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Working Papers
RESEARCH DEPARTMENT

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WP 26-55

PUBLISHED
November 2026

ISSN: 1962-5361

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DOI: https://doi.org/10.21799/frbp.wp.2026.55

Fiscal Policy as a Nominal Anchor*

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October 28, 2026

Abstract

When aggregate demand is non-Ricardian, fiscal policy can anchor the long-run price level even while satisfying the government's intertemporal budget constraint for every possible price path. Therefore, it is possible to resolve the price-level indeterminacy problem under an interest rate rule without resorting to either of the most popular (but theoretically controversial) selection devices: (i) the Taylor principle coupled with ad hoc restrictions on explosive inflation paths, or (ii) the fiscal theory of the price level. I prove this point in a broad class of macroeconomic environments and illustrate it analytically in a Blanchard–Yaari-style overlapping-generations economy.

Keywords: Indeterminacy; Taylor principle; fiscal theory of the price level; Ricardian equivalence
JEL classification: E10, E40, E52

*I thank Adrien Auclert, Francesco Bianchi, Wei Cui (discussant), Sebastian Merkel, Gastón Navarro (discussant), Christian Wolf, and seminar participants at the Philadelphia Fed, the CESifo MMI Conference, the SAET Conference, and the Fed System Macro Conference for many helpful comments and discussions. **Disclaimer:** This Philadelphia Fed working paper represents preliminary research circulated for discussion. The views expressed are solely those of the author and do not necessarily reflect the views of the Federal Reserve Bank of Philadelphia or the Federal Reserve System. Any errors or omissions are the author's responsibility. Nothing in this paper should be treated as legal advice. Philadelphia Fed working papers are available at philadelphiafed.org.

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1 Introduction

What anchors the nominal price level in a modern fiat-money economy? Since Sargent and Wallace (1975), it is well-known that the price level can be indeterminate when the central bank implements monetary policy via an interest rate rule. The Fisher equation, written in logs, illustrates the central problem:

$$i_t = r_t + \mathbb{E}_t[p_{t+1} - p_t] \quad \Rightarrow \quad p_t = \sum_{s=0}^{\infty} \mathbb{E}_t[r_{t+s} - i_{t+s}] + \underbrace{\lim_{T\to\infty} E_t[p_{t+T}]}_{p_\infty}. \tag{1}$$

Interest rate policy can pin down relative prices across time, but some other ingredient is needed to pin down the long-run level of prices p_∞. An account of the nominal anchor, therefore, is crucial in understanding price-level stability and the ability of monetary and fiscal policies to control inflation.

A standard approach to this problem argues that an independent central bank can anchor price-level expectations by committing to aggressively adjust the nominal rate in response to inflation. If the central bank obeys the *Taylor principle* (Taylor, 1993), $i_t = i_t^* + \phi(p_t - p_{t-1})$ with $\phi > 1$, a boundedness criterion selects an equilibrium:[1] there typically exists a unique bounded path of prices consistent with equilibrium. In such settings, fiscal policy is often entirely irrelevant to equilibrium outcomes (*Ricardian equivalence*). This irrelevance follows from two ingredients: fiscal policy is assumed to passively adjust taxes to stabilize debt (a *Ricardian fiscal rule*, in the terminology of Woodford, 2001b), and household demand is invariant to the timing of taxes (*Ricardian households*).

The *fiscal theory of the price level* (FTPL) instead argues that fiscal policy plays a central role in price-level determination (see Leeper, 1991; Sims, 1994; and Woodford, 1995). Off-equilibrium, the fiscal authority need not adjust planned surpluses to debt levels (a *non-Ricardian fiscal rule*), since the monetary authority stands ready to print money to honor the government's nominal debt at par. In equilibrium, the price level satisfies the "government debt valuation equation": the aggregate real value of government debt must equal the present value of real government surpluses. Expectations of future prices are anchored by the fiscal backing of government debt.

Both of these approaches have been subject to theoretical controversies. Opponents of the Taylor principle approach contend that it is simply a threat to blow up the economy (Cochrane, 2005), or that a central bank would not be able to commit to an explosive price path (Neumeyer and Nicolini, 2025). Critics of the FTPL counter that the FTPL permits the government to run infeasible fiscal policies that violate its budget constraint (Buiter, 2002); that it is simply a different threat to blow up the economy (Kocherlakota and Phelan, 1999); and that, in heterogeneous-agent models, it is neither necessary nor sufficient to select an equilibrium (Hagedorn, 2024).

I demonstrate that in a large class of economies featuring *non-Ricardian households*, fiscal policy can provide a nominal anchor even when it is required to satisfy the government's intertemporal budget constraint off-equilibrium, i.e., the central bank is free to act independently. Just as in the FTPL, fiscal policy governs the price level while monetary policy determines inflation. At the same

[1] The boundedness assumption is typically motivated by arguments that explosive prices could imply infeasible explosive real outcomes (Schmitt-Grohé and Uribe, 2000) or by off-equilibrium changes in the monetary regime aimed at stabilizing prices (Wallace, 1981; Obstfeld and Rogoff, 1983; Christiano and Rostagno, 2001; Benhabib, Schmitt-Grohé, and Uribe, 2002; Atkeson, Chari, and Kehoe, 2010).

time, my approach circumvents the theoretical conundrums that have plagued the earlier literature. Specifically, I show that:

1. Fiscal policy can determine the long-run price level if an aggregate *transversality condition* (TVC) holds for aggregate demand. This condition corresponds roughly to economies in which rational bubbles are not possible in equilibrium, or $R > G$.

2. If household demand is non-Ricardian (in a "strong" sense that I define later), fiscal policy can accomplish this goal even if fiscal policy is Ricardian. On the other hand, if demand is Ricardian, then it is necessary to resort to non-Ricardian fiscal policy.

In this perspective, the debate about whether fiscal policy can stabilize long-run prices is an empirical question ("Do we observe bubbles?") rather than a philosophical one ("Is the FTPL logically coherent?"). The controversy over the FTPL is relevant only in the knife-edge case of Ricardian households.[2]

The general environment nests a wide variety of models studied in the literature, including overlapping-generations models, models with bonds in the utility function, behavioral models, and even some heterogeneous-agent New Keynesian (HANK) models. The basic intuition is that, in non-Ricardian economies, changes in the *timing* of government surpluses can affect aggregate demand. A temporary increase in government debt tends to be expansionary, so fiscal policy can target a long-run price level by committing to run deficits when prices are below target and raise surpluses when prices are above target. This strategy is exactly analogous to how fiscal policy would peg the price level in the FTPL.

I begin by introducing a general class of infinite-horizon economies with a single consumption good and a single type of asset: nominal government bonds. The environment is characterized by an aggregate consumption function (Farhi and Werning, 2019; Auclert, Rognlie, and Straub, 2024), an aggregate supply function, a generalized Taylor rule for the nominal rate, and a fiscal rule for taxes and debt issuance. The TVC is a property of the consumption function that restricts the private sector's long-run accumulation of assets. Demand is Ricardian if aggregate consumption depends only on the present value of taxes, whereas under my notion of *strong non-Ricardian demand* (SNRD), the *timing* of taxes matters, and deficits tend to stimulate demand. This property holds in many models in which bonds are effectively "net wealth" for the private sector, in the sense that the marginal propensity to consume (MPC) out of assets exceeds the MPC out of future taxes.

I study whether there exists a Ricardian fiscal rule that can select among the possible equilibria. Proposition 2 formalizes the results outlined above, and the subsequent analysis develops the underlying intuition.

When demand is Ricardian, attempting to stimulate demand by issuing debt that will be backed by future taxes is ineffective—the present value of taxes is unchanged. Fiscal policy can provide a nominal anchor only through *unbacked* changes in government debt, which are the essence of the FTPL. When demand is non-Ricardian, by contrast, it is no longer necessary to resort to the FTPL:

[2]Recent empirical evidence suggests household demand is non-Ricardian, see Eichenbaum, Guerreiro, and Obradovic (2026).

fiscal policy can control aggregate demand entirely through *fully backed* changes in government debt. The government can attempt to raise the price level by *temporarily* running deficits even when the private sector anticipates the additional taxes that must be levied in the future.

What might go wrong when the TVC fails?[3] Consider an economy with $R < G$. Under any fiscal rule that keeps the present value of surpluses finite, the debt-to-output ratio must go to zero (relative to GDP) in the long run. This class of fiscal rules, which includes both Ricardian fiscal rules and FTPL-style rules with an exogenous present value of surpluses, therefore cannot anchor the long-run price level.

These results illustrate that, in models where SNRD holds, there is effectively an equivalence between the classes of equilibria implemented by Ricardian and non-Ricardian fiscal rules. When the TVC holds, either type of rule can implement a desired equilibrium. When the TVC fails, then it is possible that neither type of rule can do so.

This equivalence sheds light on debates about the interactions between fiscal and monetary policy. For fiscal policy to provide a nominal anchor, all that is required is a failure of Ricardian equivalence. Previous work has highlighted various fragilities in an approach that breaks Ricardian equivalence via a non-Ricardian *fiscal regime*: for example, Niepelt (2004) questions whether the FTPL is compatible with rationality; McCallum and Nelson (2006) argue that FTPL equilibria are not learnable (in the sense of Evans and Honkapohja, 2001); and Angeletos and Lian (2023) highlight that when households are Ricardian, the nominal anchor hinges on strong common knowledge assumptions (a point further elaborated in Angeletos, Lian, Wolf, and Zhang, 2026). These criticisms may cast doubt on the tight connection between the government budget and inflation in the FTPL. Nevertheless, I demonstrate that a broad message of the FTPL is robust — when aggregate demand is non-Ricardian, fiscal policy can play a key role in determining the price level, regardless of how monetary policy is conducted.

I then turn to a specific example: a standard Blanchard–Yaari overlapping-generations model with an exogenous endowment that grows at rate G. An in-depth analysis illustrates concretely how fiscal policy can anchor the price level in a familiar model and resolves several theoretical loose ends left by the general discussion. Households have log utility, die with a constant probability each period, and receive individual income that may grow more slowly than G. Markets are incomplete because households cannot trade with yet-to-be-born generations, breaking Ricardian equivalence. When individual income growth is low relative to G, excess demand for savings raises the possibility of bubble equilibria in which $R < G$. When the government balances its budget each period, the price level is indeterminate for all t, as in canonical monetary models with complete markets.

I show that, in this model, fiscal policy can select any *non-bubble* equilibrium with $R > G$ in the long run. Agents have log utility, so they consume a constant fraction of their total wealth, consisting of bond holdings plus the present value of future income net of taxes. With overlapping generations, bonds represent net wealth for households: some of the taxes required to repay bondholders will

[3]Proposition 2 does not show that the TVC is a *necessary* condition for fiscal policy to act as a nominal anchor. In fact, there are economies in which the TVC fails and fiscal policy can nevertheless select an equilibrium. Here I instead illustrate an intuition about why fiscal policy might fail to anchor the price level in *some* models where the TVC fails, as in the OLG model of Section 4.

fall on future generations. The government runs deficits or surpluses as required to target a given level of nominal wealth and, therefore, a level of nominal demand consistent with a long-run price level target. There is an analogy with money-supply rules. In the quantity theory, $MV = PY$ with constant V implies that the government can target nominal income with a money-supply rule. Here, by contrast, nominal income PY is a constant fraction of aggregate nominal wealth, so policy targets nominal wealth instead. The main result would generalize even if the marginal propensity to consume depended on the path of interest rates. Such a fiscal policy cannot, however, select a bubble equilibrium with $R < G$, which here corresponds to an equilibrium with sufficiently high inflation. In such an equilibrium, the value of government debt exceeds the present value of taxes, so the government cannot target it while committing to repay its debt in the long run.

Related literature. It would be impossible to do justice to the vast literature on determinacy in monetary economies under interest rate targets, starting with Sargent and Wallace (1975). I therefore focus on the areas most closely related to the paper and cite surveys and books that provide useful points of departure.

My paper builds on an extensive body of work on interactions between monetary and fiscal policy. Sargent and Wallace (1981) provide one of the seminal results on these interactions, demonstrating that even under a money-supply rule, a central bank cannot control inflation if it is eventually forced to monetize government debt. Canzoneri, Cumby, and Diba (2011), and later Cochrane (2023), survey results on the FTPL. Motivated by this literature, Bianchi, Faccini, and Melosi (2023) develop a structural model of "fiscal inflations" and fit it to U.S. data. Bassetto and Sargent (2020) provide a broader perspective on monetary–fiscal coordination, covering both economies with commodity standards and modern fiat-money economies. I study monetary-fiscal interactions in a general environment in which I focus on non-Ricardian *aggregate demand* rather than a non-Ricardian *fiscal regime*.

The strand of the literature closest to my paper studies determinacy in such economies. Relative to earlier work, my main contribution is to provide a *general characterization* of how a Ricardian fiscal rule can select a *globally* unique equilibrium. Hagedorn (2024) demonstrates that in a canonical Huggett model, if fiscal policy controls the growth rate of nominal liabilities, there exists a unique price level in a *steady-state* equilibrium with positive debt (a point also noted by Kaplan, 2025). The mechanism for price determinacy in my results, which runs through aggregate demand rather than the government debt valuation equation, is similar. However, the result in Hagedorn (2024) does not carry over to the broader class of economies I study or to global determinacy – to provide a nominal anchor, fiscal policy generally must actively manage surpluses in response to price-level fluctuations.[4] Rachel and Ravn (2025) take a different approach: in an overlapping generations HANK economy, they highlight how breaking Ricardian equivalence introduces new interactions between monetary and fiscal policy, which in turn alters the conditions required for *local* determinacy

[4] Canzoneri and Diba (2005) also consider local and global determinacy in a model where bonds and money are valued for their liquidity. Equilibria can be unique when money demand takes a particular form.

under the boundedness criterion.[5]

Another body of work studies the interactions between fiscal policy and *inflation* in economies with non-Ricardian households, while the nominal anchor is provided by the Taylor principle or some other conventional equilibrium selection device (Nisticò, 2012; Kaplan, Nikolakoudis, and Violante, 2023). Recently, Angeletos, Lian, and Wolf (2024) demonstrate that the OLG model I use can generate precisely the same response of inflation and output to deficits as in the canonical FTPL, provided deficits are financed by taxes far in the future. My results are complementary: they study the transmission of deficits to real activity and inflation *in equilibrium*, holding the nominal anchor fixed.[6] I instead demonstrate how a suitably designed fiscal policy can *select an equilibrium* and provide a nominal anchor.

Organization. Section 2 sets up the general environment. Section 3 states the main result and develops the intuition by linearizing around a balanced growth path. Section 4 studies a nonlinear OLG economy in depth and shows how fiscal policy can anchor the price level in a specific economy. Section 5 concludes. The Appendix contains all proofs and supplementary results.

2 A General Class of Models

I begin by specifying a general class of models that will be studied throughout the analysis. The overlapping generations model that I analyze in depth is nested in this class. I define key concepts: the aggregate transversality condition, Ricardian demand, and Ricardian fiscal policy.

2.1 Environment

Time is discrete and infinite, $t \in \{0, 1, 2, \dots\}$. There is a single consumption good. The model has three "blocks": a demand block determining the quantity of goods C_t consumed by households, a supply block determining output Y_t produced by firms, and a government policy block determining the nominal interest rate $1 + i_t$, nominal government debt B_t, and aggregate real taxes τ_t. There is no aggregate uncertainty.

The nominal price level is denoted P_t, and inflation between t and $t+1$ is defined by $1 + \pi_{t+1} = P_{t+1}/P_t$. The real interest rate on an investment from t to $t+1$ is then $R_t \equiv (1 + i_t)/(1 + \pi_{t+1})$. Throughout, it will be convenient to define

$$I_{t,t+s} \equiv \prod_{u=0}^{s-1} (1 + i_{t+u}), \qquad R_{t,t+s} \equiv \prod_{u=0}^{s-1} R_{t+u},$$

for $s > 0$, as the gross nominal and real returns, respectively, on an investment from t to $t+s$, with the convention $I_{t,t} = R_{t,t} = 1$.

[5] Cushing (1999) takes an early approach to global determinacy in an OLG model, demonstrating that a particular fiscal rule does not ensure determinacy under an interest rate peg. I derive an alternative rule that ensures determinacy.

[6] They use a device similar to the minimum-state variable (MSV) equilibrium selection in McCallum (1981).

Demand block. The demand block consists of a continuum of households that consume and save in nominal assets issued by the government. The aggregate real asset holdings of the household sector, A_t, evolve according to

$$A_{t+1} = R_t(A_t + Y_t - \tau_t - C_t). \tag{2}$$

As in Farhi and Werning (2019), Auclert, Rognlie, and Straub (2024), or Wolf (2025), I characterize optimal household behavior via an *aggregate consumption function*

$$C_t = \mathcal{C}_t(A_0, \boldsymbol{Y}, \boldsymbol{\tau}, \boldsymbol{R}), \tag{3}$$

where bold letters denote the full time path of a variable; for example, $\boldsymbol{Y} \equiv \{Y_0, Y_1, \dots\}$. Thus, the path of consumption depends on all terms in the private-sector budget constraint (2): initial private-sector asset holdings, the paths of income and taxes, and the path of real interest rates. Such a consumption function can be written down in almost *any* standard macroeconomic model—the inclusion of future realizations of income, taxes, and other variables does not imply that agents hold rational expectations about those outcomes. Indeed, Farhi and Werning (2019) show how the consumption function (3) can capture behavioral distortions.

Two types of properties of the aggregate consumption function will be central to the analysis. The first are TVC/no-Ponzi conditions on aggregate demand, which dictate the rate at which the private sector optimally accumulates assets in the long run.

Definition 1. *The aggregate consumption function* $\boldsymbol{\mathcal{C}}(\cdot)$ *satisfies an* ***aggregate transversality condition*** *(respectively,* ***no-Ponzi condition****) if, whenever* $\boldsymbol{A}$ *satisfies* (2) *and* $\boldsymbol{C}$ *satisfies* (3)*, then*

$$\lim_{T\to\infty} R_{0,T}^{-1} A_T \leq 0 \quad (\textit{resp.} \quad \lim_{T\to\infty} R_{0,T}^{-1} A_T \geq 0) \tag{4}$$

for all $(A_0, \boldsymbol{Y}, \boldsymbol{\tau}, \boldsymbol{R})$*.*

This aggregate TVC is satisfied by most standard models in which rational bubbles are not possible, including those with heterogeneous agents or behavioral distortions. On the other hand, the TVC fails in models where the aggregate supply of savings is large enough to give rise to bubbles, as in Diamond (1965). The no-Ponzi condition is satisfied in models in which the private sector cannot indefinitely roll over debt without aggregate repayment. It is typically imposed on *individual* households, but, as the OLG model presented below demonstrates, an individual no-Ponzi constraint need not imply an aggregate no-Ponzi condition.

The second key property of the consumption function dictates how aggregate demand relates to the timing of taxes. I say that *demand is Ricardian* if demand depends only on the present value of taxes rather than their timing.[7]

[7]Notice that under this definition, the aggregate consumption function is not defined when the present value of taxes diverges. Indeed, in typical representative-agent models, the consumption-savings problem does not have a well-defined solution in this case.

Definition 2. *The aggregate consumption function satisfies the* ***Ricardian demand*** *property if there exist functions* $\tilde{\mathcal{C}}_t$ *such that*

$$\mathcal{C}_t(A_0, \boldsymbol{Y}, \boldsymbol{\tau}, \boldsymbol{R}) = \tilde{\mathcal{C}}_t\left(A_0, \boldsymbol{Y}, \sum_{s=0}^{\infty} R_{0,s}^{-1}\tau_s, \boldsymbol{R}\right).$$

Demand is Ricardian in many canonical models, including canonical representative-agent monetary models and overlapping generations models with bequest motives (Barro, 1974).

The main result draws a contrast between the case of Ricardian demand, when the timing of taxes does not matter, and a notion that I term *strongly non-Ricardian demand (SNRD)*, in which the timing of taxes directly affects the timing of consumption. To formally define this property, it will be necessary to define a notion of back-loaded taxes. I say that a sequence $\boldsymbol{x}$ is more *back-loaded* than a sequence $\boldsymbol{x}'$ if, given a sequence of real interest rates $\boldsymbol{R}$, the following hold:

1. The sequences $\boldsymbol{x}$ and $\boldsymbol{x}'$ have the same present value, $\lim_{T\to\infty} \sum_{t=0}^{T} R_{0,t}^{-1}(x_t - x'_t) = 0$;
2. There exists t^* such that $x_t \leq x'_t$ for $t \leq t^*$ and $x_t > x'_t$ for $t > t^*$.

The definition of SNRD follows.

Definition 3. *The consumption function satisfies the* ***strongly non-Ricardian demand (SNRD)*** *property if, whenever* $\boldsymbol{\tau}$ *is more back-loaded than* $\boldsymbol{\tau}'$*, then* $\boldsymbol{\mathcal{C}}(A_0, \boldsymbol{Y}, \boldsymbol{\tau}, \boldsymbol{R})$ *is less back-loaded than* $\boldsymbol{\mathcal{C}}(A_0, \boldsymbol{Y}, \boldsymbol{\tau}', \boldsymbol{R})$.

