FEDERAL RESERVE BANK
PHILADELPHIA

Working Papers
RESEARCH DEPARTMENT

International Currency Dominance

Joseph Abadi
Federal Reserve Bank of Philadelphia

Jesús Fernández-Villaverde
University of Pennsylvania, NBER, CEPR, and Visiting Scholar, Federal Reserve Bank of Philadelphia Research Department

Daniel Sanches
Federal Reserve Bank of Philadelphia

WP 26-21

PUBLISHED
April 2026

ISSN: 1962-5361

DOI: https://doi.org/10.21799/frbp.wp.2026.21

International Currency Dominance*

Joseph Abadi †
Federal Reserve Bank of Philadelphia

Jesús Fernández-Villaverde ‡
University of Pennsylvania, NBER, and CEPR

Daniel Sanches §
Federal Reserve Bank of Philadelphia

December 2025

Abstract

We present a micro-founded monetary model of the world economy to study international currency competition. Our model features both "unipolar" equilibria, with a single dominant international currency, and "multipolar" equilibria, in which multiple currencies circulate internationally. Long-run equilibria are highly history-dependent and tend towards the emergence of a dominant currency. Governments can compete to internationalize their currencies by offering attractive interest rates on their sovereign debt, but large economies have a natural advantage in ensuring the dominance of their currencies. We calibrate the model to assess the quantitative importance of these mechanisms and study the dynamics of the international monetary system under counterfactual scenarios.

Keywords: dominant currency, international monetary system, strategic complementarities, history dependence.

JEL classifications: E42, E58, G21.

*We thank Gianluca Benigno (discussant), Athanasios Geromichalos, Lucas Herrenbrueck, Todd Keister, Arvind Krishnamurthy, Ricardo Lagos, Sebastian Merkel, Jonathan Payne, Ignacio Presno (discussant) and seminar participants at the 2024 Workshop on Money, Banking, Finance, and Payments, the Federal Reserve Bank of Philadelphia, Yeshiva University, the SCIEA conference, the CESifo MMI conference, the NY Fed International Roles of the Dollar conference, the Princeton Macro-Finance conference, and the Midwest Macro conference for useful comments. **Disclaimer:** This Philadelphia Fed working paper represents preliminary research that is being circulated for discussion purposes. The views expressed in these papers are solely those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Philadelphia or the Federal Reserve System. Any errors or omissions are the responsibility of the authors. No statements here should be treated as legal advice. Philadelphia Fed working papers are free to download at https://philadelphiafed.org/research-and-data/publications/working-papers.

†Email: joseph.abadi@phil.frb.org

‡Email: jesusfv@upenn.edu.

§Email: daniel.sanches@phil.frb.org

1 Introduction

Throughout history, international trade and finance have been dominated by a few currencies, despite the presence of numerous local ones. In the colonial era, the Dutch florin and the French franc circulated widely until the British pound sterling emerged as the leading global currency. The dollar later assumed this role with the Bretton Woods system, though other reserve currencies, such as the pound, the Japanese yen, and eventually the euro, also played significant roles (Eichengreen, 2019).

While contingent events, like the Bretton Woods agreement, influenced the international monetary system, macroeconomic fundamentals and government policies have been equally critical. Issuers of dominant currencies have typically been large economies with substantial fiscal capacity, such as Victorian Britain or the postwar US. Even the relatively small Dutch economy of the 17th and 18th centuries had significant capacity to issue private debt, thanks to its advanced financial markets. Once a currency becomes dominant, its international status typically persists for decades.

This paper examines how both macroeconomic fundamentals and historical contingencies shape the international circulation of currencies. In the long run, which countries' currencies are most likely to circulate globally? Can governments adopt fiscal or monetary policies to internationalize their currencies? Are "multipolar" international monetary regimes with multiple competing currencies stable, or does the world economy tend towards an equilibrium with a single dominant currency? Under what circumstances can a dominant currency lose its status?

Our approach adapts New Monetarist tools (Lagos and Wright, 2005) to a canonical two-country macroeconomic setting to address classic questions about the international monetary system (see also Matsuyama et al., 1993, and Wright and Trejos, 2001). The model emphasizes money's role as an international medium of exchange, with each country's government bonds potentially used for international trade.[1] The model incorporates three types of international monetary regimes: a "classical" equilibrium without an internationally accepted currency, a "dominant currency" equilibrium where one country's bond circulates globally, and a "multipolar" equilibrium where multiple bonds coexist as international currencies.

The model's basic idea is as follows. The demand for a currency is shaped by its *liquidity* in international trade (how widely accepted it is) and its *liquidity premium* (the cost of holding that currency relative to an illiquid asset). Our assumptions, described in greater detail below, imply that currencies may differ in their liquidity because agents always accept their domestic currency, but they do not necessarily accept foreign currency. We endogenize the liquidity of each currency in international trade by assuming that foreign agents must pay a (realistically small) fixed cost if they wish to accept it. Strategic complementarities emerge naturally from

[1]The study of which commodities serve as media of exchange dates back further. Kiyotaki and Wright (1989) introduced one of the first models examining how the medium of exchange is determined in equilibrium.

the model: an increase in the global acceptability of a country's bonds enhances demand for that currency, increasing the benefit of accepting it. Hence, dominant-currency equilibria in which only one currency circulates internationally often arise in the long run. Governments can compete to internationalize their currencies by offering attractive real returns on their nominal debt (i.e., a low liquidity premium): to do so, they must back their debt with taxes rather than seigniorage. However, countries are not on equal footing in this competition: larger countries have a natural advantage, as foreign agents expect higher chances of trading with domestic agents who accept that currency.

More concretely, the benchmark model features several countries, each populated by two agent types: buyers and sellers. Time is continuous. Agents trade continuously in a centralized market (CM), where, as in canonical international macroeconomic models, intermediate input goods sourced from each country and labor are aggregated to produce final output. Governments issue nominal bonds, set their nominal interest rates, and levy taxes on domestic agents in the CM as well. The main departure from canonical models concerns trade in intermediate goods: instead of being traded in Walrasian markets, they are traded in a search-and-matching market where money is essential (the *decentralized* market, or DM). Agents meet at random in bilateral, anonymous meetings in which sellers from each country produce input goods for buyers, who bring them back to their domestic CM. The assumption that money is essential is motivated by evidence that limits to contract enforcement in international trade create the need for cash in advance or bonds posted as collateral in trade finance (e.g., Antràs and Foley, 2015).

A critical assumption concerns the acceptability of currencies in DM trade. By default, sellers recognize only their domestic currency, which implies they are unwilling to accept foreign currency. Consequently, a country's bonds are more frequently accepted when purchasing input goods from that country. Imperfect recognizability is a typical abstraction in the literature (Lester et al., 2012) that captures various reasons individuals or firms may not accept foreign currencies, such as overhead costs of hiring staff to manage currency exposures, legal tender restrictions designed to encourage the use of domestic currency, or compliance costs.[2]

These administrative costs of accepting foreign currencies motivate our second key assumption: sellers must make a fixed-cost investment to recognize a foreign currency. When doing so, they take into consideration the discounted future gains from trade they will earn by trading with buyers who hold that currency. All else equal, sellers will be more willing to pay this cost for currencies that are widely held internationally, since doing so offers greater future opportunities for trade. Crucially, we do not assume that these costs are large: when we calibrate the model, we find that the costs paid to recognize foreign currencies are on the order of basis points of total trade quantities.

[2]This assumption is key to resolving the usual Kareken-Wallace indeterminacy (Kareken and Wallace, 1981). Alternative approaches include Gomis-Porqueras et al. (2017), who introduce counterfeiting threats in a two-country model to address indeterminacy.

We derive our analytical results in a special case of the model with two countries: Home and Foreign. We begin by characterizing buyers' portfolio decisions, taking the liquidity of each currency as given. Then, we demonstrate our key results on the complementarities in sellers' decisions and the stability of dominant currency regimes.

The key forces determining buyers' demand for each type of bond are (i) their demand for each country's input good, (ii) the probability that each currency will be accepted in trade, and (iii) the *liquidity premium* on each currency (i.e., the real return on an illiquid bond minus the real return on the corresponding government's nominal bonds). Agents are assumed to have a home bias in their demand for input goods, which naturally translates into a home bias in portfolio composition. Nevertheless, agents may tilt their portfolio towards a foreign currency if that currency has a low liquidity premium or if it is broadly accepted (e.g., if the foreign country is large or if its currency is widely recognized globally).

Three types of international monetary regimes may emerge. First, in the "classical" regime, most sellers in each country accept only domestic currency. All buyers then hold a diversified portfolio of bonds, since they wish to purchase both Home and Foreign inputs. This regime corresponds to the one studied in most cash-in-advance models (e.g., Lucas, 1982; Svensson, 1985). Second, there are "dominant currency" equilibria in which one currency is widely accepted internationally, so that currency's share in trade settlement exceeds the domestic economy's share of trade in goods (Gopinath and Stein, 2021). Agents in the dominant-currency country hold only their domestic currency, whereas agents in other countries hold the dominant currency and may hold their own currency as well. Finally, there is a "multipolar" regime in which all currencies are widely accepted. In this regime, agents hold only their own currency (given home bias in portfolio holdings), and all currencies circulate internationally.

We first examine how the international monetary regime depends on economic fundamentals and government policies, specifically the relative size of the two countries and the liquidity premium on each currency. A lower liquidity premium (i.e., a higher real rate of return on its debt) can help to internationalize a currency: high rates of return induce foreign investors to substitute towards a currency to meet their liquidity needs. Governments can target a high real rate of return on their debt by setting a high nominal interest rate relative to the growth rate of nominal liabilities: the gap between the two pins down how much of the interest bill must be tax-financed rather than covered by seigniorage. A larger country size also favors the circulation of its currency. For example, when Home is large relative to Foreign, Foreign agents are more willing to hold Home bonds, anticipating future trade opportunities with Home agents. Conversely, Home agents see fewer opportunities to trade with Foreign agents. That is, large economies can issue a dominant currency *despite* a high liquidity premium on their debt, e.g., the U.S. today or Britain at the turn of the 20th century (Chen et al., 2025).

We then turn to sellers' decisions to accept non-domestic currencies in order to study the determinants of currency dominance in the long run. The key insight emerging from the model is that there is a natural two-way strategic complementarity between buyers' decision to hold a currency and sellers' decision to accept it. Namely, the returns to accepting a foreign currency are higher if global holdings of that currency are larger, and global demand for a currency is greater if it is more widely accepted. Then, for example, a Foreign seller's decision to recognize Home currency enhances the liquidity of Home bonds, increasing the demand for Home bonds. In turn, greater global holdings of Home currency raise the benefits of recognizing it.

These strategic complementarities have crucial implications for the nature of long-run equilibria. We demonstrate that there are no stable equilibria in the "classical" regime where all agents hold a diversified portfolio of bonds (except for the trivial equilibrium where the cost of accepting non-domestic currency is so high that no agent chooses to do it). The only non-trivial stable equilibria are dominant-currency equilibria (if the cost of recognizing non-domestic currency is intermediate) or multipolar equilibria (if the cost is small). The instability of classical regime equilibria implies that the long-run outcome may be very sensitive to initial conditions: a small change in policies (e.g., one currency's liquidity premium falling by a small amount) can result in a different dominant currency in the long run. Once a dominant currency has emerged, though, it is difficult to dethrone: dominant currency equilibria are stable, so they cannot unravel from small changes in policies or fundamentals.

Finally, we calibrate the model to assess the quantitative importance of this mechanism and the dynamics of currency competition in counterfactual scenarios. We study an extended setting with three regions: the U.S., the Euro area, and the "Rest of the World" (RoW), focusing on the decisions of sellers in the RoW to accept dollars or euros. We find that the forces driving the world economy towards a dominant-currency equilibrium survive quantitatively, even in the extended setting.

Our first counterfactual experiment considers whether a dominant currency may lose its status if the world economy enters a "trade war" in which the U.S. imposes tariffs on imports and other countries retaliate via reciprocal tariffs on U.S. exports. We find that even for high and persistent tariffs (e.g., a permanent tariff of 30%), the dollar maintains its dominance. Even as trade with the U.S. declines, agents in Europe and the RoW continue to hold dollars to trade with each other. The failure of tariffs to destabilize international monetary arrangements is in line with historical evidence: dominant currencies have maintained their status across eras of protectionism and volatility in international trade. Moreover, even if the dollar were to lose its dominant status, the real effects on trade would be quite modest. Quantitatively relevant parameter values imply a liquidity premium less than 50bp and international holdings of domestic bonds not exceeding 50% of GDP. Hence, the "exorbitant privilege" can finance a stream of imports of at most 0.25% of GDP.

The second counterfactual experiment considers the fiscal version of the "Triffin dilemma"

(Bordo and McCauley, 2018), which dictates that as emerging economies grow, a single developed economy will not have the fiscal capacity to meet the entire world's demand for liquid assets, prompting a shift to a multipolar system. To capture this concern, we assume that the RoW grows faster than the U.S. and the Euro area. The dollar begins as the dominant currency, but the U.S. must lower the interest rate paid to bondholders if the interest burden on its debt exceeds its capacity to tax domestic agents. We find that although growth in the rest of the world may cause the U.S. to run up against fiscal constraints, the resulting increase in the dollar liquidity premium does not cause a broad shift away from dollars and into euros. Network externalities are strong enough that the dollar remains dominant despite the lower return on dollar assets.

Our final quantitative experiment asks whether the rise of China's economy could reshape the structure of the international monetary system. We introduce China as a fourth region in the global economy with its own currency, the renminbi. Motivated by China's recent efforts to better integrate its financial system with the global economy, we first consider a policy under which China expands its payment network (as it has done with CIPS). In our model, this policy amounts to providing agents from China with a faster matching technology. We find that absent an improved matching technology, the introduction of the renminbi in global markets (i.e., capital account liberalization) is not enough to significantly reduce the dollar's share of global trade settlement and international reserves. However, the expansion of China's payment system could increase the renminbi's share of global trade and reserves, making it a *secondary* international currency while the dollar remains dominant. Second, we consider a policy in which China's government attempts to compete by shifting out its bond supply curve, thereby cutting its liquidity premium in half. This strategy is more successful: the renminbi's share of international reserves climbs to rival the dollar's.

Our paper closely relates to the literature on how currencies compete internationally as media of exchange. Building on earlier models (Matsuyama et al., 1993, Zhou, 1997, Wright and Trejos, 2001, Head and Shi, 2003, and Camera and Winkler, 2003), we develop a tractable two-country model with divisible currency and no restrictions on portfolio holdings. We show that the model can be extended to address various international finance issues. Zhang (2014) also presents a two-country model with divisible currencies. Madison (2024) examines fiscal policy and currency substitution in a domestic economy. Relative to these papers, our contribution is to analytically characterize the *dynamics* of the international monetary system and to quantitatively evaluate scenarios that could change international payment arrangements.

Our work also engages with studies on the emergence of "dominant" currencies. Gopinath and Stein (2021) emphasize *unit of account* frictions, arguing that strategic complementarities in trade invoicing and private debt denomination drive dominance. Farhi and Maggiori (2018) and Clayton et al. (2024) analyze currency competition as *stores of value* through reputation games, while Pflueger and Yared (2024) study the interaction between reserve asset competition and a great power conflict.

Our model instead focuses on long-term competition between currencies as *media of exchange*, asking which fundamental forces lead a currency to dominate as a liquid asset. This focus reflects historical precedents, such as the Venetian ducat and Florentine florin, which were favored for their ease of storage and transport. Closest to our work in this respect are two papers. Chahrour and Valchev (2022) demonstrate how strategic complementarities in collateral choice can lead to the emergence and persistence of a dominant means of trade finance, and in a carefully calibrated DSGE model, they study the interactions between patterns of trade and the dominant currency. Coppola et al. (2023) present a stylized theoretical model in which thick market externalities can create a dominant currency for cross-border debt settlement, focusing mainly on the role of fiscal policy in determining the dominant currency. We view our contribution as complementary: we endogenize the medium of exchange in an otherwise canonical international monetary model. This permits us to study how a variety of counterfactuals related to trade, finance, and monetary policy shape the international monetary system and its dynamics in general equilibrium (both analytically and quantitatively).

Finally, our paper connects to the broader literature on the "exorbitant privilege" of the US. (Gourinchas and Rey, 2005, 2022). Gourinchas and Rey (2007) document how low returns on US debt sustain trade deficits. Caballero et al. (2008) and Maggiori (2017) model the emergence of exorbitant privilege in contexts of global financial imbalances. Jiang (2024) explains the cyclical behavior of dollar convenience yields and US debt returns, while Jiang and Richmond (2023) study a global fiscal cycle of reserve currency competition. In our model, the larger country enjoys an exorbitant privilege by internationalizing its currency while paying low interest rates. It also sustains persistent trade deficits by raising seigniorage revenues from foreign agents.

The rest of the paper is organized as follows. Section 2 introduces the model, and Section 3 derives the equilibrium conditions. Section 4 presents the main analytic results on international monetary regimes, comparative statics, and the stability of dynamic equilibria. Our calibration and counterfactual exercises are presented in Section 5. Section 6 concludes. The Appendix contains all proofs.

2 Model

This section introduces a general model of the international monetary system. Each country's government issues bonds that circulate as media of exchange both domestically and internationally. The key assumption that differentiates our model from canonical international monetary models is that agents cannot always recognize foreign-currency bonds. Instead, we endogenize the degree to which each currency is accepted internationally. We outline the model and derive its equilibrium, setting the stage for analyzing the conditions under which each currency circulates internationally. Later, we specialize to a two-country version of the model to derive analytical results and a quantitatively richer version of the model that we use to study the stability of dollar dominance under a range of counterfactual scenarios.

