt with issues such as: the interaction between interest rate

- Models yield directionally consistent outputs when given similar inputs. In some model types, the outputs are almost identical.
- Where there are discrepancies, they reflected differences in: model inputs, preprocessing (i.e., packaging transactions into a readable format), valuation, errors in model usage during testing, and misunderstandings by participants regarding application of standardized parameters.
- Substantive differences in results across models can be attributed to different approaches to valuations and correlation calculation methods. Model outputs are significantly affected by: valuation methods, changes in spreads, discount rates, and the treatment of cash flows.
- The most significant drivers of portfolio risk are credit quality (tested by subjecting portfolios to specified downgrade scenarios), asset correlation, and loss given default (LGD).
- Internal models focus on scoring methodologies and aggregate measures of default, not default probabilities and credit migrations.

We focus on the IIF/ISDA results for the middle market portfolio.

7.1 IIF/ISDA Results on the Middle Portfolio

The results for the middle markets, mortgages, and retail credit showed a range of credit risk estimates.[45] Moreover, proprietary internal models were used most often by the banks participating in the survey for the middle markets portfolio as compared to any

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and credit risk, the role of collateral and mortgage insurance, portfolio "seasoning" or diversification of tenor, and cross-country differences in securitization. See Jarrow and Turnbull (2000) for a discussion of the lack of separability of market risk and credit risk.

[44] This is adapted from IIF/ISDA (2000), pp. 2-3.

[45] In particular, the credit risk estimates for portfolios of large corporate obligations were more consistent across the models. See IIF/ISDA (2000).

other portfolio. These internal models typically focused on default only. The standardized test portfolio for middle markets was a composite of 2,500 real-world exposures, averaging £894,000 per obligor. Five percent of the total exposures came from one obligor and the next five obligors represented an additional 6 percent of the exposures. The average maturity was 2.5 years, with approximately 35 percent maturing within one year. To replicate portfolio concentration, all exposures were assumed to be in the United Kingdom. Investment grade loans represented 69 percent of the portfolio, with the remaining 31 percent below investment grade.

INSERT TABLE 6 AROUND HERE

As shown in Table 6, there were significant differences in the risk measures estimated by the different models for the middle market portfolio. KMV generated VaR estimates of 3.0 percent in contrast to the CreditMetrics estimates of 1.6 percent. Part of this discrepancy may be the result of differences in maturity assumptions; the KMV users rounded maturities of less than one year to one year, whereas CreditMetrics users left all maturities as specified in the portfolio. Moreover, the middle market portfolio was deliberately allowed to retain some flexibility for individual interpretation.[46] Thus, different assumptions about product types (e.g., receivables, letters of credit and commitments) may account for some of the variability in outputs shown in Table 6.

Examining the source of this variability, IIF/ISDA found considerable divergence in credit risk estimates for DM versus MTM models. In particular, migration risk increases estimates of unexpected losses (UL) and VaR estimates for all models in Table

[46] Table 6 understates the degree of variability in credit risk estimates, particularly for internal models. When banks undertook their own practice runs using their own parameter settings, the range of outputs increased dramatically. For example, estimates of 1 percent VAR ranged from 3.1 percent to 13.0 percent for the middle market portfolio using the banks' parameterization of their proprietary internal models.

6.[47] That is, KMV model estimates of UL increased from a range of 0.6-0.8 to a range of 1.1-1.6 (for the four banks that provided estimates). Using CreditMetrics, the impact of a shift from DM to MTM was less dramatic: UL increased from 0.4-0.5 under DM to 0.5 under MTM. Moreover, the choice of DM or MTM had no impact at all on expected losses.

The estimates of VaR showed considerable diversity across different iterations of estimation. In particular, the MTM estimates of VaR using the KMV model ranged from 1.6 to 5.3 percent. The range for the DM model was less: 2.4 - 3.0. Finally, CreditMetrics and internal models yielded more consistent estimates of VaR in the 1.5 – 1.8 percent range. Thus, for the middle market portfolio in particular, model assumptions have considerable impact on credit risk estimates.

8 Summary and Conclusion

Although we have made much progress in credit risk measurement, there is still much we do not know. Middle market obligors are particularly affected by inaccuracies and inconsistencies in credit risk measurement models. Different models produce markedly different credit risk assessments, thereby undermining the credibility of all estimates and potentially restricting middle market firms' access to credit. To alleviate this, we must produce more accurate, long-term time series databases on the credit performance of middle market firms. Along with continual modeling improvements, the state of the credit risk measurement art will most certainly improve.