Intuitively, this notion implies that government debt issuance stimulates the economy: when agents expect taxes to be postponed, they consume more in the present. In the OLG model of Section 4, this property holds because government debt is effectively net wealth for the private sector. SNRD also holds in several other standard non-Ricardian models in the literature.[8]

Supply block. The supply side consists of firms whose aggregate supply schedule satisfies

$$Y_t = \mathcal{Y}_t(\boldsymbol{P}) \quad \text{with} \quad G^{-T}Y_T \text{ bounded as } T \to \infty. \tag{5}$$

That is, output depends on the sequence of nominal prices and grows at gross rate no faster than G in the long run. This assumption is consistent with both flexible- and sticky-price specifications. In a flexible-price setting, output is typically exogenous: it may come from an exogenous endowment, as in the OLG model presented later, or from static labor–leisure optimization and firms' first-order conditions—for example, $W_t u'(Y_t) = v'(L_t)$ with $W_t = 1$ and $Y_t = L_t$. With sticky prices, one often obtains expressions relating output directly to the path of nominal prices. For example, the linearized New Keynesian Phillips curve can be written as $\hat{y}_t = (\hat{\pi}_t - \beta\hat{\pi}_{t+1})/\kappa$, where $\hat{\pi}_t \equiv \hat{p}_t - \hat{p}_{t-1}$. Kocherlakota (2022) demonstrates that the functional form (5) can capture a wide variety of

[8] Appendix D explicitly derives similar results in other non-Ricardian environments: D.1 considers a model where bonds provide liquidity services, D.2 studies a two-agent TANK model with hand-to-mouth consumers, and D.3 augments a representative-agent model with cognitive discounting, as in Gabaix (2020).

sticky-price specifications in which the short- and long-run slopes of the Phillips curve differ.[9]

Government. The government sets the nominal interest rate and issues debt financed with taxes. The nominal rate follows a generalized Taylor-type rule

$$1 + i_t = \Phi_t(\boldsymbol{P}, \boldsymbol{Y}) \quad \text{with} \quad \frac{1}{T}\sum_{t=0}^{T} |\log(1 + i_t)| \ \text{ bounded as } \ T \to \infty. \tag{6}$$

That is, the nominal rate in any period can depend on the entire sequence of prices and output.[10] This formulation allows for sophisticated rules that incorporate arbitrary lags of inflation or expectations of future macroeconomic outcomes. The bounded average assumption is a technical criterion ensuring that long-maturity nominal bond prices are defined along any path, but it is not a "fiscal dominance" assumption as in Leeper (1991).

The government begins at $t = 0$ with an outstanding portfolio of debt consisting of $\overline{B}_t$ nominal bonds that mature at time t. The initial nominal value of government debt is then

$$B_0 = \overline{B}_0 + \sum_{t=1}^{\infty} I_{0,t}^{-1} \overline{B}_t. \tag{7}$$

From $t = 0$ forward, the nominal value of the government debt portfolio evolves according to

$$B_{t+1} = (1 + i_t)(B_t - P_t \tau_t). \tag{8}$$

I define a fiscal rule as a path of taxes that, in principle, can depend on the entire path of prices, nominal rates, and output,

$$\tau_t = \mathcal{T}_t(\boldsymbol{P}, 1 + \boldsymbol{i}, \boldsymbol{Y}). \tag{9}$$

Like the path of nominal rates, the path of taxes may depend on the complete paths of the variables in the rule. Even though nominal rates are determined by the other variables through the generalized Taylor rule (6), it is important for fiscal policy to condition on nominal rates independently. This ensures that the government's intertemporal budget constraint can hold after any history, rather than only for equilibrium sequences $1 + \boldsymbol{i}$. I call a policy satisfying this constraint a *Ricardian fiscal rule*.

Definition 4. *A fiscal rule $\boldsymbol{\mathcal{T}}(\cdot)$ is **Ricardian** (respectively, **strictly Ricardian**) if*

$$\lim_{T \to \infty} I_{0,T}^{-1} B_T \leq 0 \quad (\textit{resp.} \ \ = 0) \quad \forall \quad (\boldsymbol{P}, 1 + \boldsymbol{i}, \boldsymbol{Y}). \tag{10}$$

In typical formulations of the FTPL, fiscal policy does not satisfy this constraint off equilibrium.

[9] Richer specifications of the supply curve are possible as well. For instance, the supply curve could include other variables in the consumption function (3). This would be appropriate if future income, interest rates, and other variables influenced households' optimal labor supply. What is key for the results is that taxes cannot enter the interest rate rule (6), even implicitly through output Y_t.

[10] The paths of taxes or government debt are not permitted to directly influence nominal rate decisions. I make this assumption for analytical clarity: Ricardian equivalence will then hold whenever both fiscal policy and aggregate demand are Ricardian.

Much of the theoretical debate surrounding the FTPL concerns whether this constraint is a necessary ingredient of any reasonable model of fiscal policy.

Market clearing and equilibrium. The market-clearing conditions for goods and assets are

$$C_t = Y_t \quad \forall\, t, \tag{11}$$

$$A_t = B_t/P_t \quad \forall\, t. \tag{12}$$

Equilibria with and without bubbles are defined below.

Definition 5. *An **equilibrium** consists of sequences* $\{C_t, Y_t, A_t, B_t, \tau_t, P_t, 1+i_t\}$ *such that equations* (2)–(3)*,* (5)*,* (6)–(9)*, and* (11)–(12) *hold, and* $P_t > 0$ *for all* t*. A **bubble equilibrium** satisfies the additional condition*

$$\lim_{T\to\infty} I_{0,T}^{-1} B_T > 0. \tag{13}$$

Given a consumption function $\boldsymbol{\mathcal{C}}(\cdot)$, a supply function $\boldsymbol{\mathcal{Y}}(\cdot)$, and government policy rules $(\boldsymbol{\Phi}(\cdot), \boldsymbol{\mathcal{T}}(\cdot))$, the *equilibrium set* consists of all triples $(\boldsymbol{P}^*, 1+\boldsymbol{i}^*, \boldsymbol{Y}^*)$ consistent with some equilibrium. There is *equilibrium multiplicity* when the equilibrium set contains more than one element. Fiscal policy is said to *implement* $(\boldsymbol{P}^*, 1+\boldsymbol{i}^*, \boldsymbol{Y}^*)$ if that is the only element of the equilibrium set.

3 The Main Result

Throughout this section, I consider whether fiscal policy can implement a desired equilibrium, thereby providing a nominal anchor. I state the main result and then log-linearize around a balanced growth path to illustrate the underlying intuition.

3.1 When Can Fiscal Policy Provide a Nominal Anchor?

To set the stage, I begin with a familiar irrelevance result: the *Ricardian equivalence* proposition.

Proposition 1 (Ricardian equivalence)**.** *Suppose the consumption function* $\boldsymbol{\mathcal{C}}(\cdot)$ *satisfies the Ricardian demand property. Then given two strictly Ricardian fiscal rules* $\boldsymbol{T}, \boldsymbol{T}'$*, the equilibrium set is the same under either fiscal rule.*

When both fiscal policy and aggregate demand are Ricardian, fiscal policy is powerless to stabilize the price level. Under a Ricardian fiscal policy, the present value of taxes is fixed (including off-equilibrium) by the initial real value of government debt. When demand is Ricardian, a change in the timing of taxes cannot affect demand, so fiscal policy is effectively irrelevant to equilibrium outcomes.

The main result characterizes the conditions under which fiscal policy can select an equilibrium. When the TVC holds, it is possible for fiscal policy to select an equilibrium if *either* of the key assumptions underlying Ricardian equivalence fails: either the fiscal rule *or* aggregate demand must be non-Ricardian.

Proposition 2. *Suppose that under a fiscal rule $\boldsymbol{\mathcal{T}}_0(\cdot)$, there is equilibrium multiplicity, and consider a desired equilibrium with prices $\boldsymbol{P}^*$, nominal rates $1+\boldsymbol{i}^*$, and output $\boldsymbol{Y}^*$. If the consumption function satisfies the TVC, then there exists a fiscal rule $\boldsymbol{\mathcal{T}}(\cdot)$ that uniquely implements it.*

- *If the consumption function is Ricardian and satisfies no-Ponzi, then $\boldsymbol{\mathcal{T}}(\cdot)$ cannot be strictly Ricardian.*
- *If the consumption function satisfies SNRD, then $\boldsymbol{\mathcal{T}}(\cdot)$ can be chosen to be Ricardian.*

The first route, a non-Ricardian *fiscal regime*, is well-known. Canonical formulations of the FTPL feature a representative household sector. The consumption function satisfies the TVC/no-Ponzi conditions (4), which are combined with the equilibrium condition $C_t = Y_t$ to yield a *government debt valuation equation*,

$$\frac{B_t}{P_t} = \sum_{s=0}^{\infty} R_{t,t+s}^{-1} \tau_{t+s}. \tag{14}$$

The left-hand side represents the private sector's real wealth in government bond holdings, and the right-hand side corresponds to its total tax liabilities. In equilibrium, these two must be equal for the *level* of consumption demand C_t to equal supply Y_t at all dates. Off-equilibrium, taxes are not constrained to satisfy this condition, so if, for example, the real present value of taxes is less than B_t/P_t, demand will exceed supply, so prices P_t must rise to restore equilibrium.

The second route is novel: even a Ricardian fiscal rule can select an equilibrium when the assumption of *Ricardian demand* is relaxed instead. The basic idea is that under SNRD, deficits financed by taxes in the future stimulate demand in the present but depress demand in the long run. If the price level is too low, for example, the government can run deficits, issue debt, and pay the debt back in the future to attempt to bring the price level up. The government effectively sets an upward-sloping supply curve for nominal debt, issuing more when the real value of a nominal unit, $1/P_t$, is high. This strategy successfully implements the target equilibrium if debt supply is above (below) asset demand whenever the nominal price level is too low (high). A problem arises only if the TVC does not hold: then, sometimes asset demand is on a trajectory such that

$$\lim_{T\to\infty} R_{0,T}^{-1} A_T > 0,$$

meaning that if the government tries to stimulate demand by issuing more debt, it will violate the Ricardian constraint on policy (10).

In the OLG model, I prove a partial converse to Proposition 2: when the TVC fails, then *no* fiscal policy in which taxes have finite present value can select a unique equilibrium in a stable way. When rational bubbles are possible, the size of the bubble is an additional source of indeterminacy that fiscal policy on its own cannot resolve. In Sections 3.2-3.3, I also demonstrate why the intuition behind this result is likely to carry over to other models as well.

Hence, the main results demonstrate that the question of whether fiscal policy can anchor the price level is nearly equivalent to the question of whether an aggregate TVC holds (which, in turn, typically corresponds to economies in which rational bubbles are not possible). The question of whether the government can violate an intertemporal budget constraint, on the other hand, is

secondary: it is relevant only in the subset of models where demand is Ricardian. When demand is non-Ricardian in the strong sense, the FTPL and Ricardian fiscal rules implement equivalent sets of equilibria.

On its own, Proposition 2 does not give much guidance as to whether the fiscal policies it prescribes are natural or realistic. In the rest of the paper, I demonstrate that when demand is non-Ricardian, it is often possible to anchor prices with a simple class of policies: the government cuts deficits and reduces debt when prices are above target, and it does the opposite when prices are below target. Hence, the policies look familiar to those in the FTPL literature except that they do not violate the Ricardian constraint.

In the remainder of this section, I linearize around a balanced growth path in the general model to outline the logic behind why such policies can actually work in a wide variety of models. Then, I explicitly work out an example of such a policy in a perpetual-youth OLG model to illustrate how the shortcomings and formal loose ends that arise with a linear approximation can be addressed in a full, nonlinear model.

3.2 A Decomposition of Demand in the Log-Linearized Model

I make several simplifications to streamline the exposition. First, I linearize around a balanced growth path (BGP) in which output Y_t, taxes τ_t, and real debt $b_t \equiv B_t/P_t$ grow at rate G. The nominal interest rate is constant at $1+i^*$, and gross inflation is constant at $1+\pi^*$, so the gross real interest rate is $R = (1+i^*)/(1+\pi^*)$. It is convenient to work with log deviations from the BGP, denoted by lowercase variables with hats; for example, $\hat{c}_t \equiv (C_t - C_t^*)/C_t^*$ (where C_t^* is consumption along the BGP), $\hat{b}_t \equiv (B_t - B_t^*)/B_t^*$, and so on. Throughout, I consider balanced growth paths with positive debt, $A_t = G^t A_0$ for $A_0 > 0$. Hence, the TVC implies $R > G$. I contrast this case with the possibility that $R < G$, which I refer to loosely as a failure of the TVC.

Second, I specialize to a purely *forward-looking* consumption function that depends only on current assets and future realizations of output, taxes, and real interest rates,[11]

$$C_t = \mathcal{C}(A_t, \{Y_{t+s}, \tau_{t+s}, R_{t+s}\}) \quad \text{for some function } \mathcal{C}(\cdot).$$

I further assume that $\mathcal{C}(\cdot)$ is homothetic in $(A_t, \{Y_{t+s}, \tau_{t+s}\})$ and that the MPC out of current assets is equal to the MPC out of current transfers, $\partial \mathcal{C}_t/\partial A_t = -\partial \mathcal{C}_t/\partial \tau_t$. With these assumptions, it is possible to write down the key identity relating changes in demand to fiscal policy, interest rates, and output:[12]

$$\hat{c}_t = \omega \frac{\partial \mathcal{C}}{\partial A}\hat{a}_t + \sum_{s=0}^{\infty} \left(\frac{G}{R}\right)^s \left(R^s \frac{\partial \mathcal{C}}{\partial Y_s}\hat{y}_{t+s} + \omega\left(1 - \frac{G}{R}\right) R^s \frac{\partial \mathcal{C}}{\partial \tau_s}\hat{\tau}_{t+s} \right) + \sum_{s=0}^{\infty} \frac{R}{Y^*}\frac{\partial \mathcal{C}}{\partial R_s}\hat{r}_{t+s}, \tag{15}$$

where $\omega \equiv A_t^*/Y_t^*$ denotes the debt-GDP ratio along the BGP, $\partial \mathcal{C}/\partial Y_s$ denotes the partial derivative of C_t with respect to Y_{t+s}, and so forth.

[11] Note that this is just a special class of consumption functions of the form (3) that can be derived from combining $\mathcal{C}(\cdot)$ with (2), the private sector's period budget constraint.

[12] To find the coefficient on the $\partial \mathcal{C}/\partial R_s$ term, it is necessary to use the fact that $\mathcal{C}(\cdot)$ is homothetic in $(A_t, \{Y_{t+s}, \tau_{t+s}\})$.

I characterize the conditions under which fiscal policy can "anchor" the long-run price level in the following sense. If the target price path is $P_T^* = (1+\pi^*)^T P_0^*$, the objective of fiscal policy is to anchor *nominal* consumption toward $P_t^* Y_t$. Policy accomplishes this goal if nominal demand $\hat{p}_t + \hat{c}_t$ falls below (resp. above) nominal income $\hat{p}_t + \hat{y}_t$ whenever prices are too high (resp. low) relative to the target path. That is, for some $\kappa > 0$,

$$\lim_{T\to\infty} \left(\hat{c}_T \left(\hat{b}_T - \hat{p}_T, \{\hat{y}_{T+s}, \hat{\tau}_{T+s}, \hat{r}_{T+s}\} \right) - \hat{y}_T \right) = -\kappa \lim_{T\to\infty} \hat{p}_T,$$

so that prices must converge to target in any equilibrium, $\hat{p}_T \to 0$ if $\hat{c}_t = \hat{y}_t$ for all t. Fiscal policy enters demand through both real asset holdings $\hat{a}_t = \hat{b}_t - \hat{p}_t$ and taxes $\hat{\tau}_t$. The parameter κ indexes how aggressively fiscal policy attempts to anchor nominal demand.

To illustrate how fiscal policy can anchor demand, I combine (15) with the government's flow budget constraint to derive an intuitive decomposition of aggregate demand. In doing so, it is necessary to draw out the implications of Definitions 1-4 in this linearized setting.

Ricardian fiscal rules and the TVC. The Ricardian constraint on fiscal policy and the TVC can be expressed as nominal present-value relationships that put constraints on the behavior of fiscal policy and aggregate demand, respectively. It will be useful to denote the *discounted nominal values* of variables with tildes. For example, $\tilde{y}_t \equiv \hat{p}_t + \hat{y}_t - \sum_{s=0}^{t-1} \hat{i}_s$ denotes the log deviation of discounted nominal income at t, $P_t Y_t / I_{0,t}$. This is the $t = 0$ nominal price of a claim on nominal output at time t. Similarly, $\tilde{b}_t \equiv \hat{b}_t - \sum_{s=0}^{t-1} \hat{i}_s$ denotes the log deviation of $B_t / I_{0,t}$, nominal debt at t discounted by the nominal return through t, $\tilde{\tau}_t$ denotes the log deviation of discounted nominal taxes $P_t \tau_t / I_{0,t}$, and so on.

The government's nominal flow budget constraint (8) linearizes as

$$\hat{b}_{t+1} = \hat{i}_t + \frac{R}{G}\hat{b}_t - (\frac{R}{G} - 1)(\hat{p}_t + \hat{\tau}_t),$$

which can then be written in terms of discounted nominal values and iterated forward to obtain

$$\tilde{b}_t = \underbrace{\sum_{s=0}^{\infty} (\frac{G}{R})^s (1 - \frac{G}{R}) \tilde{\tau}_{t+s}}_{\text{PV[Taxes]}} + (\frac{R}{G})^t \underbrace{\lim_{T\to\infty} (\frac{G}{R})^{t+T} \tilde{b}_{t+T}}_{\tilde{b}_\infty}. \tag{16}$$

The left-hand side is the (discounted) nominal value of debt at time t. The right-hand side consists of two components: the nominal present value of all future taxes, the sum over $P_t \tau_t / I_{0,t}$, plus an "unbacked" component of the debt, denoted $\tilde{b}_\infty$. The fiscal rule is Ricardian if all debt is backed along any path,

$$\text{Ricardian fiscal rule } \Rightarrow \ \tilde{b}_\infty = 0.$$

The aggregate TVC yields a similar relationship between assets and the present value of consumption, income, and taxes. The private sector budget constraint can be linearized in nominal

terms to obtain

$$\tilde{a}_t = \sum_{s=0}^{\infty} (\frac{G}{R})^s (\omega^{-1}(\tilde{c}_{t+s} - \tilde{y}_{t+s}) + (1 - \frac{G}{R})\tilde{\tau}_{t+s}) + (\frac{R}{G})^t \underbrace{\lim_{T\to\infty} (\frac{G}{R})^{t+T} \tilde{a}_{t+T}}_{\tilde{a}_\infty}, \tag{17}$$

and the TVC is the condition $\tilde{a}_\infty = 0$. All savings are eventually used to finance consumption or tax payments, so the private sector does not roll over savings indefinitely. This limit condition effectively imposes a restriction on the relevant domain of linearization: if nominal income or taxes diverge too quickly, it is not possible to ensure $\tilde{a}_\infty = 0$. Hence, in this setting, (discounted) nominal income and taxes must have finite present value — that is, they must be summable when discounted by the factor G/R:

$$\text{TVC} \;\Rightarrow\; R > G \;\text{ and }\; \tilde{\boldsymbol{y}}, \tilde{\boldsymbol{\tau}} \in \ell^1_{G/R} \equiv \left\{ \boldsymbol{x} \in \mathbb{R}^{\mathbb{N}} : \sum_{t=0}^{\infty} (\frac{G}{R})^t |x_t| < \infty \right\}.$$

I denote the set of such sequences by $\ell^1_{G/R}$. Typically, in models where the TVC holds, this restriction can be derived from the global formulation of the model: agents' individual optimality conditions are impossible to satisfy when the present value of income diverges. Under the TVC, then, the primitives of the model yield a natural domain for linearization, whereas the more typical boundedness criterion is usually motivated by an appeal to off-equilibrium changes in the policy regime.[13]

The demand decomposition. The expansion of aggregate demand (15) can be combined with the linearized government budget constraint (16) and the goods market clearing condition $\tilde{c}_t = \tilde{y}_t$ to obtain

$$\tilde{y}_t = \underbrace{\overbrace{\omega \sum_{s=1}^{\infty} (\frac{G}{R})^s (R^s \frac{\partial \mathcal{C}}{\partial \tau_s} - R^{s-1} \frac{\partial \mathcal{C}}{\partial \tau_{s-1}}) \tilde{b}_{t+s}}^{\text{Backed debt}} + \overbrace{(\frac{R}{G})^t \omega \frac{\partial \mathcal{C}}{\partial A} \tilde{b}_\infty}^{\text{Unbacked debt}} + \overbrace{\sum_{s=0}^{\infty} (\frac{G}{R})^s R^s \frac{\partial \mathcal{C}}{\partial Y_s} \tilde{y}_{t+s}}^{\text{Nominal income}}}_{\text{Nominal wealth}} + \underbrace{\sum_{s=0}^{\infty} \gamma_s \hat{r}_{t+s}}_{\text{MPC}(\boldsymbol{r})}, \tag{18}$$

where γ_s represent the coefficients derived from (15) for the real interest rate terms.[14] The left-hand side is nominal income, and the right-hand side is a decomposition of aggregate nominal demand.