Setting. Time is continuous and infinite, $t \in [0, \infty)$. The global economy consists of J regions indexed by j. The global population is normalized to one, with a fraction ξ^j of agents residing in region j. Each country's population is split equally between two types of agents: *buyers* and *sellers*. There is a global centralized market (CM) where trade takes place continuously in a Walrasian market: goods are traded within countries, and assets can be traded across countries. International trade in goods occurs in pairwise meetings in which agents match at random (according to a Poisson process detailed later). We refer to these meetings collectively as the decentralized market (DM). There are $N \leq J$ governments (corresponding to regions $j \in \{1, \ldots, N\}$) that conduct fiscal and monetary policy: they levy taxes on domestic agents and issue bonds in their domestic currencies. This structure permits monetary unions as well as situations where one region does not have its own currency – e.g., in our quantitative model, there is a region (the "rest of the world") where agents choose whether to accept dollars or euros. There is no aggregate uncertainty.

Technology. Production takes place in two stages. The first stage of production is in the DM, where bilateral meetings occur according to a Poisson process with rate λ. Trading partners are drawn uniformly at random from the world population, so conditional on a meeting, the probability that an agent's trading partner is from region j is ξ^j.[3] In DM meetings, a region-j seller can produce a region-i *raw input* good z_i at a linear disutility cost. A region-j buyer can convert region-i raw inputs one-for-one into an *intermediate input* good x_i^j.

The second stage of production takes place in the CM. In each region j, inputs purchased by region-j buyers from other regions are aggregated into a *composite intermediate* good X^j according to the CES production function

$$X_t^j = \left(\sum_{i=1}^{J} {\omega_i^j}^{\frac{1}{\sigma}} {x_{it}^j}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where $\sigma > 0$ is the elasticity of substitution across intermediate inputs (similar to an Armington elasticity) and $\omega_i^j \geq 0$ is region j's expenditure weight on region-i inputs, with $\sum_{i=1}^{J} \omega_i^j = 1$ for all j.[4]

Then, competitive firms aggregate intermediate input goods and labor (supplied by domestic

[3] The assumption of uniform matching probabilities is analytically convenient but not essential to the main results.

[4] When $\sigma = 1$, we take limits and set $X_t^j = \prod_{i=1}^{J} {x_{it}^j}^{\omega_i^j}$.

agents) into final output via the Cobb-Douglas production function

$$y_t^j = X_t^{j\,\alpha} \ell_t^{j\,1-\alpha}, \tag{2}$$

where $1-\alpha \in (0,1)$ is labor's share of output. Goods cannot be traded across regions or physically invested in the CM. Therefore, the resource constraint in each country is

$$c_t^j = y_t^j, \tag{3}$$

where c_t^j denotes consumption in region j.

Preferences. Agents derive utility from consumption and disutility from labor in the CM and production of raw inputs in the DM. We assume agents have CRRA preferences over consumption and suffer linear disutility from labor, as is typical in New Monetarist models. Let $\{T_n\}_{n=1}^{\infty}$ denote the (countable) set of times at which an agent enters the DM. Then, the lifetime utility of a region-j agent is:

$$U^j = \mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\left(\frac{c_t{}^{1-\gamma}-1}{1-\gamma} - \ell_t\right)dt - \sum_{k=1}^{\infty} e^{-\rho T_k} z_{T_k}\right], \tag{4}$$

where z_{T_k} denotes raw inputs produced in the DM, c_t is individual consumption of country j's final good, ℓ_t is labor supplied in the CM, $\rho > 0$ is the subjective discount rate, and $\gamma^{-1} > 0$ is the intertemporal elasticity of substitution.[5]

Assets and the CM. In the CM, there are Walrasian markets for labor, intermediate inputs, final output, and financial assets. Since goods cannot be traded across countries in the CM, the prices of inputs and final goods may differ across countries. Region $j = 1$ final output is used as the numéraire, with q_t^j denoting the real exchange rate: the price of region j's final output good relative to that of region 1. In region j, region-i intermediate goods x_i^j trade at a price $q_t^j \psi_{i,t}^j$, and wages are $q_t^j w_t^j$ (so that ψ_{it}^j and w_t^j are input prices and wages in units of domestic output).

There are two types of financial assets in the world economy: nominal bonds in currencies $n \in \{1, \ldots, N\}$ as well as privately issued illiquid, risk-free debt in zero net supply. Illiquid bonds can be thought of as financial claims that are privately issued in international markets, such as corporate debt or stocks. A bond issued by government n pays an instantaneous nominal interest rate $i_t^n dt$. The price of currency-n bonds in the CM is denoted φ_t^n, so the real return on currency-n debt is

$$r_t^n = i_t^n + \frac{\dot{\varphi}_t^n}{\varphi_t^n}, \tag{5}$$

where a dot denotes a time derivative. The real interest rate on illiquid bonds is $r_t dt$.

[5] Again, when $\gamma = 1$, we take limits and set the utility of consumption to $\log(c_t^j)$.

The DM. In the DM, buyers and sellers are anonymous, with private trading histories, making a medium of exchange essential. Buyers and sellers negotiate the terms of trade according to a Nash bargaining protocol where the buyer has a share $\theta \in [0, 1]$ of the bargaining power.

Our key assumption to break the Kareken-Wallace (1981) indeterminacy result is that sellers always recognize their domestic currency, but they cannot always recognize non-domestic currencies.[6] Specifically, only an endogenous fraction $\delta^j_{nt} \in [0, 1]$ of region-j sellers can recognize currency n, where we assume $\delta^n_{nt} = 1$ for all $n \leq N$ and t. Under this assumption, currencies will not be perfect substitutes and can therefore pay different rates of return: each currency will be more liquid when purchasing the corresponding region's input good.

Formally, imperfect recognizability means that sellers are incapable of distinguishing genuine units of non-domestic currency from counterfeits, but this assumption can stand in for several institutional frictions that make it costly for individuals or firms to accept foreign-currency payments, such as compliance or legal costs, costs of hiring employees to trade currencies and manage risk exposures, and so on. As in Lester et al. (2012), we make the analytically convenient (but strong) assumption that counterfeit currency can be produced at no cost, implying that sellers will never accept a currency that they cannot recognize.[7]

We endogenize the recognizability of each currency by assuming that sellers can make a fixed-cost investment to recognize a foreign currency going forward. Specifically, for each n, there is a technology that permits region $j \neq n$ sellers to recognize that currency. According to a Poisson event with rate ζ, a region-j seller receives an opportunity to pay a utility cost κ to obtain this technology (or maintain their existing investment).[8] The seller can then recognize currency n until the next such event. Hence, the fraction of region-j sellers who recognize currency n evolves according to

$$\dot{\delta}^j_{nt} = \left(I^j_{nt} - \zeta \delta^j_{nt}\right) dt, \tag{6}$$

where $I^j_{nt} dt \in [0, \zeta dt]$ denotes the measure of such sellers who choose to incur the utility cost at time t. The recognizability problem is limited to sellers in the DM, so agents can trade currencies without restriction in the CM, and buyers can carry any currency into the DM without having to invest in the technology.

Importantly, when we calibrate the model, we find that the utility cost κ required to generate interesting results is realistically quite small. That is, even though the administrative costs that motivate our assumption tend to be modest, even small frictions to accepting foreign currencies are enough to generate stable dominant currency regimes.

Governments. Governments pay their debts by levying lump-sum taxes τ^n_t on domestic

[6] This assumption is typical in the literature; see, e.g., Zhang (2014).

[7] Lester et al. (2012) show that the results are qualitatively unchanged when the cost of counterfeiting is positive, but such considerations substantially complicate the analysis: buyers and sellers will use probabilistic strategies for counterfeiting and accepting unrecognized currencies.

[8] These Poisson events are independently distributed across currencies n.

agents and issuing a quantity of nominal bonds B_t^n in the CM. Each government's policy consists of a path of bond issuances $\{B_t^n\}_{t=0}^{\infty}$ and a path of nominal interest rates $\{i_t^n\}_{t=0}^{\infty}$ on domestic-currency debt. We denote the growth rate of the stock of currency-n debt by $\mu_t^n \equiv \dot{B}_t^n / B_t^n$.

Government n's budget constraint, expressed in terms of the numéraire good, is:

$$\dot{b}_t^n = r_t^n b_t^n - q_t^n \tau_t^n, \tag{7}$$

where $b_t^n \equiv \varphi_t^n B_t^n$ represents the real quantity of currency-n bonds outstanding at time t. The real rate on currency-n debt, r_t^n, is determined in equilibrium, and taxes $\{\tau_t^n\}_{t=0}^{\infty}$ adjust to satisfy equation (7) at each instant.

We assume that each government n uses its monetary and fiscal policy tools (i_t^n, μ_t^n) to target a path for the *liquidity premium* l_t^n on its debt, defined as the difference between the returns r_t on an illiquid bond and the returns on currency-n assets:

$$l_t^n \equiv r_t - r_t^n.$$

The liquidity premium is the cost of foregone interest that agents incur to hold a currency. In a steady state, the real returns on currency n are $i^n - \mu^n$ (since prices increase at rate μ^n), so the liquidity premium is simply $l^n = \rho - (i^n - \mu^n)$. Note that $i^n - \mu^n$ is simply the part of the interest bill that is paid by taxes rather than debt issuance, so the greater the tax backing of the debt, the lower the liquidity premium.[9]

Remark 1 *In our model, we assume that governments are the sole issuers of liquid assets, and we refer to these assets as "bonds." Ultimately, however, what matters for currency demand is simply the liquidity premium on each currency. One could think of cash in our model as a liquid asset that pays no interest, $i^n = 0$. A metallic standard (such as the gold standard) could be thought of as a constraint that μ_t^n must adjust to target a particular path for the price level. Hence, our model can accommodate other types of publicly issued liquid assets.*

Remark 2 *We abstract away from private liquidity (e.g., bank deposits) for simplicity. Given that governments elastically supply liquidity at a given price in our model, private issuance would not affect the liquidity premium. (Of course, this would no longer be true if governments inelastically supplied liquid assets.) However, private issuance would matter in one sense: the government would no longer harvest the liquidity premium on the entire stock of domestic-currency debt, since private issuance would make up part of the debt stock.*

[9] Governments have two tools, the nominal rate and the rate of bond issuance, to target one variable, the liquidity premium. This leaves one degree of freedom. In Appendix B.2, we show that given any path of the nominal rate i_t^n (resp. any path of bond issuance μ_t^n), there exists a path of μ_t^n (resp. i_t^n) that implements the desired liquidity premium in an equilibrium. However, for a given path of policies, there may be multiple equilibria featuring different inflation rates. Studying how the government can rule out such equilibria with off-equilibrium policies is beyond the scope of this paper.

Remark 3 *In our model, all liquid assets are used for cross-region trade in goods. However, all that is required for our model's main comparative statics and analytical implications on the stability of equilibria is some source of gains from trade that is decreasing in the quantities traded. Hence, we believe our results would continue to hold if liquid assets were used for other purposes, such as trade or settlement in financial markets, in which case the parameters ω_j^i and ξ^j could be reinterpreted as determinants of country j's centrality in financial trade.*

3 Equilibrium

We now outline the model's equilibrium conditions. We first demonstrate the linearity of agents' Bellman equations, since this will be a key result in characterizing DM outcomes. Then, we solve the DM bargaining problem, which permits us to derive buyers' static portfolio decisions and sellers' dynamic decisions to invest in recognizing foreign currencies. Finally, we present the goods and asset market clearing conditions in the CM and define an equilibrium.

3.1 Bellman equations

Consider the problem of an agent in the CM. The agent's individual state variables at time t are total financial asset holdings a_t and a type η_t indicating their region of residence and the set of currencies they recognize. Hence, we write $\eta_t = (j, B)$ if the agent is a region-j buyer and $\eta_t = (j, S_t)$ if the agent is a region-j seller, where $S_t \subset \{1, ..., N\}$ denotes the set of currencies that the agent recognizes at t. The agent chooses consumption c_t, labor ℓ_t, and asset holdings $\mathbf{b}_t = (b_{1t}, \ldots, b_{Nt}, b_{It})$ (where b_{it} denotes real holdings of country-i bonds and b_{It} denotes holdings of illiquid bonds), taking prices as given. These choices are subject to short-selling constraints on government bonds, the portfolio allocation constraint

$$\sum_{n=1}^{N} b_{nt} + b_{It} = a_t, \tag{8}$$

and the flow budget constraint

$$\dot{a}_t = \sum_{n=1}^{N} r_t^n b_{nt} + r_t b_{It} + q_t^j \left(w_t^j \ell_t - c_t - \tau_t^j \right), \tag{9}$$

where τ_t^j is the tax levied by region j's government (if it has one) in the CM.

Let $W_t(a|\eta)$ represent the CM value function of an agent of type η with assets a, and let $V_t(\mathbf{b}|\eta)$ be the expected value upon entering the DM, conditional on bond portfolio $\mathbf{b}$. The value function satisfies the Hamilton-Jacobi-Bellman (HJB) equation:

$$\rho W_t(a|\eta) = \max_{c,\ell,\mathbf{b}} \frac{c^{1-\gamma}-1}{1-\gamma} - \ell + \dot{a}W_t{}'(a|\eta) + \lambda\big(V_t(\mathbf{b}|\eta) - W_t(a|\eta)\big)$$
$$+ \tilde{\zeta}^{\eta}\big(\tilde{W}_t(a|\eta) - W_t(a|\eta)\big) + \frac{d}{dt}W_t(a|\eta)$$
$$\text{s.t. } (8),\ (9),\ b_{nt} \geq 0. \tag{10}$$

Here, $W_t{}'(a|\eta)$ denotes the first derivative of $W_t(a|\eta)$ with respect to assets. The second line captures the change in a seller's value function when the seller gets an opportunity to pay the cost to recognize a currency n. The term $\tilde{\zeta}^{\eta}$ is the rate at which the agent gets an opportunity to invest in the technology to recognize a currency: it is equal to zero for buyers and $\sum_{n\neq j} \zeta$ for sellers. The term $\tilde{W}_t(a|\eta)$ is the seller's expected value in this contingency. We return to this term later, when we solve sellers' problem.

Assuming utility is quasi-linear in labor, value functions become linear in asset values, with the coefficient on asset values equal to the inverse of the domestic wage:

Proposition 4 *There exists a constant $\hat{W}_t(\eta)$ for each t and η such that:*

$$W_t(a|\eta) = \hat{W}_t(\eta) + \frac{a}{q_t^j w_t^j}.$$

As typical in the New Monetarist literature, this result is convenient because it implies that we do not need to keep track of the distribution of wealth. All agents in a region have the same marginal value of wealth, regardless of their previous trading histories. In turn, this implies all agents of the same type have identical demand curves and behave identically in the DM. Next, we solve for DM outcomes to derive the value functions $V(\boldsymbol{b}|\eta)$.

3.2 DM bargaining

To characterize bargaining outcomes in the DM, we begin by solving for the terms of trade. By Proposition 4, a region-i seller is willing to produce one unit of raw input goods (i.e., incur one unit of disutility) in exchange for a payment equal to their domestic wage $q_t^i w_t^i$. A region-j buyer values region-i raw input goods at $q_t^j \psi_{it}^j$. Therefore, there are gains from trade as long as $q_t^j \psi_{it}^j > q_t^i w_t^i$. Hence, let

$$\Psi_{it}^j \equiv \frac{q_t^j \psi_{it}^j}{q_t^i w_t^i}$$

be an indicator of the gains from trade between a region-j buyer and a region-i seller. The following proposition characterizes the outcome of Nash bargaining in this case.

Proposition 5 *The price at which a region-i seller sells raw inputs to a region-j buyer in the DM is*

$$\phi_{it}^{j}=\frac{\Psi_{it}^{j}}{1+\theta(\Psi_{it}^{j}-1)}q_t^i w_t^i. \tag{11}$$

The buyer spends all bonds that the seller can recognize if $\Psi_{it}^{j}>1$.

This solution to the Nash bargaining problem implies that the split of surplus realized between a region-j buyer with a portfolio of bonds $\mathbf{b}=(b_1,...,b_N,b_I)$ and a region-i seller who recognizes a set of currencies S is

$$\text{DM surplus } =\left(\Psi_{it}^{j}-1\right)^{+}\sum_{n\in S}b_n\times\begin{cases}\theta/q_t^j w_t^j & \text{buyer}\\ (1-\theta)/\Psi_{it}^{j}q_t^i w_t^i & \text{seller}\end{cases}.$$

This total surplus is obtained by multiplying the per unit surplus, $q_t^j\psi_{it}^j-q_t^i w_t^i$, by the quantity produced, $\sum\limits_{n\in S}b_n/\phi_{it}^j$ (i.e., the quantity of the buyer's bonds the seller can recognize divided by the price).

Averaging across agents' possible trading partners, we then have the following characterization of DM value functions.

Proposition 6 *The expected value obtained by a type-η agent with bond portfolio $\boldsymbol{b}$ in the DM is*

$$V_t(\boldsymbol{b}|\eta)=W_t(a|\eta)+\frac{1}{q_t^j w_t^j}\times\begin{cases}\theta\sum\limits_{i=1}^{J}\sum\limits_{n=1}^{N}\xi^i\delta_{nt}^i\left(\Psi_{it}^j-1\right)^{+}b_n & \textit{buyer}\\ (1-\theta)\sum\limits_{i=1}^{J}\sum\limits_{n\in S}\xi^i\left(1-\frac{1}{\Psi_{jt}^i}\right)^{+}b_{nt}^i & \textit{seller}\end{cases}. \tag{12}$$

where $a=\sum\limits_{n=1}^{N}b_n+b_I$ denotes total assets and b_{nt}^i denotes aggregate holdings of currency-n bonds by region-i buyers.

