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Interchange Fees in Payment Networks: Implications for Prices, Profits, and Welfare*

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Abstract

This paper develops a two-sided model of the payment card market with elastic consumer demand, merchant and network market power, *ad valorem* interchange fees, cardholder rewards and cash as an alternative payment method. Drawing on insights from public finance, we define a *credit card tax*—an endogenous wedge between consumer and merchant prices generated by interchange fees, rewards, and credit card adoption. We show how this tax affects equilibrium prices, platform profits and welfare. Our analysis yields a novel and policy-relevant result: Contrary to conventional wisdom, capping interchange fees can *increase* equilibrium rewards when consumer demand is relatively inelastic. This, in turn, raises credit card adoption and intensifies cross-subsidization, benefiting card users, potentially at the expense of cash users. By contrast, when demand is more elastic, fee caps reduce rewards and card usage, improving outcomes for both groups. We also characterize the conditions under which interchange fee caps enhance allocative efficiency and encourage socially desirable payment choices. Overall, the paper offers new theoretical insights into the regulation of two-sided payment markets.

Keywords: credit cards, two-sided networks, merchant competition, interchange fees, regulation

JEL codes: L13, L40, G28, E42

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1 Introduction

This paper examines the equilibrium level of the wholesale price set by a payment card network—the interchange fee—and its effects on prices, profits, and welfare in a two-sided payment market. This fee fundamentally influences the distribution of acceptance costs and benefits between merchants and consumers. In the United States, interchange fees typically range between 2% and 3% of the transaction value.[1] For decades, interchange fees have been the subject of intense policy debate. Merchants argue that interchange fees are excessive, compressing their margins and ultimately raising retail prices. Card networks and issuing banks, by contrast, maintain that interchange revenue funds cardholder rewards and other benefits, and that imposing a fee cap would harm credit card users.[2] The central policy question is therefore not only whether interchange fees are too high, but also how any regulation of these fees reshapes the allocation of costs and benefits across merchants, cash users, and credit card users.

These concerns have motivated substantial regulatory scrutiny. In the United States, the Durbin Amendment to the Dodd–Frank Act capped debit card interchange fees for large issuers, requiring that they be "reasonable and proportional" to processing costs.[3] More recently, the proposed Credit Card Competition Act of 2023 seeks to increase network competition in credit card routing, potentially putting downward pressure on fees.[4] Similar interventions, including direct fee caps, have been implemented in the European Union, Australia, and Canada.[5] The rationale for such policies is straightforward. When merchants charge a common retail price and do not surcharge by payment method, interchange fees can generate cross-subsidies across consumers and distort both payment choice and product market outcomes.

The analysis of interchange fees is complicated by the fact that payment cards operate in a classic two-sided market. A card network intermediates between cardholders and merchants, and pricing decisions on one side affect participation and behavior on the other. Rewards influence consumers' willingness to use credit rather than cash; this, in turn, changes the fraction of transactions subject to merchant fees and thus affects merchants' pricing decisions. A proper analysis of interchange fees must therefore

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[1] Interchange fees account for most of the merchant discount (on average about 2.25%, according to Wang (2025) and Egan et al. (2026)) paid by merchants to their acquiring banks for processing card transactions.

[2] Rewards average approximately 1.45% of transaction value, according to Wang (2025) and Egan et al. (2026). The industry's position is reflected in campaigns such as "Congress: Hands off my rewards!", Electronic Payments Coalition (2022).

[3] For regulatory details, see: `Federal Register (2023)`.

[4] See `S.1838 - Credit Card Competition Act of 2023`.

[5] For the European Union, see Regulation (EU) 2015/751, available at: `EUR-Lex`.

jointly determine network pricing, consumer payment choice, merchant behavior, and aggregate demand.

To study these issues, we develop a model of a two-sided payment card market in which interchange fees, cardholder rewards, merchant pricing, and consumers' choice of payment instrument are all endogenous. We consider a monopoly payment network and, to the best of our knowledge, provide the first framework that jointly incorporates four features that are central to the analysis: (i) elastic consumer demand for goods, including both a constant elasticity benchmark and a linear demand extension; (ii) merchant market power; (iii) ad valorem interchange fees and rewards; and (iv) cash as an alternative payment method. These features are important for distinct but related reasons. Elastic demand allows payment costs to affect aggregate consumption and, under linear demand, implies imperfect pass-through of payment costs into retail prices. Merchant market power determines how these costs are transmitted through prices. Ad valorem fees and rewards reflect prevailing institutional arrangements. Finally, the presence of cash both constrains the network and makes payment choice endogenous; it is also essential for capturing the distributional effects of payment pricing, since under a common retail price a higher share of credit card transactions, potentially induced by higher rewards, raises merchants' average payment costs and can increase the implicit burden embedded in retail prices for all consumers, especially cash users. Table 1 in the Literature Review Section classifies the related theoretical literature along these dimensions and highlights how our framework differs from existing models.

A central contribution of the paper is the identification of two endogenous "credit card taxes." We use this terminology to emphasize the analytical parallel between interchange-driven wedges and more familiar tax wedges, not to suggest that such taxes are necessarily undesirable in a normative sense. One tax is borne by cash users and the other by credit card users. Each is defined as the wedge between the price paid by the consumer and the price received by the merchant. These wedges arise endogenously from the interaction of interchange fees, rewards, consumer payment choices, and merchant pricing.

The tax paid by cash users depends positively on the interchange fee and on the fraction of consumers using credit cards. The reason is that, under a common retail price, a higher share of card transactions raises merchants' average payment cost and therefore the wedge between what cash users pay and what merchants receive. The tax paid by credit card users depends on the same two variables but is reduced by rewards. Thus, rewards compress the effective wedge faced by card users relative to cash users. In equilibrium, both taxes are jointly determined by the network's pricing problem, merchants' pricing decisions, and consumers' adoption of credit cards. This tax-based formulation makes transparent how the payment system redistributes surplus across consumer groups and provides a convenient way to study regulation.

In our main model, we parameterize the retail sector along two dimensions: the (constant) elasticity of demand, and the conduct parameter that captures the intensity of competition in the final-goods market. Together with consumers' relative preference for credit over cash and the marginal cost of production, these parameters jointly determine equilibrium fees, rewards, retail prices, tax wedges, aggregate demand, and network and merchant profits.

We study regulation in the form of a cap on the interchange fee. Because the cap constrains one margin of the network's pricing problem, a key intermediate question is how the network optimally adjusts cardholder rewards. We show that the answer depends on product demand elasticity: when demand is relatively inelastic, a modest cap may increase equilibrium rewards, as the network optimally emphasizes transaction volume over margin preservation; when demand is relatively elastic, the cap reduces rewards, as the network instead places greater weight on restoring margin. Thus, the claim often advanced by industry participants–that lower interchange fees necessarily imply lower rewards–is not supported by our analysis.

The main welfare effects of regulation, however, are not captured by rewards alone. What matters is how the cap affects the two taxes. For cash users, two opposing forces are at work. First, a lower interchange fee directly reduces their tax. Second, the tax may rise indirectly if the cap increases rewards and expands credit card adoption, thereby increasing the share of transactions subject to merchant fees. The net effect depends on the demand system. Under constant elasticity demand, the direct effect dominates, so the tax paid by cash users falls unambiguously. Under linear demand, however, the indirect effect can dominate when demand is initially relatively inelastic. The reason is that the cap lowers the weighted-average tax and raises aggregate consumption; under linear demand, this makes demand less elastic, so rewards become less effective at stimulating card users' spending. The network therefore responds by increasing rewards more aggressively, which induces greater card adoption and exacerbates cross-subsidization from cash to card users. As a result, the tax borne by cash users can rise. Hence, whether regulation benefits cash users depends not only on the reduction in interchange fees, but also on the endogenous response of rewards and card adoption.

For credit card users, the tax depends on the interchange fee, the fraction of consumers using credit, and the reward. Even so, the effect of regulation is sharper. We show that an interchange fee cap reduces the tax paid by credit card users in all the specifications we study. When rewards rise, cardholders benefit directly. When rewards fall, the reduction in the interchange fee, together with the induced adjustment in card adoption, more than offsets the decline in rewards. Consequently, the cap unambiguously lowers the effective wedge faced by credit card users.

Because consumer welfare depends on prices rather than taxes alone, we also study pass-through. Under constant elasticity demand or perfect competition among merchants, the price received by merchants is unaffected by the taxes, so any tax reduction is fully passed through to consumers. In those benchmark cases, a lower tax translates directly into a lower price. Under linear demand with imperfect competition, pass-through is incomplete. In that environment, a fee cap can lower the taxes while merchants adjust their prices in response, making the incidence of regulation more nuanced. The key implication is that credit card users still benefit, but cash users may not. This distinction is central for understanding the distributional consequences of interchange-fee regulation.

Our framework also clarifies the meaning of "subsidy" in this context. Much of the literature informally describes cash users as subsidizing card users whenever rewards are present. In our model, rewards imply that credit card users face a lower tax than cash users, but this is not necessarily a subsidy in the strict sense. A genuine subsidy arises only when the reward is large enough that the effective tax on card users becomes negative, so that card users pay a price below the amount received by merchants. In some cases, card users may even pay a price below marginal cost. The model therefore distinguishes clearly between a lower tax on card users and an actual subsidy to card use.

More broadly, the paper identifies three sources of inefficiency in the payment card system: network market power, merchant market power, and reward-induced distortions in payment choice. These distortions can generate both inefficient consumption and excessive use of credit cards. We show that an interchange fee cap can mitigate both inefficiencies when demand elasticity lies in an intermediate range. In that region, the cap reduces the relevant wedges, improves consumption efficiency, and moves payment choice closer to the social optimum. Outside that range, the regulation may improve one margin while worsening another, especially when higher rewards induce additional consumers to switch from cash to credit.

In addition to the regulation results, the model yields comparative statics for how the two taxes and merchant profits vary with merchant competition and with the substitutability between cash and credit. Under constant elasticity demand, greater competition among merchants lowers the tax paid by credit card users and, under some conditions, also lowers the tax paid by cash users. Under linear demand, the elasticity effect can reverse these results. Likewise, as cash and credit become closer substitutes, the network loses market power, which affects interchange fees, rewards, and the tax differential between the two payment methods. These results highlight a broader point: interchange fees cannot be understood in isolation from retail market structure and consumer payment choice.

By incorporating merchant market power, elastic product demand, ad valorem fees and rewards, and cash as an alternative payment method, our framework is closer to the institutional reality of payment card markets than standard tractable benchmarks. The cost of this added realism is that some results–particularly those involving interchange fee caps, endogenous rewards, card adoption, and pass-through–are not available in closed form and are therefore characterized numerically.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the model, and Section 4 characterizes the equilibrium. Section 5 examines the socially efficient outcome and interchange-fee regulation through a cap. Section 6 considers several extensions, including differential benefits from cash and credit, a prohibition on rewards, costly cash usage, and linear demand. Section 7 concludes. All proofs are collected in Appendix A.

2 Literature review

Over the last three decades, an extensive theoretical literature on two-sided markets has emerged, and many of those papers examine the implications for payment systems; see Rochet and Tirole (2006), Rysman (2009) and Jullien, Pavan, and Rysman (2021) for reviews of this literature. Our purpose here is simply to sketch the main findings related to our work and highlight our contributions.

Our modeling framework builds upon and extends the work of Wang (2010) and Shy and Wang (2011), both of which adopt a constant elasticity demand specification with ad valorem interchange fees and rewards. In particular, Wang (2010) assumes a perfectly competitive merchant sector and an exogenously fixed distribution of cash and credit card users. Among other findings, his analysis concludes that capping interchange fees reduces rewards (for demand elasticity greater than one—the case we focus on), lowers retail prices, increases consumption and thereby enhances consumer welfare.

Our analysis yields several important distinctions relative to Wang (2010). First, we show that a cap can in some circumstances lead to an *increase* in rewards, but the equilibrium tax imposed on both cash and credit card users declines (under a constant elasticity demand). This demonstrates that consumer benefits from regulation can persist even in a more realistic setting characterized by merchant market power and endogenous competition between payment instruments. However, we also establish that under a linear demand specification, a cap can potentially harm cash users. By incorporating merchant market power, an endogenous rate of credit card adoption by consumers and departing from constant elasticity demand, our framework generates several key theoretical predictions and policy implications that either fall outside the scope of Wang (2010) or meaningfully diverge from its conclusions.

Shy and Wang (2011) compare ‘proportional’ and ‘fixed’ transaction fees in a model where merchants possess market power, but cash is not available as an alternative payment method. Our primary innovation relative to Shy and Wang (2011) is the explicit inclusion of cash as an alternative payment mode. In our framework, each consumer chooses their preferred payment method, allowing the network to influence both the intensive and extensive margins of payment behavior. In the absence of cash, an indeterminacy arises: the interchange fee and reward cannot be uniquely determined.[6] Consequently, without cash, meaningful comparative statics cannot be derived to analyze how product market competition and other key parameters affect the interchange fee and rewards separately. In particular, the impact of interchange fee regulation cannot be assessed. Any cap imposed on the interchange fee becomes ineffective, as the network can adjust rewards to offset the cap, thereby maintaining the effective tax and thus prices and welfare, at their pre-regulation equilibrium levels.

A key feature of our framework is that regulation affects not only interchange fees and rewards, but also the endogenous adoption of credit cards. This adoption margin subsequently shapes the effective costs for both card and cash users. In the models of Wang (2010) and Shy and Wang (2011), where the share of credit transactions is fixed, or where cash is not a viable alternative, this channel is absent. When a payment network can influence not only the intensive margin of usage, but also the extensive

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[6] Wang (2010) resolves this indeterminacy by assuming that the network (or issuers collectively) faces a convex cost function for processing card payments, which uniquely determines the interchange fee and rewards, even though merchants accepting cards do not serve cash-paying customers. The assumption of convex transaction costs seems inconsistent with the decreasing average costs that are typical in payment networks. This is reinforced by recent technological innovations, such as cloud computing, that have significantly reduced the cost of the marginal transaction.

margin of adoption, it has stronger incentives to raise rewards. This is because the same increase in rewards stimulates card transactions more substantially when it also expands the base of card users. Consequently, in a model with a fixed adoption rate (Wang, 2010), or one that excludes cash (Shy and Wang, 2011), as our analysis reveals, a fee cap regulation is less likely to increase equilibrium rewards and thus less likely to generate outcomes that harm cash users.

Baxter (1983) developed the first formal model of interchange fees in a payment scheme. In that paper, Baxter makes three key assumptions: i) issuers and acquirers make no profit (perfect competition), ii) merchants do not use card acceptance strategically, i.e., to attract consumers from rival merchants who do not accept a card and iii) there is no merchant heterogeneity in the benefit of accepting cards. Schmalensee (2002) develops a model that explores the double marginalization problem that emerges when networks set an interchange fee that is paid to card issuers and the acquiring bank for the merchant separately sets a merchant discount for processing transactions.

In Rochet and Tirole (2002), the consumers receive a different benefit from transacting using cards rather than paying cash. There is a single payment network that sets an interchange fee, but it is not ad valorem. The retail market is a Hotelling model, but consumers face the choice of purchasing a fixed quantity of their preferred good. Rochet and Tirole (2011) develop a similar model, but here there is competition between networks modeled using the Hotelling framework. Wright (2004) uses a model similar to Rochet and Tirole (2002) and relaxes all three assumptions of Baxter (1983). Bedre-Defolie and Calvano (2013) show that networks oversubsidize card usage and overtax merchants.

Wang and Wright (2017, 2018) assume Bertrand competition among sellers in a market with many different goods that vary widely in their costs and values. The authors show that ad valorem fees and taxes represent an efficient form of price discrimination relative to uniform fees that disadvantage low-cost, low-value goods. Wang (2025) develops a structural empirical approach to a two-sided market of payments. He finds that interchange fee caps increase welfare by reducing rewards, retail prices, and credit card use. Low-income consumers, who rely less on credit, pay lower prices, but high-income consumers get hurt because their rewards decrease. In our model, while there is no income heterogeneity, we predict that any reduction in rewards will not completely offset the interchange fee reduction and lower card adoption due to a cap.

Rochet and Wright (2010) develop a model of credit card interchange fees that explicitly incorporates the credit functionality, distinguishing between "ordinary purchases" (where cash suffices) and "credit purchases" (where credit is necessary). They show that a monopoly card network always sets the interchange fee above the level that maximizes consumer surplus, and that a conservative regulatory cap based on merchants' net avoided costs from providing store credit can unambiguously raise consumer surplus relative to the unregulated outcome.

Edelman and Wright (2015) shows that an intermediary always chooses to impose price coherence if it has the ability to do so. In the context of payments, this is the equivalent of the "no surcharge" rule that precludes merchants from charging customers a different price based on how they choose to pay. An important qualification is that the fees examined in that paper are not ad valorem.

The prevailing methodology in most of the above papers relies on one or more of the following simplifying assumptions: (1) consumer demand is inelastic,[7] (2) fees are specific rather than ad valorem, (3) Bertrand competition is assumed in the product market and (4) cash transactions are not allowed, or the fraction of credit card users is taken to be exogenous. While these assumptions greatly facilitate analytical tractability, they abstract from several important features of real-world credit card markets. In addition, much of the literature assumes that the benefits and costs experienced by merchants and consumers from card usage are independent of each other and of the product price. Under this approach, interchange fees and rewards affect platform participation and usage decisions but do not feed back into retail prices. This separation rules out an important channel through which payment card fees influence market outcomes and welfare in practice. Our study aims to bridge these gaps by developing a model that incorporates a richer and more realistic set of features of the credit card market. Table 1 organizes the closest theoretical contributions according to the four assumptions outlined above, facilitating a direct comparison with our modeling approach.

Nevertheless, and in order to provide a tractable framework for analyzing our core mechanisms, we adopt the following two assumptions. First, we assume perfect competition among banks (Wang, 2010; Shy and Wang, 2011), which isolates the role of the card network's fee structure from effects of banking market power. Second, we model a mature market (Wang, 2010) where all merchants accept cards, allowing us to examine the endogenous outcomes, such as consumer payment choice and retail price adjustments, that follow widespread adoption, rather than the adoption decision itself.

