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# Pricing-to-Market in Business Cycle Models\*

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## Abstract

We evaluate several leading microfounded pricing-to-market (PTM) mechanisms embedded in a two-country DSGE model with volatile exchange rates driven by real and financial shocks. Across these frameworks, including the reduced-form Kimball specification, we identify a fundamental *parameterization trilemma*: models typically struggle to simultaneously match empirically plausible producer markups, muted expenditure switching (low short-run trade elasticity), and the low exchange-rate pass-through needed to account for the business-cycle dynamics of prices and quantities. We provide an analytical characterization of this trilemma and quantitatively assess each model's performance vis-à-vis a unified set of empirical benchmarks.

*Keywords:* exchange-rate pass-through, pricing-to-market, real rigidity, international comovements

*JEL codes:* F31, E32, F41

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# 1 Introduction

Recent advances in international finance provide a tractable framework for modeling exchange-rate volatility in DSGE settings (Gabaix and Maggiori, 2015; Itskhoki and Mukhin, 2021). By better capturing the environment faced by firms and households in open economies, and by linking exchange rates to finance and policy, this framework opens a wide range of applications but also raises new challenges. Once exchange rates act as major, largely independent shocks, they influence a broad set of price and quantity variables, making the exchange rate itself a major source of volatility. This paper examines how this extension affects the performance of standard international macro theories—specifically, whether these theories provide enough flexibility to generate the muted responses of prices and quantities to exchange rates that the data call for. We document a common tension across most models: jointly achieving empirically plausible producer markups, muted expenditure switching (i.e., a low short-run trade elasticity), and low exchange-rate pass-through is both difficult and necessary to match the data. We refer to this tension as the *parameterization trilemma*.

Our baseline is the canonical two-country model of Backus et al. (1995), augmented with financial-shock-driven exchange-rate volatility as in Gabaix and Maggiori (2015).<sup>12</sup> Within this setup, we embed several leading theories of incomplete pass-through: (i) the *Distribution Cost* (CD) model of Corsetti and Dedola (2005); (ii) the *Price Dispersion* (PD) model of Alessandria (2009); (iii) the *Nested CES Aggregation with Cournot Competition* (NCES) model of Atkeson and Burstein (2008) and Dornbusch (1987); (iv) the *Deep Habits* (DH) model of Ravn et al. (2007); and (v) the *Customer Capital* (CC) model of Drozd and Nosal (2012). Our selection is not exhaustive, but it spans a broad range of distinct and tractable mechanisms for price dynamics, each with a strong track record in macroeconomics and trade.<sup>3</sup> In addition to these microfounded models, we also analyze the widely used reduced-form Kimball

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<sup>1</sup>Earlier work by Devereux and Engel (2002) points to the potential of UIP shocks to explain exchange-rate dynamics. Mac Mullen and Woo (2025) discuss the contribution of financial shocks to model dynamics and emphasize the need for both financial and non-financial shocks. Our results likewise confirm the importance of real shocks—in our case, productivity shocks—for the models’ ability to match quantity dynamics.

<sup>2</sup>As shown by Itskhoki and Mukhin (2021), the exact implementation of this mechanism has little effect on exchange-rate properties.

<sup>3</sup>The survey by Burstein and Gopinath (2014) helped guide this selection. The models we consider have had a large impact on the macroeconomics and international trade literatures; examples include Corsetti et al. (2008), Nakamura and Zerom (2010),

aggregator (KA), introduced by [Kimball \(1995\)](#). In the literature, the KA model serves as a stand-alone reduced-form approach, and we aim to evaluate its potential in this regard. We confront these theories with a standard set of international business-cycle statistics for prices and quantities.

The CD, PD, and CC models were designed for international business-cycle applications. Two considerations motivate the inclusion of the DH and NCES models, which were originally developed in different contexts. We include the DH model because it has proved successful in closed-economy settings and because its PTM implications have been only partially documented ([Ravn et al., 2007](#)).<sup>4</sup> We also include it because it is related to a broader class of Phelps-Winter-type customer-capture ideas—models in which firms invest in customers by lowering prices rather than by taking costly non-price actions—in contrast to the CC model, which emphasizes non-price investment. We include the NCES model because it has become prominent in trade-related applications and has proved useful in relating pricing observations to trade facts. By embedding this model in a standard business-cycle environment, we hope to gain further insight into the plausibility of its core mechanism and to assess its applicability in a general-equilibrium setting.

Notably, our analysis abstracts from nominal frictions. While present in the data, as we see it, they are best viewed as supplementary for connecting models to monetary policy and for introducing demand shocks. Empirical evidence suggests that limited pass-through is largely a real phenomenon, since conditioning pass-through on price adjustments versus nonadjustments makes little difference in the observed pass-through patterns ([Gopinath and Itskhoki, 2011](#)). In addition, with volatile and persistent exchange rates, attributing deviations from the law of one price to nominal frictions raises questions about why firms do not exploit large profit opportunities created by exchange-rate movements. The models we consider provide competing answers to this question and our analysis isolates their quantitative potential.

In an environment with volatile exchange rates, a model’s success in matching a broad range of international business-cycle moments largely hinges on its ability to deliver *both* low exchange-rate pass-through to import prices and muted expenditure switching (i.e., a low short-run trade elasticity). Empirical es-

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[Corsetti and Pesenti \(2005\)](#), [Auer and Schoenle \(2016\)](#), [Edmond et al. \(2015\)](#), and [Boehm et al. \(2023a\)](#).

<sup>4</sup>As shown by [Ravn et al. \(2007\)](#), a specific form of government spending shock that triggers habit formation can produce incomplete pass-through.

estimates suggest that pass-through coefficients and short-run trade elasticities are *both* low ([Burstein and Gopinath, 2014](#); [Ruhl, 2008](#)). Our trilemma shows that jointly matching these two features of the data is challenging, and points to this tension as the root cause of the various ways in which the included models struggle to account for the standard slate of business-cycle statistics.

Specifically, we find that the reduced-form KA framework performs poorly on business-cycle quantity statistics because it struggles to reconcile plausible markups (gross margins) with a low short-run trade elasticity. This result reflects a more general property that, to varying degrees, also applies to the microfounded models: manipulating the curvature of the demand schedule *symmetrically* for home and foreign varieties is insufficient to match the data in open-economy settings. Importantly, this issue is not easily resolved by introducing convex adjustment costs on trade flows, because doing so interacts with the model's pricing implications. In sum, we find that the KA model neither nests nor outperforms the best-performing microfounded models in our open-economy setting.

The NCES model suffers from a related problem and, for this reason, is not well suited to general-equilibrium applications. Its performance on price statistics in isolation, however, is excellent. The habit model, by contrast, performs well on quantities but delivers pricing-to-market in reverse and implies more-than-complete pass-through. The remaining models do well in accounting for the business-cycle dynamics of quantities, but they generally fall short of delivering sufficiently low pass-through of exchange rates to prices. Among the setups we consider, the search models perform best in balancing prices and quantities, but they do so at the cost of introducing frictions that may be harder to identify, justify, and measure.

In many of the included models (KA, CD, and PD, though not NCES), the issues we highlight can be mitigated by assuming a higher markup target than the 50% value used in our baseline parameterization. Although markup measurement is challenging and the precise range of plausible markups is uncertain, models should remain consistent with profit and margin accounting within the confines of their structure. Under model-consistent measurement, we find that markups above 70% are difficult to justify on empirical grounds, yet in most cases values above that threshold are required to fully overcome the trilemma.<sup>5</sup>

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<sup>5</sup>Our conclusions differ from those of [Itskhoki and Mukhin \(2021\)](#) regarding the KA model's ability to match the data com-

The rest of the paper is organized as follows. Section 2 introduces the encompassing framework and then presents the setup of each model. Section 3 provides an analytical characterization of the trilemma. Section 4 presents the parameterization and quantitative results. Section 5 concludes.

## 2 Setup

The world economy consists of two ex ante symmetric countries, labeled *home* and *foreign*. Time is discrete and the horizon is infinite. The history of shocks up to and including period  $t$  is denoted by  $s^t := (s_0, s_1, \dots, s_t) \equiv (s^{t-1}, s_t) \in \mathcal{S}^t$ , where the initial state  $s_0$  is given. The probability of any history  $s^t$  is determined by the product measure  $\Pr(s^t) \equiv \prod_{\tau=1}^t \Pr(s_\tau | s^{\tau-1})$  induced by the shock processes and the associated conditional measure  $\Pr(s_t | s^{t-1})$ . Each country is populated by a large number of identical households, a final-goods sector with identical producers, and an intermediate-goods sector with differentiated products. Factors of production are immobile across countries. A global financial sector intermediates trade in non-contingent bonds between home and foreign households.

We begin by describing the structural components common to all models, which we refer to as the *encompassing GE environment*. We exploit the ex-ante symmetry of the two regions and omit symmetric equilibrium conditions for the foreign country.

### 2.1 Encompassing GE Environment

**Household sector.** Out of total final-good purchases  $A(s^t)$ , the representative household chooses its allocation to consumption  $c(s^t)$  and investment  $i(s^t)$ , labor supply  $l(s^t)$ , and one-period noncontingent bond holdings  $b(s^t)$  to maximize

$$\sum_{t=0}^{\infty} \beta^t \int_{S^t} u(c(s^t), l(s^t)) Pr(ds^t), \quad (1)$$

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prehensively. [Itskhoki and Mukhin \(2021\)](#) use indirect inference and treat margins and markups as free parameters, effectively assuming a substantially higher target.

subject to (for all  $s^t$ ): (i) output allocation  $c(s^t) + i(s^t) = A(s^t)$ ; (ii) capital accumulation law of motion  $k(s^t) = (1 - \delta)k(s^{t-1}) + i(s^t) - \Phi(i(s^t), k(s^{t-1}))$ ; and (iii) the budget constraint:

$$P(s^t)A(s^t) + b(s^t) = R(s^{t-1})b(s^{t-1}) + w(s^t)l(s^t) + r(s^t)k(s^{t-1}) + \Pi(s^t), \quad (2)$$

where  $P(s^t) \equiv 1$  serves as the local numéraire. The household receives income from labor  $w(s^t)l(s^t)$ , capital  $r(s^t)k(s^{t-1})$ , and maturing bonds  $R(s^{t-1})b(s^{t-1})$ , as well as profits  $\Pi(s^t)$  generated by domestic firms and an equal share of profits from the global financial sector.<sup>6</sup> Capital depreciates at a constant rate  $\delta$ , and  $\Phi(i(s^t), k(s^{t-1}))$  represents the standard quadratic adjustment cost. Prices and profits are taken as given. Ponzi schemes are ruled out.

**Financial sector.** The financial sector comprises a large number of identical short-lived arbitrageurs ( $o$ ) and noise traders ( $n$ ).<sup>7</sup> Let  $x(s^t)$  denote the real exchange rate: the relative price of a claim to foreign consumption in terms of home consumption. After any history  $s^t$ , arbitrageurs choose a zero-net-capital carry-trade position  $o(s^t)$  in home bonds and an offsetting position  $-o(s^t)/x(s^t)$  in foreign bonds. The profit from such a position is

$$\Pi_o(s^{t+1}) := \left[ R(s^t) - R^*(s^t) \frac{x(s^{t+1})}{x(s^t)} \right] o(s^t). \quad (3)$$

The key friction is that arbitrageurs can renege on repayment ex post (i.e., run away), in which case their payoff is a fraction  $\min\left(1, \frac{\Gamma}{y^{ss}}|o|\right)$  of the long position  $o$ , where  $\Gamma > 0$  is a parameter,  $|\cdot|$  denotes the absolute value, and  $y^{ss}$  is steady-state output. Since creditors anticipate this, arbitrageurs face the following ex ante constraint:

$$\frac{\mathbb{E}_{s^t} \Pi_o(s^{t+1})}{R(s^t)} \geq \frac{\Gamma}{y^{ss}} o(s^t)^2, \quad (4)$$

<sup>6</sup> $\Pi(s^t) := \Pi_A(s^t) + \Pi_I(s^t) + \frac{1}{2}\Pi_o(s^t) + \frac{1}{2}\Pi_n(s^t)$ , with each component defined in the subsequent sections.

<sup>7</sup>Our setup follows [Gabaix and Maggiori \(2015\)](#). [Itskhoki and Mukhin \(2021\)](#) propose an alternative and more general setup by modeling risk-averse  $m$ -financiers. The implications of their model and this one are similar because the final log-linearized equations are identical after remapping parameters.

where the left-hand side denotes the discounted expected profit from the carry trade and the right-hand side represents the payoff from reneging. Since  $\mathbb{E}_{s^t} \Pi_o(s^{t+1})$  is linear in  $o$ , whereas the constraint is convex in  $o$ , maximization of  $\mathbb{E}_{s^t} \Pi_o(s^{t+1})$  subject to the above financing constraint yields the following distorted Uncovered Interest Parity (UIP) condition:

$$R(s^t) = \mathbb{E}_{s^t} \left[ R^*(s^t) \frac{x(s^{t+1})}{x(s^t)} \right] + \frac{\Gamma}{y^{ss}} R(s^t) o(s^t). \quad (5)$$

Noise traders are the source of financial shocks. Like arbitrageurs, they take a random position  $n(s^t)$ , and their profit is

$$\Pi_n(s^{t+1}) := \left[ R(s^t) - R^*(s^t) \frac{x(s^{t+1})}{x(s^t)} \right] n(s^t), \quad (6)$$

where  $\log(n(s^t)/y^{ss}) = \rho_n \log(n(s^{t-1})/y^{ss}) + \varepsilon_t^n$  with  $\varepsilon_t^n \sim \mathcal{N}(0, \sigma_n^2)$ . We refer to  $\varepsilon^n$  as the (global) financial shock. The noise trader shock is the key source of exchange-rate volatility in the calibrated model.<sup>8</sup>

**Intermediate goods sector.** The intermediate-goods sector comprises monopolistically competitive firms that transform a generic intermediate input  $Y > 0$  into differentiated varieties defined on some indexing set  $\mathcal{I}(s^t) \subset \mathbb{R}_+$ —which also indexes producers and determines their mass. The production function is

$$Y(s^t) := z(s^t) k(s^{t-1})^\alpha l(s^t)^{1-\alpha},$$

<sup>8</sup>With bond-market clearing  $b(s^t) + o(s^t) + n(s^t) = 0$  and equal sharing of profits, note that the country's net foreign asset position evolves according to

$$b(s^t) = \frac{1}{2} \left[ R^*(s^{t-1}) \frac{x(s^t)}{x(s^{t-1})} + R(s^{t-1}) \right] b(s^{t-1}) + GDI(s^t) - FS(s^t).$$

Accordingly, when a shock occurs, the arbitrageurs' limited capacity to hold lopsided positions requires the real exchange rate to adjust so that the return differential induces them to clear the market.

where  $l(s^t)$  is labor input supplied by home households,  $k(s^{t-1})$  is capital supplied by home households, and  $z(s^t)$  follows the process

$$z(s^t) = \rho z(s^{t-1}) + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \sigma_z^2),$$

where  $\varepsilon_t$  is the *productivity shock*. The world economy's exogenous state vector is thus given by  $s_t = (\varepsilon_t, \varepsilon_t^*, \varepsilon_t^n)$ , and the cost of producing the generic input is

$$v(s^t) := \min_{k,l} \{w(s^t)l + r(s^t)k\} \text{ s.t. } z(s^t)k^\alpha l^{1-\alpha} = 1. \quad (7)$$

**Final goods sector.** Final goods are produced from domestic and foreign intermediates by a competitive final-goods sector. These intermediates are aggregated according to some function

$$A(s^t) := \mathcal{A}(\{d(i, s^t)\}_{i \in \mathcal{D}(s^t)}, \{f(i, s^t)\}_{i \in \mathcal{F}(s^t)}), \quad (8)$$

where  $\mathcal{D}(s^t) \subseteq \mathcal{I}(s^t)$  and  $\mathcal{F}(s^t) \subseteq \mathcal{I}^*(s^t)$  are the sets of domestic and foreign varieties being aggregated.

## 2.2 Market Structures (PTM Models)

We now turn to the market structures that give rise to the models considered below. We begin with the standard monopolistic-competition setup—which is our baseline. The baseline model incorporates monopoly distortions in the production of intermediate goods, but because it lacks the structural frictions needed to generate PTM and its business-cycle properties are identical to those of an analogous perfectly competitive model (Backus et al., 1995) in a log-linearized setup, we refer to it as *frictionless*.<sup>9</sup>

<sup>9</sup>The CES markup  $\mu = \theta/(\theta - 1)$  vanishes in log-linearized deviations from a given steady state. The presence of profits implies a different mapping from observed factor shares to the capital-share parameter  $\alpha$ , so the effect of markups operates solely through the steady-state calibration of this parameter.

**Frictionless Baseline Model (FB).** Here, (8) is the standard Armington aggregator:

$$\mathcal{A}_{CES}(\cdot) := G(d(s^t), f(s^t)) := \left[ \omega^{\frac{1}{\gamma}} d(s^t)^{\frac{\gamma-1}{\gamma}} + (1-\omega)^{\frac{1}{\gamma}} f(s^t)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}, \quad (9)$$

where

$$d(s^t) := \left[ \int_0^1 d(i, s^t)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \quad f(s^t) := \left[ \int_0^1 f(i, s^t)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \quad (10)$$

and hence  $\mathcal{I} = \mathcal{I}^* = [0, 1]$ , with  $\mathcal{D} = \mathcal{I}$  and  $\mathcal{F} = \mathcal{I}^*$ . The parameter  $\gamma > 0$  is the elasticity of substitution between the domestic and foreign composite bundles  $d(s^t)$  and  $f(s^t)$ ,  $\theta > 1$  is the elasticity of substitution between the individual varieties that comprise these bundles, and  $1/2 < \omega < 1$  is the home-bias parameter.<sup>10</sup>

Final-good supply  $A(s^t)$  is produced by a sector of identical final-goods producers that take prices as given and choose  $d(i, s^t)$  and  $f(i, s^t)$  to maximize

$$\Pi_A(s^t) := \mathcal{A}_{CES}(\cdot) - \int_0^1 [p_d(i, s^t)d(i, s^t) + p_f(i, s^t)f(i, s^t)] di. \quad (11)$$

This gives rise to the standard downstream demand system:

$$d(i, s^t) = \left[ \frac{p_d(i, s^t)}{P_d(s^t)} \right]^{-\theta} d(s^t), \quad f(i, s^t) = \left[ \frac{p_f(i, s^t)}{P_f(s^t)} \right]^{-\theta} f(s^t), \quad (12)$$

where  $P_j(s^t) := \partial_j G(d(s^t), f(s^t))$ ,  $j = d, f$ . Throughout, the notation  $f(p_d|i, s^t)$  denotes a real-valued function of the argument preceding “|.”<sup>11</sup>

The intermediate-goods sector is populated by a continuum of identical firms, each of which is a global monopolist over its variety  $i$ . Intermediate-goods producers choose the price of variety  $i$  in the domestic

<sup>10</sup> $\theta > 1$  is required to ensure finite markups.

<sup>11</sup>As for notation, we use  $\partial_x(\cdot) \equiv \frac{\partial(\cdot)}{\partial x}$  as shorthand for partial differentiation and similarly use  $d_x(\cdot) \equiv \frac{d(\cdot)}{dx}$  as shorthand for total derivatives. We use standard prime notation whenever it is unambiguous that differentiation applies to a function of a single argument. In some contexts, the notation  $f(p_d|i, s^t)$  is used to make explicit an argument that appears within a functional composition in the original expression.

market,  $p_d(i, s^t)$ , and in the export market,  $p_x(i, s^t) \equiv x(s^t) p_d^*(i, s^t)$ , maximizing

$$\begin{aligned} \Pi_I(i, s^t) := & [p_d(i, s^t) - v(s^t)] d(p_d(i, s^t) | i, s^t) \\ & + [x(s^t) p_d^*(i, s^t) - v(s^t)] d^*(p_d^*(i, s^t) | i, s^t), \end{aligned} \quad (13)$$

where  $d(p_d | i, s^t)$  corresponds to (12), and  $d^*(p_d^* | i, s^t) = \left[ \frac{p_d^*}{P_d^*(s^t)} \right]^{-\theta} d^*(s^t)$ . Under this baseline, the law of one price holds, and the home price of the domestic good is given by the standard monopoly-pricing formula

$$p_d(i, s^t) = \underbrace{x(s^t) p_d^*(i, s^t)}_{p_x(i, s^t)} = \underbrace{\frac{\theta}{\theta - 1}}_{=1+\mu} v(s^t), \quad (14)$$

and, by symmetry, the home price of the imported good is given by

$$p_f(i, s^t) = \frac{\theta}{\theta - 1} x(s^t) v^*(s^t). \quad (15)$$

**Definitions of Prices.** We refer to  $p_f$  as the *import price* and to  $p_x$  as the *export price*; in the data, these correspond to dock-level (wholesale) prices. Incomplete pass-through, when present, is reflected in how the real exchange rate  $x$  affects  $p_f$ . Finally, we define the *terms of trade* as the ratio of these two prices,  $tot := p_f/p_x$ . Incomplete or intermediate pass-through reduces the volatility of the terms of trade relative to that of the real exchange rate.

**KA Model.** As in the baseline model, here  $\mathcal{I} = \mathcal{I}^* = [0, 1]$ ,  $\mathcal{D} = \mathcal{I}$ ,  $\mathcal{F} = \mathcal{I}^*$ . However, (8) is defined implicitly by a homogeneous-of-degree-one constraint (hereafter Kimball aggregator):

$$\int_{[0,1]} \left[ \omega g \left( \omega^{-1} \frac{d(i, s^t)}{A(s^t)} \right) + (1 - \omega) g \left( (1 - \omega)^{-1} \frac{f(i, s^t)}{A(s^t)} \right) \right] di = 1, \quad (16)$$

where  $g(\cdot)$  is a real-valued function that is assumed to be (i) strictly increasing, (ii) strictly concave, and (iii) normalized such that  $g(1) = g'(1) = 1$  and  $g''(1) \in (0, 1)$ , where  $g'$  and  $g''$  denote the first and

second derivatives, respectively (differentiability is ensured almost everywhere by property (i)).<sup>12</sup>

Our specification of the aggregator follows [Itskhoki and Mukhin \(2021\)](#). In a log-linearized setting, specifying the functional form for  $g(\cdot)$  is unnecessary, since all the relevant properties of the function are captured by the curvature parameter  $g''(1)$ . This yields the most general specification for such environments.

The final-goods producer chooses final output  $A(s^t)$  to maximize static profits given by

$$\Pi_A(s^t) := A(s^t) - \mathcal{C}(A(s^t), s^t), \quad (17)$$

where

$$\mathcal{C}(A(s^t), s^t) := \min_{d(i, s^t), f(i, s^t)} \int_0^1 [p_d(i, s^t)d(i, s^t) + p_f(i, s^t)f(i, s^t)] di, \quad (18)$$

subject to (16), defines the cost. Solving the above cost minimization problem yields the downstream demand system:

$$d\left(\frac{p_d}{\lambda} | i, s^t\right) = \omega h\left(\frac{p_d}{\lambda}\right) A(i, s^t), \quad f\left(\frac{p_f}{\lambda} | i, s^t\right) = (1 - \omega) h\left(\frac{p_f}{\lambda}\right) A(i, s^t), \quad (19)$$

where  $h(x) := (g')^{-1}(x)$ , and where  $\lambda(s^t)$  denotes the Lagrange multiplier associated with the aggregation constraint in cost minimization, given by

$$1 = \int_0^1 \left[ \omega g\left(h\left(\frac{p_d(i, s^t)}{\lambda(s^t)}\right)\right) + (1 - \omega) g\left(h\left(\frac{p_f(i, s^t)}{\lambda(s^t)}\right)\right) \right] di. \quad (20)$$

Unlike in the baseline model, it is not the case that this multiplier equals  $P$ .

Since (16) exhibits constant returns to scale, average and marginal costs coincide; hence,  $\mathcal{C}(A, s^t) \equiv \mathcal{C}(1, s^t) A$ . This property, combined with the zero-profit condition, implies  $C(1, s^t) \equiv 1$ . In the symmetric steady state,  $p_d^{ss} = p_f^{ss}$ , which—together with our numéraire normalization of the final-good price

<sup>12</sup>The Kimball aggregator nests the standard CES aggregator seen in the baseline model. To see this, let  $g(y) = 1 + \frac{\gamma}{\gamma-1}(y^{1-1/\gamma} - 1)$ .

$P$ —implies  $p_d^{ss} = p_f^{ss} = \lambda^{ss} = 1$ .<sup>13</sup>

Intermediate-goods producers take the above demand system as given and choose the domestic and foreign prices of the produced variety  $i$ —denoted by  $p_d(i, s^t)$  and  $p_d^*(i, s^t)$ , respectively—to maximize

$$\Pi_I(i, s^t) := [p_d(i, s^t) - v(s^t)] d\left(\frac{p_d(i, s^t)}{\lambda(s^t)} \mid \cdot\right) + [x(s^t) p_d^*(i, s^t) - v(s^t)] f\left(\frac{p_f(i, s^t)}{\lambda(s^t)} \mid \cdot\right). \quad (21)$$

Accordingly, the home country's import price solves

$$p_f(i, s^t) = x(s^t) v^*(s^t) \frac{\gamma\left(\frac{p_f(i, s^t)}{\lambda(s^t)}\right)}{\gamma\left(\frac{p_f(i, s^t)}{\lambda(s^t)}\right) - 1}, \quad (22)$$

where

$$\gamma\left(\frac{p_f}{\lambda}\right) := -\partial_{\log p_f} \log f\left(\frac{p_f}{\lambda} \mid \cdot\right) \equiv -\partial_{\log p_f} \log h\left(\frac{p_f}{\lambda}\right) \equiv \frac{h'\left(\frac{p_f}{\lambda}\right) p_f}{h\left(\frac{p_f}{\lambda}\right) \lambda} \quad (23)$$

is demand elasticity, and where, by the definition of the aggregator and the inverse function theorem, we know  $h'(z) = 1/g''(h(z))$  (all  $z > 0$ ).

**CD Model.** This model introduces a fixed distribution cost. The final aggregator is identical to that in the baseline model (FB), but to purchase an intermediate good, the final-goods producer must pay a fixed cost  $\xi v(s^t)$ , implying the following profit maximization:

$$\Pi_A(s^t) := \mathcal{A}_{CES}(\cdot) - \int_0^1 [(p_d(i, s^t) + \xi v(s^t)) d(i, s^t) + (p_f(i, s^t) + \xi v(s^t)) f(i, s^t)] di. \quad (24)$$

<sup>13</sup>(19) evaluated at the symmetric steady state gives  $d^{ss}/A = \omega$  and  $f^{ss}/A = 1 - \omega$ . Hence, by (18),  $p_d^{ss} = p_f^{ss} = \lambda^{ss} = 1$ .

The key implication of this cost is that the downstream demand system features non-constant elasticity with respect to the price alone.<sup>14</sup>

$$d(p_d|i, s^t) = \left[ \frac{p_d + \xi v(s^t)}{P_d(s^t)} \right]^{-\theta} d(s^t), \quad f(p_f|i, s^t) = \left[ \frac{p_f + \xi v(s^t)}{P_f(s^t)} \right]^{-\theta} f(s^t), \quad (25)$$

where  $P_j(s^t) := \partial_j G(d(s^t), f(s^t))$ ,  $j = d, f$ . The intermediate-goods producer's problem is the same as in the baseline model and we omit the details. It is easy to verify that the profit-maximizing import price solves to

$$p_f(i, s^t) = \frac{\theta}{\theta - 1} x(s^t) v^*(s^t) + \frac{\xi}{\theta - 1} v(s^t). \quad (26)$$

**PD Model.** The PD model features distribution costs in the form of search frictions in matching with suppliers—which also replace monopolistic competition as the source of market power. Search is directed to country, and the frictions do not affect expenditure switching per se. The costs of search, and the fact that they are denominated in local units, give rise to endogenous markups and incomplete pass-through. Since goods delivered by each type of match are aggregated symmetrically, aggregation implies  $\mathcal{I} = \mathcal{I}^* = \mathcal{D} = \mathcal{F} = \{1\}$  and thus  $\mathcal{A}_{CES} = G(d, f)$ , where  $d$  is the total quantity delivered by matches with domestic producers and  $f$  is the quantity delivered by matches with foreign producers. Final-goods producers are all identical and behave competitively. Each producer operates the technology  $G(d, f)$  and hires measures  $h_d$  and  $h_f$  of atomless representatives (hereafter “reps”) who search for intermediate goods on the producer's behalf, and at a total search cost of  $(h_d + h_f)v(s^t)$ . Each match is capacity-constrained and delivers  $\theta^{-1}$  units of the good per period; consequently, at most  $\theta^{-1}h_j$  of the good can be purchased given the choice of  $h_j$  at  $s^t$ , where  $j = d, f$ .

By assumption, searching reps receive one price quote with probability  $q$  and two quotes with probability  $1 - q$ . Their policy is described by reservation prices,  $r_d$  and  $r_f$ , set by the final-goods producer. Since

<sup>14</sup>The fixed-cost (Leontief) specification is not strictly essential, but a low elasticity of substitution is. If the distribution technology were Cobb–Douglas in distribution services and inputs, the model would imply complete pass-through. Qualitatively, it is therefore sufficient to assume that the elasticity of substitution is less than unity; however, a higher elasticity would erode the model's ability to generate PTM.

equilibrium posted prices will never exceed reservation prices,  $d = \theta^{-1}h_d$  and  $f = \theta^{-1}h_f$ .