This equilibrium condition is a generalization of the FTPL's government debt valuation equation (14). Aggregate demand can be broken into two components: a set of terms that depend on nominal wealth (the first three terms), and a marginal propensity to consume out of wealth (the term MPC($\boldsymbol{r}$), which depends on real interest rates). There is a natural analogy to models in which

[13] One might wonder whether this restriction on the domain of admissible paths is truly different from the boundedness condition that prohibits nominal explosions in standard models. The difference is that $\tilde{\boldsymbol{y}}, \tilde{\boldsymbol{\tau}}$ are both the sum of an initial *price level* $\hat{p}_0$ plus a real variable, e.g., $\tilde{y}_t = \hat{p}_0 + \hat{y}_t - \sum_{s=0}^{t-1} \hat{r}_s$. Quickly diverging paths for these variables are then *real* rather than nominal explosions that prevent agents from optimizing.

[14] These coefficients are given by $\gamma_s \equiv \frac{R}{Y^*} \frac{\partial \mathcal{C}}{\partial R_s} + \sum_{u=s+1}^{\infty} \left(\frac{G}{R}\right)^u \left(R^u \frac{\partial \mathcal{C}}{\partial Y_u} + \omega(1 - \frac{G}{R}) R^u \frac{\partial \mathcal{C}}{\partial \tau_u}\right)$.

money plays an important role (e.g., Lucas and Stokey, 1987), where $MV = PY$ implies that in logs, nominal income $\hat{p} + \hat{y}$ is equal to the quantity of money $\hat{m}$ plus a velocity $\hat{v}(\boldsymbol{i})$ that may depend on the nominal rate.

The "nominal wealth" component of demand, in turn, can be broken down into three parts: fiscal policy, which can affect demand via (i) changes in government debt that are *fully backed* by taxes, or (ii) *unbacked* changes in debt ($\tilde{b}_\infty \neq 0$), and (iii) anticipated future nominal income. The results will focus on how fiscal policy can offset fluctuations in nominal demand stemming from changes in anticipated nominal income, which is the key source of indeterminacy in standard models.

The importance of the Ricardian demand property is immediately apparent from this decomposition. When aggregate demand depends only on the present value of taxes, the MPC out of taxes at t is equal to R^s times the MPC out of taxes at $t+s$, so

$$\text{Ricardian demand} \;\Rightarrow\; R^s \frac{\partial \mathcal{C}}{\partial \tau_s} = R^{s+1} \frac{\partial \mathcal{C}}{\partial \tau_{s+1}} \;\forall\; s.$$

Then, per (18) changes in backed debt do not affect demand at all. If the government runs deficits, the private sector anticipates that they will be exactly offset by taxes in the future. Only unbacked fluctuations in nominal debt can affect demand, which is why the FTPL is essential to link deficits and the price level in canonical representative-agent models. When both demand and the fiscal regime are Ricardian, fiscal policy is entirely irrelevant.

By contrast, when SNRD holds, deficits stimulate demand: the MPC out of taxes at t is greater than R times the MPC out of taxes at $t+1$, which is greater than R^2 times the MPC out of taxes at $t+2$, and so on:

$$\text{SNRD} \;\Rightarrow\; \left| R^s \frac{\partial \mathcal{C}}{\partial \tau_s} \right| > \left| R^{s+1} \frac{\partial \mathcal{C}}{\partial \tau_{s+1}} \right| \;\forall\; s.$$

Anticipated future taxes do not offset the effects of deficits, so backed changes in debt affect demand. Even under a Ricardian regime, fiscal policy matters for the long-run price level.

The results in the next section leverage this decomposition to explain exactly how fiscal policy can anchor the price level when either demand or the fiscal regime is non-Ricardian, and they explain what can go wrong when the TVC fails.

3.3 A Simple Rule to Anchor the Price Level

A general class of fiscal rules that can anchor the long-run price level when the TVC holds is

$$\tilde{b}_{t+1} = -\kappa_t \tilde{y}_t, \tag{19}$$

where $\boldsymbol{\kappa}$ is an exogenous sequence of suitably chosen positive constants. When nominal income is too high, the government raises surpluses and cuts debt, while it does the opposite when nominal income is too low. This general strategy is sufficient both when demand is non-Ricardian and when fiscal policy is permitted to be non-Ricardian. I begin by demonstrating this result in the case where demand is non-Ricardian, since that is the main novelty of this framework.

The case of non-Ricardian demand. When demand is non-Ricardian, it is typically sufficient to consider fiscal rules in which the feedback from nominal income to debt quantities is constant, $\tilde{b}_{t+1} = -\kappa \tilde{y}_t$ for some $\kappa > 0$.

Claim. *Suppose that SNRD and the linearized TVC hold. Then generically,*[15] *under the fiscal rule* $\tilde{b}_{t+1} = -\kappa \tilde{y}_t$ *with* $\kappa > 0$ *sufficiently large, all equilibria satisfy* $\lim_{T\to\infty} \hat{p}_T = 0$.

Some additional technical conditions and the formal proof are deferred to the Appendix, but I outline the basic idea here. Notice that under the TVC, attention is restricted to a domain where $(G/R)^t \tilde{y}_t \to 0$, so this fiscal rule is automatically Ricardian. It satisfies $\tilde{b}_\infty = 0$ for any nominal income path.

Suppose, for simplicity, that output is exogenous ($\hat{y}_t = 0$) and that monetary policy sets a constant nominal interest rate ($\hat{i}_t = 0$). Under these conditions, all of the operators in (18) are time-invariant forward convolutions, so it suffices to consider solutions in which all variables grow at a constant geometric rate, which must be less than R/G in absolute value by the TVC. Consider a path along which the nominal price level grows by factor $1 \leq \mu < R/G$ each period, $\hat{p}_t = \mu^t \hat{p}_0$.[16] Nominal income $\tilde{y}_t$ and debt $\tilde{b}_t$ then grow at the same rate, and the real interest rate is $\hat{r}_t = \hat{i}_t + \hat{p}_t - \hat{p}_{t+1} = \mu^t(1-\mu)\tilde{y}_0$. By (18), aggregate nominal demand is

$$\tilde{c}_t = \Big(\sum_{s=0}^{\infty} \big(\frac{G}{R}\mu\big)^s R^s \frac{\partial \mathcal{C}}{\partial Y_s} + (1-\mu) \sum_{s=0}^{\infty} \gamma_s \mu^s - \kappa\omega \sum_{s=1}^{\infty} \big(\frac{G}{R}\mu\big)^s \big(R^s \frac{\partial \mathcal{C}}{\partial \tau_s} - R^{s-1} \frac{\partial \mathcal{C}}{\partial \tau_{s-1}}\big) \Big) \tilde{y}_t.$$

Fiscal policy anchors nominal demand if the change in nominal income is below $\tilde{y}_t$ when it is positive and above it when it is negative. The strength of feedback κ must then satisfy

$$\kappa \geq \sup_{\mu \in [1, R/G)} \frac{\sum_{s=0}^{\infty} \big(\frac{G}{R}\mu\big)^s R^s \frac{\partial \mathcal{C}}{\partial Y_s} + (1-\mu) \sum_{s=0}^{\infty} \gamma_s \mu^s - 1}{\omega \sum_{s=1}^{\infty} \big(\frac{G}{R}\mu\big)^s \big(R^s \frac{\partial \mathcal{C}}{\partial \tau_s} - R^{s-1} \frac{\partial \mathcal{C}}{\partial \tau_{s-1}}\big)}.$$

When this condition is satisfied, there cannot exist an equilibrium in which the price level either stays constant, or grows geometrically, away from the target, so $\hat{p}_T \to 0$.

In principle, the MPCs out of income and taxes, as well as the demand response to changes in real rates, discipline how strongly fiscal policy must react in order to stabilize the long-run price level. The required response tends to be stronger when demand is nearly Ricardian — that is, when the stimulus achieved by delaying taxes is small.

Hence, fiscal policy can select a unique long-run price level (at least locally) via a typical backward-looking feedback rule. This rule is robust in the sense that the response of debt issuance to changes in nominal income does not have to be carefully calibrated. Just like a Taylor rule, as long as the feedback coefficient κ is large enough, the long-run price level is unique. Some questions remain, however: does this rule simply resolve indeterminacy in the price level by introducing a

[15] Here, "generically" means for an open, dense set of MPC sequences $\partial \mathcal{C}/\partial \tau_s$.

[16] In general, it is necessary to consider negative and complex $|\mu| \geq 1$ as well, which can generate oscillating solutions. The proof of the claim above addresses these classes of solutions, but they are omitted from the informal discussion here for brevity.

different indeterminacy, such as multiplicity in the path of real rates? This is possible, but it is not true in general. The debt issuance rule derived in the OLG model of Section 4 is a simple feedback rule of the form (19) that induces a unique equilibrium. Even in the more general setting considered in this section, there is residual indeterminacy only if there exist paths of (backed) government debt issuance that induce zero demand response, as in the case where demand is Ricardian, i.e.,

$$\exists\ \tilde{\boldsymbol{b}} \neq \mathbf{0}:\ \sum_{s=1}^{\infty} \left(\frac{G}{R}\right)^s \left(R^s \frac{\partial \mathcal{C}}{\partial \tau_s} - R^{s-1} \frac{\partial \mathcal{C}}{\partial \tau_{s-1}}\right) \tilde{b}_{t+s} = 0\ \forall\ t.$$

Formally, the degree of indeterminacy is equal to the dimension of the kernel of this linear operator in the space of debt sequences.

There are other issues as well. A common criticism of equilibrium selection via the Taylor principle is that it selects an equilibrium by threatening to blow up the economy when inflation is off-target (Cochrane, 2011), rendering it impossible for agents to optimize.[17] Another concern is that this strategy simply eliminates local indeterminacy, but that global indeterminacy will reemerge in the full nonlinear model.[18] These concerns are best addressed in the OLG setting of Section 4. For now, I proceed to analyze how non-Ricardian fiscal policy can stabilize demand.

Non-Ricardian fiscal regimes and the FTPL. The canonical cashless, representative-agent models used in much of the modern literature on monetary economics (Woodford, 2003; Galí, 2015; Cochrane, 2024) feature Ricardian demand. In such settings, the FTPL is essentially the only theory that can link the government's nominal debt-issuance policy to the long-run price level: only a non-Ricardian fiscal policy can anchor demand. The key equilibrium condition is derived by combining the TVC, $\tilde{a}_\infty = 0$, with the asset-market clearing condition $\tilde{a}_t = \tilde{b}_t$ to obtain that unbacked government debt must be zero in equilibrium, $\tilde{b}_\infty = 0$.

The usual approach to the FTPL takes government surpluses as given, which, with an exogenous real rate, determines the $t = 0$ price level through the valuation equation (14). Suppose that the nominal interest rate is constant, $\hat{\imath}_t = 0$, and that the government keeps surpluses at their BGP level regardless of how prices evolve, $\hat{\tau}_t = 0$. Nominal taxes then move one-for-one with the price level, $\tilde{\tau}_t = \hat{p}_t$, and the government flow budget constraint yields a path of nominal debt

$$\tilde{b}_T \to -\left(\frac{R}{G}\right)^T \times \underbrace{\sum_{s=0}^{\infty} \left(\frac{G}{R}\right)^s \left(1 - \frac{G}{R}\right) \hat{p}_s}_{\text{Weighted avg. } \hat{p}_t}.$$

Thus, the government debt valuation equation, $\tilde{b}_\infty = 0$, pins down the equilibrium value of a weighted average of price levels $\hat{p}_t$. The FTPL is often studied in environments with exogenous

[17] Apart from making optimization impossible, there is another sense in which this rule could destabilize demand: when nominal demand is high, the stock of government debt could contract so much that demand decreases, causing prices to fall and leading to a large increase in debt, and so on. The fiscal rule in the OLG model is explicitly constructed to rule out this type of instability via iteration.

[18] Indeed, this is precisely what happens with the "locally Ricardian" fiscal rules formulated by Woodford (2001a) in an economy with Ricardian demand.

output and a constant real interest rate, $\hat{r}_t = 0$, pinned down by the household's Euler equation, so pinning down an average price level $\hat{p}_t$ is equivalent to pinning down an initial price level $\hat{p}_0$.

It is also straightforward to design non-Ricardian fiscal rules that anchor the long-run price level directly. One such rule, which takes the form (19) with $\kappa_t = (R/G)^t$, is

$$\tilde{b}_{t+1} = -(\frac{R}{G})^t \tilde{y}_t.$$

Then $\tilde{b}_\infty = 0$ only if $\tilde{y}_T \to 0$—that is, only if nominal income converges to target in the long run. With exogenous output, equivalently, prices converge to target.

What happens when the TVC fails? It is easiest to illustrate the issue when linearizing around a BGP with $R < G$. Suppose that the government is restricted to use a fiscal policy in which the present value of taxes is finite:

$$\sum_{t=0}^{\infty} (\frac{G}{R})^t \tilde{\tau}_t \text{ is finite} \quad \Leftrightarrow \quad \tilde{b}_\infty \text{ is finite.}$$

Both Ricardian fiscal rules and typical formulations of the FTPL satisfy this criterion. Then, returning to the decomposition of demand in (18), it is almost immediate that such a policy has no effect on demand in the long run,

$$\tilde{c}_T = \omega \underbrace{(\frac{R}{G})^T}_{\to 0} \Bigg((1 - \frac{G}{R}) \underbrace{\sum_{s=0}^{\infty} (\frac{G}{R})^{T+s} R^s \frac{\partial \mathcal{C}}{\partial \tau_s} \tilde{\tau}_{T+s}}_{\to 0} + \underbrace{\frac{\partial \mathcal{C}}{\partial A} \tilde{b}_\infty}_{\text{finite}} \Bigg) + \sum_{s=0}^{\infty} (\frac{G}{R})^s R^s \frac{\partial \mathcal{C}}{\partial Y_s} \tilde{y}_{T+s} + \sum_{s=0}^{\infty} \gamma_s \hat{r}_{T+s}.$$

All of the terms involving fiscal policy go to zero. If the present value of taxes is finite, then taxes must not increase faster than the real rate R. But $R < G$, so the growth rate of taxes is slower than that of the aggregate economy, and the ratio of taxes to GDP goes to zero in the long run. If the model admits long-run nominal indeterminacy, then any such fiscal rule cannot influence the long-run price level.

Taken together, these results suggest that when demand is non-Ricardian (in the "strong" sense of Definition 3):

- If the TVC holds, *either* a Ricardian fiscal rule or an FTPL-style rule can select an equilibrium by guaranteeing that prices converge to a desired level in the long run; and
- If the TVC fails, then *neither* a Ricardian fiscal rule nor an FTPL-style rule can do so.

In this sense, in a broad class of models satisfying SNRD, one can expect Ricardian and non-Ricardian fiscal rules to be equivalent in terms of the equilibria they can implement. Both types of rules can achieve this goal through a strategy that "anchors" demand by running deficits when nominal prices are too low and surpluses when they are too high.

4 An Overlapping Generations Model

So far, the analysis has focused on the intuition behind the main result in a general class of models. This has left open some theoretical loose ends, including the question of how, exactly, to construct a fiscal policy that can select an equilibrium price path.

This section addresses these loose ends in a fully nonlinear overlapping-generations model: a simplified version of Blanchard (1985) without capital.

4.1 Environment

The aggregate endowment grows at a constant rate G,

$$Y_t = G^t Y, \tag{20}$$

where $Y > 0$ is a constant. The economy is populated by a unit mass of "perpetual youth" households who have log preferences over consumption and die with probability $1-\delta$ per period, with $\delta \in (0,1]$. Dying households are replaced by a measure $1-\delta$ of newborns to keep the total population constant. The government sets an exogenous path $1+\boldsymbol{i}$ for the nominal rate, which converges to $1+i^*$ as $T \to \infty$. All agents are assumed to have perfect foresight regarding aggregate outcomes.

The aggregate endowment and taxes are not distributed equally across households. Instead, at time t, the net income of a household born at time $s \leq t$ is

$$y_t^s - \tau_t^s = \underbrace{\frac{1-\tilde{\lambda}\delta}{1-\delta}\tilde{\lambda}^{t-s}}_{\text{cohort-}s\text{ share}} \underbrace{(Y_t - \tau_t)}_{\text{aggregate income}}.$$

Here, $\tilde{\lambda}$ is a constant such that $0 < \tilde{\lambda}\delta \leq 1$.[19] Henceforth, I denote $\lambda \equiv \tilde{\lambda}\delta$—as shown below, Ricardian equivalence corresponds to $\lambda = 1$. The expected endowment income of an individual household grows at rate $\lambda G = \delta\tilde{\lambda}G$ (the survival rate times growth conditional on survival), rather than at the aggregate growth rate G. For example, $\lambda G < 1$ can be viewed as a declining lifetime income profile that generates a demand for saving in old age.

The role of perpetual youth is to break Ricardian equivalence: when households do not live forever, they anticipate that taxes will be paid in part by future generations, so government bonds are net wealth for the private sector. The role of the life-cycle income profile ($\lambda \neq 1$) is to allow for bubbles. When λ is sufficiently small, the excess demand for savings in old age implies that the aggregate TVC (4) does not hold. As a result, there exist equilibria in which the government does not repay its debt in the long run, i.e., (13) holds in equilibrium.

The consumption-savings problem of a household born at time s is standard. As is typical in perpetual youth models, I assume the existence of annuities markets that agents can use to insure against the risk of death. The assets of households who die between t and $t+1$ are paid out to surviving households, so that those who survive from t to $t+1$ earn nominal interest $(1+i_t)/\delta$ on

[19] It is necessary to define the income process when $\delta = 1$ as well. In this case, there is a single representative household that lives forever and receives income Y_t each period.

their savings. The household's problem can then be written as

$$\max_{c_t^s, a_t^s} \sum_{t=s}^{\infty} (\tilde{\beta}\delta)^{t-s} \log c_t^s \quad \text{s.t.} \quad a_s^s \text{ given}, \lim_{T\to\infty} R_{0,T}^{-1} \delta^T a_T^s \geq 0,$$
$$a_{t+1}^s = \frac{1}{\delta} \frac{1+i_t}{1+\pi_{t+1}} (a_t^s + y_t^s - \tau_t^s - c_t^s), \tag{21}$$

where $\tilde{\beta} \in (0,1)$ is households' subjective discount factor, and c_t^s, a_t^s denote the household's consumption and real asset holdings at time $t \geq s$. Note that in addition to a flow budget constraint, households are subject to a no-Ponzi condition that defines their budget sets. Henceforth, I let $\beta \equiv \tilde{\beta}\delta$ denote households' effective discount factor.

It is well known that, with log preferences, a household consumes a constant fraction of its total wealth each period.[20]

$$C_t = \mathcal{C}(A_t, \{Y_{t+s}, \tau_{t+s}, R_{t+s}\}) = \underbrace{(1-\beta)}_{\text{MPC}} \times \underbrace{(A_t + H_t)}_{\text{wealth}}, \tag{22}$$

where $C_t = (1-\delta) \sum_{s=0}^{\infty} \delta^s c_t^{t-s}$, $A_t = (1-\delta) \sum_{s=0}^{\infty} \delta^s a_t^{t-s}$, and

$$H_t = \sum_{s=0}^{\infty} \lambda^s R_{t,t+s}^{-1} (Y_{t+s} - \tau_{t+s}).$$

Aggregate demand is the product of a marginal propensity to consume, $1-\beta$, times total wealth, which consists of financial assets plus "human wealth" H_t: the present value (to the household) of future endowment income net of taxes.

Note that in human wealth, future income and taxes are discounted at rate $\lambda^s R_{t,t+s}^{-1}$ rather than $R_{t,t+s}^{-1}$. This difference in discount rates is the source of Ricardian non-equivalence. (Later, I formally demonstrate that demand is non-Ricardian in the sense of Definition 2.) Even if households expect that government bond issuance will eventually be repaid via future taxes, an increase in current bond wealth is not completely offset by the increase in discounted future taxes. When $\lambda < 1$, future income is discounted at a rate different from $R_{t,t+s}^{-1}$ for two reasons: households may die before future income is received, and their individual endowments may grow at a different rate than the aggregate endowment even conditional on survival.

4.2 Equilibria

I characterize the equilibrium as a fixed-point equation in the sequence space

$$\mathcal{S} = \left\{ \boldsymbol{PY} \in \mathbb{R}^{\mathbb{N}} : \sum_{t=0}^{\infty} \left(\frac{\lambda}{1+i^*} \right)^t |P_t Y_t| < \infty, \quad P_t Y_t \geq 0 \ \forall t \right\}$$

[20] For completeness, I derive this result in the Appendix.

(where, with some abuse of notation, $\boldsymbol{PY}$ denotes the nominal income sequence $\{P_tY_t\}$), equipped with the norm

$$\|\boldsymbol{PY}\|_{\mathcal{S}} = \sum_{t=0}^{\infty} \left(\frac{\lambda}{1+i^*}\right)^t |P_tY_t|.$$

The domain of admissible nominal income sequences is restricted to guarantee that a solution to households' problem exists.[21] Since the nominal rate and output are exogenous, all fiscal rules of the form (9) can be written simply as functions on $\mathcal{S}$, $\tau_t = \mathcal{T}_t(\boldsymbol{PY})$.

To develop a fixed-point equation, it is necessary to derive the consumption function purely in terms of initial assets, real income, nominal interest rates, and prices, as in (3).