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[7] Elastic demand refers to demand for the final good (not participation/usage demand for the platform or payment instrument).

Table 1: Comparison with the closest theoretical literature

	Elastic demand		Merchant power	Ad valorem	Cash
Paper	CE	non-CE			
Baxter (1983)	×	×	×	×	×
Bedre-Defolie & Calvano (2013)	×	×	✓	×	✓
Edelman & Wright (2015)	×	×	✓	×	×
Rochet & Tirole (2002)	×	×	✓	×	✓
Rochet & Tirole (2011)	×	×	✓	×	✓
Rochet & Wright (2010)	×	×	✓	×	✓
Schmalensee (2002)	×	×	×	×	×
Shy & Wang (2011)	✓	×	✓	✓	×
Wang (2010)	✓	×	×	✓	×
Wang & Wright (2017, 2018)	×	×	✓	✓	×
Wright (2004)	×	×	✓	×	✓
This paper	✓	✓	✓	✓	✓

Notes: CE = constant elasticity; non-CE = non-constant elasticity. ✓ = present; × = absent.

The available empirical research on the effects of payment card regulation documents that reductions in credit card interchange fees are generally passed on to merchants and that generosity of rewards falls, but there is also evidence to the contrary (Chan et al. (2012) for the example of Australia). The evidence of a pass through of merchant cost savings into the prices consumers pay is mixed. For the U.S., this research has focused on the effects of the caps on interchange fees charged by large debit card issuers implemented by Reg ii. Mukharlyamov and Sarin (forthcoming) explore whether gasoline prices fell in geographies where gasoline retailers were relatively more exposed to reductions in debit interchange. They could not quantify a measurable effect. One aspect of of the Reg II cap was that interchange fees actually rose for small ticket purchases. Wang et al. (2014) report the result of a survey of merchants which showed a higher proportion of merchants experienced an increase in debit acceptance costs as opposed to a decrease and were more likely to increase prices than reduce them.[8] More general surveys of the pass through of cost or tax changes in different markets find intermediate values of pass through (see the Appendix to Evans and Mateus (2011)).

Although not explicitly modeled in this paper, it is plausible that interchange fees and rewards are influenced by the incentives merchants and lenders face in extending credit to consumers. For instance, Chakravorti and To (2007) demonstrate that retailers are willing to accept higher merchant discounts

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[8] It should be noted that the modal experience reported by merchants was no change in debit acceptance costs and no change in the prices they charged.

when the proportion of illiquid consumers rises. Similarly, Chatterjee et al. (2025) develops a general equilibrium framework in which interchange fees and rewards contribute to expanding credit limits and lowering interest rates.

In our model, there are two means by which the consumers incur costs for payment services–either embedded in retail prices or fees (i.e., negative rewards). In practice, there are other costs consumers may incur. For example, one apparent consequence of the introduction of interchange fee caps for debit card transactions in the U.S. was an increase in fees charged on checking accounts and a reduction in the availability of free checking (Manuszak and Wozniak (2017), Kay et al. (2018), Mukharlyamov and Sarin (forthcoming)).

Our paper is also related to Gomes and Mantovani (2025), who study fee regulation in a two-sided platform environment with price parity and a cap on seller commissions. Their analysis shows that regulating intermediary fees can have equilibrium effects beyond the directly regulated margin, with implications for prices, profits and welfare. The key distinction from our setting lies in the adjustment channel. Gomes and Mantovani (2025) emphasize platform investment and the possibility that regulation induces a reallocation toward other seller-side fees. In our model, by contrast, a cap on the interchange fee affects rewards, credit card adoption and consumption, thereby altering the implicit taxes and prices faced by cash and credit users.

3 Structure of the market

We consider an industry consisting of n firms (merchants), $j = 1, ..., n$, producing a single homogeneous product. The output of firm j is denoted by x_j and the industry output by $X = \sum_{j=1}^{n} x_j$. All the merchants have the same cost structure $C(x) = cx$, where $c > 0$ is a constant marginal cost. The consumer price is given by an inverse demand function $P(X)$, with derivative $P_X(X) < 0$ and elasticity $\varepsilon \equiv \frac{P}{XP_X} < 0$. Consumers, when purchasing goods, can use either cash or a credit card.

Payment systems operate on a two-sided model: each time a credit card is used, the merchant incurs a fee, while the consumer may benefit from a reward. We assume that there exists a monopoly payment network denoted by $\mathcal{N}$, with $N_A < n$ acquiring banks and $N_I < n$ issuing banks.[9] Banks are homo-

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[9] American Express operates as a closed-loop network, directly setting both merchant fees and consumer rewards. In contrast, open-loop networks such as Visa and MasterCard influence merchant and consumer pricing indirectly through adjustments

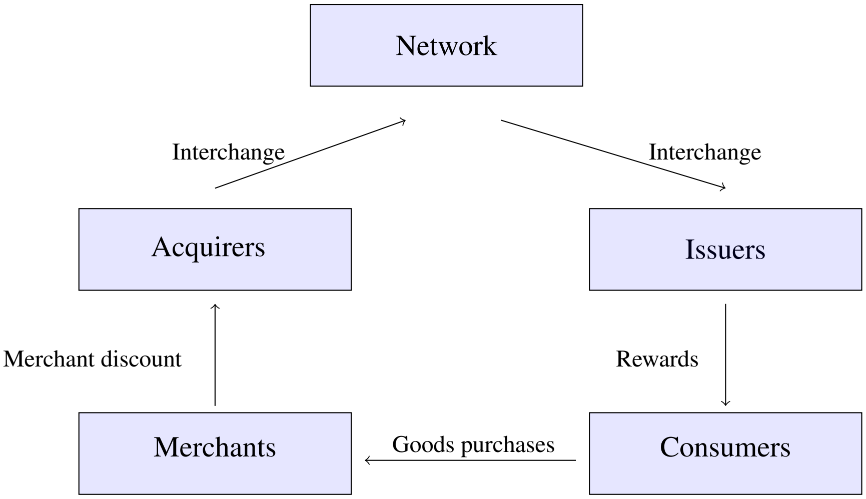

Figure 1: Payment flows in the Network

geneous and compete à la Bertrand for merchants and cardholders. In Figure 1, we present the payment flows in the network. Each acquiring bank α chooses its merchant discount m^{α} to attract merchants, and each issuing bank ι chooses a reward $R^{\iota} \leq r$ to attract cardholders and to influence the value of consumer transactions. If $R^{\iota} < 0$, then the reward becomes a fee. The network chooses the interchange fee, $i \in [0, 1]$, which becomes each acquirer's marginal cost. Part of i, denoted by r, goes to the issuing banks to fund the rewards and the rest is kept by the network.[10] For simplicity, all other fees and costs to process a transaction are assumed to be zero.[11]

The reward for the credit card is a percentage of the value of the transaction that a consumer who uses the credit card receives as cash back from the issuing bank. The acquiring bank charges a merchant discount fee, which is a percentage of the value of the transaction that is paid by the merchant to the acquiring bank when a consumer uses the credit card. We assume that the network cannot price discriminate across banks, i.e., all acquirers pay the same interchange fee to the network and all issuers receive the same fraction of the interchange fee from the network.

Consumers exhibit horizontal preferences between cash and credit, as discussed further in Section 4.4. Network fees and rewards affect the value of transactions within the payment network and can incentivize switching between the two payment modes. We assume that merchants do not impose surcharges,

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to interchange and network fees. Our modeling framework aligns with the structure and pricing mechanisms of these open-loop networks.

[10] Equivalently, the issuing banks receive the interchange fee and pay the network a network fee f, so that $r = i - f$.

[11] See Section 6.3 for an illustration of the implications of a non-zero cost to merchants of accepting cash.

meaning they cannot differentiate prices based on the payment method.[12] Furthermore, we consider a mature card market in which the extensive margin of card adoption is largely saturated (e.g., Wang (2010)). As a result, all merchants are assumed to accept both cash and credit.

We analyze a five-stage game with simultaneous and independent moves in each stage. The timing is as follows.

- Stage 1: The payment network sets the interchange fee i and determines the portion r of the fee that is passed to each issuing bank.
- Stage 2: Each acquiring bank sets its merchant discount m^{α}, and each issuing bank sets its reward R^{ι}.
- Stage 3: Each merchant chooses its output level x_j, anticipating the share of consumers who will use credit cards.
- Stage 4: Each consumer observes the rewards and the equilibrium retail price, then decides whether to use a credit card or cash.
- Stage 5: Consumers make purchases.

We solve for a subgame-perfect Nash equilibrium in pure strategies.

4 Equilibrium analysis

Although we solve the game by backward induction, the equilibrium in stage 2 is straightforward. Since acquiring and issuing banks compete à la Bertrand and are homogeneous, their equilibrium behavior is uniquely determined by the zero profit condition, regardless of the subsequent choices of merchants and consumers. We therefore present the stage 2 equilibrium first for expositional clarity.

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[12] Foster et al. (2024) find that surcharging remains relatively uncommon. Furthermore, for more than two decades (2003-2026), Australian card networks were prohibited from imposing no-surcharge rules, allowing merchants to surcharge card payments (subject to cost-of-acceptance limits). This regime was reversed in 2026 after the Reserve Bank concluded that surcharging no longer provided effective price signals and instead generated consumer confusion and enforcement difficulties in a card-dominant payments environment; see Reserve Bank of Australia (2026).

4.1 Acquiring and issuing banks

Acquiring banks compete in merchant fees m^{α}. The network interchange fee i is each acquiring bank's marginal cost. Given that acquiring banks are homogeneous and compete for merchants à la Bertrand, each acquiring bank sets the same merchant discount $m^{\alpha} = i$, for all α. Acquiring banks earn zero profits in equilibrium.

Issuing banks compete in rewards R^{ι}. Each issuing bank receives from the network part of the interchange fee $r < i$. This is the maximum amount, per dollar of transactions, that each issuing bank can give to users as a reward. (If $r < 0$, then the network fee f issuing banks pay to the network is higher than the interchange fee they receive.) Given that issuing banks are homogeneous and compete à la Bertrand for users, each issuing bank sets the same reward $R^{\iota} = r$, for all ι (it will be a fee for credit card users if $r < 0$). Issuing banks earn zero profits in equilibrium.

Therefore, in the unique equilibrium, each merchant pays a merchant discount i and each consumer receives a reward r per dollar of transaction with a credit card. The network fee, representing the per-dollar profit earned by the network, is given by $i - r$.

4.2 Consumers

Consumers make two sequential decisions. First, each consumer selects a payment mode; subsequently, they determine their level of consumption. The initial decision determines the fraction of consumers, denoted by μ, who choose to make purchases using a credit card. The determination of μ is discussed in detail in Section 4.4. Consumers who pay with cash face a price of P, whereas those who use a credit card pay a discounted price of $P \cdot (1 - r)$ (if $r > 0$).[13]

There are two goods in the model: a consumption good x and a numeraire good y, the price of which is normalized to one. Each consumer's preferences over these goods are represented by the quasi-linear utility function

$$U = \frac{kx(1/x)^{1/k}}{k-1} + y,$$

where $k > 1$. Consumers maximize their utility subject to a budget constraint, yielding a standard

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[13] We use the symbol $\cdot$ to clearly denote multiplication, as in $P \cdot (1 - r)$, to distinguish it from a functional form such as $P(X)$, particularly when the distinction is not immediately clear from context.

constant elasticity demand function with elasticity $\varepsilon = -k$. The demand for good x by a cash-paying consumer is given by $x_{ch} = \frac{1}{P^k}$, while the demand by a credit card user is $x_{cc} = \frac{1}{(P \cdot (1-r))^k}$.

Aggregate demand is then

$$X \equiv \mu x_{cc} + (1-\mu) x_{ch},$$

and the corresponding aggregate inverse demand function is

$$P(X, r, \mu) = \left(\frac{1}{X}\right)^{1/k} \left(\frac{\mu}{(1-r)^k} + 1 - \mu\right)^{1/k}. \tag{4.1}$$

4.3 Merchant competition

All merchants accept cash and credit. Also, merchants form rational expectations about credit card adoption μ. We assume that each merchant j expects a fraction μ of its sales to be paid with a credit card, while a fraction $1-\mu$ to be paid with cash, i.e., there is no expectation of a systematic sorting of cash or credit users to certain merchants, which is a reasonable assumption given that merchants are identical. Each merchant takes the rewards and the interchange fee as given and chooses its output x_j to maximize (average) profits given by

$$\begin{aligned} \pi_j &= (\mu \cdot (1-i) P(X, r, \mu) + (1-\mu) P(X, r, \mu)) x_j - c x_j \\ &= (1 - \mu i) P(X, r, \mu) x_j - c x_j, \end{aligned} \tag{4.2}$$

where $P(X, r, \mu)$, the price at the register, is given by (4.1).

In selecting its output, each merchant j conjectures that other merchants' responses will be such that $\frac{dX}{dx_j} = \lambda$, the conjectural variation λ being taken as a fixed constant throughout. The case $\lambda = 1$ corresponds to the Cournot conjecture. When $\lambda = 0$, conjectures are 'competitive' and we obtain the Bertrand outcome. When $\lambda = n$, each firm believes that all other active firms will behave exactly as it does: tacit collusion among incumbent firms then being perfect (in the sense that aggregate profits are maximized conditional on the number of firms). It will be assumed throughout that $\lambda \in [0, n]$.[14]

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[14] See Seade (1980), Bresnahan (1981) and Delipalla and Keen (1992) for similar modeling frameworks.

The first order condition of the representative merchant is (omitting arguments)

$$\frac{\partial \pi_j}{\partial x_j} = (1-\mu i)\left(P_X \frac{dX}{dx_j} x_j + P\right) - c = 0. \tag{4.3}$$

Restricting attention to symmetric equilibria, this becomes

$$\begin{aligned}(1-\mu i)\left(P_X X\gamma + P\right) = c \quad &\Rightarrow \quad P \cdot \left(1 + \frac{\gamma}{\varepsilon}\right) = \frac{c}{(1-\mu i)} \\ &\Rightarrow \quad P = \frac{c}{\left(1-\frac{\gamma}{k}\right)} \frac{1}{(1-\mu i)},\end{aligned} \tag{4.4}$$

where $\gamma \equiv \frac{\lambda}{n} \in [0,1]$.[15] As γ decreases, the merchant market becomes more competitive and price approaches marginal cost c (adjusted by the $1-\mu i$). From the merchant's perspective, a lower interchange fee has the same effect as a reduction in marginal cost and therefore the same effect on price.

Since in our case the elasticity of the slope of inverse demand, $E \equiv -\frac{P_{XX}X}{P_X}$, is $\frac{1+k}{k} < 2$, the second order condition, which is $2-\gamma E > 0$, is satisfied. The stability condition requires that $1+\gamma\cdot(1-E) > 0$ (see, for example, Seade (1980) and Delipalla and Keen (1992)), which is also satisfied. From the first-order condition (4.4), a non-negative price-cost margin implies that the elasticity must satisfy $\gamma - k < 0$, which also holds.

From equation (4.4), it follows that the average price received by merchants, P^m, is given by $(1-\mu i)P$. At this point, it is useful to introduce the expressions for the implicit tax consumers face. The tax paid by credit card users is defined as $z_{cc} \equiv \frac{1-r}{1-\mu i}$, implying that the price they pay is $P^{cc} = z_{cc}P^m$. For cash users, the tax is $z_{ch} \equiv \frac{1}{1-\mu i}$, resulting in a price of $P^{ch} = z_{ch}P^m$. An increase in the interchange fee i, or in the share of credit card users μ, raises the tax burden for all consumers. In contrast, a higher reward r reduces the tax specifically for credit card users.

The price merchants receive in the presence of a credit card is the same as the price without a credit card (i.e., using (4.4), P^m is not a function of z_{cc} or z_{ch}),

$$P^m = \frac{ck}{k-\gamma} \geq c. \tag{4.5}$$

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[15] Note that γ is similar to the conduct parameter θ in Weyl and Fabinger (2013) and Miklos-Thal and Shaffer (2021). Following most of the literature, we assume that the conduct parameter is not a function of aggregate output. For cases where the conduct parameter is a function of aggregate output, see Miklós-Thal and Shaffer (2021).

Therefore, consumers ultimately bear the full burden of a tax or receive the entire benefit of a subsidy. This outcome is a well known property of constant elasticity demand. It also implies that changes in the taxes directly reflect changes in consumer welfare. Under alternative assumptions about demand, however, a portion of the incidence of a credit card tax could fall on merchants.[16]

Efficiency dictates that $P = c$. We can have $P > c$ either because $\gamma > 0$, or $i > 0$, or both. The first source of inefficiency arises when competition in the merchant market is imperfect. The second source of inefficiency is due to the credit card tax levied by the payment network. Since $i > 0$ (otherwise network profit cannot be positive), there is the potential for double-marginalization: the first mark-up is from the merchants when they have market power and the second mark-up is from the payment network (that has market power).[17]

4.4 Endogenizing credit card and cash user shares

We assume that credit and cash are 'differentiated.' We model differentiation using the circular model of Salop (1979).[18] In particular, on a unit circumference circle, the network is located at 0, cash is located at $\frac{1}{2}$ and users/consumers are uniformly distributed on the circumference with density one. We assume that each consumer receives a fixed gross benefit $V > 0$, pays a price P^{cc} when making purchases with a credit card, a price P^{ch} from making purchases with cash and incurs a linear per-unit of distance to a payment method transportation cost $t > 0$.[19] The parameter t captures the degree of differentiation between credit and cash and hence is a measure of network market power: a higher t implies higher network market power. Thus, the consumer located at $\ell \in \left[0, \frac{1}{2}\right]$, if she uses the credit card obtains a net utility $V - P^{cc} - t\ell$ and if she uses cash obtains a net utility $V - P^{ch} - t \cdot \left(\frac{1}{2} - \ell\right)$. The fraction of consumers who use a credit card is given by

$$\mu = \frac{1}{2} + \frac{P^{ch} - P^{cc}}{t} = \frac{1}{2} + \frac{c \cdot (z_{ch} - z_{cc})}{t \cdot \left(1 - \frac{\gamma}{k}\right)}. \tag{4.6}$$

First, note that the second term of μ in (4.6) is zero when r is zero, since $z_{ch} - z_{cc} = \frac{r}{1-\mu i}$. This

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[16] See Weyl and Fabinger (2013) for a more general treatment of this point. The linear demand we consider in Section 6.4 implies an imperfect pass-through.