It can be shown that equilibrium pricing entails a unique mixed strategy described by a continuous cumulative distribution function (CDF)  $F(p|s^t)$ , and this distribution's support is an interval  $[P_l(s^t), P_h(s^t)]$ . Since the distribution of purchase prices is determined by the lowest price quote obtained by the rep, and there can be up to two quotes, the distribution of the lowest quote is<sup>15</sup>

$$H(p|s^t) = qF(p|s^t) + (1 - q) [1 - (1 - F(p|s^t))^2]. \quad (27)$$

The expected price paid by the rep is thus given by  $p_j(s^t) = \int_{P_l}^{P_h} p H_j(dp|s^t)$ , where  $j = d, f$  because search is directed by country and there are distinct price distributions  $H_d$  and  $H_f$  in the home market. Accordingly, after any history  $s^t$ , the representative final-goods producer chooses  $h_d$  and  $h_f$  to maximize static profits

$$\Pi_A(d, f|s^t) := G(d, f) - p_d(s^t)d - p_f(s^t)f - v(s^t)\theta(d + f). \quad (28)$$

Intermediate-goods producers do not observe the number of quotes that their customers (reps) receive. Specifically, if a rep receives two quotes and she has an alternate quote with a lower price in hand, no sale takes place; in contrast, if the same customer has only one quote, a sale takes place at any price up to  $P_h$ . This means that producers face a trade-off between the markup they charge and the selling probability, whereas the distribution  $F$  satisfies that the expected profit from quoting an arbitrary price is the same on its support, implying

$$(P_h(s^t) - mc(s^t)) \left[ \frac{q}{q + 2(1 - q)} \right] = (p - mc(s^t)) \left[ \frac{q + 2(1 - q)(1 - F(p|s^t))}{q + 2(1 - q)} \right], \quad (29)$$

where  $mc(s^t)$  denotes the potentially distinct marginal cost from the one that determines search costs (e.g.,  $mc(s^t) = v^*(s^t)x(s^t)$  for importers, while the search cost is determined by  $v(s^t)$ ).

<sup>15</sup> $H_d(p, s^t) := \Pr(\hat{p} \leq p) = q\Pr(\hat{p} \leq p | 1 \text{ quote}) + (1 - q)\Pr(\hat{p} \leq p | 2 \text{ quotes})$ , where  $q\Pr(\hat{p} \leq p | 1 \text{ quote}) = qF(p)$ . Let  $\hat{p}_i$  denote the  $i$ -th draw from distribution  $F$ , for  $i = 1, 2$ . Then,  $\Pr(\hat{p} \leq p | 2 \text{ quotes}) = \Pr((\hat{p}_1 \leq p \ \& \ \hat{p}_2 > p) \text{ or } (\hat{p}_1 > p \ \& \ \hat{p}_2 \leq p) \text{ or } (\hat{p}_1 \leq p \ \& \ \hat{p}_2 \leq p))$ , which simplifies to  $1 - \Pr(\hat{p}_1 > p \ \& \ \hat{p}_2 > p)$ . Since each draw is independent, we have  $\Pr(\hat{p} \leq p | 2 \text{ quotes}) = 1 - (1 - F(p))^2$ . Combining these expressions yields the result stated in the main text.

The left-hand side of (29) is the expected profit from quoting the maximum price  $P_h(s^t)$ , which results in a sale when the customer receives only a single quote. The right-hand side represents the expected profit from quoting an interior price  $P_l(s^t) \leq p < P_h(s^t)$ . Such quotes, note, result in a sale if the customer receives either one quote, or if the price  $p$  is the lower of two quotes obtained. Since each customer may receive more than one quote, the expected number of quotes per physical customer per producer is  $q + 2(1 - q)$ , which gives rise to  $q + 2(1 - q)(1 - F(p|\cdot))$  in the numerator. At the upper bound of the distribution  $P_h(s^t)$ , the final-goods producer must be indifferent between purchasing the good or aborting the search to send another rep at the expected price, hence  $\theta v(s^t) = P_h(s^t) - p_d(s^t)$ ; the lower bound solves  $F(P_l(s^t)) = 0$ .<sup>16</sup> Together, these conditions imply that the expected (average) import price paid by the final-goods producer is

$$p_f(s^t) = x(s^t) v^*(s^t) + \frac{\theta q}{1 - \theta} v(s^t). \quad (30)$$

**NCES Model.** The NCES model features a continuum of sectors on a unit interval, each containing a discrete number of  $N > 2$  home and foreign firms (varieties) that compete in a Cournot fashion (choose quantity). Let the first  $N_X < N$  goods be imported and the remainder be domestically sourced. Accordingly, aggregation involves  $\mathcal{I} = [0, 1] \times \{1, \dots, N\}$ ,  $\mathcal{D} = \mathcal{I}$ , and  $\mathcal{F} = [0, 1] \times \{1, \dots, N_X\}$ . Define a tuple  $\mathbf{i} := (i_1, i_2) \in \mathcal{I}$ , where  $i_1$  indexes the sector and  $i_2$  indexes the firm within a sector. The aggregator (8) in this model is

$$\mathcal{A}_{NCES}(\{d(\mathbf{i}, s^t)\}_{\mathbf{i} \in \mathcal{D}}, \{f(\mathbf{i}, s^t)\}_{\mathbf{i} \in \mathcal{F}}) := \left[ \int_0^1 (y(i_1, s^t))^{\frac{\gamma-1}{\gamma}} di_1 \right]^{\frac{\gamma}{\gamma-1}}, \quad (31)$$

and it involves aggregation across sectors, whereas aggregation of country-specific varieties takes place within each sector:

$$y(i_1, s^t) := \left[ \sum_{i_2=1}^N (d(i_1, i_2, s^t))^{\frac{\theta-1}{\theta}} + \sum_{i_2=1}^{N_X} (f(i_1, i_2, s^t))^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}. \quad (32)$$

<sup>16</sup>  $P_h = v + v \frac{\theta}{1-q}$ ,  $P_l = v + v \frac{q\theta}{2-3q+q^2}$ , and  $F(p) = 1 - \frac{1}{2} \frac{q}{1-q} \frac{P_h-p}{p-v}$ .

The parameter  $\gamma > 0$  is the sectoral elasticity and  $\theta$  is the firm-level elasticity, where  $\theta > \gamma$  and  $\theta > 1$ .

The presence of non-atomistic firms is the defining feature because firms partly internalize the sectoral elasticity  $\gamma$ . When this elasticity differs from the within-sector elasticity  $\theta$ , as assumed, the model gives rise to endogenous markups and generates incomplete pass-through.<sup>17</sup>

The representative final-goods producer takes prices of individual varieties as given and chooses  $d(\mathbf{i}, s^t)$  and  $f(\mathbf{i}, s^t)$  to maximize

$$\Pi_A(s^t) := \mathcal{A}_{NCES}(\cdot) - \int_0^1 \left[ \sum_{i_2} p_d(\mathbf{i}, s^t) d(\mathbf{i}, s^t) + \sum_{i_2} p_f(\mathbf{i}, s^t) f(\mathbf{i}, s^t) \right] di_1,$$

where the abbreviated  $\mathcal{A}_{NCES}(\cdot)$  is as stated above. The implied inverse downstream demand function for a generic variety  $\mathbf{i}$  is

$$p(Q|\mathbf{i}, s^t) = \left[ \frac{Q}{y(Q|\mathbf{i}, s^t)} \right]^{-\frac{1}{\theta}} \left[ \frac{y(Q|\mathbf{i}, s^t)}{A} \right]^{-\frac{1}{\gamma}}, \quad (33)$$

where  $Q$  represents the quantity demanded and  $y(Q|\cdot)$  is the sectoral aggregator defined in (32). The sectoral aggregator still depends on  $Q$  because individual firms are non-atomistic and perceive the impact of their choices on sectoral aggregates.

Intermediate-goods producers from each country are identical. Domestic producer  $\mathbf{i} = (i_1, i_2) \in \mathcal{I}$  takes (33) as given and, after history  $s^t$ , chooses quantity  $d(s^t)$  for the home market and quantity  $d^*(s^t)$  (for  $i_2 \leq N_X$ ) for the foreign market (exporters only) to maximize

$$\begin{aligned} \Pi_I(\mathbf{i}, s^t) := & \left[ p(d(s^t)|\mathbf{i}, s^t) - v(s^t) \right] d(s^t), \\ & + \mathbf{1}_{\{\mathbf{i}=(i_1, i_2): i_2 \leq N_X\}} \left[ x(s^t) p^*(d^*(s^t)|\mathbf{i}, s^t) - (1 + \tau) v(s^t) \right] d^*(s^t), \end{aligned} \quad (34)$$

where  $\tau > 0$  is an iceberg transportation cost that we use in calibration to induce additional home bias,

<sup>17</sup>In contrast to the original paper, here we exogenously fix the number of firms in the economy and assume they are identical within their respective categories: exporters or non-exporters. There are also no non-tradable goods in our formulation of this model.

beyond assuming  $N_X < N$ . We do not model entry, and so  $N, N_X$  are exogenous parameters. Some producers only sell domestically and in that case  $d^* = 0$ .

The first-order conditions imply that the import price is given by the standard monopoly pricing formula:

$$p_f(s^t) = \frac{\varepsilon_f(\mathbf{i}, s^t)}{\varepsilon_f(\mathbf{i}, s^t) - 1} (1 + \tau) x(s^t) v^*(s^t), \quad (35)$$

but with endogenous demand elasticity determined by market shares:

$$\begin{aligned} \varepsilon_f(\mathbf{i}, s^t) &:= \left[ \frac{1}{\theta} (1 - S_f(\mathbf{i}, s^t)) + \frac{1}{\gamma} S_f(\mathbf{i}, s^t) \right]^{-1}, \\ S_f(\mathbf{i}, s^t) &:= \frac{p_f(\mathbf{i}, s^t) f(\mathbf{i}, s^t)}{\sum_{i_2=1}^N p_d(\mathbf{i}, s^t) d(\mathbf{i}, s^t) + \sum_{i_2=1}^{N_X} p_f(\mathbf{i}, s^t) f(\mathbf{i}, s^t)}. \end{aligned} \quad (36)$$

The key here is that the elasticity of demand is endogenous and depends on the firm's market share,  $S_f$ , as well as the market share of the firm's group (e.g., importers). In particular, if importers constitute a minority of firms in a sector due to home bias, a symmetric shock to  $S_f$  has a bigger effect on elasticity. This feature implies that home bias enhances the model's ability to generate incomplete pass-through.

**DH Model.** Final aggregations are the same as in the baseline model, and hence  $\mathcal{I} = \mathcal{I}^* = [0, 1]$ ,  $\mathcal{D} = \mathcal{I}$ ,  $\mathcal{F} = \mathcal{I}^*$ . The key difference is that the arguments of the final aggregator are not raw quantities but habit-adjusted quantities given by

$$d(s^t) := \left[ \int_0^1 d^h(i, s^t)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \quad f(s^t) := \left[ \int_0^1 f^h(i, s^t)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \quad (37)$$

where  $d^h(i, s^t) = h_d(i, s^{t-1})^\zeta d(i, s^t)$ , and where  $h_d(i, s^t) = \rho h_d(i, s^{t-1}) + (1 - \rho) \bar{d}(i, s^t)$ . The parameter  $0 < \zeta \leq 1$  denotes the marginal productivity of habit, and habit evolves dynamically. Habit persistence is modulated by the parameter  $0 < \rho < 1$  and follows the law of motion.  $\bar{d}$  is the *average* level of purchases of good  $i$  across the economy, so as to ensure that final-goods producers do not internalize the impact of their own choices on habit formation.

Final-goods producers take habit and prices as given and choose  $d(i, s^t)$  and  $f(i, s^t)$  to maximize

$$\Pi_A(s^t) = \mathcal{A}_H(\cdot) - \int_0^1 [p_d(i, s^t) d(i, s^t) + p_f(i, s^t) f(i, s^t)] di. \quad (38)$$

Accordingly, the demand system is given by

$$d(p_d, h_d|i, s^t) = \left[ \frac{p_d}{P_d(s^t)} \right]^{-\theta} h_d^{\zeta(\theta-1)} d(s^t), \quad f(p_f, h_f|i, s^t) = \left[ \frac{p_f}{P_f(s^t)} \right]^{-\theta} h_f^{\zeta(\theta-1)} f(s^t), \quad (39)$$

where  $P_j(s^t) \equiv \partial_{j(s^t)} G(d(s^t), f(s^t))$ ,  $j = d, f$ .

Intermediate-goods producers face the above demand and operate as monopolists over their respective varieties, just as in the baseline model. But their optimization problem is dynamic because they recognize that habit formation is persistent and depends on the prices they set. Specifically, their static profit is

$$\begin{aligned} \Pi_I(i, s^t) &= [p_d(i, s^t) - v(s^t)] d(p_d(i, s^t), h_d(i, s^{t-1}) | s^t) \\ &\quad + [x(s^t) p_d^*(i, s^t) - v(s^t)] d^*(p_d^*(i, s^t), h_d(i, s^{t-1}) | s^t), \end{aligned} \quad (40)$$

where  $d(\cdot | s^t)$  is given by (39) (analogously for  $d^*$ ). Producers set prices at home,  $p_d(i, s^t)$ , and abroad,  $p_d^*(i, s^t)$ , to maximize  $\sum_{t=0}^{\infty} \int \beta^t u_c(s^t) \Pi_I(i, s^t) \mu(ds^t)$ , subject to the law of motion of habit,  $h_d(i, s^t) = \rho h_d(i, s^{t-1}) + (1 - \rho) d(i, s^t)$  (analogously for  $d^*$ ). The first-order conditions imply that the import price is

$$p_f(i, s^t) = \frac{\theta}{\theta - 1} [x(s^t) v^*(s^t) - (1 - \rho) \Delta_f(i, s^t)], \quad (41)$$

where

$$\Delta_f(i, s^t) = \beta \mathbb{E}_{s^t} \frac{u_{c^*}^*(s^{t+1})}{u_{c^*}^*(s^t)} \left[ \rho \Delta_f(i, s^{t+1}) + \underbrace{\frac{\partial_{h_f} f(p_f, h_f|i, s^{t+1})}{\partial_{p_f} f(p_f, h_f|i, s^{t+1})}}_{MRT} \frac{f(p_f, h_f|i, s^{t+1})}{x(s^{t+1})} \right] \quad (42)$$

denotes the shadow value of habit that renders the markup endogenous.<sup>18</sup> Intuitively, by inducing habit today, producers increase future demand for their goods, and hence can charge a higher price in the future while selling the same quantity. This is captured by the MRT term above, and the value of habit cumulates these discounted “dividends” along the lines of the standard asset pricing equation.

**CC Model.** This model combines search frictions in building market share with bargaining; together these two features give rise to incomplete pass-through. The PT mechanism is fundamentally different from the PD model because search cannot be directed to producers from a given country. It is up to the producers and their investment in marketing how “visible” they become to searching customers. This key feature affects both trade dynamics and surplus splitting via bargaining.

The commodity space evolves dynamically because goods are best interpreted as distinct across long-lasting matches of final-goods producers and intermediate-goods producers.<sup>19</sup> Let  $H_d(s^t)$  and  $H_d^*(s^t)$  denote the measures of active matches between home intermediate-goods producers and final-goods producers at home and abroad, respectively. The sets of available varieties are then given by  $\mathcal{I}(s^t) = [0, H_d(s^t) + H_d^*(s^t)]$ ,  $\mathcal{D}(s^t) = [0, H_d(s^t)]$ , and  $\mathcal{F}(s^t) = [0, H_f(s^t)]$ . As in the PD model, each match features a satiation point normalized to unity, that is, a match-level throughput constraint.

The aggregator in (8) is given by (9), where

$$d(s^t) = \int_{\mathcal{D}(s^t)} \min \{1, d(i, s^t)\} di \text{ and } f(s^t) = \int_{\mathcal{F}(s^t)} \min \{1, f(i, s^t)\} di.$$

Accordingly, there is residual product differentiation purely attributable to country of origin, in that  $\gamma$  will be uniquely set higher than in other models to match the long-run estimates of trade elasticity—which are much higher than the short-run estimates.

As in the PD model, the final-goods producer chooses the mass of representatives  $h(s^t)$  to search for suppliers, incurring a total cost of  $\chi h(s^t) v(s^t)$ . But unlike in the PD model, search is undirected, and the probabilities of meeting a producer from the home country (a type  $d$  producer) or from the foreign

<sup>18</sup>The Lagrange multiplier on the law of motion for habit.

<sup>19</sup>The presence of search frictions implicitly implies that customers search for specific attributes inherent to the match.

country (a type  $f$  producer) are  $\pi(s^t)$  and  $1 - \pi(s^t)$ , respectively. These probabilities are endogenous—determined by marketing capital investment, as described below—but taken as given by the final-goods producer. The instantaneous profit of the final-goods producer is

$$\Pi_A(s^t) = G(d(s^t), f(s^t)) - p_d(s^t) d(s^t) - p_f(s^t) f(s^t) - \chi v(s^t) h(s^t), \quad (43)$$

where  $p_d(s^t)$  and  $p_f(s^t)$  are the average prices of goods  $d$  and  $f$  purchased through each type of match. These prices are determined by bargaining in bilateral matches with producers, but for reasons detailed below, they do not depend on the producer's choices and can effectively be treated as given in profit maximization.

Since matches are long lasting, the final-goods producer's problem is dynamic. The first order condition determining  $h(s^t)$  (henceforth *retail demand*) can be derived as follows. Let the marginal product of a country variety be  $P_j(s^t) = \partial_{j(s^t)} G(d(s^t), f(s^t))$ ,  $j = d, f$ . The value of an active match with an intermediate-goods producer  $i \in \mathcal{I}$  from country  $j = d, f$  is

$$V_j(i, s^t) = \max\{P_j(s^t) - p_j(i, s^t), 0\} + (1 - \delta_H) \mathbb{E}_{s^t} \left[ \beta \frac{\partial_c u(s^{t+1})}{\partial_c u(s^t)} V_j(i, s^{t+1}) \right], \quad (44)$$

which cumulates the instantaneous trade surplus  $P_j(s^t) - p_j$  over the expected duration of a match that depreciates with probability  $\delta_H$  per period.  $h(s^t)$  is set optimally when its unit cost,  $\chi v(s^t)$ , equals the expected marginal benefit, and hence

$$\chi v(s^t) \geq \pi(s^t) V_d(i, s^t) + [1 - \pi(s^t)] V_f(i, s^t), \text{ with equality if } h(s^t) > 0. \quad (45)$$

On the other side of matching there is a normalized mass of intermediate-goods producers, both from the home country and from the foreign country. A representative producer  $i \in \mathcal{I}$  sells output by investing in country-specific *marketing capital*  $m$  that attracts searching reps (a fraction of  $h$ ) to her as opposed to other producers who do the same in that market. Let  $a_d(i, s^t)$  and  $a_d^*(i, s^t)$  be investments in each respective type of capital, and let  $m_d(s^t) \equiv m_d(i, s^t)$  and  $m_d^*(s^t) \equiv m_d^*(i, s^t)$  denote the average

marketing capital held by producers active in each respective market.<sup>20</sup> The marketing capital of producer  $i$  in each respective market then follows the law of motion given by

$$\begin{aligned} m_d(i, s^t) &= (1 - \delta_m) m_d(i, s^{t-1}) + a_d(i, s^t) - \Psi(a_d(i, s^t), m_d(i, s^{t-1})) \\ m_d^*(i, s^t) &= (1 - \delta_m) m_d^*(i, s^{t-1}) + a_d^*(i, s^t) - \Psi(a_d^*(i, s^t), m_d^*(i, s^{t-1})), \end{aligned} \quad (46)$$

and the fraction of searching reps that match with that producer at  $s^t$  in each respective market is

$$\frac{m_d(i, s^t)}{m_d(s^t) + m_f(s^t)} h(s^t) \text{ and } \frac{m_d^*(i, s^t)}{m_d^*(s^t) + m_f^*(s^t)} h(s^t), \quad (47)$$

where  $0 < \delta_m < 1$  is the exogenous depreciation rate and  $\Psi(a, m)$  is a convex adjustment cost referred to as the *market expansion friction*. Since matches are long lasting, the list of customers evolves according to the law of motion given by

$$\begin{aligned} H_d(i, s^t) &= (1 - \delta_H) H_d(i, s^{t-1}) + \frac{m_d(i, s^t)}{m_d(s^t) + m_f(s^t)} h(s^t), \\ H_d^*(i, s^t) &= (1 - \delta_H) H_d^*(i, s^{t-1}) + \frac{m_d^*(i, s^t)}{m_d^*(s^t) + m_f^*(s^t)} h^*(s^t), \end{aligned} \quad (48)$$

and this determines how much the producer can sell in each market via the constraints:  $d(i, s^t) \leq H_d(i, s^t)$  and  $d^*(i, s^t) \leq H_d^*(i, s^t)$ . The producer's instantaneous profit at  $s^t$  is given by

$$\begin{aligned} \Pi_I(i, s^t) &= [p_d(i, s^t) - v(s^t)] d(i, s^t) - v(s^t) a_d(i, s^t) \\ &\quad + [x(s^t) p_d^*(i, s^t) - v(s^t)] d^*(i, s^t) - x(s^t) v^*(s^t) a_d^*(i, s^t), \end{aligned} \quad (49)$$

where prices are bargained over in bilateral matches and, as shown below, are independent of the producer's choices (the producer is atomless and has no impact on aggregates). Accordingly, the producer effectively chooses investment in marketing in each market,  $a(s^t)$ ,  $a^*(s^t)$ , to maximize the present discounted value of profits:  $\sum_{t=0}^{\infty} \int \beta^t u_c(s^t) \Pi_I(i, s^t) Pr(ds^t)$  subject to (46), (48), and the sales con-

<sup>20</sup>Feasibility requires  $d(s^t) + d^*(s^t) + a_d(s^t) + a_f(s^t) + \chi h(s^t) = Y(s^t)$ .

straints  $d(i, s^t) \leq H_d(i, s^t)$ ,  $d^*(i, s^t) \leq H_d^*(i, s^t)$ . (Assuming markups are positive, the sales constraints hold with equality.)

Finally, as already noted, prices are determined by Nash bargaining and solve

$$p_j(s^t) := \arg \max_{p_j} V_j(p_j|i, s^t)^{1-\eta} W_j(p_j|i, s^t)^\eta, \quad (50)$$

where  $j = d, f$  and  $0 \leq \eta \leq 1$  is the bargaining power parameter. The value of a match to the producer is given by

$$W_j(p_j|i, s^t) = \max\{p_j - mc(s^t), 0\} + (1 - \delta_H) \mathbb{E}_{s^t} \left[ \beta \frac{\partial_c u(s^{t+1})}{\partial_c u(s^t)} W_j(p_j(s^{t+1})|i, s^{t+1}) \right], \quad (51)$$

where  $V_j(p_j|i, s^t)$  represents the match value to the final-goods producer given by (44) and  $mc_f(s^t) := v^*(s^t)x(s^t)$ ,  $mc_d(s^t) := v(s^t)$ . The bargaining equation, imposed state-by-state, implies that the import price is

$$p_f(s^t) \equiv p_f(i, s^t) = \eta P_f(s^t) + (1 - \eta) x v^*(s^t), \quad (52)$$

and, as asserted, prices are decoupled from the firms' dynamic market-share accumulation choices.

### 3 Theory: The Parameterization Trilemma

This section characterizes the theory-implied trade-offs in jointly matching three moments: (i) exchange-rate pass-through (PT) into import prices, (ii) the short-run trade elasticity (TE), and (iii) average profit margins (markups). Matching all three poses a structural challenge we refer to as the *parameterization trilemma*. After a brief review of the empirical evidence, we lay out our analytical framework and discuss each model separately in the results subsection. More detailed derivations for this section can be found in the Online Appendix.

**Exchange-rate pass-through (PT).** PT coefficients are typically estimated by regressing import prices on exchange rates at various horizons while controlling for production costs at the source. At business-cycle frequencies, real and nominal exchange rates co-move closely; consequently, short-run (up to one year) PT coefficients estimated for nominal exchange rates are informative.

Most estimates imply pass-through coefficients in the 0.2–0.6 range, meaning that a one percent real exchange-rate depreciation (an increase in  $x$ ) leads to an approximately 0.2–0.6 percent increase in import prices (for a survey, see [Burstein and Gopinath, 2014](#)). As an example, [Campa and Goldberg \(2005\)](#) use a distributed-lag model:

$$\Delta \log p_t^j = \alpha + \sum_{i=0}^4 a_i^j \Delta \log e_{t-i}^j + \sum_{i=0}^4 b_i^j \Delta \log w_{t-i}^j + c^j \Delta \log gdp_t^j + \varepsilon_t^j, \quad (53)$$

where  $p_t^j$  is the nominal dock import price in country  $j$  in quarter  $t$ ,  $w_t^j$  measures production costs in the corresponding exporting country, and  $\Delta gdp_t^j$  is GDP growth in the importing country  $j$ . [Campa and Goldberg \(2005\)](#) estimate, in nominal terms, a short-run pass-through of 0.46 ( $a_0^j$  in the regression; averaged across 23 OECD countries) and a long-run pass-through of 0.64 ( $\sum_{i=0}^4 a_i^j$  in the regression; averaged across 23 OECD countries). Some studies report significantly lower figures; for example, [Gopinath and Itskhoki \(2022\)](#) find estimates between 0.2 and 0.3. We consider a given PTM model successful if it generates a pass-through coefficient of 0.5 or lower.

We associate the empirical estimates of the business-cycle-frequency pass-through coefficient with the partial-equilibrium, on-impact elasticity of the import price with respect to a marginal change in the exchange rate around the steady state. Consistent with the empirical strategy of controlling for confounding factors in the regression, our theoretical definition of pass-through strips out the general-equilibrium (GE) feedback effects operating through marginal costs ( $v, v^*$ ), the stochastic discount factor (via  $c, c^*$ ), and the endogenous response of the prices of domestically produced goods ( $p_d$ ). Regarding expectations, we assume that the optimizing entity expects all importers to react symmetrically—an assumption that is immaterial in most frameworks, with the CC and NCES models being the exceptions. In most models, a one-time shock at  $s^t$  suffices because the PTM mechanism is static:  $x \equiv x(s^t)$  and the expected path is

$x(s^{t+\tau}) = 1$  for  $\tau = 1, 2, 3, \dots$ . The DH model extends this setup by allowing the initial exchange-rate shock to decay only gradually in expectation, with  $0 < \kappa \leq 1$  governing that persistence. This captures the idea that, on impact, agents expect the initial shock to unwind only gradually over future periods.

Formally, let  $p_f(x|s^t)$  denote the policy function of the price-setting entity, expressed as a function of the real exchange rate  $x \equiv x(s^t)$  at the state in which the one-time exchange-rate shock occurs. We present the first-order conditions underlying the policy function  $p_f(x|s^t)$  when discussing each model. Given this policy function, the theoretical pass-through coefficient (PT) is defined as

$$PT := \partial_{\log x(s^t)} \log p_f(x(s^t) | s^t) \Big|_{ss}, \quad (54)$$

where the definition of the partial derivative operator  $\partial_{\log x(\cdot)} \Big|_{ss}$  embeds the assumptions stated above.

**Trade elasticity (TE).** Trade elasticity (TE) in our analysis always refers to the short-run elasticity of imported quantities with respect to changes in import prices at business-cycle frequency, holding confounding factors constant. This concept differs from the so-called long-run trade elasticity estimates, which are typically much higher.<sup>21</sup> Time-series estimates of trade elasticity at business-cycle frequency typically fall below unity, and micro estimates that control for general-equilibrium effects are not very different from those based on aggregate time-series data. We define a model as consistent with the empirical evidence if its implied elasticity is below unity.

To extract trade elasticity from our models, we use the policy function determining the import ratio  $\frac{f}{a}(p_f|s^t)$  for a given country, expressed as a function of the import price  $p_f \equiv p_f(s^t)$  and evaluated as a log-linear slope around the deterministic steady state. Typically, this policy function is associated with the final-goods producer's problem, and this definition likewise abstracts from GE transmission channels (see above). Similarly, our definition assumes symmetry, in that all importers supplying the relevant good

<sup>21</sup>See [Ruhl \(2008\)](#) for a comprehensive review of the literature. Long-run estimates of trade elasticity are distinct because they are based on the permanent effects of trade liberalizations or on cross-country price variation. Given our focus on business-cycle analysis, we target short-run estimates. In the quantitative section, rather than relying on regression-based estimates, we follow the simple approach of [Drozd and Nosal \(2012\)](#), which uses the volatility ratio of quantities to prices. This method abstracts from correlation and yields an upper-bound estimate of the short-run trade elasticity. We target 0.7.

to the final-goods producer are expected to raise their prices symmetrically. Formally,

$$TE := \left( -\partial_{\log p_f} \log \frac{f}{d} (p_f|s^t) \Big|_{ss} \right) \equiv (PT)^{-1} \left( -\partial_{\log x(s^t)} \log \frac{f}{d} (p_f(x(s^t)|s^t)|s^t) \Big|_{ss} \right), \quad (55)$$

where the identity on the right-hand side follows from the definition of PT above. As above, the definition of the differentiation operator  $\partial_{\log x}(\cdot|_{ss})$  removes the indirect GE effects. The function  $p_f(x|s^t)$  is the same policy function that enters the definition of PT.

**Producer markups.** Under Cobb–Douglas production function and static cost minimization without frictions, gross margins provide a measure of markups over the cost of goods sold (Hall, 1988; De Loecker et al., 2020). Let capital be treated as predetermined at the firm level—consistent with how cost of goods sold (COGS) is defined in 10-K income statements. Then  $wl$  is COGS, and the cost-minimization problem reduces to  $\mathcal{C}(y) := \min_l wl$  subject to  $y = Ak^\alpha l^{1-\alpha}$ , where  $k$  is predetermined. The implied measured marginal cost is  $mc \equiv \mathcal{C}'(y)$ . The production function is Cobb–Douglas,  $\partial y/\partial l = (1 - \alpha)y/l$ , so marginal cost is  $mc = \frac{wl}{(1-\alpha)y}$ . Accordingly, the COGS-based steady-state markup measure is  $\mu_{COGS}^{ss} := \frac{P^{ss}y^{ss} - \mathcal{C}^{ss}}{\mathcal{C}^{ss}}$ . Crucially, consistent with setting the upper bound, this measure is lower than the model-based markup definition that uses marginal cost that includes rental price of capital,  $\mu^{ss} = (p_d^{ss} - v^{ss})/v^{ss}$ , and is therefore generous relative to the markup concept implied by the model and later used in the calibration.<sup>22</sup>

To get a sense of an upper bound for the markup target in the models, Table 1 reports several estimates of the gross margin based on various sources and methodologies. The first draws on the S&P Compustat North America database (Fundamentals Annual), which is sourced from SEC 10-K annual filings by public corporations.<sup>23</sup> Although affected by selection into public trading, this source comes closest to capturing variable production costs through reported Cost of Goods Sold (COGS). However, because COGS is a gross-output measure, whereas our models are stated in value-added terms, it is not directly comparable to the model-implied  $\mu_{COGS}^{ss}$ . We therefore apply an adjustment based on the share of value

<sup>22</sup>For a review of measurement and the challenges involved in identifying markups, see Nekarda and Ramey (2020).