This proposition yields two crucial results. First, a buyer's value function is linear in bond holdings of each type, which will allow us to derive independent Euler equations for each type of bond. Second, a seller's value function is additive in the set of currencies recognized in the DM, implying that from the perspective of an individual seller, decisions to recognize a currency are independent of past decisions to recognize other currencies. Both of these properties are useful in keeping individual decision problems analytically tractable. However, we will demonstrate that there are important strategic interactions between agents' decisions at the aggregate level.

3.3 Portfolio choices and currency recognizability

Having derived the DM value functions, we can proceed to solve for buyers' portfolio allocations and sellers' decisions to recognize foreign currencies.

Portfolio choices. The optimality conditions for liquid bond holdings reflect the tradeoff between the transaction benefits a currency yields in the DM and the cost of holding that currency. For an agent to hold currency n, the liquidity premium must be at least equal to the benefit of holding that currency, which from the Bellman equation (10) is $\lambda\frac{\partial(V_t(\mathbf{b}|\eta)-W_t(a|\eta))}{\partial b_n}$. Using the solution (12) for the DM value function, the Euler inequality determining region-j buyers' holdings of currency-n bonds can then be written as

$$l_t^n \geq \lambda\theta\sum_{i=1}^{J}\xi^i\delta_{nt}^i\big(\Psi_{it}^j-1\big)^+ \text{ with equality if } b_{nt}^j>0. \tag{13}$$

The demand for a currency hence depends on both its liquidity premium and its liquidity benefits in international markets. The right-hand side reflects that all else equal, agents are more willing to hold a currency n if it is widely accepted (high δ_{nt}^i). A currency that is not frequently accepted may have small enough liquidity benefits that some agents choose not to hold it at all. Indeed, in our dominant-currency equilibria, the dominant currency will be the only currency that is liquid enough in the DM that foreign agents are willing to hold it.

Sellers' decisions. We now turn to sellers' decisions to pay the cost to recognize foreign currencies. The (expected) value of a type $\eta=(j,S)$ seller who receives an opportunity to pay the cost to recognize some currency at t, $\tilde{W}_t(a|\eta)$ in (10), satisfies

$$\tilde{\zeta}^\eta\tilde{W}_t(a|j,S)=\sum_{n\neq j}\zeta\max\Big\{W_t(a|j,S\cup\{n\})-\kappa,\ W_t(a,|j,S\backslash\{n\})\Big\}. \tag{14}$$

With intensity ζ, the seller receives an opportunity to pay the cost κ to recognize currency n. A seller who pays the cost can recognize currencies $S\cup\{n\}$ thereafter, whereas a seller who does not pay the cost does not recognize n going forward. We must then derive the value $W_t(a|j,S\cup\{n\})-W_t(a,|j,S\backslash\{n\})$ of recognizing currency n.

From the Bellman equation, the flow value to a region-j seller of being able to recognize a currency is just the meeting rate, λ, times the value of being able to recognize that currency in the DM. Per (12), sellers' flow value in the DM is linear in the set of currencies accepted by the seller, where

$$v_{nt}^j=\frac{\lambda(1-\theta)}{q_t^j w_t^j}\sum_{i=1}^{J}\xi^i\big(1-\frac{1}{\Psi_{jt}^i}\big)^+ b_{nt}^i \tag{15}$$

is the flow value of accepting currency n. The value of recognizing currency n is equal to a discounted sum of future flow values v_{nt}^j. In the Appendix, we show that sellers choose to pay the cost to accept currency n whenever the present value of v_{nt}^j is high enough.

Proposition 7 *It is optimal for region-j sellers to pay the cost to recognize currency n at time* *t if and only if*

$$\kappa \leq \int_0^\infty e^{-(\rho+\zeta)s} v^j_{n,t+s} ds. \tag{16}$$

Note that the discount rate in the integral is $\rho + \zeta$, since sellers have to renew their investment with intensity ζ if they wish to continue accepting currency n. Intuitively, from (15), sellers find it worthwhile to accept a currency as long as (1) buyers hold large quantities of that currency, and (2) there are large gains from trade to be realized with such buyers. In our analytical two-country model, we demonstrate how this logic can lead to strategic complementarities in sellers' decisions.

3.4 Goods markets, asset markets, and equilibrium

We have solved buyers' and sellers' problems in terms of the equilibrium gains from trade Ψ^j_{it}, aggregate bond holdings b^j_{nt}, and real wages $q^j_t w^j_t$. Goods market clearing yields an additional equilibrium condition relating gains from trade to bond holdings. Equilibrium in international financial markets pins down wage-based real exchange rates.

Goods markets. Firms in region j purchase quantities $\{x^j_{it}\}^J_{i=1}$ of intermediate inputs and hire labor ℓ^j_t to maximize profits, taking the prices $\{\psi^j_{it}\}^J_{i=1}$ and w^j_t as given:

$$\max_{\{x^j_{it}\}^J_{i=1}, \ell^j_t} \; y^j_t - \sum_{i=1}^J \psi^j_{it} x^j_{it} - w^j_t \ell^j_t \;\; \text{s.t.} \;\; (1),\; (2).$$

This optimization problem yields the usual first-order conditions, which state that the marginal product of intermediate inputs should be set equal to their price,

$$\psi^j_{it} = \frac{\alpha y^j_t}{X^j_t} \left(\frac{x^j_{it}}{\omega^j_i X^j_t} \right)^{-\frac{1}{\sigma}}, \tag{17}$$

and that the marginal product of labor should be set equal to the wage,

$$w^j_t = \frac{(1-\alpha) y^j_t}{\ell^j_t}. \tag{18}$$

The labor supply condition derived from the Bellman equation (10) equates the marginal utility of consumption times the wage to the marginal disutility of labor:

$$w^j_t = \left(c^j_t / \xi^j \right)^\gamma, \tag{19}$$

since c^j_t / ξ^j is per capita consumption. Combining these relationships with the production function and resource constraint, (1)-(3), output can be expressed in terms of wage-based real exchange rates and gains from trade Ψ^j_{it},

$$y^j_t = \xi^j \big(\frac{\alpha^\alpha(1-\alpha)^{1-\alpha}}{{\Psi^j_{Xt}}^\alpha}\big)^{\frac{1}{\gamma}}, \tag{20}$$

where

$$\Psi^j_{Xt} \equiv \Bigg(\sum_{i=1}^J \omega^j_i \big(\frac{q^i_t w^i_t}{q^j_t w^j_t}\Psi^j_{it}\big)^{1-\sigma}\Bigg)^{\frac{1}{1-\sigma}}$$

is a price index for intermediate inputs. When the prices of intermediate inputs are high relative to wages (reflected by large gains from trade Ψ^j_{it}), aggregate production is low.

Finally, (17) can be combined with this expression for output and the terms of trade (11) to obtain an expression for aggregate DM expenditures of country-j buyers on country-i inputs.

Proposition 8 *Aggregate DM expenditures of country-j buyers on country-i inputs, $\phi^j_{it}x^j_{it}$ satisfy*

$$\phi^j_{it}x^j_{it} = K^j_t \times \omega^j_i \frac{\big(\frac{q^i_t w^i_t}{q^j_t w^j_t}\Psi^j_{it}\big)^{1-\sigma}}{1+\theta(\Psi^j_{it}-1)} \leq \lambda\xi^i \sum_{n=1}^N \delta^i_{nt} b^j_{nt} \quad \textit{with equality if} \;\; \Psi^j_{it} > 1, \tag{21}$$

where

$$K^j_t \equiv \alpha \xi^j q^j_t w^j_t {y^j_t}^{\frac{1}{\gamma}-1} {\Psi^j_{Xt}}^{\sigma-1}.$$

The right-hand side represents the maximum feasible expenditures of country-j buyers on country-i goods, since they meet such sellers with intensity $\lambda\xi^i$, and a seller accepts a currency-n bond with probability δ^i_{nt}. By Proposition 5, actual expenditures are equal to this maximum whenever there are gains from trade, since buyers spend all available bonds in that case. When combined with portfolio decisions (13) and wage-based real exchange rates $q^i_t w^i_t / q^j_t w^j_t$, this goods-market clearing condition can be used to pin down the gains from trade Ψ^j_{it} and bond holdings b^j_{nt}.

International financial market equilibrium. It remains to specify the equilibrium conditions that pin down exchange rates and the real interest rate on illiquid bonds.

Agents hold illiquid bonds only to smooth consumption over time, yielding a typical Euler equation derived from (10):

$$r_t = \rho + \frac{\dot{q}^j_t}{q^j_t} + \gamma \frac{\dot{c}^j_t}{c^j_t} \;\; \forall \;\; j \in \{1, \ldots, J\}. \tag{22}$$

The Euler equation (22), along with agents' labor-leisure optimization condition (19), yield a Backus-Smith condition for the goods-based real exchange rate,

$$\frac{q^j_t {c^j_t}^\gamma}{{c^1_t}^\gamma} = \big(\frac{\xi^i}{\xi^j}\big)^\gamma \frac{q^j w^j_t}{w^1_t} = \text{constant},$$

which in this model is equivalent to a constant wage-based real exchange rate.

The real exchange rate adjusts in equilibrium to balance countries' intertemporal budget constraints. Region j's net foreign asset position n_t^j evolves according to

$$\dot{n}_t^j = r_t n_t^j + \underbrace{\sum_{i=1}^{J} (\phi_{jt}^i x_{jt}^i - \phi_{it}^j x_{it}^j)}_{\text{net exports}} + \underbrace{l_t^j \sum_{i=1}^{J} b_{jt}^i - \sum_{n=1}^{N} l_t^n b_{nt}^j}_{\text{liquidity premia}}. \tag{23}$$

That is, region j's net saving consists of interest income earned on assets, plus net exports, plus liquidity premia paid by foreign agents to hold currency-j bonds, minus liquidity premia paid by domestic agents to hold foreign-currency bonds. The wage-based exchange rate adjusts so that the transversality condition holds for all countries:

$$\lim_{T\to\infty} \exp\big(-\int_0^T r_t dt\big) n_T^j = 0 \quad \forall \quad j \in \{1, \ldots, J\}. \tag{24}$$

World equilibrium. A *world equilibrium* consists of paths of prices $\{\Psi_{it}^j, \phi_{i,t}^j, l_t^n, r_t, w_t^j, q_{It}^j\}$, quantities $\{c_t^j, y_t^j, \ell_t^j, x_{it}^j, X_t^j, \tau_t^j\}$, bond portfolios $\{b_{nt}^j, b_{It}^j\}$, dynamics of the aggregate state variables $\{\delta_{nt}^j\}$, and net foreign asset positions $\{n_t^j\}$ such that (1)-(3), (7), (11)-(13), and (17)-(23) hold at all dates, and the dynamics of the aggregate state are consistent with (6) and (16).

4 Monetary regimes and dominant currencies

In this section, we study a two-country, two-currency version of the model with convenient parametric restrictions to derive analytical results. These restrictions are relaxed when we calibrate the model. We refer to the two countries as Home (H) and Foreign (F). We assume log utility and a Cobb-Douglas production function, $\gamma = \sigma = 1$. Further, we impose home bias in trade: for each country $j \in \{H, F\}$, the expenditure weight on domestic goods is

$$\omega_j^j = \frac{\beta \xi^j}{\beta \xi^j + \xi^{-j}},$$

where $\beta > 1$ is the home bias parameter. Hence, each country's expenditure weight on its goods is greater than its share of the global population. Finally, for simplicity, given that there are only two countries, in this section we let $\delta_{Ht} = \delta_{Ht}^F$ denote the fraction of Foreign sellers who recognize Home currency, and similarly we let $\delta_{Ft} = \delta_{Ft}^H$ be the fraction of Home sellers who recognize Foreign currency.

We first characterize the different regimes of currency circulation that may arise and comparative statics with respect to governments' monetary policies. Then, we derive our key results on the strategic complementarities in sellers' decisions to accept non-domestic currency and the stability of dominant currency regimes.

4.1 Currency circulation regimes

We begin by studying agents' Euler equations and characterizing the monetary regimes that can arise in equilibrium. The Euler equation for a country-j buyer with respect to Home bonds is

$$l_t^H \geq \lambda\theta\Big(\xi^H\big(\Psi_{Ht}^j - 1\big)^+ + \xi^F\delta_{Ht}\big(\Psi_{Ft}^j - 1\big)^+\Big), \tag{25}$$

whereas the Euler equation for Foreign bonds is

$$l_t^F \geq \lambda\theta\Big(\xi^H\delta_{Ft}\big(\Psi_{Ht}^j - 1\big)^+ + \xi^F\big(\Psi_{Ft}^j - 1\big)^+\Big), \tag{26}$$

where the superscript $(\cdot)^+$ denotes the positive part of the term in parentheses. Since currencies are imperfectly recognizable, they are not perfect substitutes. In particular, Home currency is more frequently accepted in transactions for Home inputs, whereas Foreign currency is more frequently accepted in transactions for Foreign inputs. Hence, it is possible for both currencies to circulate even if they yield different rates of return.

The Euler equations may not hold with equality because agents may simply choose not to hold one kind of currency. These are precisely the conditions under which a dominant currency can arise (e.g., if Home buyers choose to conduct all transactions in Home currency). When does this occur? The second key equilibrium condition comes from goods market clearing: per (21), expenditures by country-j buyers on country-i goods are bounded by their holdings of bonds that are accepted in country i:

$$\underbrace{\frac{\alpha\xi^j q_t^j w_t^j \omega_i^j}{1+\theta(\Psi_{it}^j - 1)}}_{\text{expenditures on } i} \leq \underbrace{\lambda\xi^i\big(\delta_{Ht}^i b_{Ht}^j + \delta_{Ft}^i b_{Ft}^j\big)}_{\text{bonds accepted in } i} \quad \text{for } j, i \in \{H, F\}. \tag{27}$$

Notice that if country-j buyers hold Home currency only and spend all of their bonds in DM meetings, then the *ratio* of expenditures on Home versus Foreign goods will be $\phi_{Ht}^j x_{Ht}^j / \phi_{Ft}^j x_{Ft}^j = \xi^H/\xi^F\delta_{Ht}$, since a fraction ξ^H of meetings are with Home sellers, and a fraction $\xi^F\delta_{Ht}$ are with Foreign sellers who accept Home currency. Similarly, if country-j buyers hold only Foreign bonds, the ratio of expenditures is $\phi_{Ht}^j x_{Ht}^j / \phi_{Ft}^j x_{Ft}^j = \xi^H\delta_{Ft}/\xi^F$.

Therefore, if country-j buyers choose to hold both types of bonds and there are gains from trade with both countries, the expenditure ratio must lie between these two extremes. On the other hand, if there are no gains from trade with Home ($\Psi_{Ht}^j = 1$), the ratio of expenditures can fall below $\xi^H\delta_{Ft}/\xi^F$, or if there are no gains from trade with Foreign ($\Psi_{Ft}^j = 1$), the expenditure ratio can rise above $\xi^H/\xi^F\delta_{Ht}$. Applying (21), we get the following proposition.

Proposition 9 *Country-j buyers hold both currencies if there exists a solution* $(\Psi^j_{j,t}, \Psi^j_{-j,t})$ *to the Euler equations (25)-(26) with*

$$\beta \frac{1+\theta(\Psi^j_{-j,t}-1)^+}{1+\theta(\Psi^j_{j,t}-1)^+} \in \left[\delta_{-j,t}, \frac{1}{\delta_{j,t}}\right]. \tag{28}$$

Otherwise, they hold domestic currency only if the domestic-currency Euler equation holds with equality and

$$\beta \frac{1+\theta(\Psi^j_{-j,t}-1)^+}{1+\theta(\Psi^j_{j,t}-1)^+} \begin{cases} = \frac{1}{\delta_{j,t}} & \Psi^j_{j,t}, \Psi^j_{-j,t} > 1 \\ > \frac{1}{\delta_{j,t}} & \Psi^j_{-j,t} = 1 \\ < \frac{1}{\delta_{j,t}} & \Psi^j_{j,t} = 1 \end{cases}. \tag{29}$$

Finally, they hold foreign currency only if the foreign-currency Euler equation holds with equality and

$$\beta \frac{1+\theta(\Psi^j_{-j,t}-1)^+}{1+\theta(\Psi^j_{j,t}-1)^+} \begin{cases} = \delta_{-j,t} & \Psi^j_{j,t}, \Psi^j_{-j,t} > 1 \\ > \delta_{-j,t} & \Psi^j_{-j,t} = 1 \end{cases}. \tag{30}$$

Figure 1 illustrates the solution to the Euler equations and the different monetary regimes that can emerge. In the left panel, Home and Foreign bonds have equal liquidity premia. Agents from each country hold both bonds, and the marginal products of Home and Foreign inputs are equalized across countries ($\Psi^H_H = \Psi^F_H$ and $\Psi^H_F = \Psi^F_F$). The right panel shows the case where Home's liquidity premium is lower than Foreign's. Foreign agents continue to hold both types of bonds, but Home agents choose to hold domestic bonds only. Since the liquidity premium on Home bonds is low, Home agents try to tilt their expenditures towards domestic goods. Once their entire portfolio consists of domestic bonds, they cannot further decrease their expenditures on Foreign goods, and in equilibrium $\Psi^H_H > \Psi^F_H$ and $\Psi^H_F < \Psi^F_F$.