[17] A more detailed discussion of the first-best outcome is provided in Section 5.1.

[18] See Jeon and Rey (2022) for a similar assumption regarding differentiation between two platforms.

[19] In Section 6.1, we allow the gross benefits of credit and cash to differ.

means rewards are the key to changing payment shares (the interchange fee i also has an effect when $r \neq 0$). Second, the importance of the transportation cost t is relative to the cost of merchandise. Third, the less competitive is the retail market, i.e., higher γ, the more sensitive is μ to changes in t, all else equal and for $r \neq 0$ and that is because product prices are higher.

Equation (4.6) should be interpreted as a reduced-form payment-choice rule. For tractability, we assume that consumers' choice between credit and cash depends on the per-unit price advantage of credit relative to cash. A fully micro-founded alternative would compare the indirect utilities associated with the two payment instruments after consumers optimally choose their consumption levels. With elastic demand, however, this would make payment adoption depend nonlinearly on prices, rewards, and aggregate consumption, rendering the equilibrium characterization considerably less tractable. The reduced-form specification preserves the key economic mechanism–that rewards make credit relatively more attractive–while allowing the model to remain analytically transparent.

4.5 Network's decisions

Using (4.1) and (4.4) the equilibrium aggregate output, for any given interchange fee and reward, is

$$\begin{aligned}
X &\equiv \mu x_{cc} + (1-\mu)x_{ch} = \frac{(k-\gamma)^k}{(ck)^k}\left(\frac{\mu \cdot (1-\mu i)^k}{(1-r)^k} + (1-\mu)(1-\mu i)^k\right) \\
&\quad \left(\text{using } z_{cc} \equiv \frac{1-r}{1-\mu i} \text{ and } z_{ch} \equiv \frac{1}{1-\mu i}\right) \\
&= \frac{(k-\gamma)^k}{(ck)^k}\left(\frac{\mu}{z_{cc}^k} + \frac{1-\mu}{z_{ch}^k}\right), \qquad (4.7)
\end{aligned}$$

where μ is given by (4.6).

The network's profit equals the interchange fee minus the rewards paid to consumers (i.e., the margin per dollar of credit card transactions, which is the network fee, $f = i - r$), multiplied by the total value of transactions made by consumers using a credit card. Formally, this is given by $(i-r)P\mu x_{cc}$, where P denotes the price paid by consumers at the register (see (4.4)). We assume that transaction processing costs are zero.[20] The network profit can therefore be expressed as follows

$$\pi_{\mathcal{N}}(i, r; \mu) = \frac{c\mu \cdot (i - r)}{\left(1 - \frac{\gamma}{k}\right)(1 - \mu i)} x_{cc}(i, r). \tag{4.8}$$

Using the credit card taxes $z_{cc} \equiv \frac{1-r}{1-\mu i}$ and $z_{ch} \equiv \frac{1}{1-\mu i}$, we express (4.8) as an explicit function of these taxes

$$\pi_{\mathcal{N}}(z_{cc}, z_{ch}; \mu) = \frac{c \cdot ((\mu z_{cc} + (1 - \mu) z_{ch}) - 1)}{\left(1 - \frac{\gamma}{k}\right)} x_{cc}(z_{cc}). \tag{4.9}$$

Expressing the network's profit in terms of the implicit taxes z_{cc} and z_{ch}, rather than the interchange fee i and reward r, simplifies the analysis by making the economic trade-offs more transparent. Although μ remains endogenous in both formulations, the tax representation directly captures the wedge between consumer prices and merchant receipts, turning the profit function into an intuitive function of a weighted average tax and credit card consumption. This clarifies how changes in these taxes affect the network's margin, consumer adoption and transaction volume without the added algebraic complexity introduced by the interactions between i, r, and μ.

The network profit function, after we substitute (4.6) into (4.9), is given by

$$\pi_{\mathcal{N}}(z_{cc}, z_{ch}) = \frac{k \left(\frac{k-\gamma}{ck}\right)^{k} c \cdot \left(\left(\left(\frac{z_{cc}}{2} + \frac{z_{ch}}{2} - 1\right) t - c(z_{cc} - z_{ch})^2\right) k - \frac{t\gamma \cdot (z_{cc} + z_{ch} - 2)}{2}\right) z_{cc}^{-k}}{t \cdot (k - \gamma)^2}. \tag{4.10}$$

The network chooses z_{cc} and z_{ch} to maximize (4.10). For example, the network can increase z_{ch} by increasing the interchange fee i and it can keep z_{cc} fixed by simultaneously increasing the reward r. The weighted average tax, $\mu z_{cc} + (1 - \mu) z_{ch}$, affects network profits. Because $z_{ch} > 1$ (because $i > 0$) the network can even subsidize credit card users by choosing $z_{cc} < 1$ and still make positive profits ($\mu z_{cc} + (1 - \mu) z_{ch} > 1$). Later in this section, we show when this will happen in equilibrium.

Using (4.9) and (4.6), let's understand the complex trade-offs the network faces when it changes the taxes for cash and credit card users. As a benchmark, observe that when there are no credit card users, $\mu = 0$, then $z_{ch} = 1$ and the network profit is zero. First, we consider the trade-off with respect to the tax for cash users. When z_{ch} increases, profits increase, holding μ fixed. However, from (4.6), the share of credit card users, μ, also increases, which reduces profits per transaction assuming $r > 0$ ($z_{ch} > z_{cc}$),

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[20] Including a constant marginal processing cost would not qualitatively affect our results.

which is confirmed in equilibrium. Since credit card users 'cost' the network relatively more than cash users, due to the rewards, any increase in the share of credit at the expense of cash lowers profits per transaction, all else equal. Finally, because $z_{ch} \equiv \frac{1}{1-\mu i}$, a higher μ reinforces the increase in z_{ch}. Second, we consider the trade-off with respect to the tax for credit card users. When the network increases z_{cc}, profits increase while holding μ and individual consumption x_{cc} fixed. Additionally, μ decreases, leading to further profit increases per transaction as discussed above. However, individual consumption using a credit card, x_{cc}, decreases, illustrating the usual trade-off between margin and volume. Finally, a lower μ mitigates the increase in z_{ch}, since $z_{cc} \equiv \frac{1-r}{1-\mu i}$.

Lemma 1 *Let* $z_{cc} > 0$ *and* $z_{ch} > 0$*, and restrict attention to the economically relevant domain in which the implied credit share satisfies* $\mu \in [0,1]$ *and the network's weighted-average tax margin,* $\mu z_{cc} + (1-\mu) z_{ch} - 1$*, is nonnegative. On this domain, the network profit function,* $\pi_{\mathcal{N}}(z_{cc}, z_{ch})$*, is quasi-concave in* (z_{cc}, z_{ch}).

Quasi-concavity implies that the unique interior maximizer identified in Proposition 1 below is also the global maximizer within the economically relevant domain.

Proposition 1 *Suppose* $t < \frac{16ck}{k-\gamma}$*, or equivalently* $t < 16P^m$*. Then, the network's profit-maximization problem has a unique interior maximizer within the economically relevant domain given by*

$$z_{cc}^* = \frac{(16c-t)k + \gamma t}{16c(k-1)} \text{ and } z_{ch}^* = \frac{(16c+3t)k^2 - (3\gamma+4)tk + 4\gamma t}{16(k-1)ck}. \tag{4.11}$$

Cash users always pay a higher tax in equilibrium, $z_{ch}^* - z_{cc}^* = \frac{t \cdot (k-\gamma)}{4ck} = \frac{t}{4} P^m > 0$, which implies that, $\mu = \frac{3}{4}$, i.e., 75% of consumers use a credit card and 25% use cash.[21] It also implies that consumers receive rewards in equilibrium, $r^* > 0$.

Using (4.11), (4.6), and $z_{cc} \equiv \frac{1-r}{1-\mu i}$ and $z_{ch} \equiv \frac{1}{1-\mu i}$, we can derive the unique equilibrium interchange fee and reward

$$i^* = \frac{12k^2 t - ((12\gamma+16)t - 64c)k + 16\gamma t}{(48c+9t)k^2 - (9\gamma+12)tk + 12\gamma t} \text{ and } r^* = \frac{4(k-\gamma)(k-1)t}{(16c+3t)k^2 - (3\gamma+4)tk + 4\gamma t}. \tag{4.12}$$

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[21] When we allow for different gross benefits between cash and credit, the equilibrium share of credit ranges between 0 and 1; see Section 6.1.

Without cash as an alternative payment mode (e.g., Shy and Wang (2011)), i and r cannot be uniquely determined.[22] In this scenario, only the tax for credit card users, $z \equiv \frac{1-r}{1-i}$, is pinned down. Consequently, meaningful comparative statics cannot be performed.

To illustrate the quantitative implications of (4.12), suppose that $\gamma = 0$ and normalize $c = 1$. We then choose k and, for each such k, calibrate t so that the model matches the empirical benchmark $r^* = 1.45\%$, consistent with the average reward rate reported by Wang (2025) and Egan et al. (2026). The required values are $t \approx 0.0651$ when $k = 10$, $t \approx 0.0617$ when $k = 20$ and $t \approx 0.0607$ when $k = 30$. The corresponding interchange fees are $i^* \approx 14.59\%$, $i^* \approx 8.02\%$, and $i^* \approx 5.83\%$, respectively.[23] Thus, holding rewards fixed at an empirically realistic level, a higher demand elasticity lowers the equilibrium interchange fee, moving it closer to observed U.S. values, which are between $2\% - 3\%$.[24]

The equilibrium network profits, after substituting (4.11) into (4.10), are given by

$$\pi_{\mathcal{N}}(z_{cc}^*, z_{ch}^*) = \frac{((16c-t)k+\gamma t)\left(\frac{16(k-\gamma)}{ck}\right)^k \left(\frac{(16c-t)k+\gamma t}{c(k-1)}\right)^{-k}}{16(k-1)(k-\gamma)}. \tag{4.13}$$

The equilibrium total merchant profit is

$$\Pi^m(z_{cc}^*, z_{ch}^*) = (P^m - c)X = \frac{c\gamma}{k-\gamma}\frac{(k-\gamma)^k}{(ck)^k}\left(\frac{3}{4(z_{cc}^*)^k} + \frac{1}{4(z_{ch}^*)^k}\right)$$

$$= \frac{c\gamma(ck)^{-k}(k-\gamma)^{k-1}\left(3 \cdot 16^k \left(\frac{(16c-t)k+\gamma t}{k-1}\right)^{-k} + \left(\frac{(16c+3t)k^2-(3\gamma+4)tk+4\gamma t}{16(k-1)ck}\right)^{-k}\right)}{4}. \tag{4.14}$$

Remark 1 (The role of transportation cost t) *In two-sided market models with cross-group network effects, a relatively high transportation cost is often needed to sustain an interior non-tipping equilib-*

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[22] Alternatively, the values of i and r can be uniquely identified–even in the absence of an endogenous determination of cash and credit card users–under the assumption that issuing banks incur a convex cost when processing transactions, Wang (2010). However, as discussed in footnote 6, this assumption is inconsistent with the declining average costs that are typical in payment networks.

[23] Related empirical evidence suggests that relatively high merchant demand elasticities are plausible in this setting. Egan et al. (2026) report that, under full pass-through of interchange fees into prices, their estimates for restaurants imply a demand elasticity of about 7.9.

[24] The fit is nonetheless imperfect for at least two reasons. First, to keep the analysis tractable, we abstract from competition among payment networks. Second, under our assumption of a mature market, we do not impose a merchant participation constraint for card acceptance. Both features would likely reduce fees in practice, so the model tends to overstate equilibrium interchange fees relative to the data. That said, the qualitative mechanisms emphasized in our analysis would remain operative in a richer and more realistic environment, even if additional forces were also present. The fit also improves if consumers attach a higher valuation to credit than to cash; see Section 6.1.

rium, e.g., Armstrong (2006). In our model, by contrast, this specific network effect is absent. Here, transportation cost has two opposing effects on the share of credit users: for given taxes, a higher t reduces switching between cash and credit, but it also increases the equilibrium tax difference between the two payment methods, since $z^*_{ch} - z^*_{cc} = t \cdot (k-\gamma)/(4ck)$. These two effects exactly offset, so that the equilibrium share is $\mu^* = 3/4$, independent of t. Thus, a higher t does not stabilize the interior outcome; instead, it widens the equilibrium tax wedge by increasing the equilibrium reward and makes corner outcomes more likely. Hence, Proposition 1 requires t to be relatively low for an interior solution.

4.6 Comparative statics

We analyze how the key parameters of our model, i.e., demand elasticity (k), merchant market power (γ), and network market power (t), influence the equilibrium outcomes, with particular attention to the implicit taxes borne by credit card and cash users. In each of the three propositions presented in this section, we vary one key parameter while holding the others constant, allowing us to isolate and examine its specific effects.

4.6.1 Effect of demand elasticity

We summarize the effect of demand elasticity in the following Proposition.

Proposition 2 *The equilibrium taxes as a function of the demand elasticity k are given as follows:*

a) *Relatively low demand elasticity: $1 < k < \gamma + \frac{16c}{t}$.*[25] *Both groups of consumers pay a tax. Credit card users receive a reward $r^* \geq 0$. Cash users pay a higher tax than credit card users, $z^*_{ch} > z^*_{cc} > 1$.*

   - *Credit card and cash users pay a higher price than the price merchants receive: $P^{ch} = z^*_{ch}P^m = \frac{(16c+3t)k^2-(3\gamma+4)tk+4\gamma t}{16(k-\gamma)(k-1)} > P^{cc} = z^*_{cc}P^m = \frac{(16c-t)k^2+\gamma kt}{16(k-1)(k-\gamma)} > P^m = \frac{ck}{k-\gamma}$.*

b) *Relatively high demand elasticity: $k > \gamma + \frac{16c}{t}$.*[26] *Cash users pay a tax, while credit card users are subsidized by receiving a relatively high reward, $z^*_{ch} > 1 > z^*_{cc}$.*

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[25] From the condition in Proposition 1, $\gamma + \frac{16c}{t} > 1$ always holds.

[26] This condition can be rewritten in terms of t as $t > \frac{16c}{k-\gamma}$. Since $\frac{16c}{k-\gamma} < \frac{16ck}{k-\gamma}$, the case we are examining is valid for relatively high values of t, among the permissible values from the condition in Proposition 1.

  - *Credit card users pay a lower price than the price merchants receive:* $P^{cc} = z^*_{cc} P^m = \frac{(16c-t)k^2+\gamma kt}{16(k-1)(k-\gamma)} < P^m = \frac{ck}{k-\gamma}$.
  - *If* $k > \overline{k}$, *with* $\overline{k} \geq \gamma + \frac{16c}{t}$, *then the price credit card users pay is even below marginal cost,* $P^{cc} < c$.[27]

c) *As demand elasticity increases (higher* k*):*

  - *the tax credit card users pay,* z^*_{cc}, *decreases,*
  - *the difference between the taxes,* $z^*_{ch} - z^*_{cc}$, *weakly increases, strictly so if* $\gamma > 0$,
  - *if* $k < \frac{4}{3}$, *the tax cash users pay,* z^*_{ch}, *decreases,*[28]
  - *if* $k > \frac{4}{3}$, *the tax cash users pay,* z^*_{ch}, *decreases if and only if* $t < \frac{16k^2c}{3\gamma k^2 - 8\gamma k + k^2 + 4\gamma}$.[29]

Some academics and practitioners have argued that cash users effectively subsidize credit card users when the latter receive rewards (see, for example, Schuh et al. (2010) and Felt et al. (2023)). In this paper, however, we adopt a different interpretation of the term. We define the credit card taxes borne by both card and cash users and show that, when rewards are positive, cash users face a higher effective tax than card users. Proposition 2 demonstrates that when demand elasticity is relatively high, credit card users are subsidized: they pay a lower price than the price received by merchants (even below marginal cost if the elasticity is really high), which corresponds to the equilibrium price in an all-cash economy-implying that merchants bear none of the tax burden. Conversely, when demand elasticity is low, credit usage imposes a tax on both consumer groups, though the burden is lighter for credit card users due to the rewards. This results in a shared, albeit unequal, distribution of the tax burden between cash and credit card users.

4.6.2 Effect of merchant market power

We summarize the effect of merchant market power in the Proposition below.

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[27] Where $\overline{k} \equiv \frac{(16c+t)\gamma+16c+\sqrt{256(\gamma+1)^2c^2+32\gamma(\gamma-1)tc+\gamma^2t^2}}{2t}$. The condition in terms of t can be expressed as $t > \frac{16c((1+\gamma)k-\gamma)}{k(k-\gamma)}$. It can be verified that $\frac{16c}{k-\gamma} < \frac{16c((1+\gamma)k-\gamma)}{k(k-\gamma)} < \frac{16ck}{k-\gamma}$. So, among all permissible values from the condition of Proposition 1, this case is valid for the highest values of t.

[28] When the gross benefits of cash and credit differ, as discussed in Section 6.1, this threshold becomes a function of the respective benefits. The same adjustment applies to the subsequent results and propositions. In this section, however, we assume equal gross benefits across payment modes, as allowing them to differ would not qualitatively affect our results.

[29] The threshold is decreasing as γ increases, if and only if $k > 2$.