<sup>23</sup>Compustat data from S&P Global Market Intelligence (2024) via Wharton Research Data Services (WRDS).

added in gross output for narrowly defined Bureau of Economic Analysis (BEA) sectors. The average gross profit markup—defined as the ratio of sales minus COGS to COGS (referred to earlier as the gross profit margin)—is estimated to be about 70% on a value-added basis.

A better data source for value-added-based models is the Bureau of Economic Analysis (BEA) input–output (I–O) tables (or, similarly, the KLEMS database), which provide broader coverage and allow one to construct profitability measures on a value-added basis. In the I–O accounts, the ratio of gross operating surplus to compensation of employees implies a margin of 40% for the aggregate economy. However, this estimate includes operational labor expenses that, on an income statement, would be classified under Selling, General, and Administrative (SG&A), and thus is closer to an operating margin than to a gross margin. Our preferred measure is the average for tradable sectors. Focusing on tradable sectors excludes the Management of Companies and Enterprises sector (NAICS 551114), which captures a large share of headquarters activities for larger firms and therefore brings the measure closer to COGS by focusing more narrowly on production labor costs.<sup>24</sup> This raises the implied estimate to 50%, which we adopt as our baseline target. We consider 70% to be the upper bound on what can plausibly be justified on empirical grounds.

### 3.1 Analytic Results

We now examine each model’s implications for the parameterization trilemma.

**KA Model.** The Kimball aggregator generates non-constant elasticity through the curvature of  $g$ , which in turn leads to incomplete pass-through. As we show, however, this is not enough.

To derive the PT coefficient for this model, we differentiate equation (22), abstracting from GE channels as assumed above, to obtain

$$PT = \partial_{\log x} \log \frac{\gamma \left( \frac{p_f(x|\cdot)}{\lambda(x|\cdot)} \right)}{\gamma \left( \frac{p_f(x|\cdot)}{\lambda(x|\cdot)} \right) - 1} \Big|_{ss} = - \frac{\gamma'(1) (\partial_{\log x} \log p_f(x|\cdot) - \partial_{\log x} \log \lambda(x|\cdot))}{(\gamma(1) - 1)\gamma(1)}. \quad (56)$$

<sup>24</sup>BEA’s industry classification is establishment-based and splits firm-level inputs when they are physically separated.

Table 1: Gross Margin Estimates for Aggregate U.S. Data, 2007–2017.

	2007	2012	2017
<i>S&amp;P 500 Compustat—with BEA SUT402 VA adjustment<sup>a</sup></i>			
All firms excluding FIRE, GOV, and NGO	72%	71%	71%
<i>S&amp;P 500 Compustat—unadjusted<sup>b</sup></i>			
All firms excluding FIRE, GOV, and NGO	47%	43%	49%
<i>BEA 402 Industry I–O Use Table<sup>c</sup></i>			
All sectors excluding FIRE, GOV, and NGO	39%	41%	40%
Only traded sectors (import share > 3%)	48%	50%	45%

<sup>a</sup>**S&P 500 Compustat—with BEA SUT402 VA adjustment:** Aggregated across all firms under NAICS codes mapped to 2017 BEA 402 sectors (excluding FIRE) and using the BEA 2017 crosswalk. The formula is:

$$\bar{\mu}_{\text{adjusted}} = \sum_{b \in \text{BEA}} \left( \frac{\sum_{i \in b} \text{Sales}_i - \sum_{i \in b} \text{COGS}_i}{\sum_{i \in b} \left( \frac{VA_b}{MAT_b + LAB_b} \cdot \text{COGS}_i \right)} \right) \omega_b,$$

where  $\frac{VA_b}{MAT_b + LAB_b}$  is the BEA 402 sector-specific ratio of value added (in basic prices) to the sum of cost of materials (MAT) and labor compensation (LAB), as reported in the I–O use tables. Index  $b$  denotes BEA sectors, and  $i$  indexes firms (NAICS codes) associated with  $b$ . Sales and COGS are from Compustat. The aggregation across sectors is weighted by BEA sectoral value added,  $\omega_b$ , where  $\sum_b \omega_b = 1$ .

<sup>b</sup>**S&P 500 Compustat—unadjusted:** We use the same formula but with  $\frac{VA_b}{MAT_b + LAB_b} \equiv 1$ .

<sup>c</sup>**BEA 402 Industry I–O Use Table:** *Margin* is the ratio of BEA’s gross operating surplus to total value added at producer prices, aggregated across all sectors excluding finance, insurance, and real estate (FIRE), federal and state government (including government-run utilities and educational institutions), non-government organizations (NGOs), and supplementary sectors (e.g., scrap, household sector, and unadjusted imports). Producer prices include taxes on production and are different from BEA’s basic prices.

Using the facts that  $x^{ss} p_d^{*ss} / \lambda^{ss} = x^{ss} = v^{ss} = 1$  and  $\partial_{\log x} \log \lambda(x|\cdot) = (1 - \omega) \partial_{\log x} \log p_f(x|\cdot)$ , we then differentiate (20) to derive

$$PT = \frac{(\gamma(1) - 1)\gamma(1)}{\omega\gamma'(1) + (\gamma(1) - 1)\gamma(1)}. \quad (57)$$

This formula shows that any PT can be achieved by varying  $\gamma'(1)$  or  $\gamma(1)$ , both of which are parameters. This is not surprising, since the Kimball model can generate arbitrary curvature of the demand curve. In a calibrated open-economy model, however, these are no longer free parameters, because they also determine the steady-state level of markups and the trade elasticity. To see this, we derive TE by differentiating the aggregate ratio  $f/d$  after substituting from (19) and using symmetry,  $p_f(x|i, s^t) \equiv p_f(x|s^t)$ , which under the differentiation rules imposed above gives

$$TE := -PT^{-1} \partial_{\log x} \log \frac{f\left(\frac{p_f(x|\cdot)}{\lambda(x|\cdot)}, \cdot\right)}{d\left(\frac{p_d}{\lambda(x|\cdot)}, \cdot\right)} \Big|_{ss} = \gamma(1) = \frac{1}{\mu^{ss}} + 1, \quad (58)$$

since, by (22), we also have  $\mu^{ss} := (p_d - v)/v|_{ss} = (\gamma(1) - 1)^{-1}$ .

To see how restrictive these relationships are in light of the empirical targets, consider  $TE = 1.5$ . The formula above then implies 200% steady-state markups ( $\mu^{ss} = 2$ ).

One might be tempted to conclude that imposing a convex adjustment cost on either the  $f/d$  ratio or on  $f$  itself would address this issue. As we discuss in the next section, however, this “fix” does not work because the convex adjustment cost also affects steady-state markups. What is missing from the KA model is a friction whose introduction would require a structural modification of the KA setup—which is beyond the scope of our analysis.

**CD Model.** While exhibiting constant elasticity with respect to the sum of the price and the local distribution cost, the CD model implies non-constant elasticity with respect to the price alone. This gives rise to incomplete pass-through.

The strength of this effect, however, is tied to the level of markups and therefore to the trilemma. To see this, note that  $\mu^{ss} := (p_d^{ss} - v)/v = \frac{\theta}{\theta-1} + \frac{\xi}{\theta-1} - 1$  by (26). Differentiating (26) with respect to  $x(s^t)$ , we

obtain

$$PT = \partial_x p_f(x|\cdot) \frac{x}{p_f(x|\cdot)} \Big|_{ss} = \frac{\theta}{\theta-1} v^* \frac{x}{\frac{\theta}{\theta-1} x v^* + \frac{\xi}{\theta-1} v} \Big|_{ss} = 1 - \frac{\frac{\xi}{\theta-1}}{\frac{\theta}{\theta-1} + \frac{\xi}{\theta-1}} = 1 - \frac{\mu^{ss} - \frac{1}{\theta-1}}{1 + \mu^{ss}}. \quad (59)$$

Consider again the target  $PT = 0.5$ . Even in the most favorable—though theoretically impossible—case with  $\theta \rightarrow \infty$ , the CD model requires a 100% markup target. Thus, the model falls short of matching the data along this dimension. (As in the baseline model,  $TE = \gamma$ , and it can be calibrated independently by choosing the value of  $\gamma$ .)

**PD Model.** The PD model implies PTM because search costs anchor the upper bound of the posted-price distribution to local search costs through the condition  $\theta v(s^t) = P_h(s^t) - p_d(s^t)$ .

As in the CD model, however, this same feature is also what generates static markups, and hence the trilemma. To see this, differentiate (30) to obtain

$$PT = 1 - \frac{\frac{\theta q}{1-q}}{1 + \frac{\theta q}{1-q}} = 1 - \frac{\mu^{ss}}{1 + \mu^{ss}}, \quad (60)$$

since, by (30), we have  $\mu^{ss} = \theta q / (1 - q)$ . Although the trade-off between PT and  $\mu^{ss}$  is less severe here, it remains restrictive given the empirical targets. According to the formula, achieving the target  $PT = 0.5$  requires exactly a 100% markup ( $\mu^{ss} = 1$ ). (As in the baseline setup,  $TE = \gamma$ .)

**NCES Model.** The NCES model gives rise to incomplete pass-through because firms are non-atomistic and internalize their effect on the sectoral price index. Since this link depends on importers' market shares, markups are endogenous and co-move with exchange rates.

Formally, differentiating the policy function in (35) under the rules of the log-linearization operator  $\partial_{\log x}(\cdot|_{ss})$ , we obtain

$$PT = 1 - \frac{(\theta - 1)\mu_f^{ss} - 1}{\theta} (-\partial_{\log x} \log S_f(x|\cdot)|_{ss}), \quad (61)$$

where  $\partial_{\log x} \log S_f(x|\cdot)|_{ss}$  denotes the response of importers' market share.

This link to quantities is what gives rise to the trilemma because market-share sensitivity maps directly into the trade elasticity. In this case, it is trade elasticity, rather than static markups, that becomes the constraining force, much as in the KA model.

To see this, differentiate (35) to obtain

$$\begin{aligned} -\partial_{\log x} (\log S_f(x|\cdot))|_{ss} &= -\partial_{\log x} \log \left( \frac{p_f(x|\cdot)}{p_d} \right) \Big|_{ss} + \left( -\partial_{\log x} \log \left( \frac{f(p_f(x|\cdot)|\cdot)}{d(p_d|\cdot)} \right) \Big|_{ss} \right) \\ &\quad + \partial_{\log x} \log \left( N + N_X \frac{p_f(x|\cdot)f(p_f(x|\cdot), \cdot)}{p_d d(p_d|\cdot)} \right) \Big|_{ss}, \end{aligned} \quad (62)$$

where  $p_f(x|\cdot)$  corresponds to (35). By the definition of PT in (54), the first two terms on the right-hand side of (62) correspond to  $-PT + \theta PT$ . As for the last term, note that the approximation  $d \log(1 + z)/dz \leq 1$  holds for any  $z \geq 0$ , and side-by-side division of the demand equations in (33) gives

$$\partial_{\log x} \frac{p_f(x|\cdot)f(p_f(x|\cdot), \cdot)}{p_d d(p_d|\cdot)} \Big|_{ss} = \partial_{\log x} \left( \frac{p_f(x|\cdot)}{p_d} \right)^{1-\theta} \Big|_{ss} = (1 - \theta) \left( \frac{p_f^{ss}}{p_d^{ss}} \right)^{1-\theta} PT, \quad (63)$$

$$-\partial_{\log x} \log \frac{f(p_f(x|\cdot), \cdot)}{d(p_d|\cdot)} \Big|_{ss} = \partial_{\log x} \theta \log \left( \frac{p_f(x|\cdot)}{p_d} \right) \Big|_{ss} = \theta PT. \quad (64)$$

Accordingly, by (55), we have  $TE = \theta$ . Plugging all these relations into (62), including  $TE = \theta$ , it follows that

$$-\partial_{\log x} \log S_f(x|\cdot) \Big|_{ss} \leq -PT + TE \times PT + \frac{N_X}{N} (1 - TE) PT \left( \frac{p_f^{ss}}{p_d^{ss}} \right)^{1-\theta} < (TE - 1) PT, \quad (65)$$

and, returning to (61), we thus have<sup>25</sup>

$$PT \geq 1 - \frac{(TE - 1)\mu_f^{ss} - 1}{TE} (TE - 1) PT \text{ and hence } PT \geq \left( 1 + \frac{(TE - 1)\mu_f^{ss} - 1}{\theta} (TE - 1) \right)^{-1}. \quad (66)$$

The above inequality captures the essence of the trilemma in this model:  $\theta$  determines both TE and PT.

To get a sense of how severe this constraint is, consider a 50% markup target ( $\mu^{ss} = 1.5$ ). By the formula

<sup>25</sup>We use the fact that  $(\theta - 1)\mu_f^{ss} - 1 > 0$  by (35). To see this, note that, by (35) and (36),  $\mu_f^{ss} > 1/(\theta - 1)$  since  $\theta > \gamma$ .

above, achieving  $PT = 0.5$  requires  $TE = \theta \geq 5.4$ . If we raise the markup target to 100% ( $\mu^{ss} = 2.0$ ), it still requires  $TE \geq 3.4$ , well above the range seen in the data. As in the KA model, a convex adjustment cost imposed on market shares cannot resolve this issue, because dampening  $\partial \log S_f(x|\cdot)|_{ss}$  would also undo PTM.

**DH Model.** As discussed in the model section, the novel implication of the DH setup is that the value of habit affects markups on imported goods:

$$p_f(i, s^t)/x(s^t) = \frac{\theta}{\theta - 1} [v^*(s^t) - (1 - \rho)\Delta_f(i, s^t)] \quad (67)$$

$$\Delta_f(i, s^t) = \beta \mathbb{E}_{s^t} \mathbb{M}_{t+1} \left[ \rho \Delta_f(i, s^{t+1}) + \frac{\zeta(\theta - 1)}{\theta} \frac{p_f(i, s^{t+1})}{x(s^{t+1})} \frac{f(i, s^{t+1})}{h_f(i, s^t)} \right] \quad (68)$$

where  $\mathbb{M}_{t+1} := u_{c^*}^*(s^{t+1})/u_{c^*}^*(s^t)$  is the foreign stochastic discount factor. This induces PTM independently of the model-implied TE and  $\mu^{ss}$ . The difficulty, however, is that the habit mechanism, when combined with persistent shocks, implies more-than-complete pass-through in response to the shocks of the baseline model. (The response to transitory shocks is different, as we also discuss.)

To see this, let us define the growth rate of habit as  $g_h(s^{t+1}) := h_f(i, s^{t+1})/h_f(i, s^t)$ . This is convenient because habit growth follows quantity growth.

Using the habit accumulation law, it is clear that the quantity-to-habit ratio is  $\frac{f(i, s^{t+1})}{h_f(i, s^t)} = \frac{g_h(s^{t+1}) - \rho}{1 - \rho}$ .

Substituting for  $\Delta_f$  in (68) using (67) gives a difference equation in  $p_f^*(s^t) := p_f(i, s^t)/x(s^t)$ :

$$p_f^*(s^t) = \frac{\theta}{\theta - 1} (v^*(s^t) - \beta \rho \mathbb{E}_{s^t} \mathbb{M}_{t+1}^* v^*(s^{t+1})) + \beta \mathbb{E}_{s^t} \mathbb{M}_{t+1}^* [p_f^*(s^{t+1}) (\rho - \zeta g_h(s^{t+1}) + \zeta \rho)]. \quad (69)$$

Log-linearizing this difference equation around the deterministic steady state with respect to  $p_f^*(s^t)$  and  $p_f^*(s^{t+1})$ , effectively setting  $d \log v^*(s^t) = d \log v^*(s^{t+1})$ , and solving it forward, gives

$$d \log p_f^*(s^t) = -\beta \zeta \sum_{\tau=0}^{\infty} [\beta(\rho - \zeta(1 - \rho))]^\tau \mathbb{E}_{s^t} d \log g_h(s^{t+\tau+1}). \quad (70)$$

Using the fact that  $d \log p_f = d \log p_f^* + d \log x$ , it's now easy to see that

$$PT = 1 - \beta \zeta \sum_{\tau=0}^{\infty} [\beta(\rho - \zeta(1 - \rho))]^{\tau} \partial_{\log x} \left( \mathbb{E}_{st} \log g_h(s^{t+\tau+1}) \Big|_{ss} \right). \quad (71)$$

Why does this imply PTM in reverse for persistent shocks? To see this, consider a persistent exchange-rate depreciation at home (an increase in  $x$ , that is, an appreciation from the foreign country's perspective). Following such a shock, habit gradually declines because foreign sellers raise their prices, implying  $\partial_{\log x} \log g_h < 0$  for several periods, after which it reverses course and converges back to the steady state. But since earlier periods receive greater weight because of discounting in the formula above, the summation term becomes negative when the shock is sufficiently persistent, implying  $PT > 1$ .<sup>26</sup>

**CC Model.** Incomplete pass-through arises in the CC model because the adjustment of market shares and quantities is slowed by the market-expansion friction. This friction causes match surplus to vary with exchange rates, which in turn generates variation in producer markups because bargaining splits the surplus proportionally. Since steady-state markups are determined by the level of search costs on the retailer side and are therefore independent of the market-expansion friction, the model is able to address the trilemma. Quantitatively, however, its additional features in an environment with volatile exchange rates are not fully neutral and have adverse implications for matching the behavior of measured TFP. For parameterizations targeting significantly more volatile real exchange rates, this endogenous wedge can become problematic—as we discuss.

Deriving pass-through and trade elasticity in this model is cumbersome because the model is dynamic. A simplified static version of the core mechanism is therefore useful for understanding how it works. To

<sup>26</sup>Consider a permanent shock. In that case,  $\partial_{\log x} \log g_h < 0$  throughout, and hence  $PT > 1$ . Shock persistence is essential. For example, following a one-time shock, habit recovery begins immediately (at  $s^{t+1}$ ), implying  $\partial_{\log x} \log g_h > 0$  throughout and hence  $PT < 1$ . Since exchange rates are highly persistent in the data, the first property is likely to dominate, leading to more-than-complete pass-through. We confirm in the quantitative section that this is indeed the case. (TE and  $\mu^{ss}$  are as in the baseline model.)

that end, consider  $\delta_H = 1$  and  $\delta_m = 1$ , in which case the law of motion for marketing capital is

$$m_f(i, s^t) = a_f(i, s^t) - \psi a_f^{ss} \left( \frac{a_f(i, s^t)}{a_f^{ss}} - 1 \right)^2. \quad (72)$$

The key difference is that, unlike in the full model, the adjustment cost depends on the deviation of marketing expenditures from their steady-state value rather than from their previous-period value. The importer's problem is therefore also static and reduces to

$$\max_{a_f(i, s^t)} (p_f(s^t)/x(s^t) - v(s^t)) h_d(s^t) \frac{m_f(i, s^t)}{\bar{m}_f(s^t) + \bar{m}_d(s^t)} - v^*(s^t) a_f(i, s^t), \quad (73)$$

subject to (72).

To derive PT, we use the pricing formula for the representative importer stated in (52) and combine it with the equation for the marginal retail valuation of the foreign good,  $P_f = \partial_f G(d, f)$ . Given that  $G(\cdot)$  is constant returns to scale and prices are determined by Nash bargaining, this gives

$$\log p_f(x|\cdot) = \log \left( \eta \partial_f G \left( \frac{d}{f}(x|\cdot), 1 \right) + (1 - \eta) x v^* \right),$$

and, after log-linearization, we obtain

$$PT = \partial_{\log x} \log p_f(x|\cdot) = \omega \gamma^{-1} \left( \underbrace{-\partial_{\log x} \log \frac{f}{d}(x|\cdot)}_{TE \times PT} \right) \frac{\eta + \mu^{ss}}{1 + \mu^{ss}} + \frac{1 - \eta}{1 + \mu^{ss}}, \quad (74)$$

and hence

$$PT = (1 - \eta) \left( 1 + \mu^{ss} - \frac{TE}{\gamma} (\eta + \mu^{ss}) \omega \right)^{-1}. \quad (75)$$

To the extent that  $TE/\gamma$  is sufficiently low, it is clear that the CC model can deliver incomplete pass-through. At the extreme case in which  $\frac{TE}{\gamma} = 0$ , PT is determined solely by markups and bargaining power, and it can match the data target. For example, under our calibrated value  $\eta = 0.5$ , PT is below

0.5 for all markup values, implying  $PT = 0.33$  for 50% markups; in the full model, it would then rise over time because the model is dynamic. While the formula involves TE, PT is largely independent of  $\gamma$  because the convex cost parameter in the accumulation of marketing capital affects TE. To see this, note that  $\frac{f}{d} = \frac{m_f(i, s^t)}{m_d(i, s^t)}$ , and so the elasticity of the ratio  $\frac{f}{d}$  with respect to an exchange-rate shock depends on the response of the importer's marketing capital  $m_f(i, s^t)$ . Given the profit-maximization problem above, after log-linearization around the deterministic steady state, (55) implies

$$TE \times PT = -\partial \log \frac{m_f(i, s^t)}{m_d(i, s^t)} = -\frac{\gamma(\eta + \mu^{ss})}{\gamma\mu^{ss}(1 - \omega + 2\psi) + \omega(\eta + \mu^{ss})}, \quad (76)$$

and, by (75), we obtain

$$TE = \frac{\gamma(1 + \mu^{ss})(\eta + \mu^{ss})}{\gamma(1 - \eta)\mu^{ss}(1 - \omega + 2\psi) + \omega(1 + \mu^{ss})(\eta + \mu^{ss})}. \quad (77)$$

As noted, the parameter  $\psi$  controls the value of TE implied by the model. This additional degree of freedom allows the CC model to escape the grip of the trilemma. The key structural assumption behind it is the presence of search-and-matching frictions, which place a hard constraint on how many units can be traded within a match and isolate quantities from prices in the bargaining equation. Without this feature, the conflict between matching trade elasticity and markups would reemerge.

Two additional considerations regarding the parameterization of this model are worth noting, as they represent a distinct manifestation of the trilemma. They are not binding in our calibration, but they may be problematic under a significantly more volatile exchange-rate target.

First, the model requires a high value of  $\gamma$  because this parameter governs the responsiveness of retail prices to the import share, and retail prices in turn affect trade surpluses. This can be justified because long-run trade elasticity estimates in the literature are higher than short-run elasticity estimates, but there is also a quantitative restriction. The model's performance deteriorates sharply when  $\gamma$  is 3 or 4, which, according to some estimates, could be a binding constraint (Boehm et al., 2023b).

More importantly, the model implies an endogenous wedge between measured TFP and the assumed

productivity process ( $z$ ). This follows from the presence of investment in intangible marketing capital. In particular, high exchange-rate volatility tends to generate a negative correlation between home and foreign measured TFP, which our calibration offsets by imposing a positive correlation on the residuals of the process for  $z$ . But this approach has a natural limit: the correlation cannot exceed 1. In practice, matching exchange-rate volatility of 5% or more becomes problematic.

## 4 Quantitative Results

This section evaluates the quantitative implications of the trilemma. We aim for a uniform set of targets and consistent methodology across models. We HP-filter the model-generated quarterly series with  $\lambda = 1600$  and set the period to one quarter. The parameter values are listed in Tables 2 and 3.

### 4.1 Parameterization

Table 2: Parameter Values in the Models: Structural.

All models	$\sigma$	$\beta$	$\alpha$	$\delta$	$\Gamma$				
	2	0.99	0.36	0.025	0.01				
Benchmark Model	$\nu$	$\omega$	$\gamma$	$\theta$	$\phi$				
	0.454	0.88	0.7	3	11.7				
Kimball Aggregator	$\nu$	$\omega$	$\gamma$	$\psi$	$\phi$				
	0.454	0.88	3	0.655	19.2				
Costly Distribution	$\nu$	$\omega$	$\gamma$	$\theta$	$\xi$	$\phi$			
	0.387	0.76	1.62	6	1.5	5			
Price Dispersion	$\nu$	$\omega$	$\gamma$	$\theta$	$q$	$\phi$			
	0.387	0.76	1.62	1.5	0.25	3.4			
Nested CES	$\nu$	$\gamma$	$\theta$	$N$	$N_X$	$\tau$	$\phi$		
	0.541	0.94	7.9	4	1	0.246	38.5		
Deep Habits	$\nu$	$\omega$	$\gamma$	$\theta$	$\zeta$	$\rho$	$\phi$		
	0.454	0.89	0.69	2.6	0.1	0.85	12		
Customer Capital	$\nu$	$\omega$	$\gamma$	$\eta$	$\delta_H$	$\delta_m$	$\chi$	$\psi$	$\phi$
	0.400	0.84	7.9	0.5	0.1	0.2	4.59	0.29	7.2

Table 3: Parameter Values in the Models: Shocks.

	$\sigma(\varepsilon_t)$	$\rho(\varepsilon_t, \varepsilon_{t-1})$	$\rho(\varepsilon_t, \varepsilon_t^*)$	$\rho(\varepsilon_t^n, \varepsilon_{t-1}^n)$	$\sigma(\varepsilon_t^n)$
Benchmark Model	0.61	0.98	0.3	0.97	24.1
Kimball Aggregator	0.61	0.98	0.3	0.97	38.2
Costly Distribution	0.60	0.98	0.37	0.97	20.6
Price Dispersion	0.61	0.98	0.33	0.97	19.9
Nested CES	0.56	0.99	0.56	0.97	104.1
Deep Habits	0.61	0.98	0.3	0.97	24.1
Customer Capital	2.13	0.95	0.52	0.97	19.0

**Common Targets and Parameters.** The utility function is  $u(c, l) = \frac{(c^\nu (\bar{l}-l)^{1-\nu})^{1-\sigma}}{1-\sigma}$  and the capital adjustment cost is  $\Phi(i, k_{-1}) = \frac{\phi}{2} k_{-1} \left( \frac{i}{k_{-1}} - \delta \right)^2$ .

When possible, we target the following moments: (i) an imports/GDP ratio of 12% in the U.S. data; (ii) 50% producer markups estimated using I–O tables for tradable sectors in the U.S.; (iii) the standard value of 30% work hours relative to time endowment; (iv) the volatility of investment relative to the volatility of GDP of 2.79 as observed in U.S. data; (v) a short-run elasticity of trade flows of 0.7, as implied by the volatility ratio estimates from [Drozd and Nosal \(2012\)](#); (vi) a cross-country correlation of measured TFP of 0.3; (vii) volatility and autocorrelation of measured TFP of 0.8 and 0.72, respectively<sup>27</sup>; (viii) the volatility of the real exchange rate of 3.6. Lacking a target for this parameter, we set the autoregressive coefficient for the financial shock arbitrarily at 0.97, which is the autoregressive coefficient of the raw real exchange-rate time series in the U.S. data.<sup>28</sup>

In addition, if a model allows for a degree of freedom to match the pass-through coefficient, we target a 0.4 mid-range estimate from the data (this applies to the KA model only). In contrast to the analytical section, we back out the PT coefficient by running a regression on model-generated data, mimicking the empirical approaches used to estimate PT in the data. The above targets pin down the productivity process parameters, the volatility of the financial shock, the adjustment costs for capital, and the model-specific

<sup>27</sup>The productivity shock process is assumed to be symmetric across countries and features correlated innovations.

<sup>28</sup>In Tables 8 and 9 in the appendix, we report results from the models under a low-trade-openness calibration, targeting imports/GDP of 3.5%, as in [Itskhoki and Mukhin \(2025\)](#). Table 7 shows the model fit. The exercise showcases the sensitivity of the models to that calibration target. The bottom line is that the low openness calibration does improve the performance of several models for international comovement statistics for consumption, employment and investment.

parameters described below.

The remaining parameters  $\beta, \sigma, \delta, \alpha$ , and  $\Gamma$  take the same values in all models. We choose standard values for these parameters. When a model *cannot* satisfy all the targets, we drop one of the targets. As a general rule, we drop the target that ensures the best performance of the model in terms of the implied PT coefficient (this is discussed below when applicable). Tables 2 and 3 give the parameter values, while additional Table 6 (end of the paper appendix) presents the calibration fit. We now describe the specifics for each model.

**Calibration: KA.** The only model-specific parameter is  $\gamma(1)$  (or, equivalently,  $g'(1)$ ), which we choose to match steady-state producer markups of 50%. The KA model satisfies all calibration targets with the exception of trade elasticity, which—for reasons discussed earlier—it cannot match for the targeted level of producer markups. We thus drop this target from the calibration procedure. As noted, the model can deliver any level of pass-through coefficient via the independent choice of  $\gamma'(1)$  (or, equivalently,  $g''(1)$ ). As noted above, we pick this parameter to match a PT coefficient of 0.4.

**Calibration: CD Model.** The model-specific parameters  $\xi$  and  $\theta$  are calibrated jointly to match producer markups of 50% and to obtain distribution costs that constitute 50% of “non-tradable” inputs in retail prices (as implied by [Burstein et al., 2003](#)). The model satisfies all calibration targets.

**Calibration: PD Model.** As in the CD model, we parameterize the additional search technology parameters  $q$  and  $\theta$  by matching producer markups of 50% and the share of search cost in final goods prices of 50%. The model satisfies all calibration targets.

