Credit Risk Modeling of Middle Markets

Linda Allen

Professor of Finance

Zicklin School of Business, Baruch College

Credit risk measurement is particularly difficult for middle market firms.

- Traditional models.
- Options Theoretic Structural Models
- Reduced Form Intensity-Based Models
- VaR Models
- Mortality Models – Credit Risk Plus
- Comparison of Models and Their Performance for a Portfolio of Middle Market Obligations

Traditional Models

- Expert Systems
  - The 5 C’s
    - Character, Capital, Capacity, Collateral, Cycle
    - Inconsistent and subjective weighting of 5 C’s.
  - Artificial Neural Networks
    - Flexible, “learns” from experience, objective
    - But: not transparent, may yield poor out of sample estimates, grows large very quickly.

Traditional Models

- Rating Systems
  - External Ratings
  - Internal Ratings: OCC, NAIC
  - Treacy & Carey (2000): 60% of internal rating systems only assess PD, not LGD
  - Inconsistency across ratings. Qualitative factors particularly important for small and medium sized firms.

Traditional Models

- Credit Scoring Models
  - 97% of banks use for credit card approvals, 70% for small business lending.
  - Similarities in models around the world (Table 1): Use financial rations measuring profitability, leverage, and liquidity.
  - But: Assumption of linearity. Use of balance sheet data. Not grounded in economic theory.

Table 1
International Survey of Credit Scoring Models

STUDIES CITED	EXPLANATORY VARIABLES
United States	
Altman (1968)	**EBIT/assets; retained earnings/ assets; working capital/assets; sales/assets; market value (MV) equity/book value of debt.**
Japan	
Ko (1982)	**EBIT/sales; working capital/debt; inventory turnover 2 years prior/inventory turnover 3 years prior; MV equity/debt; standard error of net income (4 years).**
Takahashi, et. al. (1979)	**Net worth/fixed assets; current liabilities/assets; voluntary reserves plus unappropriated surplus/assets; interest expense/sales; earned surplus; increase in residual value/sales; ordinary profit/assets; sales - variable costs.**
Switzerland	
Weibel (1973)	**Liquidity (near monetary resource asset – current liabilities)/ operating expenses prior to depreciation; inventory turnover; debt/assets.**
Germany	
Baetge, Huss and Niehaus (1988)	**Net worth/(total assets – quick assets – property & plant); (operating income + ordinary depreciation + addition to pension reserves)/assets; (cash income – expenses)/short term liabilities.**
von Stein and Ziegler (1984)	**Capital borrowed/total capital; short-term borrowed capital/output; accounts payable for purchases & deliveries / material costs; (bill of exchange liabilities + accounts payable)/output; (current assets – short-term borrowed capital)/output; equity/(total assets – liquid assets – real estate); equity/(tangible property – real estate); short-term borrowed capital/current assets; (working expenditure – depreciation on tangible property)/(liquid assets + accounts receivable – short-term borrowed capital); operational result/capital; (operational result + depreciation)/net turnover; (operational result + depreciation)/short-term borrowed capital; (operational result + depreciation)/total capital borrowed.**
England	
Marais (1979), Earl & Marais (1982)	**Current assets/gross total assets; 1/gross total assets; cash flow/current liabilities; (funds generated from operations – net change in working capital)/debt.**
Canada	
Altman and Lavallee (1981)	**Current assets/current liabilities; net after-tax profits/debt; rate of growth of equity – rate of asset growth; debt/assets; sales/assets.**
The Netherlands	

TABLE 1	(CONTINUED)
STUDIES CITED	EXPLANATORY VARIABLES
Spain	
Fernandez (1988)	**Return on investment; cash flow/current liabilities; quick ratio/ industry value; before tax earnings/sales; cash flow/sales; (permanent funds/net fixed assets)/industry value.**
Italy	
Altman, Marco, and Varetto (1994)	**Ability to bear cost of debt; liquidity; ability to bear financial debt; profitability; assets/liabilities; profit accumulation; trade indebtedness; efficiency.**
Australia	
Izan (1984)	**EBIT/interest; MV equity/liabilities; EBIT/assets; funded debt/ shareholder funds; current assets/current liabilities.**
Greece	
Gloubos and Grammatikos (1988)	**Gross income/current liabilities; debt/assets; net working capital/assets; gross income/assets; current assets/current liabilities.**
Brazil	
Altman, Baidya, & Ribeiro-Dias,1979	**Retained earnings/assets; EBIT/assets; sales/assets; MV equity/ book value of liabilities.**
India	
Bhatia (1988)	**Cash flow/debt; current ratio; profit after tax/net worth; interest/ output; sales/assets; stock of finished goods/sales; working capital management ratio.**
Korea	
Altman, Kim and Eom (1995)	**Log(assets); log(sales/assets); retained earnings/assets; MV of equity/liabilities.**
Singapore	
Ta and Seah (1981)	**Operating profit/liabilities; current assets/current liabilities; EAIT/paid-up capital; sales/working capital; (current assets – stocks – current liabilities)/EBIT; total shareholders' fund/liabilities; ordinary shareholders' fund/capital used.**
Finland	
Suominen (1988)	**Profitability: (quick flow – direct taxes)/assets; Liquidity: (quick assets/total assets); liabilities/assets.**
Uruguay	
Pascale (1988)	**Sales/debt; net earnings/assets; long term debt/total debt.**
Turkey	
Unal (1988)	**EBIT/assets; quick assets/current debt; net working capital/sales; quick assets/inventory; debt/assets; long term debt/assets.**

Notes: Whenever possible, the explanatory variables are listed in order of statistical importance (e.g., the size of the coefficient term) from highest to lowest. Source: Altman and Narayanan (1997).

Structural Models: The Link Between Loans and Optionality - Merton (1974)

- Payoff on pure discount bank loan with face value=0B secured by firm asset value.
  - Firm owners repay loan if asset value (upon loan maturity) exceeds principal + interest payment.
  - If asset value < OB (loan repayment amount), then default. Bank receives assets.

Using Option Valuation Models to Value Risky Loans

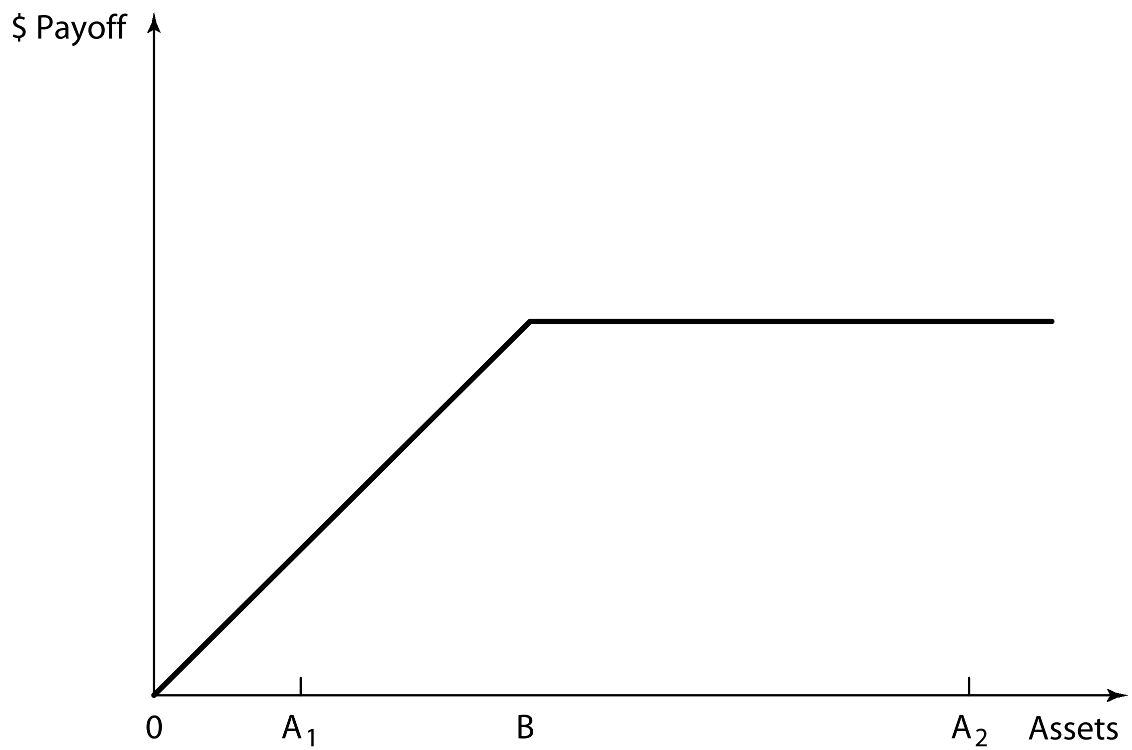
- Loan payoff = payoff to the writer of a put option on the firm's assets held by shareholders.
- Value of put option on stock = $f(S, X, r, \sigma, \tau)$ where

S=stock price, X=exercise price, r=risk-free rate, σ=equity volatility, τ=time to maturity.