Proposition 3 *As merchant market power decreases (lower γ):*

- *a) the equilibrium interchange fee, $i^* > 0$, decreases if and only if product demand elasticity is relatively low, $k < \frac{4}{3}$,*
- *b) the equilibrium reward, $r^* > 0$, increases,*
- *c) the equilibrium tax of credit card users, z^*_{cc}, decreases,*
- *d) the equilibrium tax of cash users, z^*_{ch}, decreases if and only if product demand elasticity is low, $k < \frac{4}{3}$,*
- *e) equilibrium network profits, $\pi_{\mathcal{N}}(z^*_{cc}, z^*_{ch})$, increase and*
- *f) aggregate merchant equilibrium profits, $\Pi^m(z^*_{cc}, z^*_{ch})$, are non-monotonic in merchant market power: as γ falls, they decrease for sufficiently low γ and increase for γ close to one.*

The effect of merchant competition arises from the influence of γ on the fraction of credit card users, μ. It can be understood as follows. From (4.6), as γ decreases, μ also decreases for a fixed $z_{ch} - z_{cc} > 0$. Intuitively, in a more competitive merchant market, prices are lower. Since taxes are ad valorem, a tax differential in favor of credit provides a smaller advantage to credit relative to cash. Therefore, the network has an incentive to increase $z_{ch} - z_{cc} > 0$ to restore its market share. This is achieved by decreasing z_{cc} and increasing z_{ch} or decreasing z_{ch} but at a lower rate.

Aggregate merchant profits can increase as competition in the product market intensifies. This occurs because the network, which competes with cash, lowers the tax on credit card users, who, in the base case of equal gross benefits between cash and credit, constitute the majority of consumers, thereby encouraging increased consumption. In other words, there is a range of values for the market conduct parameter in which the double marginalization problem is first order in economic significance. This result contrasts with the findings of Shy and Wang (2011), who show that aggregate merchant profits monotonically decrease as the market becomes more competitive. Higher consumption also benefits the network profits.

4.6.3 Effect of network market power

We summarize the effect of differentiation between cash and credit, which is used as a proxy for network market power, in the Proposition below (most parts of the proof are straightforward and are omitted).

Proposition 4 *As network market power decreases (lower t):*

a) the equilibrium tax cash users pay, z_{ch}^*, decreases if and only if product demand elasticity is above a threshold, $k > \frac{4}{3}$,

b) the equilibrium interchange fee i^* decreases if and only if product demand elasticity is above a threshold, $k > \frac{4}{3}$,

c) the equilibrium tax credit card users pay, z_{cc}^*, increases,

d) the equilibrium reward, r^*, decreases,

e) the difference between the taxes, $z_{ch}^* - z_{cc}^*$, decreases,

f) the equilibrium network profit, $\pi_{\mathcal{N}}(z_{cc}^*, z_{ch}^*)$, decreases, and

g) aggregate merchant equilibrium profits decrease when $\gamma > 0$ and are unchanged when $\gamma = 0$.

The finding that the tax for credit card users increases as network market power decreases might seem counterintuitive. One might expect that, as the network faces stronger competition from cash, it would increase the rewards offered to credit card users, thereby lowering the tax for these consumers. The intuition is as follows. From (4.6), and given that $z_{ch} > z_{cc}$, a lower t increases the share of credit card users, holding the taxes fixed. As we have already discussed, this leads to lower network profits per transaction (because credit card users cost the network relatively more than cash users). The network lowers the rewards and the difference between the two taxes to mitigate this negative effect. Aggregate consumption falls negatively impacting network and merchant profits.

5 Social efficiency and regulation

5.1 Social efficiency

Relative to the social optimum, our model identifies three primary sources of inefficiency. First, network market power leads to inefficiency, as the monopoly network sets an interchange fee above the marginal cost of processing transactions, which is assumed to be zero. Second, merchant market power introduces further inefficiency. Together, these two sources create a double marginalization problem, both contributing to distorted consumption patterns. Third, consumer behavior may be inefficiently influenced by rewards, which can incentivize the use of a payment method that deviates from the consumer's preferred choice in the absence of cross-subsidization. Specifically, in our model, cash and cards are imperfect substitutes, with consumer preferences represented along a Salop circle. Rewards can induce consumers to incur transportation costs that exceed the socially optimal level.

To address these distortions, a social planner would set $i = r = 0$, or equivalently $z_{cc} = z_{ch} = 1$, resulting in an equal (50%) distribution of consumers between credit and cash; see (4.6).[30] Additionally, the planner would set the price P equal to the marginal cost c.

In what follows, we do not explicitly compare the relative importance of the effects of i and r on consumption to their effects on transportation costs. But we do show how regulation affects them both and show when an interchange fee cap could improve welfare in both dimensions. We emphasize that this is not always the case.

5.2 Regulation

We examine a regulatory intervention that imposes a cap, reducing the interchange fee from its private equilibrium level.[31] Our primary focus is on the implications of this policy for consumer welfare and overall social efficiency. Since the regulation caps i directly, we need to express the network's profit function, given in equation (4.10), as a function of i. Using the expression for μ, given by (4.6), and

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[30] In Section 6.1, we allow for different valuations of cash and credit and, in that case, the socially optimal mix of payments could be different from 50%. In addition, if we allow for a differential in the physical cost of accepting cash versus cards, this too would affect the planner's decision; see Section 6.3 for an illustration

[31] In this section, we are presenting results for the constant elasticity demand specification. In Section 6.4, we will revisit our results under the assumption of linear demand.

$z_{ch} \equiv \frac{1}{1-\mu i}$ and $z_{cc} \equiv \frac{1-r}{1-\mu i}$, we solve for i and r

$$i = \frac{2t \cdot (z_{ch} - 1)(\gamma - k)}{z_{ch} \cdot (t \cdot (\gamma - k) - 2ck \cdot (z_{ch} - z_{cc}))}, \quad r = 1 - \frac{z_{cc}}{z_{ch}}.$$

Next, we invert this system to express z_{ch} and z_{cc} as functions of i and r

$$z_{ch}(i,r) = \frac{t \cdot (k - \gamma)}{4cikr} \left((2 - i) - \sqrt{(2 - i)^2 - \frac{16cikr}{t \cdot (k - \gamma)}} \right) \text{ and } z_{cc}(i,r) = (1 - r)\, z_{ch}(i,r). \quad (5.1)$$

We substitute (5.1) into the network profit function (4.10), which now becomes $\pi_{\mathcal{N}}(i,r)$ (similar to (4.8), but with endogenous μ). From the first-order condition of the network profit with respect to r, we derive the effect of i on the equilibrium reward, evaluated at the equilibrium values given by (4.12)

$$\frac{dr^*}{di} = -\frac{\frac{\partial^2 \pi_{\mathcal{N}}(i^*,r^*)}{\partial i \partial r}}{\frac{\partial^2 \pi_{\mathcal{N}}(i^*,r^*)}{\partial r^2}}. \quad (5.2)$$

At the interior optimum, the second-order condition for the network's choice of r requires $\frac{\partial^2 \pi_{\mathcal{N}}(i^*,r^*)}{\partial r^2} < 0$. Therefore, if $\frac{\partial^2 \pi_{\mathcal{N}}(i^*,r^*)}{\partial i \partial r} > 0$, implying that i and r are complementary in equilibrium, a marginal cap on the interchange fee would lead the network to reduce the reward. Conversely, if $\frac{\partial^2 \pi_{\mathcal{N}}(i^*,r^*)}{\partial i \partial r} < 0$, implying that i and r are substitutable in equilibrium, the network would increase the equilibrium reward. This subtlety turns out to be important when thinking about policy and welfare.

The effect of the regulation on the equilibrium reward constitutes an intermediate step, but it is not synonymous with changes in consumer welfare. To assess welfare implications, it is necessary to examine the impact on equilibrium taxes. We differentiate equations (5.1) with respect to i, yielding the partial derivatives $\frac{\partial z^*_{ch}}{\partial i}$ and $\frac{\partial z^*_{cc}}{\partial i}$. These expressions incorporate the direct effect of i, as well as the indirect effect on the equilibrium reward, captured by $\frac{dr^*}{di}$, and are evaluated at the equilibrium defined by equation (4.12).

Given the complexity of the underlying expressions, tractable analytical solutions are not available. We therefore characterize the effects of regulation using numerical simulations. To assess the robustness of the findings, we conduct extensive sensitivity analyses over a broad range of admissible parameter configurations, including many beyond those reported in the main text. Across all cases examined, the qualitative patterns remain unchanged. Throughout this section, we therefore present our main findings as results rather than propositions, to reflect that they are established numerically rather than through general analytical proofs. Accordingly, these findings should be interpreted as robust numerical regularities for the class of parameter values we study.

To assess the impact of i on r^*, we developed a MATLAB script, which is available upon request, that iterates over numerous admissible parameter configurations of (t, c, γ) and, for each configuration, identifies a unique threshold value of demand elasticity $k > 1$ separating two regimes. Below this threshold, a marginal reduction in the interchange fee raises the equilibrium reward; above it, the reward falls.[32] By continuity, the sign of the derivative remains unchanged on a non-degenerate interval $(\underline{i}, i^*]$ around the unregulated equilibrium, so the result applies to a neighborhood of modest fee caps rather than only to an infinitesimal change; see Ronnen (1991) for a similar argument. We summarize the findings below.

Result 1 *There exists an interval $(\underline{i}, i^*]$, such that if the interchange fee is set within $(\underline{i}, i^*]$, where i^* is the unregulated equilibrium interchange fee, the equilibrium rewards will increase if and only if demand elasticity is lower than a threshold.*

In policy debates over credit card interchange fees, the claim that reducing these fees necessarily reduces cardholder rewards is often treated as axiomatic. In our model, however, we find that this holds only when demand is relatively elastic. When demand is less elastic, a reduction in interchange fees can actually lead to an increase in rewards. This result is not limited to the case of constant elasticity demand; it also holds under a linear demand specification (see Section 6.4).[33]

The core intuition is as follows. A forced reduction in the interchange fee, i, lowers the network's margin, $i - r$. The network can respond by either lowering the reward, r, to partially restore the margin or increasing r to stimulate consumption. When demand elasticity is relatively low, a reduction in i does not lead by itself to a significant increase in consumption. In this case, the network finds it profitable to offset part of the lost revenue by encouraging additional spending, from existing and 'new' cardholders,

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[32] For each admissible parameter configuration, the script evaluates $\frac{dr^*}{di}$, given by (5.2), at the unregulated equilibrium and identifies the threshold value of k at which its sign changes. For example, when $t = c = 1$, the threshold values are $k \approx 5.0938$ when $\gamma = 1$, $k \approx 4.8424$ when $\gamma = 0.5$, and $k \approx 4.6164$ when $\gamma = 0$.

[33] There is some empirical evidence that interchange fee caps do not uniformly reduce credit card rewards. While the Reserve Bank of Australia's reforms were expected to diminish the generosity of rewards programs, banks responded in varied ways. Some introduced new premium cards with higher interchange fees to support more generous rewards, while others enhanced rewards on existing products–particularly through the addition of American Express companion cards, which increased reward rates without raising annual fees (Chan et al. (2012)).

through an increase in r.[34] By contrast, when demand elasticity is high, consumption responds strongly to the lower price induced by a lower i. This allows the network to recoup some of its lost revenue by lowering r to partially restore its margin, while still benefiting from the higher transaction volume.

The impact of an interchange fee on consumption depends critically on the equilibrium credit card taxes borne by consumers (recall that the price merchants receive is not affected by the tax, so any tax change is passed on to consumers). These taxes, $z_{ch} \equiv \frac{1}{1-\mu i}$ and $z_{cc} \equiv \frac{1-r}{1-\mu i}$, are determined by three variables: i and μ for all consumers and additionally r for credit card users. The regulation directly reduces i; the central question is how μ and r adjust in response. A decrease in μ–that is, a reduction in the proportion of consumers using credit cards–unambiguously lowers the tax burden on *all* consumers. Moreover, μ and r are positively related, all else equal: a lower reward decreases the fraction of credit card users. Consequently, when equilibrium rewards decrease, cash users are unambiguously better off (because both i and μ decrease and hence $1 - \mu i$ increases), whereas credit card users face two additional opposing forces: higher $1 - r$, but also higher $1 - \mu i$. Conversely, when equilibrium rewards increase, both types of users experience conflicting effects, since i and μ move in opposite directions; however, credit card users are more likely to benefit. In the remaining of the section we examine the net effect of the regulation in greater detail.

First, we examine the sign of the partial derivatives $\frac{\partial z^*_{cc}}{\partial i}$ and $\frac{\partial z^*_{ch}}{\partial i}$. As with Result 1, we have developed a MATLAB script, available upon request, that iterates over numerous configurations of (t, c, γ, k). For each configuration, $\frac{\partial z^*_{ch}}{\partial i}$ and $\frac{\partial z^*_{cc}}{\partial i}$, evaluated at $i = i^*$, are strictly positive, indicating that an interchange fee cap reduces taxes across all levels of demand elasticity and for all consumer types. We summarize the findings below.

Result 2 *There exists an interval $(\underline{i}, i^*]$, such that if the interchange fee is set within $(\underline{i}, i^*]$, where i^* is the unregulated equilibrium interchange fee, the equilibrium taxes and prices for credit and cash users decrease.*

Thus, even for the case when a reduction in interchange fees reduces rewards, which happens under relatively high demand elasticity, credit card users are not made worse off because, on net, the credit card tax falls, reducing the prices they ultimately pay.

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[34] This relationship holds under a modest reduction in the interchange fee. In contrast, a substantial decrease would erode the margin and necessitate a downward adjustment of the reward by the network. Despite this, the qualitative result–that a lower interchange fee may lead to higher rewards–continues to hold even for reductions such as 10% (see Section 6.4).

The regulation induces a reallocation of surplus from the payment network to consumers and merchants. Because the net price received by merchants is independent of the credit card tax, the tax reduction does not affect their per-unit revenue. However, by lowering the final price faced by consumers, the regulation stimulates the transaction volume. This increase in aggregate consumption raises total merchant profits whenever $\gamma > 0$. Thus, surplus is transferred from the network to both sides of the market. In the limiting case where $\gamma = 0$, merchant profits are zero and the transfer accrues solely to consumers.

Next, we turn to the effect of an interchange fee cap on optimal payment choice. To do that, we compare the magnitudes of $\frac{\partial z_{cc}^*}{\partial i}$ and $\frac{\partial z_{ch}^*}{\partial i}$. As with Results 1 and 2, we have developed a MATLAB script, which is available upon request, that iterates over numerous admissible parameter configurations of (t, c, γ). For each configuration, it identifies a unique threshold value of demand elasticity $k > 1$ separating two regimes: $\frac{\partial z_{cc}^*}{\partial i} > \frac{\partial z_{ch}^*}{\partial i}$ if and only if k is below a threshold k.[35] This comparison is important because it determines whether the gap $z_{ch}^* - z_{cc}^* = \frac{r^*}{1-\mu^* i^*} > 0$ narrows or widens following the regulation. This gap, in turn, affects μ, which–as discussed in Section 5.1–is socially excessive, primarily due to $r^* > 0$. Under the assumptions of the model, the gap narrows if and only if demand elasticity exceeds a threshold. Therefore, for high values of demand elasticity, the regulation helps mitigate the inefficiency associated with excessive card usage. In contrast, when elasticity is low, the gap widens, thereby exacerbating the social inefficiency. We summarize below.

Result 3 *There exists an interval $(\underline{i}, i^*]$, such that if the interchange fee is set within $(\underline{i}, i^*]$, where i^* is the unregulated equilibrium interchange fee, the inefficiency associated with excessive credit card usage is mitigated if and only if demand elasticity exceeds a threshold.*

When the equilibrium taxes fall, the effect on consumption efficiency is, in general, ambiguous. For cash users, since $z_{ch}^* > 1$, consumption moves closer to the first-best outcome (which, as discussed in Section 5.1, is $z_{cc} = z_{ch} = 1$ and $P = c$). For credit card users, the effect depends on the level of z_{cc}^*, which can be either greater or less than 1 (see Proposition 2). If $z_{cc}^* > 1$–a situation that arises when demand elasticity is relatively low ($k < \gamma + \frac{16c}{t}$)–consumption moves toward the efficient outcome. Conversely, if $z_{cc}^* < 1$, which occurs when demand elasticity is relatively high ($k > \gamma + \frac{16c}{t}$), a lower tax for credit card users may lead to a deviation from the first-best. As discussed in Proposition 2, for values of k even higher than $\gamma + \frac{16c}{t}$, that is $k > \overline{k}$, credit card users are subsidized to the extent that they pay a

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[35] For example, setting $t = c = 1$, $k \approx 4.5543$ when $\gamma = 1$, $k \approx 4.2637$ when $\gamma = 0.5$, and $k = 4$ when $\gamma = 0$.

price below the marginal cost of production c, so their consumption is inefficiently high. A lower tax in this case exacerbates this inefficiency.

Result 4 *There exists an interval* $(\underline{i}, i^*]$*, such that if the interchange fee is set within* $(\underline{i}, i^*]$*, where* i^* *is the unregulated equilibrium interchange fee, the consumption inefficiency is mitigated if demand elasticity is relatively low, that is* $k < \overline{k}$*.*

According to Result 3, the social inefficiency arising from excessive credit card usage is alleviated under an interchange fee cap regulation when the demand elasticity is relatively high. Conversely, Result 4 demonstrates that the inefficiency associated with suboptimal consumption levels is mitigated when the demand elasticity is relatively low. This raises the question of whether a single regulatory intervention can simultaneously address both types of inefficiencies. The answer is affirmative: there exists a range of values for k within which an interchange fee cap regulation mitigates both forms of social inefficiency. Specifically, by setting $t = c = 1$, when $\gamma = 0$, the relevant interval is $k \in (4, 16)$; when $\gamma = 0.5$, the interval expands to $k \in (4.2637, 24.17)$; and when $\gamma = 1$, it further extends to $k \in (4.5543, 32.5)$. We summarize below.