[47]The one exception is for the internal models. However, the low amount of credit risk estimates for the internal model shown in Table 6 stems from the model's assumption that it takes a certain amount of time for a loan to migrate to default.

Table 1
International Survey of Credit Scoring Models

STUDIES CITED	**EXPLANATORY VARIABLES**
United States	
Altman (1968)	EBIT/assets; retained earnings/ assets; working capital/assets; sales/assets; market value (MV) equity/book value of debt.
Japan	
Ko (1982)	EBIT/sales; working capital/debt; inventory turnover 2 years prior/inventory turnover 3 years prior; MV equity/debt; standard error of net income (4 years).
Takahashi, et. al. (1979)	Net worth/fixed assets; current liabilities/assets; voluntary reserves plus unappropriated surplus/assets; interest expense/sales; earned surplus; increase in residual value/sales; ordinary profit/assets; sales - variable costs.
Switzerland	
Weibel (1973)	Liquidity (near monetary resource asset – current liabilities)/ operating expenses prior to depreciation; inventory turnover; debt/assets.
Germany	
Baetge, Huss and Niehaus (1988)	Net worth/(total assets – quick assets – property & plant); (operating income + ordinary depreciation + addition to pension reserves)/assets; (cash income – expenses)/short term liabilities.
von Stein and Ziegler (1984)	Capital borrowed/total capital; short-term borrowed capital/output; accounts payable for purchases & deliveries / material costs; (bill of exchange liabilities + accounts payable)/output; (current assets – short-term borrowed capital)/output; equity/(total assets – liquid assets – real estate); equity/(tangible property – real estate); short-term borrowed capital/current assets; (working expenditure – depreciation on tangible property)/(liquid assets + accounts receivable – short-term borrowed capital); operational result/capital; (operational result + depreciation)/net turnover; (operational result + depreciation)/short-term borrowed capital; (operational result + depreciation)/total capital borrowed.
England	
Marais (1979), Earl & Marais (1982)	Current assets/gross total assets; 1/gross total assets; cash flow/current liabilities; (funds generated from operations – net change in working capital)/debt.
Canada	
Altman and Lavallee (1981)	Current assets/current liabilities; net after-tax profits/debt; rate of growth of equity – rate of asset growth; debt/assets; sales/assets.
The Netherlands	
Bilderbeek (1979)	Retained earnings/assets; accounts payable/sales; added value/ assets; sales/assets; net profit/equity.
van Frederikslust (1978)	Liquidity ratio (change in short term debt over time); profitability ratio (rate of return on equity).

TABLE 1	(CONTINUED)
STUDIES CITED	**EXPLANATORY VARIABLES**
Spain	
Fernandez (1988)	Return on investment; cash flow/current liabilities; quick ratio/ industry value; before tax earnings/sales; cash flow/sales; (permanent funds/net fixed assets)/industry value.
Italy	
Altman, Marco, and Varetto (1994)	Ability to bear cost of debt; liquidity; ability to bear financial debt; profitability; assets/liabilities; profit accumulation; trade indebtedness; efficiency.
Australia	
Izan (1984)	EBIT/interest; MV equity/liabilities; EBIT/assets; funded debt/ shareholder funds; current assets/current liabilities.
Greece	
Gloubos and Grammatikos (1988)	Gross income/current liabilities; debt/assets; net working capital/assets; gross income/assets; current assets/current liabilities.
Brazil	
Altman, Baidya, & Ribeiro-Dias,1979	Retained earnings/assets; EBIT/assets; sales/assets; MV equity/ book value of liabilities.
India	
Bhatia (1988)	Cash flow/debt; current ratio; profit after tax/net worth; interest/ output; sales/assets; stock of finished goods/sales; working capital management ratio.
Korea	
Altman, Kim and Eom (1995)	Log(assets); log(sales/assets); retained earnings/assets; MV of equity/liabilities.
Singapore	
Ta and Seah (1981)	Operating profit/liabilities; current assets/current liabilities; EAIT/paid-up capital; sales/working capital; (current assets – stocks – current liabilities)/EBIT; total shareholders' fund/liabilities; ordinary shareholders' fund/capital used.
Finland	
Suominen (1988)	Profitability: (quick flow – direct taxes)/assets; Liquidity: (quick assets/total assets); liabilities/assets.
Uruguay	
Pascale (1988)	Sales/debt; net earnings/assets; long term debt/total debt.
Turkey	
Unal (1988)	EBIT/assets; quick assets/current debt; net working capital/sales; quick assets/inventory; debt/assets; long term debt/assets.