**Calibration: NCES Model.** The presence of large firms is key to generating PTM. In the original paper, there is a large number of firms (40), calibrated to the data, but the larger size of exporting firms is what matters. In our specification, due to assumed representativeness, all firms are of equal size. We thus set  $N = 5$ , and make the extreme assumption that only one firm exports ( $N_X = 1$ ). This parameter

combination is consistent with the ratio  $(\frac{N_x}{N+N_x})$  in [Atkeson and Burstein \(2008\)](#), and it gives rise to the strongest performance on the pass-through side (there is only one exporting firm, which maximizes its size). The difference between  $\gamma$  and  $\theta$  is critical for the model’s implications. As explained, there is no degree of freedom for matching trade elasticity, which we drop from the set of calibration targets. Instead, we set the high (trade) elasticity  $\theta = 7.9$  and calibrate the sectoral elasticity to match producer markups of 50%. The latter target implies endogenous and much higher short-run trade elasticity TE than our target.

**Calibration: DH Model.** For the habit parameters  $\zeta$  and  $\rho$ , we adopt the values from [Ravn et al. \(2006\)](#) (relative habit specification), setting  $\zeta = 0.1$  and  $\rho = 0.85$ . We calibrate the elasticity  $\theta$  to match steady-state markups of 50%. The model satisfies all calibration targets.

**Calibration: CC Model.** We parameterize the matching process dynamics, following [Drozd and Nosal \(2012\)](#), by setting the depreciation rate  $\delta_H = 0.1$ , and the bargaining power to the axiomatic (symmetric) value of  $\eta = 0.5$ . We set the functional form for the adjustment cost of customer capital to  $\Psi(a, m_{-1}) = \frac{\psi}{2} m_{-1} \left( \frac{a}{m_{-1}} - \delta_m \right)^2$ . We calibrate the search cost  $\chi$ , the adjustment-cost parameter for customer capital  $\psi$ , and the depreciation rate for marketing capital  $\delta_m$  by jointly matching the following moments: producer markups of 50%, the volatility ratio (measuring short-run trade elasticity) of 0.7, and the marketing expenditures-to-GDP ratio of 7%.<sup>29</sup> Finally, we set the long-run trade elasticity to  $\gamma = 7.9$ .<sup>30</sup> The model satisfies all calibration targets.

## 4.2 Results

Tables 4 and 5 report price and quantity statistics. As the first table shows, all PTM models generate some degree of incomplete pass-through; most fall short of the empirical target of 40% that we set. Importantly, the pass-through coefficient implied by the models directly translates into correlation of

<sup>29</sup>See [He et al. \(2024\)](#) for additional evidence on this target.

<sup>30</sup>See [Drozd and Nosal \(2012\)](#) for how we measure the volatility ratio.

terms of trade ( $p_f/p_x$ ) with the exchange rate ( $x$ ). In the model-generated data, measured without noise, the generated correlations are usually 1 or -1 and mostly depend on the degree of pass-through: for pass-through coefficients into import prices strictly below 50%, we get -1; for pass-through coefficients into import prices strictly above 50%, we get 1. Hence, matching pass-through of below 0.5 and negative correlation are the same prediction. Given that, not surprisingly, the two models that generate sub-0.5 pass-through, the KA model and the CC model, get a negative correlation, while the rest imply a positive correlation. This pattern suggests that our target of 40% for pass-through based on the micro estimates may be too aggressive for the aggregate statistic. A target above but close to  $PT = 0.5$  would suffice to yield a positive correlation between  $tot$  and  $x$ .<sup>31</sup> In Table 4, section F, we also report the measure of trade elasticity (the Volatility Ratio) and the correlation of relative prices and quantities (ratio of domestic absorption to imports and their relative prices). As reported above, the KA and NCES models cannot match the short-run Volatility Ratio of 0.7, while the rest of the models match this calibration target. In terms of correlations, all models except the CC model imply a perfect correlation between the relative price and relative quantity, which means that the OLS estimate of the elasticity in these models coincides with the Volatility Ratio. In the CC model, we obtain a small and slightly negative correlation, while the data estimate is also small (0.02), with a  $p$ -value of 0.20, and is therefore not statistically different from zero. We discuss model-specific performance next.

**KA Model.** The model can match any degree of pass-through. However, as discussed in Section 3, with 50% markups it implies a trade elasticity that is far too high. This hurts the performance of this model on the quantity side vis-à-vis the baseline model. In particular, the model fails to capture the international comovement of quantities seen in the data, and implies counterfactual negative comovement of consumption, investment, and employment.

We find that modifying the setup by introducing a convex adjustment cost on quantities is not a fix in this case.<sup>32</sup> Specifically, in a setup where we have introduced a convex adjustment cost incorporated into the

<sup>31</sup>Gopinath et al. (2020), in regression analysis, find the coefficient of terms of trade on nominal exchange rate to be small and statistically not significant, ranging from positive to negative point estimates, depending on the specification. This is consistent with pass-through close to 0.5 in the model.

<sup>32</sup>The same comment applies to the NCES model and for the same reason, although we do not consider it in that case.

Table 4: International Prices: Comovement and Relative Volatility<sup>a</sup>

Statistic	Predictions of the PTM Theories									
	Data <sup>b</sup>	Baseline Model	Kimball Model	Costly Distribution	Price Dispersion	Nested CES	Deep Habits	Customer Capital		
<b>A. Correlations</b>										
$p_f, x$	0.69	1.00	1.00	1.00	1.00	1.00	1.00	0.99		
$p_d, x$	-0.18	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-0.53		
$p_f/p_x, x$	0.61	1.00	-1.00	1.00	1.00	1.00	1.00	-0.98		
$x, x_{-1}$	0.90	0.72	0.71	0.72	0.72	0.71	0.72	0.67		
<b>B. Standard Deviations relative to <math>x^c</math></b>										
$x$	3.60	3.60	3.60	3.60	3.60	3.60	3.60	3.60		
$p_f$	0.61	1.16	0.40	0.87	0.70	0.63	1.19	0.36		
$p_d$	0.13	0.16	0.05	0.12	0.10	0.09	0.17	0.05		
$p_f/p_x$	0.41	1.32	0.20	0.74	0.40	0.25	1.38	0.29		
<b>D. X-Rate Pass-through</b>										
	0.40	1.16	0.40	0.87	0.70	0.63	1.19	0.36		
<b>E. Markup</b>										
	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50		
<b>F. Trade Elast. and <math>\text{corr}(f/D, p_D/p_f)</math></b>										
Correlation	0.02	1.00	1.00	1.00	1.00	1.00	1.00	-0.12		
Volatility Ratio	0.70	0.70	3.00	0.70	0.70	7.90	0.70	0.70		

<sup>a</sup>All reported statistics are based on logged and Hodrick-Prescott filtered quarterly time series (with a smoothing parameter  $\lambda = 1600$ ).

<sup>b</sup>Data for the US, 1980:1-2004:1.

<sup>c</sup>Ratio of corresponding standard deviation to the standard deviation of the real exchange rate  $x$ .

Table 5: Quantities - Comovement and Relative Volatility<sup>a</sup>

Predictions of the PTM Theories									
Statistic	Data <sup>b</sup>	Baseline Model	Kimball Model	Costly Distribution	Price Dispersion	Nested CES	Deep Habits	Customer Capital	
<i>A. Correlations</i>									
<i>domestic with foreign</i>									
<i>Measured TFP<sup>c</sup></i>	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
GDP	0.40	0.33	0.25	0.42	0.42	-0.73	0.33	0.55	
Consumption	0.25	-0.07	-0.42	0.21	0.24	-0.93	-0.05	0.47	
Employment	0.21	0.29	-0.95	0.29	0.40	-1.00	0.48	0.54	
Investment	0.23	-0.39	-0.61	-0.10	0.02	-0.96	-0.42	0.36	
<i>GDP with</i>									
Consumption	0.83	0.69	0.42	0.86	0.92	-0.79	0.69	0.98	
Employment	0.85	0.83	0.38	0.71	0.78	0.91	0.91	0.82	
Investment	0.93	0.56	0.27	0.71	0.80	-0.82	0.55	0.98	
Net exports	-0.49	-0.64	-0.38	-0.79	-0.86	0.80	-0.64	-0.99	
<i>Terms of trade with</i>									
Net exports	-0.17	0.56	-0.74	0.41	0.30	0.98	0.55	-0.26	
<i>B. Standard deviations</i>									
<i>relative to GDP<sup>d</sup></i>									
GDP	1.33	0.86	0.86	0.90	0.92	1.76	0.86	1.55	
<i>Measured TFP</i>									
Consumption	0.60	0.93	0.93	0.89	0.87	0.46	0.92	0.52	
Investment	0.74	0.98	1.29	0.73	0.68	1.91	1.01	0.61	
Employment	2.79	2.79	2.79	2.79	2.79	2.79	2.79	2.79	
Net exports	0.81	0.13	0.42	0.25	0.26	1.68	0.13	1.27	
Net exports	0.30	1.28	1.54	1.16	1.12	2.06	1.31	1.05	

<sup>a</sup>All reported statistics are based on logged and Hodrick-Prescott filtered quarterly time series (with a smoothing parameter  $\lambda = 1600$ ).

<sup>b</sup>US data for the period 1980:1-2004:1.

<sup>c</sup>Calculated using the actual national accounting formulas; due to time varying markups measured TFP slightly differs from the TFP coefficient fed into the model.

<sup>d</sup>Ratio of corresponding standard deviation to the standard deviation of *GDP*.

modified Kimball aggregator of the form:

$$\int_{[0,1]} \left[ \begin{array}{c} \omega g \left( \frac{d(i,s^t)}{\omega A(s^t)} \right) + (1-\omega) g \left( \frac{f(i,s^t)}{(1-\omega)A(s^t)} \right) \\ - \frac{\phi_K}{2} \omega \left( \frac{d(i,s^t)/f(s^t)}{\omega/(1-\omega)} - 1 \right)^2 - \frac{\phi_K}{2} (1-\omega) \left( \frac{f(i,s^t)/d(s^t)}{(1-\omega)/\omega} - 1 \right)^2 \end{array} \right] di = 1, \quad (78)$$

the convex cost parameter enters the expression for steady-state markups, which in this case are given by  $\mu_{ss} = \frac{1}{v} - 1$ , where  $v = \frac{\theta-1}{\theta} - \phi_K$ , and where  $\theta$  is the fundamental trade elasticity that gives markups without the adjustment cost.<sup>33</sup> As the expression shows, increasing the adjustment cost increases the steady-state markup, requiring an increase in the effective elasticity (increasing  $\theta$ ) to hit the same markup target. Therefore, hitting the markup target and the elasticity target requires conflicting parameter changes, which hurts the overall parameterization. In our numerical results, we found that this implies extreme values of the elasticity  $\theta$  and the cost  $\phi_K$ , and hence we do not report the results here.

Intuitively, the monopolistically competitive producers set prices subject to a perceived slope of the demand curve and choose both prices (markups) and quantities accordingly. Their problem is static and so the same is true in the steady state. Therefore, if an adjustment cost friction is introduced, producers internalize the cost of adjustment, both in the steady state for markups and over the business cycle for the dynamics of quantities.

Our conclusion is that short of complicated dynamic adjustment frictions that introduce a form of habit in the model, the KA model does not have a simple fix using standard adjustment cost formulations.

**CD Model.** This model performs as well on quantities as the baseline, but the pass-through coefficient is too high, 0.87 versus 0.4 in the data. For this reason, the model underperforms in terms of price statistics. As for quantities, the CD model implies a counterfactual correlation of net exports with the terms of trade. However, most models struggle in this respect, as is evident from Table 5.

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<sup>33</sup>In this version of the model, we use a parametric  $g(\cdot)$  function given by  $g(x) \equiv \frac{\phi}{1+\psi} ((1+\psi)x - \psi)^{\frac{1}{\phi}} - \frac{\phi}{1+\psi} + 1$ , which gives the implicit definition of  $\theta$  as  $\phi \equiv \frac{\theta(1+\psi)}{\theta(1+\psi)-1}$ , with  $\frac{\theta}{\theta-1}$  being the targeted steady-state gross markup in the absence of adjustment cost.

**PD Model.** The *Price Dispersion* model implies incomplete pass-through and improves relative to the CD model, but still falls short of the data. The implied coefficient is 0.7 versus 0.4 in the data. As noted, this is better than the *Costly Distribution* model, which structurally implies similar pricing formulas but is more constraining in terms of calibration (as shown in Section 3).

As for quantities, the model matches TE and performs marginally better than the *Costly Distribution* model. It similarly implies a counterfactual positive (but smaller) correlation between net exports and the terms of trade. Notably, both models outperform the Kimball model because they allow for TE to be chosen independently of pricing implications and match the target we set.

**NCES Model.** The *Nested CES* model is one of the strongest performers on price statistics. It generates an incomplete pass-through of 0.63, which comes close to our target of 0.4. [Atkeson and Burstein \(2008\)](#) provide a more detailed parameterization of this model in a partial equilibrium setting and show that the model matches the price data well under a much more detailed calibration. But, as noted in Section 3, the model's performance comes at a steep cost on the quantity side. This is because the model's parameterization implies high TE, failing to satisfy our target. Not surprisingly, the model implies large and negative international comovement of output, consumption, employment, and investment. Despite the presence of capital accumulation, it gives rise to a positive correlation between net exports and output.

**DH Model.** As conjectured in Section 3, amid highly persistent exchange rate fluctuations, the DH model generates PTM, but in a direction opposite to the data.<sup>34</sup> This is confirmed in Table 4. On the quantity side, the model performs reasonably well, although it implies a counterfactual negative international correlation of consumption and investment and a positive comovement between net exports and the terms of trade.

**CC Model.** The CC model adds a degree of freedom by incorporating bargaining frictions and constraints on quantities traded within matches. Under symmetry, bargaining results in 50–50 surplus split-

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<sup>34</sup>The persistence of exchange rates in the models falls short of what is observed in the data. It cannot be matched even under shocks close to a unit root, because endogenous arbitrage through quantities dampens persistence.

ting, which naturally allows the model to come close to matching the observed degree of incomplete pass-through (by choosing the bargaining power, one could match it exactly). The model with symmetric bargaining gives a pass-through of 0.36, close to but below our data target of 0.4. However, this model is dynamic, and over time pass-through rises, eventually restoring the law of one price, so a single reference number is insufficient to evaluate its performance. In the data, pass-through estimates are considerably larger at lower frequencies. On the quantity side, the model outperforms the FB baseline. It generates the correct signs for international comovement, as well as a positive correlation of the terms of trade and net exports. However, the calibration of the model is more delicate with financial shocks because the presence of marketing investment endogenously affects measured TFP, lowering the correlation between measured TFP across countries. This can be seen in Table 3, where the assumed correlation of productivity shocks is 0.52 to match the measured TFP correlation of 0.3. The model fails to deliver a positive correlation between  $tot$  and  $x$ , which is implied by the fact that it delivers  $PT < 0.5$  (see the discussion above). For the model to deliver a higher degree of pass-through, one would need to set  $\eta < 0.5$ .

#### 4.2.1 Robustness and Sensitivity

We conclude the discussion of the quantitative predictions of the models by considering the performance of the models for each type of shock separately, to show which shocks matter for price performance and which drive quantity dynamics.

Generally, the analytical results of Section 3 characterize the behavior of prices and quantities in the model relative to movements in the real exchange rate and, as such, should not depend on the specific shock driving the model's responses. We verify this hypothesis quantitatively in Tables 10–13. There, we report the price and quantity statistics from the model when only one shock is active. In terms of prices, overall performance does not change in a significant way, with the exception of exchange-rate volatility, which is too small when model dynamics are driven only by productivity shocks—a known issue. Importantly, the pass-through coefficient remains almost identical across shocks, which confirms the point above.

On the quantity side, as expected, the performance of the models differs sharply by shock type. In line with the intended purpose of introducing the financial shock, it generates high volatility of the real exchange rate. However, it is unable to generate quantity statistics consistent with the data, especially for international comovement, and it implies negative comovement for most variables. The model with productivity shocks only performs reasonably well on quantities.

In summary, we conclude that the trilemma fleshed out in Section 3 is a general result independent of the driving process.

## 5 Conclusions

We show that matching the empirically low degree of pass-through and capturing the salient features of international business cycle comovement in quantities is challenging in the presence of volatile exchange rates. The difficulty arises from the limited flexibility of existing frameworks to insulate prices and quantities from exchange-rate volatility while remaining quantitatively disciplined. We document these challenges, termed the *parameterization trilemma*, analytically and quantitatively, for a set of leading PTM frictions, embedded within a standard international business cycle framework. Among the models considered, search models achieve the best balance between price and quantity performance. Search-based (PD) and costly-distribution (CD) mechanisms generate solid quantity dynamics, though the CD model is notably weaker on prices. The reduced-form Kimball aggregator, while tractable, neither nests nor outperforms the microfounded alternatives in open-economy settings. Finally, the NCES model, while intuitive, relies too heavily on home and foreign goods being very substitutable and therefore inherits the main quantitative challenges of the one-good BKK model of 1992.

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## A Additional Results

Table 6: Steady state calibration: baseline.

Predictions of the PTM Theories									
Statistic	Data	Baseline Model	Kimball Model	Costly Distribution	Price Dispersion	Nested CES	Deep Habits	Customer Capital	
Imports-to-GDP ratio	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12
Producer markup	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
Elasticity of trade	0.70	0.70	3.00	0.70	0.70	7.90	0.70	0.70	0.70
Investment-to-GDP ratio	0.20	0.17	0.17	0.21	0.21	0.17	0.17	0.20	0.20

Table 7: Steady state calibration: low trade openness calibration.

		Predictions of the PTM Theories							
Statistic	Data	Baseline Model	Kimball Model	Costly Distribution	Price Dispersion	Nested CES	Deep Habits	Customer Capital	
Imports-to-GDP ratio	0.12	0.035	0.035	0.035	0.035	0.035	0.035	0.035	
Producer markup	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	
Elasticity of trade	0.70	0.70	3.00	0.70	0.70	7.90	0.70	0.70	
Investment-to-GDP ratio	0.20	0.17	0.17	0.21	0.21	0.17	0.17	0.20	

Table 8: International Prices: Comovement and Relative Volatility<sup>a</sup> for low trade openness calibration.

Statistic	Predictions of the PTM Theories							
	Data <sup>b</sup>	Baseline Model	Kimball Model	Costly Distribution	Price Dispersion	Nested CES	Deep Habits	Customer Capital
<b>A. Correlations</b>								
$p_f, x$	0.69	1.00	1.00	1.00	1.00	1.00	1.00	0.99
$p_d, x$	-0.18	-1.00	-1.00	-1.00	-1.00	-1.00	-0.95	-0.12
$p_f/p_x, x$	0.61	1.00	-1.00	1.00	1.00	1.00	1.00	-0.98
$x, x_{-1}$	0.90	0.72	0.72	0.72	0.72	0.71	0.72	0.71
<b>B. Standard Deviations relative to <math>x^c</math></b>								
$x$	3.60	3.60	3.60	3.60	3.60	3.60	3.60	3.60
$p_f$	0.61	1.04	0.40	0.82	0.67	0.88	1.05	0.34
$p_d$	0.13	0.04	0.01	0.03	0.02	0.03	0.04	0.04
$p_f/p_x$	0.41	1.08	0.20	0.64	0.35	0.77	1.11	0.34
<b>D. X-Rate Pass-through</b>								
	0.40	1.04	0.40	0.82	0.67	0.88	1.05	0.33
<b>E. Markup</b>								
	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
<b>F. Trade Elast. and <math>\text{corr}(f/D, p_D/p_f)</math></b>								
Correlation	0.02	1.00	1.00	1.00	1.00	1.00	1.00	-0.16
Volatility Ratio	0.70	0.70	3.00	0.70	0.70	7.90	0.70	0.70

<sup>a</sup>All reported statistics are based on logged and Hodrick-Prescott filtered quarterly time series (with a smoothing parameter  $\lambda = 1600$ ).

<sup>b</sup>Data for the US, 1980:1-2004:1.

<sup>c</sup>Ratio of corresponding standard deviation to the standard deviation of the real exchange rate  $x$ .

Table 9: Quantities: Comovement and Relative Volatility<sup>a</sup> for low trade openness calibration.

Statistic	Predictions of the PTM Theories							
	Data <sup>b</sup>	Baseline Model	Kimball Model	Costly Distribution	Price Dispersion	Nested CES	Deep Habits	Customer Capital
<i>A. Correlations</i>								
<i>domestic with foreign</i>								
<i>Measured TFP<sup>c</sup></i>	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
GDP	0.40	0.31	0.31	0.33	0.33	0.05	0.31	0.30
Consumption	0.25	0.38	0.22	0.33	0.31	-0.61	0.41	0.24
Employment	0.21	0.39	0.38	0.40	0.40	-1.00	0.36	0.24
Investment	0.23	0.28	0.11	0.30	0.30	-0.77	0.28	0.24
<i>GDP with</i>								
Consumption	0.83	0.98	0.96	0.98	0.99	0.02	0.98	0.99
Employment	0.85	0.98	0.98	0.97	0.97	0.54	0.98	0.85
Investment	0.93	0.97	0.91	0.98	0.98	-0.11	0.97	0.99
Net exports	-0.49	-0.98	-0.94	-0.99	-0.99	0.02	-0.98	-1.00
<i>Terms of trade with</i>								
Net exports	-0.17	-0.06	-0.25	-0.06	-0.07	0.87	-0.11	-0.03
<i>B. Standard deviations</i>								
<i>relative to GDP<sup>d</sup></i>								
GDP	1.33	0.95	0.93	1.01	1.01	0.92	0.95	1.65
<i>Measured TFP</i>								
Consumption	0.60	0.84	0.86	0.79	0.79	0.87	0.84	0.48
Investment	0.74	0.63	0.71	0.53	0.54	1.53	0.65	0.58
Employment	2.79	2.79	2.79	2.79	2.79	2.79	2.79	2.79
Net exports	0.81	0.25	0.22	0.33	0.33	1.18	0.26	1.24
Net exports	0.30	0.99	1.06	1.00	1.01	1.75	1.01	1.02

<sup>a</sup>All reported statistics are based on logged and Hodrick-Prescott filtered quarterly time series (with a smoothing parameter  $\lambda = 1600$ ).

<sup>b</sup>US data for the period 1980:1-2004:1.

<sup>c</sup>Calculated using the actual national accounting formulas; due to time varying markups measured TFP slightly differs from the TFP coefficient fed into the model.

<sup>d</sup>Ratio of corresponding standard deviation to the standard deviation of *GDP*.

Table 10: International Prices: Comovement and Relative Volatility<sup>a</sup>, baseline simulation with just productivity shocks.

Statistic	Predictions of the PTM Theories									
	Data <sup>b</sup>	Baseline Model	Kimball Model	Costly Distribution	Price Dispersion	Nested CES	Deep Habits	Customer Capital		
<b>A. Correlations</b>										
$p_f, x$	0.69	1.00	1.00	1.00	1.00	1.00	1.00	0.79		
$p_d, x$	-0.18	-1.00	-1.00	-1.00	-1.00	-1.00	-0.99	-0.03		
$p_f/p_x, x$	0.61	1.00	-1.00	1.00	1.00	1.00	1.00	-0.74		
$x, x_{-1}$	0.90	0.72	0.72	0.73	0.73	0.73	0.73	0.90		
<b>B. Standard Deviations relative to <math>x^c</math></b>										
$x$	3.60	1.35	0.43	1.20	1.18	0.07	1.48	0.58		
$p_f$	0.61	1.16	0.40	0.87	0.70	0.63	1.17	0.44		
$p_d$	0.13	0.16	0.05	0.12	0.10	0.09	0.16	0.27		
$p_f/p_x$	0.41	1.32	0.20	0.74	0.40	0.25	1.33	0.42		
<b>D. X-Rate Pass-through</b>										
	0.40	1.16	0.40	0.87	0.70	0.63	1.17	0.35		
<b>E. Markup</b>										
	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50		
<b>F. Trade Elast. and <math>\text{corr}(f/D, p_D/p_f)</math></b>										
Correlation	0.02	1.00	1.00	1.00	1.00	1.00	1.00	0.18		
Volatility Ratio	0.70	0.70	3.00	0.70	0.70	7.90	0.70	0.31		

<sup>a</sup>All reported statistics are based on logged and Hodrick-Prescott filtered quarterly time series (with a smoothing parameter  $\lambda = 1600$ ).

<sup>b</sup>Data for the US, 1980:1-2004:1.

<sup>c</sup>Ratio of corresponding standard deviation to the standard deviation of the real exchange rate  $x$ .

Table 11: Quantities: Comovement and Relative Volatility<sup>a</sup>, baseline simulation with just productivity shocks.

Statistic	Predictions of the PTM Theories							
	Data <sup>b</sup>	Baseline Model	Kimball Model	Costly Distribution	Price Dispersion	Nested CES	Deep Habits	Customer Capital
<i>A. Correlations</i>								
<i>domestic with foreign</i>								
<i>Measured TFP<sup>c</sup></i>	0.30	0.30	0.30	0.33	0.32	0.56	0.30	0.69
GDP	0.40	0.34	0.36	0.43	0.42	0.59	0.34	0.73
Consumption	0.25	0.71	0.36	0.55	0.43	0.58	0.75	0.55
Employment	0.21	0.80	0.95	0.93	0.88	-0.99	0.73	0.55
Investment	0.23	0.64	0.45	0.59	0.50	0.62	0.67	0.59
<i>GDP with</i>								
Consumption	0.83	0.98	1.00	1.00	1.00	1.00	0.97	0.99
Employment	0.85	0.95	0.73	0.92	0.93	-0.38	0.96	0.85
Investment	0.93	0.98	1.00	0.99	0.99	1.00	0.98	0.99
Net exports	-0.49	-0.98	-1.00	-1.00	-1.00	-1.00	-0.97	-0.99
<i>Terms of trade with</i>								
Net exports	-0.17	-0.39	0.56	-0.46	-0.52	-0.45	-0.37	0.11
<i>B. Standard deviations</i>								
<i>relative to GDP<sup>d</sup></i>								
GDP	1.33	0.86	0.82	0.90	0.92	0.72	0.86	1.46
<i>Measured TFP</i>								
Consumption	0.60	0.93	0.97	0.88	0.86	1.01	0.93	0.48
Investment	0.74	0.72	0.88	0.65	0.63	1.00	0.75	0.63
Employment	2.79	1.70	1.51	2.11	2.30	1.00	1.66	2.72
Net exports	0.81	0.11	0.07	0.20	0.23	0.02	0.12	1.34
	0.30	0.89	0.99	0.96	0.98	1.00	0.90	1.05

<sup>a</sup>All reported statistics are based on logged and Hodrick-Prescott filtered quarterly time series (with a smoothing parameter  $\lambda = 1600$ ).

<sup>b</sup>US data for the period 1980:1-2004:1.

<sup>c</sup>Calculated using the actual national accounting formulas; due to time varying markups measured TFP slightly differs from the TFP coefficient fed into the model.

<sup>d</sup>Ratio of corresponding standard deviation to the standard deviation of *GDP*.

Table 12: International Prices: Comovement and Relative Volatility<sup>a</sup>, baseline simulation with just financial shocks.

Statistic	Predictions of the PTM Theories							
	Data <sup>b</sup>	Baseline Model	Kimball Model	Costly Distribution	Price Dispersion	Nested CES	Deep Habits	Customer Capital
<b>A. Correlations</b>								
$p_f, x$	0.69	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$p_d, x$	-0.18	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-0.96
$p_f/p_x, x$	0.61	1.00	-1.00	1.00	1.00	1.00	1.00	-0.99
$x, x_{-1}$	0.90	0.72	0.71	0.72	0.71	0.71	0.72	0.66
<b>B. Standard Deviations relative to <math>x^c</math></b>								
$x$	3.60	3.34	3.57	3.39	3.40	3.60	3.28	3.55
$p_f$	0.61	1.16	0.40	0.87	0.70	0.63	1.19	0.36
$p_d$	0.13	0.16	0.05	0.12	0.10	0.09	0.17	0.03
$p_f/p_x$	0.41	1.32	0.20	0.74	0.40	0.25	1.39	0.29
<b>D. X-Rate Pass-through</b>								
	0.40	1.16	0.40	0.87	0.70	0.63	1.19	0.36
<b>E. Markup</b>								
	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
<b>F. Trade Elast. and <math>\text{corr}(f/D, p_D/p_f)</math></b>								
Correlation	0.02	1.00	1.00	1.00	1.00	1.00	1.00	-0.12
Volatility Ratio	0.70	0.70	3.00	0.70	0.70	7.90	0.70	0.71

<sup>a</sup>All reported statistics are based on logged and Hodrick-Prescott filtered quarterly time series (with a smoothing parameter  $\lambda = 1600$ ).

<sup>b</sup>Data for the US, 1980:1-2004:1.

<sup>c</sup>Ratio of corresponding standard deviation to the standard deviation of the real exchange rate  $x$ .

Table 13: Quantities: Comovement and Relative Volatility<sup>a</sup>, baseline simulation with just financial shocks.