Proposition 3. *In the OLG model, the consumption function satisfies*

$$P_tC_t = \mathcal{G}_t(\boldsymbol{PY}) \equiv (1-\beta)I_{0,t} \times \left(\beta^t B_0 + \sum_{s=0}^{\infty} \gamma_{s,t} I_{0,s}^{-1} P_s(Y_s - \tau_s)\right), \tag{23}$$

where the constants $\gamma_{s,t}$ are defined by

$$\gamma_{s,t} = \frac{1}{1-\beta\lambda}\left[\lambda(1-\beta)\beta^t\lambda^s + (1-\lambda)\Big(\mathbf{1}\{s<t\}\beta^{t-s} + \mathbf{1}\{s\geq t\}\lambda^{s-t}\Big)\right].$$

An equilibrium is a solution to the fixed-point equation

$$P_tY_t = \mathcal{G}_t(\boldsymbol{PY}) \quad \forall\ t. \tag{24}$$

From this formulation, it is clear that demand is Ricardian if and only if $\lambda = 1$. Then, demand depends only on asset holdings net of the present value of taxes rather than on the timing of taxes. Furthermore, the TVC holds whenever $\beta < \lambda$. Otherwise, as discussed in the previous section, bubble equilibria are possible, as will be illustrated shortly.

Corollary 1. *The consumption function (23) satisfies the Ricardian demand property if and only if $\lambda = 1$. It satisfies the TVC if and only if $\beta < \lambda$.*

Equilibria obey a difference equation for nominal income, P_tY_t. Taking first differences of (24), nominal output obeys a *modified Euler equation*:

$$P_{t+1}Y_{t+1} = \frac{\beta}{\lambda}(1+i_t)P_tY_t - \frac{(1-\lambda)(1-\beta)}{\lambda}B_{t+1}. \tag{25}$$

There are two differences from a standard Euler equation with log preferences, $P_{t+1}Y_{t+1} = \beta(1+i_t)P_tY_t$. First, there is an additional factor of λ in the denominator of the first term. When $\lambda < \beta$, this allows for price paths along which the real interest rate is less than G, the growth rate of the economy. Second, there is a term involving government debt, meaning the path of debt matters for the real rate in equilibrium. Both of these differences emerge only when Ricardian equivalence fails, $\lambda < 1$.

[21] Of course, it is necessary to restrict the fiscal rule as well to guarantee a solution exists.

Any pair of paths $(\boldsymbol{PY}, \boldsymbol{B})$ satisfying (25) and $\boldsymbol{PY} \in \mathcal{S}$ corresponds to an equilibrium. If $\lambda < 1$, each equilibrium path $\boldsymbol{PY}$ corresponds to a unique path of debt $\boldsymbol{B}$. The following proposition characterizes the set of equilibria with nonnegative government debt.

Proposition 4. *For each non-negative path $\boldsymbol{B}$, there exists a continuum of equilibria, indexed by the initial level of nominal income*

$$P_0 Y_0 \geq \frac{(1-\lambda)(1-\beta)}{\lambda} \sum_{s=1}^{\infty} (\frac{\lambda}{\beta})^s I_{0,s}^{-1} B_s.$$

When this inequality holds exactly, then the long-run inflation rate is

$$\lim_{T\to\infty} \frac{P_{T+1}}{P_T} = \frac{\beta}{\lambda} \frac{1+i^*}{G} \lim_{T\to\infty} \left(1 - \frac{(\frac{\lambda}{\beta})^{T+1} I_{0,T+1}^{-1} B_{T+1}}{\sum_{s=T+1}^{\infty} (\frac{\lambda}{\beta})^s I_{0,s}^{-1} B_s}\right).$$

Otherwise, the long-run inflation rate is

$$\lim_{T\to\infty} \frac{P_{T+1}}{P_T} = \frac{\beta}{\lambda} \frac{1+i^*}{G}.$$

For each possible path of debt, there are multiple equilibria. The one with the lowest initial price level also has the lowest long-run inflation rate, which is determined by the long-run growth of the nominal debt stock. When nominal debt grows at rate G^B, the long-run inflation rate is

$$1 + \pi = \frac{G^B}{G} \in (0, \frac{\beta}{\lambda} \frac{1+i^*}{G}),$$

so the long-run real interest rate satisfies $R > \lambda G/\beta$. All other equilibria have a higher long-run inflation rate, which results in a long-run real interest rate of $R = \lambda G/\beta$. The long-run real interest rate can be below G precisely when the aggregate TVC fails. Note that bubble equilibria, in which nominal debt grows at a rate faster than $1 + i^*$, can exist only when $\lambda < \beta$. This is precisely the condition under which the aggregate transversality condition fails. This multiplicity is illustrated in Figure 1.

The problem of selecting an equilibrium can then be viewed as follows. Given a desired path of nominal debt $\boldsymbol{B}^*$, there are many possible price paths $\boldsymbol{P}$. The objective is to design a fiscal rule (which may deviate from $\boldsymbol{B}^*$ off-path) that implements a desired $\boldsymbol{P}^*$ as the unique equilibrium outcome. I turn to this problem in the next section.

4.3 A Ricardian Rule for Equilibrium Selection

To build intuition for how a Ricardian rule can uniquely implement an equilibrium, it is helpful to write the fixed-point equilibrium equation (24) more simply. Combining aggregate demand (22) with

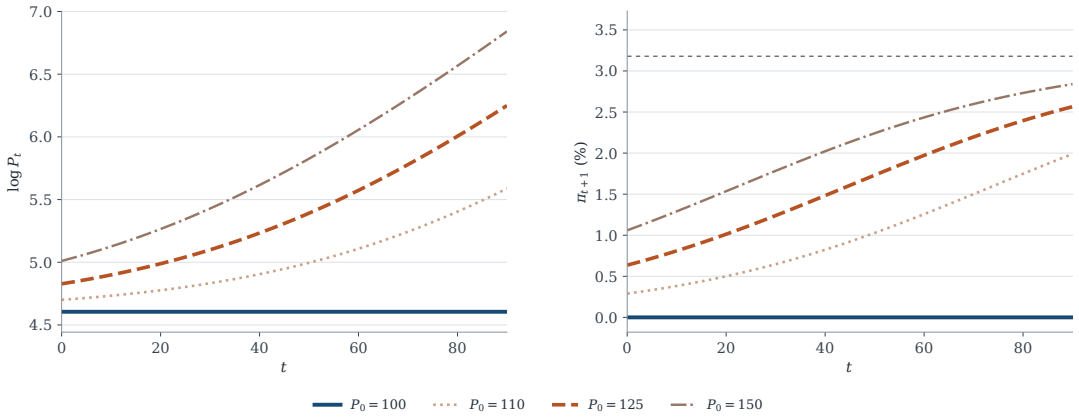


Figure 1: An illustration of equilibrium multiplicity given a path of nominal government debt that grows with output at rate $G = 1.01$. Parameter values are $\beta = 0.9, \lambda = 0.95$, and $1 + i_t = 1.1$ for all t. Each line corresponds to the path following a different initial price level, with $P_0 = 100$ being the path with the minimum long-run inflation.

the market-clearing conditions (11)–(12), the equilibrium conditions can alternatively be written as

$$P_t Y_t = (1-\beta)\Bigg(\sum_{s=0}^{\infty} \lambda^s I_{t,t+s}^{-1} P_{t+s} Y_{t+s} + \underbrace{B_t - \sum_{s=0}^{\infty} \lambda^s I_{t,t+s}^{-1} P_{t+s} \tau_{t+s}}_{\text{net bond wealth}}\Bigg). \tag{26}$$

In equilibrium, nominal income must be equal to nominal demand, which is the MPC times nominal wealth (the right-hand side of (26)). Nominal wealth, in turn, consists of two components: the present value to households of future labor income plus net wealth from bond holdings.

When demand is Ricardian ($\lambda = 1$), a Ricardian fiscal rule guarantees that net bond wealth is equal to zero even off-equilibrium. For all possible price vectors, taxes must adjust so that their present value is equal to B_t/P_t. However, when demand is non-Ricardian ($\lambda < 1$), then net bond wealth need not be zero even under a Ricardian fiscal rule.

This property suggests that a Ricardian rule could implement a unique equilibrium through a "nominal wealth targeting" strategy. Suppose the government wishes to implement an equilibrium price sequence $\boldsymbol{P}^*$. The fiscal rule should be chosen so that total nominal wealth always equals $P_t^* Y_t/(1-\beta)$. That is, for *any* price sequence $\boldsymbol{P}$,

$$B_t - \sum_{s=0}^{\infty} \lambda^s I_{t,t+s}^{-1} P_{t+s} \tau_{t+s} = \frac{P_t^* Y_t}{1-\beta} - \sum_{s=0}^{\infty} \lambda^s I_{t,t+s}^{-1} P_{t+s} Y_{t+s}. \tag{27}$$

The following lemma demonstrates that a class of fiscal rules implementing a desired equilibrium exists, although these rules may or may not be Ricardian. For a particular strength of feedback from the nominal price level to nominal debt, it satisfies (27) exactly.

Lemma 1. *Let $\boldsymbol{PY}^*$ be an equilibrium nominal income sequence, and let $\boldsymbol{B}^*$ be the associated path of nominal debt. If $\lambda < 1$, then a fiscal rule uniquely implementing $(\boldsymbol{PY}^*, \boldsymbol{B}^*)$ exists. The path of*

nominal debt $\boldsymbol{B}$ *satisfies*

$$\frac{B_{t+1}}{1+i_t} = \frac{B^*_{t+1}}{1+i_t} + \kappa(P^*_t Y_t - P_t Y_t), \quad \kappa \geq \frac{1}{1-\lambda}. \tag{28}$$

When $\kappa = 1/(1-\lambda)$*, this fiscal rule satisfies the nominal wealth target (27).*

The logic underlying the lemma is straightforward. When public debt is net wealth for the private sector, deficits affect demand. When prices fall below target ($P_t < P^*_t$), the government attempts to drive them higher by increasing deficits, whereas it cuts deficits when prices are too high ($P_t > P^*_t$).

When is this fiscal rule Ricardian? Intuitively, ruling out inflation above target is not a problem: in this scenario, the government cuts deficits rather than issuing additional debt that may need to be rolled over forever. On the other hand, ruling out deflation under the rule (28) might require permanent deficits. It turns out that the rule is Ricardian whenever inflation in the target equilibrium $\boldsymbol{P}^*$ is sufficiently low, or, equivalently, whenever the long-run real interest rate $R = (1+i^*)/(1+\pi)$ is greater than the growth rate G.

Proposition 5. *Let* $\boldsymbol{P}^*$ *be an equilibrium price vector. The fiscal rule* (28) *uniquely implements* $\boldsymbol{P}^*$*, and it is Ricardian if and only if*

$$\lim_{T\to\infty} \frac{P^*_{T+1}}{P^*_T} < \frac{1+i^*}{G}.$$

When $\kappa = 1/(1-\lambda)$*, the fixed-point mapping* $\mathcal{G}$ *induced by this fiscal rule is a contraction on* $\mathcal{S}$ *with modulus* $\lambda(1-\beta)/(1-\beta\lambda)$*.*

The policy of targeting nominal wealth is conceptually similar, in certain respects, to monetarist prescriptions based on the quantity theory of money (Friedman, 1970). In the quantity theory, the relationship $MV = PY$, together with stable velocity V, implies that the government can target nominal income by controlling the money supply. Here, (22) provides a similar stable relationship between nominal income and nominal wealth. Fiscal policy can exploit this relationship to select an equilibrium.[22]

Proposition 5 implies that the government does not require detailed knowledge of the structure of the economy or of agents' expectations of the future to anchor the price level. Structural parameters, such as the marginal propensity to consume out of wealth, do not enter the debt-issuance strategy. Forward-looking terms, such as agents' forecasts of future prices or their perceptions of their discounted nominal wealth W_t, are also absent. The fiscal rule (28) selects an equilibrium as long as the feedback from the price level to debt issuance is strong enough. In this sense, the fiscal strategy resembles the Taylor principle, which dictates how *monetary policy* can instead stabilize inflation if it reacts strongly enough to deviations from target.

[22]The assumption that agents have log utility and hence a constant MPC out of nominal wealth is analytically convenient, but it does not drive the main results. Even if households had CRRA utility with $\sigma \neq 1$, fiscal policy could use a similar strategy to select an equilibrium, as demonstrated in Appendix D.4.

Another consequence of Proposition 5 is worth emphasizing. The nominal-wealth-targeting strategy implies that the mapping $\boldsymbol{PY} \mapsto \mathcal{G}(\boldsymbol{PY})$ in the fixed-point equation (24) is a contraction. Starting from any candidate nominal-income path $\boldsymbol{PY}$, repeated applications of $\mathcal{G}$ therefore converge to the target equilibrium $\boldsymbol{PY}^*$. Because $\mathcal{G}$ gives demand as a function of the path of nominal prices, this property is analogous to cobweb stability in partial-equilibrium supply–demand diagrams. In this sense, the equilibrium selected by the nominal-wealth-targeting rule is robust rather than unstable, unlike the equilibrium-selection mechanism associated with Taylor rules.

The nominal-wealth-targeting strategy is only one fiscal rule that can uniquely implement a desired equilibrium. Although it cannot uniquely select an equilibrium with $R < G$ while remaining Ricardian, another Ricardian fiscal rule might do so. I turn to whether such rules exist in Section 4.6.

4.4 Monetary-Fiscal Interactions

Fiscal policy's ability to anchor the price level invites a reconsideration of a classic question: to what extent does the fiscal regime affect monetary policy's ability to control inflation?

In a seminal result, Sargent and Wallace (1981) demonstrate that if the fiscal authority is "dominant", then at best, monetary policy can control inflation only in the short run. If the government does not raise taxes to repay its debt, the central bank must monetize it, raising seigniorage revenue to cover deficits *along the equilibrium path*. The FTPL goes further: the government debt valuation equation (14) links government surpluses to inflation in equilibrium, even in cashless economies where the central bank sets interest rates independently (Woodford, 2001a). The FTPL is occasionally interpreted as the central bank printing cash to redeem government debt at par *off-equilibrium* (Cochrane, 2005).

Proposition 5 delivers a stronger result. When demand is non-Ricardian, fiscal policy can anchor the long-run price level even when the central bank operates independently both on- and off-equilibrium.

To illustrate this point, consider the announced sequence of interest rates $1 + \boldsymbol{i}$ at $t = 0$ to be an unanticipated change from a balanced growth path (BGP) with nominal rate $1 + i^*$, and nominal income and debt that grow at rate $(1 + \pi^*)G$, satisfying

$$1 + \pi^* = \frac{\beta}{\lambda + (1-\beta)(1-\lambda)\overline{B}/\overline{PY}} \frac{1 + i^*}{G},$$

where $\overline{B}/\overline{PY}$ denotes the BGP debt-to-output ratio. Henceforth, let $\overline{B}_t = (1 + \pi^*)^t G^t B_0$ and $\overline{PY}_t = (1+\pi^*)^t G^t \overline{PY}$ denote the paths that nominal debt and output would have taken, respectively, absent a change in interest rates at $t = 0$.

One natural way for fiscal policy to anchor the price level would be to ensure that in the long run, nominal income returns to the balanced growth path $\overline{PY}_t$. This anchor determines the long-run price level, whereas changes in the nominal rate are free to influence the price level in the short run.

To accomplish this goal, nominal debt issuance should be set according to

$$\frac{B_{t+1}}{1+i_t} = \frac{\overline{B}_{t+1}}{1+i_t} + \kappa(\overline{PY}_t - P_tY_t), \quad \kappa \geq \frac{1}{1-\lambda}. \tag{29}$$

A slight difference from the rule (28) derived in the previous section is that under the sequence of nominal rates $1+\boldsymbol{i}$, $(\overline{\boldsymbol{PY}}, \overline{\boldsymbol{B}})$ is not an equilibrium. Hence, nominal income is not equal to $\overline{PY}_t$ at all times. Instead, to first order, it follows

$$\log \frac{P_tY_t}{\overline{PY}_t} \approx -\beta \sum_{s=0}^{\infty} \left(\frac{\lambda G}{1+i^*}\right)^s \log\left(\frac{1+i_{t+s}}{1+i^*}\right) \quad \text{for } \kappa = \frac{1}{1-\lambda}.$$

When the long-run price level is fixed by fiscal policy, changes in nominal rates have conventional effects. An increase in the nominal rate must result in higher inflation as long as real interest rates do not increase by a larger amount. With a fixed long-run price level, it must be that prices fall in the short run. Intuitively, fiscal policy keeps long-run nominal income stable, and an interest rate hike devalues claims on future income and therefore decreases nominal wealth. An interest rate hike then decreases prices in the short run, whereas a cut increases them.

A different scheme instead fixes the initial price level P_0 and permits the future path of prices to evolve along with the nominal rate. Let $B_t^* = (I_{0,t}/(1+i^*)^t)\overline{B}_t, PY_t^* = (I_{0,t}/(1+i^*)^t)\overline{PY}_t$. Nominal debt issuance should satisfy

$$\frac{B_{t+1}}{1+i_t} = \frac{B_{t+1}^*}{1+i_t} + \kappa(PY_t^* - P_tY_t), \quad \kappa \geq \frac{1}{1-\lambda}. \tag{30}$$

This is precisely the same type of rule as in (28), except the target paths of debt and income are contingent on the path of the nominal rate. Therefore, this policy selects a unique equilibrium in which nominal income satisfies

$$\frac{P_tY_t}{I_{0,t}} = \frac{\overline{PY}_t}{(1+i^*)^t} \quad \Rightarrow \quad P_tY_t = \frac{I_{0,t}}{(1+i^*)^t}\overline{PY}_t.$$

This strategy stands in contrast to the typical prescription that the government should simply attempt to stabilize debt in response to changes in interest rates. The fiscal authority instead responds to an increase in the nominal rate by increasing its long-run targets for the levels of nominal debt and income. As a result, the effects of an interest rate hike are *neo-Fisherian*: a temporary increase in rates increases both the price level and inflation. Figure 2 compares the effects of the two anchoring schemes.

Away from the case of Ricardian demand, then, it is important to understand how the fiscal anchor responds to changes in interest rates. When monetary policy operates according to an interest rate rule, fiscal policy determines how exactly changes in interest rates influence inflation and the price level in the long run, even when the fiscal regime is Ricardian. Monetary policy affects the substitution of nominal income across periods, but fiscal policy affects the overall level of nominal income. If the long-run price level target changes in response to a movement in interest

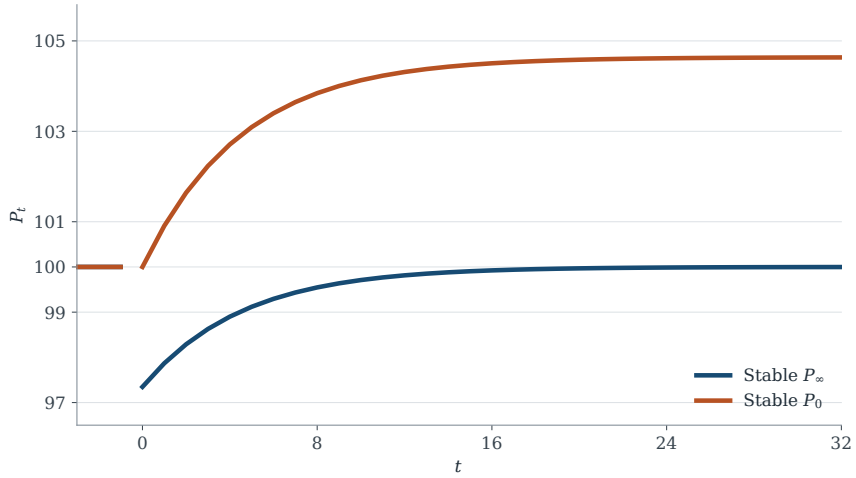


Figure 2: A comparison of the path of prices under two fiscal regimes. Along the BGP, nominal debt and output grow at 1% per period, and the nominal rate is $i^* = 10\%$. At $t = 0$, the government announces a 100 basis-point temporary interest rate hike that decays at geometric rate $\rho = 0.8$. The blue line displays the path of prices under fiscal rule (29), which pegs the long-run price level. The red line displays the path under rule (30), which pegs P_0. Parameter values are $\beta = 0.9, \lambda = 0.95$, and $\kappa = 20$.

rates, monetary policy is powerless on its own to control the price level or inflation. Fiscal policy must anchor the long-run level of nominal income for monetary policy to be effective.

4.5 Alternative Monetary Policies

One might wonder whether fiscal policy can control the long-run price level in this model simply because monetary policy has been "passive" so far: the central bank sets the nominal rate independently of all endogenous variables. In fact, fiscal policy can select an equilibrium even when monetary policy follows typical prescriptions aimed at stabilizing prices.

I illustrate this point by extending the model to include money. The central bank follows either an interest rate targeting rule satisfying the Taylor principle or a money growth rule. I outline results with finite money demand and then take the cashless limit, as in Woodford (1998).

To add money to the model, I assume the flow utility of an individual household takes the form

$$u(c, m) = (1 - \alpha)\log(c) + \alpha \log(m),$$

where $m_t^s = M_t^s / P_t$ denotes the real balances of a generation-s household carried from period t to $t + 1$ and $\alpha \in (0, 1)$ parameterizes the expenditure share on real balances.[23] The period budget constraint is

$$a_{t+1}^s = \frac{1}{\delta} \frac{1 + i_t}{1 + \pi_{t+1}} (a_t^s + y_t^s - \tau_t^s - c_t^s - \frac{i_t}{1 + i_t} m_t^s).$$

[23]The Cobb-Douglas functional form is not essential. The Appendix demonstrates that all of the key results continue to hold for utility functions of the form $\log(X(c, m))$, where $X(.,.)$ is a homothetic aggregator implying a bounded expenditure share on real balances.