Notes: Whenever possible, the explanatory variables are listed in order of statistical importance (e.g., the size of the coefficient term) from highest to lowest. Source: Altman and Narayanan (1997).

Table 2
One-Year Transition Probabilities for
BBB-Rated Borrower

AAA	0.02%	
AA	0.33	
A	5.95	
BBB	86.93 <-------------------------------	Most likely to stay
BB	5.30	in the same class
B	1.17	
CCC	0.12	
Default	0.18	

Source: Gupton, et. al., CreditMetrics-Technical Document, J.P. Morgan, April 2,1997, p. 11.

Table 3
Value of the Loan at the End of Year 1,
Under Different Ratings (Including First Year Coupon)

Year-End Rating	Value (millions)
AAA	$109.37
AA	109.19
A	108.66
BBB	107.55
BB	102.02
B	98.10
CCC	83.64
Default	51.13

Source: Gupton, et. al., CreditMetrics-Technical Document, J.P. Morgan, April 2,1997, p. 10.

Table 4
Risk Drivers in Algorithmics Mark-to-Future

Risk Exposure	*Risk Factors*	*Time Horizon*	**Type of Scenarios**	**Number of Scenarios**
Market Risk	50-1,000 interest rates, foreign exchange rates, equity prices, commodity prices	1 – 10 days	Historical, Monte Carlo simulation	100-10,000
Counterparty Credit Risk	50-100 interest rates, foreign exchange rates, equity prices, commodity prices	1 – 30 years	Monte Carlo simulation, Extreme value analysis	10-5,000
Portfolio Credit Risk	50-200 systemic market & credit factors, interest rates, exchange rates, equity & commodity prices, macroeconomic factors	1 – 10 years	Monte Carlo simulation, Extreme value analysis	5-5,000
Asset/Liability Management	20-100 interest rates, foreign exchange rates	6 months – 30 years	Historical, Monte Carlo simulation	5-5,000

Source: Dembo, et. Al. (2000), p. 11.

Table 5
Comparison of Different Credit Risk Measurement Models

	CreditMetrics	**Credit Risk Plus**	**Merton OPM KMV/Moody's**	**Reduced Form** KPMG/Kamkura
Definition of Risk	MTM	DM	MTM or DM	MTM
Risk Drivers	Asset Values	Expected Default Rates	Asset Values	Debt and Equity Prices
Data Requirements	Historical Transition Matrix, Credit Spreads & Yield Curves, LGD, Correlations, Exposures	Default Rates and Volatility, Macroeco Factors, LGD, Exposures	Equity Prices, Credit Spreads, Correlations, Exposures	Debt and Equity Prices, Historical Transition Matrix, Correlations, Exposures
Characterization of Credit Events	Credit Migration	Actuarial Random Default Rate	Distance to Default: Structural and Empirical	Default Intensity
Volatility of Credit Events	Constant or Variable	Variable	Variable	Variable
Correlation of Credit Events	Multivariate Normal Asset Returns	Independence assumption or correlation with expected default rate	Multivariate Normal Asset Returns	Poisson Intensity Processes with Joint Systemic Factors
Recovery Rates	Random (Beta distribution)	Constant Within Band	Constant or Random	Constant or Random
Numerical Approach	Simulation or Analytic	Analytic	Analytic and Econometric	Econometric
Interest Rates	Constant	Constant	Constant	Stochastic
Risk Classification	Ratings	Exposure Bands	Empirical EDF	Ratings or Credit Spreads

Table 6
Summary of IIF/ISDA Results for the Middle Market Portfolio

MODEL	Exposure GBP millions	Expected Loss %	Unexpected Loss %	1% VAR
Median Values	2,276	0.6	N/A	2.4
CreditMetrics	2,276	0.6	0.4	1.6
KMV	2,276	0.6	0.7	3.0
Internal Models	2,276	0.4 - 0.7	0.3 - 1.1	2.3 - 6.6
DM Models:				
CreditMetrics	2,276-2,350	0.6	0.4-0.5	1.6
KMV	2,276	0.6	0.6-0.8	2.4-3.0
MTM Models:				
CreditMetrics	2,283	0.6	0.5	1.7-1.8
KMV	2,213-2,276	0.5-0.8	1.1-1.6	1.6-5.3
Internal Models	2,276	0.1	0.7	1.5

Source: IIF/ISDA Study, Chapter I, pp. 21-23. The results assume that all assets in the portfolio are carried at market value.