Statistic	Predictions of the PTM Theories							
	Data <sup>b</sup>	Baseline Model	Kimball Model	Costly Distribution	Price Dispersion	Nested CES	Deep Habits	Customer Capital
<i>A. Correlations</i>								
<i>domestic with foreign</i>								
<i>Measured TFP<sup>c</sup></i>	0.30	0.53	0.97	-1.00	-1.00	-1.00	-0.67	-1.00
GDP	0.40	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00
Consumption	0.25	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00
Employment	0.21	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00
Investment	0.23	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00
<i>GDP with</i>								
Consumption	0.83	-0.52	-0.94	-0.61	-0.40	-0.99	-0.36	0.93
Employment	0.85	0.54	0.96	-0.79	-0.65	0.99	0.54	0.96
Investment	0.93	-0.56	-0.94	-0.72	-0.57	-0.97	-0.40	0.89
Net exports	-0.49	0.54	0.94	0.68	0.51	0.99	0.37	-0.91
<i>Terms of trade with</i>								
Net exports	-0.17	1.00	-1.00	0.99	0.99	1.00	0.99	-0.91
<i>B. Standard deviations</i>								
<i>relative to GDP<sup>d</sup></i>								
GDP	1.33	0.07	0.25	0.07	0.05	1.60	0.07	0.49
<i>Measured TFP</i>								
Consumption	0.60	0.00	0.00	1.87	2.11	0.21	0.00	0.78
Investment	0.74	7.73	3.41	4.46	4.71	2.05	7.92	0.44
Employment	2.79	25.82	8.34	24.10	29.82	3.03	26.05	3.37
Net exports	0.81	0.81	1.45	1.86	2.50	1.85	0.57	0.30
	0.30	10.82	4.26	8.63	10.03	2.22	10.98	1.04

<sup>a</sup>All reported statistics are based on logged and Hodrick-Prescott filtered quarterly time series (with a smoothing parameter  $\lambda = 1600$ ).

<sup>b</sup>US data for the period 1980:1-2004:1.

<sup>c</sup>Calculated using the actual national accounting formulas; due to time varying markups measured TFP slightly differs from the TFP coefficient fed into the model.

<sup>d</sup>Ratio of corresponding standard deviation to the standard deviation of *GDP*.

# Online Appendix

## Pricing-to-Market in Business Cycle Models\*

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### Abstract

This document contains the Online Appendix for the paper “**Pricing-to-Market in Business Cycle Models.**”

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# 1 Omitted Analytic Details from Section 3

This appendix provides detailed derivations of the expressions used in Section 3 of the paper.

The key defining object is the partial differentiation operator  $\partial_{\log x}(\cdot|_{ss})$  used in the definitions of TE and PT. This operator focuses on the direct impact of a (one-time) i.i.d. exchange rate shock and treats general equilibrium prices as constant. As stated in the text: 1) The pass-through coefficient describes the log-linear coefficient on the reaction function of an importer who assumes that all other importers respond to the shock but none of the aggregate local or foreign prices/marginal costs change. 2) Trade elasticity considers the reaction function of a representative (atomless) final good producer who assumes that all importers from whom it sources imported goods raise prices symmetrically, and none of the local producers change their price. The goal is to preserve the decision rule of the optimizing price setter, while assuming away all general equilibrium channels that may affect decisions indirectly.

We start by deriving the demand system for the CES baseline model. This part is standard, but the derivation here is helpful in the context of the next setup: the KA model.

## 1.1 Frictionless Baseline (FB)

**Preliminaries:** Since the final sector is consolidated into a single representative competitive firm, the core of the problem is the cost minimization problem of “producing”  $A$  units of a homogeneous final good (consumption/investment good):

$$P(s^t) A(s^t) = \min_{d_i, f_i} \int_0^1 [p_d(i, s^t) d(i, s^t) + p_f(i, s^t) f(i, s^t)] \quad (1)$$

subject to

$$A(s^t) = \underbrace{\left[ \omega^{\frac{1}{\gamma}} \left( \underbrace{\left[ \int_0^1 d(i, s^t)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}}_{d(s^t)} \right)^{\frac{\gamma-1}{\gamma}} + (1-\omega)^{\frac{1}{\gamma}} \left( \underbrace{\left[ \int_0^1 f(i, s^t)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}}_{f(s^t)} \right)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}}_{G(d(s^t), f(s^t))}. \quad (2)$$

Given constant returns to scale aggregation, it is convenient to rewrite the problem in terms of shares

$$s_d(i, s^t) = \frac{d(i, s^t)}{\omega A(s^t)}, \quad s_f(i, s^t) = \frac{f(i, s^t)}{(1-\omega) A(s^t)}, \quad (3)$$

which gives the following:

$$1 \equiv P(s^t) = \min_{s_{fi}, s_{di}} \int_0^1 [p_d(i, s^t) \omega s_d(i, s^t) + p_f(i, s^t) (1-\omega) s_f(i, s^t)] \quad (4)$$

subject to

$$\left[ \omega \left( \left[ \int_0^1 s_d(i, s^t)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \right)^{\frac{\gamma-1}{\gamma}} + (1-\omega) \left( \left[ \int_0^1 s_f(i, s^t)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \right)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}} = 1, \quad (5)$$

where  $\lambda(s^t)$  is the Lagrange multiplier imposed on the above constraint and  $P(s^t) \equiv 1$  by the numeraire normalization introduced in the text. (Throughout, we use “ $\equiv$ ” to indicate an identity.)

To avoid the confusion with differentiating integrals, we consider a perturbation from the optimum towards some arbitrary perturbation function  $h$ :  $s_d(i, s^t) + \varepsilon_d(i, s^t) h_d(i, s^t)$ , where  $s_d(i, s^t)$  is the optimum and  $\varepsilon_d(i, s^t) \in \mathbb{R}$  is the perturbation scale parameter (same for good  $f$ ). Note that optimality implies that there is no perturbation function that lowers the value of the program, which means that there is no gradient of improvement at  $\varepsilon = 0$ . To derive the first order conditions, we differentiate the following:

$$\min_{\varepsilon_{fi}, \varepsilon_{di}} \int_0^1 [p_d(i, s^t) \omega (s_d(i, s^t) + \varepsilon_{di} h_d(i, s^t)) + p_f \dots]. \quad (6)$$

s.t.

$$(\lambda(s^t)) \left[ \omega \left( \left[ \int_0^1 (s_d(i, s^t) + \varepsilon_{di} h_d(i, s^t))^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \right)^{\frac{\gamma-1}{\gamma}} + \dots \right]^{\frac{\gamma}{\gamma-1}} = 1. \quad (7)$$

Taking the derivative with respect to  $\varepsilon_{di} \equiv \varepsilon_d(i, s^t) \in \mathbb{R}$  and evaluating it around at  $\varepsilon_{di} = 0$  gives:

$$\begin{aligned} \varepsilon_{di} : \int_0^1 \omega p_d(i, s^t) \omega h_d(i, s^t) di \\ = \lambda(s^t) \omega \left[ \int_0^1 (s_d(\cdot))^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}-1} \left( \left[ \int_0^1 (s_d(\cdot))^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \right)^{\frac{-1}{\gamma}} \int_0^1 s_d(\cdot)^{\frac{-1}{\theta}} h_d(i, s^t) di. \end{aligned}$$

Let  $h_d(i, s^t)$  be a Dirac-delta function that spikes at an arbitrary point. We drop the last integral, divide by  $h(\cdot)$  on both sides, and obtain the first-order condition:

$$\begin{aligned} \varepsilon_{di} : \omega p_d(i, s^t) = \\ = \lambda(s^t) \omega s_d(i, s^t)^{\frac{-1}{\theta}} \left[ \int_0^1 (s_d(i, s^t))^{\frac{\theta-1}{\theta}} di \right]^{\frac{1}{\theta-1}} \left( \left[ \int_0^1 (s_d(i, s^t))^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \right)^{\frac{-1}{\gamma}}. \end{aligned}$$

An analogous condition applies to good  $f$  and the remaining first-order condition is the constraint in (5).

Plugging in for  $s_d$  from (3) and using the fact that, by definition, we know

$$d(s^t) := \left( \int_0^1 (d(i, s^t))^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}},$$

and hence

$$p_d(i, s^t) = \lambda(s^t) s_d(i, s^t)^{\frac{-1}{\theta}} d(s^t)^{\frac{1}{\theta} - \frac{1}{\gamma}} \omega^{\frac{1}{\gamma} - \frac{1}{\theta}} A(s^t)^{\frac{1}{\gamma} - \frac{1}{\theta}},$$

and

$$p_d(i, s^t) = \omega^{\frac{1}{\gamma}} \lambda(s^t) d(i, s^t)^{\frac{-1}{\theta}} d(s^t)^{\frac{1}{\theta} - \frac{1}{\gamma}} A(s^t)^{\frac{1}{\gamma}}.$$

The above equations imply the following state-contingent demand function (note: notation  $d(p_d|i, s^t)$  indicates that the stated object is a  $i, s^t$ -specific function of variable  $p_d \in \mathbb{R}$ ):

$$d(p_d|i, s^t) = \omega^{\frac{\theta}{\gamma}} \left( \frac{p_d(i, s^t)}{\lambda(s^t)} \right)^{-\theta} d(s^t)^{1 - \frac{\theta}{\gamma}} A(s^t)^{\frac{\theta}{\gamma}}. \quad (8)$$

In terms of the shares, we express demand as follows:

$$s_d(p_d|i, s^t) := \frac{d(p_d|i, s^t)}{\omega A(s^t)} = \left( \frac{p_d(i, s^t)}{\lambda(s^t)} \right)^{-\theta} \omega^{\frac{\theta}{\gamma} - 1} d(s^t)^{1 - \frac{\theta}{\gamma}} A(s^t)^{\frac{\theta}{\gamma} - 1}.$$

(Analogous conditions apply to good  $f$  but with  $1 - \omega$  replacing  $\omega$ .)

It is easy to verify by plugging in

$$G_d(d, f) = d^{\frac{-1}{\gamma}} \left[ \omega^{\frac{1}{\gamma}} d^{\frac{\gamma-1}{\gamma}} + (1 - \omega)^{\frac{1}{\gamma}} f^{\frac{\gamma-1}{\gamma}} \right]^{\frac{1}{\gamma-1}}$$

that the demand expression stated in the text is identical to the one derived above, since

$$\begin{aligned} \left( \frac{p_d(i, s^t)}{G_d(s^t)} \right)^{-\theta} d(s^t) &= \omega^{\frac{\theta}{\gamma}} p_d(i, s^t)^{-\theta} d(s^t)^{1 - \frac{\theta}{\gamma}} \left[ \omega^{\frac{1}{\gamma}} d(s^t)^{\frac{\gamma-1}{\gamma}} + (1 - \omega)^{\frac{1}{\gamma}} f(s^t)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\theta}{\gamma-1}} \\ &= \omega^{\frac{\theta}{\gamma}} p_d(i, s^t)^{-\theta} d(s^t)^{1 - \frac{\theta}{\gamma}} A(s^t)^{\frac{\gamma-1}{\gamma}}. \end{aligned}$$

To close the system, we add the price index equation. Since

$$\begin{aligned} d(s^t) &:= \left( \int_0^1 (d(i, s^t))^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} = \left( \int_0^1 \left( \omega^{\frac{\theta}{\gamma}} \left( \frac{p_d(i, s^t)}{\lambda(s^t)} \right)^{-\theta} A(s^t)^{\frac{\theta}{\gamma}} \right)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\gamma}{\theta-1}} \\ &= \omega \lambda(s^t)^{\gamma} A(s^t) \left( \int_0^1 p_d(i, s^t)^{1-\theta} di \right)^{\frac{\gamma}{\theta-1}}, \end{aligned}$$

it is clear that

$$\frac{d(s^t)}{A(s^t)} = \omega \lambda(s^t)^{\gamma} \left( \int_0^1 p_d(i, s^t)^{1-\theta} di \right)^{\frac{\gamma}{\theta-1}},$$

and that (5) gives

$$\left[ \omega^{\frac{1}{\gamma}} \left( \frac{d(s^t)}{A(s^t)} \right)^{\frac{\gamma-1}{\gamma}} + (1 - \omega)^{\frac{1}{\gamma}} \left( \frac{f(s^t)}{A(s^t)} \right)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}} = 1.$$

Accordingly,

$$\lambda(s^t) = \left[ \omega \left( \int_0^1 p_d(i, s^t)^{1-\theta} di \right)^{\frac{1-\gamma}{1-\theta}} + (1-\omega) \left( \int_0^1 p_f(i, s^t)^{1-\theta} di \right)^{\frac{\gamma-1}{\theta-1}} \right]^{\frac{1}{1-\gamma}}. \quad (9)$$

This completes the characterization of this system, since we know that, by constant returns to scale and the envelope theorem,  $\lambda(s^t) \equiv P(s^t) \equiv 1$ . However, since this step will be needed in the next model, we derive it explicitly. To that end, we consider the objective function in the cost minimization problem above and plug in for  $s_d(i, s^t)$ ,  $s_f(i, s^t)$  from the first order conditions to derive

$$1 \equiv P(s^t) = \int_0^1 \left[ \omega^{\frac{\theta}{\gamma}} p_d(i, s^t)^{1-\theta} \lambda(s^t)^\theta \left( \frac{d(s^t)}{A(s^t)} \right)^{1-\frac{\theta}{\gamma}} + (1-\omega)^{\frac{\theta}{\gamma}} p_f(i, s^t)^{1-\theta} \lambda(s^t)^\theta \left( \frac{f(s^t)}{A(s^t)} \right)^{1-\frac{\theta}{\gamma}} \right].$$

Plugging in for  $d(s^t)/A(s^t)$ ,  $f(s^t)/A(s^t)$  as above, we have

$$1 \equiv \lambda(s^t)^\theta \int_0^1 \left[ \omega^{\frac{\theta}{\gamma}} p_d(i, s^t)^{1-\theta} \left( \frac{d(s^t)}{A(s^t)} \right)^{1-\frac{\theta}{\gamma}} + (1-\omega)^{\frac{\theta}{\gamma}} p_f(i, s^t)^{1-\theta} \left( \frac{f(s^t)}{A(s^t)} \right)^{1-\frac{\theta}{\gamma}} \right],$$

and hence

$$1 \equiv \lambda(s^t)^\gamma \left[ \omega \left( \int_0^1 p_d(i, s^t)^{1-\theta} di \right)^{\frac{\gamma-\theta}{\theta-1}} \int_0^1 p_d(i, s^t)^{1-\theta} + \dots \right].$$

By (9), we obtain

$$1 \equiv \lambda(s^t)^\gamma \left[ \omega \left( \int_0^1 p_d(i, s^t)^{1-\theta} di \right)^{\frac{\gamma-1}{\theta-1}} + \dots \right] = \lambda(s^t)^\gamma \lambda(s^t)^{1-\gamma} = \lambda(s^t). \quad (10)$$

In the deterministic symmetric steady state, note that  $p_d^{ss} = p_f^{ss} = 1$  and  $d^{ss} = f^{ss} = A^{ss}$ . The pricing equation implies that the gross profit margin and the markup in this model are given by

$$\mu^{ss} = \frac{\theta}{\theta-1} - 1. \quad (11)$$

We next derive PT and TE as defined in the text.

**PT:** Consider the formula for the import price derived above. Under the stated assumptions for the partial differentiation operator  $\partial_{\log x}(\cdot|_{ss})$  (discussed above), we need to differentiate the expression:

$$p_f(x|i, s^t)|_{ss} = \frac{\theta}{\theta-1} \frac{x(s^t)}{v^*(s^t)}|_{ss},$$

where  $v^*(s^t) = v^*$  is treated as constant under  $\partial_{\log x}(\cdot|_{ss})$ . We obtain the expected result:

$$PT := \partial_{\log x} \log p_f(x|i, s^t)|_{ss} = x \frac{\partial_x p_f(x|i, s^t)}{p_f(x|i, s^t)}|_{ss} = 1. \quad (12)$$

**TE:** The definition of trade elasticity in the text is

$$TE := \frac{\partial_{\log x} \log \frac{d(s^t)}{f(s^t)}|_{ss}}{PT}, \quad (13)$$

where  $d(s^t), f(s^t)$  are quantities chosen by the representative (small) final good producer who faces a symmetric shock (recall that all importers from whom this producer buys goods raise the price, none of the domestic producers do and marginal cost is assumed constant under the linearization operator). We assume the import price that the importer faces is  $p_f(x|i, s^t)$  and the home price is  $p_d(x, s^t) \equiv p_d$ . The relevant equation characterizing the pricing policy of the final goods producer is thus given by

$$\frac{G_d(d(p_d|i, s^t), f(p_f(x, s^t)|i, s^t))}{G_f(d(p_d|i, s^t), f(p_f(x, s^t)|i, s^t))} \left( \frac{d(p_d|i, s^t)}{d(s^t)} \frac{f(p_f(x|s^t)|i, s^t)}{f(s^t)} \right)^{-\frac{1}{\theta}} = \frac{p_d}{p_f(x, s^t)}$$

where  $d(p_d|i, s^t), f(p_f(x, s^t)|i, s^t)$  are the demand functions derived above. Plugging for  $G$  from (5) and using symmetry  $p_f(x|s^t) \equiv p_f(x|i, s^t)$ , we obtain

$$\left( \frac{d(p_d|s^t)}{f(p_f(x|s^t)|s^t)} \right)^{-\frac{1}{\gamma}} = \frac{p_d}{p_f(x|s^t)} \Leftrightarrow \frac{d(s^t)}{f(s^t)} = \left( \frac{p_d}{p_f(x|s^t)} \right)^{-\gamma},$$

and hence, as claimed in text,

$$TE := \frac{\partial_{\log x} \log \frac{d(s^t)}{f(s^t)}|_{ss}}{PT} = \frac{\partial_{\log x} \log \left( \frac{p_d}{p_f(x|s^t)} \right)^{-\gamma}|_{ss}}{PT} = \frac{\gamma \partial_{\log x} \log (p_f(x|i, s^t))|_{ss}}{PT} = \gamma. \quad (14)$$

## 1.2 KA Model

**Preliminaries:** We begin by deriving the KA demand system, following the same steps as above. We similarly rewrite the problem in terms of shares:

$$P(s^t) = \min_{s_{fi}, s_{di}} \int_0^1 [p_d(i, s^t) \omega s_d(i, s^t) + p_f(i, s^t) (1 - \omega) s_f(i, s^t)] \quad (15)$$

s.t.

$$\int_0^1 [\omega g(s_d(i, s^t)) + (1 - \omega) g(s_f(i, s^t))] di = 1, \quad (\lambda(s^t)), \quad (16)$$

where  $\lambda(s^t)$  is the Lagrange multiplier associated with the constraint and shares  $s_d, s_f$  which we defined in (3). However, unlike in the CES model, the function  $g$  is not homogeneous of degree one, and it is no longer true that  $\lambda(s^t)$  is  $P(s^t) \equiv 1$ . To preserve the problem of the optimizing entity we need to consider how this object is affected by the exchange rate. This is the key complication relative to the baseline model.

It is easy to verify that the first-order conditions give

$$s_d : p_d(i, s^t) = \lambda(s^t) g'(s_d(i, s^t)) \quad (17)$$

$$\rightarrow s_d(i, s^t) = h\left(\frac{p_d(i, s^t)}{\lambda(s^t)}\right) \quad (18)$$

$$s_f : p_f(i, s^t) = \lambda(s^t) g'(s_f(i, s^t)) \quad (19)$$

$$\rightarrow s_f(i, s^t) = h\left(\frac{p_f(i, s^t)}{\lambda(s^t)}\right) \quad (20)$$

$$\lambda : 1 = \int_0^1 [\omega g(s_d(i, s^t)) + (1 - \omega) g(s_f(i, s^t))] di \quad (21)$$

where, as in the text, we define  $h(\cdot) := g'^{-1}(\cdot)$ . For later use, note that the inverse function theorem implies

$$h'(x)|_{x=1} = \left((g')^{-1}\right)'(x)|_{x=1} = \left(g''\left((g')^{-1}(x)\right)\right)^{-1}|_{x=1} = \frac{1}{g''(1)}, \quad (22)$$

and so in a linearized system  $g''(1)$  is the key parameter that summarizes the curvature of the demand function (around the deterministic steady state).

Plugging in for shares, the KA model's demand system is given by

$$d(p_d|i, s^t) = \omega h\left(\frac{p_d}{\lambda(s^t)}\right) A(s^t), \quad (23)$$

$$f(p_f|i, s^t) = (1 - \omega) h\left(\frac{p_f}{\lambda(s^t)}\right) A(s^t). \quad (24)$$

As in the baseline model, the Lagrange multiplier is implicitly defined by the constraint, which gives

$$1 = \int_0^1 \left[ \omega g\left(h\left(\frac{p_d(i, s^t)}{\lambda(s^t)}\right)\right) + (1 - \omega) g\left(h\left(\frac{p_f(i, s^t)}{\lambda(s^t)}\right)\right) \right] di. \quad (25)$$

Furthermore, plugging into the objective function, we know

$$1 \equiv P(s^t) = \int_0^1 \left[ p_d(i, s^t) \omega h\left(\frac{p_d(i, s^t)}{\lambda(s^t)}\right) + p_f(i, s^t) (1 - \omega) h\left(\frac{p_f(i, s^t)}{\lambda(s^t)}\right) \right]. \quad (26)$$

Together, the above two equations define  $\lambda(s^t)$ . In the symmetric steady state,  $p_d^{ss}(i) = p_f^{ss}(i) = p^{ss}$  and normalization in the text implies  $g(1) = g'(1) = 1$  and  $h(1) = 1$ ; hence,  $\lambda^{ss} = P^{ss} = 1$ ,  $p^{ss}/\lambda^{ss} = 1$ . Furthermore, since  $f^{ss} = (1 - \omega) h(1) A^{ss} = (1 - \omega) A^{ss}$ , the share of the foreign good in expenditures is  $1 - \omega$  in  $A^{ss}$ .

For later use, it is convenient to define the elasticity of demand for an individual good  $i$ :

$$\gamma\left(\frac{p_d}{\lambda}\right) := -\frac{\partial \log d\left(\frac{p_d}{\lambda}|i, s^t\right)}{\partial \log p_d} = -\frac{\partial\left(d\left(\frac{p_d}{\lambda}|i, s^t\right)\right)}{\partial p_d} \frac{p_d}{d(p_d|i, s^t)}, \quad (27)$$

which, by (23), in the steady state implies

$$\gamma(1) = \gamma\left(\frac{p_d}{\lambda}\right)|_{ss} = -h'\left(\frac{p_d}{\lambda(s^t)}\right) \frac{1}{\lambda} \frac{p_d}{h\left(\frac{p_d}{\lambda}\right)}|_{ss} = \frac{h'(1)}{h(1)} = \frac{1}{g''(1)}. \quad (28)$$

Accordingly, as noted above and in text,  $g''(1)$  is the key parameter of the log-linearized equilibrium system. It determines the (local) curvature of the demand function and hence varying demand elasticity. The standard monopolistic pricing formula implies that

$$p_f(i, s^t) = x(s^t) v^*(s^t) \frac{\gamma\left(\frac{p_f(i, s^t)}{\lambda(s^t)}\right)}{\gamma\left(\frac{p_f(x, s^t)}{\lambda(s^t)}\right) - 1}. \quad (29)$$

We omit the explicit derivation of this pricing formula as it immediately follows from the producer problem.

**PT:** As noted, the partial differentiation operator  $\partial_{\log x}(\cdot|_{ss})$  assumes invariance of the price index and home variables with respect to the exchange rate. However, as also noted above,  $\lambda(x|s^t)$  is endogenous and hence depends on  $x$ . This is consistent with the idea that we want to capture the policy function of the affected price setter.

The relevant pricing formula for the operator is equation (29):

$$p_f(x|i, s^t) = xv^*(s^t) \frac{\gamma\left(\frac{p_f(x|i, s^t)}{\lambda(x, s^t)}\right)}{\gamma\left(\frac{p_f(x|i, s^t)}{\lambda(x|s^t)}\right) - 1},$$

where  $v^*(s^t)$  is assumed independent of  $x$  under the operator  $\partial_{\log x}(\cdot|_{ss})$ . Under symmetry

$$(p_f(x|i, s^t) \equiv p_f(x|s^t)),$$

and so the Kimball aggregator after plugging in for prices gives

$$1 \equiv \int_0^1 \left[ \omega g\left(h\left(\frac{p_d}{\lambda(x, s^t)}\right)\right) + (1 - \omega) g\left(h\left(\frac{p_f(x|s^t)}{\lambda(x, s^t)}\right)\right) \right] di.$$

Differentiating the above expression under the rules of the operator  $\partial_{\log x}(\cdot|_{ss})$  yields

$$\begin{aligned} 0 &\equiv \int_0^1 \left[ \omega g'\left(h\left(\frac{p_d}{\lambda(x|s^t)}\right)\right) h'\left(\frac{p_d}{\lambda(x|s^t)}\right) \frac{p_d}{\lambda(x|s^t)^2} \partial_{\log x} \lambda(x|s^t) \right] di \\ &\quad - \int_0^1 \left[ (1 - \omega) g'\left(h\left(\frac{p_f(x|s^t)}{\lambda(x|s^t)}\right)\right) h'\left(\frac{p_f(x|s^t)}{\lambda(x|s^t)}\right) \frac{\lambda(x, s^t) \partial_{\log x} p_f(x|s^t) - p_f(x|s^t) \partial_{\log x} \lambda(x|s^t)}{\lambda(x|s^t)^2} \right] di, \end{aligned}$$

and hence

$$\begin{aligned} 0 &\equiv \int_0^1 \left[ \omega g'\left(h\left(\frac{p_d}{\lambda(x|s^t)}\right)\right) h'\left(\frac{p_d}{\lambda(x|s^t)}\right) \frac{p_d}{\lambda(x|s^t)} \partial_{\log x} \log \lambda(x|s^t) \right] di \\ &\quad - \int_0^1 \left[ (1 - \omega) g'\left(h\left(\frac{p_f(x|s^t)}{\lambda(x|s^t)}\right)\right) h'\left(\frac{p_f(x|s^t)}{\lambda(x|s^t)}\right) \frac{p_f(x|s^t)}{\lambda(x|s^t)} (\partial_{\log x} \log p_f(x|s^t) - \partial_{\log x} \log \lambda(x|s^t)) \right] di. \end{aligned}$$

Evaluating the above expression around the steady state implies

$$0 \equiv \int_0^1 [\omega g''(1) \partial_{\log x} \lambda(x|s^t)|_{ss}] di \\ - \int_0^1 [(1-\omega) g''(1) (\partial_{\log x} \log p_f(x|s^t)|_{ss} - \partial_{\log x} \log \lambda(x|s^t)|_{ss})] di.$$

Since the expression under the integral does not depend on  $i$ , we obtain

$$\omega \partial_{\log x} \log \lambda(x|s^t)|_{ss} \equiv (1-\omega) (\partial_{\log x} \log p_f(x|s^t)|_{ss} - \partial_{\log x} \log \lambda(x|s^t)|_{ss})$$

and thus

$$\partial_{\log x} \log \lambda(x|s^t)|_{ss} \equiv (1-\omega) \partial_{\log x} \log p_f(x|s^t)|_{ss}.$$

Accordingly, PT coefficient is

$$PT = \partial_{\log x} \log p_f(x|i, s^t)|_{ss} = \partial_{\log x} \log \left( xv^*(s^t) \frac{\gamma\left(\frac{p_f(x|i, s^t)}{\lambda(x|s^t)}\right)}{\gamma\left(\frac{p_f(x|i, s^t)}{\lambda(x|s^t)}\right) - 1} \right)$$

which simplifies to

$$PT = \partial_{\log x} \log p_f(x|i, s^t) = \frac{(\partial_x p_f(x|i, s^t))}{dx} \frac{x}{p_f(x|i, s^t)} + 1 = \partial_{\log x} \log \frac{\gamma\left(\frac{p_f(x|i, s^t)}{\lambda(x|s^t)}\right)}{\gamma\left(\frac{p_f(x|i, s^t)}{\lambda(x|s^t)}\right) - 1} + 1 = \\ = 1 + \frac{\gamma'(\cdot)(\gamma(\cdot) - 1) - \gamma(\cdot)\gamma'(\cdot)(\gamma(\cdot) - 1)}{(\gamma(\cdot) - 1)^2} \frac{x}{\gamma(\cdot)} \frac{\lambda(\cdot)\partial_{\log x} p_f(\cdot) - p_f(\cdot)\partial_{\log x} \lambda(\cdot)}{\lambda(\cdot)^2}.$$

To simplify it further, we use the fact that  $\partial_{\log x} (\lambda(\cdot)|_{ss}) \equiv (1-\omega) \partial_{\log x} p_f(\cdot)$  (derived above) and obtain

$$\partial_{\log x} \log p_f(x|i, s^t) = 1 + \left( \frac{p_f(\cdot)}{\lambda(\cdot)} (\partial_{\log x} \log p_f(\cdot) - \partial_{\log x} \log \lambda(\cdot)) \right) |_{ss} \\ \times \frac{\gamma'(1)(\gamma(1) - 1) - \gamma(1)\gamma'(1)(\gamma(1) - 1)}{(\gamma(1) - 1)^2} \frac{1}{\gamma(1)}$$

hence

$$\partial_{\log x} \log p_f(x|i, s^t)|_{ss} = 1 + \frac{\gamma'(1)(\gamma(1) - 1) - \gamma(1)\gamma'(1)}{(\gamma(1) - 1)\gamma(1)} \omega \partial_{\log x} \log p_f(x|i, s^t)|_{ss}, \\ 1 = \frac{1}{\partial_{\log x} \log p_f(x|i, s^t)|_{ss}} + \frac{\gamma'(1)(\gamma(1) - 1) - \gamma(1)\gamma'(1)}{(\gamma(1) - 1)\gamma(1)} \omega,$$

and thus

$$\frac{1}{\partial_{\log x} \log p_f(x|i, s^t)|_{ss}} = \frac{(\gamma(1) - 1)\gamma(1) - \gamma'(1)(\gamma(1) - 1)\omega + \gamma(1)\gamma'(1)\omega}{(\gamma(1) - 1)\gamma(1)}.$$

To conclude, we have shown that

$$PT = \partial_{\log x} \log p_f(x|i, s^t)|_{ss} = \frac{(\gamma(1) - 1)\gamma(1)}{\gamma'(1)\omega + (\gamma(1) - 1)\gamma(1)}. \quad (30)$$