Result 5 *There exists a range of intermediate demand elasticities, such that a modest interchange fee regulation not only benefits all consumers by reducing credit card taxes and prices but also enhances overall social welfare by promoting consumption closer to the first-best level and curbing excessive card usage.*

Overall, these results imply that the effects of an interchange fee cap operate through equilibrium adjustments in rewards, card adoption, and the prices paid by cash and credit users, rather than through the reduction in the fee alone. A modest cap lowers the effective tax and hence the price paid by both consumer groups, although rewards may either increase or decrease depending on demand elasticity. The welfare gains are greatest at intermediate demand elasticities, where the cap improves both consumption efficiency and payment choice.

6 Extensions

6.1 Cash and credit have different benefits

We extend our baseline model to account for differential benefits (or costs) associated with cash and credit transactions. Specifically, we assume that a consumer derives a gross benefit of V_{ch} when purchasing with cash, and a gross benefit of V_{cc} when purchasing with credit. Define the difference in benefits as $\Delta \equiv V_{cc} - V_{ch}$. When credit offers greater non-pecuniary benefits than cash, $\Delta > 0$; conversely, when cash is more beneficial, $\Delta < 0$. The mass of consumers who choose to use a credit card, as originally defined in equation (4.6), is now re-expressed as follows:

$$\mu = \frac{1}{2} + \frac{\Delta + P^{ch} - P^{cc}}{t} = \frac{1}{2} + \frac{\Delta}{t} + \frac{c \cdot (z_{ch} - z_{cc})}{t \cdot \left(1 - \frac{\gamma}{k}\right)}.$$

There are no changes in the other parts of the model. The same quasi-concavity argument as in Lemma 1 applies on the economically relevant nonnegative-margin domain. The interior solution is economically meaningful provided $\Delta > \frac{t}{2} - \frac{2\sqrt{ctk}}{\sqrt{k-\gamma}}$ together with the maintained condition that the implied credit share μ lies in $[0, 1]$. The equilibrium taxes are as follows

$$\begin{aligned} z^*_{cc} &= \frac{(-k+\gamma)t^2 + 4((4c+\Delta)k - \Delta\gamma)t - 4\Delta^2(k-\gamma)}{16(k-1)ct} \\ z^*_{ch} &= \frac{16ck^2t + (k-\gamma)(t-2\Delta)(2k\Delta + 3kt - 4t).}{16ctk(k-1)} \end{aligned}$$

The equilibrium share of credit is $\mu = \frac{\Delta}{2t} + \frac{3}{4}$. Note that $\mu \in (0, 1)$ if and only if $\Delta \in \left(-\frac{3t}{2}, \frac{t}{2}\right)$. The difference in equilibrium taxes is $z^*_{ch} - z^*_{cc} = \frac{(k-\gamma)(t-2\Delta)}{4ck}$ and is decreasing in Δ.

The equilibrium interchange fee and rewards are as follows

$$\begin{aligned} i^* &= \frac{4t\left[16ckt + (k-\gamma)(t-2\Delta)(2k\Delta + 3kt - 4t)\right]}{(2\Delta + 3t)\left[16ck^2t + (k-\gamma)(t-2\Delta)(2k\Delta + 3kt - 4t)\right]} \\ r^* &= \frac{4t(k-1)(k-\gamma)(t-2\Delta)}{(\gamma - k)(2\Delta - t)(2k\Delta + 3kt - 4t) + 16ck^2t}. \end{aligned} \tag{6.1}$$

Using the same values of k and the same normalizations as in the calibration discussed after (4.12), set $\mu = 0.85$, which implies $\Delta = 0.2t$ from $\mu = \frac{\Delta}{2t} + \frac{3}{4}$. Recalibrating t so that the model continues to match the empirical benchmark $r^* = 1.45\%$, we obtain $t \approx 0.1087$ when $k = 10$, $t \approx 0.1030$ when $k = 20$, and $t \approx 0.1012$ when $k = 30$. The corresponding interchange fees are $i^* \approx 13.04\%$, $i^* \approx 7.25\%$, and $i^* \approx 5.31\%$, respectively. Thus, allowing for $\Delta > 0$ improves the fit, but only modestly: for these benchmark values of k, the implied interchange fee / merchant discount rate remains well above the observed U.S. benchmark of roughly 2.5%.

The effect of Δ on the equilibrium taxes is given by

$$\frac{dz^*_{cc}}{d\Delta} = \frac{(k-\gamma)(t-2\Delta)}{4(k-1)ct} \text{ and } \frac{dz^*_{ch}}{d\Delta} = -\frac{((t+2\Delta)k-2t)(k-\gamma)}{4(k-1)tck}.$$

Credit card users pay a higher tax when the benefit of credit increases relative to cash, since $\frac{dz^*_{cc}}{d\Delta} > 0$. At the same time, the equilibrium reward r^* is decreasing in Δ and satisfies $r^* = 0$ at $\Delta = \frac{t}{2}$, when all consumers use credit. For cash users, $\frac{dz^*_{ch}}{d\Delta} > 0$ if and only if $(t + 2\Delta)k < 2t$, or equivalently, when $t + 2\Delta > 0$, if $k < \frac{2t}{t+2\Delta}$. Thus, the tax on cash users increases only when demand elasticity is sufficiently low. The effect of Δ on the interchange fee is more nuanced: as stated in Proposition 5, i^* decreases when $k(t + 2\Delta) > 2t$, while otherwise the sign of $\frac{\partial i^*}{\partial \Delta}$ is ambiguous.

The equilibrium profit of the network is given by

$$\pi_{\mathcal{N}} = \frac{16^k \left(\left(-\frac{t^2}{16} + \left(c + \frac{\Delta}{4}\right)t - \frac{\Delta^2}{4}\right)k + \frac{\gamma(t-2\Delta)^2}{16}\right)\left(\frac{(-4\Delta^2+4\Delta t+16ct-t^2)k+\gamma(t-2\Delta)^2)}{(k-1)ct}\right)^{-k}\left(\frac{k-\gamma}{ck}\right)^k}{t(k-\gamma)(k-1)}.$$

It can verified that $\pi_{\mathcal{N}}$ is decreasing in Δ, for all $\Delta \in \left(\frac{t}{2} - \frac{2\sqrt{ctk}}{\sqrt{k-\gamma}}, \frac{t}{2}\right)$.[36] When the benefit of credit, relative to cash, increases, the network is worse off.

We summarize in the following Proposition.

Proposition 5 *As the benefits to consumers of credit, relative to cash, increase (or the cost of cash relative to credit increases), i.e., Δ increases, the following hold:*

a) The equilibrium share of credit card users, $\mu = \frac{\Delta}{2t} + \frac{3}{4}$, increases,

b) the equilibrium tax for credit card users, z^*_{cc}, increases,

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[36] Specifically, $\pi_{\mathcal{N}}$ is convex in Δ, is minimized at $\Delta = \frac{t}{2}$ and has an asymptote at $\Delta = \frac{t}{2} - \frac{2\sqrt{ctk}}{\sqrt{k-\gamma}}$.

c) the equilibrium tax for cash users, z^*_{ch}, increases if and only if $(t+2\Delta)k < 2t$. Equivalently, when $t + 2\Delta > 0$, this condition is $k < \frac{2t}{t+2\Delta}$,

d) the tax differential $z^*_{ch} - z^*_{cc}$ is decreasing and becomes zero when all consumers use credit, that is when $\Delta = \frac{t}{2}$,

e) the equilibrium reward, r^*, is monotonically decreasing and becomes zero when all consumers use credit, that is when $\Delta = \frac{t}{2}$,

f) when $k(t + 2\Delta) > 2t$, the equilibrium interchange fee i^* is decreasing in Δ. Equivalently, if $t + 2\Delta > 0$, this condition is $k > \frac{2t}{t+2\Delta}$. Otherwise, the sign of $\partial i^*/\partial\Delta$ is ambiguous, and

g) the equilibrium profit of the network is decreasing.

Equilibrium taxes on credit card users increase for two primary reasons. First, as the benefits associated with credit usage rise, the payment network faces reduced competitive pressure from cash transactions, allowing it to lower the rewards offered to users. Second, a higher proportion of credit card users leads to an increase in the effective tax borne by all consumers. While the network may attempt to mitigate this effect by reducing the interchange fee, such adjustments are insufficient to fully counterbalance the underlying forces. A key implication of higher taxes on credit card users–who constitute the majority of consumers–is a decline in aggregate consumption, which in turn reduces the network's overall profitability.

6.2 Would cash users be better off if rewards were prohibited?

Many scholars and industry experts argue that interchange fees are elevated, in part because they help finance rewards for credit card users. Consequently, consumers who pay with cash may be disadvantaged. To explore this issue, we force $r = 0$, which implies a single tax $z \equiv \frac{1}{1-\mu i}$ that applies uniformly to all consumers. From equation (4.6), it follows that $\mu = \frac{1}{2}$. Substituting into equation (4.9), the network's profit function becomes

$$\pi_{\mathcal{N}} = \frac{(k-\gamma)^k (z-1)c}{(ck)^k z^k \left(1 - \frac{\gamma}{k}\right)},$$

which is maximized at

$$z^* = \frac{k}{k-1}.$$

Next, using the equilibrium tax for cash users when rewards are permitted, as given by equation (4.11), we find:

$$z_{ch}^{*} - z^{*} = \frac{(k-\gamma)t \cdot (3k-4)}{16(k-1)ck}, \tag{6.2}$$

which is positive if and only if $k > \frac{4}{3}$.[37] We summarize in the Proposition below.

Proposition 6 *Suppose rewards for credit card transactions are prohibited, i.e.,* $r = 0$. *In that case, the equilibrium tax on cash users decreases, relative to when rewards are allowed, if and only if demand elasticity is relatively high, that is,* $k > \frac{4}{3}$.

Under conditions of relatively high demand elasticity, our model predicts that prohibiting credit card rewards results in a lower effective tax on cash users, thereby improving their welfare. Conversely, when demand elasticity is low, cash users are better off when rewards are permitted. The underlying mechanism is as follows: banning rewards reduces the prevalence of credit card usage (from $\mu = \frac{3}{4}$ to $\mu = \frac{1}{2}$), which, all else equal, lowers the cross-subsidization burden borne by cash users. However, in response to the decline in transaction volume, the card network may raise the interchange fee to recoup lost revenue. If the increase in the interchange fee is limited, the net effect is a reduction in the equilibrium tax on cash users. When demand elasticity is high, the network faces stronger disincentives to raise fees substantially, as doing so would significantly suppress aggregate consumption.

6.3 Merchants incur a cash cost – An Illustration

In our baseline model, it becomes apparent that the introduction of credit reduces welfare by exposing consumers to taxation. This conclusion, however, rests on two key simplifying assumptions in our model: (i) the use of cash incurs no cost and (ii) the gross benefit from both payment methods, denoted by V, is identical. In this extension, we relax the first assumption.

Suppose now that merchants incur a cost χ associated with accepting, safeguarding and transporting cash, including potential losses due to employee theft. Accordingly, the profit function of merchant j,

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[37] As in Proposition 3, the threshold is not fixed at $\frac{4}{3}$; rather, it becomes a function of the gross benefits when the two payment modes differ in this respect, as modeled in Section 6.1. In this section, we deliberately abstract from such differences in gross benefits, as allowing for them would not qualitatively alter our results.

originally given in (4.2), is modified as follows

$$\begin{aligned}\pi_j &= (\mu \cdot (1-i)P(X,r,\mu) + (1-\mu)(1-\chi)P(X,r,\mu))\, x_j - cx_j \\ &= (1 - \mu i - (1-\mu)\chi)\, P(X,r,\mu)x_j - cx_j.\end{aligned}$$

Following the same steps as in the analysis in Section 4.3, the tax expressions become

$$z_{ch} \equiv \frac{1}{1-\mu i-(1-\mu)\chi}, \quad z_{cc} \equiv \frac{1-r}{1-\mu i-(1-\mu)\chi}. \tag{6.3}$$

In the absence of credit (i.e., $\mu = 0$), cash users face a tax of $z \equiv \frac{1}{1-\chi}$. When $\chi > 0$, the network's profit function becomes significantly more complex than in (4.10), precluding closed-form solutions. Consequently, we rely on numerical simulations. Setting $\chi = 0.03$ implies a 3% cost of using cash, resulting in an all-cash benchmark tax of

$$z^{\text{cash-only}} = \frac{1}{1-\chi} \approx 1.0309.$$

We further assume $t = 0.2$, $c = 2$, $\gamma = 0.5$, and $k = 35$. In equilibrium, we find $\mu^* \approx 0.9114$, indicating that approximately 91% of consumers adopt credit. The corresponding equilibrium tax rates are

$$z^*_{cc} \approx 1.0286 \qquad \text{and} \qquad z^*_{ch} \approx 1.0691.$$

Thus,

$$z^*_{cc} < z^{\text{cash-only}} < z^*_{ch}.$$

Therefore, relative to the no-credit all-cash benchmark, the approximately 91% of consumers who use credit face a lower tax burden, while cash users face a higher tax burden. Naturally, for alternative parameter values–particularly when k is low–the introduction of credit may render all consumers worse off.

The purpose of this extension is to demonstrate that, within our model, the introduction of credit can make the vast majority of consumers better off when cash handling entails a reasonable cost, even if the gross benefit of the two payment modes, i.e., the V in the indirect utility, is identical. We do not assess the plausibility of the parameter values under which this outcome arises.

6.4 Linear demand

While the assumption of constant elasticity simplifies the analysis and facilitates the derivation of several novel results, it is not without its limitations. A well known drawback is that it leads to a one-for-one increase in the price at the register in response to a rise in credit card taxes. This implies perfect pass-through and that the entire tax incidence falls on consumers, a result that may limit the policy relevance of our findings.

To address this concern, albeit at the expense of some clarity and tractability, we adopt an alternative framework with linear product demand. This specification offers a balance of analytical tractability and a demand elasticity that varies with aggregate output. This variation introduces an *elasticity effect*, the direction of which is determined by the sign of the derivative of the demand elasticity with respect to output. Under linear demand, this sign is positive; that is, market demand becomes less elastic as aggregate output increases. In this environment, the incidence of a credit card cost is shared between merchants and consumers, except in the case of a perfectly competitive merchant market.

This result aligns with the taxonomy of demand functions proposed by Mrázová and Neary (2019), who define a demand function as *subconvex* if $\log p$ is concave in $\log x$. This is equivalent to a demand function that is less convex than a constant elasticity function and one whose (absolute) elasticity of demand decreases with output, precisely the case for linear demand. Subconvexity is sometimes called "Marshall's Second Law of Demand". In contrast, *superconvex* demand, where elasticity increases with output, can lead to tax over-shifting, whereby consumers bear more than 100% of a tax. Such an outcome is often considered less plausible, further justifying our focus on the linear, subconvex case for a more realistic analysis of pass-through.[38]

Our main focus in this extension is on the welfare implications, particularly the counterparts of Results 1 and 2 in Section 5.2, which examine the effects of an interchange fee cap regulation.

Suppose individual demand is linear $x = 1 - P$. The inverse aggregate demand, accounting for rewards r and the share of credit card users μ, is given by

$$P(X, \mu, r) = \frac{1 - X}{\mu \cdot (1 - r) + (1 - \mu)}. \tag{6.4}$$

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[38] Mrázová and Neary (2017, 2019) provide a comprehensive discussion of how superconvexity and subconvexity determine competition effects and relative pass-through.

Combining (6.4) with (4.4), we derive the equilibrium aggregate consumption as a function of the two taxes

$$X(z_{ch}, z_{cc}; \mu) \equiv \mu x_{cc} + (1-\mu)x_{ch} = \frac{1 - cz_{ch} \cdot (1-\mu) - cz_{cc}\mu}{1+\gamma}. \tag{6.5}$$

Using (4.6) and the demand elasticity, $\varepsilon = -\frac{1-X}{X}$, we derive the share of credit card users[39]

$$\mu(z_{ch}, z_{cc}; X) = \frac{1}{2} + \frac{P^{ch} - P^{cc}}{t} = \frac{1}{2} + \frac{c \cdot (z_{ch} - z_{cc})}{1 - \frac{\gamma X}{1-X}}. \tag{6.6}$$

We solve (6.6) and (6.5) simultaneously with respect to X and μ, to obtain $X(z_{ch}, z_{cc})$ and $\mu(z_{ch}, z_{cc})$, either analytically (when $\gamma = 0$) or numerically (when $\gamma > 0$). The profit of the network, from (4.9), is

$$\pi_{\mathcal{N}}(z_{ch}, z_{cc}; X, \mu) = \frac{c \cdot ((\mu z_{cc} + (1-\mu)z_{ch}) - 1)}{\left(1 - \frac{\gamma X}{1-X}\right)} x_{cc}(z_{cc}), \tag{6.7}$$

where $x_{cc} = 1 - cz_{cc}$. We substitute $X(z_{ch}, z_{cc})$ and $\mu(z_{ch}, z_{cc})$ into (6.7). We restrict the marginal cost to be less than the maximum willingness to pay, i.e., $c < 1$. For individual consumption to be strictly positive, we require $x_{cc} > 0$ and $x_{ch} > 0$, which is equivalent to: $z_{cc} < \frac{1}{c}$ and $z_{ch} < \frac{1}{c}$. These conditions imply positive aggregate consumption $X > 0$. Strictly positive network profits additionally require the weighted-average tax to exceed one: $z_{ch} \cdot (1-\mu) + z_{cc}\mu > 1$.

6.4.1 Equilibrium in the absence of a credit card

We digress briefly in this subsection to derive the equilibrium for the benchmark case without credit. By setting $\mu = 0$ and $z_{ch} = 1$ in (6.5), we obtain the following expressions

$$\widetilde{X} = \frac{1-c}{1+\gamma}, \qquad \widetilde{P} = \frac{\gamma + c}{1+\gamma}, \qquad \widetilde{\Pi} = \frac{\gamma(1-c)^2}{(1+\gamma)^2} \tag{6.8}$$

where $\widetilde{X}$ is the aggregate equilibrium output, $\widetilde{P}$ is the equilibrium price all consumers pay and all merchants receive and $\widetilde{\Pi}$ is the equilibrium merchant aggregate profit, in the absence of the payment network. This benchmark equilibrium will serve as a point of comparison when we later compute the tax incidence due to the credit card.