Value of default option on risky loan = $f(A, B, r, \sigma_A, \tau)$ where

A=market value of assets, B=face value of debt, r=risk-free rate, σ_A=asset volatility, τ=time to debt maturity.

Figure 4.1 The payoff to a bank lender



Problem with Valuation of Risky Loan Put Option

- A and σ_A are not observable.
- Model equity as a call option on a firm. (Figure 1)
- Equity valuation = $E = h(A, \sigma_A, B, r, \tau)$

Need another equation to solve for A and σ_A:

$$\sigma_E = g(\sigma_A)$$

Can solve for A and σ_A to obtain a Distance to Default = $(A\text{-}B)/\sigma_A$

Empirical EDF is calculated by KMV from database. A DD = 4 translates into an EDF = 1%

KMV EDFs range from 0.03% - 20%

Figure 4.3 Equity as a call option on a firm.

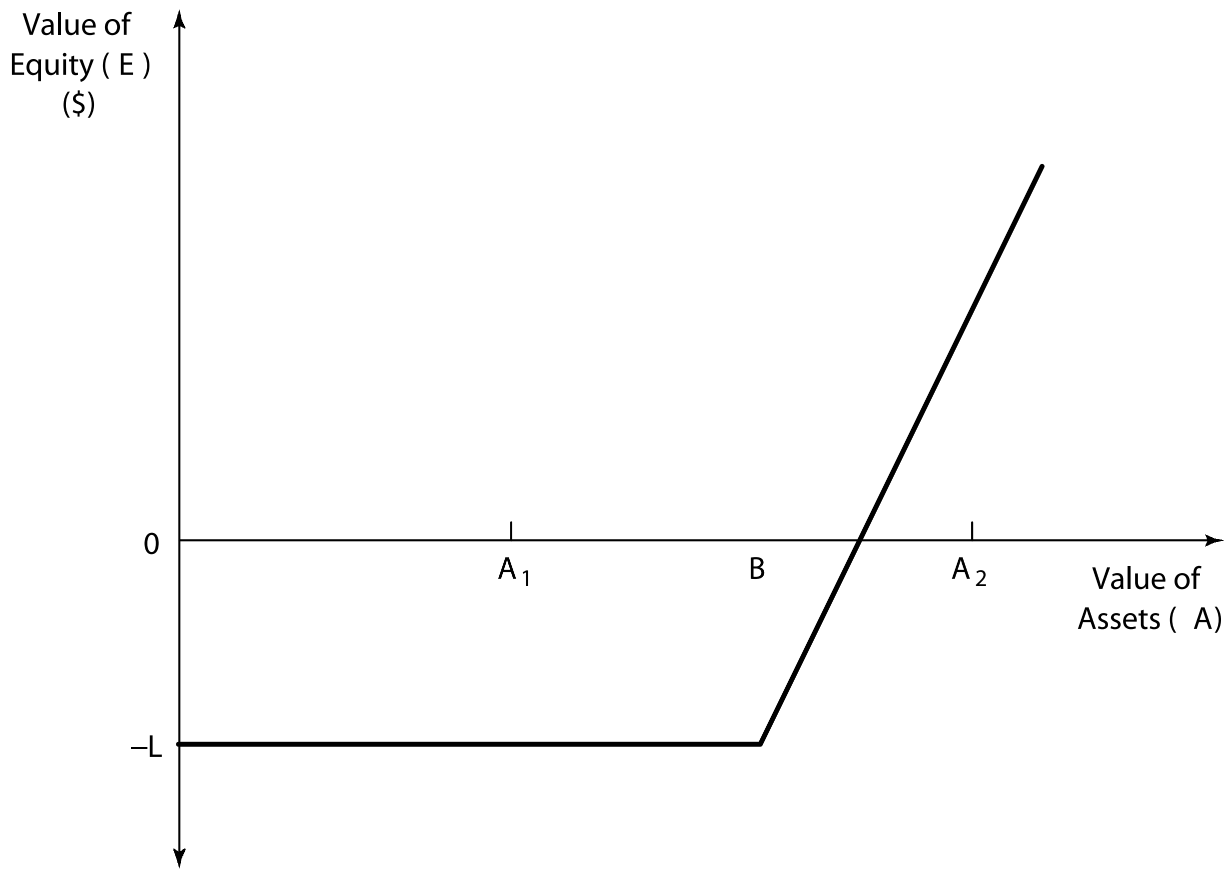
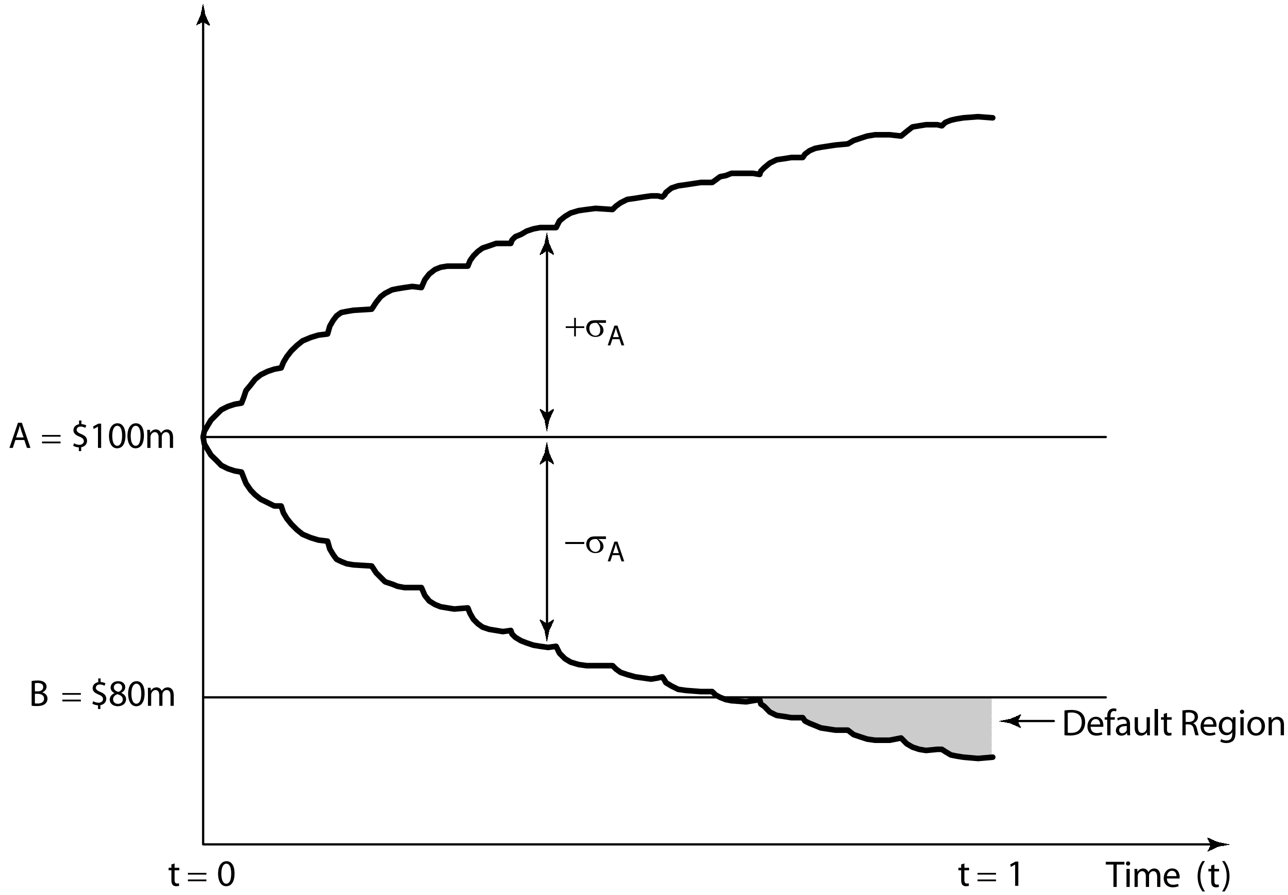
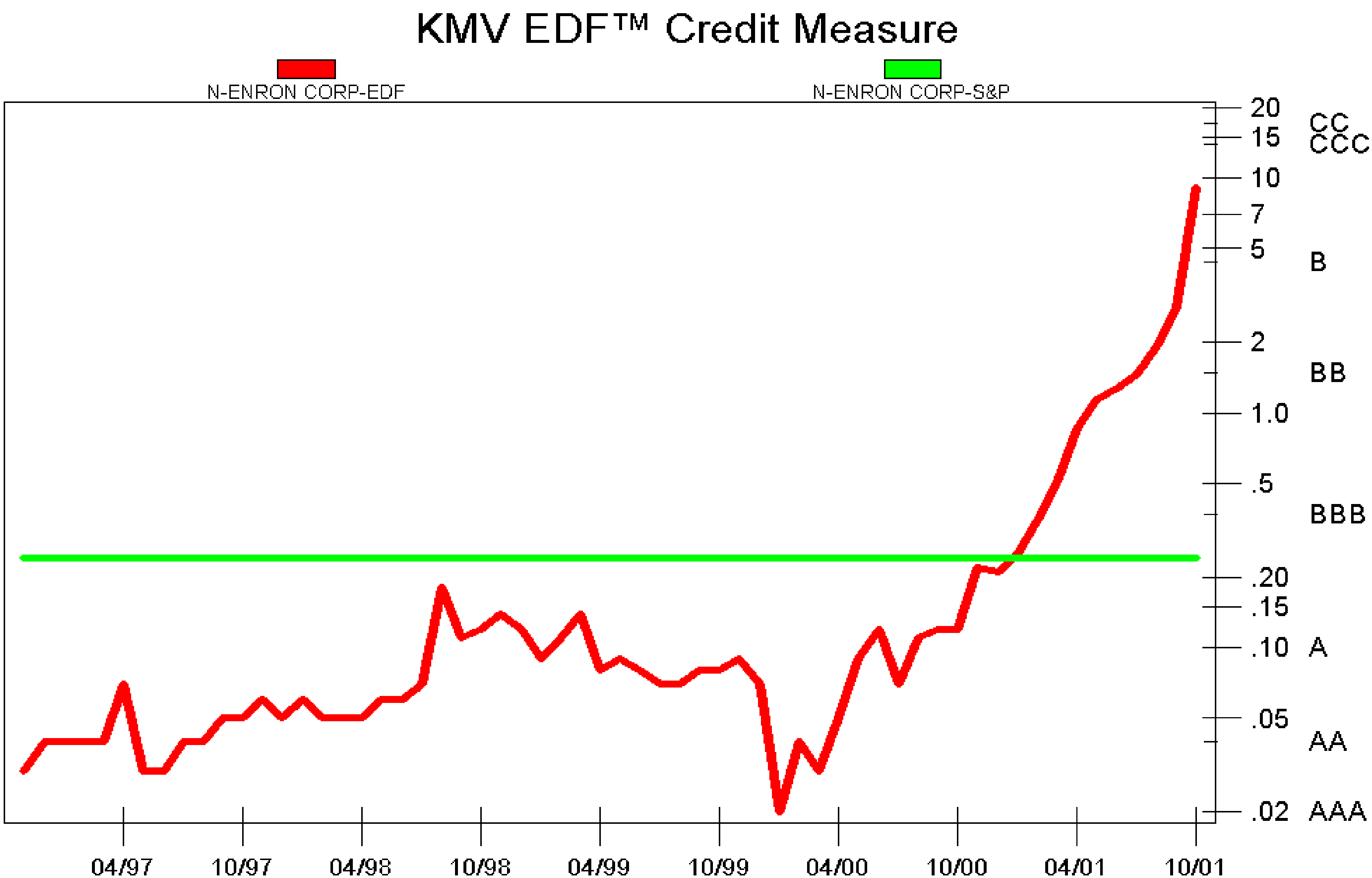