Aggregate consumption demand is still a constant MPC times aggregate wealth,

$$C_t = (1-\alpha)(1-\beta)\big(A_t + \sum_{s=0}^{\infty} R_{t,t+s}^{-1}\lambda^s(Y_{t+s} - \tau_{t+s})\big), \tag{31}$$

and money demand satisfies

$$\frac{i_t}{1+i_t}\frac{M_t}{P_t} = \frac{\alpha}{1-\alpha}C_t. \tag{32}$$

That is, with Cobb-Douglas utility, forgone interest on real balances is a constant multiple of consumption expenditures.

Monetary policy follows either a Taylor rule with a zero lower bound,

$$\log(1+i_t) = \max\Big\{\epsilon, \log(1+\hat{i}_t) + \phi\big(\log P_t - \log P_{t-1}\big)\Big\}, \tag{33}$$

where $1+\hat{i}_t \to 1+i^*$ is an exogenous sequence of nominal rate targets and $\epsilon > 0$ is a lower bound close to zero,[24] or a money growth rule,

$$\log M_{t+1} = \log M_t + \log(1+\mu_t), \tag{34}$$

where $1+\mu_t \to 1+\mu^*$ is an exogenous sequence of money growth targets. The consolidated government flow budget constraint is

$$B_{t+1} = (1+i_t)\big(B_t - P_t\tau_t - (M_t - M_{t-1})\big). \tag{35}$$

Instead of (10), the Ricardian constraint on fiscal policy applies to the entire portfolio of government liabilities, $\lim_{T\to\infty} I_{0,T}^{-1}(B_T + M_{T-1}) \leq 0$.[25]

Combining the aggregate demand condition (31) with the government flow budget constraint (35) and the market-clearing conditions $C_t = Y_t$ and $A_t = (B_t + M_{t-1})/P_t$, equilibria can again be characterized by sequences $(\boldsymbol{PY}, \boldsymbol{B}, \boldsymbol{M}, 1+\boldsymbol{i})$ satisfying the modified Euler equation

$$P_{t+1}Y_{t+1} = \frac{\beta}{\lambda}(1+i_t)P_tY_t - \frac{(1-\beta)(1-\lambda)(1-\alpha)}{\lambda}(B_{t+1}+M_t) \quad \text{where } \sum_{t=0}^{\infty} I_{0,t}^{-1}\lambda^t|P_tY_t| < \infty, \tag{36}$$

and (32), where monetary policy is determined by either (33) or (34). Note that the differences from (25) are the inclusion of money balances in the second term on the right-hand side and the factor of $1-\alpha$ in that term, which parameterizes the expenditure weight on goods.

Just as in the case without money, there are multiple equilibria when the fiscal rule is "passive" in the sense that it specifies an exogenous path for the nominal present value of government liabilities, $(B_t + M_{t-1})/I_{0,t}$. For any such rule, there exists a solution to (36) for any sufficiently large value

[24] With non-satiation in real balances, money demand becomes infinite when the nominal interest rate is exactly zero.

[25] This definition of a Ricardian fiscal policy in an economy with money is consistent with that in Woodford (1995). While others, like Christiano and Rostagno (2001) use the alternative notion $\lim_{T\to\infty} I_{0,T}^{-1}B_T \leq 0$, the definition used here is appropriate if one wants an irrelevance result like Proposition 1 to hold.

of initial nominal income P_0Y_0. Absent a suitable fiscal regime, neither a Taylor rule nor a money growth rule can independently select a desired price level.

The issue with a Taylor rule is well-understood and is covered extensively by Neumeyer and Nicolini (2025). When inflation is above target, a Taylor rule dictates an increase in the nominal rate, which for a given real rate must increase future expected inflation, so the price level can explode upwards. Nothing in the model forbids this outcome. When inflation is below target, the Taylor rule dictates a decrease in the nominal rate until, eventually, it ends up at the lower bound (a key theme of Benhabib, Schmitt-Grohé, and Uribe, 2001), and inflation remains low but stable thereafter.

A money growth rule also fails to pin down the price level. A natural intuition is that the price level should track the money supply, but this logic typically fails unless money demand takes a very specific form. Woodford (1994), for example, provides an overview of price-level indeterminacy in a Lucas-Stokey cash-in-advance economy with a money growth rule. In this setting, the indeterminacy is particularly stark: (32) demonstrates that for any level of nominal income $P_tY_t < \frac{1-\alpha}{\alpha}M_t$, there exists an interest rate that clears the market. Therefore, M_t puts an upper bound on the price level but not a lower bound. With a passive fiscal rule, there can exist a continuum of equilibria in which nominal income grows slower than the money stock. In the long run, the economy enters a liquidity trap: the nominal rate tends to zero and real balances explode.

What if the government instead follows an analogue of the rule (28) proposed in the cashless economy? This strategy can succeed in selecting *any* path of nominal income and therefore the price level consistent with equilibrium.

Proposition 6. *Suppose that, under monetary policy (33) or (34), there exists an equilibrium with sequences of nominal income, bond holdings, money holdings, and nominal interest rates* $(\boldsymbol{PY}^*, \boldsymbol{B}^*, \boldsymbol{M}^*, 1+\boldsymbol{i}^*)$. *Then the fiscal rule*

$$\frac{B_{t+1}+M_t}{I_{0,t+1}} = \frac{B^*_{t+1}+M^*_t}{I^*_{0,t+1}} + \kappa\left(\frac{PY^*_t}{I^*_{0,t}} - \frac{P_tY_t}{I_{0,t}}\right), \quad \kappa \geq \frac{1}{(1-\lambda)(1-\alpha)}, \tag{37}$$

uniquely implements that equilibrium. The rule is Ricardian if $\lim_{T\to\infty}(B^*_T + M^*_{T-1})/I^*_{0,T} \leq 0$ *and* $\lim_{T\to\infty}\frac{1}{T}\log R^*_{0,T} > \log G$.

In the cashless limit $\alpha \to 0$, this is the exact same rule as in the benchmark economy. Therefore, the results from the benchmark extend approximately to economies with small amounts of money demand where the central bank follows more typical monetary policy rules. Again, the proposed fiscal rule is Ricardian as long as two conditions hold: government debt must satisfy the Ricardian constraint *on the target path*, and the long-run real interest rate must be greater than G. The logic underlying this fiscal rule also remains identical: when demand is non-Ricardian, the timing of taxes has wealth effects that fiscal policy can exploit to control the price level. This feature of demand does not depend on monetary policy.

Even under more realistic monetary policy rules, then, an independent central bank cannot control the price level on its own. The long-run price level is determined not by the money stock, but rather by the growth rate of the government's aggregate nominal liabilities, even though the

government raises sufficient revenues to repay its debt both on- and off-equilibrium. Monetary policy determines money holdings and the specific path taken by the price level towards the long-run target.

4.6 An Impossibility Result

So far, the results have demonstrated that a nominal-wealth-targeting strategy can control the long-run price level when the TVC holds. This section proves a partial converse: when the TVC fails—that is, when bubbles are possible—no well-behaved fiscal rule can select a given price path as a stable equilibrium outcome, in a sense defined below. I consider a broad class of fiscal rules for which the present value of deficits is finite along all paths. A fiscal rule $\mathcal{T}$ has *finite-present-value deficits* if

$$-\sum_{t=0}^{\infty} I_{0,t}^{-1} \min\{0, P_t \tau_t\} < \infty \quad \forall\, \boldsymbol{PY} \in \mathcal{S}. \tag{38}$$

This class of rules, of course, nests Ricardian rules, which require the present value of taxes to be at least B_0 along all paths. It also includes FTPL-style rules: in the canonical FTPL, a fiscal rule is defined by a (finite) present value of surpluses, and P_0 adjusts so that the real value of debt is equal to that present value.

The preceding analysis shows that any equilibrium with $R > G$ can be uniquely implemented by the nominal-wealth-targeting fiscal rule. Moreover, that rule induces a contraction mapping $\boldsymbol{PY} \mapsto \mathcal{G}(\boldsymbol{PY})$. It is possible to prove a partial converse for equilibria with $R \leq G$.

Proposition 7. *Let $\boldsymbol{P}^*$ be an equilibrium price vector such that*

$$\lim_{T\to\infty} \frac{P_{T+1}^*}{P_T^*} \geq \frac{1+i^*}{G}.$$

Then there does not exist a fiscal rule $\mathcal{T}$ with finite-present-value deficits that uniquely implements $\boldsymbol{P}^*$ through a contraction mapping $\boldsymbol{PY} \mapsto \mathcal{G}(\boldsymbol{PY})$.

The basic idea is simple. In the desired equilibrium, the present value of income $\boldsymbol{PY}^*$ is infinite because $R \leq G$. Consider a candidate income path $\boldsymbol{PY}$ with low enough inflation that the present value of income is finite, $R > G$. When deficits have finite present value, then *net* income $P_t(Y_t - \tau_t)$ has finite present value as well. Hence, *demand* $\mathcal{G}(\boldsymbol{PY})$ has finite present value. Therefore, starting from a candidate nominal income vector with finite present value, repeated applications of $\mathcal{G}$ remain in that space. There is no way for repeated applications to converge to the desired equilibrium.

Intuitively, if the government is limited in its ability to run deficits, it cannot raise inflation to arbitrarily high levels. Under a Ricardian rule, the government cannot even increase the present value of demand. There is, however, an asymmetry between preventing and creating high inflation. As defined here, a Ricardian rule does not prevent the government from running *surpluses* with arbitrarily large present value. Hence, even when the TVC fails ($\beta \geq \lambda$), a Ricardian fiscal rule can select equilibria with $R > G$. This asymmetry matters only when the TVC fails: a limit on the present value of surpluses would not affect the viability of the nominal-wealth-targeting rule when

$\beta < \lambda$.

One qualification of Proposition 7 is worth mentioning. Given any equilibrium, it is possible to design a fiscal rule with finite-present-value deficits that uniquely implements it, provided that the mapping $\boldsymbol{PY} \mapsto \mathcal{G}(\boldsymbol{PY})$ need not be a contraction. Unique implementation when $R \leq G$ is therefore possible only in a somewhat pathological way: the resulting equilibrium is not cobweb-stable.

5 Conclusion

Discussions of price-level determinacy under an interest rate rule have long centered on either the Taylor principle or non-Ricardian fiscal policy. Both equilibrium-selection mechanisms have generated extensive theoretical debate. This paper shows that, when Ricardian equivalence fails in the relevant sense, fiscal policy can provide a long-run nominal anchor while satisfying the government's intertemporal budget constraint for every possible price path. Monetary policy can then focus on short-run inflation dynamics without relying on an explosive-path restriction or an FTPL-style fiscal regime.

The analysis identifies the aggregate transversality condition as the central dividing line. When it holds, the timing of fully backed taxes can stabilize aggregate demand and select the price level. The overlapping-generations model makes this mechanism explicit through a nominal-wealth-targeting rule and shows when that rule is Ricardian and dynamically stable. The same logic applies to other non-Ricardian environments, including models with behavioral distortions or bonds in the utility function.

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A Proof of the Main Result

Proof of Propositions 1 and 2. I begin by setting up some key concepts and notation. Observe that the supply curve depends on $\boldsymbol{P}$ only, and then the interest rate rule depends only on $\boldsymbol{P}$ and $\boldsymbol{Y}$. Then, if fiscal policy can uniquely implement the target sequence $\boldsymbol{P}^*$ of nominal prices, it automatically implements the desired sequences of interest rates and output.

It will be useful to define the (normalized) time-t nominal price to a claim on real output at $t+s$ (i.e., the nominal price of a real bond):

$$Q_{t,t+s} \equiv \frac{P_t}{R_{t,t+s}} = \frac{P_{t+s}}{I_{t,t+s}}. \tag{39}$$

The target value of $Q_{0,t}$ is

$$\tilde{Q}_{0,t} \equiv \frac{P_t^*}{I_{0,t}^*}. \tag{40}$$

I let $\boldsymbol{Y}(\boldsymbol{Q}), 1+\boldsymbol{i}(\boldsymbol{Q})$ denote output and nominal interest rates when price sequence $\boldsymbol{Q}$ is realized.

Finally, I define the *asset-demand correspondence*. Let

$$\boldsymbol{A} \in \mathcal{A}(\boldsymbol{Q}) \quad \Longleftrightarrow \quad \boldsymbol{\mathcal{C}}\big(A_0, \boldsymbol{Y}(\boldsymbol{Q}), \boldsymbol{\tau}(\boldsymbol{A}, \boldsymbol{Q}), 1+\boldsymbol{r}(\boldsymbol{Q})\big) = \boldsymbol{Y}(\boldsymbol{Q}), \tag{41}$$

where

$$\tau_t(\boldsymbol{A}, \boldsymbol{Q}) = A_t - \frac{A_{t+1}}{1+r_t(\boldsymbol{Q})}.$$

That is, the asset demand correspondence consists of all sequences of asset holdings $\boldsymbol{A}$ such that at interest rate $1+\boldsymbol{r}(\boldsymbol{Q})$, with taxes as implied by $\boldsymbol{A}$ and $1+\boldsymbol{r}(\boldsymbol{Q})$, the goods market-clearing condition holds.

Consider the following strategy to select an equilibrium. Let $\boldsymbol{\Delta}$ be some sequence such that $\boldsymbol{\Delta} = 0$ if and only if $\boldsymbol{Q} = \tilde{\boldsymbol{Q}}$. Define the asset supply rule

$$\frac{\boldsymbol{B}}{\boldsymbol{P}}(\boldsymbol{Q}) = \boldsymbol{A}(\boldsymbol{Q}) + \boldsymbol{\Delta}(\boldsymbol{Q}). \tag{42}$$

where $\boldsymbol{A}(\boldsymbol{Q})$ is an arbitrary element of $\mathcal{A}(\boldsymbol{Q})$. By definition, $\boldsymbol{Q}$ is an equilibrium if and only if $\boldsymbol{A}(\boldsymbol{Q}) + \boldsymbol{\Delta}(\boldsymbol{Q}) \in \mathcal{A}(\boldsymbol{Q})$ as well. Hence, it suffices to find $\boldsymbol{\Delta}$ such that this never occurs when $\boldsymbol{Q} \neq \tilde{\boldsymbol{Q}}$.

Let $\boldsymbol{\tau} = \boldsymbol{\tau}(\boldsymbol{A}(\boldsymbol{Q}), \boldsymbol{Q})$ and $\boldsymbol{\tau}' = \boldsymbol{\tau}(\boldsymbol{A}(\boldsymbol{Q}) + \boldsymbol{\Delta}(\boldsymbol{Q}), \boldsymbol{Q})$. It is simple to show that

$$\tau_t - \tau_t' = \frac{\Delta_{t+1}(\boldsymbol{Q})}{1+r_t} - \Delta_t(\boldsymbol{Q}) \quad \text{with} \quad \Delta_0(\boldsymbol{Q}) = 0. \tag{43}$$

Hence,

$$\lim_{T\to\infty} \sum_{t=0}^{T} R_{0,t}^{-1}(\tau_t - \tau_t') = \lim_{T\to\infty} R_{0,T}^{-1} \Delta_T(\boldsymbol{Q}). \tag{44}$$

Henceforth, I let $\hat{t}$ be the first t at which $Q_t \neq \tilde{Q}_t$.

Case 1: Ricardian demand and TVC/no-Ponzi hold. If Ricardian equivalence holds, choose Δ_t such that $R_{0,T}^{-1}\Delta_T(\boldsymbol{Q}) = \Psi > 0$ when $Q_{\hat{t}} < \tilde{Q}_{\hat{t}}$ and $R_{0,T}^{-1}\Delta_T(\boldsymbol{Q}) = \Psi < 0$ when the opposite holds. (Note: to make the rule continuous, Ψ can be a function of the distance between actual and target

nominal prices.)

Together, the TVC and no-Ponzi imply

$$\lim_{T\to\infty}\sum_{t=0}^{T}R_{0,t}^{-1}C_t = A_0 + \lim_{T\to\infty}\sum_{t=0}^{T}R_{0,t}^{-1}(Y_t-\tau_t). \tag{45}$$

Combining this with the equilibrium conditions $C_t = Y_t$ and $A_0 = B_0/P_0$, as well as the government flow budget constraint, we obtain

$$\lim_{T\to\infty} R_{0,T}^{-1}B_T/P_T = 0 \tag{46}$$

in any equilibrium. Hence, the only possible equilibrium is $\boldsymbol{Q} = \tilde{\boldsymbol{Q}}$ by construction. Of course, $\tilde{\boldsymbol{Q}}$ is already an equilibrium because then $\boldsymbol{B}/\boldsymbol{P}$ is an element of $\mathcal{A}(\tilde{\boldsymbol{Q}})$ by (42).

Now I turn to the necessity of a non-strictly-Ricardian policy. Suppose $\boldsymbol{\mathcal{T}}$ is strictly Ricardian. Then for all $\boldsymbol{Q}$, $\lim_{T\to\infty} R_{0,T}^{-1}B_T/P_T = 0$. With Ricardian equivalence imposed on the consumption function, the equilibrium condition becomes

$$\begin{aligned}\boldsymbol{Y}(\boldsymbol{Q}) &= \tilde{\boldsymbol{\mathcal{C}}}\Big(\lim_{T\to\infty} R_{0,T}^{-1}(\boldsymbol{Q})\frac{B_T}{P_T}(\boldsymbol{Q}), \boldsymbol{Y}(\boldsymbol{Q}), 1+\boldsymbol{r}(\boldsymbol{Q})\Big)\\ &= \tilde{\boldsymbol{\mathcal{C}}}\Big(0, \boldsymbol{Y}(\boldsymbol{Q}), 1+\boldsymbol{r}(\boldsymbol{Q})\Big),\end{aligned}$$

independently of the fiscal rule $\boldsymbol{\mathcal{T}}$. Thus, if an equilibrium with $\boldsymbol{Q}\neq\tilde{\boldsymbol{Q}}$ exists, a choice of a Ricardian fiscal rule cannot eliminate that equilibrium.

Case 2: SNRD holds. Now consider a sequence $\boldsymbol{Q}$ such that $Q_{\hat{t}} < \tilde{Q}_{\hat{t}}$. (The opposite case is symmetric.) Set taxes so that $\Delta_{\hat{t}}(\boldsymbol{Q}) > 0$, $\Delta_t(\boldsymbol{Q}) = 0$ for $t < \hat{t}$ and $\Delta_t(\boldsymbol{Q}) \geq \Delta_{t+1}(\boldsymbol{Q})/(1+r_t(\boldsymbol{Q}))$ for $t > \hat{t}$, with $\lim_{T\to\infty} R_{0,T}^{-1}(\boldsymbol{Q})\Delta_T(\boldsymbol{Q}) = 0$. (Such sequences clearly exist since, for instance, we can simply take $\Delta_t(\boldsymbol{Q}) = 0$ for $t > \hat{t}$.) The sequence $\boldsymbol{\tau}$ is more front-loaded than $\boldsymbol{\tau}'$ by construction (see (43)). Therefore, $\boldsymbol{\mathcal{C}}(.,\boldsymbol{\tau},.)$ is more back-loaded than $\boldsymbol{\mathcal{C}}(.,\boldsymbol{\tau}',.)$, and hence the only equilibrium is $\boldsymbol{Q} = \tilde{\boldsymbol{Q}}$ by the arguments above.

The TVC implies that this fiscal rule is Ricardian:

$$\begin{aligned}\lim_{T\to\infty} R_{0,T}^{-1}(\boldsymbol{Q})\frac{B_T}{P_T}(\boldsymbol{Q}) &= \lim_{T\to\infty} R_{0,T}^{-1}(\boldsymbol{Q})(A_T(\boldsymbol{Q})+\Delta_T(\boldsymbol{Q}))\\ &\leq \underbrace{\lim_{T\to\infty} R_{0,T}^{-1}(\boldsymbol{Q})A_T(\boldsymbol{Q})}_{\leq 0 \text{ by TVC}} + \underbrace{\lim_{T\to\infty} R_{0,T}^{-1}(\boldsymbol{Q})|\Delta_T(\boldsymbol{Q})|}_{=0}.\end{aligned}$$

The TVC applies in the first term because $\boldsymbol{A}(\boldsymbol{Q})$ denotes asset holdings in some *equilibrium* with prices $\boldsymbol{Q}$. (Note: This presumes the existence of such an equilibrium. If no such equilibrium exists, then it does not matter how fiscal policy is defined at $\boldsymbol{Q}$, though.) □

B Derivations for Section 3

This section derives the key formulas in Section 3.

B.1 Linearizing the Model

The first step is to derive the ratios of variables along the BGP, which are necessary to find the coefficients in the linearizations of the government and private sector budget constraints. On the BGP,

$$B^*_{t+1} = (1+i^*)(B^*_t - P^*_t\tau^*_t),$$

where B^*_t/P^*_t grows at rate G and P^*_t grows at rate $1+\pi^*$. Then

$$\frac{B^*_{t+1}}{B^*_t} = G(1+\pi^*), \quad \frac{P^*_t\tau^*_t}{B^*_t} = 1 - \frac{G}{R}.$$

The BGP debt-to-GDP ratio is defined by the constant $\omega \equiv B^*_t/P^*Y^*_t > 0$.