**TE:** Partial differentiation using  $\partial_{\log x}(\cdot)|_{ss}$  treats the home price as a constant. Using symmetry and plugging in  $p_f(x|i, s^t) \equiv p_f(x|s^t)$  for the import price, we thus need to calculate

$$TE := \frac{\partial_{\log x(s^t)} \log \frac{d(s^t)}{f(s^t)}|_{ss}}{PT} = \frac{\partial_{\log x} \log \frac{(1-\omega)h\left(\frac{p_f(x|s^t)}{\lambda(x, s^t)}\right)}{\omega h\left(\frac{p_d}{\lambda(x, s^t)}\right)}|_{ss}}{PT},$$

where, recall,  $h'(x)|_{x=1} = g''(1)$ ,  $h(1) = 1$ , and hence

$$\gamma(1) = \frac{h'(1)}{h(1)} = \frac{1}{g''(1)}.$$

The numerator of the expression for TE gives

$$\partial_{\log x} \log \frac{(1-\omega)h\left(\frac{p_f(x|s^t)}{\lambda(x|s^t)}\right)}{\omega h\left(\frac{p_d}{\lambda(x|s^t)}\right)}|_{ss} = \partial_{\log x} \log h\left(\frac{p_f(x|s^t)}{\lambda(x|s^t)}\right) - \partial_{\log x} \log h\left(\frac{p_d}{\lambda(x|s^t)}\right).$$

Using the fact that  $\partial_{\log x} \lambda(x|s^t)|_{ss} \equiv (1-\omega)\partial_{\log x} p_f(x|s^t)$  and  $h'(x)|_{x=1} = g''(1)^{-1}$ , the two terms on the right-hand side imply

$$\partial_{\log x} \log h\left(\frac{p_f(x|s^t)}{\lambda(x|s^t)}\right)|_{ss} = \frac{h'\left(\frac{p_f(\cdot)}{\lambda(\cdot)}\right)}{h\left(\frac{p_f(\cdot)}{\lambda(\cdot)}\right)} \frac{\partial_{\log x} \log p_f(\cdot) - \partial_{\log x} \log \lambda(\cdot)}{\lambda(\cdot)}|_{ss} = \omega g''(1) \partial_{\log x} \log p_f(x|s^t)$$

which gives

$$\partial_{\log x} \log h\left(\frac{p_d}{\lambda(x|s^t)}\right)|_{ss} = \frac{h'\left(\frac{p_d}{\lambda(\cdot)}\right) - \partial_{\log x} \log \lambda(\cdot)}{h\left(\frac{p_d}{\lambda(\cdot)}\right) \lambda(\cdot)}|_{ss} = -(1-\omega) \partial_{\log x} \log p_f(x|s^t).$$

Accordingly, as claimed in text, we have now shown that

$$TE = \frac{\omega g''(1)^{-1} \partial_{\log x} \log p_f(x|i, s^t) + (1-\omega) g''(1)^{-1} \partial_{\log x} \log p_f(x|i, s^t)}{PT} = g''(1)^{-1} = \gamma(1). \quad (31)$$

### 1.3 CD Model

**Preliminaries:** The pricing equations are derived analogously to the baseline model.

**PT:** As in the baseline model, we differentiate the import price under the assumption of invariant foreign marginal

cost  $v^*$ , which gives

$$\partial_{\log x} \log p_f(x|i, s^t)|_{ss} = \partial_x p_f(\cdot) \frac{x}{p_f(\cdot)}|_{ss} = \frac{\theta}{\theta-1} v^* \frac{x}{\frac{\theta}{\theta-1} x v^* + \frac{\xi}{\theta-1} v}|_{ss} = 1 - \frac{\frac{\xi}{\theta-1}}{\frac{\theta}{\theta-1} + \frac{\xi}{\theta-1}}.$$

Simplifying, we obtain

$$PT = \frac{\frac{\theta}{\theta-1}}{\frac{\theta}{\theta-1} + \frac{\xi}{\theta-1}} = \frac{1 + \mu^{ss} - \frac{\xi}{\theta-1}}{1 + \mu^{ss}} = 1 - \frac{\frac{\xi}{\theta-1}}{1 + \mu^{ss}}, \quad (32)$$

where, using the pricing formulas stated in the text, we know that the markup is given by

$$\mu^{ss} := \frac{p_d^{ss}}{v} - 1 = \frac{\theta}{\theta-1} + \frac{\xi}{\theta-1} - 1. \quad (33)$$

Accordingly, as stated in text, we derive

$$PT = 1 - \frac{\mu^{ss} - \frac{1}{\theta-1}}{1 + \mu^{ss}}, \quad (34)$$

since, by the above,

$$\frac{\xi}{\theta-1} = \mu^{ss} - \frac{\theta}{\theta-1} + 1 = \mu^{ss} - \frac{1}{\theta-1}. \quad (35)$$

**TE:** The final aggregating firm problem is the same as in the baseline model, hence  $TE = \gamma$ .

## 1.4 PD Model

**Preliminaries:** To solve for the key pricing formula, we need to solve the system of equations comprising the conditions discussed in text. To that end, note the following: 1) The distribution  $F(p) := Pr(P \leq p)$  of quoted prices by foreign importers operating in the home country is defined on the interval  $[P_l, P_h]$  and satisfies

$$(p - v^*x)(q + 2(1-q)(1 - F(p))) = (P_h - v^*x)q, \text{ all } p \in [P_l, P_h],$$

where  $v^*x$  is the marginal cost of producing the good; hence

$$F(p) = 1 - q \frac{P_h - p}{2(1-q)(p - v^*x)}. \quad (36)$$

2) The lower bound of the distribution satisfies  $F(P_l) = 0$ , hence

$$F(P_l) = 1 - q \frac{P_h - P_l}{2(1-q)(P_l - v^*x)} = 0 \rightarrow P_l = \frac{qP_h + 2(1-q)v^*x}{2-q}.$$

3) The upper bound  $P_h$  satisfies that the (local) searching entity is indifferent between instructing reps who receive a single highest quote  $P_h$  to abort their purchase and replace them by new reps who purchase at the expected price  $\bar{p}$ , implying

$$\theta v = P_h - \bar{p}, \quad (37)$$

where, here,  $\theta v$  is the home country search cost for a unit of output (recall that each rep brings  $\theta^{-1}$  units). As explained in text, the average price  $\bar{p}$  solves

$$\bar{p} = \int_{P_l}^{P_h} p \frac{dH(p)}{dp} dp \quad (38)$$

where

$$\begin{aligned} H(p) &= qF(p) + (1-q) \left( 1 - (1-F(p))^2 \right) \\ &= \frac{(p(2-q) + P_h q - 2v^*x)(p(2-q) - 2(1-q)v^*x - P_h q)}{4(1-q)(p - v^*x)^2}. \end{aligned}$$

Integrating the above expression by parts gives the average price

$$\bar{p} = v^*x + q(P_h - v^*x) \quad (39)$$

and hence the upper bound

$$\theta v = P_h - \bar{p} \rightarrow P_h = v^*x + \frac{\theta v}{1-q}. \quad (40)$$

The above conditions imply that the import price—the average purchase price  $p_f := \bar{p}$  by a rep of the imported good  $f$  implied by (38)—is

$$p_f = v + q(P_h - v) = v^*x + q \frac{\theta v}{1-q}. \quad (41)$$

**PT:** As in the baseline model and CD model above, we differentiate the import price under the assumption of invariant foreign marginal cost  $v^*$ :

$$\partial_{\log x} \log p_f(x|i, s^t)|_{ss} = \partial_x p_f(\cdot) \frac{x}{p_f(\cdot)}|_{ss} = v^* \frac{x}{xv^* + \frac{\theta q}{1-q}v} = 1 - \frac{\frac{\theta q}{1-q}}{1 + \frac{\theta q}{1-q}}.$$

The steady-state markup is

$$\mu^{ss} := \frac{p_d^{ss}}{v} - 1 = \frac{\theta q}{1-q}, \quad (42)$$

and so

$$PT = 1 - \frac{\mu^{ss}}{1 + \mu^{ss}}. \quad (43)$$

**TE:** As for TE, divide the Armington-demand first-order conditions for goods  $d$  and  $f$  to obtain  $\log(p_f/p_d) = \log((1-\omega)^{1/\gamma}/\omega^{1/\gamma}) - \gamma^{-1} \log(f/d)$  and note that the definition of TE implies  $TE = \gamma$ . Notes: Additional derivations and automated log-linearization of the above expressions can be found in the replication package *Mathematica* notebook TheoryResultsSection3\_CS.nb.

## 1.5 NCES Model

**Preliminaries:** The key here is that a nonatomistic monopolistic producer sells the good to the final goods producer who aggregates goods according to a nested CES: the outer CES aggregates over a continuum of sectors indexed by  $i_1$  and the inner CES aggregates over individual nonatomistic firms within the sector, indexed by  $i_2$ . The aggregator and nonatomistic structure imply that the demand function faced by the producer involves both the sectoral elasticity  $\gamma$  and the within-sector elasticity  $\theta$ , where we assume  $\theta > \gamma, \theta > 1$ .

Let  $\mathbf{i} = (i_1, i_2)$  and assume type-identical allocation / policy (or importers and all domestic producers within a sector are symmetric and make symmetric choices). The profit maximization of an importer is

$$\begin{aligned} \Pi_f(\mathbf{i}, s^t) &= \max_{f(\mathbf{i}, s^t), p_f(\mathbf{i}, s^t)} [p_f(\mathbf{i}, s^t) - (1 + \tau)v^*(s^t) x(s^t)] f(\mathbf{i}, s^t), \\ \text{s.t.} & \\ &\left(\frac{y(\mathbf{i}, s^t)}{Y(s^t)}\right)^{-\frac{1}{\gamma}} = \frac{P_f(\mathbf{i}, s^t)}{P(s^t)} \\ &\left(\frac{f(\mathbf{i}, s^t)}{y(\mathbf{i}, s^t)}\right)^{-\frac{1}{\theta}} = \frac{p_f(\mathbf{i}, s^t)}{P_f(\mathbf{i}, s^t)}, \\ &y(\mathbf{i}, s^t) = \left(\sum_{i_2=1}^N d(\mathbf{i}, s^t)^{\frac{\theta-1}{\theta}} + \sum_{i_2=1}^{N_X} f(\mathbf{i}, s^t)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}, \end{aligned}$$

where  $\tau \geq 0$  is iceberg cost,  $p_f(\mathbf{i}, s^t)$  is the price set by the within-sector producer,  $P_f(\mathbf{i}, s^t)$  is the sectoral price,  $y(\mathbf{i}, s^t)$  is sectoral output,  $P(s^t) \equiv 1$  is the numéraire of final consumption, and  $Y(s^t)$  is the final consumption/investment ( $C + I$ ) demand by the final producer.

To derive the first-order conditions, we substitute for prices into the objective function, which gives

$$\left[ \left(\frac{f(\mathbf{i}, s^t)}{y(\mathbf{i}, s^t)}\right)^{-\frac{1}{\rho}} \left(\frac{y(\mathbf{i}, s^t)}{Y(s^t)}\right)^{-\frac{1}{\gamma}} - (1 + \tau)v^*(s^t) x(s^t) \right] f(\mathbf{i}, s^t),$$

and plug in the last constraint to obtain unconstrained maximization:

$$\left[ f(\mathbf{i}, s^t)^{-\frac{1}{\rho}} \left(\sum_{i_2=1}^N d(\mathbf{i}, s^t)^{\frac{\theta-1}{\theta}} + \sum_{i_2=1}^{N_X} f(\mathbf{i}, s^t)^{\frac{\theta-1}{\theta}}\right)^{\frac{(\frac{1}{\theta}-\frac{1}{\gamma})\rho}{\theta-1}} Y(s^t)^{\frac{1}{\gamma}} - (1 + \tau)\frac{v^*(s^t)}{x(s^t)} \right] f(\mathbf{i}, s^t).$$

The first-order condition with respect to  $f(\mathbf{i}, s^t)$  is

$$\begin{aligned} &\left(-\frac{1}{\rho} + 1\right) f(\mathbf{i}, s^t)^{-\frac{1}{\rho}} \left(\sum_{i_2=1}^N d(\mathbf{i}, s^t)^{\frac{\theta-1}{\theta}} + \sum_{i_2=1}^{N_X} f(\mathbf{i}, s^t)^{\frac{\theta-1}{\theta}}\right)^{\frac{1-\frac{\theta}{\gamma}}{\rho-1}} Y(s^t)^{\frac{1}{\gamma}} = \\ &-f(\mathbf{i}, s^t)^{-\frac{1}{\theta}+1} \frac{1-\frac{\theta}{\gamma}}{\theta} \left(\sum_{i_2=1}^N d(\mathbf{i}, s^t)^{\frac{\theta-1}{\theta}} + \sum_{i_2=1}^{N_X} f(\mathbf{i}, s^t)^{\frac{\theta-1}{\theta}}\right)^{\frac{1-\frac{\theta}{\gamma}}{\theta-1}-1} f(\mathbf{i}, s^t)^{-\frac{1}{\theta}} Y(s^t)^{\frac{1}{\gamma}} + (1 + \tau)v^*(s^t) x(s^t) \end{aligned}$$

and hence

$$\begin{aligned} &\left(-\frac{1}{\theta} + 1\right) p_f(\mathbf{i}, s^t) = \\ &-S_f(\mathbf{i}, s^t) \frac{1-\frac{\theta}{\gamma}}{\theta} p_f(\mathbf{i}, s^t) + (1 + \tau)v^*(s^t) x(s^t) \end{aligned}$$

where

$$S_f(\mathbf{i}, s^t) := \frac{p_f(\mathbf{i}, s^t) f(\mathbf{i}, s^t)}{P_f(\mathbf{i}, s^t) y(\mathbf{i}, s^t)}$$

is the importer's market share. Simplifying, we obtain the key pricing equation of the model

$$\left(1 - \frac{1}{\theta} [1 - S_f(\mathbf{i}, s^t)] - \frac{1}{\gamma} S_f(\mathbf{i}, s^t)\right) p_f(\mathbf{i}, s^t) = (1 + \tau) v^*(s^t) x(s^t),$$

and hence

$$p_f(\mathbf{i}, s^t) = \frac{\varepsilon_f(\mathbf{i}, s^t)}{\varepsilon_f(\mathbf{i}, s^t) - 1} (1 + \tau) v^*(s^t) x(s^t),$$

where

$$\varepsilon_f(\mathbf{i}, s^t) := \left[ \frac{1}{\theta} (1 - S_f(\mathbf{i}, s^t)) + \frac{1}{\gamma} S_f(\mathbf{i}, s^t) \right]^{-1}$$

is demand elasticity. In what follows, we assume there is  $N$  symmetric home firms within each sector and a smaller  $N_X < N$  symmetric importers. We set  $\tau = 0$  (analytic section only).

**PT:** We log-linearize the pricing equation given by

$$p_f(x|\mathbf{i}, s^t) = \frac{1}{1 - \left(\frac{1}{\theta} (1 - S_f(x|\mathbf{i}, s^t)) + \frac{1}{\gamma} S_f(x|\mathbf{i}, s^t)\right)} v^*(s^t) x, \quad (44)$$

which is equivalent to the one stated in the text after multiplying the numerator and the denominator by  $\frac{1}{\theta} (1 - S_f) + \frac{1}{\gamma} S_f$ . Log-linearization of this expression under the rules of the operator  $\partial_{\log x}(\cdot|_{ss})$  gives

$$PT := \partial_{\log x} \log p_f(x|\mathbf{i}, s^t)|_{ss} = 1 - \frac{(\theta - 1)\mu_f^{ss} - 1}{\theta} (-\partial_{\log x} \log S_f(x|\mathbf{i}, s^t)|_{ss}). \quad (45)$$

(The operator  $\partial_{\log x}(\cdot|_{ss})$  assumes that  $p_d(\mathbf{i}, s^t)$  is unaffected by the exchange rate and so  $d(\mathbf{i}, s^t)$  is treated as a constant.)

Using representativeness and symmetry,  $p_f(x|\mathbf{i}, s^t) \equiv p_f(x|s^t)$  (all  $\mathbf{i}$  from the foreign country), and so

$$S_f(x|\mathbf{i}, s^t) := \frac{\frac{p_f(x|\mathbf{i}, s^t) f(x|\mathbf{i}, s^t)}{p_d d(x|\mathbf{i}, s^t)}}{N + N_X \frac{p_f(x|s^t) f(x|s^t)}{p_d d(x|s^t)}}.$$

Taking logs and applying the operator, we thus obtain

$$\begin{aligned} -\partial_{\log x} (\log S_f(x|s^t)|_{ss}) &= -\partial_{\log x} \log \left( \frac{p_f(x|s^t)}{p_d} \right) |_{ss} + \left( -\partial_{\log x} \log \left( \frac{f(x|s^t)}{d(x|s^t)} \right) |_{ss} \right) \\ &\quad + \partial_{\log x} \log \left( N + N_X \frac{p_f(x|s^t) f(x|s^t)}{p_d d(x|s^t)} \right) |_{ss}, \end{aligned} \quad (46)$$

where  $f(x|s^t)$  denotes the relevant quantity consistent with the producer problem under the assumptions of the

operator  $\partial_{\log x} (\cdot) |_{ss}$  and symmetry. Dividing the demand equations side by side,

$$d = \left( \frac{p_d}{P_d(\mathbf{i}, s^t)} \right)^{-\theta} y(\mathbf{i}, s^t),$$

$$f(x|s^t) = \left( \frac{p_f(x|s^t)}{P_f(\mathbf{i}, s^t)} \right)^{-\theta} y(\mathbf{i}, s^t),$$

we note that

$$\frac{f(x|s^t)}{d(x|s^t)} = \left( \frac{p_f(x|s^t)}{p_d} \right)^{-\theta}, \quad (47)$$

and hence

$$\frac{p_f(x|s^t) f(x, s^t)}{p_d d(x|s^t)} = \left( \frac{p_f(x|s^t)}{p_d} \right)^{1-\theta}.$$

Accordingly,

$$\partial_{\log x} \frac{p_f(x|s^t) f(x|s^t)}{p_d d(x|s^t)} |_{ss} = \partial_{\log x} \left( \frac{p_f(x|s^t)}{p_d} \right)^{1-\theta} |_{ss} = (1-\theta) \left( \frac{p_f^{ss}}{p_d^{ss}} \right)^{1-\theta} PT.$$

Eliminating the constants and plugging in for  $f/d$  from (47), we obtain

$$-\partial_{\log x} \log S_f(x|s^t) |_{ss} = -\partial_{\log x} \log(p_f(x|s^t)) |_{ss} + (\theta \partial_{\log x} \log(p_f(x|s^t)) |_{ss}) \quad (48)$$

$$+ \partial_{\log x} \log \left( 1 + \frac{N_X}{N} \left( \frac{p_f(x|s^t)}{p_d} \right)^{1-\theta} \right) |_{ss}.$$

Using the known approximation for logs,  $d \log(1+x)/dx \leq 1$  (for all  $z \geq 0$ ), and given  $\theta > 1$ , we obtain

$$-\partial_{\log x} \log S_f(x|s^t) |_{ss} \leq (\theta - 1) \left( PT - \frac{N_X}{N} \left( \frac{p_f^{ss}}{p_d^{ss}} \right)^{1-\theta} \right) \leq (\theta - 1) PT.$$

Plugging in to (45), we derive

$$PT \geq 1 - \frac{(\theta - 1)\mu_f^{ss} - 1}{\theta} (\theta - 1) PT$$

and assuming  $(\theta - 1)\mu_f^{ss} - 1 > 0$ , we obtain

$$PT \geq \left( 1 + \frac{(\theta - 1)\mu_f^{ss} - 1}{\theta} (\theta - 1) \right)^{-1}. \quad (49)$$

As for  $(\theta - 1)\mu_f^{ss} - 1 > 0$ , note that, since  $\theta > \gamma > 0$  and  $S_f^{ss} > 0$ , (44) implies

$$\mu_f^{ss} = \frac{\frac{1}{\theta} (1 - S_f^{ss}) + \frac{1}{\gamma} S_f^{ss}}{1 - \left( \frac{1}{\theta} (1 - S_f^{ss}) + \frac{1}{\gamma} S_f^{ss} \right)} \quad (50)$$

$$> \frac{\frac{1}{\theta} (1 - S_f^{ss}) + \frac{1}{\theta} S_f^{ss}}{1 - \left( \frac{1}{\theta} (1 - S_f^{ss}) + \frac{1}{\theta} S_f^{ss} \right)} \quad (51)$$

$$= \frac{\frac{1}{\theta}}{1 - \frac{1}{\theta}} = \frac{1}{\theta - 1}, \quad (52)$$

hence  $(\theta - 1)\mu_f^{ss} - 1 > 1 - 1 > 0$ .

**TE:** The formula for TE follows from the first-order condition of the producer:

$$\frac{f(x|s^t)}{d} = \left( \frac{p_f(x|s^t)}{p_d} \right)^{-\theta}. \quad (53)$$

Plugging in to the formula for TE, we obtain

$$TE = \frac{\partial_{\log x} \log \frac{d(s^t)}{f(s^t)} \Big|_{ss}}{PT} = \theta \frac{\partial_{\log x} \log \left( \frac{p_f(x|s^t)}{p_d} \right)}{PT} = \theta \frac{\partial_{\log x} \log (p_f(x|s^t))}{PT} = \theta. \quad (54)$$

Notes: Additional derivations and automated log-linearization of the above expressions can be found in the replication package *Mathematica* notebook TheoryResultsSection3\_NCES.nb.

## 1.6 DH Model

**Preliminaries:** The importer's profit maximization is

$$\Pi_f(i, s^t) = \sum_{t=0}^{\infty} \sum_{s^t} \pi(s^t) \beta^t u_c^*(s^t) [(p_f^*(i, s^t) - v^*(s^t)) f^*(i, s^t) + (p_f(i, s^t)/x(s^t) - v^*(s^t)) f(i, s^t)].$$

s.t.

$$f(i, s^t) = \left( \frac{p_f(i, s^t)}{P_f(s^t)} \right)^{-\theta} h_f(i, s^{t-1})^{\zeta(\theta-1)} f(s^t), \quad (\psi_f(i, s^t) \beta^t u_c^*(s^t) \pi(s^t))$$

$$f^*(i, s^t) = \left( \frac{p_f^*(i, s^t)}{P_f^*(s^t)} \right)^{-\theta} h_f^*(i, s^{t-1})^{\zeta(\theta-1)} f^*(s^t), \quad (\psi_f^*(i, s^t) \beta^t u_c^*(s^t) \pi(s^t)) \quad (55)$$

$$h_f(i, s^t) = \rho h_f(i, s^{t-1}) + (1 - \rho) f(i, s^t), \quad (\Delta_f(i, s^t) \beta^t u_c^*(s^t) \pi(s^t)) \quad (56)$$

$$h_f^*(i, s^t) = \rho h_f^*(i, s^{t-1}) + (1 - \rho) f^*(i, s^t), \quad (\Delta_f^*(i, s^t) \beta^t u_c^*(s^t) \pi(s^t)), \quad (57)$$

where Lagrange multipliers are defined as noted in brackets next to the constraints.

The first-order conditions are

$$\begin{aligned}
f^h & : p_f(i, s^t)/x(s^t) - v^*(s^t) = \psi_f(i, s^t) - (1 - \rho)\Delta_f(i, s^t) \\
f^* & : p_f^*(i, s^t) - v^*(s^t) = \psi_f^*(i, s^t) - (1 - \rho)\Delta_f^*(i, s^t) \\
p_f & : f(i, s^t) = \theta \frac{f(i, s^t)}{p_f(i, s^t)/x(s^t)} \psi_f(i, s^t) \\
& \rightarrow p_f(i, s^t)/x(s^t) \frac{1}{\theta} = \psi_f(i, s^t) \\
p_f^* & : f^*(i, s^t) = \theta \frac{f^*(i, s^t)}{p_f^*(i, s^t)} \psi_f^*(i, s^t) \\
& \rightarrow p_f^*(i, s^t) \frac{1}{\theta} = \psi_f^*(i, s^t) \\
h_f(i, s^t) & : \Delta_f(i, s^t) = \beta \mathbb{E}_{s^t} \frac{u_c^*(s^{t+1})}{u_c^*(s^t)} \left[ \rho \Delta_f(i, s^{t+1}) + \psi_f(i, s^{t+1}) f(i, s^{t+1}) \frac{\zeta(\theta - 1)}{h_f(i, s^t)} \right] \\
h_f^*(i, s^t) & : \Delta_f^*(i, s^t) = \beta \mathbb{E}_{s^t} \frac{u_c^*(s^{t+1})}{u_c^*(s^t)} \left[ \rho \Delta_f^*(i, s^{t+1}) + \psi_f^*(i, s^{t+1}) f^*(i, s^{t+1}) \frac{\zeta(\theta - 1)}{h_f^*(i, s^t)} \right],
\end{aligned}$$

where  $\mathbb{E}_{s^t}$  denotes the conditional expectation on state  $s^t$ . Substituting out for the multipliers, we derive

$$p_f(i, s^t)/x(s^t) = \frac{\theta}{\theta - 1} [v^*(s^t) - (1 - \rho)\Delta_f(i, s^t)] \quad (58)$$

$$\Delta_f(i, s^t) = \beta \mathbb{E}_{s^t} \frac{u_c^*(s^{t+1})}{u_c^*(s^t)} \left[ \rho \Delta_f(i, s^{t+1}) + \frac{\zeta(\theta - 1)}{\theta} \frac{p_f(i, s^{t+1})}{x(s^{t+1})} \frac{f(i, s^{t+1})}{h_f(i, s^t)} \right] \quad (59)$$

$$x(s^t)p_f^*(i, s^t) = \frac{\theta}{\theta - 1} [v^*(s^t) - (1 - \rho)\Delta_f^*(i, s^t)] \quad (60)$$

$$\Delta_f^*(i, s^t) = \beta \mathbb{E}_{s^t} \frac{u_c^*(s^{t+1})}{u_c^*(s^t)} \left[ \rho \Delta_f^*(i, s^{t+1}) + \frac{\zeta(\theta - 1)}{\theta} p_f^*(i, s^{t+1}) \frac{d^*(i, s^{t+1})}{h_f^*(i, s^t)} \right]. \quad (61)$$

As noted in text, the demand function faced by the foreign producer (one of the constraints in the optimization above) is

$$f(p_f, h_f | i, s^{t+1}) := \left( \frac{p_f}{P_f(s^t)} \right)^{-\theta} h_f^{\zeta(\theta-1)} f(s^{t+1}),$$

and so

$$\underbrace{\frac{\partial_{h_f} f(\cdot)}{\partial_{p_f} f(\cdot)}}_{MRT} \frac{f(i, s^{t+1})}{x(s^{t+1})} = \frac{\zeta(\theta - 1)}{\theta} \frac{p_f(i, s^{t+1})}{x(s^{t+1})} \frac{f(i, s^{t+1})}{h_f(i, s^t)}.$$

Accordingly, the law of motion for the value of habit can be equivalently expressed as

$$\Delta_f(i, s^t) = \beta \mathbb{E}_{s^t} \frac{u_{c^*}^*(s^{t+1})}{u_{c^*}^*(s^t)} \left[ \rho \Delta_f(i, s^{t+1}) + \underbrace{-\frac{\partial_{h_f} f(p_f, h_f|i, s^{t+1})}{\partial_{p_f} f(p_f, h_f|i, s^{t+1})}}_{MRT} \frac{f(p_f, h_f|i, s^{t+1})}{x(s^{t+1})} \right]. \quad (62)$$

**PT:** Define

$$g_h(s^{t+1}) := \frac{h_f(i, s^{t+1})}{h_f(i, s^t)},$$

which, given the habit equation,

$$h_f(i, s^{t+1}) = \rho h_f(i, s^t) + (1 - \rho) f(i, s^{t+1}), \quad (63)$$

implies

$$\frac{f(i, s^{t+1})}{h_f(i, s^t)} = \frac{g_h(s^{t+1}) - \rho}{1 - \rho}. \quad (64)$$

Given the pricing equation,

$$\frac{p_f(i, s^t)}{x(s^t)} = \frac{\theta}{\theta - 1} [v^*(s^t) - (1 - \rho) \Delta_f(i, s^t)],$$

we can express the value of habit as

$$\Delta_f(i, s^t) = -\frac{\frac{p_f(i, s^t)}{x(s^t)} \frac{\theta - 1}{\theta} - v^*(s^t)}{1 - \rho}.$$

We next plug in to the law of motion for  $\Delta_f$  above and obtain

$$\begin{aligned} -\frac{\frac{p_f(i, s^t)}{x(s^t)} \frac{\theta - 1}{\theta} - v^*(s^t)}{1 - \rho} &= \beta \mathbb{E}_{s^t} \frac{u_{c^*}^*(s^{t+1})}{u_{c^*}^*(s^t)} \left[ \rho \left( -\frac{\frac{p_f(i, s^{t+1})}{x(s^{t+1})} \frac{\theta - 1}{\theta} - v^*(s^{t+1})}{1 - \rho} \right) \right] \\ &+ \beta \mathbb{E}_{s^t} \frac{u_{c^*}^*(s^{t+1})}{u_{c^*}^*(s^t)} \left[ \frac{\zeta(\theta - 1)}{\theta} \frac{p_f(i, s^{t+1})}{x(s^{t+1})} \frac{g_h(s^{t+1}) - \rho}{1 - \rho} \right], \end{aligned}$$

hence

$$\begin{aligned} \frac{p_f(i, s^t)}{x(s^t)} &= \frac{\theta}{\theta - 1} \left( v^*(s^t) - \beta \mathbb{E}_{s^t} \frac{u_{c^*}^*(s^{t+1})}{u_{c^*}^*(s^t)} \rho v^*(s^{t+1}) \right) + \beta \mathbb{E}_{s^t} \frac{u_{c^*}^*(s^{t+1})}{u_{c^*}^*(s^t)} \left[ \rho \frac{p_f(i, s^{t+1})}{x(s^{t+1})} \right] \\ &- \beta \mathbb{E}_{s^t} \frac{u_{c^*}^*(s^{t+1})}{u_{c^*}^*(s^t)} \left[ \zeta \frac{p_f(i, s^{t+1})}{x(s^{t+1})} (g_h(s^{t+1}) - \rho) \right], \end{aligned}$$

and

$$\begin{aligned} \frac{p_f(i, s^t)}{x(s^t)} &= \frac{\theta}{\theta - 1} \left( v^*(s^t) - \beta \mathbb{E}_{s^t} \frac{u_{c^*}(s^{t+1})}{u_{c^*}(s^t)} \rho v^*(s^{t+1}) \right) \\ &+ \beta \mathbb{E}_{s^t} \frac{u_{c^*}(s^{t+1})}{u_{c^*}(s^t)} \left[ \frac{p_f(i, s^{t+1})}{x(s^{t+1})} (\rho - \zeta g_h(s^{t+1}) + \zeta \rho) \right]. \end{aligned} \quad (65)$$

We use the fact that  $p_f^*(i, s^t) := \frac{p_f(i, s^t)}{x(s^t)}$ , plug in to the above and derive the following difference equation

$$p_f^*(x(s^t) | s^t) = \frac{\theta}{\theta - 1} (v^*(s^t) - \beta \mathbb{E}_{s^t} \dots) + \beta \mathbb{E}_{s^t} \frac{u_{c^*}(s^{t+1})}{u_{c^*}(s^t)} [p_f^*(x(s^{t+1}) | s^{t+1}) (\rho - \zeta g_h(s^{t+1}) + \zeta \rho)].$$

We log-linearize this equation with respect to  $(p_f^*(i, s^t), p_f^*(i, s^{t+1}), g_h(s^{t+1}))$  around the deterministic steady state and obtain

$$d \log p_f^*(i, s^t) = -\beta \zeta \mathbb{E}_{s^t} d \log g_h(s^{t+1}) + \beta (\rho - \zeta (1 - \rho)) \mathbb{E}_{s^t} d \log p_f^*(i, s^{t+1}).$$

We solve the implied difference equation forward to obtain

$$d \log p_f^*(i, s^t) = -\beta \zeta \sum_{t=0}^{\infty} \beta^t (\rho - \zeta (1 - \rho))^t \mathbb{E}_{s^t} d (\log g_h(x | s^{t+1}) |_{ss}).$$

Finally, using the fact that

$$d \log p_f^*(i, s^t) = d \log p_f(i, s^t) - d \log x,$$

we divide both sides by  $d \log x$  and derive

$$PT \equiv \partial_{\log x} (\log p_f(x | s^t)) = \frac{\partial \log p_f(x | s^t)}{\partial \log x} = 1 - \beta \zeta \sum_{t=0}^{\infty} \beta^t (\rho - \zeta (1 - \rho))^t \mathbb{E}_{s^t} \partial_{\log x} (\log g_h(x | s^{t+1}) |_{ss}).$$

Note that  $0\zeta < 1$  and  $0 < \rho < 1$ , and so  $\rho - \zeta (1 - \rho) > 0$ .