Figure 4.4 Calculating the theoretical EDF




KMV EDF™ Credit Measure
N-ENRON CORP-EDF
N-ENRON CORP-S&P
20
15
10
7
5
2
1.0
.5
.20
.15
.10
.05
.02
CC
CCC
B
BB
BBB
A
AA
AAA
04/97
10/97
04/98
10/98
04/99
10/99
04/00
10/00
04/01
10/01

KMV's Private Firm Model

- Need equity prices to obtain KMV EDF scores. For private firms, must simulate "equity" values:
  - Calculate EBITDA for private firm_j in industry_i
  - Divide industry_i average equity market value by industry_i average EBITDA
  - Multiply the industry multiple by EBITDA for firm j to obtain the simulated "equity" MV
  - Assets = simulated "equity" + BV of debt

The Reduced Form Intensity-Based Approach

- Merton's OPM took a structural approach to modeling default: default occurs when the market value of assets fall below debt value
- Reduced form models: Decompose risky debt prices to estimate the stochastic default intensity function. No structural explanation of why default occurs.

Understanding a Basic Intensity Process
Duffie & Singleton (1998)

- $1 - PD(t) = e^{-ht}$ where h is the default intensity. Expected time to default is 1/h.
- A rated firm: h=.001: expected to default once every 1,000 years.
- B rated firm: h=.05: expected to default once every 20 years.
- If have a portfolio with 1,000 A rated loans and 100 B rated loans, then there are 6 expected defaults per year = (1000*.001)+(100*.05)=6

Disentangling PD from LGD

- Intensity-based models specify stochastic functional form for PD using CS = PD x LGD
  - Jarrow & Turnbull (1995): Fixed LGD, exponentially distributed default process.
  - Das & Tufano (1995): LGD proportional to bond values.
  - Jarrow, Lando & Turnbull (1997): LGD proportional to debt obligations.
  - Duffie & Singleton (1999): LGD and PD functions of economic conditions
  - Unal, Madan & Guntay (2001): LGD a function of debt seniority.
  - Jarrow (2001): LGD determined using equity prices.

Kamakura's Risk Manager

- Based on Jarrow (2001).
- Decomposes risky debt and equity prices to estimate PD and LGD processes.
- Fundamental explanatory variables: ROA, leverage, relative size, excess return over market index return, monthly equity volatility.
- Type 1 error rate of 18.68%.

Bond Prices: $B = B[t, T, i, \lambda(t, X(t)), \delta(t, X(t)), \gamma(t,T,X(t)), \mu, S(t,X(t))]$
Equity Prices: $\xi = \xi[t, T, i, \lambda(t, X(t)), \mu, S(t,X(t))]$

where t is the current period; T is the bond's time to maturity; i is the stochastic default-free interest rate process; $\lambda(t, X(t))$ is the default intensity process, i.e., the risk neutral PD; $\delta(t, X(t))$ is the recovery rate (1 – LGD); $\gamma(t,T,X(t))$ is the liquidity premium; μ is a stock market bubble factor; and $S(t,X(t))$ is the liquidating dividend on equity in the event of bond default.