The government's (nonlinear) flow budget constraint (8) can be written as

$$B_t = P_t\tau_t + \frac{1}{1+i_t}B_{t+1},$$

which linearizes to

$$\begin{aligned}\hat{b}_t &= \frac{P^*_t\tau^*_t}{B^*_t}(\hat{p}_t + \hat{\tau}_t) + \frac{B^*_{t+1}/(1+i^*)}{B^*_t}(\hat{b}_{t+1} - \hat{i}_t)\\ &= (1-\frac{G}{R})(\hat{p}_t + \hat{\tau}_t) + \frac{G}{R}(\hat{b}_{t+1} - \hat{i}_t).\end{aligned}$$

Subtracting $\sum_{s=0}^{t-1}\hat{i}_t$ from both sides,

$$\Rightarrow \hat{b}_t - \sum_{s=0}^{t-1}\hat{i}_s = (1-\frac{G}{R})(\hat{p}_t + \hat{\tau}_t - \sum_{s=0}^{t-1}\hat{i}_s) + \frac{G}{R}(\hat{b}_{t+1} - \sum_{s=0}^{t}\hat{i}_s),$$

which can be iterated forward to obtain (16).

An analogous calculation yields the linearized private sector budget constraint (17). Under the assumption that the coefficients $R^s\partial\mathcal{C}/\partial Y_s, R^s\partial\mathcal{C}/\partial\tau_s$ are bounded, the sums in that equation are well-defined as long as $\tilde{\boldsymbol{y}}, \tilde{\boldsymbol{\tau}} \in \ell^1_{G/R}$. The analysis focuses on the case in which the present value of taxes is finite, so there is no need to consider $\tilde{\boldsymbol{\tau}}$ outside of this space.

What is key for the remainder of the results is that $\tilde{a}_\infty \neq 0$ if $\tilde{\boldsymbol{y}}$ grows at a geometric rate $\mu \in \mathbb{C}$ with $|\mu| \geq R/G$. Consider such a sequence, and ignore all disturbances to taxes and the real rate. Then (17) yields

$$\begin{aligned}\tilde{a}_{t+1} &= \omega^{-1}(\frac{R}{G}-1)(\tilde{y}_t - \tilde{c}_t) + \frac{R}{G}\tilde{a}_t\\ &= \omega^{-1}(\frac{R}{G}-1)\Big(1 - \sum_{s=0}^{\infty}(\frac{G}{R})^s R^s\frac{\partial\mathcal{C}}{\partial Y_s}\mu^s\Big)\mu^t\tilde{y}_0 + \frac{R}{G}\tilde{a}_t\\ &= \omega^{-1}(\frac{R}{G}-1)\Big(1 - \sum_{s=0}^{\infty}(\frac{G}{R})^s R^s\frac{\partial\mathcal{C}}{\partial Y_s}\mu^s\Big)\Big(\sum_{u=0}^{t}(\frac{R}{G})^{t-u}\mu^u\Big)\tilde{y}_0.\end{aligned}$$

In general, then, $\tilde{a}_T \not\to 0$ except in the knife-edge case where $1 - \sum_{s=0}^{\infty} (\frac{G}{R})^s R^s \frac{\partial \mathcal{C}}{\partial Y_s} \mu^s$. Nominal income paths growing at rate R/G then typically cause a violation of the linearized TVC, so they are excluded from the analysis.

A one-unit increase in $\tilde{a}_0$, with no change in $\tilde{\boldsymbol{y}}$, $\tilde{\boldsymbol{\tau}}$, or $\hat{\boldsymbol{r}}$, results in $\tilde{a}_T = \left((1 - \frac{\partial \mathcal{C}}{\partial A})\frac{R}{G}\right)^T$, so $\tilde{a}_\infty \to 0$ as long as $\partial \mathcal{C}/\partial A \in (0, 1]$. Hence, no other restrictions need to be imposed on the relevant domain.

It remains to derive (15) and (18). Here it is necessary to use the assumption that $\mathcal{C}$ is homogeneous of degree one in assets, output, and taxes. Notice that under this assumption, the partial derivatives of $\mathcal{C}$ with respect to the real rate (evaluated along the BGP) satisfy

$$\left.\frac{\partial \mathcal{C}}{\partial R_{t+s}}\right|_{A_t, \{Y_{t+s}, \tau_{t+s}, R\}} = G^t \left.\frac{\partial \mathcal{C}}{\partial R_{t+s}}\right|_{A_0, \{Y_t, \tau_t, R\}},$$

so with some abuse of notation I define $\partial \mathcal{C}/\partial R_s$ to be this derivative evaluated along the BGP at $t = 0$. I also denote the partial derivatives of $\mathcal{C}(\cdot)$ with respect to Y_{t+s} and τ_{t+s} by $\partial \mathcal{C}/\partial Y_s, \partial \mathcal{C}/\partial \tau_s$, respectively. Another implication of homogeneity is

$$1 = \omega \frac{\partial \mathcal{C}}{\partial A} + \sum_{s=0}^{\infty} (\frac{G}{R})^s \left(R^s \frac{\partial \mathcal{C}}{\partial Y_s} + (1 - \frac{G}{R}) \omega R^s \frac{\partial \mathcal{C}}{\partial \tau_s} \right), \tag{47}$$

(using the equilibrium relationships $Y_t^* = C_t^*$ and $\tau_t^* = (1 - G/R)A_t^*$ along the BGP). This equation is a consequence of the fact that a parallel proportional shift in $A, \boldsymbol{Y}$, and $\boldsymbol{\tau}$ results in a proportional shift in consumption.

Then, the function $\mathcal{C}(\cdot)$ linearizes as

$$\hat{c}_t = \omega \frac{\partial \mathcal{C}}{\partial a} \hat{a}_t + \sum_{s=0}^{\infty} (\frac{G}{R})^s (R^s \frac{\partial \mathcal{C}}{\partial Y_s} \hat{y}_{t+s} + (1 - \frac{G}{R}) \omega R^s \frac{\partial \mathcal{C}}{\partial \tau_s} \tau_{t+s}) + \sum_{s=0}^{\infty} \frac{R}{Y^*} \frac{\partial \mathcal{C}}{\partial R_s} \hat{r}_{t+s},$$

where Y^* denotes BGP output at $t = 0$. The term in r uses the fact that $\partial \mathcal{C}_t/\partial R_{t+s}$ grows at rate G. The factor G^t cancels with the division of both sides by $Y_t^* = G^t Y^*$. Writing this equation in terms of the present value of output and taxes,

$$\begin{aligned} \tilde{c}_t =& \omega \frac{\partial \mathcal{C}}{\partial a} \tilde{a}_t + \sum_{s=0}^{\infty} (\frac{G}{R})^s (R^s \frac{\partial \mathcal{C}}{\partial Y_s} \tilde{y}_{t+s} + (1 - \frac{G}{R}) \omega R^s \frac{\partial \mathcal{C}}{\partial \tau_s} \tilde{\tau}_{t+s}) \\ &+ \sum_{s=0}^{\infty} \left(\frac{R}{Y^*} \frac{\partial \mathcal{C}}{\partial R_s} + \sum_{u=s+1}^{\infty} (\frac{G}{R})^u (R^u \frac{\partial \mathcal{C}}{\partial Y_u} + (1 - \frac{G}{R}) \omega R^u \frac{\partial \mathcal{C}}{\partial \tau_u}) \right) \hat{r}_{t+s}. \end{aligned} \tag{48}$$

It is possible to derive this equation simply by subtracting $\sum_{s=0}^{t-1} \hat{\imath}_t$ from both sides and using the summation identity (47). Henceforth, the coefficient on $\hat{r}_{t+s}$ in the second line will be denoted γ_s, as in the body of the text. Furthermore, this expression clarifies how the Ricardian restriction on demand relates to the derivatives of the consumption function: demand is invariant to changes in the timing of taxes, keeping their present value $(1 - G/R) \sum_{s=0}^{\infty} (G/R)^s \tilde{\tau}_{t+s}$ invariant, if and only if

$R^s \partial \mathcal{C}/\partial \tau_s$ is constant as a function of s.

Then, substituting in the equilibrium relationship $\tilde{a}_t = \tilde{b}_t$ and replacing $\tilde{b}_t$ by the decomposition (16),

$$\tilde{c}_t = \omega(1 - \frac{G}{R}) \sum_{s=0}^{\infty} (\frac{G}{R})^s (\frac{\partial \mathcal{C}}{\partial A} + R^s \frac{\partial \mathcal{C}}{\partial \tau_s}) \tilde{\tau}_{t+s} + (\frac{R}{G})^t \omega \frac{\partial \mathcal{C}}{\partial A} \tilde{b}_\infty + \sum_{s=0}^{\infty} (\frac{G}{R})^s R^s \frac{\partial \mathcal{C}}{\partial Y_s} \tilde{y}_{t+s} + \sum_{s=0}^{\infty} \gamma_s \hat{r}_{t+s}.$$

Finally, substituting $(1 - G/R)\tilde{\tau}_t = \tilde{b}_t - (G/R)\tilde{b}_{t+1}$ yields (18).

B.2 Equilibrium Under a Simple Class of Fiscal Rules

This section considers a simple class of fiscal rules,

$$\tilde{b}_{t+1} = \frac{R}{G} \theta \tilde{b}_t - \kappa \tilde{y}_t, \quad \theta \in [0, 1], \ \kappa > 0, \tag{49}$$

and studies whether these rules can resolve long-run price-level indeterminacy.

Wealth effects. The analysis focuses on the nominal wealth effects induced by this policy. Note that under this fiscal rule, nominal debt at $t + s$ can be written as

$$\tilde{b}_{t+s} = (\frac{R}{G}\theta)^s \tilde{b}_t - \kappa \sum_{u=0}^{s-1} (\frac{R}{G}\theta)^{s-1-u} \tilde{y}_{t+u}.$$

Hence, letting $\Delta_s \equiv R^s \partial \mathcal{C}/\partial \tau_s - R^{s-1} \partial \mathcal{C}/\partial \tau_{s-1}$,

$$\begin{aligned} \sum_{s=1}^{\infty} \Delta_s (\frac{G}{R})^s \tilde{b}_{t+s} &= \sum_{s=1}^{\infty} \Delta_s \theta^s \tilde{b}_t - \kappa \sum_{s=1}^{\infty} \Delta_s (\frac{G}{R})^s \sum_{u=0}^{s-1} (\frac{R}{G}\theta)^{s-1-u} \tilde{y}_{t+u} \\ &= \sum_{s=1}^{\infty} \Delta_s \theta^s \tilde{b}_t - \kappa \frac{G}{R} \sum_{s=0}^{\infty} (\frac{G}{R})^s \tilde{y}_{t+s} \sum_{u=0}^{\infty} \Delta_{s+1+u} \theta^u. \end{aligned}$$

Starting from the initial state $\tilde{b}_0 = 0$, the nominal wealth terms in aggregate demand can be collected to obtain

$$\begin{aligned} (\boldsymbol{W}_y \tilde{\boldsymbol{y}})_t - \kappa (\boldsymbol{W}_f \tilde{\boldsymbol{y}})_t = &\sum_{s=0}^{\infty} (\frac{G}{R})^s (R^s \frac{\partial \mathcal{C}}{\partial Y_s} - \kappa \frac{G}{R} \sum_{u=0}^{\infty} \Delta_{s+1+u} \theta^u) \tilde{y}_{t+s} \\ &- \kappa (\frac{R}{G})^{t-1} (\sum_{s=0}^{\infty} \Delta_s \theta^s) \sum_{s=0}^{t-1} \theta^s (\frac{G}{R})^{t-1-s} \tilde{y}_{t-1-s}, \end{aligned} \tag{50}$$

where $\boldsymbol{W}_y$ represents direct wealth effects from nominal income and $\boldsymbol{W}_f$ represents wealth effects due to the fiscal response.

It is now possible to demonstrate why sufficiently persistent debt induces a strong demand response. Suppose nominal income grows by factor $\mu \in (0, R/G)$ each period, $\tilde{y}_t = \mu^t \tilde{y}_0$. Then the

nominal demand response is

$$(\boldsymbol{W}_y\tilde{\boldsymbol{y}})_t - \kappa(\boldsymbol{W}_f\tilde{\boldsymbol{y}})_t = \mu^t\tilde{y}\Big(\sum_{s=0}^{\infty}(\frac{G}{R})^s\mu^s(R^s\frac{\partial\mathcal{C}}{\partial Y_s} - \kappa\frac{G}{R}\sum_{u=0}^{\infty}\Delta_{s+1+u}\theta^u) - \frac{\kappa}{\mu}(\sum_{s=0}^{\infty}\Delta_s\theta^s)(\frac{1-(\theta/\mu)^t}{1-\theta/\mu})\Big).$$

As $\theta \to 1$, the expression on the right hand eventually becomes negative for *any* $\kappa > 0$, $\mu \in (0,1)$. Hence, a sufficiently persistent debt issuance rule is sufficient to offset the nominal wealth effect induced by any income path that grows at a constant rate.

Closing the model. To keep the model forward-looking, I assume that the nominal interest rate is a constant, $\hat{\imath}_t = 0$, and that output is given by a purely forward-looking Phillips curve that depends only on inflation,

$$\hat{y}_t = (\boldsymbol{\mathcal{Y}}\boldsymbol{\pi})_t \equiv \sum_{s=0}^{\infty}\frac{1+\pi^*}{Y^*}\frac{\partial\mathcal{Y}}{\partial\pi_s}\hat{\pi}_{t+s+1}.$$

I assume that the price-level sequence can be recovered from the path of nominal income, i.e., that the operator

$$\boldsymbol{I} + \boldsymbol{\mathcal{Y}}(\boldsymbol{S} - \boldsymbol{I})$$

has a bounded inverse, where $\boldsymbol{S}$ is the left shift operator, $(\boldsymbol{S}\boldsymbol{x})_t = x_{t+1}$ for a sequence $\boldsymbol{x}$. Then the real rate can be recovered from the sequence of nominal income as well,

$$\hat{\boldsymbol{r}} = (\boldsymbol{I} - \boldsymbol{S})\Big(\boldsymbol{I} + \boldsymbol{\mathcal{Y}}(\boldsymbol{S} - \boldsymbol{I})\Big)^{-1}\tilde{\boldsymbol{y}}.$$

It can be written as a forward convolution, $\hat{r}_t = \sum_{s=0}^{\infty}\chi_s\tilde{y}_{t+s}$. Hence, the component of demand attributable to fluctuations in real rates can be written

$$\mathrm{MPC}_t(\hat{\boldsymbol{r}}) = (\boldsymbol{M}\tilde{\boldsymbol{y}})_t \equiv \sum_{s=0}^{\infty}\gamma_s\sum_{u=0}^{\infty}\chi_u\tilde{y}_{t+s+u}.$$

Combining this expression for the MPC with the nominal wealth effects summarized by the operators $\boldsymbol{W}_y$, $\boldsymbol{W}_f$ in (50), an equilibrium sequence of nominal income is $\tilde{\boldsymbol{y}} \in \ell^1_{G/R}$ satisfying the condition

$$\boldsymbol{0} = (\boldsymbol{W}_y + \boldsymbol{M} - \boldsymbol{I} - \kappa\boldsymbol{W}_f)\tilde{\boldsymbol{y}}. \tag{51}$$

C Derivations and Proofs for the OLG Model

I begin by deriving the aggregate consumption function (22) for the OLG model and then prove the results stated in that section.

C.1 Aggregate Demand

Individual demand and aggregation. A consumption–savings plan (c^t_{t+s}, a^t_{t+s}) solves (21) if and only if the following conditions hold:

$$1 = \tilde{\beta}(1 + r_{t+s})\frac{c^t_{t+s}}{c^t_{t+s+1}}, \tag{52}$$

$$\lim_{T\to\infty} R^{-1}_{t,t+T}\delta^T a^t_{t+T} \leq 0. \tag{53}$$

The transversality condition (53), together with the no-Ponzi condition in the household's problem, implies that individual consumption satisfies the intertemporal budget constraint

$$\sum_{u=0}^{\infty} R^{-1}_{t+s,t+s+u}\delta^u c^t_{t+s+u} = a^t_{t+s} + \sum_{u=0}^{\infty} R^{-1}_{t+s,t+s+u}\delta^u (y^t_{t+s+u} - \tau^t_{t+s+u}). \tag{54}$$

The individual Euler equation (52) yields $c^t_{t+s+u} = \tilde{\beta}^u R_{t+s,t+s+u} c^t_{t+s}$. Combining these relationships and substituting $y^t_{t+s+u} - \tau^t_{t+s+u} = (\tilde{\lambda}^{s+u}/(1-\tilde{\lambda}\delta))(Y_{t+s+u} - \tau_{t+s+u})$ gives

$$\frac{c^t_{t+s}}{1-\tilde{\beta}\delta} = a^t_{t+s} + \sum_{u=0}^{\infty} R^{-1}_{t+s,t+s+u}\left(\frac{(\tilde{\lambda}\delta)^{s+u}}{1-\tilde{\lambda}\delta}\right)(Y_{t+s+u} - \tau_{t+s+u}). \tag{55}$$

Aggregating across households, rearranging, and substituting $\beta = \tilde{\beta}\delta, \lambda = \tilde{\lambda}\delta$,

$$C_t = (1-\beta)\left(A_t + \sum_{s=0}^{\infty} R^{-1}_{t,t+s}\lambda^s (Y_{t+s} - \tau_{t+s})\right),$$

as in (22).

C.2 Proofs

Proof of Proposition 3. I begin by defining some notation. Let $X_t = Y_t - \tau_t$, and for any variable Z_t, denote $z_t \equiv P_t Z_t / I_{0,t}$. Then one can write

$$c_t = (1-\beta)(a_t + h_t), \tag{56}$$

where (a_t, h_t) follow

$$\begin{pmatrix} a_{t+1} \\ h_{t+1} \end{pmatrix} = M \begin{pmatrix} a_t \\ h_t \end{pmatrix} + \begin{pmatrix} 1 \\ -\lambda^{-1} \end{pmatrix} x_t, \tag{57}$$

where

$$M = \begin{pmatrix} \beta & -(1-\beta) \\ 0 & \lambda^{-1} \end{pmatrix},$$

with boundary conditions

$$a_0 \text{ given}, \tag{58}$$

$$\lim_{T\to\infty} \lambda^T h_T = 0. \tag{59}$$

I solve this system of equations via an eigenvector decomposition. The eigenvectors of M are u_β with eigenvalue β and u_λ with eigenvalue λ^{-1}, where

$$u_\beta = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad u_\lambda = \begin{pmatrix} 1-\beta \\ \beta - \lambda^{-1} \end{pmatrix}.$$

Note that with these definitions,

$$\begin{pmatrix} 1 \\ -\lambda^{-1} \end{pmatrix} = \frac{\beta(1-\lambda)}{1-\beta\lambda} u_\beta + \frac{1}{1-\beta\lambda} u_\lambda.$$

Let $(a_t, h_t) = c_{\beta t} u_\beta + c_{\lambda t} u_\lambda$, where

$$a_t = c_{\beta t} + (1-\beta) c_{\lambda t},$$

$$h_t = (\beta - \lambda^{-1}) c_{\lambda t}.$$

With this notation, the system (57) can be written as

$$c_{\beta,t+1} = \beta c_{\beta t} + \frac{\beta(1-\lambda)}{1-\beta\lambda} x_t, \tag{60}$$

$$c_{\lambda,t+1} = \lambda^{-1} c_{\lambda t} + \frac{x_t}{1-\beta\lambda}. \tag{61}$$

The boundary conditions (58)–(59) yield

$$c_{\beta t} = \beta^t c_{\beta 0} + \frac{1-\lambda}{1-\beta\lambda} \sum_{s=0}^{t-1} \beta^{t-s} x_s = \beta^t \left(a_0 + \frac{(1-\beta)\lambda}{1-\beta\lambda} h_0 \right) + \frac{1-\lambda}{1-\beta\lambda} \sum_{s=0}^{t-1} \beta^{t-s} x_s, \tag{62}$$

$$c_{\lambda t} = -\frac{\lambda}{1-\beta\lambda} \sum_{s=0}^{\infty} \lambda^s x_{t+s} = -\frac{\lambda}{1-\beta\lambda} h_t \tag{63}$$

The consumption rule (56) then yields

$$\begin{aligned} c_t &= (1-\beta)\left(\beta^t (a_0 + \frac{(1-\beta)\lambda}{1-\beta\lambda} h_0) + \frac{1-\lambda}{1-\beta\lambda} \sum_{s=0}^{t-1} \beta^{t-s} x_s + \frac{1-\lambda}{1-\beta\lambda} h_t \right) \\ &= (1-\beta)\left(\beta^t a_0 + \frac{1}{1-\beta\lambda} \sum_{s=0}^{\infty} (\lambda(1-\beta)\beta^t \lambda^s + (1-\lambda)(\mathbf{1}\{s<t\}\beta^{t-s} + \mathbf{1}\{s \geq t\}\lambda^{s-t})) x_s \right). \end{aligned} \tag{64}$$

Furthermore, asset holdings satisfy

$$\begin{aligned} a_t &= \beta^t (a_0 + \frac{(1-\beta)\lambda}{1-\beta\lambda} h_0) + \frac{1-\lambda}{1-\beta\lambda} \sum_{s=0}^{t-1} \beta^{t-s} x_s - \frac{(1-\beta)\lambda}{1-\beta\lambda} h_t \\ &= \beta^t a_0 + \frac{(1-\beta)\lambda}{1-\beta\lambda} \sum_{s=0}^{\infty} (\beta^t \lambda^s + \mathbf{1}\{s<t\}\beta^{t-s} - \mathbf{1}\{s \geq t\}\lambda^{s-t}) x_s. \end{aligned} \tag{65}$$

□

Proof of Corollary 1. It is clear from (22) that when $\lambda = 1$, then C_t depends only on $A_t - \sum_{s=0}^{\infty} R_{t,t+s}^{-1}\tau_{t+s}$, not on the timing of taxes. When $\lambda < 1$, on the other hand, then consumption is no longer solely a function of the present value of taxes.