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[39] For simplicity, and without significant loss of generality, we set the transportation cost parameter to $t = 1$.

6.4.2 Perfectly competitive merchant market, $\gamma = 0$

When the merchant market is perfectly competitive ($\gamma = 0$), the model yields tractable closed-form solutions, which we present as a useful benchmark (but still, results about the effects of the regulation are numerical). In this limiting case, $\frac{\gamma}{\varepsilon} = \frac{\gamma X}{1-X} = 0$ and the price received by merchants remains unaffected by the tax. Consequently, the final price paid by consumers reflects a one-to-one pass-through of the tax. Even in this limiting setting, however, our framework yields a new result, relative to constant elasticity demand, regarding the effect of a regulation on the equilibrium taxes, that we discuss below. In the next subsection, we generalize the analysis by allowing for imperfect competition ($\gamma > 0$) and hence imperfect pass-through.

The simultaneous solution of (6.6) and (6.5) yields

$$\begin{aligned} X(z_{ch}, z_{cc}) &= 1 + c^2(z_{ch} - z_{cc})^2 - \frac{c \cdot (z_{ch} + z_{cc})}{2} \\ \mu(z_{ch}, z_{cc}) &= \frac{1}{2} + c \cdot (z_{ch} - z_{cc}). \end{aligned} \tag{6.9}$$

After substituting (6.9) into (6.7), we derive the network profit as a function of the two taxes

$$\pi_{\mathcal{N}}(z_{cc}, z_{ch}) = \frac{c \cdot (2(z_{ch} - z_{cc})^2 c - z_{ch} - z_{cc} + 2)(c z_{cc} - 1)}{2}. \tag{6.10}$$

We differentiate $\pi_{\mathcal{N}}(z_{cc}, z_{ch})$ with respect to z_{cc} and z_{ch}. The following three sets of roots satisfy the first order conditions (with equality)

$$\begin{aligned} z_{cc} &= \frac{1}{c} \text{ and } z_{ch} = \frac{5 \pm \sqrt{17 - 16c}}{4c} \\ z_{cc} &= \frac{16c + 15}{32c} \text{ and } z_{ch} = \frac{16c + 23}{32c}. \end{aligned}$$

Only the last set of roots also satisfies the second order conditions. Hence, the taxes that maximize the network profit, subject to $X(z_{ch}, z_{cc}) \in (0, 1)$ and $\mu(z_{ch}, z_{cc}) \in [0, 1]$, are given by

$$z_{cc}^* = \frac{16c + 15}{32c} \text{ and } z_{ch}^* = \frac{16c + 23}{32c}. \tag{6.11}$$

The equilibrium aggregate consumption is $X^* = \frac{15}{32} - \frac{c}{2}$, so the interior solution is valid for $X^* > 0 \Rightarrow c < \frac{15}{16} = 0.9375$.

Using (6.11), together with $z_{ch} \equiv \frac{1}{1-\mu i}$ and $z_{cc} \equiv \frac{1-r}{1-\mu i}$, we derive the equilibrium interchange fee and rewards

$$i^* = \frac{4(23-16c)}{3(23+16c)} \text{ and } r^* = \frac{8}{23+16c}. \tag{6.12}$$

Note that $i^* < 1 \Leftrightarrow c > \frac{23}{112} \approx 0.205$. Therefore, the equilibrium is interior if and only if $c \in \left(\frac{23}{112}, \frac{15}{16}\right)$.

An interchange fee cap regulation. We will now express the network profit function in terms of i and r. We use $\mu(z_{ch}, z_{cc})$, from (6.9), together with $z_{ch} \equiv \frac{1}{1-\mu i}$ and $z_{cc} \equiv \frac{1-r}{1-\mu i}$ and solve with respect to z_{ch} and z_{cc}. We obtain the following two sets of solutions

$$\begin{aligned} z^1_{ch}(i,r) &= \frac{-i+2+\sqrt{-16cir+i^2+4(1-i)}}{4cir} \\ z^1_{cc}(i,r) &= -\frac{4\left(\left(\frac{i}{8}-\frac{1}{4}\right)\sqrt{-16cir+i^2+4(1-i)}+cir-\frac{i^2}{8}+\frac{i}{2}-\frac{1}{2}\right)(-1+r)}{cir(i-2-\sqrt{-16cir+i^2+4(1-i)})} \end{aligned}$$

and

$$\begin{aligned} z^2_{ch}(i,r) &= \frac{-i+2-\sqrt{-16cir+i^2+4(1-i)}}{4cir} \\ z^2_{cc}(i,r) &= -\frac{4\left(\left(-\frac{i}{8}+\frac{1}{4}\right)\sqrt{-16cir+i^2+4(1-i)}+cir-\frac{i^2}{8}+\frac{i}{2}-\frac{1}{2}\right)(-1+r)}{cir(i-2+\sqrt{-16cir+i^2+4(1-i)})}. \end{aligned}$$

We begin by substituting the two candidate solution sets into Equation (6.10). To simulate the effect of an interchange fee cap, we exogenously reduce the interchange fee i by 10% from its benchmark equilibrium level defined in Equation (6.12). We then solve the model by maximizing profit with respect to the reward r.[40] Our analysis identifies the second solution set as the economically relevant one.

Given the complexity of the resulting expressions, we proceed numerically. We examine how the equilibrium changes by varying the cost parameter c. This parameter influences aggregate consumption X, which in turn determines the demand elasticity. It is through this channel that c ultimately governs

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[40] We confirmed that our qualitative conclusions are robust to smaller reductions in the interchange fee, such as 1% or 0.1%.

the direction of the change in the equilibrium reward.

We present illustrative results for three different values of c in Table 2. Most of these results are consistent with those derived under the constant elasticity demand, with one notable exception. Under a linear demand, when elasticity is relatively low (i.e., $c = 0.4$), an interchange fee cap can make cash users worse off by increasing the tax they pay.[41]

It is important to note that the numerical results reported in Table 2, and in Tables 3 and 4 later, should be viewed as illustrating the direction of effects and relative impacts rather than their precise magnitudes (see also the discussion on this issue right after equation (4.12)).

Table 2: Equilibrium Effects of a 90% Interchange Fee Cap under Linear Demand and $\gamma = 0$

	$c = 0.4$		$c = 0.45$		$c = 0.5$	
Variables	Unregulated (i^*)	Regulated ($0.9i^*$)	Unregulated (i^*)	Regulated ($0.9i^*$)	Unregulated (i^*)	Regulated ($0.9i^*$)
Interchange Fee (i)	0.7528	0.6776	0.6976	0.6278	0.6452	0.5806
Reward (r)	0.2721	*0.3715*	0.2649	*0.2979*	0.2581	0.2538
Cash Tax (z_{ch})	2.2969	*2.3613*	2.0972	1.9008	1.9375	1.7141
Credit Tax (z_{cc})	1.6719	1.4841	1.5417	1.3345	1.4375	1.2791
Credit Share (μ)	0.7500	0.8508	0.7500	0.7548	0.7500	0.7175
Network Profit ($\pi_{\mathcal{N}}$)	0.1097	0.1000	0.0938	0.0851	0.0791	0.0725
Consumption (X)	0.2690	0.3540	0.2440	0.3370	0.2188	0.2990
Elasticity (ε)	-2.72	-1.82	-3.10	-1.97	-3.57	-2.34

Notes: Regulation caps the interchange fee at 90% of its unregulated equilibrium value. *Red italicized entries* indicate cases in which the regulated value of rewards or taxes exceeds the corresponding unregulated value.

Key Observations and Intuition from Table 2

- Effect on Rewards (r): The network's strategic response to the cap on i depends on demand elasticity.
  - Relatively Low Elasticity ($c = 0.4$, $\varepsilon \approx -2.72$ and $c = 0.45$, $\varepsilon \approx -3.10$): A lower i in itself does not significantly boost total consumption. The network finds it more profitable to increase r to stimulate spending, partially offsetting the lost revenue from the lower fee.
  - Relatively High Elasticity ($c = 0.5$, $\varepsilon \approx -3.57$): A lower i in itself triggers a substantial

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[41] The simulations underlying Table 2 were produced with MATLAB code. The code computes equilibrium outcomes across a wide range of parameter configurations and all results are qualitatively similar to those presented in the table. The code is available from the authors upon request.

increase in total consumption. The network lowers r to recoup some margin, relying on the volume increase to protect its profits.

These results align with the findings and intuition under the constant elasticity demand specification (see Result 1).

- Effect on Credit Users: The price for credit users is determined by the tax $z_{cc} \equiv \frac{1-r}{1-\mu i}$.[42] A change in z_{cc} is driven by changes in i, r and μ:
  - Direct Effect of $\downarrow i$: A lower interchange fee directly increases the denominator $(1 - \mu i)$, which lowers z_{cc} (a beneficial force).
  - Indirect Effect via r:
    - If r increases (low elasticity), the numerator $(1 - r)$ decreases, which further lowers z_{cc} (reinforcing the benefit).
    - If r decreases (high elasticity), the numerator $(1 - r)$ increases, which raises z_{cc} (a countervailing force).
  - Indirect Effect via μ: A change in the credit share μ affects the denominator.
    - If μ increases (low elasticity), due primarily to higher rewards, it diminishes the beneficial effect of lower i in the denominator.
    - If μ decreases (high elasticity), due primarily to lower rewards, it amplifies the beneficial effect of lower i in the denominator.

  Net Effect: The results show that the direct, beneficial effect of the lower interchange fee dominates the potentially countervailing movements in r and μ in all cases, leading to a lower final tax z_{cc} and thus a lower price for credit users. These results are consistent with the ones derived under the constant elasticity demand (see Result 2).

- Effect on Cash Users: The price for cash users is determined by the tax $z_{ch} \equiv \frac{1}{1-\mu i}$. A change in z_{ch} is driven by changes in i and μ:
  - Direct Effect of $\downarrow i$: A lower interchange fee directly increases the denominator $(1 - \mu i)$, which lowers z_{ch} (a beneficial force).

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[42] The price paid by credit card users is given by $P^{cc} = z_{cc} P^m$. Since $P^m = c$, and hence is unaffected by z_{cc}, it is sufficient to focus on z_{cc} to determine the direction of the change in the price paid by credit card users. The same reasoning applies to cash users.

  - Indirect Effect via μ: The change in credit share μ is critical.
    - If μ increases substantially (low elasticity, $c = 0.4$), due to higher rewards, it decreases the denominator $(1 - \mu i)$, which raises z_{ch}. The regulation harms the cash users.
    - If μ is stable or decreases (higher elasticity, $c = 0.45, 0.5$), due to lower rewards, the countervailing force through μ is weak or becomes reinforcing. This allows the direct benefit of lower i to dominate, lowering z_{ch} and benefiting cash users.

  Under the constant elasticity demand specification, we find no evidence that cash users incur a welfare loss following the regulation (see Result 2). In contrast, under linear demand, such losses can occur. The intuition is as follows.

  Under linear demand, a decline in the weighted-average tax increases aggregate consumption, which in turn makes demand less elastic. This mechanism does not arise under constant elasticity demand, where elasticity remains unchanged as consumption increases. As a result, when initial demand is relatively inelastic, the payment network has a stronger incentive under linear demand to increase rewards in order to stimulate consumption by credit card users and offset the induced decline in elasticity. The larger reward, in turn, encourages greater credit card adoption and may leave cash users worse off. It is worth emphasizing that pass-through remains complete, so this result is driven entirely by the elasticity effect. Once $\gamma > 0$, however, an additional channel emerges through merchants' pricing responses: because demand becomes less elastic, merchants raise the prices they charge in order to capture part of the benefit from the reduction in the weighted-average tax.

- Effect on Credit Share (μ): The share of credit users, $\mu = \frac{1}{2} + c \cdot (z_{ch} - z_{cc})$, changes primarily with the tax differential $(z_{ch} - z_{cc})$.
  - When r increases (low elasticity), the gap $(z_{ch} - z_{cc})$ widens, making credit more attractive and increasing μ.
  - When r decreases (high elasticity), the gap $(z_{ch} - z_{cc})$ narrows, making credit less attractive and decreasing μ.
- Network Profit ($\pi_{\mathcal{N}}$): As expected, the interchange fee cap reduces the network's profit in all scenarios, as it directly constrains a key instrument and forces a suboptimal (from the network's perspective) adjustment of r and μ.

As in the constant elasticity demand model, a regulation that caps the interchange fee impacts the three endogenous variables—i, r, and μ—which in turn influences the effects of both forms of taxation. Our analysis under linear demand reveals that, while such a regulation can improve welfare for both consumer groups (as was the case under constant elasticity), it cannot worsen welfare for both. This latter result is new; in the constant elasticity model, both groups were always made better off.

The outcome highlights a clear trade-off. A deterioration for cash users—which occurs when rewards rise under relatively inelastic demand, exacerbating the cross-subsidization—typically benefits credit card users. Conversely, when rewards fall under relatively elastic demand, the reduction in card adoption mitigates cross-subsidization and benefits cash users. Credit-card users may still benefit because the direct reduction in the interchange fee lowers their effective tax.

In summary, given that merchant profits remain at zero, the regulation reallocates surplus away from the payment network. This surplus is transferred either to all consumers, if they are collectively better off, or, in scenarios where cash users are made worse off, to credit card users, effectively resulting in a transfer from both the network and cash users to cardholders.

6.4.3 Imperfectly competitive merchant market, $\gamma > 0$

The payment network chooses z_{cc} and z_{ch} to maximize its profit, as given by equation (6.7). Under imperfect competition, incomplete pass-through implies that the incidence of an interchange fee cap depends on the intensity of product market competition. Inspecting (6.7) reveals how merchant market power, captured by γ, interacts with demand elasticity through the price. Specifically, $-\frac{\gamma}{\varepsilon} = -\frac{\gamma X}{1-X}$, indicating that the impact of elasticity becomes more pronounced as γ increases—at $\gamma = 0$, this effect vanishes.[43] This introduces a novel *elasticity effect* not present under constant elasticity demand or linear demand with perfect competition.

When $\varepsilon' > 0$ (as with linear, subconvex, demand), a decrease in aggregate output X makes demand more elastic. In this setting, in addition to the effects already identified by our analysis so far, an increase in the network's taxes z_{cc} and z_{ch} generates a negative effect on marginal network profitability: a higher tax reduces aggregate output X, which in turn increases demand elasticity, leading merchants to lower

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[43] This is intuitive. In a perfectly competitive market, demand elasticity has no effect on pricing, as price is driven to marginal cost regardless of elasticity. In contrast, for a monopolist, market elasticity plays a crucial role in determining price.

their prices and thereby reducing the network's revenue. A lower γ–reflecting stronger product market competition–weakens this effect. Consequently, the network optimally responds by increasing its tax.[44]

To measure the extent to which consumers bear the burden of the credit card tax, we define the tax incidence for each payment group. Let $\widetilde{P}$ denote the equilibrium price in an all-cash economy (the benchmark without credit cards, see (6.8)). The incidence for credit card and cash users is then given by

$$\tau_{cc} \equiv \frac{P^{cc} - \widetilde{P}}{P^{cc} - P^{m}}, \qquad \tau_{ch} \equiv \frac{P^{ch} - \widetilde{P}}{P^{ch} - P^{m}}.$$

A value of τ near one implies that consumers bear nearly all of the tax burden, meaning the credit card tax is passed through almost entirely to consumers. Conversely, a value near zero implies that merchants bear most of the burden.

As in the perfect competition case, our results are numerical. Following the structure of Table 2 for the case of $\gamma = 0$, we present, in Tables 3 and 4 below, equilibrium values–both unregulated and regulated–for different levels of γ, in order to assess the impact of merchant market power.[45]

Many of the main findings obtained under perfect competition, regarding the impact of an interchange fee cap regulation, carry over when we allow for imperfect competition. In particular, an interchange fee cap continues to raise the equilibrium reward when demand elasticity is relatively low (i.e., $c = 0.4$ and $c = 0.45$) and to reduce the reward when elasticity is relatively high (i.e., $c = 0.5$). Moreover, across all cases, all consumers face lower credit card taxes following the regulation.

There are, however, new results stemming from imperfect pass-through. When the network lowers its taxes, merchants respond by raising their price, an optimal response to the less elastic demand, making the net effect on consumer prices, in general, ambiguous. From Tables 3 and 4, we see that credit card users pay a lower price, but cash users pay a higher price when $c = 0.4$, even though the taxes decline in all instances.[46] Cash users are worse off when card adoption increases significantly after the regulation,

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[44] This mechanism is closely related to the effect of competition on markups discussed in Mrázová and Neary (2017). As the authors note on page 3840: "Hence, if globalization reduces incumbent firms' sales in their home markets, it is associated with a higher elasticity and so a lower markup if and only if demand is subconvex."

[45] These tables are computed using the same MATLAB code as Table 2. For space considerations, we report results only for two representative values of γ, as all additional simulations yield qualitatively similar outcomes. The code is available from the authors upon request. One caveat applies: we focus on values of c and γ that yield an interior equilibrium. For instance, when c and/or γ are high, cash users are priced out of the market; when c is low, the equilibrium interchange fee exceeds 100%.