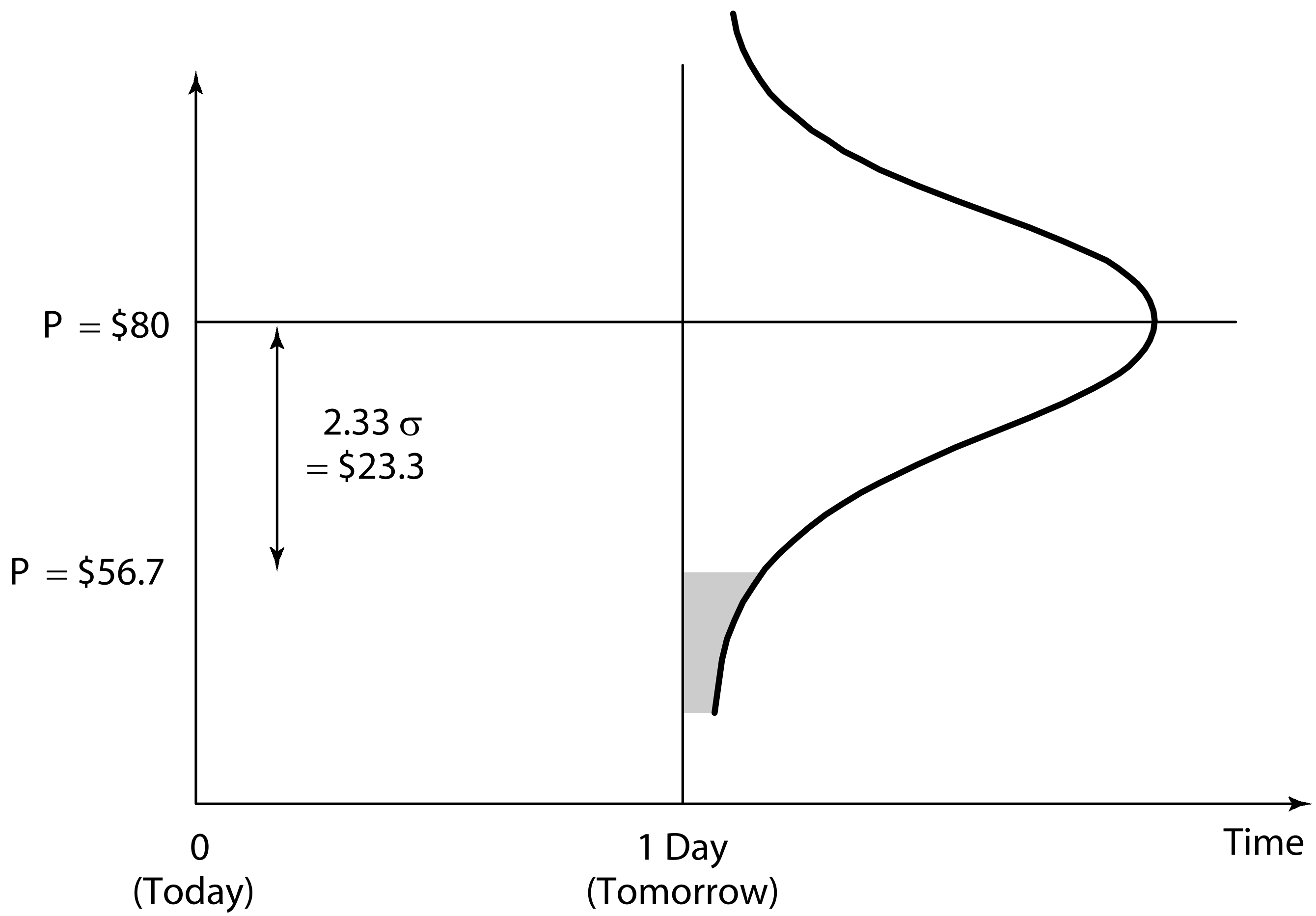
Noisy Risky Debt Prices

- US corporate bond market is much larger than equity market, but less transparent
- Interdealer market not competitive – large spreads and infrequent trading: Saunders, Srinivasan & Walter (2002)
- Noisy prices: Hancock & Kwast (2001)
- More noise in senior than subordinated issues: Bohn (1999)
- In addition to credit spreads, bond yields include:
  - Liquidity premium
  - Embedded options
  - Tax considerations and administrative costs of holding risky debt

The Concept of VAR

- Example of VAR applied to market risk.
- Market price of equity = \$80 with estimated daily standard deviation = \$10.
- If tomorrow is a “bad day” (1 in 100 worst) then what is the value at risk?
- If normally distributed, then the cutoff is 2.33σ below the mean = \$80 – 2.33(10) = \$56.70. 99% VAR = \$23.30

Figure 6.1 The VAR of traded equity.



CreditMetrics

- What is VAR is next year is a “bad” year?
- Since most loans are not publicly traded, then we do not observe the price or the standard deviation.
- Consider VAR for an individual loan.

Rating Migration

Table 2

Table 6.1 One-Year Transition Probabilities for BBB-Rated Borrower

AAA	0.02%	
AA	0.33	
A	5.95	
BBB	86.93 <------------------------------	Most likely to stay in the same class
BB	5.30	
B	1.17	
CCC	0.12	
Default	0.18	

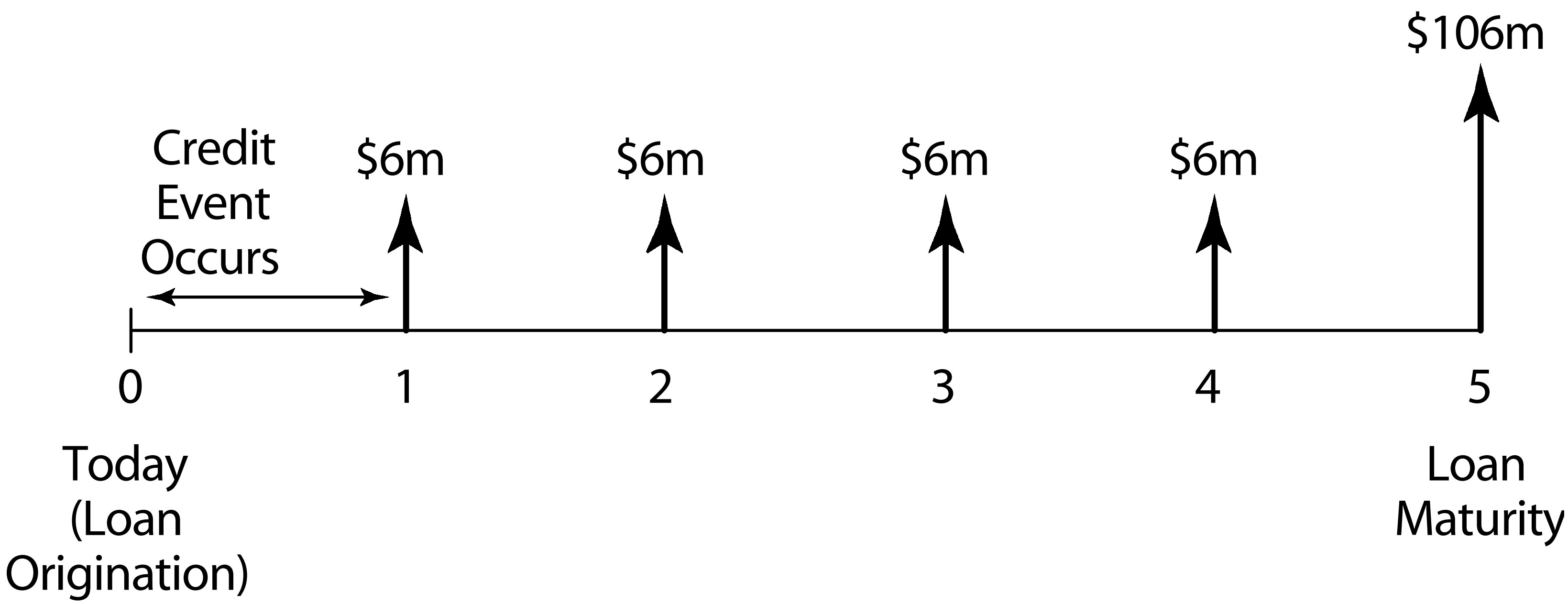
Source: Gupton, et. al., CreditMetrics-Technical Document, J.P. Morgan, April 2,1997, p. 11.

Valuation

- Consider BBB rated \$100 million face value loan with fixed 6% annual coupon and 5 years until maturity. Cash flow diagram.

$$P = 6 + \frac{6}{(1+r_1+s_1)} + \frac{6}{(1+r_2+s_2)^2} + \frac{6}{(1+r_3+s_3)^3} + \frac{106}{(1+r_4+s_4)^4} \qquad (6.1)$$

Figure 6.2 Cash flows on the five-year BBB loan.
(Credit Events are: upgrades, downgrades, or defaults.)



Valuation at the End of the Credit Horizon (in 1 year)

- Calculate one year forward zero yield curves plus credit spread (see Appendix 6.1)

Table 6.2
One Year Forward Zero Curves Plus Credit Spreads
By Credit Rating Category (%)

Category	**Year 1**	**Year 2**	**Year 3**	**Year 4**
AAA	3.60	4.17	4.73	5.12
AA	3.65	4.22	4.78	5.17
A	3.72	4.32	4.93	5.32
BBB	4.10	4.67	5.25	5.63
BB	5.55	6.02	6.78	7.27
B	6.05	7.02	8.03	8.52
CCC	15.05	15.02	14.03	13.52

Source: Gupton, et. al., CreditMetrics-Technical Document, J.P. Morgan, April 2,1997, p. 27.

Using the Forward Yield Curves to Value the Risky Loan

- Under the credit event that the loan's rating improves to A, the value at the end of yr 1:

$$P = 6 + \frac{6}{(1.0372)} + \frac{6}{(1.0432)^2} + \frac{6}{(1.0493)^3} + \frac{106}{(1.0532)^4} = \$108.66$$