Turning to the TVC, consider (65). The TVC holds if and only if, for any admissible path of net income $\boldsymbol{x}$, $a_T \to 0$ as $T \to \infty$. Recall that admissible paths $\boldsymbol{x}$ satisfy $\lambda^T x_T \to 0$ as $T \to \infty$. Hence, x_t cannot grow faster than λ^{-1}. This, in turn, implies that a_t cannot grow faster than β/λ. Thus, the TVC holds when $\lambda > \beta$ and fails when $\lambda \leq \beta$. □

Proof of Proposition 4. Solve (25) backwards to obtain

$$I_{0,t}^{-1} P_t Y_t = (\frac{\beta}{\lambda})^t \Big(P_0 Y_0 - \frac{(1-\lambda)(1-\beta)}{\lambda} \sum_{s=1}^{t} (\frac{\lambda}{\beta})^s I_{0,s}^{-1} B_s \Big). \tag{66}$$

Therefore, there exists an equilibrium path for any $P_0 Y_0 \geq \frac{(1-\lambda)(1-\beta)}{\lambda} \sum_{s=1}^{\infty} (\frac{\lambda}{\beta})^s I_{0,s}^{-1} B_s$, since the price level cannot be negative for any t. When the inequality holds strictly, we have that $I_{0,T}^{-1} P_T Y_T$ grows at rate β/λ in the long run. On the other hand, when this is an equality,

$$\begin{aligned} \lim_{T\to\infty} \frac{I_{0,T+1}^{-1} P_{T+1} Y_{T+1}}{I_{0,T}^{-1} P_T Y_T} &= \frac{\beta}{\lambda} \Big(1 - \frac{\frac{(1-\lambda)(1-\beta)}{\lambda} I_{0,T+1}^{-1} B_T}{I_{0,T}^{-1} P_T Y_T} \Big) \\ &= \frac{\beta}{\lambda} \Big(1 - \frac{I_{0,T+1}^{-1} B_T}{\sum_{s=1}^{\infty} (\frac{\lambda}{\beta})^s I_{0,s}^{-1} B_s} \Big), \end{aligned}$$

as desired. □

Proof of Lemma 1. I begin by proving the general claim that a feedback rule of the form (28) with $\kappa \geq 1/(1-\lambda)$ induces a unique equilibrium.

Under this specification, the modified Euler equation (25) yields

$$\begin{aligned} \lambda \frac{P_{t+1} Y_{t+1}}{1+i_t} &= \beta P_t Y_t - (1-\beta)(1-\lambda)(\frac{B_{t+1}^*}{1+i_t} + \kappa(P_t^* Y_t - P_t Y_t)) \\ &= \lambda \frac{P_{t+1}^* Y_{t+1}}{1+i_t} + (\beta + (1-\beta)(1-\lambda)\kappa)(P_t Y_t - P_t^* Y_t) \\ \Rightarrow \frac{P_{t+1} Y_{t+1} - P_{t+1}^* Y_{t+1}}{1+i_t} &= \frac{\beta + (1-\beta)(1-\lambda)\kappa}{\lambda} (P_t Y_t - P_t^* Y_t). \end{aligned}$$

The second line follows from applying the Euler equation to the sequence $(\boldsymbol{PY}^*, \boldsymbol{B}^*)$. Since the $t = 0$ present value of nominal income cannot diverge faster than λ^{-1}, this equation implies that as long as $\beta + (1-\beta)(1-\lambda)\kappa \geq 1$, it must be that $P_t Y_t = P_t^* Y_t$ for all t. Hence $\kappa \geq 1/(1-\lambda)$ suffices to select a unique equilibrium, as desired.

Now, I prove the claims relevant to the case $\kappa = 1/(1-\lambda)$.

Claim. *Under the rule* (28)*,*

$$P_t(H_t^Y - H_t^\tau) = P_t^*(H_t^{Y*} - H_t^{\tau*}) - \frac{1+i^*}{1-\lambda}(P_{t-1}^* - P_{t-1})G^{t-1}Y,$$

where

$$H_t^Y = \sum_{s=0}^{\infty} \lambda^s \frac{R_{0,t}}{R_{0,t+s}} G^{t+s}Y, \quad H_t^\tau = \sum_{s=0}^{\infty} \lambda^s \frac{R_{0,t}}{R_{0,t+s}} \tau_{t+s}$$

denote human wealth attributable to income and taxes, and $H_t^{Y*}, H_t^{\tau*}$ denote the values of these quantities under the target equilibrium.

Proof. Nominal taxes satisfy

$$\begin{aligned} P_t\tau_t &= B_t - \frac{B_{t+1}}{1+i_t} \\ &= B_t^* - \frac{B_{t+1}^*}{1+i_t} - \frac{1}{1-\lambda}\Big(((1+i_{t-1})P_{t-1}Y_{t-1} - P_tY_t) - ((1+i_{t-1})P_{t-1}^*Y_{t-1} - P_t^*Y_t)\Big) \\ &= P_t^*\tau_t^* - \frac{1}{1-\lambda}\Big(((1+i_{t-1})P_{t-1}Y_{t-1} - P_tY_t) - ((1+i_{t-1})P_{t-1}^*Y_{t-1} - P_t^*Y_t)\Big). \end{aligned}$$

Taking λ-discounted nominal present values of both sides and rearranging,

$$\begin{aligned} P_tH_t^\tau - P_t^*H_t^{\tau*} &= -\frac{1}{1-\lambda}\Big(((1+i_{t-1})P_{t-1}Y_{t-1} - (1-\lambda)P_tH_t^Y) \\ &\qquad - ((1+i_{t-1})P_{t-1}^*Y_{t-1} - (1-\lambda)P_t^*H_t^{Y*})\Big) \\ &= \frac{1+i_{t-1}}{1-\lambda}(P_{t-1}^*Y_{t-1} - P_{t-1}Y_{t-1}) - (P_t^*H_t^{Y*} - P_tH_t^Y), \end{aligned}$$

as desired. □

Note that then the private sector flow-budget constraint implies

$$\begin{aligned} P_{t+1}A_{t+1} &= (1+i^*)P_t(A_t + Y_t - \tau_t - C_t) \\ &= (1+i^*)\beta P_tA_t + (1+i^*)P_t(Y_t - \tau_t - (1-\beta)H_t) \\ &= (1+i^*)\beta P_tA_t + (1+i^*)(P_tH_t - \frac{\lambda}{1+i^*}P_{t+1}H_{t+1} - (1-\beta)P_tH_t) \\ &= (1+i^*)\beta P_t(A_t + H_t) - \lambda P_{t+1}H_{t+1} \end{aligned}$$

Then $P_tC_t = (1-\beta)P_t(A_t + H_t)$ implies

$$P_{t+1}C_{t+1} = (1+i^*)\beta P_tC_t + (1-\beta)(1-\lambda)P_{t+1}H_{t+1}. \tag{67}$$

Then, under this fiscal rule,

$$\begin{aligned}
P_{t+1}C_{t+1} - P^*_{t+1}Y_{t+1} &= (1+i^*)\beta(P_tC_t - P^*_tY_t) + (1-\beta)(1-\lambda)(P_{t+1}H_{t+1} - P^*_{t+1}H^*_{t+1}) \\
&= (1+i^*)\beta(P_tC_t - P^*_tY_t) + (1-\beta)(1+i^*)(P_t - P^*_t)G^tY \\
&= (1+i^*)^{t+1}\beta^{t+1}(P_0C_0 - P^*_0Y_0) \\
&\quad + (1-\beta)\sum_{s=0}^{t}(1+i^*)^{t+1-s}\beta^{t-s}G^sY(P_s - P^*_s).
\end{aligned}$$

Recall that the policy is constructed so that $P_0C_0 = P^*_0Y_0$. Then

$$P_tC_t - P_tY_t = (1-\beta)\sum_{s=0}^{t-1}(1+i^*)^{t-s}\beta^{t-1-s}(P_sY_s - P^*_sY_s) - (P_tY_t - P^*_tY_t). \tag{68}$$

□

Proof of Proposition 5. The fiscal rule (28) is Ricardian whenever

$$\begin{aligned}
0 &\geq \lim_{T\to\infty} I^{-1}_{0,T}B_T \\
&= \lim_{T\to\infty}\left(I^{-1}_{0,T}B^*_T + \frac{1}{1-\lambda}(I^{-1}_{0,T-1}P^*_{T-1}Y_{T-1} - I^{-1}_{0,T-1}P_{T-1}Y_{T-1})\right)
\end{aligned}$$

But the quantity on the right-hand side is bounded above by

$$\lim_{T\to\infty} I^{-1}_{0,T}B^*_T + \frac{1}{1-\lambda}\lim_{T\to\infty} I^{-1}_{0,T-1}P^*_{T-1}Y_{T-1},$$

which is equal to zero for any target equilibrium such that $1 + r_T > G$ in the long run.

Next, I demonstrate that the excess-demand mapping (68) is a contraction on $\mathcal{S}$. Specifically, the mapping $\boldsymbol{PY} \mapsto \boldsymbol{PC}$ can be written as the operator

$$\boldsymbol{T}(\boldsymbol{PY}) = \boldsymbol{P^*Y} + \boldsymbol{\Delta}(\boldsymbol{PY} - \boldsymbol{P^*Y}),$$

where

$$(\boldsymbol{\Delta x})_t \equiv (1-\beta)\sum_{s=0}^{t-1}(1+i^*)^{t-s}\beta^{t-1-s}x_s.$$

Then

Claim. *The map $\boldsymbol{\Delta}$ is a contraction in the $\mathcal{S}$-norm with modulus $\lambda(1-\beta)/(1-\beta\lambda)$.*

Proof. Let $\boldsymbol{x} \in \mathcal{S}$. We have

$$\begin{aligned}
\|\boldsymbol{\Delta x}\|_{\mathcal{S}} &= \sum_{t=1}^{\infty}\left(\frac{\lambda}{1+i^*}\right)^t(1-\beta)\left|\sum_{s=0}^{t-1}(1+i^*)^{t-s}\beta^{t-1-s}x_s\right| \\
&\leq \sum_{t=1}^{\infty}\lambda^t(1-\beta)\sum_{s=0}^{t-1}(1+i^*)^{-s}\beta^{t-1-s}|x_s|
\end{aligned}$$

$$= \sum_{s=0}^{\infty}(1-\beta)(1+i^*)^{-s}\beta^{-(s+1)}|x_s| \sum_{t=s+1}^{\infty}(\beta\lambda)^t$$
$$= \frac{\lambda(1-\beta)}{1-\beta\lambda}\sum_{s=0}^{\infty}\left(\frac{\lambda}{1+i^*}\right)^s|x_s| = \frac{\lambda(1-\beta)}{1-\beta\lambda}\|\boldsymbol{x}\|_{\mathcal{S}}.$$

□

□

Proof of Proposition 6. See Appendix C.3. □

Proof of Proposition 7. Retaining the convention from earlier that lowercase letters denote quantities normalized by nominal present value, an equilibrium is summarized by a single fixed-point equation on sequences,

$$\boldsymbol{py} = \underbrace{(1-\beta)a_0\boldsymbol{\beta}^t + \boldsymbol{A}(\boldsymbol{py} - \boldsymbol{\tau}(\boldsymbol{py}))}_{\boldsymbol{T}(\boldsymbol{py})}, \tag{69}$$

where $\boldsymbol{\beta}^t \equiv (1, \beta, \beta^2, \dots)$ and $\boldsymbol{A}$ is the operator defined by $(\boldsymbol{Ax})_t = (1-\beta)\sum_{s=0}^{\infty}\gamma_{s,t}x_s$.

Let $(\ell^1)^+ \subset (\ell^1_\lambda)^+$ denote the subset of sequences $\boldsymbol{x}$ such that $\sum_{t=0}^{\infty}|x_t| < \infty$. I prove the following claims.

1. The coefficients of $\boldsymbol{A}$ satisfy $(1-\beta)\sum_{t=0}^{\infty}\gamma_{s,t} = 1$ for all s.

$$(1-\beta)\sum_{t=0}^{\infty}\gamma_{s,t} = \frac{1-\beta}{1-\beta\lambda}\left(\sum_{t=0}^{\infty}(1-\beta)\beta^t\lambda^{s+1} + \sum_{t=s+1}^{\infty}(1-\lambda)\beta^{t-s} + \sum_{t=0}^{s}(1-\lambda)\lambda^{s-t}\right)$$
$$= \frac{1-\beta}{1-\beta\lambda}\left(\lambda^{s+1} + \frac{\beta(1-\lambda)}{1-\beta} + (1-\lambda)\frac{1-\lambda^{(s+1)}}{1-\lambda}\right)$$
$$= \frac{1-\beta}{1-\beta\lambda}\left(1 + \frac{\beta(1-\lambda)}{1-\beta}\right) = 1.$$

2. Then, let the present value function $\boldsymbol{PV}$ on $(\ell^1)^+$ be defined by

$$\boldsymbol{PV}(\boldsymbol{x}) = \sum_{t=0}^{\infty}x_t.$$

   The above implies $\boldsymbol{PV}(\boldsymbol{Ax}) = \boldsymbol{PV}(\boldsymbol{x})$ for all $\boldsymbol{x} \in (\ell^1)^+$. Then for all $\boldsymbol{py} \in (\ell^1)^+$,

$$\boldsymbol{PV}(\boldsymbol{T}(\boldsymbol{py})) = a_0 + \boldsymbol{PV}(\boldsymbol{py} - \boldsymbol{\tau}(\boldsymbol{py}))$$

   if $\boldsymbol{\tau}(\boldsymbol{py})$ is finite.

3. Hence, if $\boldsymbol{\mathcal{T}}$ is finite-PV, then our assumptions imply that for all $\boldsymbol{y} \in (\ell^1)^+$, we have $\boldsymbol{\tau}(\boldsymbol{py}) \in (\ell^1)^+$. The function $\boldsymbol{T}$ then maps $(\ell^1)^+$ to itself.

Since $\boldsymbol{T}$ maps $(\ell^1)^+$ to itself, it cannot be a contraction mapping towards an equilibrium $\boldsymbol{py}^*$ with $\lim_{T\to\infty}(\lambda/\beta)^T \boldsymbol{py}_T^* > 0$, since $\boldsymbol{py}^*$ lies outside the closure of $(\ell^1)^+$. □

C.3 Derivation of Results for Section 4.5

Putting money in the utility function. An individual household's flow utility is

$$u_t(c, m) = \log(X(c, m|\alpha)),$$

where c is consumption, m denotes real balances, $X(.,.|\alpha)$ is a concave and homothetic aggregator, and *alpha* is a parameter that determines money demand. The expenditure-minimization problem faced by a household within a period is

$$q_t = \min_{c,m} \; c + \frac{i_t}{1+i_t} m \;\; \text{s.t.} \;\; X(c, m|\alpha_t) = 1. \tag{70}$$

Write $X(c, m|\alpha) = cx(m/c)$ (suppressing the dependence on α). Then the first-order conditions are

$$\begin{aligned}(c) &: 1 = \xi(x(\frac{m}{c}) - \frac{m}{c}x'(\frac{m}{c})),\\ (m) &: \frac{i_t}{1+i_t} = \xi x'(\frac{m}{c}),\end{aligned}$$

where ξ is the Lagrange multiplier on the constraint $cx(m/c) = 1$. These conditions can be combined to obtain

$$g(\frac{m}{c}) \equiv \frac{x(\frac{m}{c})}{x'(\frac{m}{c})} - \frac{m}{c} = \frac{1+i_t}{i_t}. \tag{71}$$

Inverting this equation,

$$\frac{m}{c} = g^{-1}(\frac{1+i_t}{i_t}). \tag{72}$$

Then it is possible to derive the expenditure shares on consumption and real balances: multiplying the FOCs by c and m, respectively, and adding them,

$$q_t = \lambda c x(\frac{m}{c}) = \lambda. \tag{73}$$

The expenditure shares on goods and real balances, $1 - \theta(i_t)$ and $\theta(i_t)$, satisfy

$$\theta(i_t|\alpha) = \frac{m}{c}\frac{x'(\frac{m}{c})}{x(\frac{m}{c})}. \tag{74}$$

I impose the following assumption in what follows:

Assumption 1. *The expenditure share $\theta(i_t|\alpha)$ on real balances is bounded as a function of i_t, with the bound tending to zero as $\alpha_t \to 0$.*

Aggregate demand and the Euler equation. It is simple to show that with log utility, *aggregate expenditures* on consumption and forgone interest on real balances are simply a constant

marginal propensity to consume times total aggregate wealth,

$$P_t C_t + \frac{i_t}{1+i_t} M_t = (1-\beta)\big(B_t + M_{t-1} + \sum_{s=0}^{\infty} I_{t,t+s}^{-1} \lambda^s (P_{t+s} Y_{t+s} - P_{t+s}\tau_{t+s})\big). \tag{75}$$

Denote total nominal expenditures by $E_t \equiv P_t C_t + \frac{i_t}{1+i_t} M_t$. The Euler equation can be written as

$$\frac{\lambda}{1+i_t} E_{t+1} = \beta E_t - (1-\beta)(1-\lambda)\frac{B_{t+1}+M_t}{1+i_t}. \tag{76}$$

Monetary policy. Monetary policy is set according to one of two rules, either a Taylor rule for the interest rate,

$$\log(1+i_t) = \max\Big\{\epsilon, \log(1+\hat{i}_t) + \phi(\log P_t - \log P_{t-1})\Big\}, \tag{77}$$

where $1+\hat{i}_t$ is a target interest rate that converges to $1+i^*$ in the long run, or a money growth rule,

$$\log M_{t+1} = \log M_t + \log(1+\mu_t), \tag{78}$$

where $1+\mu_t$ is a money growth rate that converges to $1+\mu^*$.

The fiscal rule. The government flow budget constraint is (35). Let

$$J_t \equiv \prod_{s=0}^{t-1} \frac{1+i_s}{1+\hat{i}_s},$$

and consider the fiscal rule

$$\frac{B_{t+1}+M_t}{1+i_t} = J_t \frac{B^*_{t+1}+M^*_t}{1+\hat{i}_t} + \kappa(J_t E^*_t - E_t). \tag{79}$$

Plugging this fiscal rule into the Euler equation (76) and dividing both sides by $I_{0,t}$,

$$\lambda(e_{t+1} - e^*_{t+1}) = \big(\beta + (1-\beta)(1-\lambda)\kappa\big)(e_t - e^*_t), \tag{80}$$

where $e_t \equiv E_t/I_{0,t}$, $e^*_t \equiv E^*_t/I^*_{0,t}$ and $\theta_t \equiv \theta(i_t)$, $\theta^*_t \equiv \theta(\hat{i}_t)$. The only positive solution satisfying $\sum_{t=0}^{\infty} \lambda^t e_t < \infty$ is $e_t = e^*_t$ for all t. With the assumption that the expenditure share on real balances is bounded, this is also the only solution such that the discounted value of nominal income, $\sum_{t=0}^{\infty} I_{0,t}^{-1} \lambda^t P_t Y_t < \infty$. As $\alpha \to 0$, $E_t/I_{0,t} \to E^*_t/I^*_{0,t}$ implies $P_t Y_t/I_{0,t} \to P^*_t Y_t/I^*_{0,t}$, as desired.

D Extensions to other environments

This Appendix demonstrates how the main result of Proposition 2 works in nonlinear economies beyond the OLG environment of Section 4.

D.1 A Model with Bonds in the Utility Function

Consider an endowment economy with output $Y_t = G^t Y_0$. There is a representative agent with log utility over consumption and real bond holdings,

$$U(\boldsymbol{C}, \frac{\boldsymbol{B}}{\boldsymbol{P}}) = \sum_{t=0}^{\infty} \beta^t (\alpha \log C_t + (1-\alpha) \log \frac{B_t}{P_t}). \tag{81}$$

The agent maximizes utility subject to the nominal flow budget constraint

$$P_t C_t + \frac{B_{t+1}}{1+i_t} + \frac{B^I_{t+1}}{1+i^I_t} \leq B_t + B^I_t + P_t Y_t - P_t \tau_t \tag{82}$$

and a no-Ponzi condition. Here, B_t denotes holdings of government bonds that pay a nominal interest rate $1 + i_t$ (which is set exogenously for all t), whereas B^I_t denotes holdings of illiquid, privately issued nominal bonds that pay an interest rate $1 + i^I_t$ and are in zero net supply. Note that the agent obtains utility only from government bonds, so in equilibrium, it will not necessarily be the case that $i_t = i^I_t$.