[46] When $\gamma = 0$ and $c = 0.4$ (Table 2), the tax for cash users increases post-regulation, so they pay a higher price. Under imperfect competition, the network cuts the tax, but merchants increase their prices in response; as a result, the tax does not

Table 3: Equilibrium Effects of a 90% Interchange Fee Cap under Linear Demand and $\gamma = 0.1$

	$c = 0.4$		$c = 0.45$		$c = 0.5$	
Variables	Unregulated	Regulated	Unregulated	Regulated	Unregulated	Regulated
	(i^*)	($0.9i^*$)	(i^*)	($0.9i^*$)	(i^*)	($0.9i^*$)
Interchange Fee (i)	0.7433	0.6689	0.6891	0.6202	0.6379	0.5741
Reward (r)	0.2673	*0.3504*	0.2605	*0.2916*	0.2541	0.2510
Cash Tax (z_{ch})	2.2596	2.2570	2.0698	1.8909	1.9172	1.7095
Credit Tax (z_{cc})	1.6557	1.4661	1.5306	1.3395	1.4300	1.2804
Credit Share (μ)	0.7500	0.8326	0.7500	0.7596	0.7500	0.7229
Network Profit ($\pi_{\mathcal{N}}$)	0.1128	0.1041	0.0960	0.0883	0.0807	0.0746
Consumption (X)	0.2521	0.3278	0.2278	0.3069	0.2037	0.2731
Elasticity (ε)	-2.9664	-2.0507	-3.3901	-2.2583	-3.9086	-2.6621
Agg. Mer. Profit (Π)	0.0035	0.0067	0.0031	0.0064	0.0027	0.0053
Price cc users pay (P^{cc})	0.6854	0.6165	0.7097	0.6307	0.7338	0.6652
Price ch users pay (P^{ch})	0.9354	*0.9491*	0.9597	0.8903	0.9838	0.8881
Tax Incidence, cc (τ_{cc})	0.8505	0.8263	0.8524	0.8176	0.8535	0.8219
Tax Incidence, ch (τ_{ch})	0.9222	*0.9356*	0.9268	*0.9305*	0.9313	0.9296

Notes: Regulation caps the interchange fee at 90% of its unregulated equilibrium value. Entries shown in *red italics* indicate cases in which the regulated value of rewards, taxes, prices paid by consumers, or tax incidence exceeds the corresponding unregulated value.

Table 4: Equilibrium Effects of a 90% Interchange Fee Cap under Linear Demand and $\gamma = 0.2$

	$c = 0.4$		$c = 0.45$		$c = 0.5$	
Variables	Unregulated	Regulated	Unregulated	Regulated	Unregulated	Regulated
	(i^*)	($0.9i^*$)	(i^*)	($0.9i^*$)	(i^*)	($0.9i^*$)
Interchange Fee (i)	0.7351	0.6616	0.6820	0.6138	0.6317	0.5685
Reward (r)	0.2321	*0.3356*	0.2567	*0.2856*	0.2507	0.2481
Cash Tax (z_{ch})	2.2287	2.1943	2.0470	1.8787	1.9004	1.7039
Credit Tax (z_{cc})	1.6425	1.4579	1.5216	1.3422	1.4239	1.2811
Credit Share (μ)	0.7500	0.8227	0.7500	0.7620	0.7500	0.7266
Network Profit ($\pi_{\mathcal{N}}$)	0.1154	0.1075	0.0980	0.0909	0.0821	0.0764
Consumption (X)	0.2370	0.3038	0.2135	0.2821	0.1904	0.2514
Elasticity (ε)	-3.22	-2.29	-3.68	-2.54	-4.25	-2.98
Agg. Mer. Profit (Π)	0.0063	0.0116	0.0055	0.0108	0.0047	0.0090
Price cc users pay (P^{cc})	0.7005	0.6389	0.7240	0.6555	0.7471	0.6867
Price ch users pay (P^{ch})	0.9505	*0.9617*	0.9740	0.9175	0.9971	0.9133
Tax Incidence, cc (τ_{cc})	0.7317	0.6923	0.7347	0.6812	0.7363	0.6858
Tax Incidence, ch (τ_{ch})	0.8597	*0.8820*	0.8678	*0.8759*	0.8758	0.8745

Notes: Regulation caps the interchange fee at 90% of its unregulated equilibrium value. Entries shown in *red italics* indicate cases in which the regulated value of rewards, taxes, prices paid by consumers, or tax incidence exceeds the corresponding unregulated value.

due to higher equilibrium reward, such that z_{ch} does not decrease sufficiently relative to the increase in

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decrease much and the price cash users pay ultimately increases.

P^m.

Merchants are better off after the regulation, as they benefit from both a higher markup, $P^m - c$, and higher aggregate consumption X resulting from the decline in the weighted average price consumers pay (under constant elasticity demand, only the consumption effect was present). Thus, under imperfect competition, a regulation redistributes surplus away from the network—and sometimes from cash users—toward credit card users and merchants. Although the regulation makes both credit card users and merchants better off, the tax incidence for credit card users decreases. For cash users, when tax incidence decreases, the price they pay decreases (i.e., for $c = 0.5$). But a lower final price does not imply a lower tax incidence, as when $c = 0.45$, the tax incidence for cash users increases although the price they pay decreases after regulation. This is because the merchants benefit relatively more from the tax burden reduction.

Also, as expected, the tax incidence for consumers decreases as the merchant market becomes less competitive (when $\gamma = 0$, $\tau_{cc} = \tau_{ch} = 1$, consumers bear all the tax burden).

The effect of stronger product market competition (i.e., lower γ) is higher equilibrium taxes. This result contrasts with the constant elasticity case (see Proposition 3), where more intense product market competition lowers the tax paid by credit card users and also lowers the tax paid by cash users whenever demand elasticity is relatively low. The difference arises from the elasticity effect, which dominates the effect we identified in the constant elasticity case—an effect that is also at work here. Under linear demand, increased product market competition weakens the elasticity effect. Consequently, as we have discussed above, the network becomes less inclined to reduce taxes as competition intensifies, leading to higher equilibrium taxes in more competitive product markets.

7 Conclusion

This paper shows how interchange fees, rewards, and endogenous card adoption jointly determine the wedges between the prices consumers pay and the prices merchants receive. These wedges affect aggregate demand, merchant profits, and the distribution of surplus across cash users, credit card users, merchants, and the payment network. By endogenizing interchange fees, rewards and the share of credit card users, in a setting with merchant market power and elastic demand, the analysis identifies the channels through which payment network pricing shapes product market outcomes.

A central implication of the paper is that the effects of interchange fee regulation are more nuanced than is often suggested in policy discussions. A cap on the interchange fee does not uniformly reduce cardholder rewards. Because the cap constrains one margin of the network's pricing problem, the network may respond either by lowering rewards to partially restore margin or by increasing rewards to expand transaction volume. Which response is optimal depends on product demand elasticity. When demand is relatively inelastic, a modest cap may increase equilibrium rewards; when demand is relatively elastic, the cap reduces them. Thus, the claim often advanced by industry participants that lower interchange fees necessarily imply lower rewards is not supported by our analysis.

The welfare effects of regulation are likewise sensitive to the demand environment. In the constant elasticity benchmark, a fee cap lowers the implicit tax borne by both cash and credit users, so both groups benefit. Under linear demand, however, the incidence of regulation becomes less uniform. Credit card users continue to gain, but cash users may be worse off when demand is relatively inelastic, because higher rewards induce greater card adoption and strengthen the cross-subsidization embedded in retail prices. More generally, once demand elasticity varies with output, the effect of regulation depends not only on the direct reduction in merchant fees, but also on how rewards, card adoption, and retail prices adjust in equilibrium.

The cap lowers the profit of the payment network but the resulting redistribution of surplus depends on the demand specification and on merchant conduct. Under constant elasticity demand, surplus is reallocated from the network to consumers and, when merchants earn positive profits, to merchants as well. Under linear demand, by contrast, the redistribution can be less even: when cash users experience a welfare loss, the gains accrue to credit card users and, under imperfect competition, also accrue to merchants.

More broadly, the analysis identifies three distinct sources of inefficiency in payment markets: network market power, merchant market power, and reward-induced distortions in payment choice. In the baseline model, these forces can jointly generate inefficient consumption and excessive reliance on credit cards. Over an intermediate range of demand elasticities, an interchange fee cap mitigates both distortions, improving consumption efficiency while also reducing the overuse of credit. This interpretation, however, reflects an important baseline assumption: merchants incur no cost from handling cash. Under that assumption, positive rewards distort payment choice toward credit, so card use is excessive in the welfare sense. If merchant cash-handling costs were sufficiently high, however, this conclusion would no longer necessarily follow, since a shift from cash to cards could then be socially desirable.

The paper also yields implications for the role of product market competition. In the constant elasticity benchmark, greater merchant competition lowers the tax borne by credit card users and also lowers the tax borne by cash users when demand elasticity is sufficiently low. Under linear demand, by contrast, stronger competition can raise equilibrium taxes because the elasticity effect weakens the network's incentive to reduce them. This mechanism also implies that aggregate merchant profits need not decline monotonically with competition. Over some range, stronger competition alleviates double marginalization sufficiently to raise merchant profits.

These findings also relate to the well-known *waterbed effect*, whereby regulation of one price margin induces offsetting adjustments elsewhere in the pricing structure, (Genakos and Valletti, 2011, 2015). Our contribution is to characterize the direction of that adjustment in a two-sided payment market. In particular, the network's response through rewards, and the resulting effects on card adoption and transaction volume, depend systematically on demand elasticity. The model therefore generates testable implications for how product-market competition, substitution between cash and credit, and demand elasticity affect rewards, taxes, prices, and welfare.

At least two extensions remain for future work. The analysis assumes a monopoly payment network and Bertrand competition among issuing and acquiring banks. Allowing for competition between payment networks, or for market power at the banking level, would substantially complicate the analysis but could yield additional insights into the equilibrium determination of fees, rewards, and the effects of regulation.

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A Appendix: Proofs of the Lemma and Propositions

A.1 Proof of Lemma 1

We prove quasi-concavity for the network profit function, (4.9) or (4.10). Let

$$x \equiv z_{cc}, \qquad y \equiv z_{ch}, \qquad A \equiv \frac{ck}{(k-\gamma)t} > 0.$$

From equation (4.6), the credit-card share as a function of the two taxes is

$$\mu(x,y) = \frac{1}{2} + A(y-x).$$

Substituting this expression into the weighted-average tax margin gives

$$\begin{aligned} M(x,y) &\equiv \mu(x,y)x + \big(1-\mu(x,y)\big)y - 1 \\ &= \left(\frac{1}{2} + A(y-x)\right)x + \left(\frac{1}{2} - A(y-x)\right)y - 1 \\ &= \frac{x+y}{2} - A(y-x)^2 - 1. \end{aligned}$$

Therefore, up to a positive multiplicative constant, the network's profit can be written as

$$\pi_{\mathcal{N}}(x,y) = x^{-k}M(x,y), \qquad M(x,y) = \frac{x+y}{2} - A(y-x)^2 - 1.$$

Consider the economically relevant domain

$$D \equiv \{(x,y) : x > 0,\ y > 0,\ 0 \le \mu(x,y) \le 1,\ M(x,y) \ge 0\}.$$

The function $M(x,y)$ is concave in (x,y), since it is the sum of a linear function and the negative of a positive multiple of the square $(y-x)^2$. Hence the set $\{(x,y) : M(x,y) \ge 0\}$ is convex. Moreover, the constraints $x > 0$, $y > 0$, and $0 \le \mu(x,y) \le 1$ are convex constraints, because $\mu(x,y)$ is affine in (x,y). Therefore, D is convex.

For any scalar α, define the upper contour set

$$U_\alpha \equiv \{(x,y) \in D : \pi_{\mathcal{N}}(x,y) \ge \alpha\}.$$

If $\alpha \le 0$, then $U_\alpha = D$, because $M(x,y) \ge 0$ on D and $x^{-k} > 0$. Hence U_α is convex. Now suppose $\alpha > 0$. The condition $\pi_{\mathcal{N}}(x,y) \ge \alpha$ is equivalent to

$$x^{-k}M(x,y) \ge \alpha \quad \iff \quad M(x,y) - \alpha x^k \ge 0,$$

since $x > 0$. Define

$$\Phi_\alpha(x,y) \equiv M(x,y) - \alpha x^k.$$

Because $M(x,y)$ is concave and x^k is convex on $x > 0$ for $k > 1$, the term $-\alpha x^k$ is concave for $\alpha > 0$. Therefore, $\Phi_\alpha(x,y)$ is concave. It follows that $\{(x,y) : \Phi_\alpha(x,y) \ge 0\}$ is convex. Hence

$$U_\alpha = D \cap \{(x,y) : \Phi_\alpha(x,y) \ge 0\}$$

is convex. Thus, all upper contour sets of $\pi_{\mathcal{N}}$ are convex, and $\pi_{\mathcal{N}}$ is quasi-concave on D.

A.2 Proof of Proposition 1

Let

$$x \equiv z_{cc}, \qquad y \equiv z_{ch}, \qquad A \equiv \frac{ck}{(k-\gamma)t} > 0, \qquad d \equiv y - x.$$

From the proof of Lemma 1, the network profit is, up to a positive multiplicative constant,

$$\pi_{\mathcal{N}}(x,y) = x^{-k}M(x,y), \qquad M(x,y) = \frac{x+y}{2} - A(y-x)^2 - 1.$$

Then

$$M(x,y) = x + \frac{d}{2} - Ad^2 - 1, \qquad C(d) \equiv 1 - \frac{d}{2} + Ad^2, \qquad M(x,y) = x - C(d),$$

and the objective can be written as

$$\pi_{\mathcal{N}}(x,d) = x^{-k}\big(x - C(d)\big).$$

The nonnegative-margin condition $M(x,y) \ge 0$ is equivalent to $x \ge C(d)$.

For a fixed d, the first-order condition with respect to x is

$$\frac{\partial \pi_{\mathcal{N}}}{\partial x} = x^{-k-1}\left[(1-k)x + kC(d)\right] = 0, \qquad x(d) = \frac{k}{k-1}C(d).$$

Since $k > 1$, this satisfies $x(d) > C(d)$ whenever $C(d) > 0$, and therefore yields a strictly positive network margin. Assume now that $A > 1/16$. Since $C(d)$ is strictly convex and has minimum

$$C(d^*) = 1 - \frac{1}{16A},$$

this condition implies $C(d) > 0$ for all d. Hence, the conditional maximizer is well defined for every feasible d. Substituting $x(d) = kC(d)/(k-1)$ into the objective shows that the maximized value, conditional on d, is proportional to $C(d)^{1-k}$. Because $k > 1$, maximizing $C(d)^{1-k}$ is equivalent to minimizing $C(d)$. Since

$$C(d) = 1 - \frac{d}{2} + Ad^2$$

is strictly convex in d, it has a unique minimizer:

$$C'(d) = -\frac{1}{2} + 2Ad = 0, \qquad d^* = \frac{1}{4A}, \qquad C(d^*) = 1 - \frac{1}{16A}.$$

Therefore, an economically meaningful interior solution with $x^* > 0$ requires

$$C(d^*) > 0 \quad \Longleftrightarrow \quad 1 - \frac{1}{16A} > 0 \quad \Longleftrightarrow \quad A > \frac{1}{16}.$$

Using $A = ck/((k-\gamma)t)$, this condition becomes

$$\frac{ck}{(k-\gamma)t} > \frac{1}{16} \quad \Longleftrightarrow \quad t < \frac{16ck}{k-\gamma}.$$

Since $P^m = ck/(k-\gamma)$, this condition can equivalently be written as $t < 16P^m$.

Under this condition, the unique interior maximizer is obtained by substituting $d^* = 1/(4A)$ into $x(d) = kC(d)/(k-1)$. Hence

$$x^* = \frac{k}{k-1}\left(1 - \frac{1}{16A}\right) = \frac{16ck - (k-\gamma)t}{16c(k-1)}.$$

Recalling that $x = z_{cc}$, this gives

$$z_{cc}^* = \frac{16ck - (k-\gamma)t}{16c(k-1)} = \frac{(16c-t)k + \gamma t}{16c(k-1)}.$$

Moreover,

$$d^* = y^* - x^* = \frac{1}{4A} = \frac{(k-\gamma)t}{4ck},$$

so

$$z_{ch}^* = z_{cc}^* + \frac{(k-\gamma)t}{4ck} = \frac{(16c+3t)k^2 - (3\gamma+4)tk + 4\gamma t}{16(k-1)ck}.$$

Finally, the implied equilibrium share of credit-card users is

$$\mu^* = \frac{1}{2} + Ad^* = \frac{1}{2} + A\frac{1}{4A} = \frac{3}{4},$$

which lies strictly between zero and one. Also,

$$M(x^*, y^*) = x^* - C(d^*) = \frac{C(d^*)}{k-1} > 0$$

under $t < 16ck/(k-\gamma)$. Hence, the solution lies in the interior of the economically relevant domain. Since the conditional maximization over x is unique for each d, and $C(d)$ has a unique minimizer, the interior maximizer is unique.

The candidate also satisfies the second-order condition: the Hessian of $\pi_{\mathcal{N}}(x,d) = x^{-k}(x - C(d))$, evaluated at (x^*, d^*), is negative definite under $A > 1/16$ and $k > 1$. Since the transformation from (x,y) to (x,d) is linear and nonsingular, the second-order condition also holds in the original tax variables (z_{cc}, z_{ch}).

A.3 Proof of Proposition 2

We first characterize when credit-card users pay a tax or receive a subsidy. Using the equilibrium taxes in (4.11), we have

$$z^*_{cc} - 1 = \frac{16c - (k-\gamma)t}{16c(k-1)}.$$

Since $k > 1$, it follows that

$$z^*_{cc} > 1 \iff 16c > (k-\gamma)t \iff k < \gamma + \frac{16c}{t}, \qquad z^*_{cc} < 1 \iff k > \gamma + \frac{16c}{t}.$$

Thus, when $1 < k < \gamma + 16c/t$, both groups of consumers pay a tax, with $z^*_{ch} > z^*_{cc} > 1$. Since $P^m = ck/(k-\gamma) > 0$, the corresponding prices satisfy

$$P_{ch} = z^*_{ch}P^m > P_{cc} = z^*_{cc}P^m > P^m.$$

Using $P^m = ck/(k-\gamma)$, this gives

$$P_{cc} = z^*_{cc}P^m = \frac{(16c-t)k^2 + \gamma kt}{16(k-1)(k-\gamma)}, \qquad P_{ch} = z^*_{ch}P^m = \frac{(16c+3t)k^2 - (3\gamma+4)tk + 4\gamma t}{16(k-\gamma)(k-1)}.$$

By contrast, when $k > \gamma + 16c/t$, we have $z^*_{cc} < 1$, so credit-card users are subsidized in the sense that $P_{cc} = z^*_{cc}P^m < P^m$. Cash users still pay a tax. To see this, note that z^*_{ch} is affine in t. At $t = 0$, $z^*_{ch} = k/(k-1) > 1$, while at the upper admissible value $t = 16ck/(k-\gamma)$, which follows from Proposition 1, $z^*_{ch} = 4$. Hence $z^*_{ch} > 1$ throughout the admissible region. Therefore, $z^*_{ch} > 1 > z^*_{cc}$ when $k > \gamma + 16c/t$.