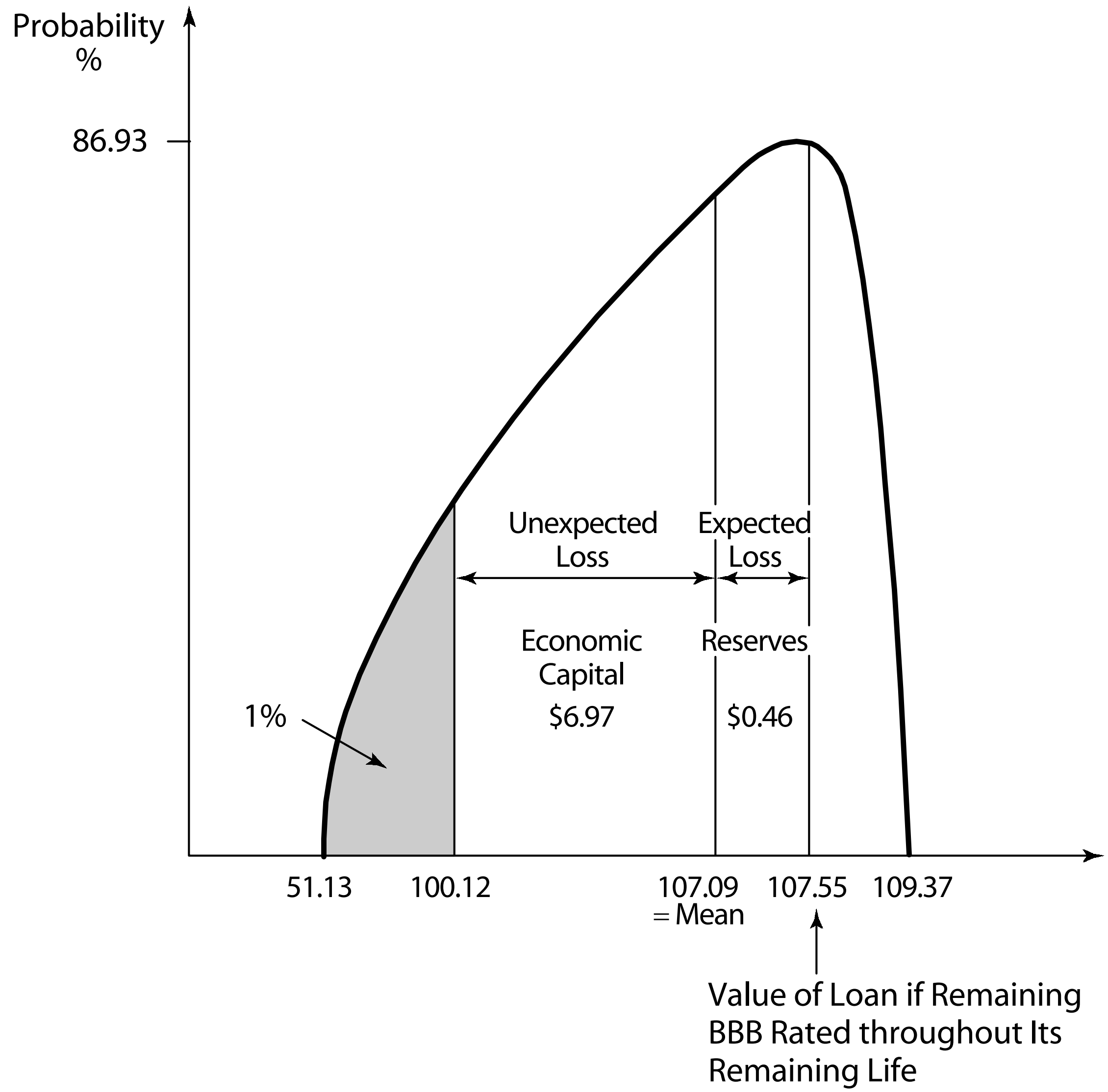
- Must repeat 8 times for each possible credit migration.

Value of the Loan at the End of Year 1 Under Different Ratings Table 3

Year-End Rating	Value (millions)
AAA	$109.37
AA	109.19
A	108.66
BBB	107.55
BB	102.02
B	98.10
CCC	83.64
Default	51.13

Source: Gupton, et. al., CreditMetrics-Technical Document, J.P. Morgan, April 2,1997, p. 10.

Figure 6.3 Actual distribution of loan values on five year BBB loan at the end of year 1 (Including first year coupon payment).



Algorithmics MtF

- Scenario-based model of credit risk, market risk, & operational risk.
- Systematic risk parameters drive creditworthiness.
- Can create 5 to 20 extreme scenarios using 50 to 200 systemic market and credit factors to conduct stress tests over 1-10 years.

Table 4 Risk Drivers in MtF

Risk Exposure	*Risk Factors*	*Time Horizon*	**Type of Scenarios**	**Number of Scenarios**
Market Risk	**50-1,000 interest rates, foreign exchange rates, equity prices, commodity prices**	1 – 10 days	Historical, Monte Carlo simulation	100-10,000
Counterparty Credit Risk	50-100 interest rates, foreign exchange rates, equity prices, commodity prices	1 – 30 years	Monte Carlo simulation, Extreme value analysis	10-5,000
Portfolio Credit Risk	50-200 systemic market & credit factors, interest rates, exchange rates, equity & commodity prices, macroeconomic factors	1 – 10 years	Monte Carlo simulation, Extreme value analysis	5-5,000
Asset/Liability Management	20-100 interest rates, foreign exchange rates	6 months – 30 years	Historical, Monte Carlo simulation	5-5,000

Source: Dembo, et. Al. (2000), p. 11.

An Example of Credit Risk Stress Testing Using MtF

- BB rated swap obligation. Fig. 4 shows the creditworthiness index (CWI) for 2 scenarios: 1 (default in 3 yrs) and 2 (no default in 10 yrs).
- Choose scenario of systemic risk factors (say, S&P500 and 6 mo. T-bill rates over the next 10 years). Call it S9.
- Measure the PD conditional on these risk factors from historical relationship between BB rated debt and S&P 500 and 6-mo. T-bill rates. One of the curves in Fig. 12.3 corresponds to the conditional PD for scenario S9.
  - Conditional PD incorporates both systematic & idiosyncratic risk factors. Ex. If systematic risk explains 5% of CWI variance, then the conditional 5-yr PD under S9 is 11.4%. If systematic risk explains 80%, then the conditional 5-yr PD under S9 is 44.4%.
- Repeat using other systemic risk factors to derive a probability distribution of conditional PD.

Figure 12.1 Merton model of default.
Source: Dembo et al. (2000), p. 68.

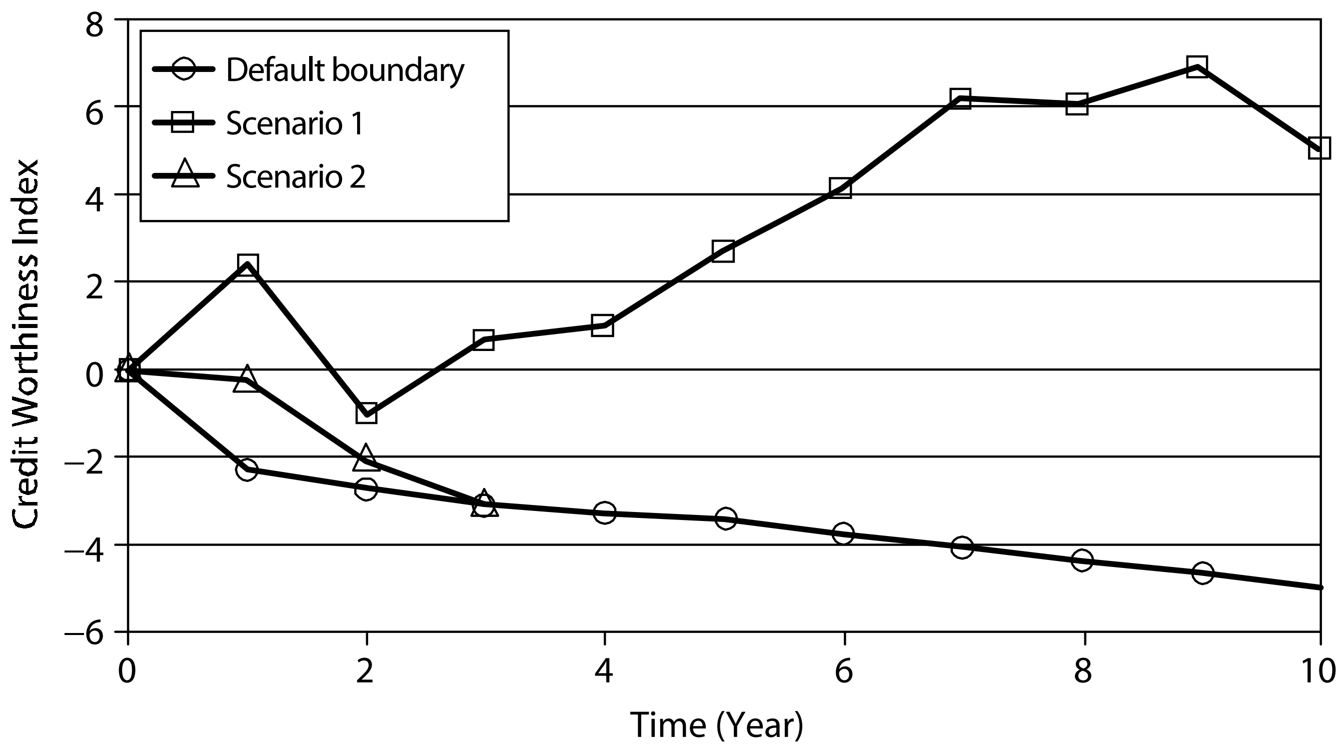
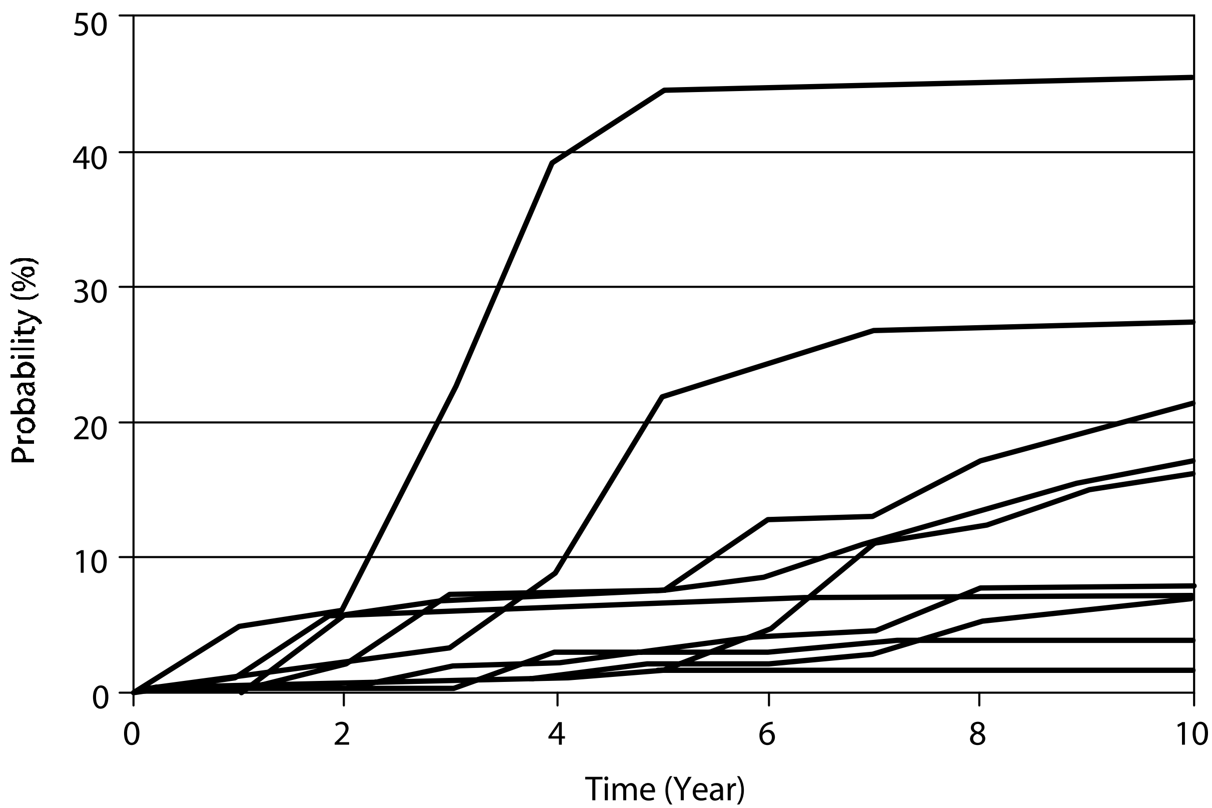