To write the aggregate consumption function in the form (3), it is first necessary to solve for the equilibrium in the illiquid asset market. The market-clearing condition is $B^I_t = 0$, and the nominal rate must satisfy the Euler equation

$$1 = \frac{\beta}{G} \frac{P_t(1+i^I_t)}{P_{t+1}}. \tag{83}$$

Henceforth, let $R^I \equiv G/\beta$ denote the *real* interest rate on illiquid bonds.

The aggregate consumption function in this model satisfies

$$C_t = \alpha(1-\beta)(\frac{B_t}{P_t} + H^Y_t - H^\tau_t), \tag{84}$$

where for a quantity X_t, H^X_t is defined as the present value of $\{X_{t+s}\}$ discounted by the illiquid risk-free rate:

$$H^X_t \equiv \sum_{s=0}^{\infty} {R^I}^{-s} X_{t+s}.$$

Furthermore, observe that $H^Y_t = Y_t/(1-\beta)$.

I construct a fiscal rule that pegs nominal demand at a target level $P^*_t Y_t$, i.e.,

$$\frac{B_t}{P_t} - H^\tau_t = \frac{P^*_t}{P_t} \frac{Y_t}{\alpha(1-\beta)} - \frac{Y_t}{1-\beta}. \tag{85}$$

A fiscal rule satisfying this equation requires

$$\begin{aligned} \frac{B_t}{P_t} - H^\tau_t &= \frac{1}{R_t} \frac{B_{t+1}}{P_{t+1}} - \frac{1}{R^I} H^\tau_{t+1} \\ &= (1 - \frac{R_t}{R^I}) \frac{1}{R_t} \frac{B_{t+1}}{P_{t+1}} + \frac{1}{R^I} \left(\frac{B_{t+1}}{P_{t+1}} - H^\tau_{t+1} \right) \end{aligned}$$

$$= (1 - \frac{R_t}{R^I})\frac{1}{R_t}\frac{B_{t+1}}{P_{t+1}} + \frac{1}{R^I}\left(\frac{P^*_{t+1}}{P_{t+1}}\frac{Y_{t+1}}{\alpha(1-\beta)} - \frac{Y_{t+1}}{1-\beta}\right),$$

Combining these relationships and rearranging,

$$(1 - \frac{R_t}{R^I})\frac{1}{R_t}\frac{B_{t+1}}{P_{t+1}} = \left(\frac{P^*_t}{P_t} - \beta R_t \frac{P^*_{t+1}}{(1+i_t)P_t}\right)\frac{Y_t}{\alpha(1-\beta)} - Y_t. \tag{86}$$

To check whether this rule is Ricardian, note that under this fiscal rule,

$$\begin{aligned} R_{0,t}^{-1}\frac{B_t}{P_t} &= I_{0,t}^{-1}\frac{P_t}{P_0}\left(\frac{(\frac{P^*_t}{P_t} - \frac{1}{\beta}\frac{P^*_{t+1}}{P_{t+1}})\frac{Y_t}{\alpha(1-\beta)} - Y_t}{1 - \frac{R_t}{R^I}}\right) \\ &\leq \frac{I_{0,t}^{-1}P^*_t}{(1-\frac{R_t}{R^I})P_0} \end{aligned}$$

Therefore, assuming that $R > G$ in the target equilibrium, this rule is Ricardian *unless* $R_T \to R^I$ as $T \to \infty$. This issue is easy to solve: if, at $t = 0$, the central bank observes that $R_T \to R^I$ as $T \to \infty$, then it just issues a constant real quantity of bonds $B_t/P_t = \overline{b}$ for all t. This cannot be an equilibrium either, since $R_T \to R^I$ requires $B_T/P_T \to \infty$ in equilibrium, since the liquidity premium on bonds goes to zero.

D.2 A Heterogeneous-Agent Model

This section considers a flexible-price model in which the demand block consists of two types of agents, as in TANK models (e.g., Debortoli and Galí, 2024). A fraction θ of households are "Ricardian" and trade in frictionless financial markets, while the remaining fraction $1 - \theta$ are "hand-to-mouth" households who simply consume their net income in each period.[26] Output Y_t grows exogenously at gross rate G and is distributed equally across agents. Taxes τ_t are distributed equally across the population as well. The nominal rate sequence $1 + \boldsymbol{i}$ is exogenous and converges to a limit $1 + i^*$ in the long run.

Ricardian households solve the problem

$$\max_{C^R_t} \sum_{t=0}^{\infty} \beta^t \log C^R_t \quad \text{s.t.} \quad A_{t+1} = R_t(A_t + \theta(Y_t - \tau_t) - C_t), \text{ no-Ponzi.}$$

Their aggregate demand, as in the benchmark OLG model with $\lambda = 1$, is just a constant fraction $1 - \beta$ of their total financial and human wealth. The aggregate demand C^H_t of hand-to-mouth households is just $(1 - \theta)(Y_t - \tau_t)$. Combining these demands, writing them in nominal terms, and imposing the equilibrium conditions $B_t = P_t A_t, C^R_t + C^H_t = Y_t$, the equilibrium equation is

$$P_t Y_t = (1 - \beta)(B_t + \theta(H^Y_t - H^\tau_t)) + (1 - \theta)(P_t Y_t - P_t \tau_t), \tag{87}$$

[26] It is often possible to micro-found this assumption by assuming that these households face a borrowing constraint and have a much higher discount factor than Ricardian households, rendering hand-to-mouth consumption optimal.

where

$$H_t^Y \equiv \sum_{s=0}^{\infty} I_{t,t+s}^{-1} P_{t+s} Y_{t+s}, \quad H_t^\tau \equiv \sum_{s=0}^{\infty} I_{t,t+s}^{-1} P_{t+s} \tau_{t+s},$$

denote the present value of nominal income and taxes, respectively. (The requirement that these present values exist puts a restriction on the relevant domain for $\boldsymbol{P}$.)

With the government budget constraint (8), it is possible to rearrange (87) as

$$\frac{B_{t+1}}{I_{0,t+1}} = \beta \frac{B_t}{I_{0,t}} + \frac{\theta}{1-\theta} \frac{P_t Y_t - (1-\beta) H_t^Y}{I_{0,t}} \tag{88}$$

when the sequence of debt $\boldsymbol{B}$ is Ricardian, so that $B_t = H_t^\tau$ for all t.

Then consider the fiscal rule

$$\frac{B_{t+1}}{1+i_t} = \beta B_t + \frac{1}{1-\theta}(PY_t^* - (1-\beta)(1-\theta)H_t^Y - \theta P_t Y_t), \tag{89}$$

where $\boldsymbol{P}Y^*$ is some desired path of nominal income. This rule is feasible for the fiscal authority if it observes the prices of real claims due at all future dates, $\{R_{t,t+s}^{-1}\}_{s\geq 0}$.

Proposition 8. *The fiscal rule (89) uniquely implements an equilibrium in which nominal income is $\boldsymbol{P}\boldsymbol{Y}^*$. This rule is Ricardian for all $\boldsymbol{P}\boldsymbol{Y}^*$ with finite present value.*

Proof. Plugging (89) into the left-hand side of (88) and rearranging,

$$PY_t^* = P_t Y_t.$$

Notice that

$$\lim_{T\to\infty} \frac{B_{T+1}}{I_{0,T+1}} = \lim_{T\to\infty} \beta \frac{B_T}{I_{0,T}} + \frac{1}{1-\theta} \frac{PY_T^* - (1-\beta)(1-\theta)H_T^Y - \theta P_T Y_T}{I_{0,T}}$$

The second set of terms on the right-hand side is summable over t and tends to zero as $t \to \infty$. Then, for any $\epsilon > 0$, there exists $\hat{T}$ such that these terms sum to less than ϵ over $t > \hat{T}$. Then $\hat{T}'$ can be chosen so that $|B_{t+1}/I_{0,t+1}| \leq \beta^{\hat{T}'-\hat{T}} B_{\hat{T}}/I_{0,\hat{T}} + \epsilon$ is arbitrarily small as well, so the policy is Ricardian. □

D.3 A Model with Behavioral Discounting

This section considers a model in which agents hold behavioral expectations. As in Gabaix (2020), agents' beliefs are *anchored* towards a given path of future outcomes. When a future outcome changes, agents do not fully internalize that news, and the degree of mis-perception increases at longer horizons.

I consider an endowment economy, $Y_t = G^t Y_0$, with a representative agent. The nominal rate is constant at $1 + i^*$. The agent solves the problem

$$\max_{C_t, A_t} \tilde{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \log C_t \right] \text{ s.t. } A_{t+1} = R_t(A_t + Y_t - \tau_t - C_t), \tag{90}$$

where $\tilde{E}_t[\cdot]$ denotes the agent's *subjective expectations at time t*. The solution to this problem is

$$C_t = (1-\beta)(A_t + \tilde{H}_t), \tag{91}$$

where $\tilde{H}_t$ is the agent's perceived present value of income,

$$\begin{aligned}\tilde{H}_t \equiv & \tilde{E}_t\Big[\sum_{s=0}^{\infty} R_{t,t+s}^{-1}(Y_{t+s} - \tau_{t+s})\Big] \\ &= \sum_{s=0}^{\infty}\Big(\lambda^s R_{t,t+s}^{-1}(Y_{t+s} - \tau_{t+s}) + (1-\lambda^s)R^{*-s}(Y_{t+s}^* - \tau_{t+s}^*)\Big),\end{aligned}$$

where R^*, Y_t^*, and τ_t^* are the real interest rate, output, and taxes along a given balanced growth path where real debt grows at rate G, $B_t^*/P_t^* = G^t b_0$. Here, the parameter $\lambda \in (0,1)$ determines how strongly anchored agents' expectations are towards BGP outcomes. When λ is small, then agents put a high weight on BGP outcomes relative to true outcomes. Note, also, that the weight $1-\lambda^s$ on the BGP grows with the horizon. There is nominal indeterminacy in this economy because cognitive discounting applies to real, rather than nominal, outcomes.

In nominal terms, the equation characterizing an equilibrium can be written as

$$\begin{aligned}P_t Y_t &= (1-\beta)\Big(B_t + \sum_{s=0}^{\infty}\lambda^s(1+i^*)^{-s}(P_{t+s}Y_{t+s} - P_{t+s}\tau_{t+s}) + P_t\sum_{s=0}^{\infty}R^{*-s}(Y_{t+s} - \tau_{t+s}^*)\Big) \\ &= (1-\beta)\Big(B_t + \sum_{s=0}^{\infty}\lambda^s(1+i^*)^{-s}(P_{t+s}Y_{t+s} - P_{t+s}\tau_{t+s}) + P_tY_t\sum_{s=0}^{\infty}(\frac{G}{R^*})^s(1-\lambda^s)(Y_0 - \tau_0^*)\Big) \\ &= \frac{1-\beta}{1-(1-\beta)H^*}\Big(B_t + \sum_{s=0}^{\infty}\lambda^s(1+i^*)^{-s}(P_{t+s}Y_{t+s} - P_{t+s}\tau_{t+s})\Big),\end{aligned}$$

where $H_t^* \equiv \sum_{s=0}^{\infty}(G/R^*)^s(1-\lambda^s)(Y_0 - \tau_0^*)$. If the government wishes to target a nominal income path $\boldsymbol{PY}^*$, it should set $\boldsymbol{B}$ so that for all t,

$$B_t - \sum_{s=0}^{\infty}(1+i^*)^{-s}\lambda^s P_{t+s}\tau_{t+s} = \frac{PY_t^*}{1-\tilde{\beta}} - \sum_{s=0}^{\infty}(1+i^*)^{-s}\lambda^s P_{t+s}Y_{t+s}, \tag{92}$$

where $\tilde{\beta} \equiv (1-\beta)/(1-(1-\beta)H^*)$. It is simple to show that a fiscal rule satisfying this equation is

$$\frac{B_{t+1}}{1+i^*} = \frac{B_{t+1}^*}{1+i^*} + \frac{1}{1-\lambda}(PY_t^* - P_tY_t), \tag{93}$$

just as in Proposition 5. This policy is Ricardian whenever $B_T^*/(1+i^*)^T \to 0$ and $PY_T^*/(1+i^*)^T \to 0$.

D.4 The OLG Model with CRRA Preferences

This section extends the results of the OLG model in Section 4 to a setting in which households have arbitrary CRRA preferences. Individual households solve problem (21), but their flow utility

is given by

$$u(c_t^s) = \frac{c_t^{s\,1-\sigma} - 1}{1-\sigma}$$

instead of $\log c_t^s$, where $1/\sigma > 0$ is the intertemporal elasticity of substitution.[27] All other aspects of the model remain unchanged.

The individual Euler equation is

$$1 = \tilde{\beta} R_t \left(\frac{c_{t+1}^s}{c_t^s}\right)^{\frac{1}{\sigma}},$$

so summing individual demands as in Appendix C.1, aggregate demand satisfies

$$C_t = \left(\sum_{s=0}^{\infty} \delta^{s(1-\sigma^{-1})} \beta^{s\sigma^{-1}} R_{t,t+s}^{\sigma^{-1}-1}\right)\left(A_t + \sum_{s=0}^{\infty} R_{t,t+s}^{-1}(Y_{t+s} - \tau_{t+s})\right). \tag{94}$$

The first term is the MPC out of wealth, which reduces to $1-\beta$ in the case of log preferences, $\sigma = 1$. The second is total wealth, as in the benchmark model.

Combining aggregate demand with the equilibrium conditions $C_t = Y_t$ and $A_t = B_t/P_t$, and writing the resulting equilibrium equation in nominal terms,

$$P_t Y_t \left(\sum_{s=0}^{\infty} q^s I_{t,t+s}^{\sigma^{-1}-1} (P_{t+s}Y_{t+s})^{1-\sigma^{-1}}\right) = \underbrace{B_t + \sum_{s=0}^{\infty} I_{t,t+s}^{-1}(P_{t+s}Y_{t+s} - P_{t+s}\tau_{t+s})}_{W_t}, \tag{95}$$

where W_t, the right-hand side, is nominal wealth, and

$$q \equiv \delta^{1-\sigma^{-1}} \beta^{\sigma^{-1}} G^{\sigma^{-1}-1}.$$

Let $py_t \equiv P_t Y_t / I_{0,t}, w_t \equiv W_t / I_{0,t}$ denote nominal income and wealth normalized by the nominal return through time t. Then (95) can be rewritten as

$$w_t = \sum_{s=0}^{\infty} q^s \left(\frac{py_t}{py_{t+s}}\right)^{\sigma^{-1}} py_{t+s}. \tag{96}$$

With this setup, it is possible to demonstrate that the main content of Proposition 5 carries over to this setting with $\sigma \neq 1$.

Proposition 9. *Let $(\boldsymbol{PY}^*, \boldsymbol{B}^*)$ be the sequences of nominal income and nominal bond holdings realized in some equilibrium of the OLG economy with CRRA preferences. Consider the fiscal rule*

$$\frac{B_{t+1}}{1+i_t} = \frac{B_{t+1}^*}{1+i_t} + \frac{1}{1-\lambda}(PY_t^* - P_t Y_t). \tag{97}$$

This rule is Ricardian whenever $\lim_{T\to\infty} B_T^*/I_{0,T} \leq 0$ and $\lim_{T\to\infty} PY_T^*/I_{0,T} \leq 0$.

Proof. **Step 1.** I begin by demonstrating that the proposed fiscal rule fixes (normalized) nominal wealth w_t at the same level as in the desired equilibrium, w_t^*. Let $b_t \equiv B_t/I_{0,t}$ denote normalized bond holdings, $h_t^\tau \equiv H_t^\tau / I_{0,t}$ denote the normalized present value of taxes, and $h_t^y \equiv H_t^Y / I_{0,t}$ denote

[27] In the limit $\sigma \to 1$, $u(.)$ converges to $\log(.)$.

the normalized present value of income, where H_t^τ and H_t^Y are defined as before. With these definitions,

$$w_t = b_t - h_t^\tau + h_t^y$$

and

$$h_t^\tau = \tau_t + \lambda h_{t+1}^\tau,$$

with an analogous formula for h_t^y.

Under the proposed fiscal rule, normalized nominal taxes $p\tau_t \equiv b_t - b_{t+1}$ satisfy

$$p\tau_t = \begin{cases} p\tau_t^* + \frac{1}{1-\lambda}(\Delta py_t - \Delta py_t^*) & t > 0 \\ p\tau_0^* + \frac{1}{1-\lambda}(py_0 - py_0^*) + b_0 & t = 0 \end{cases}, \tag{98}$$

where $\Delta py_t \equiv py_t - py_{t-1}$ is the change in normalized nominal income from $t-1$ to t. Then, taking the λ-discounted present value of taxes,

$$h_t^\tau = h^{\tau *}_t + h_t^y - h^{y*}_t + b_t - b_t^*. \tag{99}$$

Combining this expression with the definition of w_t,

$$w_t = b_t^* - h^{\tau *}_t + h^{y*}_t = w_t^*, \tag{100}$$

as desired.

Step 2. Next, I demonstrate that there exists a unique path of nominal income satisfying (96) with w_t^* on the left-hand side. Notice that (96) can be rewritten as

$$w_t^* = py_t + q(\frac{py_t}{py_{t+1}})^{\sigma^{-1}} w_{t+1}^* \quad \text{with} \quad \lim_{T\to\infty} q^T (\frac{py_t}{py_T})^{\sigma^{-1}} w_T^* = 0. \tag{101}$$

Observe that, rearranging this equation,

$$1 - \frac{py_t}{w_t^*} = q(\frac{py_t}{py_{t+1}})^{\sigma^{-1}} (\frac{w_{t+1}^*}{w_t^*}).$$

Then, the terminal condition can be rewritten as

$$w_t^* \prod_{s=0}^{\infty} (1 - \frac{py_{t+s}}{w_{t+s}^*}) = 0 \quad \Leftrightarrow \quad \sum_{s=0}^{\infty} \frac{py_{t+s}}{w_{t+s}^*} = \infty. \tag{102}$$

The summation condition can be derived from the inequalities $x \leq -\log(1-x) \leq \frac{x}{1-x}$ for $x \in (0,1)$.

Now, suppose there are two equilibrium paths, py_t^* and py_t, and define their ratio by $r_t \equiv py_t/py_t^*$. The recurrence (101) can be rearranged to obtain

$$r_{t+1} = \left(1 - \frac{\frac{py_t^*}{w_t^*} - \frac{py_t}{w_t^*}}{1 - \frac{py_t}{w_t^*}}\right)^\sigma r_t. \tag{103}$$

The solution $\boldsymbol{r}$ goes monotonically to zero if $r_0 < 1$ and to infinity if $r_0 > 1$. An unbounded solution is ruled out by the fact that py_t/w_t^* must lie below one. It then suffices to consider $r_0 < 1$. Using

the fact that $1-(1-x)^\sigma \geq \min\{1,\sigma\}x$ for $x \in [0,1]$,

$$\begin{aligned} r_t - r_{t+1} &\geq \min\{1,\sigma\}\frac{\frac{py_t^*}{w_t^*} - \frac{py_t}{w_t^*}}{1-\frac{py_t}{w_t^*}}r_t \\ &\geq \min\{1,\sigma\}(\frac{py_t^*}{w_t^*} - \frac{py_t}{w_t^*})r_t \\ &= \min\{1,\sigma\}(1-r_t)\frac{py_t}{w_t^*} \geq \min\{1,\sigma\}(1-r_0)\frac{py_t}{w_t^*}. \end{aligned}$$

Hence, if $r_0 < 1$,

$$\begin{aligned} \sum_{t=0}^{\infty}\frac{py_t}{w_t^*} &\leq \frac{1}{\min\{1,\sigma\}}\sum_{t=0}^{\infty}\frac{r_t - r_{t+1}}{1-r_0} \\ &= \frac{1}{\min\{1,\sigma\}}\frac{r_0}{1-r_0} < \infty, \end{aligned}$$

so this candidate solution does not satisfy the terminal condition (102). Given the sequence $\boldsymbol{w}^*$, the equilibrium path of nominal income $\boldsymbol{py}^*$ is therefore unique. □

Note that the rule (97) is precisely the same as (28) with $\kappa = 1/(1-\lambda)$. Therefore, the nominal wealth targeting strategy in the benchmark model continues to work in the model with CRRA preferences.

Remark. *The proof of Proposition 9 is particularly simple in the continuous-time limit. Then*

$$w_t^* = \int_0^\infty e^{-qs}(\frac{py_t}{py_{t+s}})^{\sigma^{-1}}py_{t+s}ds,$$

which can be solved in closed form.

Let $z_t = w_t^*/py_t^{\sigma^{-1}}$, and note that

$$qz_t = py_t^{1-\sigma^{-1}} + \dot{z}_t.$$

Then

$$\begin{aligned} \frac{d}{dt}(z_t^\sigma) &= \sigma z_t^{\sigma-1}\dot{z}_t \\ &= \sigma z_t^{\sigma-1}(qz_t - py_t^{1-\sigma^{-1}}) \\ &= \sigma q z_t^\sigma - \sigma {w_t^*}^{\sigma-1}. \end{aligned}$$

Rearranging and solving forward,

$$z_t^\sigma = \sigma\int_0^\infty e^{-q\sigma s}{w_{t+s}^*}^{\sigma-1}ds,$$

which implies

$$py_t = \frac{w_t^*}{\sigma \int_0^\infty e^{-q\sigma s} \left(\frac{w_{t+s}^*}{w_t^*}\right)^{\sigma-1} ds}.$$