**TE:** TE derivation is derived analogously as in the baseline model:  $TE = \gamma$ . We omit the details.

Notes: Additional derivations and automated log-linearization of the above expressions can be found in the replication package *Mathematica* notebook TheoryResultsSection3\_DH.nb.

## 1.7 CC Model

**Preliminaries:** Consider the simplified ‘‘analytic’’ setup from Section 3. The key equations are: 1) the bargaining equation

$$p_f(x | i, s^t) = \eta \partial_f G \left( \frac{d(s^t)}{f(x | s^t)}, 1 \right) + (1 - \eta) x v^*, \quad (66)$$

where  $G(d, f)$  corresponds to (5), 2) the endogenous market-share determination equation

$$\frac{f(x | s^t)}{d(s^t)} = \frac{m_f(x | s^t)}{\bar{m}_d(s^t)}, \quad (67)$$

3) and the first-order condition associated with the analytic maximization stated in Section 3 of the paper,

$$\max_{a_f(i, s^t)} (p_f(s^t)/x(s^t) - v(s^t)) h_d(s^t) \frac{m_f(i, s^t)}{\bar{m}_f(s^t) + \bar{m}_d(s^t)} - v^*(s^t) a_f(i, s^t), \quad (68)$$

subject to

$$m_f(i, s^t) = a_f(i, s^t) - \psi a_f^{ss} \left( \frac{a_f(i, s^t)}{a_f^{ss}} - 1 \right)^2. \quad (69)$$

The above, after plugging in the steady state value for  $v^* = v^{ss} = \eta/(\eta + \mu^{ss})$ —consistent with the partial differentiation to obtain PT—gives

$$\left( \frac{p_f(s^t)}{x(s^t)} - \frac{\eta}{\eta + \mu^{ss}} \right) \left( 1 - 2\Psi \frac{a_f(s^t) - a_f^{ss}}{a_f^{ss}} \right) = (\bar{m}_d + \bar{m}_f) \frac{\eta}{\eta + \mu^{ss}}, \quad (70)$$

where  $a_f^{ss} = m_f^{ss}$  is the steady-state value associated with markup  $\mu^{ss}$  that solves  $p_f^{ss} = v^{ss} (1 + \mu^{ss})$ . In the steady state, we assume  $d^{ss}/f^{ss} = \omega/(1 - \omega)$ , which gives

$$a_f^{ss} = m_f^{ss} = \mu^{ss} (1 - \omega), \quad a_d^{ss} = m_d^{ss} = \mu^{ss} \omega.$$

The term  $\mu^{ss}$  ensures that the convex adjustment cost does not bind in the steady state. While the model is static, this emulates the long-run equilibrium in the full model.

**PT:** We start from the bargaining equations, which, in log-linear form around the symmetric steady state and under the rules of the operator  $\partial_{\log x} (\log \cdot |_{ss})$ , is

$$\gamma \left( \eta + (1 + \mu^{ss}) \underbrace{\partial_{\log x} (\log p_f(x|i, s^t)|_{ss})}_{PT} - 1 \right) = \frac{\omega}{1 - \omega} (\eta + \mu^{ss}) \left( \underbrace{-\partial_{\log x} \log \left( \frac{f(x|s^t)}{d(s^t)} \right) |_{ss}}_{TE \times PT} \right),$$

where, by definition,

$$TE := (PT)^{-1} \underbrace{-\partial_{\log x} \log \left( \frac{f(x|s^t)}{d(s^t)} \right) |_{ss}}_{TE \times PT}. \quad (71)$$

It is now easy to calculate that the above equations give the expression in text:

$$PT = \frac{1 - \eta}{1 + \mu^{ss} - \frac{TE}{\gamma} (\eta + \mu^{ss}) \omega}. \quad (72)$$

**TE:** We log-linearize the constraint (69), the first-order condition in (70), the market-share equation in (67) and the bargaining equation in (66), all around the steady state under the rules of the operator  $\partial_x (\cdot |_{ss})$ . The resulting log-linearized equations, in the same sequence, are:

$$\partial_{\log x} \log (\bar{m}_f(x|i, s^t)|_{ss}) = \partial_{\log x} \log (a_f(x|i, s^t)|_{ss})$$

$$(1 + \mu^{ss}) \left( \underbrace{\partial_{\log x} (\log p_f(x|i, s^t)|_{ss})}_{PT} - 1 \right) = \partial_{\log x} \log (a_f(x|i, s^t)|_{ss}) (1 + 2\psi - \omega),$$

$$\underbrace{-\partial_{\log x} \log \left( \frac{f(x|s^t)}{d(s^t)} \Big|_{ss} \right)}_{TE \times PT} = \partial_{\log x} \log (\bar{m}_f(x|i, s^t)|_{ss}),$$

and bargaining equation

$$\gamma \left( (1 + \mu^{ss}) \underbrace{\partial_{\log x} (\log p_f(x|i, s^t)|_{ss})}_{PT} - (1 - \eta) \right) = (\eta + \mu^{ss}) \omega \underbrace{\left( -\partial_{\log x} \log \left( \frac{f(x|s^t)}{d(s^t)} \Big|_{ss} \right) \right)}_{TE \times PT}.$$

Solving the above system yields the expression from text:

$$TE = \frac{\gamma(1 + \mu^{ss})(\eta + \mu^{ss})}{\gamma(1 - \eta)\mu^{ss}(1 - \omega + 2\psi) + \omega(1 + \mu^{ss})(\eta + \mu^{ss})}.$$

Notes: Additional derivations and automated log-linearization of the above expressions can be found in the replication package *Mathematica* notebook TheoryResultsSection3\_CC.nb.<sup>1</sup>

## 2 Log-Linearized First-Order Conditions

Unless indicated otherwise, all variables showing up in the linearized equations are defined as proportional deviations from the steady state: i.e., for any variable  $X(s^t)$ , we define  $X_t \equiv dX(s^t)/X^{ss}$ , where  $X^{ss}$  is the steady state value of  $X(s^t)$ . We make exceptions for variables that can take non-positive values (e.g., net foreign assets), which we scale by steady state final goods production: i.e., for any such variable  $Y(s^t)$ , we define  $Y_t \equiv dY(s^t)/y^{ss}$ , where  $y^{ss} \equiv d^{ss} + d^{*,ss}$ . We also define the steady state share of hours worked in time endowment as  $\mathcal{L} \equiv l^{ss}/\bar{l}$ .

### 2.1 Frictionless Baseline (FB)

Marginal utility

$$u_{c,t} = (\nu(1 - \sigma) - 1) c_t - (1 - \nu)(1 - \sigma) \frac{\mathcal{L}}{1 - \mathcal{L}} l_t \quad (73)$$

$$u_{c,t}^* = (\nu(1 - \sigma) - 1) c_t^* - (1 - \nu)(1 - \sigma) \frac{\mathcal{L}}{1 - \mathcal{L}} l_t^* \quad (74)$$

Capital accumulation

$$k_t = (1 - \delta)k_{t-1} + \delta i_t \quad (75)$$

$$k_t^* = (1 - \delta)k_{t-1}^* + \delta i_t^* \quad (76)$$

Demand for final goods

$$\gamma P_{d,t} + d_t = \omega d_t + (1 - \omega) f_t \quad (77)$$

<sup>1</sup>Note: in the file,  $m$  corresponds to  $a$  above,  $\theta$  corresponds to  $\eta$  and  $\phi$  corresponds to  $\psi$ .

$$\gamma P_{d,t}^* + d_t^* = \omega f_t^* + (1 - \omega)d_t^* \quad (78)$$

$$\gamma P_{f,t} + f_t = \omega d_t + (1 - \omega)f_t \quad (79)$$

$$\gamma P_{f,t}^* + f_t^* = \omega f_t^* + (1 - \omega)d_t^* \quad (80)$$

Consumption-leisure choice

$$w_t = \frac{\mathcal{L}}{1 - \mathcal{L}} l_t + c_t \quad (81)$$

$$w_t^* = \frac{\mathcal{L}}{1 - \mathcal{L}} l_t^* + c_t^* \quad (82)$$

Optimal bond holdings

$$u_{c,t} = R_t + u_{c,t|t+1} \quad (83)$$

$$u_{c,t}^* = R_t^* + u_{c,t|t+1}^* \quad (84)$$

Optimal investment

$$u_{c,t} + \phi\delta (i_t - k_{t-1}) = u_{c,t|t+1} + [1 - \beta(1 - \delta)] r_{t|t+1} + \beta\phi\delta (i_{t|t+1} - k_t) \quad (85)$$

$$u_{c,t}^* + \phi\delta (i_t^* - k_{t-1}^*) = u_{c,t|t+1}^* + [1 - \beta(1 - \delta)] r_{t|t+1}^* + \beta\phi\delta (i_{t|t+1}^* - k_t^*) \quad (86)$$

Capital-labor choice

$$k_{t-1} - l_t = w_t - r_t \quad (87)$$

$$k_{t-1}^* - l_t^* = w_t^* - r_t^* \quad (88)$$

Marginal cost

$$v_t = (1 - \alpha)w_t + \alpha r_t - z_t \quad (89)$$

$$v_t^* = (1 - \alpha)w_t^* + \alpha r_t^* - z_t^* \quad (90)$$

Price setting

$$P_{d,t} = v_t \quad (91)$$

$$P_{f,t} = x_t + v_t^* \quad (92)$$

$$P_{d,t}^* = v_t - x_t \quad (93)$$

$$P_{f,t}^* = v_t^* \quad (94)$$

Resource constraint

$$\omega d_t + (1 - \omega)d_t^* = z_t + \alpha k_{t-1} + (1 - \alpha)l_t \quad (95)$$

$$(1 - \omega)f_t + \omega f_t^* = z_t^* + \alpha k_{t-1}^* + (1 - \alpha)l_t^* \quad (96)$$

Final goods market clearing, where  $\frac{i^{ss}}{y^{ss}} = \frac{\theta-1}{\theta} \frac{\delta\alpha}{\frac{1}{\beta}-1+\delta}$

$$\left(1 - \frac{i^{ss}}{y^{ss}}\right) c_t + \frac{i^{ss}}{y^{ss}} i_t = \omega d_t + (1 - \omega)f_t \quad (97)$$

$$\left(1 - \frac{i^{ss}}{y^{ss}}\right) c_t^* + \frac{i^{ss}}{y^{ss}} i_t^* = \omega f_t^* + (1 - \omega)d_t^* \quad (98)$$

UIP condition

$$R_t^* - R_t + x_{t|t+1} - x_t = -\Gamma (b_t + n_t) \quad (99)$$

Net foreign assets

$$b_t = \frac{1}{\beta} b_{t-1} + (1 - \omega) (d_t^* - f_t + x_t + P_{d,t}^* - P_{f,t}) \quad (100)$$

## 2.2 Kimball Aggregation (KA)

Marginal utility

$$u_{c,t} = (\nu(1 - \sigma) - 1) c_t - (1 - \nu)(1 - \sigma) \frac{\mathcal{L}}{1 - \mathcal{L}} l_t \quad (101)$$

$$u_{c,t}^* = (\nu(1 - \sigma) - 1) c_t^* - (1 - \nu)(1 - \sigma) \frac{\mathcal{L}}{1 - \mathcal{L}} l_t^* \quad (102)$$

Capital accumulation

$$k_t = (1 - \delta) k_{t-1} + \delta i_t \quad (103)$$

$$k_t^* = (1 - \delta) k_{t-1}^* + \delta i_t^* \quad (104)$$

Demand for final goods, where  $\gamma = \gamma(1)$

$$\gamma P_{d,t} + d_t = \omega d_t + (1 - \omega) f_t \quad (105)$$

$$\gamma P_{d,t}^* + d_t^* = \omega f_t^* + (1 - \omega) d_t^* \quad (106)$$

$$\gamma P_{f,t} + f_t = \omega d_t + (1 - \omega) f_t \quad (107)$$

$$\gamma P_{f,t}^* + f_t^* = \omega f_t^* + (1 - \omega) d_t^* \quad (108)$$

Consumption-leisure choice

$$w_t = \frac{\mathcal{L}}{1 - \mathcal{L}} l_t + c_t \quad (109)$$

$$w_t^* = \frac{\mathcal{L}}{1 - \mathcal{L}} l_t^* + c_t^* \quad (110)$$

Optimal bond holdings

$$u_{c,t} = R_t + u_{c,t|t+1} \quad (111)$$

$$u_{c,t}^* = R_t^* + u_{c,t|t+1}^* \quad (112)$$

Optimal investment

$$u_{c,t} + \phi \delta (i_t - k_{t-1}) = u_{c,t|t+1} + [1 - \beta(1 - \delta)] r_{t|t+1} + \beta \phi \delta (i_{t|t+1} - k_t) \quad (113)$$

$$u_{c,t}^* + \phi \delta (i_t^* - k_{t-1}^*) = u_{c,t|t+1}^* + [1 - \beta(1 - \delta)] r_{t|t+1}^* + \beta \phi \delta (i_{t|t+1}^* - k_t^*) \quad (114)$$

Capital-labor choice

$$k_{t-1} - l_t = w_t - r_t \quad (115)$$

$$k_{t-1}^* - l_t^* = w_t^* - r_t^* \quad (116)$$

Marginal cost

$$v_t = (1 - \alpha) w_t + \alpha r_t - z_t \quad (117)$$

$$v_t^* = (1 - \alpha) w_t^* + \alpha r_t^* - z_t^* \quad (118)$$

Price setting, where  $\psi = \frac{1+\gamma-h^v(1)/\gamma}{2\gamma-h^v(1)/\gamma}$

$$P_{d,t} = (1 - \psi)v_t \quad (119)$$

$$P_{f,t} = (1 - \psi)(x_t + v_t^*) \quad (120)$$

$$P_{d,t}^* = (1 - \psi)(v_t - x_t) \quad (121)$$

$$P_{f,t}^* = (1 - \psi)v_t^* \quad (122)$$

Resource constraint

$$\omega d_t + (1 - \omega)d_t^* = z_t + \alpha k_{t-1} + (1 - \alpha)l_t \quad (123)$$

$$(1 - \omega)f_t + \omega f_t^* = z_t^* + \alpha k_{t-1}^* + (1 - \alpha)l_t^* \quad (124)$$

Final goods market clearing, where  $\frac{i^{ss}}{y^{ss}} = \frac{\gamma-1}{\gamma} \frac{\delta\alpha}{\frac{1}{\beta}-1+\delta}$

$$\left(1 - \frac{i^{ss}}{y^{ss}}\right) c_t + \frac{i^{ss}}{y^{ss}} i_t = \omega d_t + (1 - \omega) f_t \quad (125)$$

$$\left(1 - \frac{i^{ss}}{y^{ss}}\right) c_t^* + \frac{i^{ss}}{y^{ss}} i_t^* = \omega f_t^* + (1 - \omega) d_t^* \quad (126)$$

UIP condition

$$R_t^* - R_t + x_{t|t+1} - x_t = -\Gamma(b_t + n_t) \quad (127)$$

Net foreign assets

$$b_t = \frac{1}{\beta} b_{t-1} + (1 - \omega)(d_t^* - f_t + x_t + P_{d,t}^* - P_{f,t}) \quad (128)$$

## 2.3 Costly Distribution (CD)

Marginal utility

$$u_{c,t} = (\nu(1 - \sigma) - 1) c_t - (1 - \nu)(1 - \sigma) \frac{\mathcal{L}}{1 - \mathcal{L}} l_t \quad (129)$$

$$u_{c,t}^* = (\nu(1 - \sigma) - 1) c_t^* - (1 - \nu)(1 - \sigma) \frac{\mathcal{L}}{1 - \mathcal{L}} l_t^* \quad (130)$$

Capital accumulation

$$k_t = (1 - \delta)k_{t-1} + \delta i_t \quad (131)$$

$$k_t^* = (1 - \delta)k_{t-1}^* + \delta i_t^* \quad (132)$$

Demand for final goods

$$\gamma P_{d,t} + d_t = \omega d_t + (1 - \omega) f_t \quad (133)$$

$$\gamma P_{d,t}^* + d_t^* = \omega f_t^* + (1 - \omega) d_t^* \quad (134)$$

$$\gamma P_{f,t} + f_t = \omega d_t + (1 - \omega) f_t \quad (135)$$

$$\gamma P_{f,t}^* + f_t^* = \omega f_t^* + (1 - \omega) d_t^* \quad (136)$$

Consumption-leisure choice

$$w_t = \frac{\mathcal{L}}{1 - \mathcal{L}} l_t + c_t \quad (137)$$

$$w_t^* = \frac{\mathcal{L}}{1 - \mathcal{L}} l_t^* + c_t^* \quad (138)$$

Optimal bond holdings

$$u_{c,t} = R_t + u_{c,t|t+1} \quad (139)$$

$$u_{c,t}^* = R_t^* + u_{c,t|t+1}^* \quad (140)$$

Optimal investment

$$u_{c,t} + \phi\delta (i_t - k_{t-1}) = u_{c,t|t+1} + [1 - \beta(1 - \delta)] r_{t|t+1} + \beta\phi\delta (i_{t|t+1} - k_t) \quad (141)$$

$$u_{c,t}^* + \phi\delta (i_t^* - k_{t-1}^*) = u_{c,t|t+1}^* + [1 - \beta(1 - \delta)] r_{t|t+1}^* + \beta\phi\delta (i_{t|t+1}^* - k_t^*) \quad (142)$$

Capital-labor choice

$$k_{t-1} - l_t = w_t - r_t \quad (143)$$

$$k_{t-1}^* - l_t^* = w_t^* - r_t^* \quad (144)$$

Marginal cost

$$v_t = (1 - \alpha)w_t + \alpha r_t - z_t \quad (145)$$

$$v_t^* = (1 - \alpha)w_t^* + \alpha r_t^* - z_t^* \quad (146)$$

Price setting

$$p_{d,t} = v_t \quad (147)$$

$$p_{f,t} = \frac{1}{(1 + \xi)(1 - \xi)} \left( v_t^* + x_t + \frac{\xi}{\theta} v_t \right) \quad (148)$$

$$p_{d,t}^* = \frac{1}{(1 + \xi)(1 - \xi)} \left( v_t - x_t + \frac{\xi}{\theta} v_t^* \right) \quad (149)$$

$$P_{f,t}^* = v_t^* \quad (150)$$

Final goods prices, where  $v^{ss} = \frac{\theta-1}{\theta(1+\xi)}$

$$P_{d,t} = (1 - \xi v^{ss}) p_{d,t} + \xi v^{ss} v_t \quad (151)$$

$$P_{f,t} = (1 - \xi v^{ss}) p_{f,t} + \xi v^{ss} v_t \quad (152)$$

$$P_{d,t}^* = (1 - \xi v^{ss}) p_{d,t}^* + \xi v^{ss} v_t^* \quad (153)$$

$$P_{f,t}^* = (1 - \xi v^{ss}) p_{f,t}^* + \xi v^{ss} v_t^* \quad (154)$$

Resource constraint

$$\omega d_t + (1 - \omega) d_t^* + \xi [\omega d_t + (1 - \omega) f_t] = (1 + \xi) [z_t + \alpha k_{t-1} + (1 - \alpha) l_t] \quad (155)$$

$$(1 - \omega) f_t + \omega f_t^* + \xi [\omega f_t^* + (1 - \omega) d_t^*] = (1 + \xi) [z_t^* + \alpha k_{t-1}^* + (1 - \alpha) l_t^*] \quad (156)$$

Final goods market clearing, where  $\frac{i^{ss}}{y^{ss}} = \frac{\theta-1}{\theta} \frac{\delta\alpha}{\frac{1}{\beta}-1+\delta}$

$$\left( 1 - \frac{i^{ss}}{y^{ss}} \right) c_t + \frac{i^{ss}}{y^{ss}} i_t = \omega d_t + (1 - \omega) f_t \quad (157)$$

$$\left(1 - \frac{i^{ss}}{y^{ss}}\right) c_t^* + \frac{i^{ss}}{y^{ss}} l_t^* = \omega f_t^* + (1 - \omega) d_t^* \quad (158)$$

UIP condition

$$R_t^* - R_t + x_{t|t+1} - x_t = -\Gamma (b_t + n_t) \quad (159)$$

Net foreign assets

$$b_t = \frac{1}{\beta} b_{t-1} + (1 - \xi v^{ss}) (1 - \omega) (d_t^* - f_t + x_t + p_{d,t}^* - p_{f,t}) \quad (160)$$

## 2.4 Price Dispersion (PD)

Marginal utility

$$u_{c,t} = (\nu(1 - \sigma) - 1) c_t - (1 - \nu)(1 - \sigma) \frac{\mathcal{L}}{1 - \mathcal{L}} l_t \quad (161)$$

$$u_{c,t}^* = (\nu(1 - \sigma) - 1) c_t^* - (1 - \nu)(1 - \sigma) \frac{\mathcal{L}}{1 - \mathcal{L}} l_t^* \quad (162)$$

Capital accumulation

$$k_t = (1 - \delta) k_{t-1} + \delta i_t \quad (163)$$

$$k_t^* = (1 - \delta) k_{t-1}^* + \delta i_t^* \quad (164)$$

Demand for final goods, where  $v^{ss} = \frac{1-q}{1-q+\theta}$

$$\gamma [(1 - \theta v^{ss}) P_{d,t} + \theta v^{ss} v_t] + d_t = \omega d_t + (1 - \omega) f_t \quad (165)$$

$$\gamma [(1 - \theta v^{ss}) P_{d,t}^* + \theta v^{ss} v_t^*] + d_t^* = \omega f_t^* + (1 - \omega) d_t^* \quad (166)$$

$$\gamma [(1 - \theta v^{ss}) P_{f,t} + \theta v^{ss} v_t] + f_t = \omega d_t + (1 - \omega) f_t \quad (167)$$

$$\gamma [(1 - \theta v^{ss}) P_{f,t}^* + \theta v^{ss} v_t^*] + f_t^* = \omega f_t^* + (1 - \omega) d_t^* \quad (168)$$

Consumption-leisure choice

$$w_t = \frac{\mathcal{L}}{1 - \mathcal{L}} l_t + c_t \quad (169)$$

$$w_t^* = \frac{\mathcal{L}}{1 - \mathcal{L}} l_t^* + c_t^* \quad (170)$$

Optimal bond holdings

$$u_{c,t} = R_t + u_{c,t|t+1} \quad (171)$$

$$u_{c,t}^* = R_t^* + u_{c,t|t+1}^* \quad (172)$$

Optimal investment

$$u_{c,t} + \phi \delta (i_t - k_{t-1}) = u_{c,t|t+1} + [1 - \beta(1 - \delta)] r_{t|t+1} + \beta \phi \delta (i_{t|t+1} - k_t) \quad (173)$$

$$u_{c,t}^* + \phi \delta (i_t^* - k_{t-1}^*) = u_{c,t|t+1}^* + [1 - \beta(1 - \delta)] r_{t|t+1}^* + \beta \phi \delta (i_{t|t+1}^* - k_t^*) \quad (174)$$

Capital-labor choice

$$k_{t-1} - l_t = w_t - r_t \quad (175)$$

$$k_{t-1}^* - l_t^* = w_t^* - r_t^* \quad (176)$$

Marginal cost

$$v_t = (1 - \alpha)w_t + \alpha r_t - z_t \quad (177)$$

$$v_t^* = (1 - \alpha)w_t^* + \alpha r_t^* - z_t^* \quad (178)$$

Price setting

$$P_{d,t} = v_t \quad (179)$$

$$P_{f,t} = \frac{1 - q}{1 - q + \theta q} (x_t + v_t^*) + \frac{\theta q}{1 - q + \theta q} v_t \quad (180)$$

$$\frac{1 - q}{1 - q + \theta q} (v_t - x_t) + \frac{\theta q}{1 - q + \theta q} v_t^* \quad (181)$$

$$P_{f,t}^* = v_t^* \quad (182)$$

Resource constraint

$$\omega d_t + (1 - \omega)d_t^* + \theta [\omega d_t + (1 - \omega)f_t] = (1 + \theta) [z_t + \alpha k_{t-1} + (1 - \alpha)l_t] \quad (183)$$

$$(1 - \omega)f_t + \omega f_t^* + \theta [\omega f_t^* + (1 - \omega)d_t^*] = (1 + \theta) [z_t^* + \alpha k_{t-1}^* + (1 - \alpha)l_t^*] \quad (184)$$

Final goods market clearing, where  $\frac{i^{ss}}{y^{ss}} = \frac{(1-q)(1+\theta)}{1-q+\theta} \frac{\delta\alpha}{\frac{1}{\beta}-1+\delta}$

$$\left(1 - \frac{i^{ss}}{y^{ss}}\right) c_t + \frac{i^{ss}}{y^{ss}} i_t = \omega d_t + (1 - \omega)f_t \quad (185)$$

$$\left(1 - \frac{i^{ss}}{y^{ss}}\right) c_t^* + \frac{i^{ss}}{y^{ss}} i_t^* = \omega f_t^* + (1 - \omega)d_t^* \quad (186)$$

UIP condition

$$R_t^* - R_t + x_{t|t+1} - x_t = -\Gamma (b_t + n_t) \quad (187)$$

Net foreign assets

$$b_t = \frac{1}{\beta} b_{t-1} + (1 - \theta v^{ss}) (1 - \omega) (d_t^* - f_t + x_t + P_{d,t}^* - P_{f,t}) \quad (188)$$

## 2.5 Nested CES (NCES)

Marginal utility

$$u_{c,t} = (\nu(1 - \sigma) - 1) c_t - (1 - \nu)(1 - \sigma) \frac{\mathcal{L}}{1 - \mathcal{L}} l_t \quad (189)$$

$$u_{c,t}^* = (\nu(1 - \sigma) - 1) c_t^* - (1 - \nu)(1 - \sigma) \frac{\mathcal{L}}{1 - \mathcal{L}} l_t^* \quad (190)$$

Capital accumulation

$$k_t = (1 - \delta)k_{t-1} + \delta i_t \quad (191)$$

$$k_t^* = (1 - \delta)k_{t-1}^* + \delta i_t^* \quad (192)$$

Demand for final goods, where  $\frac{i^{ss}}{y^{ss}} = \left( \frac{\varepsilon_d^{ss}-1}{\varepsilon_d^{ss}} \tilde{\omega} + (1-\tilde{\omega}) \frac{\varepsilon_f^{ss}-1}{\varepsilon_f^{ss}} \right) \frac{\delta\alpha}{\frac{1}{\beta}-1+\delta}$  and  $\tilde{\omega}$  is an auxiliary parameter that can be mapped onto  $\tau$  via  $1+\tau = \left( \frac{1-\tilde{\omega}}{\tilde{\omega}} \frac{N}{N_X} \right)^{\frac{1}{1-\theta}} \frac{\varepsilon_d^{ss}}{\varepsilon_d^{ss}-1} \frac{\varepsilon_f^{ss}-1}{\varepsilon_f^{ss}}$

$$\theta P_{d,t} + d_t = \left( 1 - \frac{i^{ss}}{y^{ss}} \right) c_t + \frac{i^{ss}}{y^{ss}} i_t \quad (193)$$

$$\theta P_{d,t}^* + d_t^* = \left( 1 - \frac{i^{ss}}{y^{ss}} \right) c_t^* + \frac{i^{ss}}{y^{ss}} i_t^* \quad (194)$$

$$\theta P_{f,t} + f_t = \left( 1 - \frac{i^{ss}}{y^{ss}} \right) c_t + \frac{i^{ss}}{y^{ss}} i_t \quad (195)$$

$$\theta P_{f,t}^* + f_t^* = \left( 1 - \frac{i^{ss}}{y^{ss}} \right) c_t^* + \frac{i^{ss}}{y^{ss}} i_t^* \quad (196)$$