Figure 12.3 Ten senarios on conditional default pobabilities or a second counter party.
Source: Dembo et al. (2000), p. 70.



CSFP Credit Risk Plus

- Default mode model
- CreditMetrics: default probability is discrete (from transition matrix). In CreditRisk +, default is a continuous variable with a probability distribution.
- Default probabilities are independent across loans.
- Loan portfolio's default probability follows a Poisson distribution. See Fig.8.1.
- Variance of PD = mean default rate.
- Loss severity (LGD) is also stochastic in Credit Risk +.

Figure 8.1 Comparison of credit risk plus and CreditMetrics.

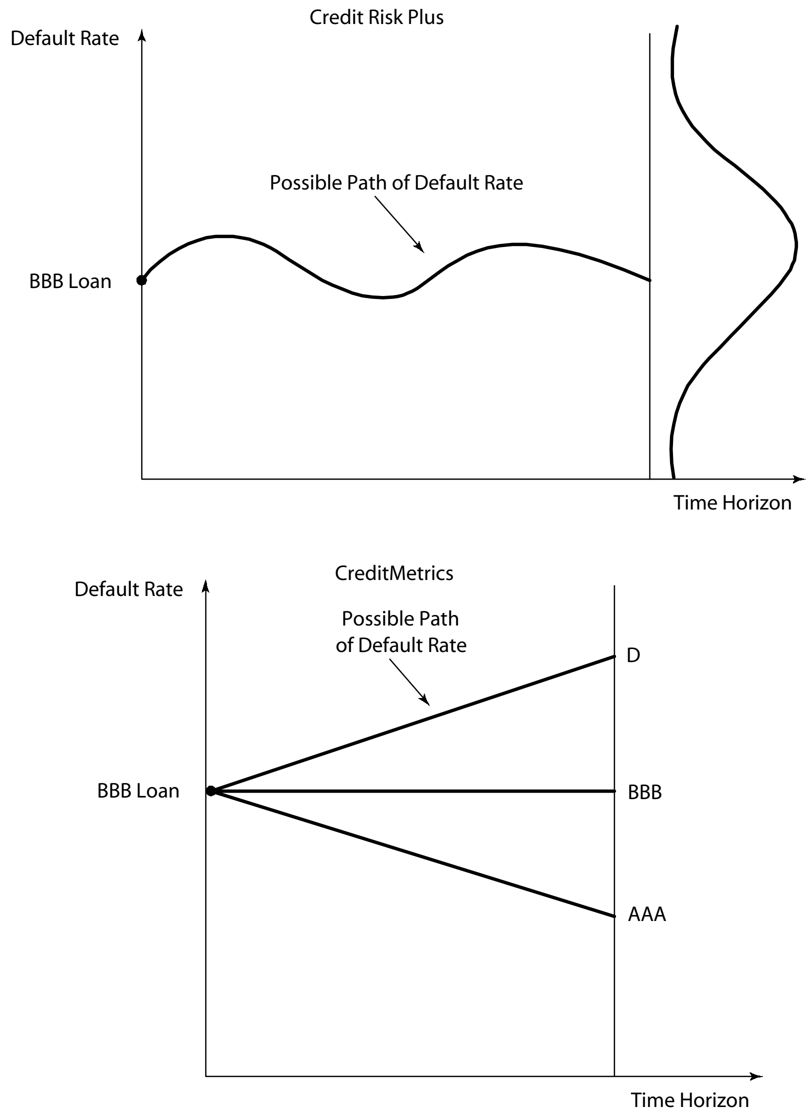
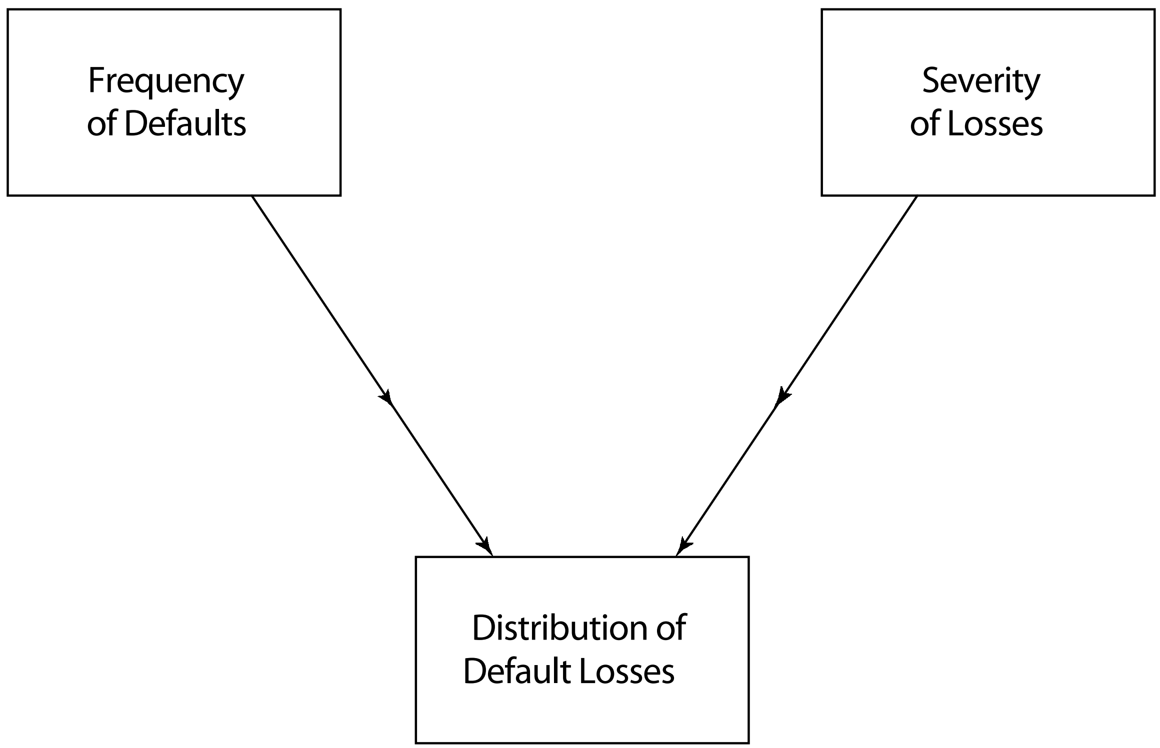