Next, credit-card users pay a price below marginal cost if and only if $P_{cc} < c$. Using

$$P_{cc} = \frac{(16c-t)k^2 + \gamma kt}{16(k-1)(k-\gamma)},$$

the equation $P_{cc} = c$ is equivalent to

$$(16c-t)k^2 + \gamma kt = 16c(k-1)(k-\gamma).$$

Solving this quadratic equation for k, the relevant threshold is

$$\bar{k} \equiv \frac{(16c+t)\gamma + 16c + \sqrt{256(\gamma+1)^2c^2 + 32\gamma(\gamma-1)tc + \gamma^2t^2}}{2t}.$$

Therefore,

$$P_{cc} < c \iff k > \bar{k}.$$

Moreover, $\bar{k} \geq \gamma + 16c/t$, so this case arises only within the high-elasticity region in which credit-card users are already subsidized relative to the merchant price.

We now turn to the comparative statics with respect to k. Differentiating z^*_{cc} with respect to k, we obtain

$$\frac{\partial z^*_{cc}}{\partial k} = -\frac{16c - (1-\gamma)t}{16c(k-1)^2}, \qquad \frac{\partial z^*_{cc}}{\partial k} < 0 \iff t < \frac{16c}{1-\gamma}.$$

For $\gamma < 1$, the condition in Proposition 1 implies

$$t < \frac{16ck}{k-\gamma} < \frac{16c}{1-\gamma}.$$

For $\gamma = 1$, the derivative is directly negative. Hence $\partial z^*_{cc}/\partial k < 0$ throughout the admissible region.

For the difference between the taxes,

$$z^*_{ch} - z^*_{cc} = \frac{t(k-\gamma)}{4ck}, \qquad \frac{\partial(z^*_{ch} - z^*_{cc})}{\partial k} = \frac{\gamma t}{4ck^2} \geq 0,$$

with strict inequality whenever $\gamma > 0$. Thus, the tax difference is weakly increasing in k, and strictly increasing if $\gamma > 0$.

Finally, differentiating z^*_{ch} with respect to k, we obtain

$$\frac{\partial z^*_{ch}}{\partial k} = \frac{-16ck^2 + \left(3\gamma k^2 - 8\gamma k + k^2 + 4\gamma\right)t}{16ck^2(k-1)^2}.$$

Therefore,

$$\frac{\partial z^*_{ch}}{\partial k} < 0 \iff t < \frac{16k^2c}{3\gamma k^2 - 8\gamma k + k^2 + 4\gamma}.$$

Moreover,

$$\frac{16k^2c}{3\gamma k^2 - 8\gamma k + k^2 + 4\gamma} > \frac{16ck}{k-\gamma} \iff k < \frac{4}{3}.$$

Hence, when $k < 4/3$, the admissibility condition in Proposition 1 implies $\partial z^*_{ch}/\partial k < 0$. When $k > 4/3$, the sign is ambiguous, and $\partial z^*_{ch}/\partial k < 0$ if and only if

$$t < \frac{16k^2c}{3\gamma k^2 - 8\gamma k + k^2 + 4\gamma}.$$

A.4 Proof of Proposition 3

Using the equilibrium taxes in (4.11), differentiation with respect to γ gives

$$\frac{\partial z^*_{cc}}{\partial \gamma} = \frac{t}{16c(k-1)} > 0.$$

Hence, as merchant market power decreases, i.e., as γ decreases, the equilibrium tax paid by credit-card users decreases.

For cash users,

$$\frac{\partial z^*_{ch}}{\partial \gamma} = -\frac{t(3k-4)}{16ck(k-1)}, \qquad \frac{\partial z^*_{ch}}{\partial \gamma} > 0 \iff k < \frac{4}{3}.$$

Thus, as γ decreases, the equilibrium tax paid by cash users decreases if and only if $k < 4/3$.

The difference between the two taxes is

$$z^*_{ch} - z^*_{cc} = \frac{t(k-\gamma)}{4ck}, \qquad \frac{\partial (z^*_{ch} - z^*_{cc})}{\partial \gamma} = -\frac{t}{4ck} < 0.$$

Hence, as γ decreases, the tax difference increases.

Next, using the equilibrium interchange fee and reward in equation (4.12), define

$$Q \equiv (16c + 3t)k^2 - (3\gamma + 4)tk + 4\gamma t.$$

Under the interiority condition in Proposition 1, $Q > 0$. Differentiating i^* with respect to γ gives

$$\frac{\partial i^*}{\partial \gamma} = -\frac{64ckt(k-1)(3k-4)}{3Q^2}, \qquad \frac{\partial i^*}{\partial \gamma} > 0 \iff k < \frac{4}{3}.$$

Therefore, as γ decreases, the equilibrium interchange fee decreases if and only if $k < 4/3$. Similarly,

$$\frac{\partial r^*}{\partial \gamma} = -\frac{64ck^2t(k-1)}{Q^2} < 0.$$

Thus, as γ decreases, the equilibrium reward increases.

We next consider equilibrium network profits. Using equation (4.13), differentiating $\pi_{\mathcal{N}}(z^*_{cc}, z^*_{ch})$ with respect to γ yields

$$\frac{\partial \pi_{\mathcal{N}}(z^*_{cc}, z^*_{ch})}{\partial \gamma} = -\frac{ck\left(\frac{16(k-\gamma)}{k}\right)^k}{c^k\left(\frac{16ck-(k-\gamma)t}{c(k-1)}\right)^k (k-\gamma)^2} < 0,$$

where the inequality follows from $k > \gamma$ and from the interiority condition in Proposition 1, which implies $16ck - (k-\gamma)t > 0$. Therefore, as γ decreases, equilibrium network profits increase.

For aggregate merchant profits, from (4.14) we can write

$$\Pi^m = c(ck)^{-k}\gamma(k-\gamma)^{k-1}\left[\frac{3}{4}(z^*_{cc})^{-k} + \frac{1}{4}(z^*_{ch})^{-k}\right].$$

Let

$$S(\gamma) \equiv \frac{3}{4}(z^*_{cc})^{-k} + \frac{1}{4}(z^*_{ch})^{-k}.$$

Then

$$\Pi^m = c(ck)^{-k}\gamma(k-\gamma)^{k-1}S(\gamma).$$

Since $\Pi^m = 0$ at $\gamma = 0$ and $S(0) > 0$, we have

$$\left.\frac{\partial \Pi^m}{\partial \gamma}\right|_{\gamma=0} = c(ck)^{-k} k^{k-1} S(0) > 0.$$

Thus, aggregate merchant profits increase for sufficiently low γ.

It is also useful to note that $S(\gamma)$ is decreasing in γ. Using

$$\frac{\partial z^*_{cc}}{\partial \gamma} = \frac{t}{16c(k-1)}, \qquad \frac{\partial z^*_{ch}}{\partial \gamma} = -\frac{t(3k-4)}{16ck(k-1)},$$

we obtain

$$S'(\gamma) = -\frac{3kt}{64c(k-1)}(z^*_{cc})^{-k-1} + \frac{(3k-4)t}{64c(k-1)}(z^*_{ch})^{-k-1}.$$

If $k \leq 4/3$, then $3k - 4 \leq 0$, so $S'(\gamma) < 0$. If $k > 4/3$, then $z^*_{ch} > z^*_{cc} > 0$ and $3k - 4 < 3k$, which again implies $S'(\gamma) < 0$. Hence $S(\gamma)$ is strictly decreasing in γ.

Differentiating Π^m gives

$$\frac{\partial \Pi^m}{\partial \gamma} = c(ck)^{-k}(k-\gamma)^{k-2}\left[k(1-\gamma)S(\gamma) + \gamma(k-\gamma)S'(\gamma)\right].$$

At $\gamma = 1$, the first term in brackets vanishes and $S'(1) < 0$, so

$$\left.\frac{\partial \Pi^m}{\partial \gamma}\right|_{\gamma=1} = c(ck)^{-k}(k-1)^{k-1} S'(1) < 0.$$

Therefore, aggregate merchant profits increase for sufficiently low γ and decrease near $\gamma = 1$.

A.5 Proof of Proposition 4

Using the equilibrium taxes in (4.11), differentiation with respect to t gives

$$\frac{\partial z^*_{cc}}{\partial t} = \frac{\gamma - k}{16c(k-1)} < 0.$$

Hence, as network market power decreases, i.e., as t decreases, the equilibrium tax paid by credit-card users increases.

For cash users,

$$\frac{\partial z^*_{ch}}{\partial t} = -\frac{(\gamma - k)(3k-4)}{16ck(k-1)}.$$

Since $k > \gamma$, it follows that

$$\frac{\partial z^*_{ch}}{\partial t} > 0 \iff k > \frac{4}{3}.$$

Therefore, as t decreases, the equilibrium tax paid by cash users decreases if and only if $k > 4/3$.

The difference between the two taxes is

$$z^*_{ch} - z^*_{cc} = \frac{t(k-\gamma)}{4ck}, \qquad \frac{\partial (z^*_{ch} - z^*_{cc})}{\partial t} = \frac{k-\gamma}{4ck} > 0.$$

Hence, as t decreases, the difference between the taxes decreases.

Next, using the equilibrium interchange fee and reward in equation (4.12), define

$$Q \equiv (16c + 3t)k^2 - (3\gamma + 4)tk + 4\gamma t.$$

Under the interiority condition in Proposition 1, $Q > 0$. Differentiating i^* with respect to t gives

$$\frac{\partial i^*}{\partial t} = -\frac{64ck(\gamma - k)(k-1)(3k-4)}{3Q^2}.$$

Since $k > \gamma$, we have

$$\frac{\partial i^*}{\partial t} > 0 \iff k > \frac{4}{3}.$$

Therefore, as t decreases, the equilibrium interchange fee decreases if and only if $k > 4/3$. Similarly,

$$\frac{\partial r^*}{\partial t} = -\frac{64ck^2(\gamma - k)(k-1)}{Q^2} > 0.$$

Thus, as t decreases, the equilibrium reward decreases.

We next consider equilibrium network profits. Differentiating equation (4.13) with respect to t gives

$$\frac{\partial \pi_{\mathcal{N}}(z_{cc}^*, z_{ch}^*)}{\partial t} = \frac{1}{16}\left(\frac{16(k-\gamma)}{ck}\right)^k \left(\frac{(16c-t)k+t\gamma}{c(k-1)}\right)^{-k} > 0,$$

where the inequality follows from $k > \gamma$ and from the interiority condition in Proposition 1, which implies

$$(16c-t)k + t\gamma = 16ck - (k-\gamma)t > 0.$$

Therefore, as t decreases, equilibrium network profits decrease.

It remains to consider aggregate merchant profits. From equation (4.14), aggregate merchant profits can be written as a positive multiplicative constant, independent of t, times

$$\frac{3}{4}(z_{cc}^*)^{-k} + \frac{1}{4}(z_{ch}^*)^{-k}.$$

Using (4.11), write

$$z_{cc}^* = \frac{k}{k-1} - Bt, \qquad z_{ch}^* = \frac{k}{k-1} + B\frac{3k-4}{k}t, \qquad B \equiv \frac{k-\gamma}{16c(k-1)} > 0.$$

Therefore,

$$\frac{\partial}{\partial t}\left[\frac{3}{4}(z_{cc}^*)^{-k} + \frac{1}{4}(z_{ch}^*)^{-k}\right] = kB\left[\frac{3}{4}(z_{cc}^*)^{-k-1} - \frac{3k-4}{4k}(z_{ch}^*)^{-k-1}\right].$$

If $k \le 4/3$, then $3k-4 \le 0$, so the expression in brackets is strictly positive. If $k > 4/3$, then $z_{ch}^* > z_{cc}^* > 0$, and hence

$$(z_{ch}^*)^{-k-1} < (z_{cc}^*)^{-k-1}.$$

Moreover, $(3k-4)/k < 3$. It follows that the expression in brackets is again strictly positive. Hence, for $\gamma > 0$,

$$\frac{\partial \Pi^m}{\partial t} > 0.$$

Therefore, as t decreases, aggregate merchant equilibrium profits decrease. If $\gamma = 0$, aggregate merchant profits are identically zero.

A.6 Proof of Proposition 5

From $\mu^* = \frac{3}{4} + \frac{\Delta}{2t}$, we have $\partial\mu^*/\partial\Delta = 1/(2t) > 0$, so the share of credit users increases in Δ.

Differentiating z_{cc}^* yields,

$$\frac{\partial z_{cc}^*}{\partial \Delta} = \frac{(k-\gamma)(t-2\Delta)}{4(k-1)ct}.$$

For $\Delta < t/2$, we have $(t-2\Delta) > 0$ and $k > \gamma$, hence $\partial z_{cc}^*/\partial\Delta > 0$.

Differentiating z_{ch}^* yields

$$\frac{\partial z_{ch}^*}{\partial \Delta} = -\frac{((t+2\Delta)k - 2t)(k-\gamma)}{4(k-1)tck}.$$

Thus $\partial z_{ch}^*/\partial\Delta > 0$ if and only if

$$(t+2\Delta)k < 2t.$$

Equivalently, when $t + 2\Delta > 0$, this condition can be written as

$$k < \frac{2t}{t+2\Delta}.$$

From $z_{ch}^* - z_{cc}^* = \frac{(k-\gamma)(t-2\Delta)}{4ck}$,

$$\frac{\partial}{\partial \Delta}(z_{ch}^* - z_{cc}^*) = -\frac{(k-\gamma)}{2ck} < 0.$$

Hence the tax differential decreases in Δ and equals zero at $\Delta = t/2$.

Differentiating r^* with respect to Δ gives

$$\frac{\partial r^*}{\partial \Delta} = -\frac{8kt(k-\gamma)(k-1)\left[16ckt + (k-\gamma)(t-2\Delta)^2\right]}{\left[16ck^2t + (k-\gamma)(t-2\Delta)(2k\Delta + 3kt - 4t)\right]^2} < 0.$$

Hence r^* is decreasing in Δ, and $r^* = 0$ at $\Delta = t/2$.

For $i^*(\Delta)$, write

$$i^*(\Delta) = \frac{4tN(\Delta)}{(2\Delta + 3t)S(\Delta)},$$

where

$$N(\Delta) = 16ckt + (k - \gamma)B(\Delta), \qquad S(\Delta) = 16ck^2t + (k - \gamma)B(\Delta),$$

and

$$B(\Delta) = (t - 2\Delta)(2k\Delta + 3kt - 4t).$$

Since

$$S(\Delta) - N(\Delta) = 16ckt(k - 1) > 0,$$

the log derivative is

$$\frac{\partial \log i^*(\Delta)}{\partial \Delta} = (k - \gamma)B'(\Delta)\left(\frac{1}{N(\Delta)} - \frac{1}{S(\Delta)}\right) - \frac{2}{2\Delta + 3t}.$$

Moreover,

$$B'(\Delta) = 2[2t - k(t + 2\Delta)].$$

Under the interiority conditions, $N(\Delta) > 0$, $S(\Delta) > 0$, and $2\Delta + 3t > 0$. Hence, if

$$k(t + 2\Delta) > 2t,$$

then $B'(\Delta) < 0$, both terms in the log derivative are negative, and therefore

$$\frac{\partial i^*(\Delta)}{\partial \Delta} < 0.$$

Equivalently, when $t + 2\Delta > 0$, this condition can be written as

$$k > \frac{2t}{t + 2\Delta}.$$

If $k(t + 2\Delta) \leq 2t$, the first term in the log derivative is nonnegative while the second term remains negative, so the sign of $\partial i^*/\partial \Delta$ is ambiguous.

It remains to show that equilibrium network profit is decreasing in Δ. Let $x \equiv z_{cc}$ and $d \equiv z_{ch} - z_{cc}$. Since

$$\mu = \frac{1}{2} + \frac{\Delta}{t} + Ad, \qquad A = \frac{ck}{(k - \gamma)t},$$

the weighted-average tax margin can be written as

$$\mu z_{cc} + (1 - \mu)z_{ch} - 1 = x + d(1 - \mu) - 1 = x - \left[1 - \left(\frac{1}{2} - \frac{\Delta}{t}\right)d + Ad^2\right].$$

Define

$$C_\Delta(d) \equiv 1 - \left(\frac{1}{2} - \frac{\Delta}{t}\right)d + Ad^2.$$

Then, up to a positive multiplicative constant,

$$\pi_N(x, d) = x^{-k}\big[x - C_\Delta(d)\big].$$

The first-order condition with respect to x implies

$$(1 - k)x + kC_\Delta(d) = 0,$$

or equivalently

$$x - C_\Delta(d) = \frac{x}{k}.$$

Therefore, evaluated at the equilibrium,

$$\pi_N^* \propto \frac{1}{k}(z_{cc}^*)^{1-k}.$$

Because

$$\frac{\partial z_{cc}^*}{\partial \Delta} = \frac{(k - \gamma)(t - 2\Delta)}{4(k - 1)ct} > 0$$

for $\Delta < t/2$, and because $1 - k < 0$, it follows that

$$\frac{\partial \pi_N^*}{\partial \Delta} < 0.$$

Thus equilibrium network profit is decreasing in Δ.