This analysis can be used to characterize how the international monetary regime depends on policies (l^H, l^F) and the model's primitives. Define

$$\tilde{l}^j_t \equiv \frac{l^j_t - \delta_{jt} l^{-j}_t}{\xi^j \left(1-\delta_{Ht}\delta_{Ft}\right)}$$

to be the *effective* liquidity premium associated with purchasing country-j goods – this is a measure of the cost of carry for a bond portfolio that purchases one unit of country-j goods and zero units of the other country's goods, adjusting for the probability of matching with a country-j seller. The portfolio consists of a long position in currency j and a short position in the other currency. The effective liquidity cost depends both on currency j's liquidity premium and on the probability that currency j is accepted, which is increasing in country j's size and the fraction of foreign agents who recognize that currency. Note that if country-i agents hold both types of

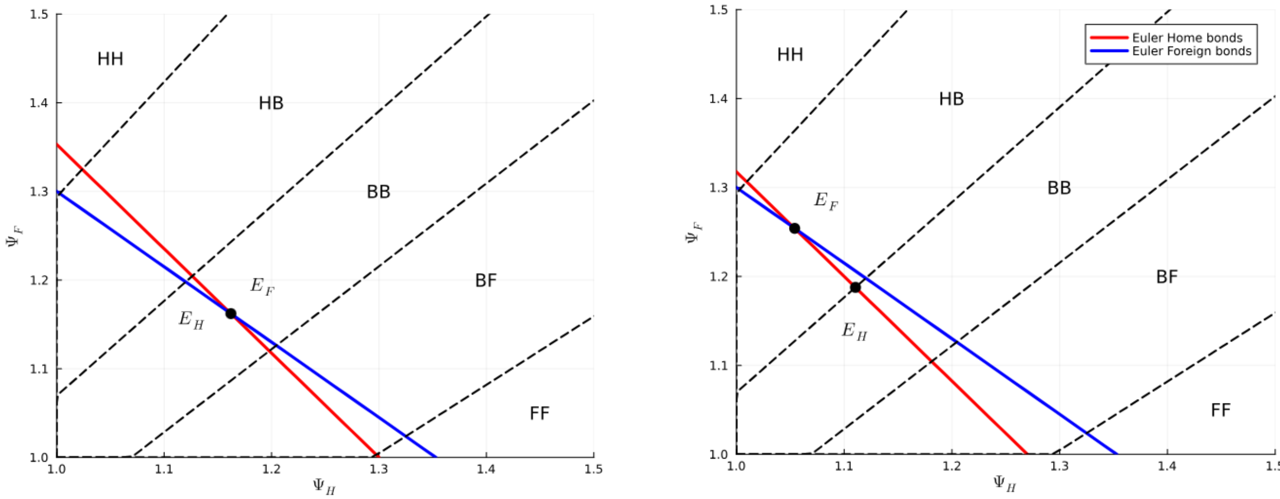

Figure 1: The solution to the Euler equations is plotted for two parameter configurations. Both panels have $\lambda = 0.2, \theta = 1.0, \xi = 0.5, \delta_H = \delta_F = 0.15, \beta = 1.1, l^F = 0.03$. The first letter in each region marks the currency held by Home, and the second marks the currency held by Foreign ("H" for Home, "B" for both, or "F" for Foreign). The left panel has $l^H = 0.03$, and in the right panel l^H decreases to 0.027. The equilibrium shifts from one in which both countries hold both currencies to one in which Home holds only its domestic currency. In the left panel, (Ψ_H^F, Ψ_F^F) is at the point E_F, whereas (Ψ_H^H, Ψ_F^H) is at the point E_H.

bonds, then $\Psi_{Ht}^i = \Psi_{Ht}^*$ and $\Psi_{Ft}^i = \Psi_{Ft}^*$, where

$$\Psi_{jt}^* \equiv 1 + \frac{\tilde{l}_t^j}{\lambda\theta}. \tag{31}$$

Intuitively, the higher $\tilde{l}_t^j$, the more costly it is for buyers to purchase country-j goods, and the lower the quantity actually purchased. Then, Proposition 9 can be used to demonstrate the following.

Proposition 10 *Define*

$$G(\tilde{l}_t^H, \tilde{l}_t^F) = \frac{1 + \theta(\Psi_{Ft}^* - 1)}{1 + \theta(\Psi_{Ht}^* - 1)} = \frac{1 + \tilde{l}_t^F/\lambda}{1 + \tilde{l}_t^H/\lambda}.$$

Then

- *Home agents hold both bonds when* $G(\tilde{l}_t^H, \tilde{l}_t^F) \in \left[\frac{\delta_{Ft}}{\beta}, \frac{1}{\beta\delta_{Ht}}\right]$, *hold only Home bonds when* $G(\tilde{l}_t^H, \tilde{l}_t^F) > \frac{1}{\beta\delta_{Ht}}$, *and hold only Foreign bonds when* $G(\tilde{l}_t^H, \tilde{l}_t^F) < \frac{\delta_{Ft}}{\beta}$.
- *Foreign agents hold both bonds when* $G(\tilde{l}_t^H, \tilde{l}_t^F) \in \left[\beta\delta_{Ft}, \frac{\beta}{\delta_{Ht}}\right]$, *hold only Home bonds when* $G(\tilde{l}_t^H, \tilde{l}_t^F) > \frac{\beta}{\delta_{Ht}}$, *and hold only Foreign bonds when* $G(\tilde{l}_t^H, \tilde{l}_t^F) < \beta\delta_{Ft}$.

This proposition provides comparative statics demonstrating how patterns of international currency circulation depend on the model's parameters and government policies.

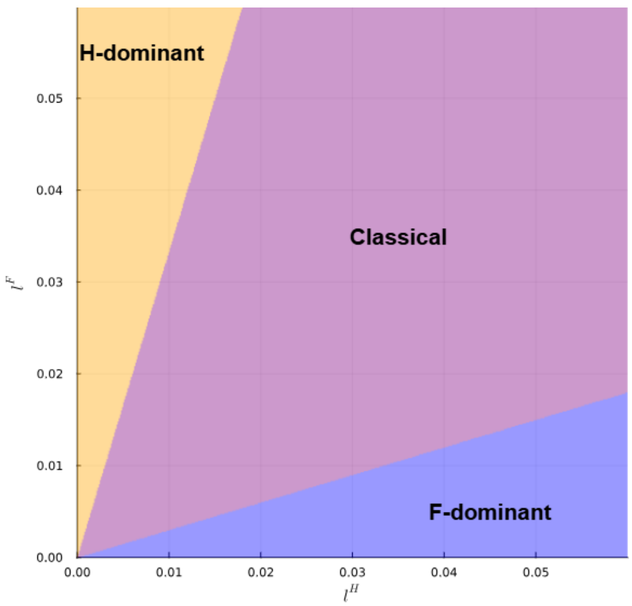

Figure 2: An illustration of how the international monetary regime depends on liquidity premia. The first letter indicates the currency held by Home ("H" for Home, "F" for Foreign, or "B" for both), and the second letter indicates the currency held by Foreign. We use parameters $\lambda = 0.2, \theta = 1.0, \xi = 0.5, \delta_H = \delta_F = 0.7, \beta = 1.3$.

A country's bonds are more attractive when their liquidity premium is lower (lower l^j), when the issuing country is larger (higher ξ^j), or when its currency is more frequently accepted by foreign sellers (higher δ_{jt}). These factors all contribute to a lower effective cost of holding a country's bonds – what matters to buyers is not only the foregone interest on liquid bonds, but also the probability that those bonds are widely recognized and accepted by sellers. Hence, it is possible for one currency to dominate international commerce even if both currencies have similar liquidity premia. Figure 2 shows how the monetary regime depends on liquidity premia.

Agents also have a greater incentive to hold foreign bonds when home bias is smaller (smaller β). When home bias is large, agents mainly want to purchase domestic inputs. Hence, they can nearly achieve their optimal expenditures on domestic and foreign goods by holding domestic bonds only. However, when home bias is smaller, this is no longer the case. One way of viewing this result is that when the global economy is more integrated, agents will have greater incentives to hold a diversified portfolio of currencies.

Furthermore, note as the fraction of agents who recognize foreign currencies goes to zero ($\delta_H, \delta_F \to 0$), all agents tend to hold both currencies. In this limit, the economy behaves just like traditional "cash-in-advance" models where a country's currency is *required* to purchase its goods (Lucas, 1982; Svensson, 1985). Conversely, in the limit where all sellers accept both currencies ($\delta_H, \delta_F \to 1$), no agent holds both currencies. Instead, when the liquidity premia on the two currencies are sufficiently close, agents hold only their domestic currency because they recognize that foreign sellers will accept it anyway.

Given these results, we classify the patterns of portfolio holdings into four different international monetary regimes. There are six possible patterns, which we label by HH, HB, BB, BF, FF, and HF, where the first letter indicates the currency held by Home agents and the second indicates that held by Foreign agents (H for Home, F for Foreign, and B for both). We classify equilibria as follows:

1. **"Classical" regime (BB):** All agents hold both currencies, just as in cash-in-advance models where each country's sellers accept only the domestic currency. In the limit where no agent accepts non-domestic currency, each currency's share of international trade settlement equals the corresponding country's export share.

2. **"Multipolar" regime** (HF): Agents hold only their domestic currency, and both currencies are accepted by a sufficiently large fraction of sellers in each country. In the limit where all sellers accept both currencies, each currency's share of international trade settlement equals the corresponding country's import share.

3. **"Home dominant" regime** (HH, HB): Home agents hold only domestic currency, whereas Foreign agents may hold both currencies. Home currency's share of global trade settlement exceeds Home's share of imports or exports.

4. **"Foreign dominant" regime** (BF, FF): Foreign agents hold only domestic currency, whereas Home agents may hold both currencies. Foreign currency's share of global trade settlement exceeds Foreign's share of imports or exports.

Our results above indicate how the international monetary regime depends on the model's primitives and government policies. No single factor determines on its own whether a currency is dominant, so our model does not predict, for example, that dominant-currency countries will have especially low liquidity premia on their debt. In fact, there is some debate in the literature on whether dominant-currency bonds have *high* liquidity premia: Chen et al. (2025) argue that they do, whereas Diamond and van Tassel (2025) argue that the "convenience yield" on US debt is neither especially high nor especially low.

There are several factors in our model that determine whether a currency is dominant: the country's size, its monetary policy (via the real return on its government bonds), and its acceptability by foreigners in international trade. The empirical literature has highlighted factors that are quite similar to those in our model: for instance, Eichengreen et al. (2018) highlight country size, monetary policy "credibility" (i.e., the inflation rate), and a factor they call "inertia," which is meant to capture strategic complementarities and history-dependence in global firms' and banks' decisions to accept a dominant currency. So far, we have demonstrated how fundamental factors shape patterns of currency circulation. In the next section, we uncover strategic complementarities in *sellers'* decisions to accept non-domestic currencies that emerge naturally from our model.

4.2 The stability of dominant currencies

So far, we have studied buyers' portfolio choice at a moment in time, taking as given sellers' decisions to accept non-domestic currencies. We have demonstrated that when a currency tends to be dominant (all else equal) when it is more liquid than the other currency in international markets. In this section, we study the sellers' problem in detail. We characterize the conditions under which sellers' decisions to accept non-domestic currency are strategic complements, leading one currency to become more liquid internationally than the other. This permits us to show that dominant currency equilibria – where only one of the two currencies is accepted in international trade – are stable.

When deciding whether to invest in the technology to accept non-domestic currency, sellers consider the expected benefits to accepting foreign currency until the next Poisson event. Recall from (15) that the *flow* value of accepting currency $-j$ for country-j sellers at time t is

$$v^j_{-j,t} = \frac{\lambda(1-\theta)}{q^j_t c^j_t}\bigg(\xi^{-j}\big(1-\frac{1}{\Psi^{-j}_{j,t}}\big)b^{-j}_{-j,t} + \xi^j\big(1-\frac{1}{\Psi^j_{j,t}}\big)b^j_{-j,t}\bigg). \tag{32}$$

The seller invests if the expected discounted benefit of accepting foreign currency j, V^j_t, exceeds the investment cost κ, where

$$V^j_t \equiv \int_0^\infty e^{-(\rho+\zeta)s} v^j_{-j,t+s} ds. \tag{33}$$

At an *interior* steady state (δ_H, δ_F), $V^H = V^F = \kappa$. However, steady states where either V^H or V^F are at a corner are also possible: for instance, a steady state in which no Home agent recognizes Foreign currency has $V^H < \kappa$ and $\delta_F = 0$.

A dominant currency can emerge if decisions to accept foreign currency are strategic *complements*, that is, if one seller's decision to accept foreign currency enhances the incentives for all other sellers to do so as well. Strategic complementarities raise the possibility of self-fulfilling equilibria where sellers in one country all invest in the technology because they expect others to do so as well. By contrast, if decisions to accept foreign currency are strategic *substitutes*, the international monetary system will tend to feature "multipolar" equilibria: as long as both countries run similar monetary policies, both currencies will eventually be accepted internationally (to some degree).

To understand the sources of strategic complementarity or substitutability in currency acceptance decisions, consider the value to Foreign sellers of recognizing Home currency at time t. From (32), there are two key considerations: (1) the gains from trade between buyers and Foreign sellers, and (2) global demand for Home bonds. We can characterize how each of these depend on the fraction of Foreign sellers who recognize Home currency.

Lemma 11 *The gains from trade between country-j buyers and country-i sellers, Ψ^j_{it}, are*

- *Increasing in the fraction of country-i sellers who recognize foreign currency j, δ_{jt}, if country-j buyers hold both currencies at t;*
- *Decreasing in δ_{jt} if country-j buyers hold only their domestic currency at time t.*

The intuition behind this lemma is straightforward. If, for example, Home buyers hold both currencies, an increase in the fraction of Foreign sellers who recognize Home currency will induce substitution from Foreign-currency to Home-currency bonds. These buyers then transact domestically more often, reducing the gains from trade in domestic meetings. For these buyers to continue to hold Foreign bonds at the same liquidity premium, the gains from trade with Foreign sellers must rise. On the other hand, if Home buyers hold only their domestic currency, an increase in the fraction of sellers who accept Home currency increases their demand for those bonds. In turn, Home buyers trade greater quantities with Foreign sellers, reducing the gains from trade that are realized.

The second component that is relevant to Foreign sellers' investment decision is the global demand for Home bonds. Naturally, both countries' demand for Home bonds is increasing in the fraction of Foreign sellers who recognize them. Then, in the regime where both countries hold both currencies, decisions to accept foreign currency are strategic complements, since both the gains from trade and bond demand are increasing in the fraction of sellers who invest.

Proposition 12 *Fix some level of the relative wage $q^F w^F / q^H w^H$ and $(l^H_t, l^F_t, \delta_{Ht}, \delta_{Ft})$ such that buyers in both countries hold both currencies. Then the value of accepting currency j at time t, v^{-j}_{jt}, is*

- *Locally increasing in the fraction of foreign sellers who recognize it, δ_{jt};*
- *Locally decreasing in the fraction of domestic sellers who recognize foreign currency, $\delta_{-j,t}$.*

Figure 3 illustrates these strategic complementarities. The arrows indicate the extent to which Home and Foreign sellers benefit from adopting the technology to accept foreign currency net of the cost. Small differences in initial conditions can lead to large differences in long-run outcomes: there is a long-run steady state where Foreign agents accept Home currency, and there is another long-run steady state where Home agents accept Foreign currency. The interior steady state where each country accepts the other's currency is unstable.

In fact, this instability is a general phenomenon. Well-known results in dynamical systems theory[10] demonstrate that when such strategic complementarities are present (as they are in the "classical" monetary regime here), equilibria cannot be stable. The following proposition characterizes the types of stable long-run equilibria that are possible.

[10] For a summary, see Hirsch and Smith (2006).

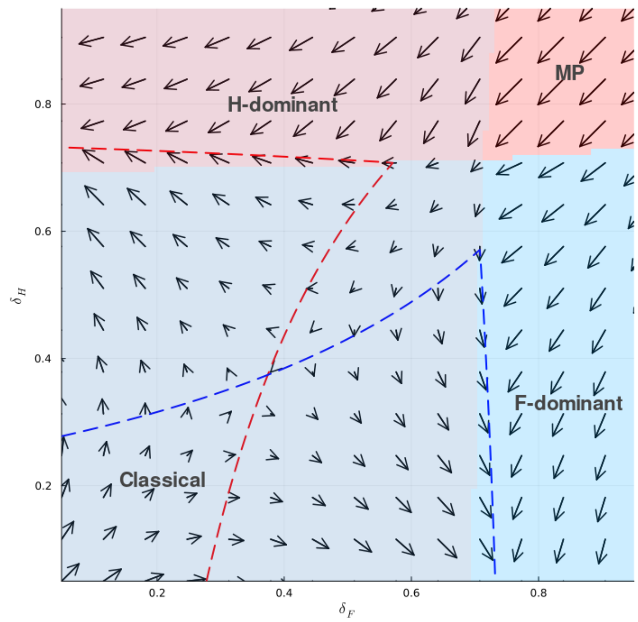

Figure 3: A phase diagram indicating the incentives to accept each currency as a function of (δ_F, δ_H). The arrows are of length $(v_F^H - \kappa, v_H^F - \kappa)$. The blue dashed line indicates the locus where $v_F^H = \kappa$, and the red dashed line indicates the locus where $v_H^F = \kappa$. The dark red is the multipolar regime, the gray region is the classical regime, and light red (blue) is the H-(F-)dominant regime. We set $l^H = l^F = 0.02$, $\lambda = 0.2, \theta = 0.5, \xi^H = 0.5, \beta = 1.1$, and $\kappa = 0.01$.