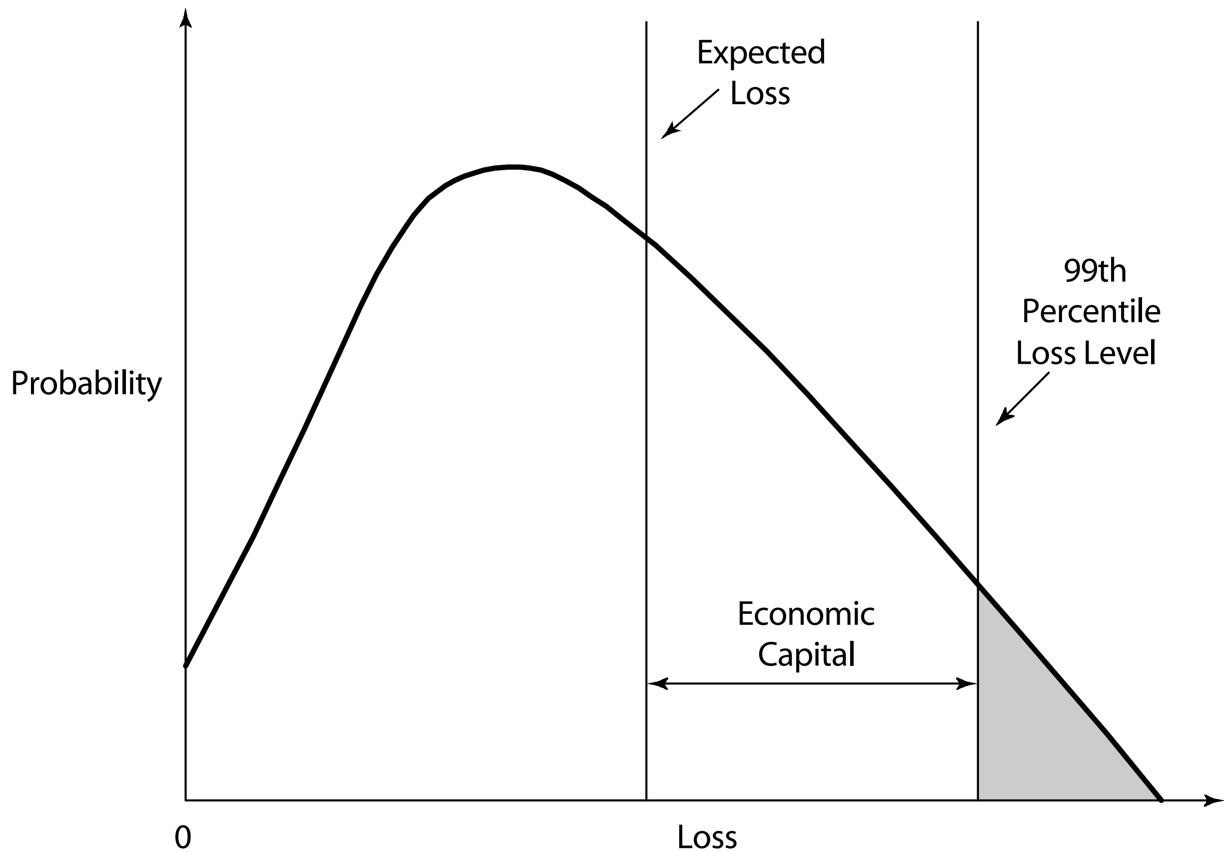
Figure 8.2 The CSFP credit risk plus model.



Distribution of Losses

- Combine default frequency and loss severity to obtain a loss distribution.
- Loss distribution is close to normal, but with fatter tails.
- Mean default rate of loan portfolio equals its variance. (property of Poisson distrib.)

Figure 8.4 Capital requirement under the CSFP credit risk plus model.



Performance Tests - Comparing Four Models:

- Options pricing models: KMV and Moody's
- Reduced form models: KPMG & Kamakura
- CreditMetrics
- Credit Risk Plus

Definition of Risk

	CreditMetrics	**CreditPortfolio View**	**Credit Risk Plus**	**Merton OPM KMV/Moody's**	**Reduced Form KPMG/Kamkura**
Definition of Risk	MTM	MTM or DM	DM	MTM or DM	MTM

Risk Drivers

	CreditMetrics	**CreditPortfolio View**	**Credit Risk Plus**	**Merton OPM KMV/Moody's**	**Reduced Form KPMG/Kamkura**
Risk Drivers	Asset Values	Macroeconomic Factors	Expected Default Rates	Asset Values	Debt and Equity Prices

Data Requirements

	CreditMetrics	CreditPortfolio View	Credit Risk Plus	Merton OPM KMV/Moody's	Reduced Form KPMG/Kamkura
Data Requirements	Historical Transition Matrix, Credit Spreads & Yield Curves, LGD, Correlations, Exposures	Historical Transition Matrix, Macroeconomic Variables, Credit Spreads, LGD, Exposures	Default Rates and Volatility, Macroeco Factors, LGD, Exposures	Equity Prices, Credit Spreads, Correlations, Exposures	Debt and Equity Prices, Historical Transition Matrix, Correlations, Exposures

Characterization of Credit Events

	CreditMetrics	**CreditPortfolio View**	**Credit Risk Plus**	**Merton OPM KMV/Moody's**	**Reduced Form KPMG/Kamkura**
Characterization of Credit Events	Credit Migration	Migration Conditional on Macroeconomic Factors	Actuarial Random Default Rate	Distance to Default: Structural and Empirical	Default Intensity

Volatility of Credit Events

	CreditMetrics	**CreditPortfolio View**	**Credit Risk Plus**	**Merton OPM KMV/Moody's**	**Reduced Form KPMG/Kamkura**
Volatility of Credit Events	Constant or Variable	Variable	Variable	Variable	Variable

Correlation of Credit Events

	CreditMetrics	**CreditPortfolio View**	**Credit Risk Plus**	**Merton OPM KMV/Moody's**	**Reduced Form KPMG/Kamkura**
Correlation of Credit Events	Multivariate Normal Asset Returns	Macroeconomic Factor Loadings	Independence assumption or correlation with expected default rate	Multivariate Normal Asset Returns	Poisson Intensity Processes with Joint Systemic Factors

Recovery Rates

	CreditMetrics	CreditPortfolio View	Credit Risk Plus	Merton OPM KMV/Moody's	Reduced Form KPMG/Kamkura
Recovery Rates	Random (Beta distribution)	Random	Constant Within Band	Constant or Random	Constant or Random

Numerical Approach

	CreditMetrics	**CreditPortfolio View**	**Credit Risk Plus**	**Merton OPM KMV/Moody's**	**Reduced Form KPMG/Kamkura**
Numerical Approach	Simulation or Analytic	Simulation	Analytic	Analytic and Econometric	Econometric

Interest Rates

	CreditMetrics	CreditPortfolio View	Credit Risk Plus	Merton OPM KMV/Moody's	Reduced Form KPMG/Kamkura
Interest Rates	Constant	Constant	Constant	Constant	Stochastic

Risk Classification

	CreditMetrics	**CreditPortfolio View**	**Credit Risk Plus**	**Merton OPM KMV/Moody's**	**Reduced Form KPMG/Kamkura**
Risk Classification	Ratings	Ratings	Exposure Bands	Empirical EDF	Ratings or Credit Spreads

The IIF/ISDA (2000) Study

- Participants: 25 commercial banks in 10 countries.
- Estimated CreditMetrics, CreditPortfolio View, Credit Risk Plus, and KMV's Portfolio Manager, as well as the banks' own internal models.

Conclusions of IIF/ISDA Study

- Models yield similar results when given similar inputs.
- Discrepancies are due to: model inputs, preprocessing (IT formats), valuation, errors in model usage during testing and standardized parameters.
- Most important differences come from assumptions about: valuation methods and correlations.
- Most significant risk drivers are credit quality, asset correlation and LGD.
- Internal models use aggregate default measures not PD and credit migrations.

IIF/ISDA Results on the Middle Market Portfolio

- Greater range of credit risk estimates than for corporate portfolio.
- 2,500 obligors, averaging £894,000 per exposure.
- 5% of total exposures come from 1 obligor; next 5 obligors represent an additional 6% of exposures.
- All exposures in the UK.