Figure 1

Figure 4.3 Equity as a call option on a firm.

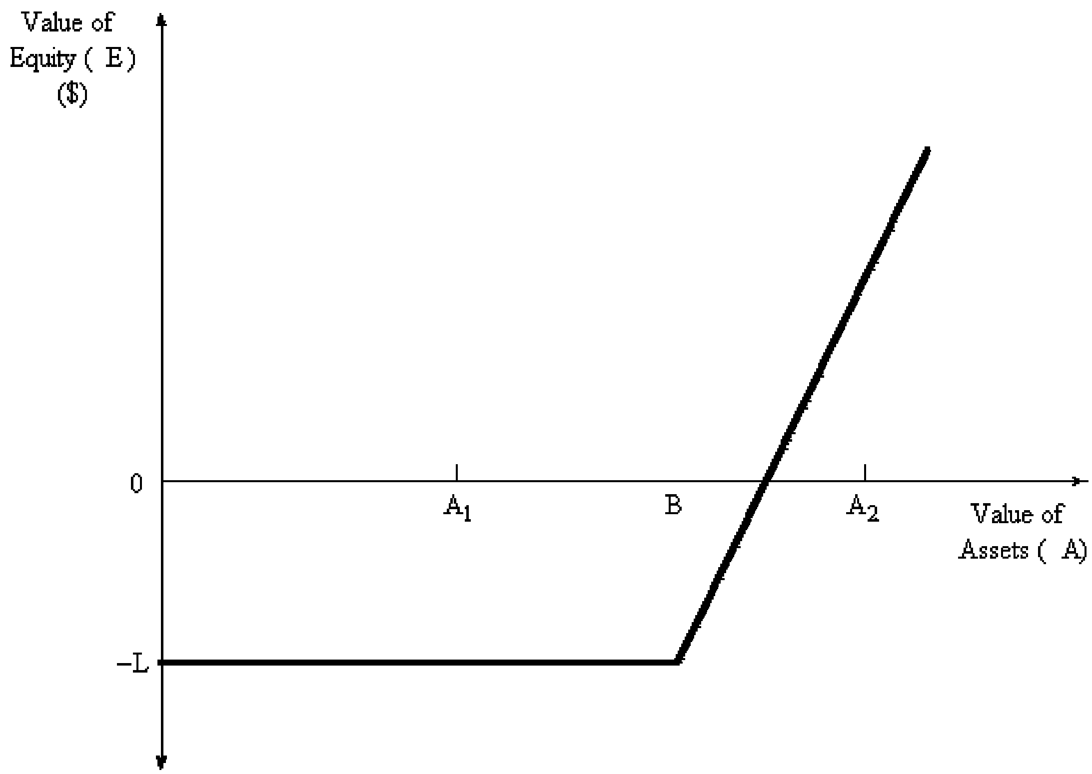


Figure 2

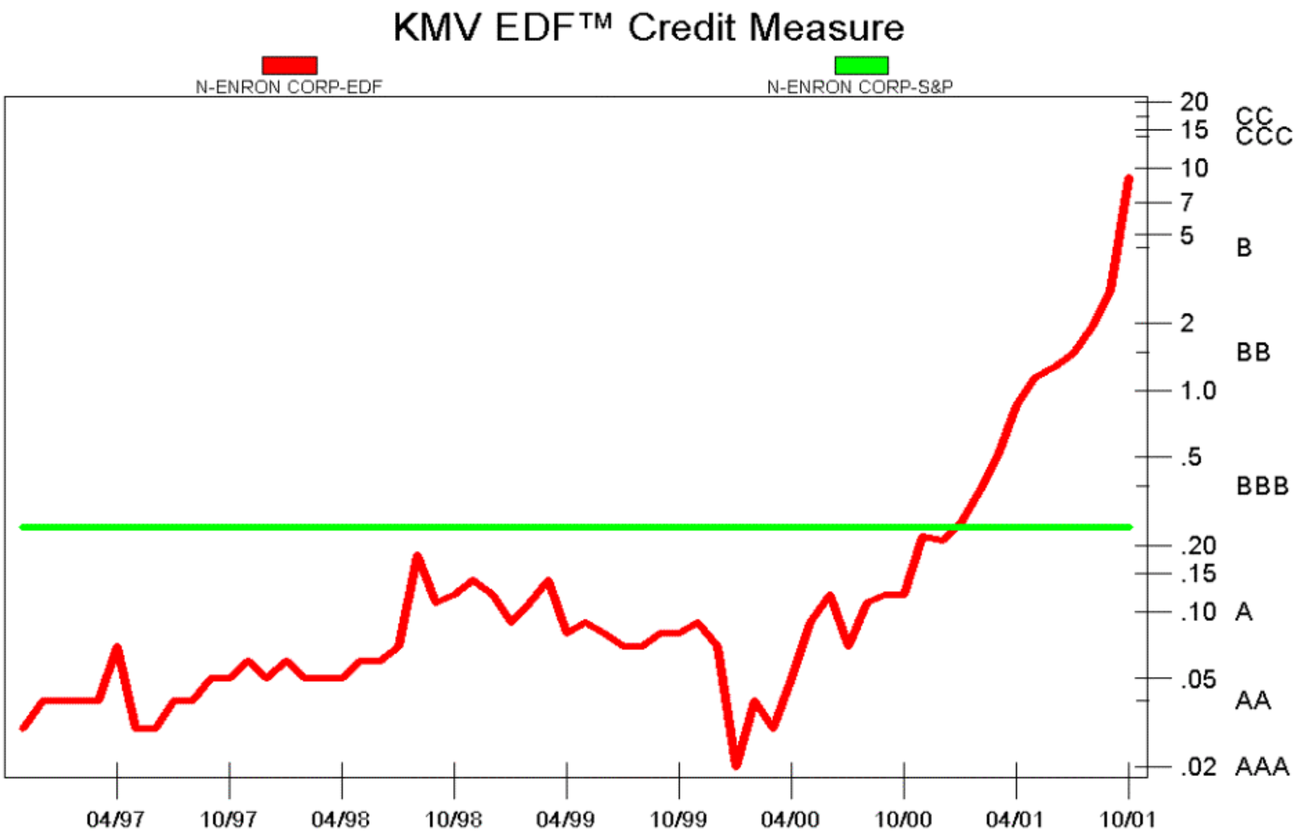

KMV EDF™ Credit Measure
N-ENRON CORP-EDF
N-ENRON CORP-S&P
20
15
10
7
5
2
1.0
.5
.20
.15
.10
.05
.02
CC
CCC
B
BB
BBB
A
AA
AAA
04/97
10/97
04/98
10/98
04/99
10/99
04/00
10/00
04/01
10/01

Figure 3

Figure 6.3 Actual distribution of loan values on five year BBB loan at the end of year 1 (Including first year coupon payment).