Proposition 13 *There are three possible types of stable long-run equilibria* (δ_H, δ_F)*:*

1. *A trivial equilibrium in the classical regime with* $\delta_H = \delta_F = 0$*, where sellers in each country accept only their domestic currency;*

2. *An equilibrium where one currency* j *is dominant and no country-*j *seller recognizes non-domestic currency,* $\delta_{-j} = 0$*;*

3. *A multipolar equilibrium where agents in each country hold only domestic currency.*

The stability analysis also gives some indication of how the international monetary system can be expected to evolve over time. During transitions to a long-run equilibrium, small changes in initial conditions or policies can have large effects on which currency is eventually selected as dominant. However, once a currency is established as dominant, small perturbations – such as a temporary change in policy or a moderate exogenous shock – are not likely to change the international monetary regime, since dominant-currency equilibria are stable. Instead, a large shock is likely required to dethrone a dominant currency if there is sufficient inertia in international payment systems (i.e., if the frequency ζ at which sellers can make their investment decisions is sufficiently low).

5 Quantitative assessment

Having laid out the theoretical case for the stability of dominant-currency regimes, we now quantify the model to assess the empirical relevance of this mechanism. We study a setting with three regions: the U.S. (US), the Euro area (EU), and the rest of the world (RoW), which consists of a continuum of infinitesimally small countries. Since each country trades with itself only in a measure zero of trades, we can ignore the presence of specific local currencies and instead focus on a setting where the only two currencies are the dollar and the euro.[11]

We begin by describing our calibration and analyzing the properties of the model's steady state. Then, we analyze the stability of international monetary regimes under various counterfactual scenarios. We consider perfect-foresight transition paths in which, at $t = 0$, agents learn of a change in the economy's parameters. We use a shooting algorithm to compute the evolution of the key state variables $\boldsymbol{\delta} = (\delta_{EU}^{US}, \delta_{US}^{EU}, \delta_{US}^{RoW}, \delta_{EU}^{RoW})$, i.e., the fraction of sellers in each region who recognize each non-domestic currency. For numerical stability, we assume that these state variables lie in a range $\delta_i^j \in [\delta_{min}, \delta_{max}] = [0.1, 0.9]$.[12] After finding the transition paths for the endogenous states, we compute economic aggregates of interest.

5.1 Calibration

We calibrate the model at an annual frequency, so that one unit of time is equal to a year. We calibrate the model by breaking up the parameter vector into two sets: *externally calibrated* parameters whose values are either standard in the literature or come from straightforward empirical targets, and *internally calibrated* parameters that are calibrated jointly to match a set of key moments in the data. We calibrate the model to a *dollar-dominant* steady state: for a given parameter vector, we find the corresponding steady state $\boldsymbol{\delta}$ such that neither U.S. nor RoW sellers pay the cost to recognize euros, $\delta_{EU}^{US} = \delta_{EU}^{RoW} = \delta_{\min}$.

Externally calibrated parameters. The externally calibrated parameters are listed in Table 1. We set population shares to $\xi^{US} = \xi^{EU} = 0.25$ and $\xi^{RoW} = 0.5$ to match the facts that (1) US and Euro zone output are roughly equal in PPP terms,[13] and (2) US GDP is roughly 25% of global GDP.[14] The annual subjective discount rate is set to $\rho = 0.01$ to target a steady-state real interest rate of 1%. The intertemporal elasticity of substitution is set to unity ($\gamma^{-1} = 1$), as is typical in models with balanced growth. The elasticity of substitution between intermediate

[11]This simplification permits us to reduce the state space. Instead of assuming away local currencies, another possibility would be to assume that local currencies account for some fraction of domestic trade but cannot be exchanged internationally (e.g., by assuming that κ is infinite for these currencies).

[12]To formalize this restriction, we could assume that a fraction δ_{min} of sellers always have access to the technology to recognize currency n, and a fraction $1 - \delta_{max}$ cannot acquire it at any cost.

[13]From the World Bank, https://data.worldbank.org/indicator/NY.GDP.MKTP.CD?locations=EU.

[14]From the World Bank, global GDP was \$111.33 trillion in 2024, and US GDP was \$29.18 trillion. See https://data.worldbank.org/indicator/ny.gdp.mktp.cd.

inputs is set to $\sigma = 2.0$, in between conventional estimates of the Armington elasticity: Broda and Weinstein (2006) estimate an elasticity of 2.9, whereas Feenstra et al. (2018) finds Armington elasticities ranging from 0.87-4.08 across sectors.[15] It remains to calibrate intermediates' share in output, α, and the expenditure weights ω_i^j on intermediate inputs.

There are two parameters that do not have standard values. The bargaining power parameter, which has no obvious empirical counterpart, is set to a benchmark value of $\theta = 0.5$ as in Drozd and Nosal (2012). The parameters governing sellers' decisions to adopt non-domestic currency do not have clear counterparts in the data, since the stylized decision in our model captures several motives to accept domestic currency in a reduced-form way. The frequency at which sellers receive an opportunity to pay a cost to recognize a currency is set to $\zeta = 0.1$, so that sellers make this decision on average once every 10 years. Transitions between dominant-currency regimes will therefore have a half-life of 10 years, roughly matching half the length of the transition from the pound sterling to the U.S. dollar during the interwar period. We calibrate the cost of investing the technology, κ, below.

We also calibrate government policies externally. For our quantitative results, what matters is the steady-state liquidity premium $l^j = r^I - (i^j - \pi^j)$ on each currency, not the nominal rate and inflation individually. The empirical counterpart of the liquidity premium in our model is the "convenience yield" on a country's bonds: the difference between the interest rate on an illiquid bond and that on a government security. Using a novel methodology that constructs synthetic illiquid bonds via stocks and portfolios of options, Diamond and van Tassel (2025) demonstrate that the convenience yields on U.S. dollar and euro bonds are both roughly 30bp, so we set $l^j = 0.003$ for each country.

Table 1: Externally Calibrated Parameters

Parameter	Description	Value
ξ^{US}	US share of global population	0.25
ξ^{EU}	Euro area share of global population	0.25
ρ	Annual subjective discount rate	0.01
γ^{-1}	Intertemporal elasticity of substitution	1.0
σ	Elasticity of substitution (intermediate inputs)	2.0
θ	Buyer bargaining power	0.5
ζ	Seller investment frequency	0.1
l^j	Liquidity premium (both currencies)	0.003

Internally calibrated parameters. Next, we calibrate the remaining parameters to moments in the data. As in our analytical model, we assume that expenditure weights are pinned

[15]The literature on international trade typically finds higher values of the Armington elasticity (close to 5) than the international macroeconomics literature.

down by population shares and a home bias parameter, so that for $j \in \{US, EU\}$,

$$\omega_j^j = \frac{\beta\xi^j}{\beta\xi^j + \sum_{i\neq j}\xi^i}, \quad \omega_i^j = \frac{\xi^i}{\beta\xi^j + \sum_{i\neq j}\xi^i} \quad \text{(for } i \neq j\text{)}.$$

On the other hand, the expenditure weights for RoW are proportional to population shares, $\omega_i^{RoW} = \xi^i$, since RoW consists of individual small countries rather than a single large country. The home bias parameter is primarily identified by import/GDP ratios in the data.

We must also calibrate the paramter α, which pins down the share of intermediates in production. A larger α implies a greater amount of international trade, so we include global trade as a percentage of GDP as a moment in our calibration.

The parameter κ is the key determinant of the fraction of global sellers who recognize dollars in a dollar-dominant steady state. Thus, our calibration will target the weighted fraction of sellers who recognize dollars, $(\xi^{EU}\delta_{US}^{EU} + \xi^{RoW}\delta_{US}^{RoW})/(\xi^{EU} + \xi^{RoW})$. To ensure numerical stability, we assume that there is a small amount of dispersion in sellers' costs of investing to accept non-domestic currency, so that the law of motion of δ^j is a smooth approximation to the non-differentiable step function in (6). Formally, we set the distribution F of sellers' investment costs κ_i so that

$$\Pr(\kappa_i \leq x) = F(x) = \frac{1}{2}\big(1 + \tanh(\frac{x-\kappa}{\epsilon})\big),$$

where $\epsilon = 5 \times 10^{-5}$ is chosen as small as possible so that our solution algorithm is stable.

Another important parameter is λ, the frequency with which agents meet in the DM. This parameter effectively governs the velocity of bonds, so it influences bond demand. The government debt/GDP ratio is hence a natural target to include to identify λ.

Finally, we treat the relative wage $q^{w,RoW}$ as a parameter and set $q^{w,EU} = 1$ to ensure symmetry between the U.S. and the Euro area. Recall that relative wages adjust in equilibrium so that countries' intertemporal budget constraints hold and that they are key determinants of the terms of trade in the DM. Hence, relative wages will affect steady-state net foreign asset positions and the balance of trade.

Table 2: Internally Calibrated Parameters

Parameter	Description	Value
β	Home bias	5.0
α	Intermediate input share	0.36
λ	Meeting rate	3.4
κ	Cost of accepting non-domestic currency	5.3×10^{-4}
$q^{w,EU}$	EU relative wage	1.00
$q^{w,RoW}$	RoW Relative wage	0.62

We calibrate the parameters $\Theta = (\beta, \alpha, \kappa, \lambda, q^{w,RoW})$ to match five moments. (1) We match the U.S. debt/GDP ratio, which we set at 50%, near the average of its values over the post-war period.[16] (2) We set the target for the fraction of non-U.S. sellers who accept dollars to 70%, motivated by the finding in Berthou et al. (2022) that 70% of French export firms invoice trade in dollars. (3) We target a U.S. import/GDP ratio of 15%, near the middle of its range for the 21st century.[17] (4) We target a U.S. trade deficit of 4%, also close to its 21st century average.[18] (5) We target a 40% trade/GDP ratio for the rest of the world, as in Chahrour and Valchev (2022).

In our calibration, we set the parameters to minimize a square loss function

$$L(\Theta) = (\hat{m}(\Theta) - m)' W (\hat{m}(\Theta) - m),$$

where $\hat{m}(\Theta)$ denotes the vector of simulated model moments under parameter vector Θ (in the dollar-dominant equilibrium), m denotes moments in the data, and W is a diagonal weighting matrix. We set the weight on all moments to be equal to 1 except for the share of RoW sellers who recognize dollars, which we set to 10. Table 2 gives the value of our calibrated parameters, and Table 3 compares the simulated model moments to their counterparts in the data. Overall, our model is able to match the data moments closely.

Importantly, we find that the cost of accepting foreign currencies, parametrized by κ, is quite modest under our calibration. In the dollar-dominant steady state, the costs paid by EU and RoW sellers to accept dollars are less than 10bp of the total volume of dollars transacted by those sellers. Hence, our main results do not rely on unrealistically severe frictions to accepting non-domestic currencies.

Table 3: Model and Data Moments

Moment	Model value	Target
U.S. Debt/GDP	51%	50%
δ_{US}^{RoW}	0.70	0.70
U.S. Imports/GDP	17%	15%
U.S. Trade Deficit/GDP	4.0%	4.0%
RoW Trade/GDP	36%	40%

5.2 The effects of a trade war

Our first policy experiment analyzes the effects of a trade war. We assume that in the CM, the U.S. levies a proportional tax τ^I on the sale of intermediate inputs purchased from all other regions, and in retaliation, the Euro area and the RoW levy a symmetric tariff on U.S. imports. These tariffs are assumed to be permanent – in this sense, our results will put an upper bound on

[16]See https://fred.stlouisfed.org/series/GFDEGDQ188S.
[17]See https://fred.stlouisfed.org/series/B021RE1Q156NBEA.
[18]See https://fred.stlouisfed.org/series/BOPGTB.

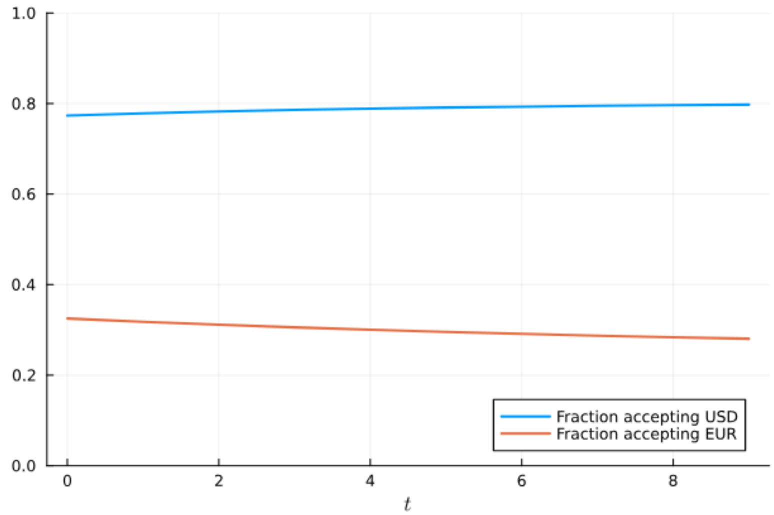

Figure 4: The fraction of sellers globally accepting dollars (blue) and euros (red) as a function of time in the trade war experiment. Import tariffs are set at $\tau^I = 0.3$.

the effects of a temporary trade war. The tariff is a tax on buyers' sales of intermediate inputs in the CM: for instance, U.S. buyers who import goods receive only $(1-\tau^I)\psi_{it}^{US}$ when selling inputs from region i. All proceeds from tariffs are distributed lump-sum to domestic agents.

We solve for the economy's transition path to a new steady state for the range of values $\tau^I \in [0, 0.3]$, so the maximum tariff we consider is 30%. In all cases, we find that the tariff does not trigger a switch to the euro as the world's dominant currency. Figure 4 illustrates the fraction of global sellers who accept dollars and euros along the transition path with $\tau^I = 0.3$.

In principle, a trade war could destabilize the dollar-dominant regime by reducing the demand for dollar bonds: in particular, buyers from the Euro area and the RoW might be expected to hold fewer dollar bonds because of the tariffs on *imports* from the U.S. In turn, lower global dollar bond holdings would incentivize sellers to stop accepting dollars in the long run.

In our model, however, despite the strong effects of tariffs on trade flows, a trade war does not reduce global dollar bond demand. Under our calibration, most buyers do not hold dollar bonds *at the margin* because they wish to purchase U.S. goods – that is, most buyers hold enough dollar bonds to fully exhaust gains from trade with U.S. sellers, $\Psi_{US}^j = 1$. Instead, buyers' incentives to hold dollars are mainly to trade with sellers in the RoW (where dollars are the only accepted currency) and the Euro area. Therefore, a trade war can actually slightly *increase* dollar demand, as expenditures switch from U.S. goods to RoW goods.

We perform a comparative statics exercise to highlight these points in Figure 5. We compute the log change in (1) dollar bond issuance and (2) U.S. imports between a steady state with zero tariffs and a steady state with tariff τ^I (for various levels of the tariff). Tariffs have large effects on real allocations: for a 30% tariff, U.S. imports are nearly 70% lower than in the no-tariff steady state. Nevertheless, dollar bond holdings are 6% *higher* in the steady state with 30% tariffs.

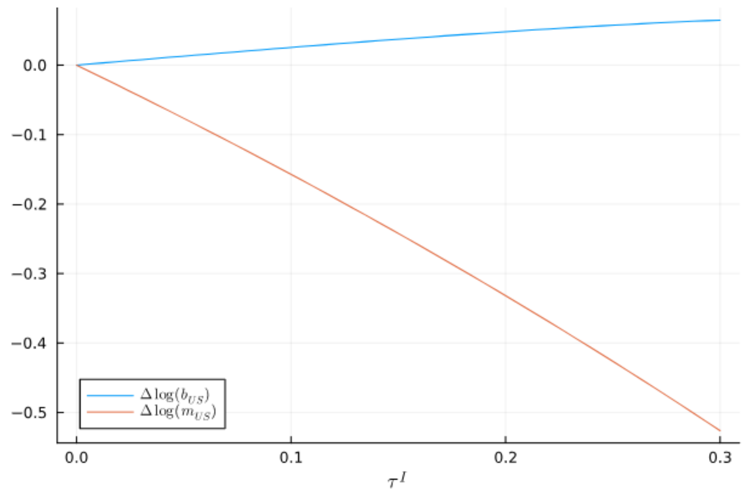

Figure 5: Comparative statics results for the trade war exercise. The blue line corresponds to the log change in steady-state dollar bond issuance from a steady state with zero tariffs to a steady state with tariff τ^I imposed on imports to and exports from the U.S. The red line plots the log change in U.S. imports between the two steady states.

Our results on the effects of tariffs and trade wars broadly match the historical evidence. For example, the world economy underwent large shifts in trade flows and policies from the 1850s until the end of World War I (e.g., the McKinley tariffs of 1890 and naval blockades during the war). Nevertheless, the pound maintained its dominance throughout this period. Similarly, the dollar emerged as a competitor to the pound in the early 1930s, when the Smoot-Hawley tariffs were in place, and dollar dominance survived the more protectionist environment following the Bretton Woods agreement.

5.3 The "fiscal" Triffin dilemma

In recent years, the rapid growth of emerging market economies and their accumulation of dollar reserves has led observers to propose a "fiscal" version of the Triffin dilemma (Farhi et al., 2011; Bordo and McCauley, 2018). As the world's supply for reserve assets grows, a dominant currency's status may become unstable because the issuing country cannot raise the taxes required to back the debt needed to satisfy the world's demand for liquid assets. Hence, the international monetary system may need to become multipolar, with several countries issuing reserve assets in different currencies.