Consumption-leisure choice

$$w_t = \frac{\mathcal{L}}{1-\mathcal{L}} l_t + c_t \quad (197)$$

$$w_t^* = \frac{\mathcal{L}}{1-\mathcal{L}} l_t^* + c_t^* \quad (198)$$

Optimal bond holdings

$$u_{c,t} = R_t + u_{c,t|t+1} \quad (199)$$

$$u_{c,t}^* = R_t^* + u_{c,t|t+1}^* \quad (200)$$

Optimal investment

$$u_{c,t} + \phi\delta (i_t - k_{t-1}) = u_{c,t|t+1} + [1 - \beta(1 - \delta)] r_{t|t+1} + \beta\phi\delta (i_{t|t+1} - k_t) \quad (201)$$

$$u_{c,t}^* + \phi\delta (i_t^* - k_{t-1}^*) = u_{c,t|t+1}^* + [1 - \beta(1 - \delta)] r_{t|t+1}^* + \beta\phi\delta (i_{t|t+1}^* - k_t^*) \quad (202)$$

Capital-labor choice

$$k_{t-1} - l_t = w_t - r_t \quad (203)$$

$$k_{t-1}^* - l_t^* = w_t^* - r_t^* \quad (204)$$

Marginal cost

$$v_t = (1 - \alpha)w_t + \alpha r_t - z_t \quad (205)$$

$$v_t^* = (1 - \alpha)w_t^* + \alpha r_t^* - z_t^* \quad (206)$$

Price setting, where  $\varepsilon_d^{ss} = \left[ \frac{1}{\theta} \left( 1 - \frac{\tilde{\omega}}{N} \right) + \frac{1}{\gamma} \frac{\tilde{\omega}}{N} \right]^{-1}$  and  $\varepsilon_f = \left[ \frac{1}{\theta} \left( 1 - \frac{1-\tilde{\omega}}{N_X} \right) + \frac{1}{\gamma} \frac{1-\tilde{\omega}}{N_X} \right]^{-1}$

$$P_{d,t} = \frac{1}{1 - \varepsilon_d^{ss}} \varepsilon_{d,t} + v_t \quad (207)$$

$$P_{f,t} = \frac{1}{1 - \varepsilon_f^{ss}} \varepsilon_{f,t} + v_t^* + x_t \quad (208)$$

$$P_{d,t}^* = \frac{1}{1 - \varepsilon_f^{ss}} \varepsilon_{d,t}^* + v_t - x_t \quad (209)$$

$$P_{f,t}^* = \frac{1}{1 - \varepsilon_d^{ss}} \varepsilon_{f,t}^* + v_t^* \quad (210)$$

Market shares

$$S_{d,t} = P_{d,t} + d_t - (\tilde{\omega} [P_{d,t} + d_t] + (1 - \tilde{\omega}) [P_{f,t} + f_t]) \quad (211)$$

$$S_{f,t} = P_{f,t} + f_t - (\tilde{\omega} [P_{d,t} + d_t] + (1 - \tilde{\omega}) [P_{f,t} + f_t]) \quad (212)$$

$$S_{d,t}^* = P_{d,t}^* + d_t^* - ((1 - \tilde{\omega}) [P_{d,t}^* + d_t^*] + \tilde{\omega} [P_{f,t}^* + f_t^*]) \quad (213)$$

$$S_{f,t}^* = P_{f,t}^* + f_t^* - ((1 - \tilde{\omega}) [P_{d,t}^* + d_t^*] + \tilde{\omega} [P_{f,t}^* + f_t^*]) \quad (214)$$

Demand elasticities, where

$$\varepsilon_{d,t} = \varepsilon_d^{ss} \frac{\tilde{\omega}}{N} \left( \frac{1}{\theta} - \frac{1}{\gamma} \right) S_{d,t} \quad (215)$$

$$\varepsilon_{d,t}^* = \varepsilon_f^{ss} \frac{1 - \tilde{\omega}}{N_X} \left( \frac{1}{\theta} - \frac{1}{\gamma} \right) S_{d,t}^* \quad (216)$$

$$\varepsilon_{f,t} = \varepsilon_f^{ss} \frac{1 - \tilde{\omega}}{N_X} \left( \frac{1}{\theta} - \frac{1}{\gamma} \right) S_{f,t} \quad (217)$$

$$\varepsilon_{f,t}^* = \varepsilon_d^{ss} \frac{\tilde{\omega}}{N} \left( \frac{1}{\theta} - \frac{1}{\gamma} \right) S_{f,t}^* \quad (218)$$

Resource constraint, where  $\bar{\omega} = \left[ 1 + \frac{1 - \tilde{\omega}}{\tilde{\omega}} \frac{\varepsilon_d^{ss}}{\varepsilon_d^{ss} - 1} \frac{\varepsilon_f^{ss} - 1}{\varepsilon_f^{ss}} \right]^{-1}$

$$\bar{\omega} d_t + (1 - \bar{\omega}) d_t^* = z_t + \alpha k_{t-1} + (1 - \alpha) l_t \quad (219)$$

$$\bar{\omega} f_t + (1 - \bar{\omega}) f_t = z_t^* + \alpha k_{t-1}^* + (1 - \alpha) l_t^* \quad (220)$$

Final goods market clearing

$$\left( 1 - \frac{i^{ss}}{y^{ss}} \right) c_t + \frac{i^{ss}}{y^{ss}} i_t = \tilde{\omega} d_t + (1 - \tilde{\omega}) f_t \quad (221)$$

$$\left( 1 - \frac{i^{ss}}{y^{ss}} \right) c_t^* + \frac{i^{ss}}{y^{ss}} i_t^* = \tilde{\omega} f_t^* + (1 - \tilde{\omega}) d_t^* \quad (222)$$

UIP condition

$$R_t^* - R_t + x_{t|t+1} - x_t = -\Gamma (b_t + n_t) \quad (223)$$

Net foreign assets

$$b_t = \frac{1}{\beta} b_{t-1} + (1 - \tilde{\omega}) (d_t^* - f_t + x_t + P_{d,t}^* - P_{f,t}) \quad (224)$$

## 2.6 Deep Habits (DH)

Marginal utility

$$u_{c,t} = (\nu(1 - \sigma) - 1) c_t - (1 - \nu)(1 - \sigma) \frac{\mathcal{L}}{1 - \mathcal{L}} l_t \quad (225)$$

$$u_{c,t}^* = (\nu(1 - \sigma) - 1) c_t^* - (1 - \nu)(1 - \sigma) \frac{\mathcal{L}}{1 - \mathcal{L}} l_t^* \quad (226)$$

Capital accumulation

$$k_t = (1 - \delta) k_{t-1} + \delta i_t \quad (227)$$

$$k_t^* = (1 - \delta)k_{t-1}^* + \delta i_t^* \quad (228)$$

Consumption-leisure choice

$$w_t = \frac{\mathcal{L}}{1 - \mathcal{L}} l_t + c_t \quad (229)$$

$$w_t^* = \frac{\mathcal{L}}{1 - \mathcal{L}} l_t^* + c_t^* \quad (230)$$

Optimal bond holdings

$$u_{c,t} = R_t + u_{c,t|t+1} \quad (231)$$

$$u_{c,t}^* = R_t^* + u_{c,t|t+1}^* \quad (232)$$

Optimal investment

$$u_{c,t} + \phi\delta (i_t - k_{t-1}) = u_{c,t|t+1} + [1 - \beta(1 - \delta)] r_{t|t+1} + \beta\phi\delta (i_{t|t+1} - k_t) \quad (233)$$

$$u_{c,t}^* + \phi\delta (i_t^* - k_{t-1}^*) = u_{c,t|t+1}^* + [1 - \beta(1 - \delta)] r_{t|t+1}^* + \beta\phi\delta (i_{t|t+1}^* - k_t^*) \quad (234)$$

Capital-labor choice

$$k_{t-1} - l_t = w_t - r_t \quad (235)$$

$$k_{t-1}^* - l_t^* = w_t^* - r_t^* \quad (236)$$

Marginal cost

$$v_t = (1 - \alpha)w_t + \alpha r_t - z_t \quad (237)$$

$$v_t^* = (1 - \alpha)w_t^* + \alpha r_t^* - z_t^* \quad (238)$$

Price setting

$$p_{d,t} = \left(1 + \frac{\zeta(1 - \rho)\beta}{1 - \beta\rho}\right) v_t - \frac{\zeta(1 - \rho)\beta}{1 - \beta\rho} \Delta_{d,t} \quad (239)$$

$$p_{f,t} - x_t = \left(1 + \frac{\zeta(1 - \rho)\beta}{1 - \beta\rho}\right) v_t^* - \frac{\zeta(1 - \rho)\beta}{1 - \beta\rho} \Delta_{f,t} \quad (240)$$

$$p_{d,t}^* + x_t = \left(1 + \frac{\zeta(1 - \rho)\beta}{1 - \beta\rho}\right) v_t - \frac{\zeta(1 - \rho)\beta}{1 - \beta\rho} \Delta_{d,t}^* \quad (241)$$

$$p_{f,t}^* = \left(1 + \frac{\zeta(1 - \rho)\beta}{1 - \beta\rho}\right) v_t^* - \frac{\zeta(1 - \rho)\beta}{1 - \beta\rho} \Delta_{f,t}^* \quad (242)$$

Composite (habit-adjusted) goods

$$d_t = \tilde{d}_t + \zeta h_{d,t-1} \quad (243)$$

$$f_t = \tilde{f}_t + \zeta h_{f,t-1} \quad (244)$$

$$d_t^* = \tilde{d}_t^* + \zeta h_{d,t-1}^* \quad (245)$$

$$f_t^* = \tilde{f}_t^* + \zeta h_{f,t-1}^* \quad (246)$$

Demand for intermediate goods

$$\tilde{d}_t = -\theta(p_{d,t} - P_{d,t}) - \zeta(1 - \varphi)h_{d,t-1} + d_t \quad (247)$$

$$\tilde{f}_t = -\theta(p_{f,t} - P_{f,t}) - \zeta(1 - \varphi)h_{f,t-1} + f_t \quad (248)$$

$$\tilde{d}_t^* = -\theta (p_{d,t}^* - P_{d,t}^*) - \zeta(1 - \varphi)h_{d,t-1}^* + d_t^* \quad (249)$$

$$\tilde{f}_t^* = -\theta (p_{f,t}^* - P_{f,t}^*) - \zeta(1 - \varphi)h_{f,t-1}^* + f_t^* \quad (250)$$

Habit formation

$$h_{d,t} = \rho h_{d,t-1} + (1 - \rho) \tilde{d}_t \quad (251)$$

$$h_{f,t} = \rho h_{f,t-1} + (1 - \rho) \tilde{f}_t \quad (252)$$

$$h_{d,t}^* = \rho h_{d,t-1}^* + (1 - \rho) \tilde{d}_t^* \quad (253)$$

$$h_{f,t}^* = \rho h_{f,t-1}^* + (1 - \rho) \tilde{f}_t^* \quad (254)$$

Shadow price of habits

$$\Delta_{d,t} = u_{c,t|t+1} - u_{c,t} + \beta\rho\Delta_{d,t|t+1} + (1 - \beta\rho) (p_{d,t|t+1} + \tilde{d}_{t|t+1} - h_{d,t}) \quad (255)$$

$$\Delta_{f,t} = u_{c,t|t+1}^* - u_{c,t}^* + \beta\rho\Delta_{f,t|t+1} + (1 - \beta\rho) (p_{f,t|t+1} - x_{t|t+1} + \tilde{f}_{t|t+1} - h_{f,t}) \quad (256)$$

$$\Delta_{d,t}^* = u_{c,t|t+1} - u_{c,t} + \beta\rho\Delta_{d,t|t+1}^* + (1 - \beta\rho) (p_{d,t|t+1}^* + x_{t|t+1} + \tilde{d}_{t|t+1}^* - h_{d,t}^*) \quad (257)$$

$$\Delta_{f,t}^* = u_{c,t|t+1}^* - u_{c,t}^* + \beta\rho\Delta_{f,t|t+1}^* + (1 - \beta\rho) (p_{f,t|t+1}^* + \tilde{f}_{t|t+1}^* - h_{f,t}^*) \quad (258)$$

Demand for composite goods, where  $\frac{i^{ss}}{y^{ss}} = \frac{\theta-1}{\theta} \left(1 + \frac{\zeta(1-\rho)\beta}{(1-\beta\rho)}\right) \frac{\delta\alpha}{\frac{1}{\beta}-1+\delta}$

$$\gamma P_{d,t} + d_t = \left(1 - \frac{i^{ss}}{y^{ss}}\right) c_t + \frac{i^{ss}}{y^{ss}} i_t \quad (259)$$

$$\gamma P_{d,t}^* + d_t^* = \left(1 - \frac{i^{ss}}{y^{ss}}\right) c_t^* + \frac{i^{ss}}{y^{ss}} i_t^* \quad (260)$$

$$\gamma P_{f,t} + f_t = \left(1 - \frac{i^{ss}}{y^{ss}}\right) c_t + \frac{i^{ss}}{y^{ss}} i_t \quad (261)$$

$$\gamma P_{f,t}^* + f_t^* = \left(1 - \frac{i^{ss}}{y^{ss}}\right) c_t^* + \frac{i^{ss}}{y^{ss}} i_t^* \quad (262)$$

Resource constraint, where  $\tilde{\omega} = \frac{\left(\frac{\omega}{1-\omega}\right)^{\frac{1}{1+\zeta(1-\gamma)}}}{\left(\frac{\omega}{1-\omega}\right)^{\frac{1}{1+\zeta(1-\gamma)}} + 1}$

$$\tilde{\omega}\tilde{d}_t + (1 - \tilde{\omega})\tilde{d}_t^* = z_t + \alpha k_{t-1} + (1 - \alpha)l_t \quad (263)$$

$$\tilde{\omega}\tilde{f}_t^* + (1 - \tilde{\omega})\tilde{f}_t = z_t^* + \alpha k_{t-1}^* + (1 - \alpha)l_t^* \quad (264)$$

Final goods market clearing

$$\left(1 - \frac{i^{ss}}{y^{ss}}\right) c_t + \frac{i^{ss}}{y^{ss}} i_t = \frac{1}{1 + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{1+\zeta(1-\gamma)}}} d_t + \frac{1}{1 + \left(\frac{\omega}{1-\omega}\right)^{\frac{1}{1+\zeta(1-\gamma)}}} f_t \quad (265)$$

$$\left(1 - \frac{i^{ss}}{y^{ss}}\right) c_t^* + \frac{i^{ss}}{y^{ss}} l_t^* = \frac{1}{1 + \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{1+\zeta(1-\gamma)}}} f_t^* + \frac{1}{1 + \left(\frac{\omega}{1-\omega}\right)^{\frac{1}{1+\zeta(1-\gamma)}}} d_t^* \quad (266)$$

UIP condition

$$R_t^* - R_t + x_{t|t+1} - x_t = -\Gamma (b_t + n_t) \quad (267)$$

Net foreign assets

$$b_t = \frac{1}{\beta} b_{t-1} + (1 - \tilde{\omega}) \left( \tilde{d}_t^* - \tilde{f}_t + x_t + p_{d,t}^* - p_{f,t} \right) \quad (268)$$

## 2.7 Customer Capital (CC)

Marginal utility

$$u_{c,t} = (\nu(1 - \sigma) - 1) c_t - (1 - \nu)(1 - \sigma) \frac{\mathcal{L}}{1 - \mathcal{L}} l_t \quad (269)$$

$$u_{c,t}^* = (\nu(1 - \sigma) - 1) c_t^* - (1 - \nu)(1 - \sigma) \frac{\mathcal{L}}{1 - \mathcal{L}} l_t^* \quad (270)$$

Capital accumulation

$$k_t = (1 - \delta) k_{t-1} + \delta i_t \quad (271)$$

$$k_t^* = (1 - \delta) k_{t-1}^* + \delta i_t^* \quad (272)$$

Demand for final goods

$$\gamma P_{d,t} + d_t = \omega d_t + (1 - \omega) f_t \quad (273)$$

$$\gamma P_{d,t}^* + d_t^* = \omega f_t^* + (1 - \omega) d_t^* \quad (274)$$

$$\gamma P_{f,t} + f_t = \omega d_t + (1 - \omega) f_t \quad (275)$$

$$\gamma P_{f,t}^* + f_t^* = \omega f_t^* + (1 - \omega) d_t^* \quad (276)$$

Consumption-leisure choice

$$w_t = \frac{\mathcal{L}}{1 - \mathcal{L}} l_t + c_t \quad (277)$$

$$w_t^* = \frac{\mathcal{L}}{1 - \mathcal{L}} l_t^* + c_t^* \quad (278)$$

Optimal bond holdings

$$u_{c,t} = R_t + u_{c,t|t+1} \quad (279)$$

$$u_{c,t}^* = R_t^* + u_{c,t|t+1}^* \quad (280)$$

Optimal investment

$$u_{c,t} + \phi \delta (i_t - k_{t-1}) = u_{c,t|t+1} + [1 - \beta(1 - \delta)] r_{t|t+1} + \beta \phi \delta (i_{t|t+1} - k_t) \quad (281)$$

$$u_{c,t}^* + \phi \delta (i_t^* - k_{t-1}^*) = u_{c,t|t+1}^* + [1 - \beta(1 - \delta)] r_{t|t+1}^* + \beta \phi \delta (i_{t|t+1}^* - k_t^*) \quad (282)$$

Capital-labor choice

$$k_{t-1} - l_t = w_t - r_t \quad (283)$$

$$k_{t-1}^* - l_t^* = w_t^* - r_t^* \quad (284)$$

### Marginal cost

$$v_t = (1 - \alpha)w_t + \alpha r_t - z_t \quad (285)$$

$$v_t^* = (1 - \alpha)w_t^* + \alpha r_t^* - z_t^* \quad (286)$$

Price setting (optimal bargaining), where  $\mathcal{U} = \frac{\chi\eta}{1-\eta}[1 - (1 - \delta_H)\beta] + 1$

$$\mathcal{U}p_{d,t} = (\mathcal{U} - 1 + \eta)P_{d,t} + (1 - \eta)v_t \quad (287)$$

$$\mathcal{U}p_{f,t} = (\mathcal{U} - 1 + \eta)P_{f,t} + (1 - \eta)(v_t^* + x_t) \quad (288)$$

$$\mathcal{U}p_{d,t}^* = (\mathcal{U} - 1 + \eta)P_{d,t}^* + (1 - \eta)(v_t - x_t) \quad (289)$$

$$\mathcal{U}p_{f,t}^* = (\mathcal{U} - 1 + \eta)P_{f,t}^* + (1 - \eta)v_t^* \quad (290)$$

### Marketing capital

$$m_{d,t} = (1 - \delta_m)m_{d,t-1} + \delta_m a_{d,t} \quad (291)$$

$$m_{f,t} = (1 - \delta_m)m_{f,t-1} + \delta_m a_{f,t} \quad (292)$$

$$m_{d,t}^* = (1 - \delta_m)m_{d,t-1}^* + \delta_m a_{d,t}^* \quad (293)$$

$$m_{f,t}^* = (1 - \delta_m)m_{f,t-1}^* + \delta_m a_{f,t}^* \quad (294)$$

### Optimal marketing capital conditions

$$\begin{aligned} v_t + \psi\delta_m(a_{d,t} - m_{d,t-1}) &= \beta(1 - \delta_m)(v_{t|t+1} + u_{c,t|t+1} - u_{c,t}) + \beta\psi\delta_m(a_{d,t|t+1} - m_{d,t}) \\ &+ [1 - (1 - \delta_m)\beta](h_t + W_{d,t} - \omega m_{d,t} - (1 - \omega)m_{f,t}) \end{aligned} \quad (295)$$

$$\begin{aligned} v_t - x_t + \psi\delta_m(a_{f,t} - m_{f,t-1}) &= \beta(1 - \delta_m)(v_{t|t+1} - x_{t|t+1} + u_{c,t|t+1}^* - u_{c,t}^*) + \beta\psi\delta_m(a_{f,t|t+1} - m_{f,t}) \\ &+ [1 - (1 - \delta_m)\beta](h_t + W_{f,t} - \omega m_{d,t} - (1 - \omega)m_{f,t}) \end{aligned} \quad (296)$$

$$\begin{aligned} v_t^* + x_t + \psi\delta_m(a_{d,t}^* - m_{d,t-1}^*) &= \beta(1 - \delta_m)(v_{t|t+1}^* + x_{t|t+1} + u_{c,t|t+1} - u_{c,t}) + \beta\psi\delta_m(a_{d,t|t+1}^* - m_{d,t}^*) \\ &+ [1 - (1 - \delta_m)\beta](h_t^* + W_{d,t}^* - (1 - \omega)m_{d,t}^* - \omega m_{f,t}^*) \end{aligned} \quad (297)$$

$$\begin{aligned} v_t^* + \psi\delta_m(a_{d,t}^* - m_{d,t-1}^*) &= \beta(1 - \delta_m)(v_{t|t+1}^* + u_{c,t|t+1}^* - u_{c,t}^*) + \beta\psi\delta_m(a_{f,t|t+1}^* - m_{f,t}^*) \\ &+ [1 - (1 - \delta_m)\beta](h_t^* + W_{d,t}^* - (1 - \omega)m_{d,t}^* - \omega m_{f,t}^*) \end{aligned} \quad (298)$$

### Customer list

$$d_t = (1 - \delta_H)d_{t-1} - \delta_H[\omega m_{d,t} + (1 - \omega)m_{f,t}] + \delta_H(m_{d,t} + h_t) \quad (299)$$

$$f_t = (1 - \delta_H)f_{t-1} - \delta_H[\omega m_{d,t} + (1 - \omega)m_{f,t}] + \delta_H(m_{f,t} + h_t) \quad (300)$$

$$d_t^* = (1 - \delta_H)d_{t-1}^* - \delta_H[(1 - \omega)m_{d,t}^* + \omega m_{f,t}^*] + \delta_H(m_{d,t}^* + h_t^*) \quad (301)$$

$$f_t^* = (1 - \delta_H)f_{t-1}^* - \delta_H[(1 - \omega)m_{d,t}^* + \omega m_{f,t}^*] + \delta_H(m_{f,t}^* + h_t^*) \quad (302)$$

Optimal customer list conditions

$$W_{d,t} = \frac{1 - (1 - \delta_H)\beta}{\mathcal{U} - 1} (\mathcal{U}p_{d,t} - v_t) + (1 - \delta_H)\beta (u_{c,t|t+1} - u_{c,t} + W_{d,t|t+1}) \quad (303)$$

$$W_{f,t} = \frac{1 - (1 - \delta_H)\beta}{\mathcal{U} - 1} (\mathcal{U}[p_{f,t} - x_t] - v_t^*) + (1 - \delta_H)\beta (u_{c,t|t+1}^* - u_{c,t}^* + W_{f,t|t+1}) \quad (304)$$

$$W_{d,t}^* = \frac{1 - (1 - \delta_H)\beta}{\mathcal{U} - 1} (\mathcal{U}[p_{d,t}^* + x_t] - v_t) + (1 - \delta_H)\beta (u_{c,t|t+1} - u_{c,t} + W_{d,t|t+1}^*) \quad (305)$$

$$W_{f,t}^* = \frac{1 - (1 - \delta_H)\beta}{\mathcal{U} - 1} (\mathcal{U}p_{f,t}^* - v_t^*) + (1 - \delta_H)\beta (u_{c,t|t+1}^* - u_{c,t}^* + W_{f,t|t+1}^*) \quad (306)$$

Retailer value

$$J_{d,t} = \frac{1}{\chi} \left( \frac{\mathcal{U} - 1 + \eta}{\eta} P_{d,t} - \mathcal{U}p_{d,t} \right) + (1 - \delta_H)\beta (u_{c,t|t+1} - u_{c,t} + J_{d,t|t+1}) \quad (307)$$

$$J_{f,t} = \frac{1}{\chi} \left( \frac{\mathcal{U} - 1 + \eta}{\eta} P_{f,t} - \mathcal{U}p_{f,t} \right) + (1 - \delta_H)\beta (u_{c,t|t+1} - u_{c,t} + J_{f,t|t+1}) \quad (308)$$

$$J_{d,t}^* = \frac{1}{\chi} \left( \frac{\mathcal{U} - 1 + \eta}{\eta} P_{d,t}^* - \mathcal{U}p_{d,t}^* \right) + (1 - \delta_H)\beta (u_{c,t|t+1}^* - u_{c,t}^* + J_{d,t|t+1}^*) \quad (309)$$

$$J_{f,t}^* = \frac{1}{\chi} \left( \frac{\mathcal{U} - 1 + \eta}{\eta} P_{f,t}^* - \mathcal{U}p_{f,t}^* \right) + (1 - \delta_H)\beta (u_{c,t|t+1}^* - u_{c,t}^* + J_{f,t|t+1}^*) \quad (310)$$

Free entry by retailers

$$\omega J_{d,t} + (1 - \omega)J_{f,t} = v_t \quad (311)$$

$$(1 - \omega)J_{d,t}^* + \omega J_{f,t}^* = v_t^* \quad (312)$$

Resource constraint, where  $\tilde{\mathcal{W}} = \frac{\delta_m \delta_H W^{ss}}{1 - (1 - \delta_m)\beta} \frac{\mathcal{U} - 1 + \eta}{\eta}$

$$\omega d_t + (1 - \omega)d_t^* + \tilde{\mathcal{W}}(\omega a_{d,t} + (1 - \omega)a_{f,t}) + \chi \delta_H h_t = (1 + \tilde{\mathcal{W}} + \chi \delta_H)(z_t + \alpha k_{t-1} + (1 - \alpha)l_t) \quad (313)$$

$$(1 - \omega)f_t + \omega f_t^* + \tilde{\mathcal{W}}(\omega a_{f,t}^* + (1 - \omega)a_{d,t}^*) + \chi \delta_H h_t^* = (1 + \tilde{\mathcal{W}} + \chi \delta_H)(z_t^* + \alpha k_{t-1}^* + (1 - \alpha)l_t^*) \quad (314)$$

Final goods market clearing, where  $\frac{i^{ss}}{y^{ss}} = \left(1 + \tilde{\mathcal{W}} + \chi \delta_H\right) \frac{\eta}{\mathcal{U} - 1 + \eta} \frac{\delta \alpha}{\frac{1}{\beta} - 1 + \delta}$

$$\left(1 - \frac{i^{ss}}{y^{ss}}\right) c_t + \frac{i^{ss}}{y^{ss}} i_t = \omega d_t + (1 - \omega)f_t \quad (315)$$

$$\left(1 - \frac{i^{ss}}{y^{ss}}\right) c_t^* + \frac{i^{ss}}{y^{ss}} i_t^* = \omega f_t^* + (1 - \omega)d_t^* \quad (316)$$

UIP condition

$$R_t^* - R_t + x_{t|t+1} - x_t = -\Gamma(b_t + n_t) \quad (317)$$

Net foreign assets, where  $\mathcal{M} = \eta \frac{\mathcal{U}-1}{\mathcal{U}-1+\theta} \frac{\delta_H}{1-(1-\delta_H)\beta} \frac{\delta_m}{1-(1-\delta_m)\beta}$

$$b_t = \frac{1}{\beta} b_{t-1} + (1-\omega) \frac{\eta \mathcal{U}}{\mathcal{U}-1+\eta} (d_t^* - f_t + x_t + p_{d,t}^* - p_{f,t}) + (1-\omega) \mathcal{M} (v_t + a_{f,t} - v_t^* - x_t - a_{d,t}^*) \quad (318)$$

### 3 Data Sources and Replication

**Data sources.** Data for the model section come from the replication package of Drozd and Nosal (2012), available online from the *American Economic Review*. Most statistics are taken from that paper, and we use their estimate of the TFP process. Original data sources are: OECD Main Economic Indicators ([https://www.oecd.org/en/publications/main-economic-indicators\\_16097319.html](https://www.oecd.org/en/publications/main-economic-indicators_16097319.html)), Bureau of Labor Statistics (import and export price indices, <https://www.bls.gov/mxp/>), International Monetary Fund Direction of Trade Statistics (<https://data.imf.org/en/datasets/IMF.STA:IMTS>, currently IMTS, previously DOTS), and Bureau of Economic Analysis (BEA) National Income and Product Accounts and input–output tables (<https://www.bea.gov/industry/input-output-accounts-data>).

In addition, as cited in the paper, we use the S&P Compustat North America (Fundamentals Annual, 2024) dataset accessed via Wharton Research Data Services (WRDS), and Bureau of Economic Analysis input–output tables at the detail level (BEA 402 Industry I–O Use Tables, benchmark years 2007, 2012, and 2017). The NAICS crosswalk files are in the same folder and were downloaded from the BEA website in January 2025 (`2017_to_2022_NAICS.xlsx`, `2022_to_2017_NAICS.xlsx`).

**Replication of Table 1 (markups).** Compustat: Run the Jupyter notebook `compustat_markups.ipynb` in the replication package (Data folder). Due to proprietary data restrictions, the folder does not include the S&P 500 Compustat file. The code contains an automated download from WRDS. Prior to the download, WRDS credentials need to be entered when prompted. I–O tables: See Excel file `2017_402_sectoral_markups.xlsx` (sheet `Table_1`, Data folder). The file is linked, and the source data come from the 2017 SUT tables downloaded from the BEA website in January 2025 (`Use_SUT_Framework_2017.xlsx`).

**Replication of Tables 4– (model results).** Run the Dynare codes from the replication package. Instructions are in `README.txt`.