Table 6: Summary of Results on the Middle Market Portfolio

MODEL	Exposure GBP millions	Expected Loss %	Unexpected Loss %	1% VAR
Median Values	2,276	0.6	N/A	2.4
CreditMetrics	2,276	0.6	0.4	1.6
KMV	2,276	0.6	0.7	3.0
Internal Models	2,276	0.4 - 0.7	0.3 - 1.1	2.3 - 6.6
DM Models:				
CreditMetrics	2,276-2,350	0.6	0.4-0.5	1.6
KMV	2,276	0.6	0.6-0.8	2.4-3.0
MTM Models:				
CreditMetrics	2,283	0.6	0.5	1.7-1.8
KMV	2,213-2,276	0.5-0.8	1.1-1.6	1.6-5.3
Internal Models	2,276	0.1	0.7	1.5

Table 6: Comparing Expected Losses (EL), Unexpected Losses (UL) and VaR For the Middle Market Portfolio

- KMV estimated VaR at 3% and CreditMetrics at 1.6%
- Impact of DM vs. MTM
  - UL
    - KMV: 0.6-0.8 for DM and 1.1-1.6 for MTM
    - CreditMetrics: 0.4-0.5 for DM and 0.5 for MTM
  - VaR
    - KMV: 2.4-3.0 for DM and 1.6-5.3 for MTM
    - CreditMetrics: 1.6 for DM and 1.7-1.8 for MTM
  - EL: no significant impact on estimates. 0.5-0.8. But, significant difference for internal models 0.1.

Other comparative studies find that:

- Structurally, all credit risk models are similar.
- Where they differ is in their parameter assumptions. This makes a crucial difference.
- Smaller discrepancies in estimates occur when homogenous sub-portfolios (eg, high and low credit quality) are evaluated separately. That is, there are greater discrepancies across models in evaluating credit risk of low quality portfolios than for high quality portfolios. Fig. 9.1 – 9.3.

Figure 9.1 CreditMetrics vs. KMV's Portfolio Manager: Entire portfolio.

Source: Koyluoglu, Bangia and Garside (1999).

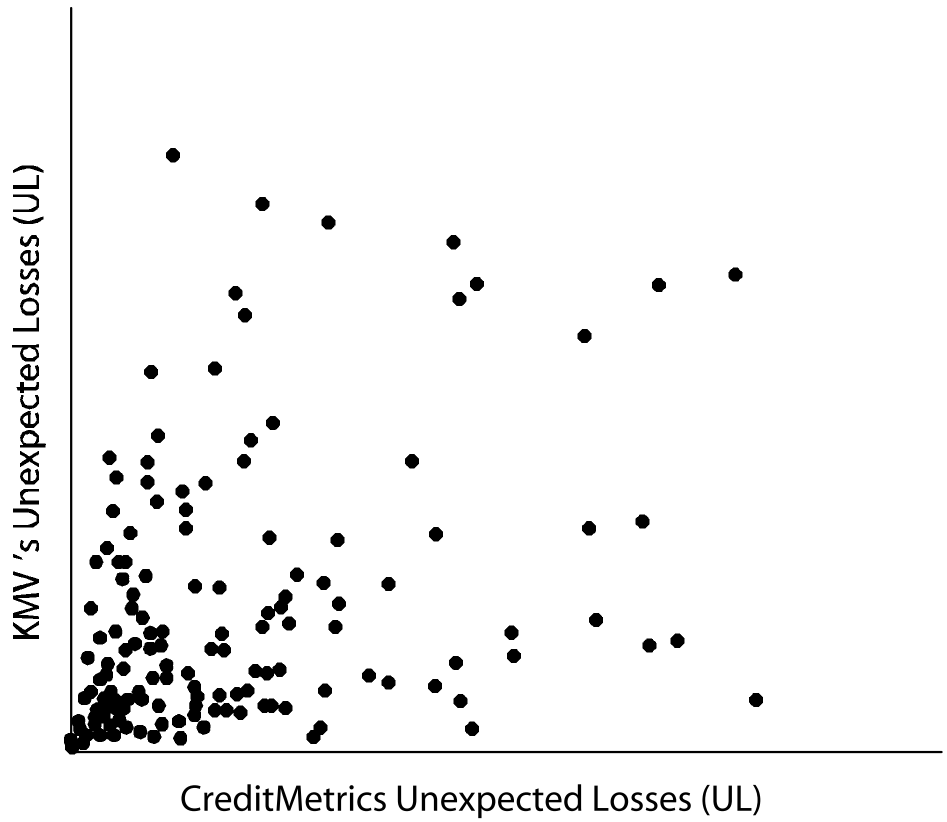


Figure 9.2 Unexpected losses for high credit quality portfolio.

Source: Koyluoglu, Bangia and Garside (1999).

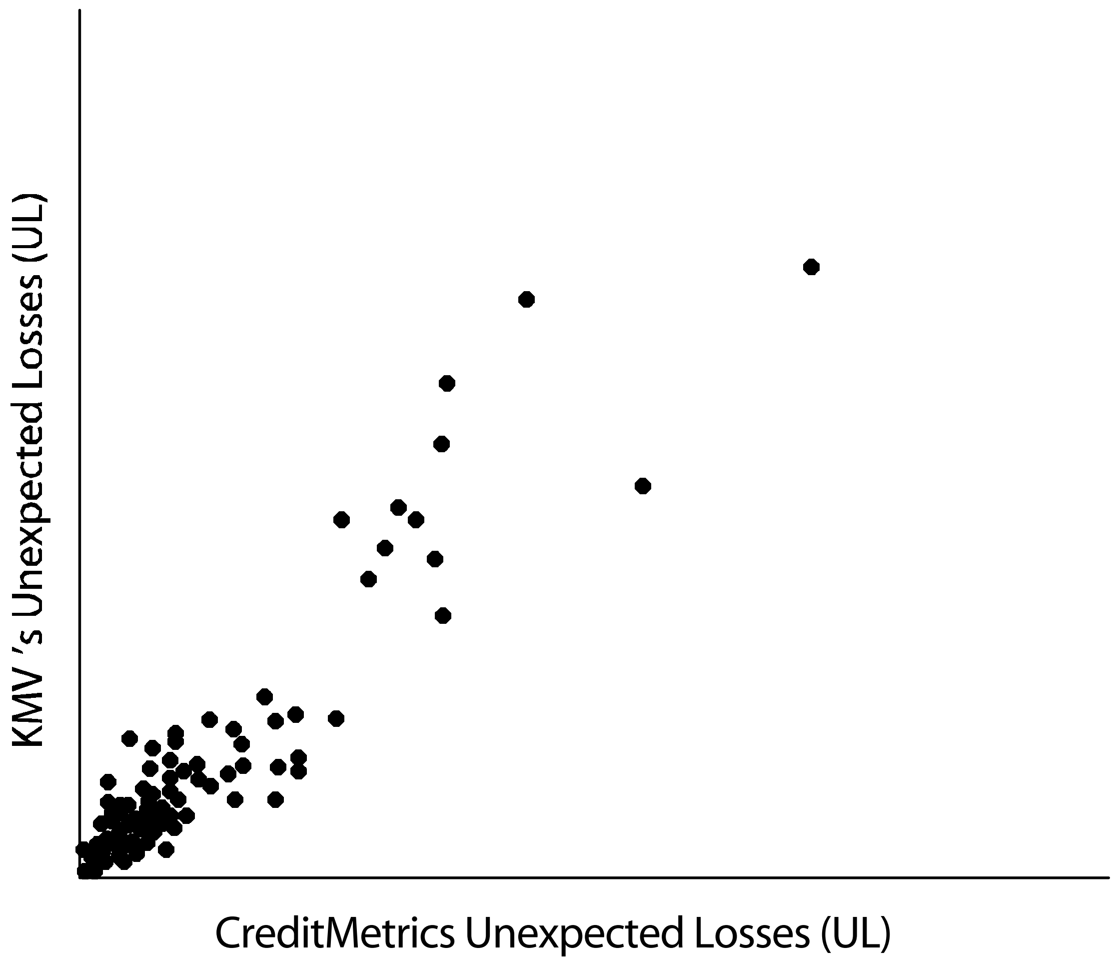


Figure 9.3 Unexpected losses for low credit quality portfolio.

Conclusions: The Road Ahead

- Modeling Improvements
  - Disentangle PD and LGD and consider correlations between PD, LGD and EAD
  - Integrate measurement of credit risk, market risk and operational risk.
- Data Improvements
  - Parameter Mistakes Can Be Costly – eg., BIS II Standardized Internal Ratings Based Model could have increased capital to 12%
  - Would improve backtesting and stress testing of models.