We study a counterfactual in our model to assess this hypothesis. Since the primary concern is that growth in the rest of the world may outstrip the U.S.'s ability to supply a global safe asset, in our experiment, we introduce region-specific TFP growth and assume that productivity grows in RoW while remaining constant in the U.S. and the EU. Specifically, we let a_t^j denote labor-augmenting TFP in region j, so that the aggregate production function becomes $y_t^j = {X_t^j}^{\alpha}(a_t^j \ell_t^j)^{1-\alpha}$, and the utility cost of producing one unit of inputs in the DM is $1/a_t^j$. We assume

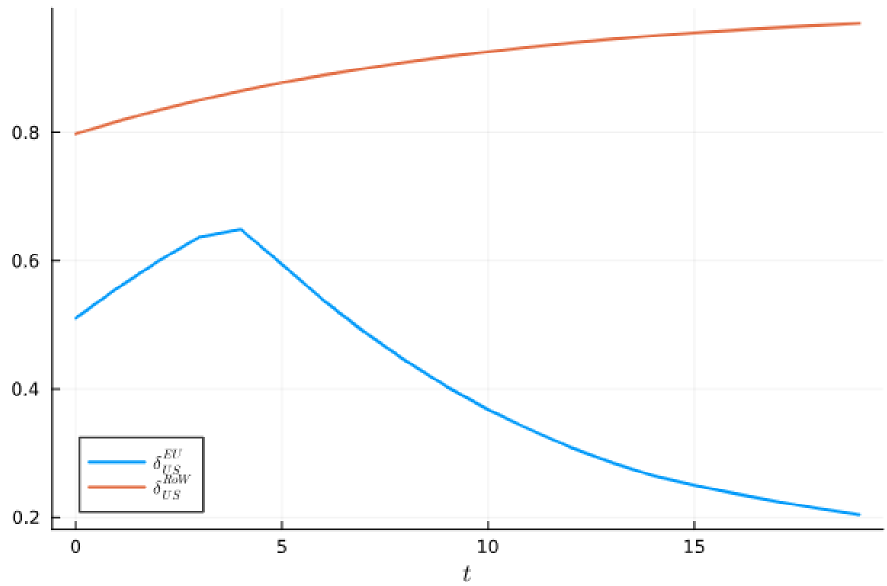

Figure 6: Dynamics in the "fiscal Triffin" experiment. The red (blue) line corresponds to the fraction of agents in RoW (EU) who accept dollars as a function of t.

that $a_0^j = 1$ in all regions initially, but in RoW, productivity grows linearly up to some value $\overline{a}^{RoW}$ over the course of $T = 20$ years.

The fiscal Triffin dilemma stems from the limited fiscal capacity of the U.S. and the Euro area. Each region's government can raise at most $\overline{\tau}$ per capita in taxes. When a country's debt is about to exceed the fiscally sustainable level, the government targets a higher liquidity premium (i.e., decreases the return on its debt) to ensure it will be able to repay its debt. Specifically, if l^{j*} is a government's target liquidity premium in the absence of fiscal constraints, it sets

$$l_t^j = \begin{cases} l^{j*} & \overline{\tau}^j \geq (r_t^I - l^{j*})b_t^j \\ r_t^I - \frac{\overline{\tau}^j}{b_t^j} & \text{otherwise} \end{cases} \tag{34}$$

Fiscal capacity $\overline{\tau}$ is set so that the U.S. hits its fiscal constraint whenever its debt issuance is 50% greater than in the initial steady state. Our main interest is in assessing whether the dollar can maintain its dominance even if the dollar debt exceeds the point at which the initial rate of return is sustainable.

We find that the rapid growth of the rest of the world does indeed lead the U.S. to run up against its fiscal capacity constraint. Nevertheless, the dollar remains the world's dominant currency in the sense that sellers in the rest of the world continue to accept only dollars, not euros. Figure 6 illustrates how widely accepted the dollar is over time. The fraction of sellers in RoW who accept dollars, δ_{US}^{RoW}, actually increases over time, while the fraction in the EU first increases and the declines.

Dollar debt expands to roughly 125% of initial US GDP. The liquidity premium on US bonds

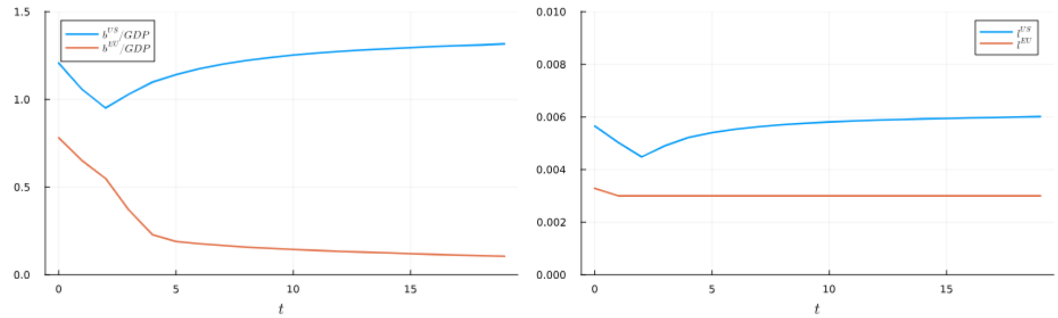

Figure 7: Dynamics of bond issuance and liquidity premia in each currency for the "fiscal Triffin" experiment. The left plot illustrates the quantity of bonds issued by the U.S. and the EU over time, normalized by each country's initial GDP. The right plot shows the liquidity premium on each type of bond.

thus doubles from 30 to 60bp to keep the debt sustainable. On the other hand, euro debt falls over time. (See Figure 7.)

What accounts for the increase in dollar bond demand that persists despite the increase in the dollar liquidity premium? News of future growth in RoW causes RoW's real exchange rate to appreciate vis-a-vis the U.S. and the Euro area. In turn, this increases real wages in RoW, so bond demand expands. The increase in bond demand actually causes the U.S. to hit its fiscal limit immediately at $t = 0$, raising the liquidity premium.

Despite the high initial liquidity premium on dollar bonds, buyers in the U.S. and RoW continue to prefer dollars because they are far more liquid in international markets: at $t = 0$, holding dollars is necessary for any buyer who wants to trade with the U.S. or RoW sellers. There is little additional incentive for any agent outside the euro area to increase euro bond holdings, since substitution from dollars to euros reduces the cost of trading only with Euro area sellers. Therefore, it continues to be valuable to accept dollars, and sellers do not switch to accepting euros. In this scenario, *inertia* is crucially important in maintaining the dollar's dominance. It does not benefit buyers to switch over to euros when the liquidity premium jumps because, at that point, few sellers accept euros, but then sellers have no incentive to pay the cost to move towards accepting euros (which, in the aggregate, could happen only gradually over time).

5.4 The rise of China

In recent years, observers have asked whether the continued growth of China's economy could lead to the rise of the renminbi as an international currency. To pursue this goal, China has introduced several policies aimed at integrating both its trade and financial system with other countries: it has invested in trade partnerships with regional partners, expanded its CIPS settlement system, and taken measures to liberalize its capital account.

To analyze whether China could be successful in internationalizing its currency, we introduce

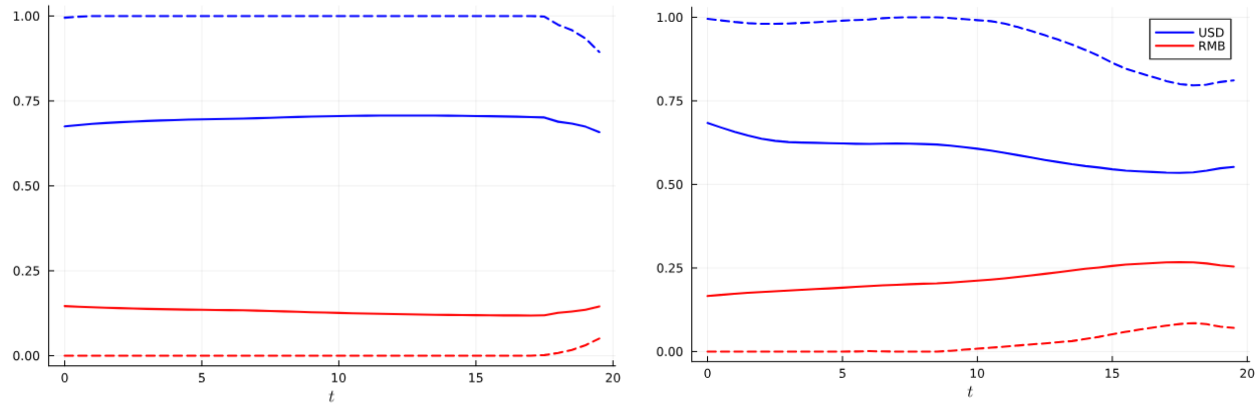

Figure 8: Shares of USD and RMB in global trade (solid lines) and international reserves (dashed lines) over time after China is introduced in the global economy. The left panel displays the results for the case $\nu = 1$, when China does not expand its international payment network, and the right panel displays the results for $\nu = 2$.

it into the model as a fourth country. Since China's economy is similar in size to the U.S. and Europe, we set its share of the global population equal to that of those two regions, $\xi^{China} = 0.25$. Just like the U.S. and the Euro area, China is assumed to have a home bias in goods. China's relative wage is assumed to be equal to that of RoW, as is the fraction of its sellers who recognize dollars and euros. We consider a scenario in which China has already fully liberalized its capital account, so RMB bonds can be traded freely across borders, and domestic and foreign investors earn identical yields. We also assume that the liquidity premium on the renminbi is identical to that on other currencies, in line with the evidence in Diamond and van Tassel (2025) that most countries' government bonds have similar convenience yields. (As we will show, capital account liberalization on its own is not sufficient to internationalize the renminbi.) When China is introduced in the global economy, only a fraction $\delta_{\min}$ of sellers in other countries recognize its bonds.

We first consider an experiment in which China promotes renminbi settlement by expanding opportunities to use its payment network. In our model, this roughly corresponds to increasing the rate at which agents from other countries meet agents from China in the DM. We assume that the intensity of meetings with agents from China is $\nu \times \lambda\xi^{China}$, where $\nu \geq 1$. We compare a situation in which China does not expand its payment network, $\nu = 1.0$, to a case in which it does so aggressively, $\nu = 2.0$.

Figure 8 plots the shares of (1) global trade (solid lines) and (2) international reserves (dashed lines) in USD and RMB for a period of $T = 20$ years after China is introduced into the global economy. (The introduction of China can be thought of as liberalization of its capital account, so that foreigners can hold its bonds.)

The introduction of RMB bonds to international markets, by itself, does not lead to broad international adoption of the renminbi. This is simply due to its lack of liquidity: only $\xi^{China} = 25\%$

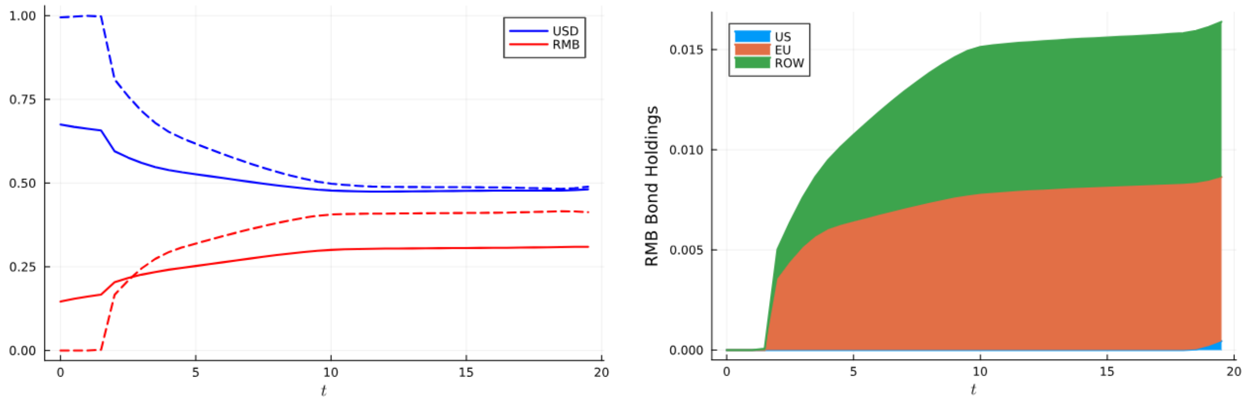

Figure 9: Results for the experiment in which China cuts its liquidity premium permanently to 15bp at $t = 0$. Left panel: The share of the dollar (blue) and renminbi (red) in global trade (solid lines) and international reserves (dashed lines) over time. Right panel: renminbi bonds held by the U.S. (blue), Euro area (orange), and RoW (green).

of global sellers accept renminbi by default, and a large fraction of those also accept dollars. Therefore, buyers do not have strong incentives to hold Chinese bonds in their portfolios, since they rarely provide additional opportunities for trade. Trade in renminbi is entirely driven by trades within China and a small fraction of trades by Chinese buyers in other countries. International reserves are equal to zero because it is not optimal for foreign buyers to hold Chinese bonds. The fraction of sellers who recognize renminbi does not increase appreciably over time.

The renminbi acquires a stronger international role when China expands its payment network. When buyers meet Chinese sellers more frequently, they have stronger incentives to hold China's bonds. The inclusion of renminbi bonds in buyers' portfolios induces sellers to pay the cost to accept those bonds, gradually increasing the fraction of trade conducted in renminbi over time. In turn, buyers tilt their portfolios further towards China's bonds. It takes a long time for the transition to occur, however, and even then, the dollar remains by far the dominant currency in international transactions: after 20 years, renminbi constitute only about 10% of international reserves versus about 80% for the dollar.

Is there any way that the renminbi can become more competitive in international markets? Next, we consider a policy in which China's government targets a lower liquidity premium on its bonds. In our model, this corresponds to expanding the real supply of government debt. Hence, this policy could be thought of as corresponding to measures that expand the available stock of Chinese debt for foreign investors. Specifically, we assume that at $t = 0$, China permanently cuts its target liquidity premium in half to 15bp.

Figure 9 plots the results. The dollar's share falls in both global trade and international reserves falls over time, and the renminbi's rises as it becomes more accepted in international trade. In fact, after $T = 20$ years, the renminbi share of reserves is close to the dollar's. Hence, a sustained decrease in the renminbi's liquidity premium can make it a competitor to the dollar.

The right panel displays the composition of renminbi bond holdings. The increase in renminbi reserves is attributable to roughly equal increases in the Euro area and the RoW, whereas buyers in the U.S. continue to hold dollars almost exclusively. There is a sense in which the international monetary system becomes fragmented: agents in the Euro area and RoW hold dollars to trade with the U.S. and renminbi to trade with China, but the U.S. and China do not hold significant quantities of each others' bonds.

6 Conclusion

We have developed a micro-founded monetary model to examine which assets serve as international media of exchange. Governments compete to internationalize their currencies by increasing the real interest rates on their bonds. Larger countries have a natural advantage in establishing themselves as dominant currency issuers. However, this advantage may diminish as the large country's share of global trade declines. These model predictions align well with historical evidence from the 19th and 20th centuries.

Our model is well-suited to study transitions in the dominant currency regime. When we endogenize the acceptability of currencies, we find that the "classical" equilibria studied by the early literature in international finance – where a country's currency is required to purchase its goods – are unstable. Instead, the economy often tends towards a dominant-currency regime. Small changes in initial conditions can have large effects on the structure of the international monetary regime, so there is a significant role for history-dependence. However, once a dominant currency is established, it is unlikely to be dethroned by a small shock or policy change.

References

Antràs, P. and Foley, C. F. (2015). Poultry in motion: A study of international trade finance practices. *Journal of Political Economy*, 123(4):853–901.

Berthou, A., Horny, G., and Mésonnier, J.-S. (2022). The real effects of invoicing exports in dollars. *Journal of International Economics*, 135.

Bordo, M. and McCauley, R. (2018). Triffin: Dilemma or myth? NBER Working Paper 24195.

Broda, C. and Weinstein, D. (2006). Globalization and the gains from variety. *Quarterly Journal of Economics*, 121(2):541–585.

Caballero, R., Farhi, E., and Gourinchas, P.-O. (2008). An equilibrium model of "global imbalances" and low interest rates. *American Economic Review*, 98(1):358–393.

Camera, G. and Winkler, J. (2003). International monetary trade and the law of one price. *Journal of Monetary Economics*, 50(7):1531–1553.

Chahrour, R. and Valchev, R. (2022). Trade finance and the durability of the dollar. *Review of Economic Studies*, 89(4):1873–1910.

Chen, Z., Jiang, Z., Lustig, H., van Nieuwerburgh, S., and Xiaolan, M. (2025). Exorbitant privilege gained and lost: Fiscal implications. *Journal of Political Economy*, 133(12):3713–3761.

Clayton, C., dos Santos, A., Maggiori, M., and Schreger, J. (2024). International currency competition. Working paper.

Coppola, A., Krishnamurthy, A., and Xu, C. (2023). Liquidity, debt denomination, and currency dominance. NBER Working Paper 30984.

Diamond, W. and van Tassel, P. (2025). Risk-free rates and convenience yields around the world. *Journal of Finance*, (forthcoming).

Drozd, L. and Nosal, J. (2012). Understanding international prices: Customers as capital. *American Economic Review*, 102(1):364–395.

Eichengreen, B. (2019). *Globalizing Capital: A History of the International Monetary System*. Princeton University Press, 3rd edition.

Eichengreen, B., Mehl, A., and Chitu, L. (2018). *How Global Currencies Work: Past, Present, and Future*. Princeton University Press.

Farhi, E., Gourinchas, P.-O., and Rey, H. (2011). *Reforming the International Monetary System*. CEPR.