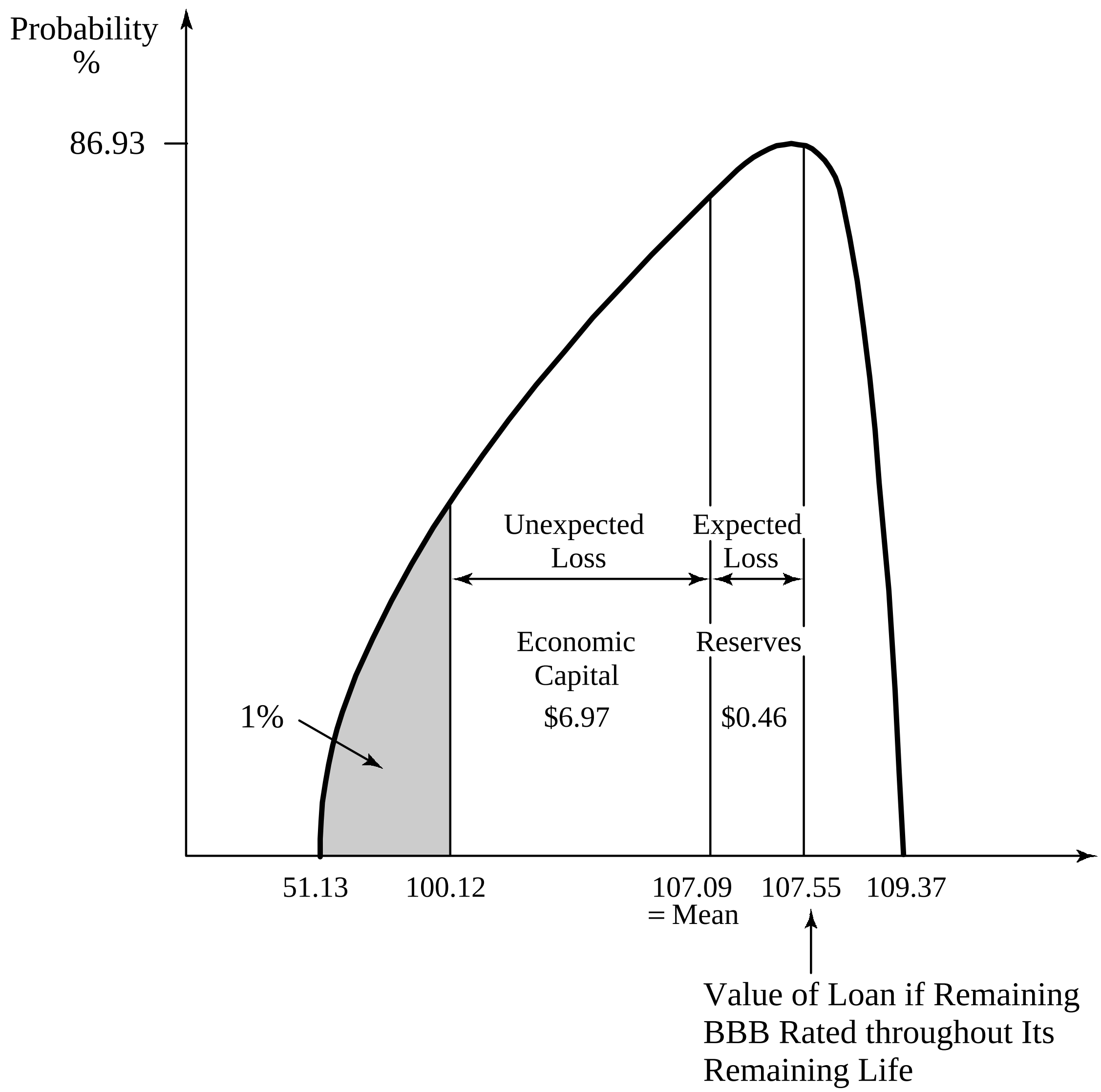
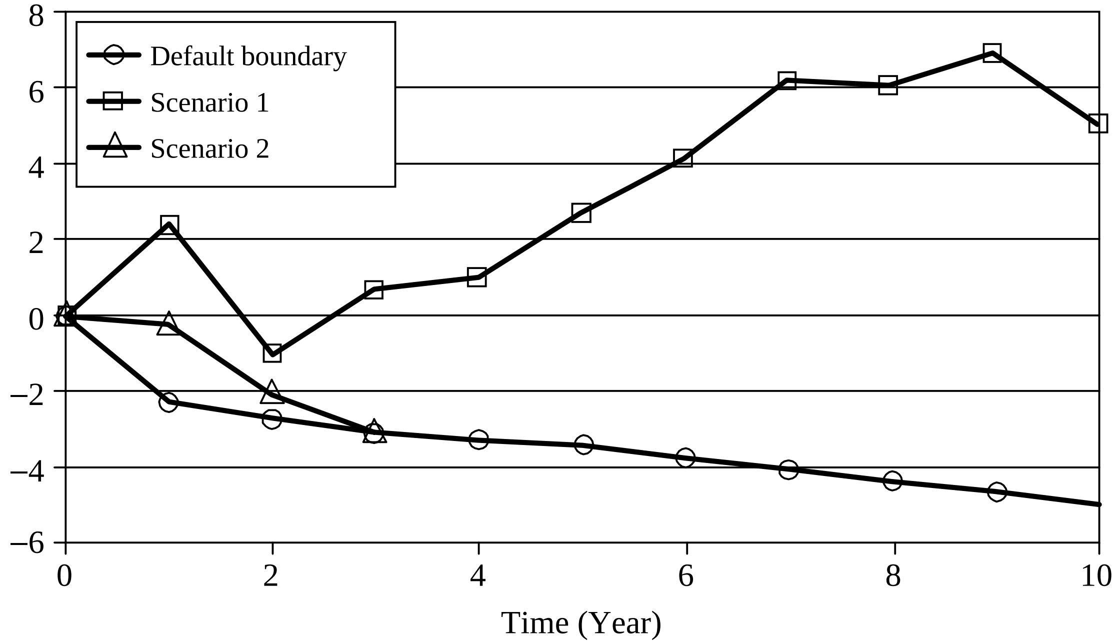


Figure 4

Figure 12.1 Merton model of default.
Source: Dembo et al. (2000), p. 68.



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