Farhi, E. and Maggiori, M. (2018). A model of the international monetary system. *Quarterly Journal of Economics*, 133(1):295–355.

Feenstra, R., Luck, P., Obstfeld, M., and Russ, K. (2018). In search of the Armington elasticity. *Review of Economics and Statistics*, 100(1):135–150.

Gomis-Porqueras, P., Kam, T., and Waller, C. (2017). Nominal exchange rate determinacy under the threat of counterfeiting. *American Economic Journal: Macroeconomics*, 9(2):256–273.

Gopinath, G. and Stein, J. (2021). Banking, trade, and the making of a dominant currency. *Quarterly Journal of Economics*, 136(2):783–830.

Gourinchas, P.-O. and Rey, H. (2005). From world banker to world venture capitalist: U.S. external adjustment and the exorbitant privilege. In Clarida, R., editor, *G7 Current Account Imbalances*. University of Chicago Press.

Gourinchas, P.-O. and Rey, H. (2007). International financial adjustment. *Journal of Political Economy*, 115(4):665–703.

Gourinchas, P.-O. and Rey, H. (2022). Exorbitant privilege and exorbitant duty. CEPR Discussion Paper No. DP16944.

Head, A. and Shi, S. (2003). A fundamental theory of exchange rates and direct currency trades. *Journal of Monetary Economics*, 50(7):1555–1591.

Hirsch, M. and Smith, H. (2006). Monotone dynamical systems. In Cañada, A., Drábek, P., and Fonda, A., editors, *Handbook of Differential Equations: Ordinary Differential Equations*, volume 2, pages 239–357. North-Holland.

Jiang, Z. (2024). Exorbitant privilege: A safe-asset view. Available at SSRN: https://ssrn.com/abstract=4600477.

Jiang, Z. and Richmond, R. (2023). Reserve asset competition and the global fiscal cycle. Available at SSRN: https://ssrn.com/abstract=4636643.

Kareken, J. and Wallace, N. (1981). On the indeterminacy of equilibrium exchange rates. *Quarterly Journal of Economics*, 96(2):207–222.

Kiyotaki, N. and Wright, R. (1989). On money as a medium of exchange. *Journal of Political Economy*, 97(4):927–954.

Lagos, R. and Wright, R. (2005). A unified framework for monetary theory and policy analysis. *Journal of Political Economy*, 113(3):463–484.

Lester, B., Postlewaite, A., and Wright, R. (2012). Information, liquidity, asset prices, and monetary policy. *Review of Economic Studies*, 79(3):1209–1238.

Lucas, R. E. (1982). Interest rates and currency prices in a two-country world. *Journal of Monetary Economics*, 10(3):335–359.

Madison, F. (2024). A microfounded approach to currency substitution and government policy. *Journal of Economic Theory*, 219(105847).

Maggiori, M. (2017). Financial intermediation, international risk sharing, and reserve currencies. *American Economic Review*, 107(10):3038–3071.

Matsuyama, K., Kiyotaki, N., and Matsui, A. (1993). Toward a theory of international currency. *Review of Economic Studies*, 60(2):283–307.

Pflueger, C. and Yared, P. (2024). Global hegemony and exorbitant privilege. NBER Working Paper 32775.

Svensson, L. E. (1985). Currency prices, terms of trade, and interest rates: A general equilibrium asset-pricing cash-in-advance approach. *Journal of International Economics*, 18(1-2):17–41.

Wright, R. and Trejos, A. (2001). International currency. *B.E. Journal of Macroeconomics*, 1(0).

Zhang, C. (2014). An information-based theory of international currency. *Journal of International Economics*, 93:286–301.

Zhou, R. (1997). Currency exchange in a random search model. *Review of Economic Studies*, 64(2):289–310.

A Proofs

Proof of Proposition 4. The first-order condition of the HJB equation (10) with respect to labor ℓ is

$$1 = \nu_t^{j,\eta} q_t^j w_t^j,$$

where $\nu_t^{j,\eta}$ is the Lagrange multiplier on the budget constraint (9). But $\nu_t^{j,\eta}$ is just the marginal value of wealth $W_t{}'(a|\eta)$, as usual. Hence,

$$W_t{}'(a|\eta) = \frac{1}{q_t^j w_t^j}, \tag{35}$$

which immediately implies the linear form of the value function in the proposition, as desired. ■

Proof of Proposition 5. A buyer who purchases z_i inputs worth $q_t^j \psi_{it}^j$ gets utility $\frac{\psi_{it}^j}{w_t^j}$. On the other hand, the seller incurs disutility z_i. Let m be the transfer made from the buyer to the seller, resulting in a utility cost $\frac{m}{q_t^j w_t^j}$ for the buyer and a benefit $\frac{m}{q_t^i w_t^i}$ for the seller. If the buyer has a total quantity of bonds $\overline{m}$ that the seller recognizes, the Nash bargaining solution satisfies

$$\max_{z_i,m} \; \theta \log\big(\frac{\psi_{it}^j}{w_t^j} z_i - \frac{m}{q_t^j w_t^j}\big) + (1-\theta)\log\big(\frac{m}{q_t^i w_t^i} - z_i\big) \;\; \text{s.t.} \;\; z_i \geq 0, m \leq \overline{m}.$$

After simplifying, the first-order condition with respect to z_i yields

$$z_i = \theta \frac{m}{q_t^i w_t^i} + (1-\theta)\frac{m}{q_t^j \psi_{it}^j}.$$

Hence, the terms of trade $\phi_{it}^j \equiv m/z_i$ satisfy (11), since $\Psi_{it}^j = q_t^j \psi_{it}^j / q_t^i w_t^i$. ■

Proof of Proposition 7. From Proposition 6, the flow value obtained by a country-j seller in the DM who can accept currency n is given by (32). Use (10) to subtract $W(a|(j, S\backslash\{n\}))$ from $W(a|(j, S\cup\{n\}))$ and obtain

$$(\rho+\zeta)\big(W(a|(j,S\cup\{n\})) - W(a|(j,S\backslash\{n\}))\big) = v_{nt}^j + \frac{d}{dt}\big(W(a|(j,S\cup\{n\})) - W(a|(j,S\backslash\{n\}))\big), \tag{36}$$

noting that savings and consumption decisions are identical regardless of the set of currencies that the seller accepts. This equation can be integrated forward to obtain

$$W(a|(j,S\cup\{n\})) - W(a|(j,S\backslash\{n\})) = \int_0^\infty e^{-(\rho+\zeta)t} v_{n,t+s}^j ds. \tag{37}$$

Combining this result with the maximization problem in (14), we obtain the decision rule (16). ■

Proof of Proposition 8. See Appendix B.1, where we derive the result in the more general model used for calibration and counterfactuals. ■

Proof of Proposition 9. Given $\Psi^j_{j,t}$ and $\Psi^j_{-j,t}$, (21) implies that the ratio of expenditures on domestic versus foreign inputs is

$$\frac{\phi^j_{j,t} x^j_{j,t}}{\phi^j_{-j,t} x^j_{-j,t}} = \frac{\omega^j}{1-\omega^j} \frac{1+\theta(\Psi^j_{-j,t}-1)}{1+\theta(\Psi^j_{jt}-1)}. \tag{38}$$

If country j buyers hold both types of bonds, this ratio cannot be greater than $\frac{\xi^j}{\delta_{jt}\xi^{-j}}$, since they are able to spend domestic bonds in a fraction ξ^j of meetings and foreign bonds in a fraction $\xi^{-j}\delta_{jt}$. Similarly, the ratio cannot be less than $\frac{\delta_{-j,t}\xi^j}{\xi^{-j}}$. Given that $\omega^j = \beta\xi^j / \left(\beta\xi^j + \xi^{-j}\right)$, this restriction on the ratio of expenditures is equivalent to

$$\beta \frac{1+\theta(\Psi^j_{-j,t}-1)}{1+\theta(\Psi^j_{jt}-1)} \in \left[\delta_{-j,t}, \frac{1}{\delta_{jt}}\right] \tag{39}$$

if country-j buyers hold both bonds.

Next, suppose there is no solution to the Euler equation in which country-j buyers hold both types of bonds. If they hold domestic bonds only, there are three possible cases. In the first case, country j has not exhausted all possible gains from trade with either country, so $\Psi^j_{jt} > 1$ and $\Psi^j_{-jt} > 1$. Thus, buyers spend all of their bond holdings in each meeting, meaning the ratio of expenditures on domestic versus foreign inputs is exactly $\xi^j/\delta_{jt}\xi^{-j}$. On the other hand, if gains from trade with foreign sellers are equal to zero ($\Psi^j_{-j,t} = 1$), then this ratio is higher, since not all bonds are spent in meetings with foreign sellers but all bonds are spent in meetings with domestic sellers. Finally, if gains from trade with domestic sellers are equal to zero ($\Psi^j_{j,t} = 1$), the ratio is lower. Therefore, again applying (21) to write aggregate expenditures in terms of gains from trade and the definition of ω^j, we obtain (29).

A similar logic yields (30). The only difference is that there is no case in which gains from trade with domestic agents are equal to zero ($\Psi^j_{j,t} = 1$), but gains from trade with foreign agents are not ($\Psi^j_{-j,t} > 1$). Suppose otherwise. The locus of possible combinations ($\Psi^j_{j,t}$, $\Psi^j_{-j,t}$) such that an agent holds foreign bonds only satisfies

$$\Psi^j_{-j,t} - 1 = \frac{\delta_{-j,t}}{\beta}(\Psi^j_{j,t}-1) - \frac{1}{\theta}(1 - \frac{\delta_{-j,t}}{\beta}). \tag{40}$$

Suppose there is a solution to the Euler equation when $\Psi^j_{j,t} = 1$. Then $\Psi^j_{-j,t} < 0$, since $\delta_{-j,t} \leq 1$ and $\beta \geq 1$ by assumption. Therefore, such a solution is not possible. ■

Proof of Proposition 10. Henceforth, define

$$\tilde{l}_t^H = \frac{l_t^H - \delta_{Ht} l_t^F}{1 - \delta_{Ht}\delta_{Ft}}, \quad \tilde{l}_t^F = \frac{l_t^F - \delta_{Ft} l_t^H}{1 - \delta_{Ht}\delta_{Ft}}. \tag{41}$$

Suppose that (25)-(26) both hold for country-j buyers. Observe that these equations can be rearranged to obtain

$$\tilde{l}_t^H = \lambda\theta\xi(\Psi_{Ht}^j - 1)^+, \tilde{l}_t^F = \lambda\theta(1-\xi)(\Psi_{Ft}^j - 1)^+,$$

which in turn implies $\Psi_{it}^j = \Psi_{it}^*$, as defined in (31).

Proposition 9 demonstrated that if $G(\tilde{l}_t^H, \tilde{l}_t^F) < \frac{\delta_{Ft}}{\beta}$, then there does not exist a solution of the Euler equation in which Home agents hold both bonds. Instead, they hold Home bonds only.

Instead, we look for a solution to the Euler equation in which Home agents hold Foreign bonds only. Proposition 9 showed that we either have $\Psi_{Ft}^H = 1$, in which case the Euler equation implies

$$\Psi_{Ht}^H = 1 + \frac{l_t^F}{\lambda\theta\xi}, \tag{42}$$

or we have $\Psi_{Ht}^H > 1$ and $\Psi_{Ft}^H > 1$, and (40) holds with $j = H$. In this case, the solution to the Euler equation has

$$\Psi_{Ht}^H - 1 = \frac{\beta}{\delta_{Ft}}(\Psi_{Ft}^H - 1) + \frac{1}{\theta}\big(\frac{\beta}{\delta_{Ft}} - 1\big),$$

$$l_t^F = \lambda\theta\bigg(\xi\delta_{Ft}(\Psi_{Ht}^H - 1) + (1-\xi)(\Psi_{Ft}^H - 1)\bigg).$$

The solution to the Euler equation can be found by solving these two equations for the two unknowns Ψ_{Ht}^H, Ψ_{Ft}^H.

Similar arguments apply to all of the other inequalities in the proposition. ■

Proofs of Lemma 11 and Proposition 12. We will analyze Home agents' demand for bonds in each regime. By a symmetry argument, this will permit us to characterize properties of Foreign agents' bond demand as well.

Some preliminaries: Here we derive some preliminary results that will be useful throughout the analysis. First, define

$$D = 1 - \delta_H\delta_F. \tag{43}$$

We now present some derivatives: for $j \in \{H, F\}$,

$$\frac{\partial \tilde{l}^j}{\partial \delta_{-j}} = \frac{\delta_j \tilde{l}^j}{D}, \tag{44}$$

$$\frac{\partial \tilde{l}^j}{\partial \delta_j} = -\frac{\tilde{l}^{-j}}{D}, \tag{45}$$

$$\frac{\partial D}{\partial \delta_{-j}} = -\delta_j. \tag{46}$$

Also,

$$\frac{\partial \Psi_H^*}{\partial \tilde{l}^H} = \frac{1}{\lambda\theta\xi}, \quad \frac{\partial \Psi_H^*}{\partial \tilde{l}^F} = 0, \tag{47}$$

$$\frac{\partial \Psi_F^*}{\partial \tilde{l}^H} = 0, \quad \frac{\partial \Psi_H^*}{\partial \tilde{l}^F} = \frac{1}{\lambda\theta(1-\xi)}. \tag{48}$$

Case 1: Home agents hold both bonds: In this regime, we have

$$\Psi_H^H = \Psi_H^* \quad \text{and} \quad \Psi_F^H = \Psi_F^*. \tag{49}$$

Bond demand is given by

$$b_H^H = \frac{1}{\lambda D}\left(\frac{\phi_H^H x_H^H}{\xi} - \delta_F \frac{\phi_F^H x_F^H}{1-\xi}\right), \tag{50}$$

$$b_F^H = \frac{1}{\lambda D}\left(\frac{\phi_F^H x_F^H}{1-\xi} - \delta_H \frac{\phi_H^H x_H^H}{\xi}\right), \tag{51}$$

where

$$\phi_H^H x_H^H = \frac{\alpha\omega^H}{1+\theta(\Psi_H^* - 1)}, \quad \phi_F^H x_F^H = \frac{\alpha(1-\omega^H)}{1+\theta(\Psi_F^* - 1)} \tag{52}$$

denote expenditures on Home and Foreign goods, respectively.

Derivatives with respect to δ_F: We have

$$\begin{aligned}\frac{\partial \Psi_H^*}{\partial \delta_F} &= \frac{\partial \Psi_H^*}{\partial \tilde{l}^H}\frac{\partial \tilde{l}^H}{\partial \delta_F} \\ &= \frac{1}{\lambda\theta\xi}\frac{\delta_H \tilde{l}^H}{D} \geq 0,\end{aligned}$$

$$\begin{aligned}\frac{\partial \Psi_F^*}{\partial \delta_F} &= \frac{\partial \Psi_F^*}{\partial \tilde{l}^F}\frac{\partial \tilde{l}^F}{\partial \delta_F} \\ &= -\frac{1}{\lambda\theta(1-\xi)}\frac{\tilde{l}^H}{D} \leq 0.\end{aligned}$$

To obtain the derivative of bond demand, we first differentiate expenditures:

$$\begin{aligned}\frac{\partial \phi_H^H x_H^H}{\partial \delta_F} &= \frac{\partial \phi_H^H x_H^H}{\partial \Psi_H^*}\frac{\partial \Psi_H^*}{\partial \delta_F} \\ &= -\frac{\alpha\omega^H\theta}{(1+\theta(\Psi_H^*-1))^2}\frac{1}{\lambda\theta\xi}\frac{\delta_H \tilde{l}^H}{D} \\ &= -\frac{\delta_H}{D}\frac{\tilde{l}^H/\lambda\xi}{1+\tilde{l}^H/\lambda\xi}\phi_H^H x_H^H < 0,\end{aligned}$$

$$\begin{aligned}\frac{\partial \phi_F^H x_F^H}{\partial \delta_F} &= \frac{\partial \phi_F^H x_F^H}{\partial \Psi_F^*}\frac{\partial \Psi_F^*}{\partial \delta_F} \\ &= -\frac{\alpha(1-\omega^H)\theta}{(1+\theta(\Psi_F^*-1))^2}\frac{1}{\lambda\theta(1-\xi)}\Big(-\frac{\tilde{l}^H}{D}\Big) \\ &= \frac{1}{D}\frac{\tilde{l}^H/\lambda(1-\xi)}{1+\tilde{l}^F/\lambda(1-\xi)}\phi_F^H x_F^H > 0.\end{aligned}$$

Then, the derivative of bond demand is

$$\begin{aligned}\frac{\partial b_H^H}{\partial \delta_F} &= -\frac{\partial D/\partial \delta_F}{D}b_H^H + \frac{\partial \phi_H^H x_H^H/\partial \delta_F}{\lambda\xi D} \\ &\quad - \frac{1}{\lambda(1-\xi)D}\Big(\phi_F^H x_F^H + \delta_F\frac{\partial \phi_F^H x_F^H}{\partial \delta_F}\Big) \\ &= \frac{\delta_H}{D}b_H^H - \frac{\delta_H}{D}\frac{\tilde{l}^H/\lambda\xi}{1+\tilde{l}^H/\lambda\xi}\frac{\phi_H^H x_H^H}{\lambda\xi D} \\ &\quad - \left(\frac{\phi_F^H x_F^H}{\lambda(1-\xi)D} + \frac{\delta_F}{D}\frac{\tilde{l}^H/\lambda(1-\xi)}{1+\tilde{l}^F/\lambda(1-\xi)}\frac{\phi_F^H x_F^H}{\lambda(1-\xi)D}\right).\end{aligned}$$

To conclude that $\frac{\partial b_H^H}{\partial \delta_F} \leq 0$, it suffices to show that $\phi_F^H x_F^H \geq \lambda(1-\xi)\delta_H b_H^H$. In this regime, we have

$$\frac{\phi_H^H x_H^H}{\phi_F^H x_F^H} \leq \frac{\xi}{(1-\xi)\delta_H} \tag{53}$$

from results in the paper. Then, from the definition of b_H^H,

$$\begin{aligned} b_H^H &\leq \frac{1}{\lambda D}\left(\frac{\phi_F^H x_F^H}{(1-\xi)\delta_H} - \delta_F\frac{\phi_F^H x_F^H}{1-\xi}\right) \\ \Leftrightarrow \lambda(1-\xi)Db_H^H &\leq D\phi_F^H x_F^H \\ \Leftrightarrow \lambda(1-\xi)b_H^H &\leq \phi_F^H x_F^H,\end{aligned}$$

as desired.

Next, take derivatives of Home agents' demand for Foreign bonds:

$$\begin{aligned}\frac{\partial b_F^H}{\partial \delta_F} &= -\frac{\partial D/\partial \delta_F}{D} b_F^H + \frac{\partial \phi_F^H x_F^H/\partial \delta_F}{\lambda(1-\xi)D} - \delta_H \frac{\partial \phi_H^H x_H^H/\partial \delta_F}{\lambda \xi D} \\ &= \frac{\delta_H}{D} b_F^H + \frac{1}{D} \frac{\tilde{l}^H/\lambda(1-\xi)}{1+\tilde{l}^F/\lambda(1-\xi)} \frac{\phi_F^H x_F^H}{\lambda(1-\xi)D} + \frac{\delta_H^2}{D} \frac{\tilde{l}^H/\lambda\xi}{1+\tilde{l}^H/\lambda\xi} \frac{\phi_H^H x_H^H}{\lambda\xi D} \\ &> 0.\end{aligned}$$

We conclude that in this regime,

$$\frac{\partial}{\partial \delta_F}\left(1-\frac{1}{\Psi_F^H}\right) b_H^H < 0, \tag{54}$$

$$\frac{\partial}{\partial \delta_F}\left(1-\frac{1}{\Psi_H^H}\right) b_F^H > 0. \tag{55}$$

Derivatives with respect to δ_H: This case is completely symmetric. We have

$$\frac{\partial}{\partial \delta_H}\left(1-\frac{1}{\Psi_F^H}\right) b_H^H > 0, \tag{56}$$

$$\frac{\partial}{\partial \delta_H}\left(1-\frac{1}{\Psi_H^H}\right) b_F^H < 0. \tag{57}$$

Case 2: Home agents hold Home bonds only: In this regime, there are two sub-cases based on whether $\beta\delta_H$ is greater than or less than one. Of course, regardless of which sub-case applies, we have

$$b_F^H = 0. \tag{58}$$

A.1 Sub-case 1: $\beta\delta_H \geq 1$

In this case, the gains from trade in each type of meeting are

$$\Psi_F^H = 1 + \frac{\left(l^H - \lambda\xi\left(\beta\delta_H - 1\right)\right)^+}{\lambda\theta(\beta\xi + 1 - \xi)\delta_H}, \tag{59}$$

$$\Psi_H^H = 1 + \frac{\max\left\{\frac{\beta\xi}{\beta\xi+1-\xi} l^H + \frac{1-\xi}{\beta\xi+1-\xi}\lambda\xi\left(\beta\delta_H - 1\right), l^H\right\}}{\lambda\theta\xi}. \tag{60}$$

Bond demand is

$$b_H^H = \frac{\phi_H^H x_H^H}{\lambda\xi} = \frac{\alpha\omega^H}{\lambda\xi} \frac{1}{1+\theta(\Psi_H^H - 1)}. \tag{61}$$

It is immediately apparent from these expressions that $\frac{\partial \Psi_F^H}{\partial \delta_H} \leq 0$, $\frac{\partial \Psi_H^H}{\partial \delta_H} \geq 0$, and hence $\frac{\partial b_H^H}{\partial \delta_H} \leq 0$. Moreover, none of these quantities depend on δ_F (since Home agents do not hold Foreign bonds). Therefore,

$$\frac{\partial}{\partial \delta_H}(1 - \frac{1}{\Psi_F^H})b_H^H \leq 0, \tag{62}$$

$$\frac{\partial}{\partial \delta_F}(1 - \frac{1}{\Psi_F^H})b_H^H = 0. \tag{63}$$

A.2 Sub-case 2: $\beta\delta_H < 1$

This sub-case is similar to the previous one with minor alterations:

$$\Psi_H^H = 1 + \frac{\left(l^H - \lambda(1-\xi)\left(\frac{1}{\beta} - \delta_H\right)\right)^+}{\lambda\theta\left(\xi + \frac{1-\xi}{\beta}\right)}, \tag{64}$$

$$\Psi_F^H = 1 + \frac{\max\left\{\frac{1-\xi}{\beta\xi+1-\xi}l^H + \frac{\beta\xi}{\beta\xi+1-\xi}\lambda(1-\xi)\left(\frac{1}{\beta} - \delta_H\right), l^H\right\}}{\lambda\theta(1-\xi)\delta_H}. \tag{65}$$

Now we have

$$b_H^H = \frac{\phi_F^H x_F^H}{\lambda(1-\xi)\delta_H} = \frac{\alpha(1-\omega^H)}{\lambda(1-\xi)\delta_H}\frac{1}{1+\theta(\Psi_F^H - 1)}. \tag{66}$$

Again, it is clear that $\frac{\partial \Psi_F^H}{\partial \delta_H} \leq 0$, $\frac{\partial \Psi_H^H}{\partial \delta_H} \geq 0$, and $\frac{\partial b_H^H}{\partial \delta_H} \leq 0$. We therefore obtain (62) and (63).

Case 3: Home agents hold Foreign bonds only: In this regime, we have

$$b_H^H = 0 \tag{67}$$

and quantities do not depend on δ_H, the fraction of Foreign agents who accept Home bonds. The gains from trade are

$$\Psi_F^H = 1 + \frac{\left(l^F - \lambda\xi\left(\beta - \delta_F\right)\right)^+}{\lambda\theta(\beta\xi + 1 - \xi)}, \tag{68}$$

$$\Psi_H^H = 1 + \frac{\max\left\{\frac{\beta\xi}{\beta\xi+1-\xi}l^F + \frac{1-\xi}{\beta\xi+1-\xi}\lambda\xi\left(\beta - \delta_F\right), l^F\right\}}{\lambda\theta\xi\delta_F}. \tag{69}$$

Bond holdings are

$$b_F^H = \frac{\phi_H^H x_H^H}{\lambda\xi\delta_F} = \frac{\alpha\omega^H}{\lambda\xi\delta_F}\frac{1}{1+\theta(\Psi_H^H - 1)}. \tag{70}$$

It is immediate that $\frac{\partial \Psi_F^H}{\partial \delta_F} \geq 0$, $\frac{\partial \Psi_H^H}{\partial \delta_F} \leq 0$, and $\frac{\partial b_F^H}{\partial \delta_F} \leq 0$. Thus, we have

$$\frac{\partial}{\partial \delta_H}(1 - \frac{1}{\Psi_H^H})b_F^H = 0, \tag{71}$$

$$\frac{\partial}{\partial \delta_F}(1 - \frac{1}{\Psi_H^H}) b_F^H \leq 0. \tag{72}$$

■

Proof of Proposition 13. We proceed by classifying the possible equilibria in each regime. In what follows, it will be useful to define the state variable $\boldsymbol{\delta} = (\delta_H, \delta_F)$ and write the law of motion (6) as

$$\dot{\boldsymbol{\delta}} = \zeta \begin{pmatrix} \mathbf{1}\{v_H^F(\boldsymbol{\delta}) \geq \kappa\} - \delta_H \\ \mathbf{1}\{v_F^H(\boldsymbol{\delta}) \geq \kappa\} - \delta_F \end{pmatrix}. \tag{73}$$

Classical regime: By Proposition 12, the law of motion (73) describes a competitive system in the sense of Hirsch and Smith (2006): the first component is decreasing in δ_F, and the second component is decreasing in δ_H. Let $\boldsymbol{\delta}^*$ be an interior equilibrium in the classical regime. The Kamke-Müller monotonicity result implies for any $\boldsymbol{\delta}_0 = (\delta_H, \delta_F)$ that evolves according to (73) such that $\delta_H > \delta_H^*, \delta_F < \delta_F^*$, then there exists $T > 0$ such that $\boldsymbol{\delta}_t > \boldsymbol{\delta}^*$ for all $t \in [0, T]$. Hence, $\boldsymbol{\delta}^*$ cannot be stable, and the only possible stable equilibrium is $\boldsymbol{\delta}^* = (0, 0)$. (Clearly, this point is an equilibrium for sufficiently large κ.)

Dominant currency regime: We analyze the Home-dominant regime. The proof for the Foreign-dominant regime is exactly analogous.

Suppose $\boldsymbol{\delta}^*$ is an equilibrium in the Home-dominant regime. From the proof of Proposition 12, it is actually possible to demonstrate that the system (73) is competitive in this regime as well, so there can be no stable interior equilibrium. Any stable equilibrium must then lie along the boundary $\delta_F = 0$.

■

B Derivations of key equations

B.1 Goods market equilibrium

In this section, we derive (20)-(21) in a generalized version of the model. Just as in our counterfactual exercises, we allow for tariffs τ_{it}^j levied on country-j buyers who sell country-i inputs in the CM and time-varying productivity a_t^j, where the aggregate production function is

$$y_t^j = X_t^{j\,\alpha} (a_t^j \ell_t^j)^{1-\alpha}, \tag{74}$$

Furthermore, sellers' utility cost of production in the DM is $1/a_t^j$. (Productivity a_t^j is permitted to follow an arbitrary deterministic path.) A key quantity is sellers' marginal cost of production in the DM, defined as

$$mc_t^j \equiv q_t^j w_t^j / a_t^j.$$

With differential productivity, the gains from trade become

$$\Psi_{it}^j \equiv q_t^j \psi_{it}^j / mc_t^i.$$

With a tariff, the terms of trade in the DM change as well. They are given by

$$\begin{aligned}
\phi_{it}^j &= \frac{(1-\tau_{it}^j)\Psi_{it}^j}{1+\theta((1-\tau_{it}^j)\Psi_{it}^j - 1)} mc_t^i, \\
&= \frac{\frac{mc_t^i}{mc_t^j}(1-\tau_{it}^j)\Psi_{it}^j}{1+\theta((1-\tau_{it}^j)\Psi_{it}^j - 1)} mc_t^j,
\end{aligned}$$

and buyers spend all of their available bonds whenever the gains from trade net of tariffs are positive, $(1-\tau_{it}^j)\Psi_{it}^j \geq 1$.

The first-order conditions for intermediate goods (17) can be combined to obtain, via the usual CES algebra,

$$\psi_{Xt}^j = \frac{\alpha y_t^j}{X_t^j} \quad \text{where} \quad \psi_{Xt}^j \equiv \left(\sum_{i=1}^J \omega_i^j {\psi_{it}^j}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \tag{75}$$

Then, multiply (18) raised to the power $1-\alpha$ with (75) raised to the power α, use the production function (74), and multiply by q_t^j on both sides to obtain

$$\begin{aligned}
\alpha^\alpha (1-\alpha)^{1-\alpha} q_t^j {a_t^j}^{1-\alpha} &= q_t^j \big(\sum_{i=1}^J \omega_i^j {\psi_{it}^j}^{1-\sigma} \big)^{\alpha/(1-\sigma)} {w_t^j}^{1-\alpha} \\
&= {mc_t^j}^\alpha (q_t^j w_t^j)^{1-\alpha} \big(\sum_{i=1}^J \omega_i^j (\frac{mc_t^i}{mc_t^j} \Psi_{it}^j)^{1-\sigma} \big)^{\alpha/(1-\sigma)} \\
&= {a_t^j}^{1-\alpha} mc_t^j {\Psi_{Xt}^j}^\alpha,
\end{aligned}$$

where

$$\Psi_{Xt}^j \equiv \big(\sum_{i=1}^J \omega_i^j (\frac{mc_t^i}{mc_t^j} \Psi_{it}^j)^{1-\sigma} \big)^{1/(1-\sigma)}.$$

This expression can be rearranged to obtain

$$w_t^j = \frac{\alpha^\alpha (1-\alpha)^{1-\alpha} a_t^j}{{\Psi_{Xt}^j}^\alpha}, \tag{76}$$

which can be combined with the resource constraint (3) and the labor-leisure optimization condition (19) to obtain

$$y_t^j = \xi^j \left(\frac{\alpha^\alpha (1-\alpha)^{1-\alpha} a_t^j}{{\Psi_{Xt}^j}^\alpha} \right)^{\frac{1}{\gamma}}. \tag{77}$$

Then, note that the quantity of intermediates purchased by region j from region i can be re-written as

$$\begin{aligned}
x^j_{it} &= \omega^j_i X^j_t \big(\frac{\psi^j_{it}}{\psi^j_{Xt}}\big)^{-\sigma} \\
&= \omega^j_i \frac{\alpha q^j_t y^j_t}{q^j_t \psi^j_{Xt}} \big(\frac{q^j_t \psi^j_{it}}{q^j_t \psi^j_{Xt}}\big)^{-\sigma} \\
&= \omega^j_i \frac{\alpha q^j_t y^j_t}{mc^j_t \Psi^j_{Xt}} \big(\frac{\frac{mc^i_t}{mc^j_t}\Psi^j_{it}}{\Psi^j_{Xt}}\big)^{-\sigma} \\
&= \frac{\alpha q^j_t y^j_t}{mc^j_t} \times \omega^j_i {\Psi^j_{Xt}}^{\sigma-1} \big(\frac{mc^i_t}{mc^j_t}\Psi^j_{it}\big)^{-\sigma} \\
&= \frac{\alpha q^j_t w^j_t}{mc^j_t} \frac{y^j_t}{w^j_t} \times \omega^j_i {\Psi^j_{Xt}}^{\sigma-1} \big(\frac{mc^i_t}{mc^j_t}\Psi^j_{it}\big)^{-\sigma} \\
&= \frac{\alpha \xi^j q^j_t w^j_t}{mc^j_t} \times \left(\alpha^\alpha (1-\alpha)^{1-\alpha} a^j_t\right)^{\frac{1}{\gamma}-1} {\Psi^j_{Xt}}^{\sigma-1-\alpha(\frac{1}{\gamma}-1)} \times \omega^j_i \big(\frac{mc^i_t}{mc^j_t}\Psi^j_{it}\big)^{-\sigma} \\
&= \frac{K^j_t}{mc^j_t} \times \omega^j_i \big(\frac{mc^i_t}{mc^j_t}\Psi^j_{it}\big)^{-\sigma},
\end{aligned}$$

where

$$K^j_t \equiv \alpha \xi^j q^j_t w^j_t \times \left(\alpha^\alpha (1-\alpha)^{1-\alpha} a^j_t\right)^{\frac{1}{\gamma}-1} \times {\Psi^j_{Xt}}^{\sigma-1-\alpha(\frac{1}{\gamma}-1)}.$$

Putting this expression together with the terms of trade, we obtain

$$\phi^j_{it} x^j_{it} = K^j_t \omega^j_i \frac{(1-\tau^j_{it})\big(\frac{mc^i_t}{mc^j_t}\Psi^j_{it}\big)^{1-\sigma}}{1+\theta((1-\tau^j_{it})\Psi^j_{it}-1)}. \tag{78}$$

B.2 Implementation of a liquidity premium target

In this sub-section, we demonstrate how a government can use its fiscal and monetary tools (the nominal rate i^n_t and the rate of bond issuance μ^n_t) to implement a given path of the liquidity premium.

Let $\{\tilde{l}^n_t\}^N_{n=1}$ denote the set of liquidity premium targets for each currency. Let $\{\tilde{b}^n_t\}^N_{n=1}$ denote real demand for bonds in each currency at time t, and let $\tilde{r}_t$ denote the real interest rate, in some equilibrium in which liquidity premia are equal to their target values. We construct a set of policies $\{i^n_t, \mu^n_t\}^N_{n=1}$ such that there exists an equilibrium in which $l^n_t = \tilde{l}^n_t$ and $b^n_t = \tilde{b}^n_t$ for all n and t, and $r_t = \tilde{r}_t$ for all t.

Recall that $b^n_t = \phi^n_t B^n_t$, where B^n_t is nominal debt in currency n and ϕ^n_t is the real price of a currency-n bond. If such an equilibrium exists,

$$\mu_t^n = \frac{\dot{\tilde{b}}_t^n}{\tilde{b}_t^n} - \frac{\dot{\phi}_t^n}{\phi_t^n}. \tag{79}$$

Furthermore, from the definition of the liquidity premium,

$$\tilde{l}_t^n = \tilde{r}_t - \left(i_t^n - \frac{\dot{\phi}_t^n}{\phi_t^n}\right). \tag{80}$$

These are the only two equations that must hold in order for an equilibrium of the desired form to exist (maintaining the assumption that taxes can always adjust to satisfy governments' budget constraints). Combining these relationships, such an equilibrium exists whenever

$$i_t^n - \mu_t^n = \tilde{r}_t - \tilde{l}_t^n - \frac{\dot{\tilde{b}}_t^n}{\tilde{b}_t^n}. \tag{81}$$

Only the difference between i_t^n and μ_t^n is pinned down by equilibrium conditions, so there exists a μ_t^n (resp. i_t^n) satisfying these conditions for any given path of i_t^n (resp. μ_t^n), as claimed.